# Trade and the Spatial Distribution of Transport Infrastructure*

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**Abstract**

This paper constructs a theoretical model, in which transportation costs between two locations depend on cumulative infrastructure investment and governments allocate infrastructure spending over continuous space, thereby affecting the geographical pattern of transportation costs. Modelling international trade, we assume that governments set their infrastructure investment schedules in a non-cooperative fashion. We find that governments provide too little infrastructure investment because they ignore the benefits from reductions in domestic transportation costs that accrue to foreign consumers. Moreover, the distribution of infrastructure chosen by local governments is skewed towards central regions, which magnifies small discrete border frictions and creates 'border regions' within countries. French data on transportation costs are consistent with our theory.

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# 1 Introduction

Transportation costs feature prominently in modern theories of international trade and regional development. They have been shown to matter greatly for the volume of bilateral economic exchange between two places, and ease of access to markets is an important determinant of regional per capita income. A large empirical literature makes massive efforts to address the suspected endogeneity of transportation costs to economic outcomes. Yet, next to nothing is known about the processes that determine these costs. So, in their authoritative handbook chapter, Redding and Turner (2014) ask for "*further research ... examining the political economy of transport infrastructure investments*". The present paper is a first step towards filling this gap of knowledge.

The provision of transport infrastructure is one of the oldest activities that governments of all sorts have always been engaged in. The roads built by the Inca in South America or by the Romans in Europe bear testimony to this fact. Indeed, any known civilization has devoted resources to the construction of roads presumably with the objective to tap into the welfare gains from regional specialization and trade. On average, OECD countries spend about 1% of GDP on inland transportation infrastructure and maintenance; this amounts on average to about 3% of countries' public budgets. Emerging or transition economies can spend up to 10% of the budgets on.[1]

In this paper we focus on land-borne means of transportation, by far the most important mode in the EU or in NAFTA. We set up a theoretical model, in which governments allocate infrastructure spending over space, thereby affecting the geographical pattern of transportation costs. We find that the optimal infrastructure investment at some point in space is not only determined by local conditions at that point, but also – and predominantly – by the situation in other locations that produce and demand goods which transit through that point.

When we model international trade, we assume that two governments set their infrastructure investment schedules in a non-cooperative fashion. This reflects the fact that despite some international cooperation, the construction and maintenance of roads is decided by national governments. Even in Europe, in which there is some central coordination of infrastructure investment plans, member states need to cofinance all projects and they have substantial influence over the spatial allocation of spending, or the prioritization of projects.

As an outcome, governments provide too little infrastructure investment because they ignore the benefits from reductions in domestic transportation costs that accrue to foreign consumers.

[1] ITF-OECD (2012), p. 4.

More interestingly, however, the extent of underinvestment varies over space. In particular, regions close to the political border of jurisdictions suffer more strongly from underinvestment while in more central regions there can be overinvestment relative to the efficient benchmark. Therefore, trade across national borders entails higher transportation costs than trade within countries, holding distance constant.

We show analytically that investment profiles exhibit more spatial variation the larger the elasticity of substitution between investment at different points in space is, the lower the elasticity of substitution between varieties from different addresses is, and the lower the elasticity of trade costs with respect to geographical distance is.

Moreover, we show that a small border cost (say a tariff) makes the border zone wider by drawing even more investment into central locations. This mechanism may contribute towards explaining the empirical fact that international borders tend to restrict international trade more severely than what observable border costs together with plausible trade elasticities would suggest (Anderson and van Wincoop, 2003). Empirical gravity models, which proxy transportation costs by geographical distance, would therefore diagnose large border effects even if the true border costs are small.[2] We use our model to generate data on trade flows and trade costs, and apply a conventional gravity model to these data. We find that the obtained border effect overestimates the true border costs by about 20%. Hence, endogenous infrastructure investment can explain part of the border effect puzzle that has received so much attention in the literature. Our analysis has an important policy implication: Interregional coordination, which merely aims at removing the global underinvestment problem, could exacerbate underinvestment in border regions.

We extend our analysis by adding a non-contiguous country which supplies and demands goods from the contiguous two countries connected by a land-borne transportation network. We find that an increase in the economic mass of this overseas trading partner induces a reallocation of spending towards coastal regions and away from the hinterland and the landlocked country.

Our main result – the endogenous emergence of border regions and their amplification by discrete border frictions – is robust to a number of model variations. First, governments may outsource the design and operation of roads to monopolistic firms who care only about profits and not about welfare. However, the central issue leading to inefficient investment variation persists: the monopolists underinvest more in locations close to the border where the positive externality on the foreign monopolist's profits is largest. Second, we argue that allowing for

[2] In fact, they would find border effects even if true border costs were zero.

labor mobility within countries would strengthen our results

Our paper is related to at least four important strands of literature. First, it connects with papers that study the importance of geographical frictions and transportation costs for economic outcomes: trade, migration, and welfare. Typically, the literature has treated those costs as exogenous. Limao and Venables (2001) find that up to 60 percent of the cross-country variation in transport costs is due to differences in the quality and quantity of transport infrastructure and that high cost of land-borne transportation is a more relevant trade barrier than the costs of maritime transportation in many countries. Venables (2005) argues that infrastructure explains a larger share of spatial income inequality than sheer geography, while the latter certainly is a determinant of the former. Behar and Venables (2011) and Redding and Turner (2014) provide excellent surveys. While they cite empirical work on the determinants of transportation costs, they do not provide theoretical references on their endogenous emergence.

Many existing papers assume that countries (or regions) do not have a geographical extension. Recent work provides more spatial detail, but continuous to treat infrastructure as exogenous. Cosar and Demir (2014) show that the upgrading of motorways in Turkey significantly increases exports of transport-intensive goods of landlocked cities. Allen and Arkolakis (2013) incorporate realistic topographical features of geography into a spatial model of trade. They find the introduction of the US interstate highway system has reduced the costs of a coast-to-coast shipment by about a third. Duranton et al. (2013) use data on US interstate highways to show that highways within cities cause them to specialize in sectors that have high weight to value ratios. Using a multiregion general equilibrium model of trade, Donaldson (2012) and Donaldson and Hornbeck (2013) analyze the welfare gains from railroads in India and the United States, respectively. They find that improved market access through reduced transport costs creates trade and generates welfare gains, but that it also leads to trade diversion.

Second, our paper is related to a small literature that endogenizes transportation costs, usually by introducing a proper transportation sector. Using an economic geography model, Behrens and Picard (2011) show that the prices for transporting differentiated goods increase in the degree of spatial specialization of the economy and that this channel dampens core-periphery patterns. While their model has a competitive transport sector, Hummels et al. (2009) provides evidence that monopolistic market structure in the transport sector restrict trade. These papers, in contrast to ours, do not analyze the endogenous emergence of road infrastructure.

Third, our paper is related to literature that jointly considers international and intranational aspects of trade. Courant and Deardorff (1992) emphasize the importance of trade *within* countries for trade patterns across countries. Rossi-Hansberg (2005) studies the effects of small

border costs on the regional distribution of workers within a country and assesses the implications of the equilibrium population distribution on intra- versus international trade flows. However, his focus is not on infrastructure investment. Another close cousin of the present paper is the *racetrack* model of Fujita et al. (2001, ch. 6 and 17).[3] In that model, space is continuous, just as in the proposed setup. However, the two approaches differ in focus. The racetrack model endogenizes the distribution of manufacturing labor across space under conditions of increasing economies of scale. It does not say anything on the endogenous spatial distribution of the stock of transport infrastructure, nor does it shed light on how that stock shapes transport costs. In the proposed setup, production technologies exhibit constant returns to scale, and the distribution of workers is exogenous. In turn, transport costs are endogenous. While the proposed model can be generalized to allow for increasing returns to scale and worker mobility, excluding these elements makes the model straightforward to analyze.

Fourth, our paper relates to work on the border puzzle. Transport costs are usually related to geographical distance while the border effect is attributed to some lumpy cost that materializes when crossing a border. This dichotomy of variable and fixed costs enjoys empirical support, see Anderson and van Wincoop (2003). The latter authors estimate that the US-Canadian border reduces international trade relative to intranational trade by a factor of 4.7.[4] Explanations for fixed border costs abound. Among other things, they are related to informational costs (Casella and Rauch, 2003), contract enforcement costs (Anderson, 2003), exchange rates (Rose and van Wincoop, 2001). Surprisingly, border effects exist also *within* countries, where the above explanations do not help.[5] Our setup shows that border effects can arise even in the absence of explicit costs at the border.

[3]The racetrack model is discussed also as the 'seamless world model' (Krugman and Venables, 1997).

[4]Prior to Anderson and van Wincoop (2003), McCallum (1995) compares trade flows within Canada to flows between Canadian provinces and U.S. states, controlling for distance and regional GDPs. Everything else equal, crossing the border reduces trade by a factor as high as 22. For Europe, Nitsch (2000) finds that on average intranational trade is about 10 times higher than international one. Nitsch arrives at his results after controlling for cultural proximity (language), along other conventional gravity covariates. Wei (1996) constructs measures for imports of countries to themselves and compares this with imports from a statistically identical foreign country. He finds that the former magnitude is 2.5 times larger than the latter. Helliwell (1998) offers a comprehensive overview of the pre Anderson and van Wincoop state of the econometric literature. Evans (2003) decomposes cross-country price differences of traded goods into a component due to distortions and a component driven by consumer preferences. She demonstrates that the preferences effect is relatively important quantitatively. Obstfeld and Rogoff (2000) have cited this fact as a major puzzle in international macroeconomics.

[5]Okubo (2004) shows a border effect for trade between Japanese regions, Wolf (2004) for the US, and Combes et al. (2005) for their sample of French departments.

The structure of the paper is as follows. Section 2 presents some stylized facts on the transportation sector that inform our modeling choices. Section 3 presents and defends our formulation of the mapping between the spatial distribution of infrastructure investment and transport costs. Section 4 sets up the general equilibrium environment which motivates intra- and international trade and analyzes the optimal infrastructure investment schedule in a closed economy. Section 5 moves to a setting of two symmetric open economies and derives our core results on the endogenous emergence of border regions. Section 6 discusses tentative evidence. Finally, section 7 discusses several extensions while section 8 concludes.

## 2 Stylized facts

The world is not flat. Geographical obstacles such as mountains, rivers or swamps, affect the ease at which goods and people move across space. Since the most ancient civilizations, states have invested heavily in infrastructure projects to overcome geographical distance more efficiently. While military objectives have often been looming large, interregional exchange of goods always played an important role, too. Hence, there is little doubt that–along obvious geographical factors–government actions have a bearing on the spatial distribution of infrastructure, too.

The stock of transport infrastructure investment is very difficult to measure, and there are virtually no data to carry out spatial comparisons. Hence, a more indirect approach is required. For instance, one can see to what extent geographical variables explain variation in variable transportation costs across regions and interpret the residual as shaped by the quality of transport infrastructure investment. This section briefly discusses established stylized facts on the provision of public transport infrastructure and how those can be used in explaining spatial patterns in infrastructure investment.

*Fact 1: Infrastructure investment and transportation costs.* Limao and Venables (2001) use regression analysis to explain measured transportation costs as a function of infrastructure investment and other determinants. They find that about 60% of the variation in transportation costs can be explained by cross-country differences in the quantity and quality of infrastructure. They conclude that transport infrastructure is quantitatively more important in explaining transportation costs than sheer geography. More recently, Allen and Arkolakis (2013) show that transportation costs vary dramatically over the surface of the US. Moreover, the construction of interstate highways has cut transport costs in the US by two thirds.

*Fact 2: Intracontinental trade flows are mostly land-borne.* Since the border puzzle appears in trade within the European Union, NAFTA, or Japan, it is natural to take a brief look at the

Table 1: Value and tonnage of U.S. merchandise trade with Canada and Mexico by transportation mode

| | 2005 | | | | 2010 | | | |
|---|---|---|---|---|---|---|---|---|
| **Mode** | **Value[a)]** | | **Weight [b)]** | | **Value[a)]** | | **Weight [b)]** | |
| Truck | 491 | 62% | 191 | 28% | 557 | 61% | 187 | 29% |
| Rail | 116 | 15% | 141 | 21% | 131 | 14% | 134 | 21% |
| Air | 33 | 4% | 1 | 0% | 45 | 5% | 1 | 0% |
| Water | 58 | 7% | 256 | 38% | 81 | 9% | 210 | 32% |
| Pipeline | 52 | 7% | 86 | 13% | 63 | 7% | 106 | 16% |
| Other | 39 | 5% | 5 | 1% | 40 | 4% | 9 | 1% |
| Total | 790 | 100% | 679 | 100% | 918 | 100% | 646 | 100% |

**Notes.** a) current US$ billions, b) millions of tons. Source: US department of transportation, Freight Facts and Figures 2011, Table 2.8.
http://www.ops.fhwa.dot.gov/freight/freight_analysis/nat_freight_stats/docs/11factsfigures/index.htm

relative prevalence of intercontinental transport modes. Table 1 reports data for merchandise trade within NAFTA. It appears that about 60% of the value of total trade flows between NAFTA member states is transported by truck. Total land-borne traffic amounts to about three quarters of the total value of trade. Hence, while the share of air-borne traffic is certainly increasing, it is not prevalent. Not surprisingly, in terms of weight, water-borne traffic appears relatively important, reflecting the low unit-value of bulky goods transported by that mode. Transcontinental trade, in contrast, is dominated by maritime transport (about 75% of value in both the EU and the US).[6]

*Fact 3: The public input into the provision of transportation services is important.* According to the Bureau of Economic Analysis, in the US, public gross investment plus government consumption spending on transportation goods amount to about 9.4 percent of total government spending (across all levels of government) or 1.8 percent of GDP in 2004. Private gross fixed investment in transport equipment (this excludes cars used for private use) is 1.3 percent of GDP.[7] Table 2, Panel B, shows that, in 2010, about 5% of total government spending fell on the construction and maintenance of transport infrastructure. Data collected by the Bureau of Transport Statistics indicates that toll revenue finances only a small fraction of total public transport infrastructure spending in the US.

*Fact 4: Regional governments influence interregional infrastructure projects substantially.*

[6]Combes and Lafourcade (2005) note that, "in Europe, around 72% of trade volumes are shipped through the road network (against around 15% for rail, 8% for pipers and 5% for rivers)" (p. 324), which roughly corresponds to the North American pattern.

[7]NIPA tables 1.15 and 3.155.

Table 2: On the role of transportation (construction and maintenance) in public finances, USA

| | 1900 | 1950 | 2000 | 2010 |
|---|---|---|---|---|
| **A) Shares in total government spending on transportation in the USA** | | | | |
| **Federal** | 0% | 22% | 28% | 33% |
| **State** | 2% | 41% | 43% | 39% |
| **Local** | 98% | 46% | 48% | 50% |
| **Federal transfers**[a)] | 0% | -8% | -19% | -22% |
| **B) Shares of transportation in spending at different levels of government in the USA** | | | | |
| **Federal** | 0% | 3% | 3% | 3% |
| **State** | 2% | 19% | 9% | 7% |
| **Local** | 18% | 14% | 5% | 8% |
| **Total** | 10% | 7% | 5% | 5% |

**Source:** www.governmentspending.com, own calculations.
a) Grants and aids by federal government to state or local governments.

Panel A in Table 2 shows for the US that all levels of government contribute towards spending on transport infrastructure and equipment. About 90% fall on the local or state level, but federal transfers have become quite important over time. Clearly, financial and planning authority need not coincide, but the data does suggest that local governments must have important roles in the investment allocation process. In Europe, member states decide almost exclusively on transport infrastructure investment projects. There is some coordination at the Union level through the so called Trans-European Transport Network, but the budget is limited to about 1 billion Euro per year over the 2007-2013 spending period. Total spending on construction and maintenance in the EU member states to about 0.9% of EU GDP (i.e., 120 billion Euro) per year, so that the EU contribution to total spending is less than 1%.[8]

[8] http://ec.europa.eu/transport/infrastructure/tentec/tentec-portal/site/en/abouttent.htm

# 3 Modeling transportation costs

Economic geography models, pioneered amongst others by Krugman (1991), bring together monopolistic competition with Samuelson's (1952) iceberg trade costs assumption.[9] The iceberg assumption has proved convenient, because it makes the introduction of a specific transportation sector redundant: during transportation, a distance-dependent share of the output shipped from the location of production gets lost, i.e. melts away. The implicit transportation sector production function uses the good being transported as the only input. Formally, assume that economic space $S$ is unidimensional and continuous (e.g., the real line, or the circumference of a circle). For two arbitrary addresses $x$ and $z$ in $S$, a Krugman-type transportation costs specification would be

$$T(x,z) = e^{a|x-z|} \geq 1, \tag{1}$$

where the coefficient $a > 0$ is the iceberg decay parameter. $T(x,z)$ models the cost of delivering a good over the distance $|x-z|$ as an *ad-valorem* tax equivalent, where the tax income is lost in transit. In order to receive one unit of the good at $x$, $T(x,z)$ units of that good have to leave the factory at $z$. A share $1 - T(x,z)^{-1}$ of the good 'melts' in transport; the share $T(x,z)^{-1}$ arrives at $x$ when one unit of the good is shipped at $z$.

The iceberg formulation amounts to introducing a shadow transport sector, which uses the share $1 - T(x,z)^{-1}$ of a good to be shipped from $z$ to $x$ as an input. The transport service is produced at the location of the producer, using the same input mix than the good to be shipped. Given the continuous space nature of our setup, one could more generally posit that transportation services are produced in some sub-interval in $[x,z]$. In the specific economic environment proposed below, these differences do not matter, as f.o.b. prices in any location are independent of demand for the variety produced at those locations.

In this paper, transportation costs are modeled as a function of *cumulative* infrastructure investment. Public infrastructure investment refers to the process of investing some resource at specific locations $s \in S$ with the aim of reducing transportation costs.[10] We assume that the set of geographic locations, $S$, is given by an interval, $[0, \bar{s}]$, where $\bar{s}$ characterizes the geographical size of the economy. Note that an alternative way to formulate the geographic space in the

[9] Note that much of the new trade theory literature that discusses trade in differentiated goods under increasing returns to scale and monopolistic competition uses an essentially discrete formalization of trade costs. Associated empirical papers using the gravity equation do, however, resort to Krugman's specification. For a model of trade in continuous space see Krugman and Venables (1997).

[10] Since the model proposed is static, we use the term infrastructure investment and stock of infrastructure interchangeably.

economy is to assume that $S$ is a circle. However, in contrast to the linear case, the circle geography does not exhibits a natural periphery, implying that the geography does not affect the infrastructure investment. Indeed, if all the locations on a unit circle are symmetric (in terms of endowments, available technologies, etc.), the infrastructure investment is also symmetric across the locations, implying the symmetric transportation infrastructure. In contrast, in the below analysis we show that, in the case of a linear space, the symmetry of locations does not imply the symmetry of investment decisions and, therefore, the symmetry of the transport infrastructure: i.e. geography matters.

We model the effectively available stock of infrastructure over some interval $[x, z] \in S$ as a constant elasticity of substitution aggregator function

$$I(x,z) = \left[\int_x^z i(s)^{1-\delta} ds\right]^{\frac{1}{1-\delta}}, \delta > 1, x \leq z, \tag{2}$$

where $i(s)$ is the level/stock of infrastructure at location $s \in [x, z]$ and $\delta > 1$ is a constant technological parameter (which will have a precise economic interpretation later). $I(x, z)$ increases in distance. This formulation has the natural implication that spreading a constant investment budget $B$ over increasing distance $z - x$ lowers the available stock of $I(x, z)$. Note also that if $i(s)$ is equal to zero on some subset (with a positive measure) of $[x, z]$, then the available stock of infrastructure over the whole interval $[x, z]$ is zero.

The costs of transportation a product from $z$ to $x$, where $x \leq z$, are linked with the infrastructure stock as follows:[11]

$$T(x,z) = \left(1 + \frac{1}{\delta - 1} I(x,z)^{1-\delta}\right)^{\gamma}, \delta > 1, \gamma > 0, x \leq z. \tag{3}$$

A more general definition, where $x$ and $z$ are not ordered, implies that

$$\begin{aligned} T(x,z) &= \left(1 + \frac{1}{\delta - 1} |I(x,z)^{1-\delta}|\right)^{\gamma} \\ &= \left(1 + \frac{1}{\delta - 1} \left|\int_x^z i(s)^{1-\delta} ds\right|\right)^{\gamma}, \end{aligned}$$

where $|\cdot|$ means the absolute value. As can be seen, the transportation costs are symmetric in the sense that delivering a product from $x$ to $z$ costs the same as delivering a product from $z$ to $x$. Here, parameter $\gamma$ represents the effect of the total stock of infrastructure on the transportation costs, while $\delta$ stands for the elasticity of substitution between infrastructure investments at different locations (see the Lemma below). Note that for any locations $x$, $y$, and $z$, the triangle

[11] In Section 4.5, we discuss the robustness of the results in the paper to changes in the functional form of the transportation costs.

inequality holds: $T(x,y)T(y,z) \geq T(x,z)$ (the strict inequality holds, if $x$, $y$, and $z$ represent different locations).[12] That is, it is cheaper to transport products directly to a destination address, rather than through some intermediate address.

