Optimal Spatial Emissions*

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Abstract

How should carbon reduction efforts be allocated across regions? We develop a multi-sector quantitative spatial model with carbon emissions and taxes. We introduce emission allocative efficiency (EAE) to quantify potential gains from reallocating emissions to improve aggregate real income. We derive sufficient statistics that link EAE with data on inter-regional trade, labor, and carbon emissions, highlighting the role of regional heterogeneity, externalities, and interdependence in shaping optimal spatial carbon taxes. Leveraging EAE, we develop an iterative algorithm to compute high-dimensional optimal spatial carbon taxes. Applying this framework to the 2017 Chinese economy, we find that: (i) optimal carbon taxes are negatively correlated with *Katz centrality* in inter-regional trade networks; (ii) EAE, easily computable via a linear system, is strongly correlated with the ratio of optimal to observed carbon taxes; and (iii) optimal taxes increase Chinese real income by 1.37% without altering total emissions. Moreover, decentralized carbon taxes, set non-cooperatively by regional governments to meet local emission targets, result in excessive taxation in *upstream* sectors. This underscores the importance of regional coordination in carbon policy design.

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1 Introduction

Ambitious carbon reduction targets, adopted by many countries,[1] may result in reduced production efficiency and higher costs.[2] However, the question of how to spatially allocate carbon reduction efforts and the associated costs remains unresolved. This complexity arises from (i) substantial regional heterogeneity in productivity, amenities, and industrial composition; (ii) distortions and externalities that impede efficient emissions allocation through carbon markets;[3] and (iii) inter-regional linkages, such as trade, migration, and input-output linkages. Designing spatial carbon policies that enhance overall welfare requires careful consideration of these dimensions of heterogeneity, externalities, and interdependence.

To understand the optimal spatial allocation of carbon emissions, it is essential to develop a quantitative spatial model that incorporates carbon emissions and policies, along with an efficient algorithm to solve for optimal spatial policies. Currently, the literature lacks such a comprehensive model.[4] Additionally, solving for high-dimensional optimal spatial policies remains computationally challenging.[5]

To address these gaps, we develop a multi-sector quantitative spatial model incorporating carbon emissions and carbon taxes. Building on the spatial model by Allen and

[1]For example, in 2022, Australia legislated its greenhouse gas emission reduction targets, aiming to reach emission levels 43% below 2005 levels by 2030 and achieve net zero by 2050. In 2021, President Biden set the U.S. Greenhouse Gas Pollution Reduction Target, aiming to reduce net greenhouse gas emissions by 50-52% from 2005 levels by 2030. In 2020, China proposed to reduce carbon emissions rapidly by 2045 and achieve carbon neutrality by 2060.

[2]Recent studies emphasize the role of energy as a key production input, with carbon emissions being an inevitable byproduct. Consequently, carbon reduction increases energy input costs, leading to higher production costs. See, for example, Larch and Wanner (2017) and Farrokhi and Lashkaripour (2024).

[3]Classical externalities in spatial economies include the impacts of regional population size on local productivity and amenities.

[4]Farrokhi and Lashkaripour (2024) make progress in this direction by incorporating carbon emissions and policies into a quantitative trade model and characterizing optimal trade policies aimed at reducing global carbon emissions. Their framework follows the quantitative trade model with pollution emissions developed by Shapiro and Walker (2018). However, to our knowledge, there is no quantitative framework designed to understand optimal carbon policies within a country across different regions.

[5]For instance, Lashkaripour and Lugovskyy (2023) highlight "the well-known limitations of numerical optimization routines when applied to nonlinear models with many free-moving variables."

Arkolakis (2014), we integrate the specifications for carbon emissions and carbon taxes from Shapiro and Walker (2018) and Farrokhi and Lashkaripour (2024). Our model simulates a spatial economy where workers move freely across regions, influenced by variations in technology, amenities, and local agglomeration and congestion forces. Regions are interconnected through trade and input-output linkages, enabling the exchange of goods and services.

In our model, carbon emissions are treated as a factor of production, with their cost determined by government-imposed carbon taxes. When a carbon tax is applied to a specific sector in a particular region, it reduces carbon emissions in that sector and region but increases production costs. This tax also impacts carbon emissions and production costs in other sectors and regions through trade and input-output linkages.

Building on our model, we characterize carbon taxes that maximize aggregate real income while adhering to an aggregate carbon emission constraint. To achieve this, we develop a novel emission allocative efficiency (EAE) measure, which quantifies the potential real income gains from reallocating emissions. This measure evaluates the trade-off between the benefits of reducing carbon emissions and the associated losses in production efficiency for any given set of spatial carbon taxes. The optimal carbon tax for a specific sector in a particular region should be higher than the observed carbon tax if its EAE exceeds 1, and lower if its EAE is less than 1. This property of the EAE offers guidance for enhancing real income through emission reallocation without explicitly solving for optimal spatial carbon taxes.

Our analysis of emission allocative efficiency (EAE) addresses two key dimensions. First, in the one-sector version of our model, we derive analytical sufficient statistics that link EAE to inter-regional trade, labor, and emissions data. Our findings suggest that carbon taxes should be lower in regions with higher Katz centrality (KC) in the inter-regional trade network, as taxes in these regions result in larger aggregate productivity losses. Additionally, we show that optimal carbon taxes should decline in more central regions as agglomeration strengthens or congestion weakens. These sufficient statistics offer theoretical characterizations that connect optimal spatial carbon taxes with regional heterogeneity, externalities, and interdependence.