The choice of functional forms (2) and (3) proofs convenient as the problem of optimally allocating $i(s)$ *over space* resembles the problem of optimally allocating consumption spending *over time*. Moreover, the formulation (3) has properties long discussed (but rarely modelled) by transport economists (Winston, 1985, and Gramlich, 1994).

**Lemma 1** *Generalized iceberg trade costs $T(x,z)$ have the following properties:*

(i) $T(x,z) \geq 1$ *with* $T(x,x) = 1$ *and* $T(x,y)T(y,z) \geq T(x,z)$.

(ii) $T(x,z) = (1 + a|z-x|)^{\gamma}$, *if $i(s)$ is a constant $\bar{\imath}$ over the interval $[x,z]$, and* $a = \bar{\imath}^{1-\delta}/(\delta - 1)$.

(iii) $T(x,z) = \infty$, *if $i(s) = 0$ on some subset (with a positive measure) of $[x,z]$.*

(iv) $T(x,z)$ *is increasing in distance: i.e. for fixed $x$ a more distant location $z$ results in higher transportation costs. Moreover, if $i(s)$ is a differentiable function on $S$, $T(x,z)$ is convex in distance if the distance-induced increment to $T_z(x,z)$ in trade costs is not outweighed by an improvement in infrastructure. That is, if* $(\gamma - 1)[a(z)]^2 / (T(x,z))^{1/\gamma} > i'(z)\, i(z)^{-\delta}$ *where* $a(z) = i(z)^{1-\delta}/(\delta - 1)$.

(v) *The (interregional) elasticity of substitution between infrastructure investment at different locations is* $0 < 1/\delta < 1$, so *that investments at different places are gross complements.*

(vi) *Investment smoothing property: if investment costs do not vary across locations, then the cost-efficient way to achieve some exogenous target level of transportation costs involves a flat spatial investment profile*

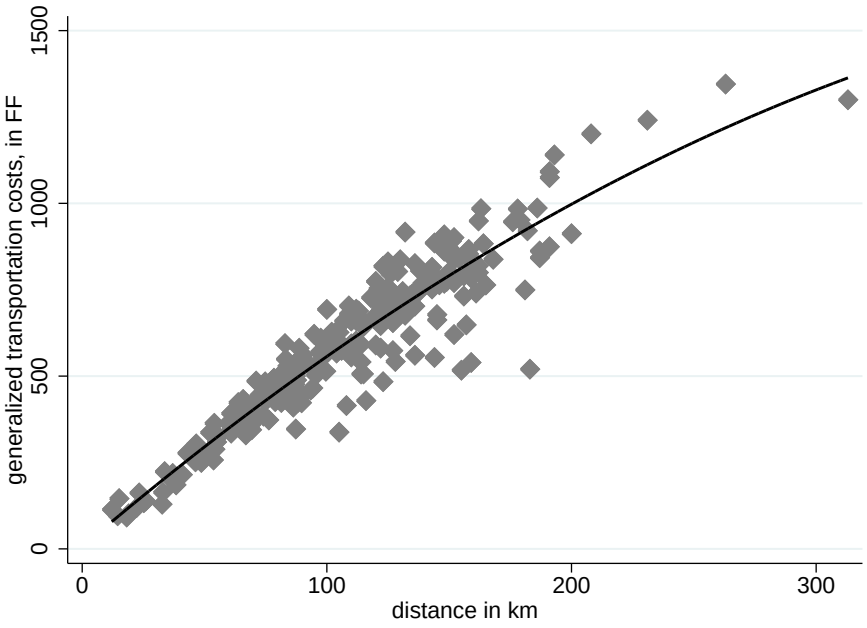
$$i(s) = \left\{ (z-x) / \left[ (\delta - 1) \left( \bar{T}^{1/\gamma} - 1 \right) \right] \right\}^{1/(\delta - 1)}.$$

*where $\bar{T} > 1$ is the target level of iceberg transportation costs.*

[12] Indeed, it is straightforward to see that for three different locations $x$, $y$, and $z$ such that $x < y < z$,

$$\begin{aligned}
T(x,y)T(y,z) &= \left(1 + \frac{1}{\delta - 1}\int_x^y i(s)^{1-\delta}\, ds\right)^{\gamma} \left(1 + \frac{1}{\delta - 1}\int_y^z i(s)^{1-\delta}\, ds\right)^{\gamma} \\
&= \left(1 + \frac{1}{\delta - 1}\int_x^y i(s)^{1-\delta}\, ds + \frac{1}{\delta - 1}\int_y^z i(s)^{1-\delta}\, ds + \frac{1}{(\delta - 1)^2}\int_x^y i(s)^{1-\delta}\, ds \int_y^z i(s)^{1-\delta}\, ds\right)^{\gamma} \\
&> \left(1 + \frac{1}{\delta - 1}\int_x^y i(s)^{1-\delta}\, ds + \frac{1}{\delta - 1}\int_y^z i(s)^{1-\delta}\, ds\right)^{\gamma} \\
&= \left(1 + \frac{1}{\delta - 1}\int_x^z i(s)^{1-\delta}\, ds\right)^{\gamma} = T(x,z).
\end{aligned}$$

Figure 1: Transportation costs and distance: a concave relationship




**Notes.** Generalized transportation costs between capitals of French departements in French Francs (1993) as a function of geographical distance in km. See Section 6 and Appendix B for more details.

**Proof** The first three properties directly follow from the definition of the trade costs in (3). The last three properties are proved in the Appendix.

One of the critiques of the Krugman specification of transportation costs (see McCann, 2005) is that, according to (1), the delivered price of a good transported from a producer to a consumer over some distance is convex with respect to distance, which is at least partly counterfactual. In particular, regressing the log of transportation costs between capitals of French *départements* on the log of geographical distance reveals an elasticity of 0.90 with a robust standard error of 0.02 (see Figure 1). The hypothesis of the elasticity being equal to unity is rejected at the 1% level.[13]

In the present framework, iceberg transportation costs are convex in distance, as long as the component of $dT(x,z)/dz$ driven by variation in infrastructure investment does not outweigh the pure distance component of $dT(x,z)/dz$ (see property (iv) in the above lemma). Hence, in general, whether $T(x,z)$ is convex in distance depends on the spatial allocation of infrastructure investment. Moreover, if $\gamma$ is assumed to be less than or equal to one, the transportation cost

[13]Using robust regression methods to punish outliers leads to an elasticity of 0.92, still different from 1.00 at the 1% level. The same holds true if one restricts the sample to distances below 200, 150 or 100 km.

function is concave in distance.

Properties (v) and (vi) exploit the isomorphism between (3) and the usual representation of utility in an optimal growth model. The parameter $\delta$ measures the ease with which infrastructure investment at some address can substitute for investment at another place. The restriction $\delta > 1$ ensures that investments at different places are gross complements: investment at some address makes investment at some other place more worthwhile, which seems realistic.[14]

# 4 Infrastructure investment in a closed economy

This section embeds the transport technology described in (3) into a specific model of inter and intra-regional trade. The model is static and features a single factor of production, labor. The economic environment combines spatial product differentiation with constant returns to scale production functions and perfect competition.[15]

## 4.1 Geographical space and goods space

As discussed above, the geographic space $S$ that constitutes the economy is understood as a continuum of locations (or: addresses) $s \in S$ organized along an interval, $[0, \bar{s}]$. At each location $s$ there is a representative household who inelastically supplies $m(s)$ units of labor. Households are immobile across space, so $m(s)$ is exogenous. We can leave the form of $m(s)$ open as long as $m(s) > 0$ for all $s \in S$ (no inhabited locations). The total endowment of labor in the economy is then equal to $\int_{s \in S} m(s)ds$, which we define as $L$.

At each location, a homogeneous agricultural and a spatially differentiated industrial good can be produced. Consumers consume both types, perceiving industrial goods produced at specific locations as imperfect substitutes. There are no costs of transporting the agricultural good. Moreover, the agricultural good serves as an input into infrastructure provision. Each location $s$ is home of consumers and producers. We denote addresses of consumers by $x$ and addresses of producers by $z$.

We assume that locations may differ with respect to the topological circumstances. In other words, we have a distribution of productivities $q(s)$ which gives the rate at which resources

[14] We do not allow for incremental transport costs incurred at address $s$ to depend on the volume of traffic transiting through $s$. Actually, in equilibrium, the contrary will hold: more traffic at $s$ will encourage the planner to invest more in infrastructure, thereby driving down the gradient of $T$. This lowers the incremental trade costs at $s$ for all units of goods that transit through $s$.

[15] It is, therefore, of the Armington (1969) type; see also Anderson and van Wincoop (2003).

are transformed into infrastructure investment goods. Specifically, infrastructure at address $s$ is produced according to a linear production function $i(s) = b(s)/q(s)$, where $b(s)$ denotes the input of the agricultural good used for infrastructure investment, and $1/q(s) > 0$ measures the rate at which that resource is transformed into infrastructure. We restrict $q(s)$ to be continuously differentiable. Feasibility of an investment policy $i(s)$ implies that

$$\int_{s \in S} q(s)\, i(s) \leq B, \tag{4}$$

where $B$ is the amount of agricultural good invested in the economy infrastructure and will be endogenously determined by government policy.

The assumption that the agricultural good can be transported freely across space can be relaxed only at the price of considerable complication. It is similar to the assumption of a costlessly tradable agricultural good in much of the economic geography literature and delivers factor price equalization across space (in nominal terms) as long as all locations produce both types of goods (which we assume). Moreover, the existence of the agricultural good as an input in infrastructure production makes thinking about a transportation technology for transferring infrastructure production inputs from one region to the other redundant.[16]

### 4.2 Consumption

The utility function of a representative household at location $x$ is a monotone transformation of a Cobb-Douglas aggregate over the homogeneous agricultural good and a Dixit-Stiglitz aggregate over industrial goods:

$$U(x) = \left[c^A(x)\right]^{\alpha\rho/((1-\alpha)(1-\rho))} \left[u(x)\right]^{\rho/(1-\rho)}, \alpha \in (0,1), 0 < \rho < 1, \tag{5}$$

where $c^A(x)$ denotes the total quantity of the agricultural good consumed at address $x$ and $u(x)$ is the subutility index attributable to spatially differentiated goods. Specifically,

$$u(x) = \left( \int_{z \in S} c^I(x,z)^\rho \, dz \right)^{1/\rho}, \tag{6}$$

where $c^I(x,z)$ is the quantity of an industrial variety produced at address $z$ and consumed at $x$.[17]

[16] An alternative/equivalent way to model infrastructure provision is to assume that the only input into infrastructure is labor, which in turn is perfectly mobile between locations (within an economy). But consumption takes place at the place of origin. This will equalize wages across locations (which is in the present model done by the presence of the homogenous agricultural good) making the presence of the agricultural good redundant.

[17] Notice that $U(x)$ is a positive monotonic transformation $U(x) = [\tilde{U}(x)]^{\rho/((1-\alpha)(1-\rho))}$ of the usual Cobb-Douglas formulation $\tilde{U}(x) = \left[c^A(x)\right]^\alpha \left[u(x)\right]^{1-\alpha}$. The transformation makes the theoretical analysis of the model more tractable (without qualitatively changing the main conclusions).

Let $Y^n(x)$ denote household $x's$ net income in terms of a numeraire to be defined below. Then, the budget constraint of household $x$ is

$$Y^n(x) = c^A(x)\, p^A(x) + \int_{z\in S} c^I(x,z)\, p^I(x,z)\, dz, \tag{7}$$

where $p^A(x)$ is the price of the agricultural good at location $x$ and $p^I(x,z)$ is the price of a variety imported from location $z$ and consumed at $x$.

The utility maximization problem implies that the demand functions for the agricultural good and a certain variety of the industrial good are respectively

$$c^A(x) = \frac{\alpha Y^n(x)}{p^A(x)} \text{ and } c^I(x,z) = (1-\alpha)\, Y^n(x)\, \frac{p^I(x,z)^{-\sigma}}{P^I(x)^{1-\sigma}}, \tag{8}$$

where $\sigma = 1/(1-\rho)$ and

$$P^I(x) = \left[\int_{z\in S} p^I(x,z)^{1-\sigma}\, dz\right]^{\frac{1}{1-\sigma}} \tag{9}$$

is the price index for industrial goods at location $x$.

The indirect utility attainable at prices $p^A(x)$, $p^I(x,z)$ and income $Y^n(x)$ can be written as

$$V(x) = \left(\alpha^{\alpha/(1-\alpha)}(1-\alpha)\right)^{\rho/(1-\rho)} \left[p^A(x)\right]^{-\alpha\rho/((1-\alpha)(1-\rho))} \left[P^I(x)\right]^{1-\sigma} \left(Y^n(x)\right)^{\rho/((1-\alpha)(1-\rho))}. \tag{10}$$

### 4.3 Production

At each location $z \in S$, the agricultural and the industrial good are produced under conditions of perfect competition. The only input of production is labor. Production functions for the two types of goods are linear $y^A(z) = bl^A(z)$ where $y^I(z) = l^I(z)$, where $b > 0$ is a productivity parameter common for all locations. Output quantities are denoted by $y^A(z)$ and $y^I(z)$, and labor inputs by $l^A(z)$ and $l^I(z)$, respectively.

We assume that workers are perfectly mobile across agricultural and industrial firms. This in turn implies that $p^A(z) = w(z)/b$ and $p^I(z) = w(z)$, where $w(z)$ is the wage rate (expressed in units of numeraire) at address $z$.

### 4.4 Equilibrium

Industrial goods bear transportation costs. We assume that there are no trade costs other than transportation costs.[18] Hence, the c.i.f. prices faced by consumers differ from the f.o.b.

[18] In section 5.3, when we consider international trade, we will introduce a discrete trade friction at the border.

(ex-factory) prices. In particular, a household at $x$ faces the price

$$p^I(x,z) = p^I(z)\,T(x,z)$$

for a variety of the industrial good imported from location $z$. In contrast, the agricultural good can be transported freely. In the paper, we impose a *non-full-specialization* (NFS) assumption: there is always a strictly positive quantity of agricultural production at each location. The NFS assumption introduces factor price equalization in terms of the agricultural good.[19] We may therefore choose the agricultural good as the *numeraire* and set $p^A(z) = 1$ for all $z \in S$. Since $p^A(z) = 1$ for any $z$, we drop the superscripts $A$ and $I$ in the following.

As the price of the agricultural good is normalized to unity, the wage rate at location $z$, $w(z)$, is equal to $b$. The gross income at location $x$ in terms of the numeraire is then $Y(x) = bm(x)$. Finally, from our assumption on technology, the c.i.f. prices of industrial goods are

$$p(x,z) = bT(x,z). \tag{11}$$

The government imposes a lump-sum tax $t$, which is assumed identical across addresses $s \in S$. Thus, the net income at location $x$, $Y^n(x)$, is $(1-t)bm(x)$. The total tax income is

$$B = bt\int_{s\in S^c} m(s)\,ds = btL.$$

Substituting the expressions for the net household income and the price index and taking into account that the price of the agricultural good is one, we can now rewrite the indirect utility function in (10) as follows:

$$V(x) = \Omega\left((1-t)m(x)\right)^{(\sigma-1)/(1-\alpha)}\left[\int_{z\in S} T(x,z)^{1-\sigma}\,dz\right], \tag{12}$$

where $\Omega \equiv \left(\alpha^{\alpha/(1-\alpha)}b^{\alpha/(1-\alpha)}(1-\alpha)\right)^{(\sigma-1)}$ is a function of exogenous parameters only. The latter can be rewritten as follows:

$$V(x) = \Omega(1-t)^{(\sigma-1)/(1-\alpha)}\tilde{m}(x)\left[\int_{z\in S} T(x,z)^{1-\sigma}\,dz\right], \tag{13}$$

where

$$\tilde{m}(x) = (m(x))^{(\sigma-1)/(1-\alpha)}.$$

Equation (13) shows that the measure of economic activity at a location, $\tilde{m}(x)$, is exactly proportional to indirect utility (defined over the entire household located at $x$). Hence, in

[19] Relaxing the NFS assumption would allow to study the interaction between infrastructure investment policies and regional specialization patterns. This is an interesting issue that raises additional complications. It is therefore left to future research.

per capita terms (with respect to $\tilde{m}(x)$), indirect welfare is independent from $\tilde{m}(x)$: Higher endowment at a location does not trigger a negative terms of trade effect as in the standard Armington model due to the presence of a linear outside sector that absorbs any variation in labor endowments across locations.

### 4.5 The choice of infrastructure investment

In this section, we characterize optimal policies $\{i^a(s), t^a\}_{s\in S}$ in a closed economy. The social planner chooses the infrastructure investment and the tax rate to maximize the total welfare in the economy. Specifically, the planner's objective function is an aggregate of the indirect utilities achieved at every address. Accordingly, ignoring the irrelevant constant, the planner problem is

$$\{i^a(x), t^a\}_{x\in S} = \arg\max \left\{ (1-t)^{(\sigma-1)/(1-\alpha)} \int_{x\in S} \tilde{m}(x)\, v(x) dx \,\middle|\, \int_{x\in S} q(x)\, i(x)\, dx \leq btL \right\}, \tag{14}$$

where

$$v(x) = \int_{z\in S} T(x,z)^{1-\sigma}\, dz. \tag{15}$$

Note that the household size at location $x$, represented by $(m(x))^{(\sigma-1)/(1-\alpha)}$, plays a role of the weight attached to the consumption utility $v(x)$. Hence, from the social planner's problem, we may expect larger infrastructure investments in more densely populated regions.

The above optimization problem can be equivalently written as follows:

$$\{i^a(x), t^a\}_{x\in S} = \arg\max \left\{ (1-t) \left( \int_{x\in S} \tilde{m}(x)\, v(x) dx \right)^{\frac{1-\alpha}{\sigma-1}} \,\middle|\, \int_{x\in S} q(x)\, i(x)\, dx \leq btL \right\}. \tag{16}$$

To guarantee the concavity of the objective function in (14), we assume that

$$\gamma(\sigma-1)(\delta-1) < 1, \text{ and} \tag{17}$$

$$\frac{1-\alpha}{\sigma-1} < 1. \tag{18}$$

The first inequality means that $T(x,z)^{1-\sigma}$ is concave with respect to infrastructure investments, implying that the consumption utility, $v(x)$, is concave in infrastructure investments as well.[20] The second inequality, which would be always met if $\sigma \geq 2$, implies that the objective function is concave in $v(x)$. As a result, the objective function is concave with respect to infrastructure investments. Note that the results derived in the paper are qualitatively robust to changes in the

[20] In brief, the idea behind is that the function, $\left(1 + \frac{1}{\delta-1} x^{1-\delta}\right)^{\gamma(1-\sigma)}$ is strictly concave on $[0,\infty)$ if and only if $\gamma(\sigma-1)(\delta-1) < 1$. This in turn implies that $T(x,z)^{1-\sigma}$ is concave with respect to infrastructure investments.

functional form of the transportation costs as soon as $T(x,z)^{1-\sigma}$ is concave in $i(s)$, $s \in [x,z]$, and has an unbounded derivative at $i(s)=0$.

Next, we characterize the solution of the planner's problem. Note that we find the solution among continuous functions on $[0,\bar{s}]$. That is, the social welfare is maximized with respect to $i^a(x)$, where $i^a(x)$ is continuous. This limitation also holds in the subsequent analysis (see Section 5). The following proposition holds.

**Proposition 1** *The optimal allocation of infrastructure spending across space and the optimal tax rate chosen by a social planner under autarky are implicitly determined by the following system of equations:*

$$i^a(x)^{\delta} = \frac{b\gamma(1-\alpha)(1-t^a)L}{q(x)}\left(\phi^L(x)+\phi^R(x)\right), \tag{19}$$

$$t^a = \frac{\int_{s\in S} q(s)\, i^a(s)\, ds}{bL}, \tag{20}$$

*where*

$$\phi^L(x) = \frac{\int_0^x \tilde{m}(s)\left(\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^t i^a(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_{s\in S}\tilde{m}(s)\,v(s)ds},$$

$$\phi^R(x) = \frac{\int_x^{\bar{s}} \tilde{m}(s)\left(\int_0^x\left(1+\frac{1}{\delta-1}\int_t^s i^a(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_{s\in S}\tilde{m}(s)\,v(s)ds}.$$

**Proof** In the Appendix.

The terms $\phi^L(x)$ and $\phi^R(x)$ represent the aggregate marginal welfare gains (from a rise in $i^a(x)$) to the left and the right of location $x$, respectively. The investment at location $x$ is higher, the larger the sum $\phi^L(x)+\phi^R(x)$ or the lower the cost of infrastructure at the location, $q(x)$. In the next proposition, we summarize some additional properties of the optimal infrastructure investment function, $i^a(x)$, and the consumption utility, $v(x)$.