Second, we propose an EAE-based iterative algorithm for solving optimal spatial carbon taxes. This algorithm integrates the linear system that solves for EAE with the non-linear system that captures equilibrium changes under exogenous shocks (the "exact-hat" algebra). By incorporating EAE, the algorithm improves computational efficiency by structuring the Jacobian matrix in the optimization process. This approach enables the efficient computation of complex, high-dimensional, and continuous optimal policies within a general equilibrium framework. The algorithm is versatile and applicable to a wide range of quantitative trade and spatial models, facilitating the solution of optimal policy problems.

We apply our framework to quantify optimal carbon taxes for 30 provinces and 15 sectors in China using a calibrated model based on 2017 data on production, trade flows, input-output linkages, population, and carbon emissions. We calculate the EAE for the observed carbon taxes in the calibrated economy and then compute optimal carbon taxes using our iterative algorithm. We find that

(i) Optimal spatial carbon taxes are significantly negatively correlated with Katz centrality within observed inter-regional trade networks. This result is consistent with our analytical characterization of optimal spatial carbon taxes in the simplified one-sector model.

(ii) Emission Allocative Efficiency (EAE), which can be easily computed using a linear system, shows a strong correlation with the ratio of optimal to observed carbon taxes. Thus, EAE offers policymakers qualitative guidance for reallocating emissions to improve real income, without calculating precise optimal spatial carbon taxes.

(iii) Implementing optimal spatial carbon taxes could increase Chinese welfare by 1.37% while maintaining total emissions unchanged. This highlights the potential welfare gains from accounting for regional heterogeneity, externalities, and interdependence in carbon reduction policy design.

Our optimal carbon taxes serve as a valuable benchmark for spatial carbon reduction policies. However, it is often impractical for the central government to enforce region-sector-specific carbon policies. Instead, the central government may allocate emission constraints to regions and delegate the authority for sectoral carbon policies to local governments. To elucidate this decentralized carbon allocation, we analyze a non-cooperative carbon tax game within our spatial model. In this scenario, each region sets its local carbon taxes to maximize real income while adhering to regional emission constraints, considering the carbon taxes in other regions as given. We find that in this non-cooperative scenario, local governments fail to internalize the impacts of their carbon taxes on production and emissions in other regions. This leads to excessively high carbon taxes on upstream sectors, resulting in efficiency losses. This result highlights the necessities for regional coordination in carbon policy design.

Related Literature–This paper relates to several strands of literature. First, it relates to quantitative explorations of policies on carbon emission. Shapiro (2021) and Garcia-Lembergman, Ramondo, Rodriguez-Clare, and Shapiro. (2024) quantify the impacts of carbon policies in the global economy. Farrokhi and Lashkaripour (2024) further consider the optimal design of carbon policies within a quantitative trade model. Our paper complements this strand of literature by focusing on the optimal design of carbon emission policies across different regions within a country, which, to the best of our knowledge, has not been extensively explored in previous studies.[6]

Second, we contribute to the characterization of optimal spatial policies. Fajgelbaum and Gaubert (2020) characterize optimal transfers in a generalized spatial framework. Henkel, Seidel, and Suedekum (2021) and Colas and Hutchinson (2021) investigate optimal taxes and fiscal transfers across different regions. Our paper enriches this body of

[6] One exception is Arkolakis and Walsh (2023). They investigate the optimal spatial allocation of electricity transmission networks and the corresponding consequences on the adoption of renewable energy. This paper departs from their work by considering generalized spatial policies on carbon emissions.

literature by bridging the gap between theoretical characterizations and the implementation of crucial spatial policies in practice.

Third, our work relates to targeting interventions in networks. Galeotti, Golub, and Goyal (2020) provide generalized theoretical results for this problem. Liu (2019) examines the impacts of industrial policies in production networks, while Lashkaripour and Lugovskyy (2023) consider optimal industrial policies in trade networks. Liu and Ma (2024) investigate innovation subsidies in knowledge networks. This paper contributes to this strand of literature by deriving sufficient statistics that can be used to characterize optimal spatial carbon taxes. Our framework can be applied to characterize a wide range of policies in networks, such as the combination of zoning and industrial policies and economic sanctions in trade and technology networks.

This paper is structured as follows. Section 2 introduces our model. Section 3 characterizes optimal spatial carbon taxes utilizing the emission allocative efficiency (EAE). In Section 4, we calibrate our model and perform counterfactual analysis for optimal spatial emissions. We conclude in Section 5.

2 Spatial Model with Carbon Emissions and Carbon Taxes

2.1 Environment

Consider a spatial economy with N regions, denoted by (i, n, k), and J sectors, denote by (j, s). Total endowment of workers is $\bar{L}$. Workers are freely mobile across regions and sectors. The representative consumer in region i has a Cobb-Douglas preference over J sectors:

$$U_i = B_i L_i^{-\beta} \prod_{j=1}^{J} \left(C_i^j\right)^{\alpha_j}, \quad \sum_{j=1}^{N} \alpha_j = 1, \tag{1}$$

where C_i^j is the consumption of sector j in region i. $B_i L_i^{-\beta}$ represents amenity in region i, where B_i is the exogenous amenity shifter, L_i is the labor in region i, and $\beta \geq 0$ captures the congestion force over space.