**Proposition 2** *The optimal infrastructure investment function, $i^a(x)$, and the consumption utility, $v(x)$, have the following properties:*

(i) *The infrastructure investments are zero at the borders of the region: $i^a(\bar{s}) = i^a(0)$.*

(ii) *If $q(x)$ is continuously differentiable, $i^a(x)$ is increasing in the neighborhood of zero and decreasing in the neighborhood of one: specifically, $(i^a(x))'_{x=0} = \infty$ and $(i^a(x))'_{x=\bar{s}} = -\infty$.*

(iii) *If there is no variation in the cost of infrastructure investment and the household size across the locations: $q(x) = q$ and $m(x) = m$ for any $x \in S$, then $v(x)$ and $i^a(x)$ are symmetric around $x = \bar{s}/2$ and have a hump shape with maximum at $x = \bar{s}/2$.*

**Proof** The first property immediately follows from the definitions of $\phi^L(x)$ and $\phi^R(x)$. Specifically, we have that $\phi^L(\bar{s}) = \phi^L(0) = \phi^R(\bar{s}) = \phi^R(0) = 0$. The last two properties are proved in the Appendix.

According to the proposition, if there is no variation in the cost of infrastructure and the household size, the optimal infrastructure is symmetric around the middle point of the $[0, \bar{s}]$-interval. That is, to maximize the social welfare, the social planner concentrates the infrastructure around the middle point. This in turn implies that the transportation costs in the middle region are lower than those at the periphery and, thereby, households located closer to the middle point have higher indirect utility.

### 4.6 Comparative statics

In this section, we explore how the parameters in the model affect the infrastructure profile chosen by the social planner. Note that the optimal infrastructure investments at location $x$ can be written as follows:

$$i^a(x) = \left( \frac{\gamma(1-\alpha)\left(bL - \int_{s\in S} q(s)\, i^a(s)\, ds\right)}{q(x)} \left(\phi^L(x) + \phi^R(x)\right) \right)^{1/\delta}, \tag{21}$$

where $\phi^L(x)$ and $\phi^R(x)$ depend on the investments at all locations $x \in [0, \bar{s}]$ (see *Proposition* 1). In other words, the infrastructure investments at location $x$ depend on that particular location, the infrastructure profile in the whole economy, and the set of parameters in the model. That is,

$$i^a(x) = i^a(x, I^a, \epsilon),$$

where $I^a$ represents the infrastructure profile in the economy and $\epsilon$ is the set of parameters in the model. Thus, the changes in $i^a(x)$ due to changes in $\epsilon$ are implicitly determined from

$$\frac{\partial i^a(x)}{\partial \epsilon} = \frac{\partial i^a(x, I^a, \epsilon)}{\partial \epsilon} + \frac{\partial i^a(x, I^a, \epsilon)}{\partial I^a} \frac{\partial I^a}{\partial \epsilon}. \tag{22}$$

The first term in the right-hand side of (22) captures the direct effect of $\epsilon$ on $i^a(x)$, while the second term stands for the indirect effects.

As can be seen, a rise in $b$ increases the right-hand side of (21) resulting in the positive direct as well as overall effect of $b$ on $i^a(x)$: there are more investments in the infrastructure in more productive (richer) economies. It is also intuitively clear that a rise in the cost of infrastructure investments, $q(x)$, in some neighborhood of $x$ reduces the investments in that neighborhood. In particular, if we assume that $q(x) = q$ for all $x$, then the impact of $q$ on $i^a(x)$ is exactly opposite to that of $b$: a rise in $q$ decreases $i^a(x)$.

Recall that the size of the economy, $L$, is given by $\int_0^{\bar{s}} m(s)ds$ and, therefore, depends on the geographical size, $\bar{s}$, and the population density, $m(x)$. To examine the impact of $L$ on the infrastructure profile, we assume that $m(x) = m$ for all $x \in [0, \bar{s}]$. In this case, $L$ is equal to $m\bar{s}$. Note that if $m(x)$ is the same at all locations, $\phi^L(x)$ and $\phi^R(x)$ do not directly depend on $m$, as it is canceled out (see *Proposition* 1). Hence, the impact of $m$ on $i^a(x)$ is exactly the same as that of $b$: that is, a rise in $m$ increases $i^a(x)$ for all $x$. The next proposition summarizes the above considerations.

**Proposition 3** *Changes in the parameters in the model have the following effects on the optimal infrastructure profile in a closed economy:.*

(i) *A rise in $b$ increases $i^a(x)$ for all $x$.*

(ii) *Assuming that $m(x) = m$ for all $x$, a rise in $m$ increases $i^a(x)$ for all $x$.*

(iii) *Assuming that $q(x) = q$ for all $x$, a rise in $q$ decreases $i^a(x)$ for all $x$.*

**Proof** The proof is based on the concept of the monotone comparative static (see, for instance, Milgrom and Shannon (1994)). The details are provided in the Appendix.

Unfortunately, the rigorous analysis of the overall effect (the direct effect plus the indirect effect) of $\bar{s}$ on the infrastructure profile is quite complex. Therefore, in this comparative static, we focus only on the *direct* effect. In particular, it is possible to show that a rise in $\bar{s}$ (from $\bar{s}'$ to $\bar{s}''$) increases $i^a(x)$ for $x$ from some left neighborhood of $\bar{s}'$. Indeed, when $\bar{s}$ rises from $\bar{s}'$ to $\bar{s}''$, $\bar{s}'$ becomes an internal location of the new geography and, therefore, has some positive infrastructure investments. This means that $i^a(\bar{s}')$ increases (as before the changes $\bar{s}'$ was the border location with zero infrastructure). By continuity, the infrastructure investments increase in some left neighborhood of $\bar{s}'$ as well. In addition, we find that if the cost of infrastructure is uniform ($q(x) = q$ for all $x$), then $i^a(x)$ rises in all locations $x \in S$. The results of numerical experiments in Section 4.6.1 suggest that if we take into account the indirect effects as well, then depending on the parameters $i^a(x)$ can decrease for $x$ sufficiently close to the left border (even if the cost of infrastructure is uniform). This implies that while the direct effects are of the first order, the indirect effects should not be completely ignored in the analysis.

**Proposition 4** *Assuming that $m(x) = m$ for all $x$, a rise in $\bar{s}$ has a positive direct effect on $i^a(x)$ for $x$ close to the right border and has an ambiguous impact on the investments at the other locations. If in addition $q(x) = q$ for all $x$, then a rise in $\bar{s}$ has a positive direct effect on $i^a(x)$ for all $x \in S$.*

**Proof** In the Appendix.

The analysis of the effects of $\gamma$, $\delta$, and $\sigma$ on the infrastructure profile is complex as well, as these parameters are included into the expressions for $\phi^L(x)$ and $\phi^R(x)$ in the non-trivial way. At the same time, the intuition, which is behind the effects, seems straightforward. A rise in $\gamma$ makes the dependence of the transportation costs on infrastructure investments stronger. As a result, $i^a(x)$ rises at all locations. A rise in $\sigma$ makes the varieties of the differentiated product more substitutable, shifting the consumption towards local varieties. As a result, the gains from infrastructure investments fall and, therefore, $i^a(x)$ falls as well for all $x$. Finally, a rise in $\delta$ makes infrastructure investments in different locations less substitutable (as $1/\delta$ falls), which in turn results in higher optimal investments at all locations. We run a number of numerical experiments that appear to be consistent with the above intuition. In the next section, we present some numerical experiments related to the comparative statics above.

### 4.6.1 Simulations

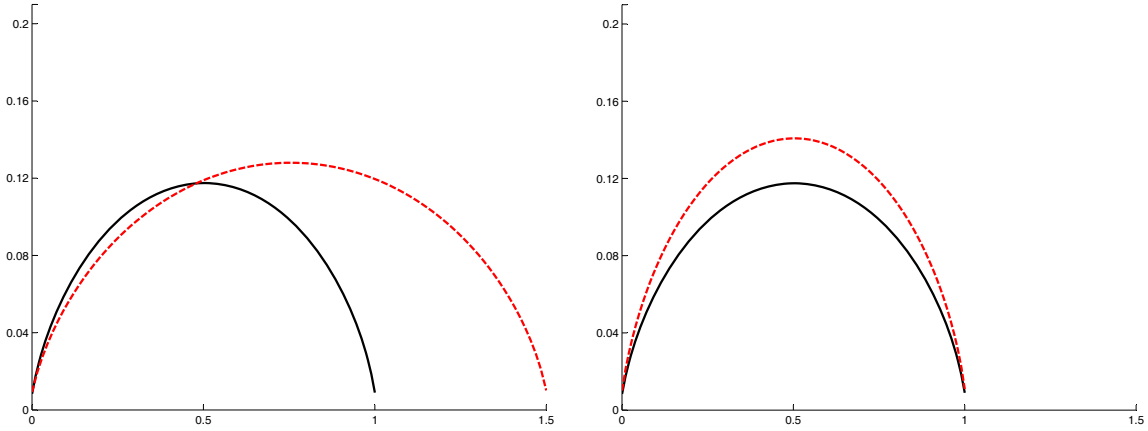
In this section, we simulate the effects of changes in the parameters on the infrastructure distribution. The benchmark parameterization is as follows. We set $\sigma$ to 2, $\delta$ to 1.5, $\gamma = 0.75$, and $\alpha = 0.5$. This guarantees that $\gamma(\sigma-1)(\delta-1) < 1$ and $(1-\alpha)/(\sigma-1) < 1$. In our simulations, we assume no variation in the cost of infrastructure $q(x)$ and location sizes $m(x)$. Specifically, we consider $q(x) = q = 1$, $m(x) = m = 1$, and $b = 1$. Finally, as the initial value of $\bar{s}$, we consider 1. Under this parameterization, the market size $L$ is equal to 1 as well. The solid black curve in all panels of Figure 2 depicts the default configuration described above. In this case, the optimal tax rate is equal to 0.08.

Next, we change the parameters in the model and examine how these changes affect the equilibrium infrastructure profile. As can be inferred from Panel (a) in Figure 2, a rise in $\bar{s}$ from 1 to 1.5 significantly increases infrastructure investments at locations that are relatively close to the initial border and slightly decreases investment at the locations that are relatively far from the initial border. This result is in line with the discussion in the previous section. In this case, the tax rate rises to 0.09 (from 0.08).

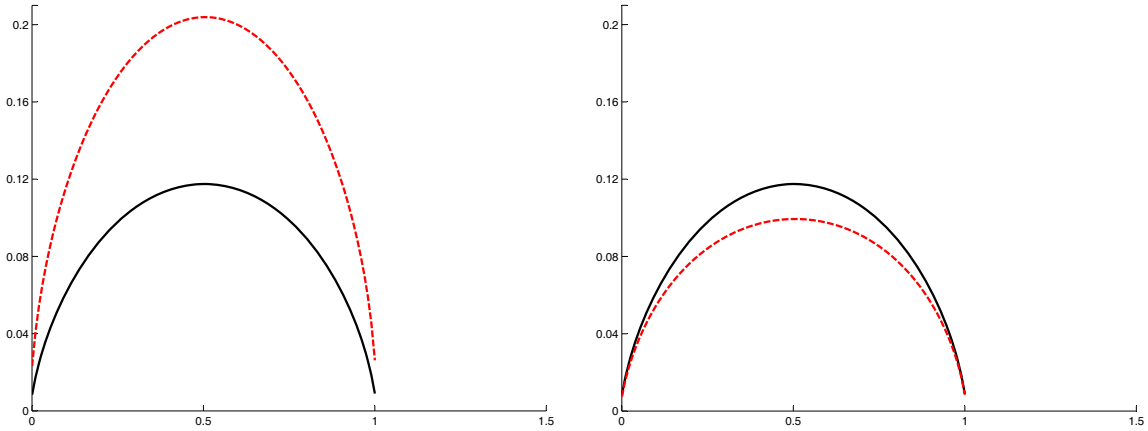
Finally, a rise in $\gamma$ (to 1) or $\delta$ (to 2) increases the investments at all locations (see panels ($b$) and ($c$), respectively), whereas an increase in $\sigma$ (to 3) reduces the investments at all locations (Panel ($d$)).

Figure 2: Closed economy equilibrium investment loci: comparative statics

(a) Moving $\bar{s}$ from 1.0 to 1.5 (b) Moving $\gamma$ from 0.75 to 1.0




(c) Moving $\delta$ from 1.5 to 2.0 (d) Moving $\sigma$ from 2 to 3




**Notes.** Solid black curve: default investment distribution ($\sigma = 2, \delta = 1.5, \gamma = 0.75, \alpha = 0.5$). Dashed red curves: distributions resulting from alternative parameterizations.

# 5 Infrastructure investment in an open economy

In this section, we present the key results of the paper. We assume that the world economy consists of two independent countries, each with its own government that decides on infrastructure investment in a non-cooperative way. However, consumers demand goods produced all over the world. Thus, we have a situation with 'globalized markets, regional politics'. As in the previous section, the world geography is linear. To simplify the analysis, we assume that countries are symmetric. In particular, the world geography is described by the $[0, 2\bar{s}]$-interval, where locations from $[0, \bar{s}]$ and $(\bar{s}, 2\bar{s}]$ represent the home and foreign country, respectively. Alternatively, one may think of the $[0, 2\bar{s}]$-interval as a closed country, with segments representing autonomous regions within that country. As before, the homogenous good is assumed to be produced at all locations in both countries, resulting in the same wage rates (equal to $b$). We also assume away any variation in the costs of infrastructure and the household sizes across locations: i.e., $q(x) = q$ and $m(x) = m$ for all $x \in [0, 2\bar{s}]$. The idea behind this assumption is to isolate a pure border effect on the equilibrium infrastructure profile.[21]

In the following analysis, the rationale behind many results is the discrepancy between the political and the economic reach of countries: while infrastructure investment decisions are limited to domestic locations, consumers demand imports from both countries and, therefore, from all locations. Since countries decide in a non-cooperative way, they do not internalize the positive externality that their investment decisions exert on consumers in other countries. This leads to global underprovision of infrastructure.

In the following, we first examine the world planner problem. Then, we formulate and solve the game between the two countries when infrastructure decisions are undertaken independently. Finally, we compare the obtained outcomes of the game to the world-planner solution.

## 5.1 World planner problem

The world planner chooses the world infrastructure profile to maximize the indirect utility of the entire world population. In this case, the space which individuals trade across and the space which infrastructure decisions are made over coincide. In other words, we have one market and one jurisdiction (that decides over infrastructure investments). Thus, the world planner problem can be written as follows:

$$\{i^w(x), t^w\}_{x\in[0,2\bar{s}]} = \arg\max_{i(x),t} \left\{ (1-t) \left( \int_0^{2\bar{s}} v(x)dx \right)^{\frac{1-\alpha}{\sigma-1}} \middle| q \int_0^{2\bar{s}} i(x)\, dx \leq 2bLt \right\}, \tag{23}$$

[21] The framework can be easily extended to the case when $q(x)$ and $m(x)$ are symmetric around $x = \bar{s}$.

where $v(x)$ is the consumption utility at address $x$ and $L$ is the population size in each country.[22] As can be seen, the world planner problem looks very similar to the social planner problem in the case of a closed economy. The only difference is that the geographical space is now given by the $[0, 2\bar{s}]$-interval rather than by the $[0, \bar{s}]$-interval. Hence, we can formulate the following proposition.[23]

**Proposition 5** *The optimal allocation of infrastructure spending across the world geography and the optimal tax rate chosen by the world planner are implicitly determined by the following system of equations:*

$$i^w(x)^\delta = \frac{2bL\gamma(1-\alpha)(1-t^w)}{q}\left(\phi^{L,w}(x)+\phi^{R,w}(x)\right), \tag{24}$$

$$t^w = \frac{q\int_0^{2\bar{s}} i^w(s)\,ds}{2bL}, \tag{25}$$

*where $\phi^{L,w}(x)$ and $\phi^{R,w}(x)$ are defined in analogy to their counterparts in Proposition 1 with the right border equal to $2\bar{s}$ and $v(s) = \int_0^{2\bar{s}} T(x,z)^{1-\sigma}\,dz$. The properties of the world planner solution are the same as those for the autarky planner case; see Proposition 2.*

**Proof** The proof is exactly the same as that for Proposition 1.

Similar to the infrastructure profile in the case of a closed economy, $i^w(x)$ is symmetric around $x = \bar{s}$ and has a hump shape, implying that the maximum of the function is achieved at $x = \bar{s}$ (this is due to the absence of variation in the costs of infrastructure and household sizes). Thus, the first-best allocation involves the highest levels of infrastructure investment around the border between the countries. Indeed, as the world planner cares about the entire world population, she finds it optimal to invest more at locations closer to the "core" of the world (as the social marginal gains at these locations are higher), which is the border. In the next section, we explore the case when infrastructure decisions are undertaken independently by each country.

## 5.2 Global economics, regional politics

In this section, we find the solution of a game between two countries that choose the infrastructure investments simultaneously and independently from each other. The game is defined

[22] Notice that the constraint in the maximization problem is

$$q\int_0^{2\bar{s}} i(x)\,dx \le bt\int_0^{2\bar{s}} m(x)\,dx = 2b\bar{s}m = 2bL,$$

where $L$ is the size of the countries.

[23] As in the previous section, we assume that the optimal infrastructure profile is chosen among continuous functions.

as $\Gamma = (I, U_i, \Theta_i)$, where $I = \{H, F\}$ represents the set of countries, $\Theta_i$ is the $i$th country's strategy set ($i \in I$), and $U_i$ is the $i$th country's payoff functional defined on $\Theta_H \times \Theta_F$. In the context of the present framework, the set of strategies of the home country, $\Theta_H$, is given by $\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]}$, where $i^H(x) \geq 0$, $\int_0^{\bar{s}} i^H(x)\,dx \leq bLt^H/q$, and $t^H \in [0,1]$. Similarly, $\Theta_F$ is the set of $\left\{i^F(x), t^F\right\}_{x\in[\bar{s},2\bar{s}]}$, where $i^F(x) \geq 0$, $\int_{\bar{s}}^{2\bar{s}} i^F(x)\,dx \leq bLt^F/q$, and $t^F \in [0,1]$. Here, $i^H(x)$ and $i^F(x)$ are continuous on $[0,\bar{s}]$ and $[\bar{s},2\bar{s}]$, respectively. Finally, the countries' payoffs are represented by the corresponding countries' total welfare functions.[24]

Since the countries are symmetric, in the following analysis we focus on the home country only. Given the infrastructure profile and the tax rate in the foreign country, the social planner at home solves the following maximization problem:

$$\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]} = \arg\max_{t,i(x)} \left\{ (1-t)\left(\int_0^{\bar{s}} v(x)dx\right)^{\frac{1-\alpha}{\sigma-1}} \middle| q\int_0^{\bar{s}} i(x)\,dx \leq bLt^H \right\}, \tag{26}$$

where

$$v(x) = \int_0^{2\bar{s}} T(x,z)^{1-\sigma}\,dz.$$

Note that as agents consume both domestic and foreign products, the consumption utility, $v(x)$, depends not only on the domestic infrastructure profile, but also on the infrastructure investments undertaken abroad (through the transportation costs). As a result, the infrastructure investments in the foreign country affect the choice of the infrastructure investments in the

[24] As the Nash equilibrium in the above game, we consider the limit of the Nash equilibrium in the corresponding discrete approximation $\Gamma_n = (I, U_{in}, \Theta_{in})$ (in other words, the equilibrium in game $\Gamma$ is considered as the limit ($n \to \infty$) of the equilibrium in game $\Gamma_n$). To formulate $\Gamma_n$, we consider a uniform partition of the $[0, 2\bar{s}]$-interval given by $\{x_i\}_{i=0..2n}$, where $x_0 = 0$, $x_n = \bar{s}$, and $x_{2n} = 2\bar{s}$. In this case, the set of strategies of the home country, $\Theta_{Hn}$ in country $i$ is given by $\left\{i^H(x_i), t^H\right\}_{i=0..n}$, where $i^H(x_i) \geq 0$, $\sum_{i=0}^{n} i^H(x_i)\triangle_n \leq bLt^H/q$, and $t^H \in [0,1]$ (here, $\triangle_n = \bar{s}/n$ is the partition size). In the same way, $\Theta_{Fn}$ is the set of $\left\{i^F(x_i), t^F\right\}_{i=[n+1,2n]}$ (without loss of generality, we assume that location $n$ belongs to the home country), where $i^F(x_i) \geq 0$, $\sum_{i=n+1}^{2n} i^F(x_i)\triangle_n \leq bLt^F/q$, and $t^F \in [0,1]$. The countries' payoffs are then the following:

$$\begin{aligned} U_{Hn} &= (1-t^H)\left(\sum_{i=0}^{n} v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}}, \\ U_{Fn} &= (1-t^F)\left(\sum_{i=n+1}^{2n} v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}}, \end{aligned}$$

where

$$v(x_i) = \sum_{j=0}^{2n} T(x_i, x_j)^{1-\sigma}\,\triangle_n\,.$$

It is straightforward to see that the game $\Gamma_n$ has a Nash equilibrium in pure strategies. This is due to the fact that the strategy sets are non-empty, convex, and compact subsets of a metric vector space and the payoff functions are continuous on $\Theta_{Hn} \times \Theta_{Fn}$ and concave in the own strategy of a player: i.e., $U_{in}$ is concave on $\Theta_{in}$.

home country, and vice versa. The next lemma describes the best response of the home social planner given a certain infrastructure profile in the foreign country.