Each sector j consists of a unit mass of varieties, aggregated by a CES function with the elasticity of substitution $\sigma_j \geq 0$. Following Shapiro and Walker (2018) and Farrokhi and Lashkaripour (2024), we regard carbon emission as a factor of production whose price is determined by carbon tax. This is a tractable way to incorporate carbon abatement costs and carbon policies into a general equilibrium framework. Specifically, we assume that each variety is produced by a firm using labor, carbon, and intermediates in a perfectly competitive market. The unit cost of variety ω of sector j produced in region i is given by

$$c_i^j(\omega) = \frac{c_i^j}{z_i^j(\omega)}, \quad c_i^j \equiv L_i^{-\psi_j} w_i^{\gamma_j^L} \prod_{s=1}^{J} (P_i^s)^{\gamma_{sj}} \left(t_i^j\right)^{\xi_j}, \quad \gamma_j^L + \sum_{s=1}^{J} \gamma_{sj} + \xi_j = 1, \tag{2}$$

where w_i is the wage in region i, P_i^s is the price index of sector s in region i, $t_i^j > 0$ is the tax rate on carbon emission in region i and sector j, and $z_i^j(\omega)$ is the productivity of variety ω. Notice that (i) $\psi_j \geq 0$ characterizes the sectoral agglomeration force,[7] and (ii) ξ_j is the share of carbon emission in producing good j which, as shown below, affects the emission intensity of sector j.

The exogenous productivity $z_i^j(\omega)$ is drawn independently from a Frechet distribution with level parameter A_i^j and shape parameter $\theta_j \geq \sigma_j$. Exporting good j from region i to n incurs an iceberg trade cost τ_{in}^j.

Finally, we assume that region i receives a share s_i of carbon tax revenue. The allocation of these transfers is critical for the efficiency of spatial carbon taxes, as emphasized by Fajgelbaum and Gaubert (2020). Our baseline specification sets $s_i = \frac{L_i}{\bar{L}}$, distributing tax revenue evenly across households, regardless of location. This specification avoids distorting labor's spatial allocation and is adopted in all quantitative exercises in this paper. We also consider $s_i = \frac{w_i L_i}{\sum_{k=1}^{N} w_k L_k}$, where revenue is distributed based on regional wage incomes, $w_i L_i$. This simplifies the equilibrium system, facilitating the derivation of

[7] This specification follows Adao, Arkolakis, and Esposito (2023) to allow productivities of different sectors respond differently to changes in local production scale.

analytical sufficient statistics for optimal spatial carbon taxes.

2.2 Equilibrium

We proceed by defining the equilibrium in our model. Let X_i^j be the total expenditure in region i on good j and X_{in}^j be the value of trade of good j from region i to n. Then

$$\lambda_{in}^j \equiv \frac{X_{in}^j}{X_n^j} = \frac{A_i^j \left(\tau_{in}^j c_i^j\right)^{-\theta_j}}{\sum_{k=1}^N A_k^j \left(\tau_{kn}^j c_k^j\right)^{-\theta_j}}. \tag{3}$$

The price indices can be expressed as

$$P_n^j = \left[\sum_{k=1}^N A_k^j \left(\tau_{kn}^j c_k^j\right)^{-\theta_j}\right]^{-\frac{1}{\theta_j}}, \quad P_n = \prod_{j=1}^J \left(P_n^j\right)^{\alpha_j}. \tag{4}$$

The wage satisfies

$$w_i L_i = \sum_{j=1}^J \gamma_j^L \sum_{n=1}^N \lambda_{in}^j X_n^j. \tag{5}$$

Final income in region i is the sum of wage income and carbon tax revenue:

$$Y_i = w_i L_i + s_i R, \quad R \equiv \sum_{k=1}^N \sum_{j=1}^J R_k^j, \quad R_k^j \equiv \xi_j \sum_{n=1}^N \lambda_{kn}^j X_n^j, \quad s_i = \frac{L_i}{\bar{L}}. \tag{6}$$

Notice that $\sum_{k=1}^N \sum_{n=1}^N \lambda_{kn}^j X_n^j = \sum_{n=1}^N X_n^j$ for all j. Therefore, $R = \sum_{j=1}^J \sum_{n=1}^N \xi_j X_n^j$.

X_i^j is the sum of final consumption and intermediate usage:

$$X_i^j = \alpha_j Y_i + \sum_{s=1}^J \gamma_{js} \sum_{n=1}^N \lambda_{in}^s X_n^s. \tag{7}$$

Welfare equalization implies that

$$B_i L_i^{-\beta} \frac{Y_i / L_i}{P_i} = W. \tag{8}$$

Since $\bar{L} = \sum_{i=1}^{N} L_i$, we have

$$\frac{L_i}{\bar{L}} = \frac{\left(B_i \frac{Y_i/L_i}{P_i}\right)^{\frac{1}{\beta}}}{\sum_{k=1}^{N} \left(B_k \frac{Y_k/L_k}{P_k}\right)^{\frac{1}{\beta}}}, \tag{9}$$

and the aggregate welfare can be measured by the weighted average of regional real income:

$$W = \frac{1}{\bar{L}^{\beta}} \left[\sum_{k=1}^{N} \left(B_k \frac{Y_k / L_k}{P_k} \right)^{\frac{1}{\beta}} \right]^{\beta}. \tag{10}$$

Finally, the aggregate emission is given by

$$E = \sum_{j=1}^{J} \sum_{i=1}^{N} E_i^j, \quad E_i^j \equiv \frac{\xi_j}{t_i^j} \sum_{n=1}^{N} \lambda_{in}^j X_n^j. \tag{11}$$

Definition Given parameters $\left(\psi_j, \beta, \theta_j, \alpha_j, \gamma_j^L, \gamma_{sj}, \xi_j; A_i^j, B_i, \tau_{in}^j; \bar{L}; t_i^j\right)$, the equilibrium consists of $\left(w_i, L_i, P_i^j, X_i^j\right)$ such that (i) (w_i) is given by labor market clearing in Equation (5); (ii) (L_i) is given by the labor allocation in Equation (9); (iii) $\left(P_i^j\right)$ is given by the price index in Equation (4); (iv) $\left(X_i^j\right)$ is given by goods market clearing in Equation (7).