**Lemma 2** *Given an infrastructure profile in the foreign country,* $\left\{i^F(x)\right\}_{x\in(\bar{s},2\bar{s}]}$*, the home social planner chooses the infrastructure investments and the tax rate,* $\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]}$*, that implicitly satisfy the following equations:*

$$\begin{aligned} i^H(x)^\delta &= \frac{bL\gamma(1-\alpha)(1-t^H)}{q}\left(\tilde{\phi}^L(x)+\tilde{\phi}^R(x)\right), \\ t^H &= \frac{q\int_0^{\bar{s}} i^H(x)\,dx}{bL}, \end{aligned}$$

*where* $\tilde{\phi}^R(x)=\phi^R(x)$ *and*

$$\tilde{\phi}^L(x)=\phi^L(x)+\frac{\int_0^x\left(\int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^H(r)^{1-\delta}\,dr+\frac{1}{\delta-1}\int_{\bar{s}}^t i^F(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}} v(s)ds},$$

*with* $\phi^L(x),\phi^R(x)$ *given in Proposition 1 (at* $m(x)=m$*).*

**Proof** In the Appendix.

As can be seen, the optimal infrastructure profile in an open economy has a similar form as that in the case of a closed economy. The main difference is the expression for $\tilde{\phi}^L(x)$ that represents the aggregate marginal welfare gains from a rise in $i^H(x)$ to the left of location $x$. The reason behind is that infrastructure investments at location $x$ affect the transportation costs from all foreign locations to the locations on the left of $x$ (as the foreign country is located on the right of $x$). This in turn leads to additional welfare gains from a rise in infrastructure investment at location $x$ compared to the closed economy case. As a result, in contrast to the closed economy case, $i^H(x)$ is strictly greater than zero at $x=\bar{s}$ if and only if $i^F(x)$ is strictly positive in some right neighborhood of $x=\bar{s}$ (meaning that the home country is not isolated from the foreign country). If $i^F(x)$ is equal to zero in some right neighborhood of $x=\bar{s}$, then $\tilde{\phi}^L(x)$ is exactly the same function as in the case of a closed economy, as in this case the transportation costs from foreign locations are infinitely high. Finally, as in the closed economy case, the infrastructure investment at location $x=0$ is zero: $i^H(0)=0$.

In the paper, we focus our analysis on the symmetric Nash equilibrium of the game. In the symmetric equilibrium, the countries' equilibrium infrastructure profiles are symmetric around $x=\bar{s}$, meaning that $i^F(x)=i^H(2\bar{s}-x)$ for $x\in[\bar{s},2\bar{s}]$. Taking into account the results stated in *Lemma* 2, we formulate the following proposition.

**Proposition 6** *In the symmetric Nash equilibrium, the equilibrium home infrastructure*

*profile and tax rate are implicitly determined by the following system of equations:*

$$\begin{aligned} i^H(x)^\delta &= \frac{bL\gamma(1-\alpha)(1-t^H)}{q}\left(\tilde{\phi}^{L,N}(x)+\tilde{\phi}^{R,N}(x)\right), \\ t^H &= \frac{q\int_0^{\bar{s}} i^H(x)\,dx}{bL}, \end{aligned}$$

*where* $\tilde{\phi}^{L,N}(x)=\tilde{\phi}^{L}(x)$ *and* $\tilde{\phi}^{R,N}(x)=\tilde{\phi}^{R}(x)$ *from Lemma 2 and* $i^F(r)=i^H(2\bar{s}-r)$.

**Proof** The proof directly follows from *Lemma* 2 and the fact that in the symmetric equilibrium $i^F(x)=i^H(2\bar{s}-x)$.

Next, we explore some properties of the equilibrium home infrastructure investments. Note that in the symmetric equilibrium, there always exists some left (right) neighborhood of $x=\bar{s}$ where $i^H(x)$ ($i^F(x)$) is strictly greater than zero. Indeed, if it is not true, then $i^H(x)$ is equal to zero around $x=\bar{s}$. This in turn means that $i^F(x)$ is equal to zero around $x=\bar{s}$ as well (due to the symmetry). As a result, there is no trade with the foreign country (because of infinitely large transportation costs) and the equilibrium infrastructure profile corresponds to the optimal infrastructure profile in the closed economy, which is, as was shown, strictly positive for any $x\in(0,\bar{s})$. This constitutes the contradiction. Thus, we can conclude that, in contrast to the closed economy case, the infrastructure investment at $x=\bar{s}$ is strictly positive due to the possibility of trade with the foreign country. Specifically,

$$i^H(\bar{s})^\delta=\frac{bL\gamma(1-\alpha)(1-t^H)}{q}\tilde{\phi}^{L,N}(\bar{s})>0.$$

We then explore the behavior of the infrastructure profile around the border. The following proposition holds.

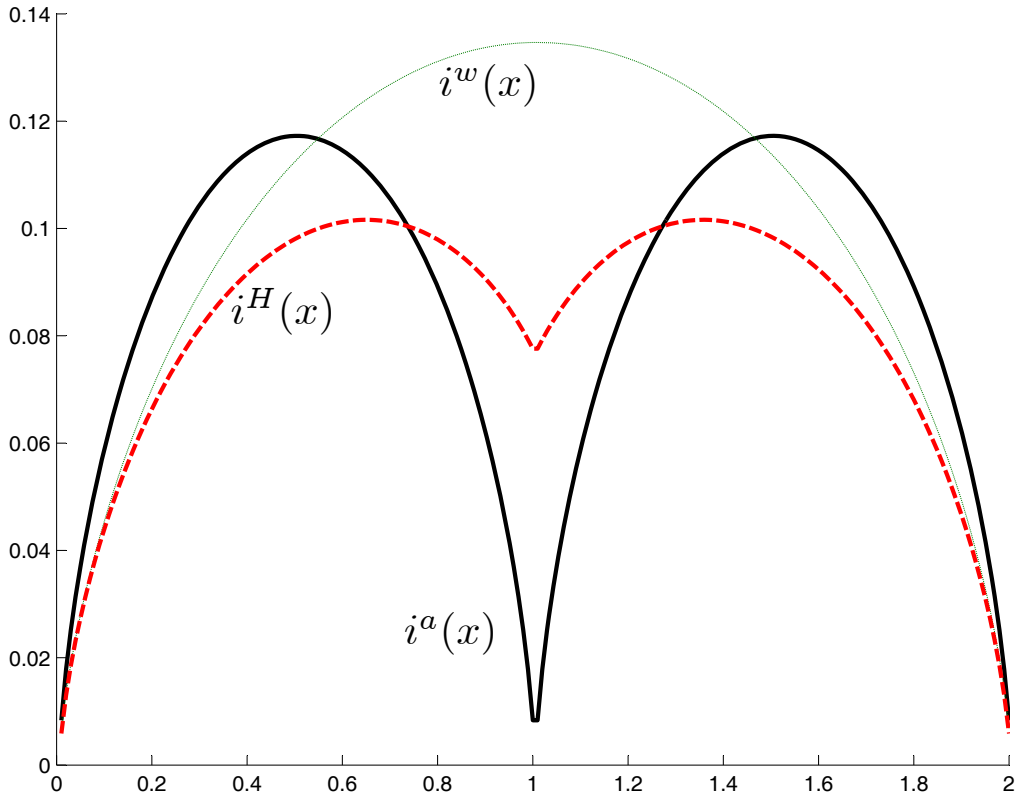
**Proposition 7** *In the symmetric Nash equilibrium,* $\left(i^H(x)\right)'_{x=0}=\infty$ *and* $\left(i^H(x)\right)'_{x=\bar{s}}$ *is negative, but finite.*

**Proof** In the Appendix.

The above proposition suggests that the presence of the foreign country skews the distribution of infrastructure investments towards the border (compared to the closed economy case), as this increases the gains from trade with the foreign country. Nevertheless, the infrastructure profile still has a hump shape (as $\left(i^H(x)\right)'_{x=0}>0$ and $\left(i^H(x)\right)'_{x=\bar{s}}<0$), meaning that the investments at the border are lower than those in some internal locations. This in turn implies the underinvestment of infrastructure at the border compared to the first best allocation (the world planner's solution) where the maximum of infrastructure investments is achieved at the border. Figure 3 illustrates this reasoning and compares the optimal autarky $i^a(x)$, world-planner

$i^w(x)$, and non-cooperative $(i^H(x), i^F(x))$ infrastructure investment schedules (we consider the parameterization used in Section 4.6.1).

Figure 3: The distribution of infrastructure investment across space




**Notes.** $i^a(x), i^w(x)$, and $i^H(x)$ refer to the autarky, central-planer, and non-cooperative optimal infrastructure investment distributions. The parameterization is as follows: $\sigma = 2$, $\delta = 1.5$, $\gamma = 0.75$, $\alpha = 0.5$, $q(x) = q = 1$, $m(x) = m = 1$, $b = 1$, and $\bar{s} = 1$.

### 5.2.1 Regional politics, global spending

We also ask the following question. What will the infrastructure profile be, if we force a social planner in each country to spend exactly the half of the global welfare maximizing amount, but leave them freedom to choose where to invest? Given this policy, the social planner faces the following optimization problem:

$$\left\{i^{H,w}(x)\right\}_{x\in[0,\bar{s}]} = \arg\max_{i(x)} \left\{ \left( \int_0^{\bar{s}} v(x)dx \right)^{\frac{1-\alpha}{\sigma-1}} \middle| q\int_0^{\bar{s}} i\left(x\right) dx \leq bLt^w \right\},$$

where $t^w$ is the tax rate that maximizes the global welfare (which is given for the social planner). It can be shown that the solution of the maximization problem above is given by

$$i^{H,w}(x)^{\delta} = \frac{\gamma\left(1-\alpha\right)\left(\int_{s\in S} v(s)ds\right)^{\frac{1-\alpha}{\sigma-1}}}{\lambda_0 q}\left(\tilde{\phi}^{L,N}(x)+\tilde{\phi}^{R,N}(x)\right),$$

where $\lambda_0$ is the corresponding Lagrange multiplier that can be found from the budget constraint

$$q\int_0^{\bar{s}} i^{H,w}\left(x\right)dx = bLt^w.$$

As a result, we have that $\left\{i^{H,w}(x)\right\}_{x\in[0,\bar{s}]}$ solves

$$i^{H,w}(x) = \frac{bLt^w}{q}\frac{\left(\left(\tilde{\phi}^{L,N}(x)+\tilde{\phi}^{R,N}(x)\right)\right)^{1/\delta}}{\int_0^{\bar{s}}\left(\left(\tilde{\phi}^{L,N}(x)+\tilde{\phi}^{R,N}(x)\right)\right)^{1/\delta}dx}.$$

As can be seen, the expression for $i^{H,w}(x)$ is quite different compared to that for $i^H\left(x\right)$. This is due to the fact that the social planner chooses the infrastructure profile given the tax rate $t^w$. In case of $i^H\left(x\right)$, the social planner chooses both infrastructure investment and the tax rate. As a result, the first order conditions appear to be different. At the same time, it is straightforward to see that $i^{H,w}(x)$ has a hump shape as well. In other words, $i^{H,w}(x)$ and $i^H\left(x\right)$ differ in levels: $i^{H,w}(x)$ tends to be higher than $i^H\left(x\right)$, as the tax rate chosen by the global planner is higher than that chosen by the local social planners ($t^w > t^H$); but have a very similar (hump) shape.

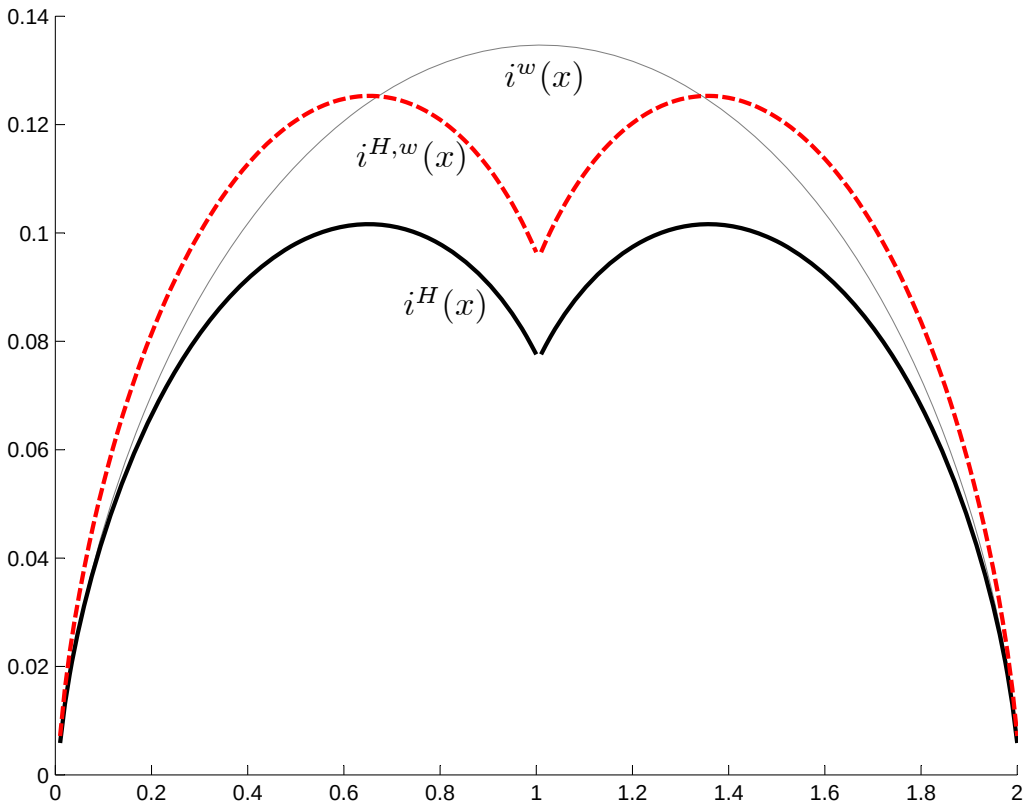
To assess by how much the discussed policy reduces infrastructure underinvestment around the border (compared to the infrastructure profile $i^H\left(x\right)$), we consider numerical simulations. In particular, we take the parameterization used to construct Figure 3 and compute the infrastructure profile $i^{H,w}(x)$. Figure 4 illustrates the results. As can be seen from the figure, $i^{H,w}(x)$ is higher than $i^H\left(x\right)$ for all $x$. As discussed above, this is due to the fact that $t^w > t^H$ (in our case, $t^w = 10\%$ and $t^H = 8\%$). As a result, in case of $i^{H,w}(x)$ the local social planner spends more on infrastructure compared to the case when she can choose the tax rate as well. At the same tome, there is still underinvestment around the border. Having the same resources the local planner tends to invest more at internal locations and less around the border compared to the global social planner.

In the next section, we explore welfare losses associated with the Nash equilibrium outcomes.

### 5.2.2 Welfare losses

In this subsection, we illustrate how misallocation of the infrastructure investments (compared to the first best, where the world planner chooses the infrastructure profile) affects welfare.

Figure 4: The distribution of infrastructure investment when the total spending on infrastructure is set to the first-best level




**Notes.** $i^H(x), i^w(x)$ refer to the non-cooperative and central-planer optimal infrastructure investment distributions. $i^{H,w}(x)$ refers to the non-cooperative optimal infrastructure investment distribution when the total spending on infrastructure is set to the first-best level. The parameterization is as follows: $\sigma = 2$, $\delta = 1.5$, $\gamma = 0.75$, $\alpha = 0.5$, $q(x) = q = 1$, $m(x) = m = 1$, $b = 1$, and $\bar{s} = 1$.

Specifically, we compare the total welfare of Home under the first best allocation with that under the Nash equilibrium. To do so, we use simulations, as the theoretical analysis is very cumbersome. Recall that the aggregate welfare at Home is

$$W = \int_0^{\bar{s}} V(x)\, dx,$$

where $V(x)$ is given by (up to a constant)

$$V(x) = (1-t)^{(\sigma-1)/(1-\alpha)} \tilde{m}(x) \left[ \int_{z \in S} T(x,z)^{1-\sigma}\, dz \right].$$

As a measure of welfare losses from misallocation, we consider a percentage change in welfare when moving from the Nash equilibrium to the first best.

Table 3: Welfare Losses

| Parameters | Welfare Losses | Compensating Variation |
|---|---|---|
| *Benchmark:* | 1.6% | 0.8% |
| $m = 100$ | 2.6% | 1.3% |
| $m = 1$ | 4% | 2% |
| $q = 0.01$ | 0.4% | 0.2% |
| $\delta = 2.3$ | 0.4% | 0.2% |
| $\gamma = 1.75$ | 2.7% | 1.4% |
| $\bar{s} = 10$ | 3.6% | 1.8% |

We consider the following *benchmark* parameterization (recall that the countries are symmetric): $\sigma = 2$, $\delta = 1.5$, $\gamma = 0.75$, $\alpha = 0.5$, $q(x) = q = 1$, $m(x) = m = 1000$, $b = 1$, and $\bar{s} = 1$. We find that under such a parameterization the aggregate welfare in the Nash equilibrium is lower than that under the first best allocation by 1.6%. In terms of compensating variation (the percentage change in the nominal GDP that corresponds to the same welfare change), the losses constitute around 0.8% of the total income.[25] We then change some parameters to see how the size of welfare losses will change. Table 3 summarizes our findings. The first column describes a certain parameterization where we change parameters compared to the benchmark parameterization. The second and third columns report the corresponding welfare changes.

As can be seen from the table, the larger is the country size, the lower are the welfare losses. For instance, a decrease in $m$ from 1000 to 1 increases the losses from 1.6% to 4%. Similar outcomes are observed, if $q$ falls or $b$ rises: the welfare losses increase. The table also shows that if infrastructure investments are less substitutable ($\delta$ rises), the welfare losses become less substantial. In particular, a rise in $\delta$ from 1.5 to 2.3 decreases the losses from 2% to 0.4%. At the same time, a rise in $\gamma$ leads to greater welfare losses. Finally, a rise in $\bar{s}$ can substantially increase the magnitude of the welfare losses as well. Specifically, a rise in $\bar{s}$ from 1 to 10 increases the losses from 1.6% to 3.6%. Note that if we force a social planner in each country to spend

[25] The losses in terms of compensating variation depend on the value of $(\sigma-1)/(1-\alpha)$, which is equal to 2 under our parameterization. In fact, the compensating variation (in percentage) is equal to $1-(1-0.016)^{(1-\alpha)/(\sigma-1)} \approx 0.008$.

exactly the half of the global welfare maximizing amount (see the previous section), but allow them to choose where to invest, welfare losses will be much smaller. For instance, in case of the benchmark specification welfare losses will be around 0.04%. This suggests that welfare losses arising in case of the Nash equilibrium are mainly due to insufficient finance (a lower tax rate), rather than the distribution of infrastructure itself.

### 5.3 Infrastructure investments and other trade costs

In this subsection, we discuss how the presence of additional trade costs (different from the transportation costs in the model) affects the equilibrium infrastructure profile. In particular, we assume that the cost of delivering one unit of a product produced at foreign location $z$ to domestic location $x$ is $\tau T(x,z)$, where $\tau > 1$ and $T(x,z)$ is modeled as before. Here, $\tau$ can be interpreted as some additional exogenous trade costs arising, for instance, from the presence of tariffs or other types of trade costs different from pure transportation costs. If $\tau$ is equal to one, then we have the framework considered above.[26]

Under the presence of additional exogenous trade costs, $i^H(x)$ solves the similar system of equations (see *Proposition* 6):

$$\begin{aligned} i^H(x)^\delta &= \frac{bL\gamma(1-\alpha)(1-t^H)}{q}\left(\tilde{\phi}^{L,N}(x,\tau)+\tilde{\phi}^{R,N}(x,\tau)\right), \\ t^H &= \frac{q\int_0^{\bar{s}} i^H(x)\,dx}{bL}, \end{aligned}$$

where $\tilde{\phi}^{L,N}(x,\tau)$ and $\tilde{\phi}^{R,N}(x,\tau)$ depend on $\tau$. Specifically, $\tau$ affects $\tilde{\phi}^{L,N}(x,\tau)$ and $\tilde{\phi}^{R,N}(x,\tau)$ through the costs of transporting products from Foreign to Home and through the aggregate consumption welfare (recall that the marginal gains from infrastructure investments are normalized by the aggregate consumption welfare, $\int_0^{\bar{s}} v(x)dx$). In the former case $\tilde{\phi}^{L,N}(x,\tau)$ is only affected, while in the latter case both $\tilde{\phi}^{L,N}(x,\tau)$ and $\tilde{\phi}^{R,N}(x,\tau)$ are affected (see the Appendix for details). Due to complexity if the analysis, we focus only on the *direct* effects of changes in $\tau$ on the infrastructure investments.

One of the implications of considering only the direct effects is that we ignore the effects of changes in the tax rate $t^H$: that is, in the considered exercise $t^H$ is fixed. This means that to some extent we explore the redistributive effects of $\tau$ on the infrastructure profile (as the total spendings on infrastructure remain unchanged). The following proposition holds:

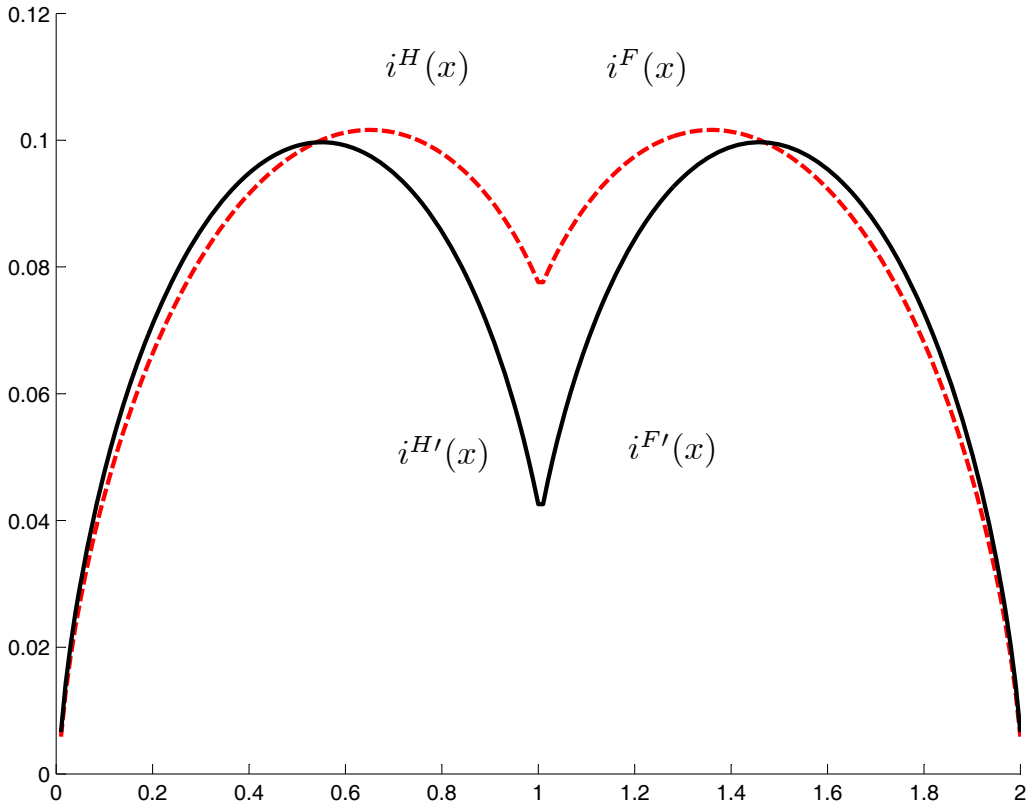
[26] Notice that the additional trade costs take place only if a product is imported from foreign locations (there are no additional trade costs for transporting between domestic locations).

**Proposition 8** *The direct effect of a rise in $\tau$ is a decrease in $i^H(x)$ in some left neighborhood of the right border and an increase in $i^H(x)$ at locations that are relatively far from that border.*

**Proof** In the Appendix.

Figure 5 provides an illustration.

Figure 5: The effect of higher discrete border costs




**Notes.** $i^H(x)$ and $i^F(x)$ refer to the non-cooperative (Nash) infrastructure investment distributions, and $i^{H\prime}(x)$ and $i^{F\prime}(x)$ show the same loci with a higher border cost.

In fact, the proposition implies that a rise in $\tau$ redistributes the infrastructure profile away from the border. This finding does not seem surprising, as a rise in the trade costs, $\tau$, reduces the gains from trade with the foreign country and, therefore, decreases the marginal returns from investing around the border, implying that $i^H(x)$ falls near the border. We believe that controlling for the indirect effects will not substantially change the above intuition and result in the similar redistribution of infrastructure investments. The numerical experiments we conduct confirms this belief.

The above results offer a natural explanation for the large border effect observed in the data. Indeed, a rise in $\tau$ not only increases the trade costs with the foreign country, but also decreases the stock of infrastructure around the border. As a result, the effect on the total transportation costs from Foreign to Home can be magnified, implying a larger border effect. In the next section, we numerically explore the role of infrastructure in explaining the border effect.

### 5.3.1 Numerical illustrations

To assess the magnitude of the effect of infrastructure distribution on the border effect, we consider the following (simulated) gravity regression used in a number of empirical trade papers to estimate the border effect:

$$\ln X_{ij} = \mu_1 \ln d_{ij} - \mu_2 B_{ij} + ex_i + im_j + \epsilon_{ij}, \tag{27}$$

where $X_{ij}$ represents trade volumes from location $i$ to $j$ generated by the model, $d_{ij}$ is the distance between the locations, $B_{ij}$ is a border dummy that equals to one if $i$ and $j$ belong to different countries, $ex_i$ and $im_j$ are exporter and importer fixed effects, and $\epsilon_{ij}$ is the error term.

In the model where transportation costs depend only on distance, one would expect the estimate of $\mu_2$ being equal to $(\sigma - 1)\ln\tau$ (recall that $\tau$ describes the additional transportation costs associated with crossing the border (see the previous section)). However, if not only distance, but also infrastructure matters for transportation costs, then the estimate of the border effect derived from a regression in (27) is upward biased. That it, one would expect the estimate of $\mu_2$ being higher than $(\sigma - 1)\ln\tau$.

To examine numerically the size of the bias, we first consider the benchmark parameterization used in Section 5.2.2 with $\tau$ equal to 1.5. Since in this case $\sigma = 2$, the estimate of $\mu_2$ using trade volumes simulated by the model without infrastructure is $\ln 1.5 \approx 0.41$. In case of trade volumes generated by the present model, we have the following outcome (the robust standard errors are in brackets):[27]

$$\ln X_{ij} = -\underset{(0.0004)}{0.07} \ln d_{ij} - \underset{(0.0005)}{0.48} B_{ij} + ex_i + im_j + \epsilon_{ij}.$$

As predicted, the estimated value of $\mu_2$ is higher than 0.41. Specifically, infrastructure investment explains 15% $(1 - 0.41/0.48)$ of the estimated border effect. Alternatively, in the presence of infrastructure investment, the border effect increases by 17% $(0.48/0.41 - 1)$. If we increase $\sigma$ from 2 to 3.4 (the value used in a number of quantitative trade papers), we have that

$$\ln X_{ij} = -\underset{(0.001)}{0.17} \ln d_{ij} - \underset{(0.001)}{1.17} B_{ij} + ex_i + im_j + \epsilon_{ij},$$

[27] The $R^2$ of the regression is around 99%.

while in the model without infrastructure the estimate of $\mu_2$ is $(\sigma-1)\ln\tau \approx 0.97$. Thus, in this case infrastructure investment increases the magnitude of the estimated border effect by around 21%.

To explore further how changes in the parameters in the model affect the role of infrastructure in explaining the border effect, we consider the following exercise. As the values of the parameters of interest we take random draws from uniform distributions. For these values we calculate the percentage change by which the presence of infrastructure increases the border effect (given by the estimate of $\mu_2$ minus $(\sigma-1)\ln\tau$). Then, we take a new draw of the parameters and calculate a new percentage change and so on. Finally, we regress the percentage change on the values of the parameters to explore the direction and the magnitude of the effects.

The set of the parameters in the effects of which we are interested in are $\gamma$, $\delta$, $\sigma$, and $\bar{s}$. We assume that $\gamma$ is a random draw from the uniform distribution on $[0.1; 2]$. To be consistent with the constraint $(\sigma-1)(\delta-1)\gamma < 1$, we take the values of $\delta$ and $\sigma$ as draws from uniform distributions on $[1; 1.1+1/\gamma]$ and $\left[2; \min\left(1+\frac{1}{\gamma(\delta-1)}, 8\right)\right]$, respectively (here we limit the minimum possible value of $\sigma$ by 2 and the maximum possible value by 8). For $\bar{s}$ we consider a uniform distribution on $[1; 2]$. The number of draws is 5000. The rest of the parameters are the following: $\alpha = 0.5$, $q(x) = q = 1$, $m(x) = m = 1000$, and $b = 1$.

The following regression describes the effects of the parameters on the size of the border effect:

$$\triangle_i = \underset{(1.88)}{-62.65} + \underset{(0.56)}{54.09}\gamma_i + \underset{(0.21)}{2}\,\delta_i + \underset{(0.31)}{11.66}\sigma_i + \underset{(0.8)}{5.8}\,\bar{s}_i + \nu_i,$$

where $\{\gamma_i, \delta_i, \sigma_i, \bar{s}_i\}$ is draw $i \in \overline{1..5000}$, $\triangle_i$ is the corresponding percentage change in the border effect, and $\nu_i$ is the error term. The mean value of $\triangle_i$ is 37% with the minimum and the maximum given by 0.3% and 124%, respectively. As can be seen, the parameters of interest positively and significantly affect the magnitude of the impact of infrastructure on the border effect. For instance, a unit rise in $\gamma$ increases the impact by 54%, while a unit rise in $\delta$ increases the magnitude only by 2%. The $R^2$ of the above regression is 0.76.

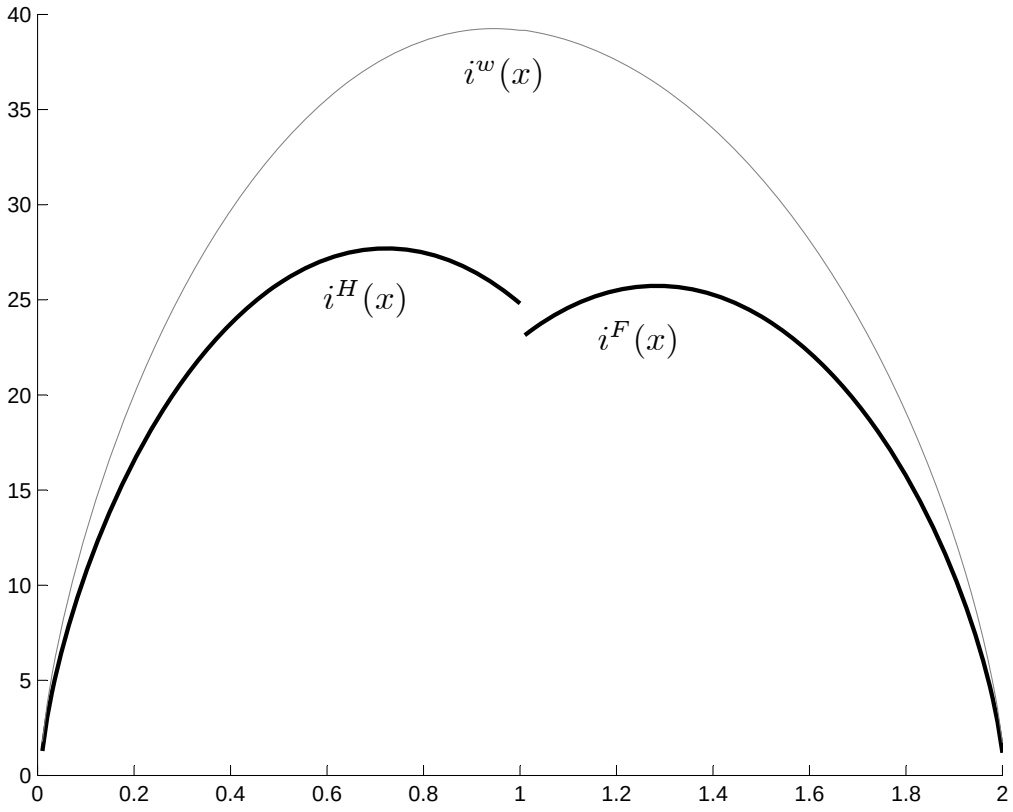
### 5.4 The role of asymmetry

In this subsection, we analyze how asymmetry between countries affects the optimal distribution of infrastructure and explore its welfare implications.

### 5.4.1 Asymmetry in country size

First, as the source of asymmetry we consider differences in the size of the countries. Specifically, in the simulations we consider the same parameterization as above, but assume that $m(x) = 1000$ for all domestic locations and $m(x) = 900$ for all foreign locations: i.e., Home is larger than Foreign. Figure 6 depicts the infrastructure profiles under the Nash equilibrium and the first best allocation. As can be inferred from the figure, the first best allocation of infrastructure is slightly skewed towards domestic locations, as they have a bigger size. In case of the asymmetric Nash equilibrium, the infrastructure profile has a hump shape in both countries with more infrastructure investments at Home (as Home is larger and, therefore, has a higher total income). Notice that in the case of the Nash equilibrium, the infrastructure investment profile has a discontinuity at the border (caused by the asymmetry between the countries).

Figure 6: The distribution of infrastructure across asymmetric countries




**Notes.** $i^H(x)$ and $i^F(x)$ refer to the non-cooperative and $i^w(x)$ to the central-planer and optimal infrastructure investment distributions.

The results of the simulations show that the welfare losses of the large country (Home) slightly

Table 4: Welfare Losses

| Parameters | Welfare Losses of Home | Welfare Losses of Foreign |
|---|---|---|
| *Benchmark:* | 1.60% | 1.60% |
| $m_H = 1000$, $m_F = 900$ | 1.59% | 1.70% |
| $m_H = 1000$, $m_F = 800$ | 1.58% | 1.78% |

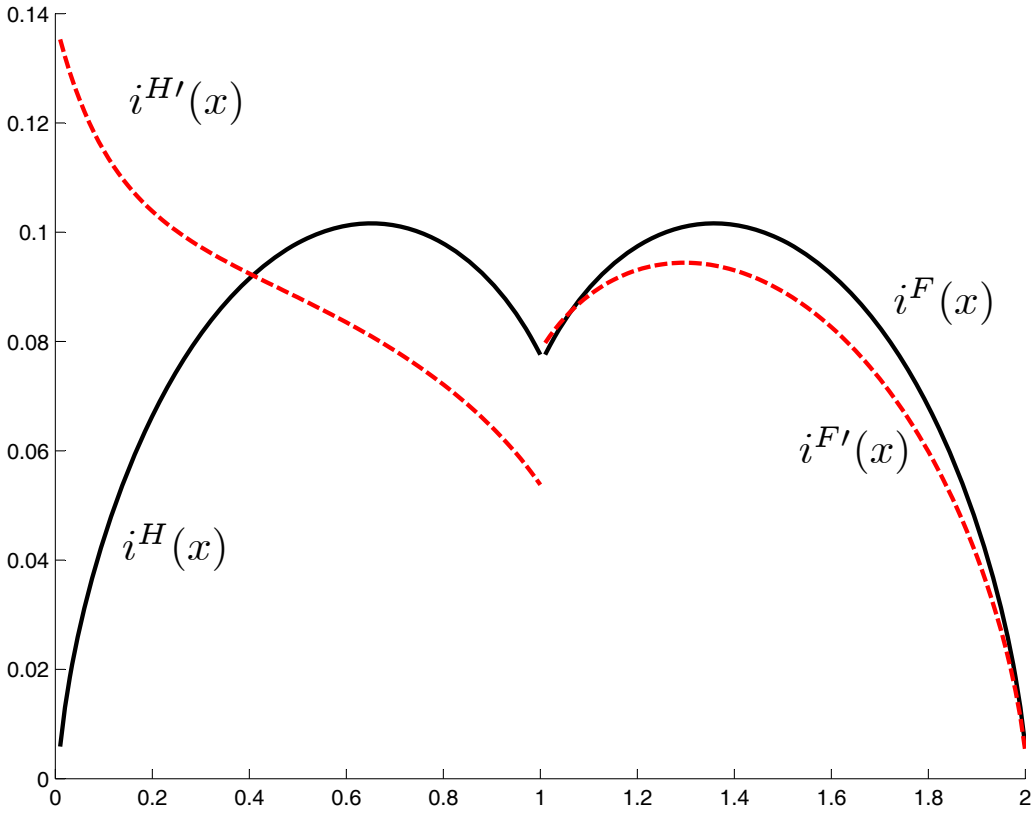
decrease (from 1.6% to 1.59%) compared to the benchmark case, while the losses of the small country (Foreign) increase from 1.6% to 1.7%. This implies that small countries lose relatively more from an inefficient distribution of infrastructure. The intuition behind is the following. In the case of the first-best allocation the smaller country benefits from the redistribution of resources across the countries enforced by the social planner, while in the case of the non-cooperative equilibrium the smaller country uses only its own resources and, therefore, looses relatively more than the large country. This intuition can be also illustrated by Figure 6. As can be seen from the figure, in the case of the first best allocation the infrastructure investments do not substantially differ across the countries: the distribution is almost symmetric. While in the case of the non-cooperative equilibrium, the level of infrastructure investments at Foreign is significantly lower than that at Home. We then further decrease the size of Foreign: $m(x) = 800$ for foreign locations. In this case, the welfare losses of Home are 1.58% and the losses of Foreign are 1.78%, which is in line with the above intuition. Table 4 summarizes the above findings.

### 5.4.2 The emergence of a third country – China

So far, we have studied a world of two countries which are connected by land-borne (intracontinental) trade. However, it is straightforward to investigate how the emergence of a third country affects the intracontinental distribution of infrastructure investment, trade, welfare, and the size of the border effect. We do this by assuming that there is a third country, call it China, which supplies and demands goods to Home and Foreign via a harbor that is located at address 0 in Home. We take China to be of the same size as Home and Foreign, but assume that its supply and demand capacity is exogenous to the outcomes in Home and Foreign. In particular, we do not model the spatial extension of China, and treat it as a mass point at location 0.

By construction, this constellation creates a costal country (Home) and a hinterland (Foreign). It also breaks symmetry between Home and Foreign, as any location in Home now is

Figure 7: Overseas trade and the distribution of intracontinental transport infrastructure investment




more central than any region in Foreign. With constant distribution of infrastructure investment, this would be true: on average, consumer prices will be lower in Home than in Foreign. However, with endogenous infrastructure investment, there will be additional effects that tend to exacerbate inequality both between Home and Foreign as well as within them.

Figure 7 shows how the intracontinental infrastructure investment distribution changes with the emergence of intercontinental trade. In Home, there will be a massive reallocation of spending towards the coastal locations where the Chinese imports crowd out varieties from locations within Home and Foreign. Investment falls rapidly with increasing distance to the harbor and is lower than in the benchmark case (intracontinental trade only) in about half of all locations in Home. In particular, Home will invest substantially less at the border with Foreign. With a dramatically more skewed distribution of investment in Home, the distribution of real wages will also become more skewed, so that trade with China makes Home a more unequal economy.

In Foreign, now a hinterland, incentives for investment are globally lower, except for locations very close to the border with Home. The reason for this is that domestic goods become less

important in the price index of consumers in Foreign, as consumers allocate spending to Chinese imports. Thus, the marginal utility gain from improving investment at most addresses within Foreign falls. Close to the border, the opposite is true because imported goods have become more relevant for welfare. Interestingly, the emergence of China increases the border effect, because it diverts investment away from it. This means that the estimated border dummy should grow larger in a standard gravity equation despite the fact that border related trade costs have not changed.

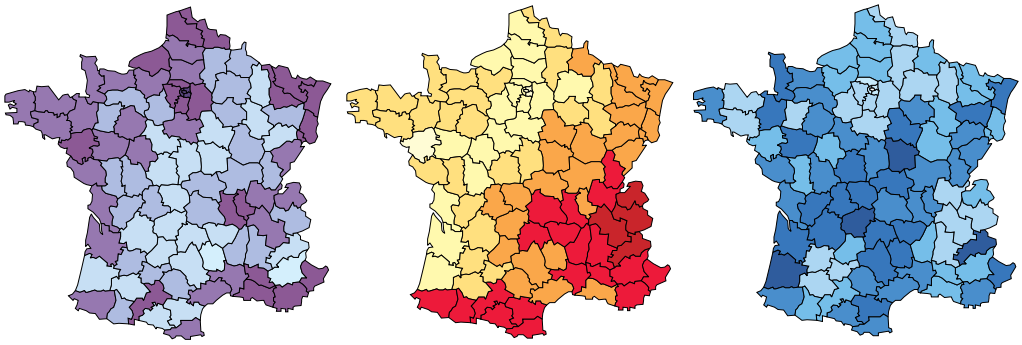
# 6 Empirical analysis

In this section we explore a data set on the spatial distribution of transportation costs in France and confront the model to the data. Reliable data on transportation costs is difficult to obtain. However, Combes and Lafourcade (2005) provide estimates of transportation costs across French départements for the year of 1993. These data are particularly interesting, because the authors adopt a comprehensive perspective on transportation costs. They report the total cost of transporting a standardized container by standardized truck from one departemental capital to another. They take account of the cost of fuel, truck depreciation, tire use, the wage bill and accommodation costs of the truck driver(s), road tolls, taxes, and insurance costs. For the purpose of the present paper, we look at variable costs, thereby excluding expenses related to the loading and unloading of the truck.