Following Dekle, Eaton, and Kortum (2008), we can express our equilibrium system in relative changes. For any variable $Z > 0$, we denote Z' as its level after changes and $\hat{Z} \equiv \frac{Z'}{Z}$. Let $\chi_{in}^j \equiv \frac{\gamma_j^L \lambda_{in}^j X_n^j}{w_i L_i}$ be the export share. Let $\delta_i^w \equiv \frac{w_i L_i}{Y_i}$ be the wage income share and $\delta_n^j \equiv \frac{1}{R} \xi_j X_n^j$ be the carbon tax revenue share. Let $\nu_{ij}^Y \equiv \frac{\alpha_j Y_i}{X_i^j}$ be final expenditure share. Let $\nu_{in}^{js} \equiv \frac{\gamma_{js} \lambda_{in}^s X_n^s}{X_i^j}$ be intermediate expenditure share.

Then given exogenous changes $\left(\hat{t}_i^j\right)$, we can derive $\left(\hat{w}_i, \hat{L}_i, \hat{P}_n^j, \hat{X}_n^j\right)$ by solving the following non-linear system:

$$
\begin{aligned}
&\hat{w}_i \hat{L}_i = \sum_{j=1}^{J} \sum_{n=1}^{N} \chi_{in}^j \hat{\lambda}_{in}^j \hat{X}_n^j, \quad \hat{\lambda}_{in}^j = \left(\hat{c}_i^j\right)^{-\theta_j} \left(\hat{P}_n^j\right)^{\theta_j}, \quad \hat{c}_i^j = \hat{L}_i^{-\psi_j} \hat{w}_i^{\gamma_j^L} \left(\hat{t}_i^j\right)^{\xi_j} \prod_{s=1}^{J} \left(\hat{P}_i^s\right)^{\gamma_{sj}} \\
&\left(\hat{P}_n^j\right)^{-\theta_j} = \sum_{i=1}^{N} \lambda_{in}^j \left(\hat{c}_i^j\right)^{-\theta_j}, \quad \hat{P}_n = \prod_{j=1}^{J} \left(\hat{P}_n^j\right)^{\alpha_j} \\
&\hat{X}_i^j = \nu_{ij}^Y \hat{Y}_i + \sum_{s=1}^{J} \sum_{n=1}^{N} \nu_{in}^{js} \hat{\lambda}_{in}^s \hat{X}_n^s, \quad \hat{Y}_i = \delta_i^w \hat{w}_i \hat{L}_i + \left(1 - \delta_i^w\right) \hat{L}_i \sum_{j=1}^{J} \sum_{n=1}^{N} \delta_n^j \hat{X}_n^j, \\
&\hat{L}_i = \frac{\left(\frac{\hat{Y}_i / \hat{L}_i}{\hat{P}_i}\right)^{\frac{1}{\beta}}}{\sum_{k=1}^{N} \iota_k \left(\frac{\hat{Y}_k / \hat{L}_k}{\hat{P}_k}\right)^{\frac{1}{\beta}}}, \quad \iota_i \equiv \frac{L_i}{\bar{L}}.
\end{aligned}
\tag{12}
$$

3 Optimal Spatial Emissions

3.1 The Central Government's Problem

The central government decides $\left(t_i^j\right)$ to maximize the aggregate real income subject to an aggregate emission constraint:

$$
\begin{aligned}
&\max_{\left(t_i^j; w_i, L_i, P_n^j, X_n^j\right)} W \equiv \frac{1}{\bar{L}^\beta} \left[\sum_{k=1}^{N} \left(B_k \frac{Y_k / L_k}{P_k} \right)^{\frac{1}{\beta}} \right]^\beta \\
&\text{s.t. } \sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\xi_j}{t_i^j} \sum_{n=1}^{N} \lambda_{in}^j X_n^j \le \bar{E} \\
&\left(w_i, L_i, P_n^j, X_n^j\right) \text{ satisfy Equation (4), (5),(9), and (7)}
\end{aligned}
\tag{13}
$$

Regarding Problem (13), two key questions merit discussion. First, why is the disutility from carbon emissions not included in the central government's objective function? The central government's problem in Equation (13) is indeed equivalent to that in a model considering disutility from carbon emissions. To see this, we re-express the central government's problem following the specification in Farrokhi and Lashkaripour (2024):

$$
\max_{\left(t_i^j;w_i,L_i,P_n^j,X_n^j\right)} \log \left\{ \underbrace{\frac{1}{\bar{L}^{\beta}} \left[\sum_{k=1}^{N} \left(B_k \frac{Y_k/L_k}{P_k} \right)^{\frac{1}{\beta}} \right]^{\beta}}_{W} \right\} - \delta_E \log \left(\underbrace{\sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\xi_j}{t_i^j} \sum_{n=1}^{N} \lambda_{in}^j X_n^j}_{E} \right) \tag{14}
$$

$$
\text{s.t.} \quad \left(w_i, L_i, P_n^j, X_n^j\right) \text{ satisfy Equation (4), (5),(9), and (7)}
$$

where $\delta_E > 0$ is the weight for aggregate emission in the central government's objective.

Problem (14) is isomorphic to Problem (13), with identical solutions once δ_E is calibrated so that the aggregate emission in Problem (14) matches the value $\bar{E}$ in Problem (13). We choose to solve Problem (13) in this paper because the aggregate emission constraint $\bar{E}$ can be directly linked to real-world policy targets.