Combes and Lafourcade (2005) use their data to calculate measures of average remoteness of regions, i.e., the average cost to reach some place from the rest of France. Not surprisingly, they document a strong core-periphery pattern. In the present context, we are more interested in the spatial gradient of transportation costs. We use the Combes-Lafourcade data to construct a proxy of the *incremental* trade costs per kilometer of transiting through a département. Appendix $B$ contains the details on the construction of that proxy. It turns out that the spatial variation in the variable cost of overcoming one unit of distance is not constant over space. Per kilometer costs of overcoming one kilometer of distance are on average 4.79 French Francs. The associated standard deviation is 2.77.

Figure 8 plots the density of population (population per square kilometer), a measure of the geographical difficulty of territory (defined in the Appendix), and average variable transportation costs incurred by transiting a département (also defined in the Appendix). Not surprisingly, population density is highest in the Paris region (about 20,000 inhabitants per square kilometer in Paris intra-muros) and somewhat lower in the closer Parisian neighborhood (Île de France).

Figure 8: Population density, difficulty of territory, and variable trade costs per km in France



**Notes.** Source of data is Combes and Lafourcade (2005). Left figure: population density; middle figure: difficulty (ruggedness) of territory; right figure: variable trade costs (transit costs) per km. Shading grows darker as respective values rise

Regions with strong urban conglomerations, such as the Lille or Lyon regions (départements Nord and Rhône, respectively) have densities of 450. Départements in border regions have above average values, while interior départements have densities below average.[28] Our measure of difficulty of territory is based on the difference between the highest and the lowest altitude above sea level in a département. This measure is naturally high in départements that in the Alps, the Pyrenees, or the Massif Central. It is low in coastal areas or along large rivers (e.g., the Loire valley).

The rightmost panel of 8 shows transportation costs per kilometer. There is no strong observed association of transportation costs to geographical or demographic features. However, costs tend to be low when the territory is easy or the population of density is high. Transportation costs are lowest in the Paris or Lyon region. However, they are also low in the strongly populated North-West.[29]

One of the predictions in the model is that all else equal, transportation costs in the middle regions are lower than those at the periphery (due to lower investments in transportation infrastructure). To confront this prediction to the data, we consider the following simple OLS regression that attempts to explain the pattern of transportation costs across space:

$$\ln T_i = \partial_0 + \partial_1 \ln D_i + \partial_2 X_i + \varepsilon_i,$$

where $T_i$ is the variable transportation costs per kilometer in département $i$, $D_i$ is the measure

[28] The average density is about 105 inhabitants per kilometer.

[29] Note that there is no transportation data for Paris intra-muros.

of the overall remoteness of the département, and $X_i$ is the set of controls. According to the theory, we expect that $\partial_1$ is greater than zero.

Table 5 reports the results of the regression. Column (1) shows that the geographical difficulty of territory explains about 16 percent of the variance in transportation costs. Column (2) adds the total income of a département that appears to have a negative impact on transportation costs: the bigger is a region, the lower are the transportation costs there. Column (3) adds population size that allows us to control for the role of per capita income. As can be seen, transportation costs are lower in richer départements (controlling for populations size). In column (4) we add land surface as a control. Controlling for population size, this effectively accounts for the role of population density. As expected, an increase in density reduces the variable transportation costs. Moreover, the R-squared of the regression surges from 33 percent to 58 percent.

Columns (5) and (6) include two measures of the overall remoteness of geographical units. Depending on the variable included, the R-squared of the regression increases to up to 61 percent. Moreover, regardless of the exact measure of remoteness, transiting in more remote regions is significantly more costly, holding geographical and demographic factors constant. This empirical finding is in line with our expectation of positive $\partial_1$.

# 7 Extensions

In this section, we discuss some potential extensions of the framework.

## 7.1 The monopolist case

Here we consider an example when infrastructure investments are made by a third party. In the analysis above, we assume that the infrastructure profile is chosen by the government. However, it can be the case that the government "outsources" infrastructure investments to a third party. Below we consider a simple example of such a situation and show that the qualitative implications of the model do not change.

We assume that there is a monopolist, who decides on the infrastructure profile. The decision is based on the profit maximization and the profits of the monopolist are given by

$$\pi\left(i^M\right) = p\int_0^{\bar{s}} \bar{R}(x)dx - \int_0^{\bar{s}} q(x)i^M(x)dx, \tag{28}$$

where $p$ is the price per unit of the traffic the monopolist receives, $\bar{R}(x)$ is the measure of the total traffic through location $x$, $i^M(x)$ is infrastructure investments at location $x$, and $q(x)$ is

Table 5: Different remoteness measures and average transit costs

| Dep. var.: Ln variable transport costs per km for transits through a département | | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| ln distance to Paris | | | | | 0.034** | |
| | | | | | (0.015) | |
| ln GTC to Paris | | | | | | 0.039** |
| | | | | | | (0.015) |
| ln geography | 0.041*** | 0.029*** | 0.032*** | 0.020** | 0.004 | 0.001 |
| | (0.013) | (0.011) | (0.010) | (0.008) | (0.011) | (0.011) |
| ln GDP | | -0.048*** | -0.186*** | -0.028 | -0.019 | -0.018 |
| | | (0.013) | (0.050) | (0.052) | (0.048) | (0.047) |
| ln population | | | 0.167*** | -0.002 | -0.006 | -0.007 |
| | | | (0.056) | (0.059) | (0.056) | (0.055) |
| ln area (km2) | | | | 0.087*** | 0.068*** | 0.067*** |
| | | | | (0.013) | (0.015) | (0.014) |
| Constant | 1.590*** | 2.026*** | 1.112*** | 1.110*** | 1.059** | 0.967*** |
| | (0.009) | (0.118) | (0.334) | (0.299) | (0.285) | (0.288) |
| $N$ | 94 | 94 | 94 | 94 | 94 | 94 |
| $adj.R^2$ | 0.158 | 0.289 | 0.333 | 0.583 | 0.609 | 0.616 |
| $F-stat.$ | 10.06 | 9.443 | 10.66 | 41.91 | 41.25 | 45.17 |
| $RMSE$ | 0.097 | 0.090 | 0.088 | 0.070 | 0.068 | 0.067 |
| Robust standard errors in parentheses, $^{***}p<0.01,^{**}p<0.05,^{*}p<0.1$. | | | | | | |
| GTC: generalized transport cost. See the text and Appendix B for more details. | | | | | | |

the cost of the investments. In the above specification, we assume that the government subsidies the transportation sector by paying the fixed price $p$ per unit of the traffic to the monopolist (for example, such a subsidy can be financed through an income tax), who conditional on that invests in the transportation infrastructure. This implies that the monopolist takes the price $p$ as given and $p$ does not affect the transportation costs and, therefore, the c.i.f. prices of products (as $p$ is paid by the government). Such an assumption removes the distortive effects of $p$ on the infrastructure profile.

Next, we describe the infrastructure profile chosen by the monopolist in case of closed and open economies.

### 7.1.1 The closed economy

In the closed economy, the total traffic through location $x$ is given by

$$\bar{R}(x) = \int_0^x \left( \int_x^{\bar{s}} R(y,z)dz \right) dy + \int_x^{\bar{s}} \left( \int_0^x R(y,z)dz \right) dy,$$

where

$$R(y,z) = (1-\alpha)(1-t^M)\frac{Y(z)T\left(y,z\right)^{1-\sigma}}{\int_0^{\bar{s}} T\left(v,z\right)^{1-\sigma} dv}.$$

Here, $R(y,z)$ is interpreted as the traffic from $y$ to $z$: the number of units of a product produced at $y$ and transported to $z$. It is determined by demand at location $z$ and the transportation costs from $y$ to $z$. Note that in general $R(y,z)$ is not equal to $R(z,y)$. Finally, $t^M$ is the income tax imposed by the government to finance the infrastructure investments and $Y(z)$ is the gross income at location $z$. We assume that the monopolist takes the income tax and the location sizes as given.

It is possible to show that in case of the interior solution, the infrastructure profile maximizing the monopolist's profit is given by

$$i^M\left(x\right)^\delta = \frac{p\gamma(1-\alpha)(1-t^M)(\sigma-1)}{q(x)}\left(\phi^{L,M}(x) + \phi^{R,M}(x)\right), \tag{29}$$

where $\phi^{L,M}(x)$ ($\phi^{R,M}(x)$) represents the aggregate marginal profits due to a rise in traffic from the right (the left) to the left (the right) of location $x$.[30] The infrastructure profile chosen by the monopolist has qualitative properties that are similar to those in the benchmark case. Specifically, $i^M(0) = i^M(\bar{s}) = 0$. If $q(x) = q$ and $Y(z) = Y$ for all $x, z \in [0, \bar{s}]$ (implying no asymmetries in the cost of infrastructure and the distribution of income), the infrastructure profile has a hump shape and is symmetric around $\bar{s}/2$.

[30] The corresponding derivations and the expressions for $\phi^{L,M}(x)$ and $\phi^{R,M}(x)$ are available in the Online Appendix.

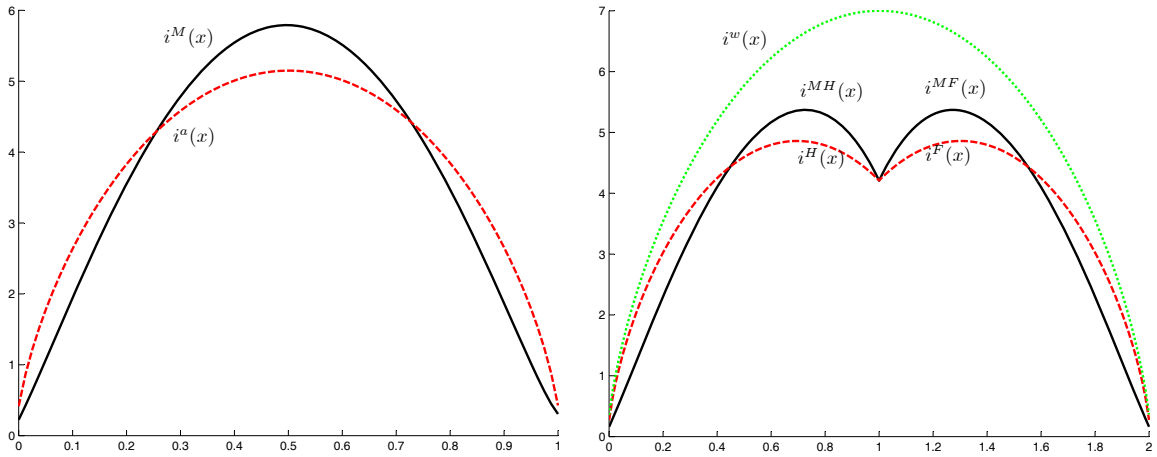
However, there is also a conceptual difference. Under certain parameters, it can be the case that the monopolist finds it optimal to put zero investments into the locations that are relatively close to the borders. As a result, there can be regions (with positive measure) that do not have transport infrastructure at all. In contrast, the social planner chooses positive infrastructure investments for all locations (except $x = 0$ and $x = \bar{s}$).

The intuition behind this outcome is as follows. A rise in the infrastructure investments in a certain region has two effects on the total traffic. On the one hand, due to better infrastructure the traffic through the region goes up, increasing the total traffic. On the other hand, the traffic through other regions goes down because of the corresponding changes in the price indices. This tends to decrease the total traffic. It appears that in some cases the negative effect prevails over the positive one in the border regions, resulting in zero infrastructure investments around the borders.

Figure 9 (panel $(a)$) simulates the infrastructure distributions (in the symmetric case) chosen by the monopolist and the social planner under the same set of the parameters in the model. In the numerical experiment, the value of $p$ is chosen such that the total cost of infrastructure investments is the same in both cases: i.e., the two distributions have the same average. As can be seen from the figure, the social planner tends to invest more around the periphery, while the monopolist invests more around the geographical center of the economy.

Figure 9: Equilibrium investment loci: social planner vs. monopolist.

(a) Autarky (b) Trade




**Notes.** Solid black curves ($i^M(x)$, $i^{MH}(x)$, and $i^{MF}(x)$) refer to investment profile resulting from non-cooperative monopolists. Dashed red curves ($i^a(x), i^H(x)$, and $i^F(x)$) refer to (non-cooperative Nash) solutions, and $i^w(x)$ is the world planner outcome.

### 7.1.2 The open economy

In the open economy, the total traffic through location $x$ is given by

$$\bar{R}(x) = \int_0^x \left( \int_x^{2\bar{s}} R(y,z)dz \right) dy + \int_x^{2\bar{s}} \left( \int_0^x R(y,z)dz \right) dy.$$

As in the benchmark case, we assume that the monopolists at Home and Foreign choose the corresponding infrastructure investments in a non-cooperative way. In the symmetric Nash equilibrium with no asymmetries in the cost of infrastructure and the location size, the domestic infrastructure profile (in case of the interior solution) is given by

$$\tilde{\imath}^M(x)^\delta = \frac{p\gamma(1-\alpha)(1-\tilde{t}^M)(\sigma-1)}{q} \left( \tilde{\phi}^{L,M}(x) + \tilde{\phi}^{R,M}(x) \right),$$

where as before $\tilde{\phi}^{L,M}(x)$ ($\tilde{\phi}^{R,M}(x)$) represents the aggregate marginal profits due to a rise in traffic from the right (the left) to the left (the right) of location $x$. Similar to the social planner solution in the open economy, the infrastructure profile is skewed towards the border with the foreign country. Moreover, numerical simulations show that the infrastructure distribution has a hump shape.[31] Finally, it is straightforward to show that the presence of costs associated with crossing the border will redistribute the infrastructure away from the border, amplifying the border effect.

Figure 9 (panel ($b$)) compares the simulated infrastructure distributions chosen by the social planner and the monopolist in the open economy (as in the previous figure, the two distributions have the same average) with the distribution chosen by the world social planner. As can be inferred from the figure, the monopolist tends to invest more around the border with the foreign country, while the social planner invests more at the periphery. This is in line with the idea outlined in the previous section that the monopolist invests more at central locations of the economy. However, simulations demonstrate that even though the monopolist invests more around the border than the social planner does, there is still substantial underinvestments in the infrastructure compared to the first best allocation.

## 7.2 Toll taxes

In reality, many countries operate toll taxes for freight traffic. The ratio of interregional highways subject to decentrally administered toll systems ranges from 6 percent to 52 percent in France. Countries such as Germany and Austria have centrally administered distance dependent road pricing for lorries. User fees may be contingent on a wide array of factors such as the situation

[31] A strict proof of a hump shape of the infrastructure distribution appears to be quite complicated.

of the environment (e.g., smog), the degree of congestion, the time at which a road is traveled (fees may be higher for travel during night or weekends), or whether an sensible region is crossed (e.g., some protected zone).

How a toll system affects the main argument in this paper depends very much on its specific design. Suppose, the government decides on the infrastructure allocation across space, but rather than taxing consumers through lump-sum taxes, it taxes road users without discriminating between home- or foreign-bound transport. This kind of taxation is of course distortionary, since it will affect goods prices of the regional and the world economy. Further assume that the fee may vary continuously on the space of addresses and that it works just as our iceberg transportation costs, albeit with the shaved transported good not lost but transferred into the governments coffers. In that case, the government has two margins of action: it sets a distribution of fees, $f(s)$, where $s \in [0, \bar{s}]$, and decides on the infrastructure allocation, $i(s)$. Total income from fees will be $B = \int_0^{\bar{s}} f(s) R(s) ds$, where $R(s)$ is the value of goods that are transported through the point $s$. Note that $R(s)$ depends on the domestic and foreign distribution of infrastructure. If the government is free to spend $B$ on whatever infrastructure distribution it prefers, and does not impose any additional tax, it will find it optimal to make fees dependent on the distance to the border, with higher fees the closer the border. Moreover, it will concentrate its spending as before in the central regions of the jurisdiction. The reason for this result is identical to the one discussed above: The government cares only about home welfare, thus taxing foreigners and transferring the receipts to home citizens is a welcome option.

If governments are not allowed to spend toll income on places other than those were the income has been generated, there will now be a direct link between infrastructure investment and transit volume, $f(s) R(s) / q(s) = i(s)$. Governments will then set $f(s)$ such that native welfare is maximized. By imposing high fees in border regions, governments tax foreign consumers more strongly than domestic ones; however, the implied high investment volumes are of little value for domestic consumers. By imposing high fees in the center, governments affect domestic consumers, but also achieve high utility for them. The implied distribution of infrastructure need not exhibit excessive spatial variation, but it still can; the exact outcome depending on underlying model parameters.

There are a number of institutional arrangements that involve the private sector into the construction and maintenance of transport infrastructure projects. Governments could sell exploitation licenses to private firms who construct roads and set fee structures $f(s)$.While this is a choice of many governments, the economic modeling poses a number of problems, since one would have to decide whether the licenses are sold to a single provider, to a consortium, or to lo-

cal monopolists, and whether those firms can commit to certain fee schedules and infrastructure investments when making their bids. At the same time, the example in the previous section illustrates that the spatial distribution of infrastructure would not necessary qualitatevely change if the private sector is involved.

### 7.3 Supranational entities

Since decentralized transport infrastructure decisions generate positive externalities leading to a global underprovision and excessive spatial variation of infrastructure investment, there is a case for a supranational entity, such as the European Union (EU), to intervene. Indeed, the EU is involved in a large project, the Trans-European-Networks (TEN), that strives to coordinate and cofinance national infrastructure provision efforts. Member states have committed themselves to construct a number of road and rail links that link European regions. The EU, in turn, cofinances these projects. The degree of cofinancing is higher in peripheral regions than in central ones. Is this policy able to internalize the externalities highlighted in the present paper? In principle, the answer is yes. In order to achieve the first-best infrastructure distribution, the EU should be allowed to tax citizens (or member state governments on their behalf) and subsidize the price of infrastructure. The problem with the above subsidization principle is that it does not necessarily ensure that the overall quantity of infrastructure provision is efficient.

### 7.4 Internal labor mobility

In the model we assume away the effects of internal labor mobility on the infrastructure profile. We believe that allowing $m(s)$ to adjust endogenously will not qualitatively change the main implications of the model. For instance, the equilibrium in a closed economy in case of internal labor mobility (associated with some adjustment costs to avoid extreme solutions) can be considered as the outcome of the following multi-stage variation of the model. The economy starts with the uniform distribution of population. In the absence of other asymmetries, the optimal distribution of infrastructure in this case is symmetric around the middle location. Then, given this infrastructure profile, agents decide about their location and, since the transportation infrastructure is better around middle locations, the distribution of population will "squeeze" around the middle point. Given the new distribution of population, the social planner adjusts the distribution of infrastructure and so on till convergence. The resulting distribution of infrastructure will be symmetric around the middle location, but more squeezed around that point compared to the case with no labor mobility (as around the middle point the distribution of population will be denser). In other words, labor mobility will magnify the concentration of

infrastructure around middle locations. In case of an open economy labor mobility will magnify the skewness of the distribution of infrastructure towards the border. Note that in a one-stage model where the social planner chooses the infrastructure profile taking into account population mobility, multiple equilibria are possible.

# 8 Conclusions

This paper develops a model where consumers demand goods from the entire world, but the world is fragmented into jurisdictions which set infrastructure investment schedules in a non-cooperative way. Governments caring only for their own welfare constituency will ignore the effects that their decisions have on foreign consumers; this basic externality leads to global underinvestment. The externality is stronger the more foreign-bound transit flows through an address, and the size of such transit is larger the closer national borders are to that address. Hence, infrastructure underinvestment is stronger in peripheral regions of jurisdictions rather than in central ones.

The local lack of infrastructure investment makes imports from other countries more expensive than imports from other regions from the same country, even if geographical distance or incomes of trading partners are the same. Our infrastructure story may therefore contribute towards unpacking trade costs and explaining the border puzzle highlighted by McCallum (1995).

The model has a range of predictions that can be put to an empirical test. For instance, the model has interesting predictions relating to the effects of preferential trade liberalization. Infrastructure investment should be skewed towards that border which is economically permeable. Subsequent waves of EU enlargement could be used to check this hypothesis. The model could also be brought to a calibration exercise. Standard new economic geography models predict economic inequality, but require the existence of natural peripheries. Our argument would allow economic inequality across space even in circumstances where no natural periphery exists, and borders have exclusively political significance. Calibration exercises of the standard models often lead to simulated inequalities statistics that are too low compared to the data. Allowing for the endogenous allocation of infrastructure across space could help improve this fit.

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# Appendix A

In the Appendix, we provide all the necessary proofs for the lemmas and propositions in the paper.

## Proof of Lemma 1

### Property $(iv)$

The behavior of $T(x,z)$ with respect to geographical distance can be checked by looking at the derivative of $T(x,z)$ with respect to $z$. Specifically, we have that

$$\begin{aligned}
T(x,z) &= \left(1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds\right)^{\gamma},\\
T_z(x,z) &= \gamma\left(1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds\right)^{\gamma-1}\frac{1}{\delta-1}\left[i(z)^{1-\delta}\right]>0,\\
T_{zz}(x,z) &= \gamma\left(1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds\right)^{\gamma-1}\left((\gamma-1)\frac{\left(\frac{1}{\delta-1}\left[i(z)^{1-\delta}\right]\right)^2}{1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds}-i(z)^{-\delta}i'(z)\right)\gtrless 0\\
&\iff \frac{(\gamma-1)}{(T(x,z))^{1/\gamma}}\left(\frac{i(z)^{1-\delta}}{\delta-1}\right)^2 \gtrless i'(z)\,i(z)^{-\delta}.
\end{aligned}$$

The left-hand side shows the effect of a marginal increase in distance on $T_z(x,z)$ under the assumption that $i(z+dz)=i(z)$. It reflects variation in trade costs due to an increase in distance, holding infrastructure constant. The right-hand side makes the opposite assumption and reports the change in $T_z(x,z)$ due to the difference in infrastructure investment between $z$ and $z+dz$, holding the sheer costs of distance constant. $T(x,z)$ is convex (as in Krugman), if the left-hand side dominates.

### Property $(v)$

We compute the elasticity of substitution between investment at two different locations $s', s'' \in [x,z]$ as follows

$$-\frac{d\ln\left[i(s')/i(s'')\right]}{d\ln\left|\frac{\partial T(x,z)}{\partial i(s')}/\frac{\partial T(x,z)}{\partial i(s'')}\right|},$$

where for any $s\in[x,z]$

$$\frac{\partial T(x,z)}{\partial i(s)} \overset{d.}{\equiv} \gamma\left(1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds\right)^{\gamma-1} i(s)^{-\delta}.$$

As a result, the elasticity of substitution is given by

$$-\frac{d\ln\left[i(s')/i(s'')\right]}{-d\delta\ln\left|i(s')/i(s'')\right|}=\frac{1}{\delta}<1.$$

### Property $(vi)$

The Lagrangian for the cost minimizing problem can be written as follows:

$$\Lambda\left(\{i(s)\},\lambda\right)=q\int_x^z i(s)\,ds+\lambda\left[1+\frac{1}{\delta-1}\int_x^z i(s)^{1-\delta}\,ds-\bar{T}^{1/\gamma}\right],$$

where $q$ is the cost of infrastructure investment at location $s$. It is straightforward to see that the first order conditions imply that, for any two locations $k,l\in[x,z]$,

$$i(k)=i(l).$$

Taking into account the budget constraint, we derive that

$$i\left(k\right)=i\left(l\right)=\left\{\left(z-x\right)/\left[\left(\delta-1\right)\left(\bar{T}^{1/\gamma}-1\right)\right]\right\}^{1/(\delta-1)}.$$

## Proof of Proposition 1

To find the optimal infrastructure investments on $[0,\bar{s}]$, we first formulate and analyze a discrete modification of the social planner's maximization problem (which approximates the actual continuous maximization problem). Remember that the social planner solves

$$\left\{i^{a}\left(x\right),t^{a}\right\}_{x\in S}=\arg\max\left\{\left(1-t\right)\left(\int_{x\in S}\tilde{m}\left(x\right)v(x)dx\right)^{\frac{1-\alpha}{\sigma-1}}\left|\int_{x\in S}q\left(x\right)i\left(x\right)dx\leq btL\right.\right\}.$$

where

$$\begin{aligned}v(x) &= \int_{z\in S}T\left(x,z\right)^{1-\sigma}dz\\ &= \int_{0}^{\bar{s}}\left(1+\frac{1}{\delta-1}\left|\int_{x}^{z}i\left(s\right)^{1-\delta}ds\right|\right)^{\gamma(1-\sigma)}dz\\ &= \int_{0}^{x}\left(1+\frac{1}{\delta-1}\int_{z}^{x}i\left(s\right)^{1-\delta}ds\right)^{\gamma(1-\sigma)}dz+\int_{x}^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_{x}^{z}i\left(s\right)^{1-\delta}ds\right)^{\gamma(1-\sigma)}dz.\end{aligned}$$

In the discrete version of the problem, we assume that the geography of the economy consists of $n$ points (transportation hubs) uniformly distributed on $[0,\bar{s}]$: $0=x_1<x_2..<x_n=\bar{s}$. We define the distance between location $x_i$ and $x_{i+1}$ by $\triangle_n$: that is, $x_2-x_1=...x_n-x_{n-1}=\triangle_n$. We also assume that the transportation costs between two locations, $x_i$ and $x_j$, depend on the level of infrastructure at the location of the producer and the intermediate locations between $x_i$ and $x_j$ (not including the location of the consumer) and the distance between the locations determined by $\triangle_n$.[32] Specifically, if a product is sent from $x_i$ to $x_j$ $(x_i<x_j)$, the transportation costs are

$$T(x_i,x_j)=\left(1+\frac{1}{\delta-1}\sum_{k=i}^{j-1}i\left(x_k\right)^{1-\delta}\triangle_n\right)^{\gamma}.$$

We then formulate the social planner's problem in the case of the discrete geography in the following way:

$$\left\{i^{a}\left(x_i\right),t^{a}\right\}_{i=1..n}=\arg\max\left\{\left(1-t\right)\left(\sum_{i=1}^{n}\tilde{m}\left(x_i\right)v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}}\right\},\tag{30}$$

subject to

$$\sum_{i=1}^{n}q\left(x_i\right)i\left(x_i\right)\triangle_n\leq btL,\tag{31}$$

$$i(x_i)\geq 0,\ i=1..n.\tag{32}$$

[32] The fact that the transportation costs between $x_i$ and $x_j$ do not include the level of infrastructure investment at the destination location (the location of the consumer) means non-symmetric transport costs. However, taking the limit $(n\to\infty)$ will imply symmetric transport costs between any locations, as in this case the contribution of the infrastructure of a single location into the total transport costs will be negligible.

Here,

$$v(x_i) = \triangle_n \left( \begin{array}{c} \sum_{k=1}^{i-1} \left( 1 + \frac{1}{\delta - 1} \sum_{j=k}^{i-1} i\left(x_j\right)^{1-\delta} \triangle_n \right)^{\gamma(1-\sigma)} + \\ \sum_{k=i+1}^{n} \left( 1 + \frac{1}{\delta - 1} \sum_{j=i+1}^{k} i\left(x_j\right)^{1-\delta} \triangle_n \right)^{\gamma(1-\sigma)} + 1 \end{array} \right), \tag{33}$$

where the first term in the brackets describes the welfare derived from trade with locations on the left side of $x_i$, the second term describes the welfare derived from trade with locations on the right side of $x_i$, and one stands for the transportation costs from $x_i$ to $x_i$ (that are normalized to unity). In this case, the social planner's problem is to choose the level of transportation infrastructure at each location $x_i$ that maximizes the social welfare. This maximization problem is the approximation of the maximization problem in the main text (see (14)). Specifically, taking the limit ($n \to \infty$), one can derive the actual maximization problem described in the main text. This in turn implies that the solution of the continuous maximization problem is the limit of the solution of the discrete problem (when $n \to \infty$).

Note that the existence of the global maximum in the above discrete maximization problem follows from the continuity of the objective function and the fact that the objective function is maximized on the compact subset of $R^n$ (which is given by the constraints in (31) and (32)). Moreover, it is straightforward to show that if $\gamma(\sigma-1)(\delta-1) < 1$ (as assumed in the paper), $\left(1 + \frac{1}{\delta-1} \sum_{j=i+1}^{k} i\left(x_j\right)^{1-\delta} \triangle_n\right)^{\gamma(1-\sigma)}$ is a concave function in $i\left(x\right) = \left\{i\left(x_j\right)\right\}_{j=1..n}$ for any $i$ and $k$. This in turn implies that $\tilde{v}(x_i)$ is concave in $i\left(x\right)$ as the sum of concave functions, but not strictly concave, as $v(x_i)$ does not depend on $i(x_i)$. However, it is straightforward to show that $\sum_{i=1}^{n} \tilde{m}\left(x_i\right) v(x_i)$ is strictly concave with respect to $i\left(x\right)$. Finally, since $\left(1-\alpha\right)/\left(\sigma-1\right)$ is assumed to be strictly less than one, the objective function in (30) is strictly concave in infrastructure investments, $i(x)$.

Next, we formulate the corresponding Lagrange function that can be written as follows:

$$\Lambda \equiv (1-t) \left( \triangle_n \sum_{i=1}^{n} \tilde{m}\left(x_i\right) v(x_i) \right)^{\frac{1-\alpha}{\sigma-1}} - \lambda_0 \left( \sum_{i=1}^{n} q\left(x_i\right) i\left(x_i\right) \triangle_n - btL \right),$$

where $\lambda_0$ is the Lagrange multiplier of the corresponding constraint. The necessary and sufficient condition for the global maximum is that

$$\begin{aligned} \frac{\partial \Lambda}{\partial i\left(x_i\right)} &= 0,\ i = 1..n, \\ \frac{\partial \Lambda}{\partial t} &= 0. \end{aligned}$$

Next, we find the expression for $\partial\Lambda/\partial i\left(x_i\right)$. Specifically,

$$\frac{\partial \Lambda}{\partial i\left(x_i\right)} = \frac{(1-t)\left(1-\alpha\right)}{\sigma-1} \triangle_n \left( \triangle_n \sum_{l=1}^{n} \tilde{m}\left(x_l\right) v(x_l) \right)^{\frac{1-\alpha}{\sigma-1}-1} \sum_{l=1}^{n} \tilde{m}\left(x_l\right) \left(v(x_l)\right)'_{i(x_i)} - \lambda_0 \triangle_n q\left(x_i\right). \tag{34}$$

One can show that

$$\begin{aligned} \left(v(x_l)\right)'_{i(x_i)} &= \left(\triangle_n\right)^2 \gamma(\sigma-1) i(x_i)^{-\delta} \sum_{k=1}^{i} \left( 1 + \frac{1}{\delta-1} \sum_{j=k}^{l-1} i\left(x_j\right)^{1-\delta} \triangle_n \right)^{\gamma(1-\sigma)-1} && \text{if } i < l, \\ \left(v(x_l)\right)'_{i(x_i)} &= \left(\triangle_n\right)^2 \gamma(\sigma-1) i(x_i)^{-\delta} \sum_{k=i}^{n} \left( 1 + \frac{1}{\delta-1} \sum_{j=l+1}^{k} i\left(x_j\right)^{1-\delta} \triangle_n \right)^{\gamma(1-\sigma)-1} && \text{if } i > l, \\ \left(v(x_l)\right)'_{i(x_i)} &= 0 && \text{if } i = l. \end{aligned}$$

As a result,

$$\begin{aligned}\frac{\partial \Lambda}{\partial i\left(x_i\right)} &= \frac{\left(\triangle_n\right)^3 \frac{\gamma(1-\alpha)(1-t)}{i(x_i)^\delta} \sum_{l=1}^{i-1} \tilde{m}\left(x_l\right)\left(\sum_{k=i}^{n}\left(1+\frac{1}{\delta-1}{}^{k}_{j=l+1} i\left(x_j\right)^{1-\delta} \triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\left(\triangle_n \sum_{l=1}^{n} \tilde{m}\left(x_l\right) v(x_l)\right)^{1-\frac{1-\alpha}{\sigma-1}}} \\ &+\frac{\left(\triangle_n\right)^3 \frac{(1-\alpha)(1-t)}{i(x_i)^\delta} \sum_{l=i+1}^{n} \tilde{m}\left(x_l\right)\left(\sum_{k=1}^{i}\left(1+\frac{1}{\delta-1}{}^{l-1}_{j=k} i\left(x_j\right)^{1-\delta} \triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\left(\triangle_n \sum_{l=1}^{n} \tilde{m}\left(x_l\right) v(x_l)\right)^{1-\frac{1-\alpha}{\sigma-1}}} \\ &-\lambda_0 \triangle_n q\left(x_i\right).\end{aligned} \tag{35}$$

Finally,

$$\begin{aligned}\frac{\partial \Lambda}{\partial t} &= -\left(\triangle_n \sum_{l=1}^{n} \tilde{m}\left(x_l\right) v(x_l)\right)^{\frac{1-\alpha}{\sigma-1}}+\lambda_0 bL=0 \iff \\ \lambda_0 &= \frac{\left(\triangle_n \sum_{l=1}^{n} \tilde{m}\left(x_l\right) v(x_l)\right)^{\frac{1-\alpha}{\sigma-1}}}{bL}.\end{aligned} \tag{36}$$

Note that since $\frac{\partial \Lambda}{\partial i(x_i)}|_{i(x_i)=0}=\infty$ (because of $\gamma(\sigma-1)(\delta-1)<1$), $i^a\left(x_i\right)$ is strictly greater than zero for all $i=1..n$. That is, the global maximum $\left\{i^a\left(x_i\right), t^a\right\}_{i=1..n}$ does not include zero investments and, therefore, solves the following system of equations

$$\begin{aligned}\frac{q(x_i) i^a\left(x_i\right)^\delta}{\triangle_n bL\gamma\left(1-\alpha\right)\left(1-t^a\right)} &= \frac{\sum_{l=1}^{i-1} \tilde{m}\left(x_l\right)\left(\sum_{k=i}^{n}\left(1+\frac{1}{\delta-1}{}^{k}_{j=l+1} i^a\left(x_j\right)^{1-\delta} \triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\sum_{i=1}^{n} \tilde{m}\left(x_i\right) v(x_i)} \\ &+\frac{\sum_{l=i+1}^{n} \tilde{m}\left(x_l\right)\left(\sum_{k=1}^{i}\left(1+\frac{1}{\delta-1}{}^{l-1}_{j=k} i^a\left(x_j\right)^{1-\delta} \triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\sum_{i=1}^{n} \tilde{m}\left(x_i\right) v(x_i)}, \\ \sum_{i=1}^{n} q\left(x_i\right) i^a\left(x_i\right) \triangle_n &= bt^a L.\end{aligned} \tag{37}$$

As the solution of the continuos maximization problem is the limit of the solution of the discrete problem (when $n \to \infty$), it is straightforward to see that the optimal infrastructure investment function $i^a\left(x\right)$ and the tax rate $t^a$ solve

$$\begin{aligned}i^a\left(x\right)^\delta &= \frac{bL\gamma\left(1-\alpha\right)\left(1-t^a\right)}{q(x)}\left(\phi^L(x)+\phi^R(x)\right), \\ t^a &= \frac{\int_0^{\bar{s}} q\left(s\right) i^a\left(s\right) ds}{b},\end{aligned}$$

where

$$\begin{aligned}\phi^L(x) &= \frac{\int_0^x \tilde{m}\left(s\right)\left(\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^t i^a\left(r\right)^{1-\delta} dr\right)^{\gamma(1-\sigma)-1} dt\right) ds}{\int_0^{\bar{s}} \tilde{m}\left(s\right) v(s) ds}, \\ \phi^R(x) &= \frac{\int_x^{\bar{s}} \tilde{m}\left(s\right)\left(\int_0^x\left(1+\frac{1}{\delta-1}\int_t^s i^a\left(r\right)^{1-\delta} dr\right)^{\gamma(1-\sigma)-1} dt\right) ds}{\int_0^{\bar{s}} \tilde{m}\left(s\right) v(s) ds}.\end{aligned}$$

## Proof of Proposition 2

### Property $(ii)$

Note that the derivative of $i^a(x)$ with respect to $x$ can be written as follows:

$$\begin{aligned} q(x)\delta i^a(x)^{\delta-1}\left(i^a(x)\right)' + q'(x)i^a(x)^{\delta} &= bL\gamma(1-\alpha)(1-t^a)\left(\left(\phi^L(x)\right)' + \left(\phi^R(x)\right)'\right) \iff \\ \left(i^a(x)\right)' &= \frac{bL\gamma(1-\alpha)(1-t^a)\left(\left(\phi^L(x)\right)' + \left(\phi^R(x)\right)'\right) - q'(x)i^a(x)^{\delta}}{q(x)\delta i^a(x)^{\delta-1}}. \end{aligned}$$

We have that

$$\begin{aligned} \left(\phi^L(x)\right)' &= \frac{\tilde{m}(x)\left(\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^t i(r)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right) - \int_0^x \tilde{m}(s)\left(1+\frac{1}{\delta-1}\int_s^x i(r)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}ds}{\int_0^{\bar{s}}\tilde{m}(s)v(s)ds}, \\ \left(\phi^R(x)\right)' &= \frac{-\tilde{m}(x)\left(\int_0^x\left(1+\frac{1}{\delta-1}\int_t^x i(r)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right) + \int_x^{\bar{s}}\tilde{m}(s)\left(1+\frac{1}{\delta-1}\int_x^s i(r)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}ds}{\int_0^{\bar{s}}\tilde{m}(s)v(s)ds}. \end{aligned}$$

It is straightforward to see that

$$\begin{aligned} \left(\phi^L(0)\right)' + \left(\phi^R(0)\right)' &> 0, \\ \left(\phi^L(1)\right)' + \left(\phi^R(1)\right)' &< 0. \end{aligned}$$

Since $q'(x)$ is continuous on $[0,\bar{s}]$ and $i^a(0) = i^a(\bar{s}) = 0$, $q'(0)i^a(0)^{\delta} = q'(\bar{s})i^a(\bar{s})^{\delta} = 0$. This in turn immediately implies that

$$\begin{aligned} \left(i^a(0)\right)' &= \infty \\ \left(i^a(\bar{s})\right)' &= -\infty. \end{aligned}$$

### Property $(iii)$

First, we show that $i^a(x)$ is symmetric around $x = \bar{s}/2$. To do so, we use the fact that the objective function of the planner's problem is strictly concave, implying that there is a unique solution of (19). Indeed, if there are two different solutions of (19), then there are two different stationary points, which is not possible with a strictly concave objective function. It is straightforward to see that if there is no variation in the cost of infrastructure investment and the household size across the locations: $q(x) = q$ and $m(x) = m$ for any $x \in [0,\bar{s}]$, symmetric investments indeed solve (19). Taking into account the uniqueness of the solution, we can conclude that the symmetric infrastructure profile delivers the global maximum of the maximization problem. Finally, if $i^a(x)$ is symmetric around $x = \bar{s}/2$, the transportation costs are symmetric and, thereby, $v(x)$ is symmetric as well (this immediately follows from the definition of $v(x)$ (see (15)).

Next, we prove that $i^a(x)$ has a hump shape with the peak at $x = \bar{s}/2$. Note that the optimal investment profile can be considered as a unique fixed point of a certain functional $J : E \longrightarrow E$. Here, $E$ is the subset of the space of symmetric (around $x = \bar{s}/2$) continuous functions on $[0,\bar{s}]$, which is determined by the constraints in the maximization problem. Specifically, the functional is given by

$$J(i^a(x)) = \left(\frac{bL\gamma(1-\alpha)(1-t^a)}{q}\left(\phi^L(x) + \phi^R(x)\right)\right)^{1/\delta},$$

where

$$\begin{aligned}\phi^L(x) &= \frac{\int_0^x \left(\int_x^{\bar{s}} \left(1+\frac{1}{\delta-1}\int_s^t i^a(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1} dt\right) ds}{\int_0^{\bar{s}} v(s)ds},\\ \phi^R(x) &= \frac{\int_x^{\bar{s}} \left(\int_0^x \left(1+\frac{1}{\delta-1}\int_t^s i^a(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1} dt\right) ds}{\int_0^{\bar{s}} v(s)ds}.\end{aligned}$$

We then show that if $i^a(x)$ has a hump shape with the peak at $x=\bar{s}/2$, then $J(i^a(x))$ has a hump shape. This in turn means that the fixed point of the functional has a hump shape. That is, the optimal investment profile has a hump shape.

Consider a function $i_1^a(x)\in E$ that has a hump shape. We define $i_2^a(x)$ as the value of the functional at $i_1^a(x)$: i.e., $i_2^a(x)=J(i_1^a(x))$. Next, we prove that $i_2^a(x)$ has a hump shape. To show this, we consider the derivative of $i_2^a(x)$. It is straightforward to see that

$$i_2^a(x)^{\delta-1}\left(i_2^a(x)\right)' = 2\frac{bL\gamma}{q\delta}\frac{(1-\alpha)(1-t^a)}{\int_0^{\bar{s}} v(s)ds}H(x),$$

where

$$H(x)=\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^z i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz-\int_0^x\left(1+\frac{1}{\delta-1}\int_z^x i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz.$$

The first thing to notice is that $H(\bar{s}/2)=0$ (which follows from the symmetry of $i_1^a(x)$). Consider any $x<\bar{s}/2$, then the function $H(x)$ can be written as follows:

$$\begin{aligned}H(x) &= \int_x^{2x}\left(1+\frac{1}{\delta-1}\int_x^z i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz-\int_0^x\left(1+\frac{1}{\delta-1}\int_z^x i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz\\ &\quad+\int_{2x}^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^z i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz,\end{aligned}$$

where

$$\int_{2x}^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^z i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz>0.$$

Note that as $i_1^a(s)$ is symmetric around $x=\bar{s}/2$ and has a hump shape at $\bar{s}/2$, for any $\triangle\in(0,x]$

$$i_1^a(x+\triangle)>i_1^a(x-\triangle).$$

This in turn means that for any $\triangle\in(0,x]$,

$$1+\frac{1}{\delta-1}\int_x^{x+\triangle} i_1^a(s)^{1-\delta}\,ds<1+\frac{1}{\delta-1}\int_{x-\triangle}^x i_1^a(s)^{1-\delta}\,ds,$$

implying that

$$\left(1+\frac{1}{\delta-1}\int_x^{x+\triangle} i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1}>\left(1+\frac{1}{\delta-1}\int_{x-\triangle}^x i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1},$$

as $\gamma(1-\sigma)-1$ is negative. As a result, it is straightforward to see that

$$\int_x^{2x}\left(1+\frac{1}{\delta-1}\int_x^z i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz>\int_0^x\left(1+\frac{1}{\delta-1}\int_z^x i_1^a(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)-1} dz.$$

That is, for any $x < \bar{s}/2$, $H(x) > 0$. Similarly, it is possible to show that for any $x > \bar{s}/2$, $H(x) < 0$. Thus, $(i_2^a(x))'$ is strictly greater than zero 0 on $x \in [0, \bar{s}/2)$ and strictly less than zero on $(\bar{s}/2, 1]$. This means that $i_2^a(x)$ has a hump shape. Therefore, the solution of (19) has a hump shape.