Second, can carbon emissions derived in Problem (13) be replicated through a carbon market, thereby obviating the need for spatial carbon policies? Fajgelbaum and Gaubert (2020) have argued that in the absence of externalities and distortions, the spatial equilibrium leads to the first best once the income of fixed factors is evenly distributed across households. In our model, carbon emissions are considered a factor of production and their income is evenly distributed across workers. Therefore, carbon emissions can be efficiently allocated through a carbon market if (i) there is no local agglomeration externality, *i.e.* $\psi_j = 0$ for all j and (ii) there is no local amenity externality, *i.e.* $\beta = 0$. This result can be summarized by the following result:

Proposition 1 (Optimal Spatial Carbon Taxes without Externalities) *Suppose that* $s_i = \frac{L_i}{\bar{L}}$. *If* $\psi_j = \beta = 0$, *then the solution to Problem* (13) *satisfies* $t_i^{j*} = t^* > 0$ *where*

$$
t^* = \sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\xi_j}{\bar{E}} \sum_{n=1}^{N} \lambda_{in}^j X_n^j. \tag{15}
$$

Proposition 1 indicates that, in the absence of externalities, optimal spatial carbon taxes are straightforward: the central government simply sets the national carbon price as in Equation (15), allowing the national carbon market to efficiently allocate emissions across space to achieve the aggregate target $\bar{E}$. We present the details of this national carbon market in Appendix A.1.

However, in the presence of agglomeration and amenity externalities, the optimal spatial emissions in Problem (13) cannot be achieved through the national carbon market. In this scenario, spatial carbon taxes can raise real income while maintaining the aggregate emission level. Our analysis will focus on this case in the sections that follow.

3.2 Emission Allocative Efficiency (EAE)

In this subsection, we characterize the solution to Problem (13), $\mathbf{t}^*$, in the case where $\beta > 0$ and $\psi_j > 0$ for some j. To this end, we develop an emission allocative efficiency (EAE) measure: for any carbon tax profile $\mathbf{t} \equiv \left(t_i^j\right)$, its corresponding EAE is defined as

$$M_i^j(\mathbf{t}) \equiv \frac{\mu}{W} \left[\underbrace{E_i^j + \sum_{s=1}^{J} \sum_{k=1}^{N} \left(-\frac{\partial \log R_k^s}{\partial \log t_i^j} \right) E_k^s}_{\text{Effect of } t_i^j \text{ on carbon emissions}} \right] \left(\underbrace{-\frac{\partial \log W}{\partial \log t_i^j}}_{\text{Effect of } t_i^j \text{ on real income}} \right)^{-1}, \tag{16}$$

where μ is the Lagrange multiplier of the aggregate carbon emission constraint in Equation (13).

By construction, $M_i^j(\mathbf{t})$ increases with the effect of t_i^j on aggregate carbon emission and decrease with (the absolute value of) the effect of t_i^j on aggregate real income W. As a result, $M_i^j(\mathbf{t})$ summarizes the key trade-off in determining carbon taxes: the increase in t_i^j would lower carbon emissions but also lower real income by raising production costs. In the following lemma, we will argue that $M_i^j(\mathbf{t})$ measures the extent to which emission reallocation would increase the aggregate real income.

Proposition 2 (Emission Allocative Efficiency) *Let* $\mathbf{t}^* \equiv \left(t_i^{j*}\right)$ *be the solution of Problem* (13). *Then there exists* $\delta > 0$ *such that for any* $\mathbf{t}$ *satisfying* $\sum_{i,j}\left[t_i^j - t_i^{j*}\right]^2 \leq \delta$, $M_i^j(\mathbf{t})$ *defined by Equation* (16) *has the following properties:*

1. $M_i^j(\mathbf{t}^*) = 1$ *for all* (i,j).
2. *If* $M_i^j(\mathbf{t}) > 1$, *then* $t_i^{j*} > t_i^j$.
3. *If* $M_i^j(\mathbf{t}) < 1$, *then* $t_i^{j*} < t_i^j$.

Proposition 2 suggests that t_i^j should increase if the benefit from carbon reduction exceeds the loss from lowering real income. Moreover, Proposition 2 implies that t_i^{j*} is higher in the region-sector pair where (i) the carbon tax can substantially reduce carbon emission, or (ii) the carbon tax has small negative effects on the aggregate welfare.

The properties of $M_i^j(\mathbf{t})$ in Proposition 2 hold exactly only if $\mathbf{t}$ is close to $\mathbf{t}^*$. However, our counterfactual analysis in Section 4.2 will show that they hold numerically in most of the (i,j)-pairs for the observed $\mathbf{t}$ in our quantification practice. Consequently, $M_i^j(\mathbf{t})$ could offer a qualitative guidance for spatial carbon policies: carbon taxes should be higher in region-sector pairs with higher $M_i^j(\mathbf{t})$ and lower in those with lower EAE.

To demonstrate the usefulness of EAE in designing spatial carbon policies, we (i) derive sufficient statistics that link EAE to observable data, and (ii) develop an EAE-based iterative algorithm to solve optimal spatial carbon taxes.

3.3 Sufficient Statistics for EAE in the One-Sector Model

In this subsection, we express the emission allocative efficiency (EAE) by model parameters $\left(\psi_j, \beta, \theta_j, \alpha_j, \gamma_j^L, \gamma_{sj}, \xi_j\right)$ and data on trade, labor, and emissions, $\left(X_{in}^j, L_i, E_i^j\right)$. In particular, we derive analytical sufficient statistics for EAE in the one-sector special case of our model to understand the structure of optimal spatial carbon policies.