Finally, we show that $v(x)$ has a hump shape as well. We have that

$$\begin{aligned} v'(x) &= \left(\int_0^x \left(1 + \frac{1}{\delta - 1}\int_z^x i(s)^{1-\delta} ds\right)^{\gamma(1-\sigma)} dz + \int_x^{\bar{s}} \left(1 + \frac{1}{\delta - 1}\int_x^z i(s)^{1-\delta} ds\right)^{\gamma(1-\sigma)} dz\right)' \\ &= \frac{\gamma(\sigma-1) i(x)^{1-\delta}}{\delta - 1}\left(\int_x^{\bar{s}} \left(1 + \frac{1}{\delta - 1}\int_x^z i(s)^{1-\delta} ds\right)^{\gamma(1-\sigma)-1} dz - \int_0^x \left(1 + \frac{1}{\delta - 1}\int_z^x i_1^a(s)^{1-\delta} ds\right)^{\gamma(1-\sigma)-1} dz\right) \\ &= \frac{\gamma(\sigma-1) i(x)^{1-\delta}}{\delta - 1} H(x). \end{aligned}$$

Taking into account the properties of $H(x)$, we can conclude that $v(x)$ has a hump shape.

## Proof of Proposition 3

Recall that the optimal infrastructure profile $i^a(x)$ maximizes the following function (up to an irrelevant constant):

$$W(I, A) = \left(1 - A\int_{x\in S} i(x)\, dx\right)\left(\int_{x\in S} v(x) dx\right)^{(1-\alpha)/(\sigma-1)},$$

where $I = \{i(x)\}_{x\in\{0,\bar{s}\}}$ and $A = q/bL$ (where $L = m\bar{s}$). The partial derivative of $W(I, A)$ with respect to $A$ is given by

$$\frac{\partial W(I, A)}{\partial A} = -\left(\int_{x\in S} i(x)\, dx\right)\left(\int_{x\in S} v(x) dx\right)^{(1-\alpha)/(\sigma-1)} < 0.$$

We say that $I' \gtrsim I''$ if and only if $i'(x) \geq i''(x)$ for all $x \in S$. It is straightforward to see that if $I' \gtrsim I''$, then

$$\frac{\partial W(I', A)}{\partial A} \leq \frac{\partial W(I'', A)}{\partial A} \text{ for any } A.$$

The latter follows from the fact that $v(x)$ is increasing in infrastructure investment. Thus, $W(I, A)$ is submodular and satisfies single crossing property. This in turn implies that a rise in $A$ decreases the optimal infrastructure profile $I^a$ (see details in Milgrom and Shannon (1994)). That is, $i^a(x)$ falls for all $x$. This proves the proposition.

## Proof of Proposition 4

If $m(x)$ is the same at all locations, then $L = m\bar{s}$ and

$$i^a(x) = \left(\frac{\gamma(1-\alpha)\left(bm\bar{s} - \int_0^{\bar{s}} q(s) i^a(s) ds\right)}{q(x)}\left(\phi^L(x) + \phi^R(x)\right)\right)^{1/\delta}.$$

In this case, the partial derivative of $i^a(x)$ with respect to $\bar{s}$ is given by (here we use the fact that $i^a(\bar{s}) = 0$)

$$\frac{\partial i^a(x)}{\partial \bar{s}} = \frac{\gamma(1-\alpha)}{q(x)\delta}\left(i^a(x)\right)^{1-\delta}\left(\left(\phi^L(x) + \phi^R(x)\right)'_{\bar{s}}\left(bm\bar{s} - \int_0^{\bar{s}} q(s) i^a(s) ds\right) + bm\left(\phi^L(x) + \phi^R(x)\right)\right).$$

The derivatives of $\phi^L(x)$ and $\phi^R(x)$ with respect to $\bar{s}$ are given by

$$\left(\phi^j(x)\right)'_{\bar{s}} = \frac{\int_0^x \left(1 + \frac{1}{\delta-1}\int_s^{\bar{s}} i^a(r)^{1-\delta} dr\right)^{\gamma(1-\sigma)-1} ds - v(\bar{s})\phi^j(x)}{\int_0^{\bar{s}} v(s) ds},$$

where $j \in \{L, R\}$. Thus,

$$\frac{\partial i^a(x)}{\partial \bar{s}} = \frac{bm\gamma(1-\alpha)}{q(x)\delta}\left(i^a(x)\right)^{1-\delta}\left(\begin{array}{c}\frac{2\bar{s}(1-t^a)\int_0^x\left(1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^a(r)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}ds}{\int_0^{\bar{s}} v(s)ds} \\ +\left(\phi^L(x)+\phi^R(x)\right)\left(1-\frac{\bar{s}v(\bar{s})(1-t^a)}{\int_0^{\bar{s}} v(s)ds}\right)\end{array}\right).$$

As can be seen, if $x$ is sufficiently close to $\bar{s}$, then $\left(\phi^j(x)\right)'_{\bar{s}} > 0$ (as $\phi^j(\bar{s}) = 0$) and, thereby, $\partial i^a(x)/\partial\bar{s} > 0$. Moreover, if the cost of infrastructure is uniform,

$$\frac{\partial i^a(x)}{\partial \bar{s}} > 0 \text{ for all } x.$$

This is due to fact that if $q(x) = q$ for all $x$, $v(x)$ is symmetric around $x = \bar{s}/2$ and has a hump shape. As a result, $v(\bar{s}) \leq v(s)$ for any $s \in S$. This in turn means that

$$\begin{array}{rcl}\int_0^{\bar{s}} v(s)ds & > & \bar{s}v(\bar{s}) \implies \\ 1-\frac{\bar{s}v(\bar{s})(1-t^a)}{\int_0^{\bar{s}} v(s)ds} & > & 0.\end{array}$$

## Proof of Lemma 2

As in the proof of Proposition 1, we first formulate and analyze a discrete modification of the social planner's maximization problem, which approximates the actual continuos maximization problem, and then take the limit. Recall that the home social planner solves

$$\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]} = \arg\max_{t,i(x)}\left\{(1-t)\left(\int_0^{\bar{s}} v(x)dx\right)^{\frac{1-\alpha}{\sigma-1}} \middle| q\int_0^{\bar{s}} i(x)\,dx \leq bLt^H\right\},$$

where

$$\begin{array}{rcl} v(x) & = & \int_0^{2\bar{s}} T(x,z)^{1-\sigma}\,dz \\ & = & \begin{array}{c}\int_0^x\left(1+\frac{1}{\delta-1}\int_z^x i(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)}dz+\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^z i(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)}dz \\ +\int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^{\bar{s}} i(s)^{1-\delta}\,ds+\frac{1}{\delta-1}\int_{\bar{s}}^z i^F(s)^{1-\delta}\,ds\right)^{\gamma(1-\sigma)}dz\end{array}\end{array}.$$

To formulate the corresponding discrete maximization problem, we consider a uniform partition of the $[0, 2\bar{s}]$-interval given by $\{x_i\}_{i=0..2n}$, where $x_0 = 0$, $x_n = \bar{s}$, and $x_{2n} = 2\bar{s}$. Then, the discrete analogue of the continuos maximization problem can be written as follows (without loss of generality, we assume that location $n$ belongs to the home country):

$$\left\{i^H(x_i), t^H\right\}_{i=0..n} = \arg\max_{t,i(x_i)}\left\{(1-t)\left(\sum_{i=0}^n v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}}\right\},$$

subject to

$$q\sum_{i=0}^n i(x_i)\triangle_n \leq bLt.$$

where

$$v(x_i) = \triangle_n \begin{array}{c}1+\sum_{k=0}^{i-1}\left(1+\frac{1}{\delta-1}\prod_{j=k}^{i-1} i(x_j)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)}+\sum_{k=i+1}^{n}\left(1+\frac{1}{\delta-1}\prod_{j=i+1}^{k} i(x_j)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)} \\ +\sum_{k=n+1}^{2n}\left(1+\frac{1}{\delta-1}\prod_{j=i+1}^{n} i(x_j)^{1-\delta}\triangle_n+\frac{1}{\delta-1}\prod_{j=n+1}^{k} i^F(x_j)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)}\end{array}$$

and $\triangle_n = 1/n$.

The first order conditions are given by

$$\begin{aligned}\frac{\partial \Lambda}{\partial i\left(x_i\right)} &= 0,\, i = 0..n,\\ \frac{\partial \Lambda}{\partial t} &= 0,\end{aligned}$$

where $\Lambda$ is the corresponding Lagrange function. It is straightforward to see that

$$\begin{aligned}\frac{\partial \Lambda}{\partial i\left(x_i\right)} &= \frac{\left(1-t\right)\left(1-\alpha\right)}{\sigma-1}\left(\sum_{i=0}^{n} v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}-1}\triangle_n \sum_{l=0}^{n}\left(v(x_l)\right)'_{i(x_i)} - \lambda_0\triangle_n\, q,\\ \frac{\partial \Lambda}{\partial t} &= -\left(\sum_{i=0}^{n} v(x_i)\triangle_n\right)^{\frac{1-\alpha}{\sigma-1}} + \lambda_0 bL.\end{aligned}$$

We have that for $i < l$,

$$\left(v(x_l)\right)'_{i(x_i)} = \left(\triangle_n\right)^2\gamma(\sigma-1)i(x_i)^{-\delta}\sum_{k=0}^{i}\left(1+\frac{1}{\delta-1}{}_{j=k}^{l-1} i\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1},$$

for $i > l$,

$$\begin{aligned}\left(v(x_l)\right)'_{i(x_i)} &= \left(\triangle_n\right)^2\gamma(\sigma-1)i(x_i)^{-\delta}\sum_{k=i}^{n}\left(1+\frac{1}{\delta-1}{}_{j=l+1}^{k} i\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1}\\ &\quad+\left(\triangle_n\right)^2\gamma(\sigma-1)i(x_i)^{-\delta}\sum_{k=n+1}^{2n}\left(1+\frac{1}{\delta-1}{}_{j=l+1}^{n} i\left(x_j\right)^{1-\delta}\triangle_n+\frac{1}{\delta-1}{}_{j=n+1}^{k} i^F\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1}.\end{aligned}$$

Finally, $\left(v(x_l)\right)'_{i(x_i)} = 0$ if $i = l$.

Thus, the optimal infrastructure profile, $\left\{i^H(x_i)\right\}_{i=0..n}$, solves the following system of equations:

$$\begin{aligned}\frac{qi^H\left(x_i\right)^{\delta}}{\triangle_n bL\gamma\left(1-\alpha\right)\left(1-t^H\right)} &= \frac{\sum_{l=0}^{i-1}\left(\sum_{k=i}^{n}\left(1+\frac{1}{\delta-1}{}_{j=l+1}^{k} i^H\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\sum_{i=0}^{n} v(x_i)}\\ &\quad+\frac{\sum_{l=0}^{i-1}\left(\sum_{k=n+1}^{2n}\left(1+\frac{1}{\delta-1}{}_{j=l+1}^{n} i^H\left(x_j\right)^{1-\delta}\triangle_n+\frac{1}{\delta-1}{}_{j=n+1}^{k} i^F\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\sum_{i=0}^{n} v(x_i)}\\ &\quad+\frac{\sum_{l=i+1}^{n}\left(\sum_{k=0}^{i}\left(1+\frac{1}{\delta-1}{}_{j=k}^{l-1} i^H\left(x_j\right)^{1-\delta}\triangle_n\right)^{\gamma(1-\sigma)-1}\right)}{\sum_{i=0}^{n} v(x_i)}.\end{aligned}$$

Taking the limit, we obtain the solution of the continuous maximization problem:

$$\begin{aligned}\frac{qi^H\left(x\right)^{\delta}}{bL\gamma\left(1-\alpha\right)\left(1-t^H\right)} &= \frac{\int_0^x\left(\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^t i^H\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}} v(s)ds}\\ &\quad+\frac{\int_x^{\bar{s}}\left(\int_0^x\left(1+\frac{1}{\delta-1}\int_t^s i^H\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}} v(s)ds}.\\ &\quad+\frac{\int_0^x\left(\int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^H\left(r\right)^{1-\delta}dr+\frac{1}{\delta-1}\int_{\bar{s}}^t i^F\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}} v(s)ds}.\end{aligned}$$

Finally, the optimal tax rate solves

$$t^H = \frac{q\int_0^{\bar{s}} i^H(s)\,ds}{bL}.$$

## Proof of Proposition 7

From the expression for $i^H(x)$, we can see that

$$\delta i^H(x)^{\delta-1}\left(i^H(x)\right)' = \frac{bL\gamma(1-\alpha)(1-t^H)}{q}\left(\tilde{\phi}^{L,N}(x)+\tilde{\phi}^{R,N}(x)\right)'.$$

It is straightforward to see that

$$\left(\tilde{\phi}^{R,N}(x)\right)'_{x=0} > 0 \text{ and } \left(\tilde{\phi}^{R,N}(x)\right)'_{x=\bar{s}} < 0.$$

Consider the derivative of $\tilde{\phi}^{L,N}(x)$ with respect to $x$. We have that

$$\begin{aligned}\left(\tilde{\phi}^{L,N}(x)\right)' &= \frac{\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^t i^H(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt}{\int_0^{\bar{s}}v(s)ds} - \frac{\int_0^{x}\left(1+\frac{1}{\delta-1}\int_s^x i^H(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}ds}{\int_0^{\bar{s}}v(s)ds}\\ &+\frac{\int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_x^{\bar{s}} i^H(r)^{1-\delta}\,dr+\frac{1}{\delta-1}\int_{\bar{s}}^t i^H(2\bar{s}-r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt}{\int_0^{\bar{s}}v(s)ds}.\end{aligned}$$

As can be inferred from the above expression, $\left(\tilde{\phi}^{L,N}(x)\right)'_{x=0} > 0$. Moreover, as

$$\int_0^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^H(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}ds = \int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_{\bar{s}}^t i^H(2\bar{s}-r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt,$$

$\left(\tilde{\phi}^{L,N}(x)\right)'_{x=\bar{s}} = 0$.

Hence, we can conclude that $\left(i^H(x)\right)'_{x=0} = \infty$ (as $i^H(0)^{\delta-1} = 0$) and $\left(i^H(x)\right)'_{x=\bar{s}}$ is negative, but finite (as $i^H(\bar{s})^{\delta-1} > 0$).

## Proof of Proposition 8

Recall that the infrastructure profile is determined by

$$i^H(x)^{\delta} = \frac{bL\gamma(1-\alpha)(1-t^H)}{q}\left(\tilde{\phi}^{L,N}(x,\tau)+\tilde{\phi}^{R,N}(x,\tau)\right).$$

It is straightforward to see that if there are additional transport costs for products produced in the foreign country, then

$$\begin{aligned}\tilde{\phi}^{L,N}(x,\tau) &= \frac{\int_0^x\left(\int_x^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^t i^H(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}}\left(\int_0^{\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds+\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_{\bar{s}}^{2\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds}\\ &+\frac{\tau^{1-\sigma}\int_0^x\left(\int_{\bar{s}}^{2\bar{s}}\left(1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^H(r)^{1-\delta}\,dr+\frac{1}{\delta-1}\int_{\bar{s}}^t i^H(2-r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}}\left(\int_0^{\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds+\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_{\bar{s}}^{2\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds},\\ \tilde{\phi}^{R,N}(x,\tau) &= \frac{\int_x^{\bar{s}}\left(\int_0^x\left(1+\frac{1}{\delta-1}\int_t^s i^H(r)^{1-\delta}\,dr\right)^{\gamma(1-\sigma)-1}dt\right)ds}{\int_0^{\bar{s}}\left(\int_0^{\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds+\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_{\bar{s}}^{2\bar{s}}T(s,z)^{1-\sigma}\,dz\right)ds}.\end{aligned}$$

Therefore, to explore the direct effect of $\tau$ on $i^H(x)$, we need to analyze how changes in $\tau$ affect $\tilde{\phi}^{L,N}(x,\tau)+\tilde{\phi}^{R,N}(x,\tau)$. Specifically, we find that the sign of $\left(\tilde{\phi}^{L,N}(x,\tau)+\tilde{\phi}^{R,N}(x,\tau)\right)'_{\tau}$ is the same as the sign of the following expression (remember that taking the derivative, we ignore all the indirect effects):

$$
\begin{aligned}
R(x) \stackrel{d}{\equiv} & \int_0^x \left( \int_x^{\bar{s}} \left( 1+\frac{1}{\delta-1}\int_s^t i^H(r)^{1-\delta}\,dr \right)^{\gamma(1-\sigma)-1} dt \right) ds \int_0^{\bar{s}} \left( \int_{\bar{s}}^{2\bar{s}} T(s,z)^{1-\sigma}\,dz \right) ds \\
& + \int_x^{\bar{s}} \left( \int_0^x \left( 1+\frac{1}{\delta-1}\int_t^s i^H(r)^{1-\delta}\,dr \right)^{\gamma(1-\sigma)-1} dt \right) ds \int_0^{\bar{s}} \left( \int_{\bar{s}}^{2\bar{s}} T(s,z)^{1-\sigma}\,dz \right) ds \\
& - \int_0^x \left( \int_{\bar{s}}^{2\bar{s}} \left( 1+\frac{1}{\delta-1}\int_s^{\bar{s}} i^H(r)^{1-\delta}\,dr + \frac{1}{\delta-1}\int_{\bar{s}}^t i^H(2\bar{s}-r)^{1-\delta}\,dr \right)^{\gamma(1-\sigma)-1} dt \right) ds \int_0^{\bar{s}} \left( \int_0^{\bar{s}} T(s,z)^{1-\sigma}\,dz \right) ds.
\end{aligned}
$$

As can be seen, $R(x)$ is negative in some left neighborhood of $x=\bar{s}$, implying that $i^H(x)$ falls for all $x$ from this neighborhood. Finally, as $\int_0^{\bar{s}} i^H(x)\,dx$ remains the same, $i^H(x)$ must rise at some locations that are relatively far from the border.

# Appendix B

In this Appendix, we provide the description of the data used to construct Table 5.

Table 6: Summary statistics for Table 5

| Variable | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| Ln variable transport costs$^{(a)}$ | 1.57 | 0.11 | 1.18 | 1.73 |
| Ln geographical difficulty$^{(b)}$ | -0.51 | 1.02 | -2.35 | 1.52 |
| Ln population | 13.03 | 0.70 | 11.20 | 14.74 |
| Ln area in square km | 8.53 | 0.70 | 5.16 | 9.23 |
| Remoteness measures | | | | |
| Ln distance to Paris | 0.33 | 0.17 | 0.01 | 0.66 |
| Ln trade costs weighted distance to Paris$^{(c)}$ | 5.25 | 0.84 | 2.48 | 6.24 |
| Ln distance to rest of France$^{(d)}$ | 7.84 | 0.15 | 7.58 | 8.27 |
| Ln trade costs weighted distance to rest of France$^{(e)}$ | 0.42 | 0.19 | 0.08 | 0.90 |

Construction of variables.
(a) Miren Lafourcade has kindly provided access to generalized trade cost data for France départements for the year of 1993. Those data are constructed as the (more disaggregated, but unfortunately confidential) data that Combes et Lafourcade (2005) describe. The data contains trade costs département by département. One can recover total variable transport costs by subtracting the costs of loading and unloading the truck of FF 60. To obtain a measure of transit costs, we average total variable transport costs per kilometer between neighboring départements, using the neighbors' area as weights.
(b) Geographical difficulty is the measured by the difference between the points of highest and lowest altitude above sea level in a département.
(c) Trade cost weighted distance to Paris is the generalized trade cost index (including fixed costs, as reported by Combes and Lafourcade, 2005) for transportation from or to Paris to or from the respective département.
(d) Distance to rest of France is computed as the average unweighted distance of some département to all the other départements.
(e) Repeats the exercise conducted for (d), but uses generalized trade costs instead of distance (as reported by Combes and Lafourcade, 2005).