Consider there is only one sector, *i.e.* $J = 1$ and there is no roundabout production, *i.e.* $\gamma_{js} = 0$ for all (j, s). We omit subscript/superscript j for all variables in this case. The EAE in this case can be expressed as

$$M_i(\mathbf{t}) = \frac{\mu\xi}{W}\left[E_i + \sum_{k=1}^{N}\left(-\frac{\partial \log R_k}{\partial \log t_i}\right)E_k\right]\left(-\frac{\partial \log W}{\partial \log t_i}\right)^{-1}. \tag{17}$$

We define several parameters and matrices. Let $\breve{\theta} \equiv \frac{\theta}{1+\theta(2-\xi)}$, $\delta_1 \equiv 1 + [1 + \theta(1-\xi)]\beta - \theta\psi$, and $\delta_2 \equiv (1-\xi) - \theta(1-\xi)\beta + (1+\theta)\psi$. Let $\boldsymbol{\iota} \equiv [\iota_i]$ be a column vector, $\mathbf{e}$ be a column vector with all ones, and $\mathbf{I}$ be the identity matrix. Let $\chi_{in} \equiv \frac{X_{in}}{\sum_{k=1}^{N} X_{ik}}$, and $\boldsymbol{\chi} = [\chi_{in}]$ with i denoting the rows and n denoting the columns.

Proposition 3 (EAE and Katz Centrality in the One-Sector Model) *Suppose that* $s_i = \frac{w_i L_i}{\sum_{k=1}^{N} w_k L_k}$. *The real income effect of carbon taxes:*

$$\left[\frac{\partial \log W}{\partial \log t_1}, \ldots, \frac{\partial \log W}{\partial \log t_N}\right] = -\frac{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\boldsymbol{\chi}\right]}{\frac{\theta}{\breve{\theta}\delta_1}\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}\mathbf{e}} \tag{18}$$

The effects of carbon taxes on carbon tax revenues

$$\left[\frac{\partial \log R_k}{\partial \log t_i}\right] = \left[1 + \breve{\theta}\left(\beta + \psi - \frac{1}{\theta}\right)\right]\left[\frac{\partial \log L_k}{\partial \log t_i}\right] - \breve{\theta}\xi\mathbf{I}, \tag{19}$$

where $\left[\frac{\partial \log L_k}{\partial \log t_i}\right] = -\left[\mathbf{I} - \frac{1}{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}\mathbf{e}}\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}\mathbf{e}\boldsymbol{\iota}'\right]\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\boldsymbol{\chi}\right]$.

According to Proposition 3, the EAE in the one-sector model, $M_i(\mathbf{t})$ in Equation (17), can be expressed in terms of data on $\boldsymbol{\chi}$, $\boldsymbol{\iota}$, and (E_i) and parameter values on $(\breve{\theta}, \delta_1, \delta_2)$. The detailed proof to Proposition 3 is presented in Appendix A.3.1. Under our baseline allocation of carbon tax revenue, *i.e.* $s_i = \frac{L_i}{L}$, the sufficient statistics for the EAE are similar to those in Proposition 3 but much less tractable and interpretable. We present the details of this case in Appendix A.3.2.

Two points merit further discussion. First, the vector $\boldsymbol{\iota}' \left[\mathbf{I} - \frac{\delta_2}{\delta_1}\boldsymbol{\chi}\right]^{-1}$ represents the Katz Centrality (KC) in inter-regional trade networks, a centrality measure that captures the rich heterogeneity and spatial interdependence of regions. KC reflects the impact of each region's production costs on aggregate real income. Proposition 3 shows that regions with higher KC tend to exhibit lower emission allocative efficiency (EAE). Intuitively, carbon taxes should be lower in regions where increases in production costs have a more significant negative effect on aggregate real income. Katz Centrality is commonly used to assess the influence of nodes in a network. The contribution of Proposition 3 lies in linking KC with EAE and demonstrating its usefulness in determining optimal spatial carbon taxes.

Second, the attenuation parameter of the Katz Centrality (KC), $\frac{\delta_2}{\delta_1}$, can be interpreted structurally within the context of our spatial model. Specifically,

$$\frac{\delta_2}{\delta_1} = \frac{(1-\xi) - \theta(1-\xi)\beta + (1+\theta)\psi}{1 + [1+\theta(1-\xi)]\,\beta - \theta\psi}, \tag{20}$$

which rises with agglomeration externalities ψ and falls with congestion forces β. This parameter encapsulates the role of productivity and amenity externalities in shaping optimal spatial carbon taxes. As agglomeration strengthens or congestion weakens, optimal carbon taxes should decrease in regions more central to trade networks, as taxes in these regions lead to larger reductions in aggregate real income.

3.4 EAE-Based Iterative Algorithm to Solve for $\left(t_i^{j*}\right)$

In this subsection, we characterize the emission allocative efficiency (EAE) in our full model. Instead of analytical sufficient statistics, we express $\left(\frac{\partial \log R_k^s}{\partial \log t_i^j}\right)$ and $\left(\frac{\partial \log W}{\partial \log t_i^j}\right)$ in our full model as the solution to a linear recursive system.

Without loss of generality, we normalize $\sum_{i=1}^N w_i L_i = 1$. For any variables $Z > 0$, we denote $\tilde{Z} \equiv d \log Z$. Then $\left(\tilde{w}_i, \tilde{L}_i, \tilde{P}_i^j, \tilde{X}_i^j\right)$ can be computed by solving:

$$\tilde{w}_i + \tilde{L}_i = \sum_{j=1}^{J} \sum_{n=1}^{N} \chi_{in}^j \left(\tilde{\lambda}_{in}^j + \tilde{X}_n^j\right), \quad \tilde{\lambda}_{in}^j = -\theta_j \tilde{c}_i^j + \theta_j \tilde{P}_n^j, \quad \tilde{c}_i^j = -\psi_j \tilde{L}_i + \gamma_j^L \tilde{w}_i + \xi_j \tilde{t}_i^j + \sum_{s=1}^{J} \gamma_{sj} \tilde{P}_i^s$$
$$\tilde{X}_i^j = \nu_{ij}^Y \tilde{Y}_i + \sum_{s=1}^{J} \sum_{n=1}^{N} \nu_{in}^{js} \left(\tilde{\lambda}_{in}^s + \tilde{X}_n^s\right), \quad \tilde{Y}_i = \delta_i^w \left(\tilde{w}_i + \tilde{L}_i\right) + \left(1 - \delta_i^w\right) \left(\tilde{L}_i + \sum_{j=1}^{J} \sum_{n=1}^{N} \delta_n^j \tilde{X}_n^j\right)$$
$$\tilde{P}_n^j = \sum_{i=1}^{N} \lambda_{in}^j \tilde{c}_i^j, \quad \tilde{P}_n = \sum_{j=1}^{J} \alpha_j \tilde{P}_n^j$$
$$\tilde{L}_i = \frac{1}{\beta} \left(\tilde{Y}_i - \tilde{L}_i - \tilde{P}_i\right) - \frac{1}{\beta} \sum_{k=1}^{N} \iota_k \left(\tilde{Y}_k - \tilde{L}_k - \tilde{P}_k\right) \tag{21}$$

The welfare effects of carbon taxes can then be derived by:

$$\tilde{W} = \sum_{k=1}^{N} \iota_k \left(\tilde{Y}_k - \tilde{L}_k - \tilde{P}_k\right). \tag{22}$$

The impacts of carbon taxes on carbon tax revenues can be expressed as

$$\tilde{R}_i^j = \sum_{n=1}^{N} \frac{\xi_j \lambda_{in}^j X_n^j}{R_i^j} \left(\tilde{\lambda}_{in}^j + \tilde{X}_n^j\right). \tag{23}$$

Therefore, for any spatial economy where we can observe $\left(\psi_j, \beta, \theta_j, \alpha_j, \gamma_j^L, \gamma_{sj}, \xi_j\right)$ and $\left(X_{in}^j, L_i, E_i^j\right)$, $M_i^j(\mathbf{t})$ defined by Equation (16) can be derived by Equation (21), (22), and (23).

We then combine the linear system in Equation (21) with the nonlinear "exact-hat" algebra (12) to develop the following algorithm for solving optimal spatial carbon taxes:

Algorithm 4 (Optimal Spatial Carbon Taxes) $\mathbf{t}^*$ *that solves the problem in Equation* (13) *can be calculated as follows:*

1. *Guess* $\mathbf{t}^* \in \mathbb{R}_{++}^{N \times J}$.

2. *Solve for* $\left(X_{in}^j, \iota_i\right)$ *under* $\mathbf{t}^*$ *by the "exact-hat algebra" in Equation* (12).

3. *Solve the linear system* (21).

4. *Calculate* $M_i^j(\mathbf{t}^*)$ *using Equation* (22), (23), *and* (16).

5. *Update* t_i^{j*} *by* $t_i^{j*} M_i^j(\mathbf{t}^*)$.

6. *Repeat Step 1-5 until* $t_i^{j*} = t_i^{j*} M_i^j(\mathbf{t}^*)$ *for all* (i, j).

7. *Adjust the level of* $\mathbf{t}^*$ *to bind the aggregate emission constraint in Equation* (13).

Notably, the linear system (16) offers sufficient statistics for the Jacobian matrix of Problem (13). Therefore, Algorithm 4 effectively derives the Jacobian matrix under any given $\mathbf{t}$ utilizing sufficient statistics embedded in the linear system (16), achieving the efficiency of Newton's method with analytical Jacobian matrix.

4 Quantifying Optimal Spatial Carbon Taxes

4.1 Data and Calibration

Our quantitative analysis requires values on $\left(\psi_j, \beta, \theta_j, \alpha_j, \gamma_j^L, \gamma_{sj}; \xi_j, t_i^j; X_{in}^j\right)$. We consider $N = 30$ Chinese provinces and $J = 15$ sectors in 2017. We calibrate (θ_j, ψ_j) from Lashkaripour and Lugovskyy (2023). Notably, we rescale the scale elasticities so that its average is equal to 0.05, consistent with the estimate in Adao et al. (2023). We report the calibrated values of (θ_j, ψ_j) in the first two columns of Table 1.

We calibrate $\beta = \frac{2}{3}$ from Tombe and Zhu (2019). We calibrate $\left(\alpha_j, \gamma_j^L, \gamma_{sj}\right)$ using Chinese aggregate input-output table for 2017. We obtain $\left(X_{in}^j\right)$ directly from Chinese inter-provincial input-output table for 2017.

Table 1: Calibration of $(\theta_j, \psi_j, \xi_j)$

Sector	Description	θ_j	ψ_j	ξ_j
1	Agriculture&Mining	6.227	0.0254	0.0203
2	Food	2.303	0.0697	0.0064
3	Textiles, Leather&Footwear	3.359	0.0397	0.0083
4	Wood	3.896	0.0406	0.0032
5	Paper	2.646	0.0567	0.0153
6	Petroleum	1.200	0.2163	0.0368
7	Chemicals	3.966	0.0411	0.0262
8	Rubber&Plastic	5.157	0.0248	0.0041
9	Minerals	5.283	0.0296	0.0883
10	Basic&Fabricated Metals	3.004	0.0371	0.0635
11	Machinery	7.75	0.0213	0.0077
12	Electrical&Optical Equipment	1.235	0.0979	0.0034
13	Transport Equipment	2.805	0.0229	0.0041
14	N.E.C.&Recycling	6.169	0.0270	0.0054
15	Services	10	0.0000	0.0070
	Simple Average	4.33	0.05	0.02

We calibrate $\left(\xi_j, t_i^j\right)$ combining Chinese inter-provincial input-output table and Carbon Emission Account&Datasets (CEADs). In particular, we have

$$\frac{\xi_j}{t_i^j} = \frac{E_i^j}{\sum_{n=1}^N X_{in}^j}. \tag{24}$$

We normalize $\frac{1}{N}\sum_{i=1}^N t_i^j = 1$ for all j. We then get ξ_j and t_i^j separately. To this end, we attribute all sectoral variations in emission intensities to (ξ_j). Notably, we adjust the unit of E_i^j so that $\frac{1}{J}\sum_{j=1}^J \xi_j = 0.02$, consistent with the estimate in Shapiro and Walker (2018). We report the calibrated values of (ξ_j) in the last column of Table 1.

4.2 Optimal Spatial Carbon Taxes

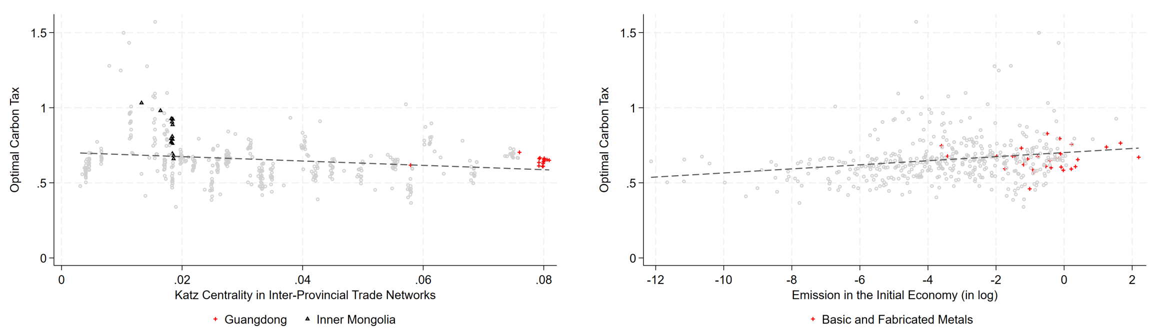
In this subsection, we derive and characterize optimal spatial carbon taxes, $\left(t_i^{j*}\right)$, in our calibrated economy. Using a personal computer without parallel computation, it takes about three hours for our EAE-based algorithm to solve for $\left(t_i^{j*}\right)_{i=1,\dots,N;J=1,\dots,J}$ where $N = 30$ and $J = 15$. Optimal spatial carbon taxes generate significant welfare gains in China, increasing real income by 1.37% without altering aggregate carbon emissions. This result underscores the importance of considering the spatial dimension in designing optimal carbon policies.

To understand the variations in optimal spatial carbon taxes across regions and sectors, we link them with Katz centrality (KC) of region i and sector j in the observed inter-provincial trade networks of sector j, defined as

$$\left[KC_1^j, \ldots, KC_N^j\right] \equiv \boldsymbol{\iota}' \left[\mathbf{I} - \frac{\delta_{2j}}{\delta_{1j}} \chi_j\right]^{-1}, \tag{25}$$

where $\chi_j \equiv \left[\chi_{in}^j\right]$ is the inter-provincial export share matrix of sector j.

Proposition 3 shows that, in the one-sector model with $s_i = \frac{w_i L_i}{\sum_{k=1}^N w_k L_k}$, optimal spatial carbon taxes are negatively correlated with Katz centrality within the observed inter-regional trade network. Panel (a) of Figure 1 confirms that this negative correlation also holds in our full model. To maximize the aggregate real income, the central government should reallocate more emissions to province-sectors with greater influence in domestic production networks. For instance, $\left(t_i^{j*}\right)$ values are generally higher in Inner Mongolia, which is peripheral in inter-province trade networks, compared to Guangdong, a key production hub in China.



(a) Katz Centrality (b) Initial Emission

Figure 1: Optimal Spatial Carbon Taxes in the 2017 Chinese Economy

(Notes: The dash line is the linear fit line. Katz centrality is defined in Equation (25).)

We then link optimal spatial carbon taxes $\left(t_i^{j*}\right)$ with province-sector emissions in the calibrated economy, E_i^j. Equation (16) indicates that E_i^j reflects the direct impacts of carbon taxes on carbon emissions. Panel (b) of Figure 1 demonstrates a positive correlation between $\left(t_i^{j*}\right)$ and E_i^j. Higher carbon taxes on high-emission province-sectors result in significant carbon reduction gains. For instance, $\left(t_i^{j*}\right)$ are particularly high for the basic and fabricated metal sectors, which are major emission contributors in most provinces.

We further investigate the variation in optimal spatial carbon taxes across regions and sectors by regressing $\left(t_i^{j*}\right)$ on KC_i^j and $\log\left(E_i^j\right)$. Column (1) of Table 2 shows that $\left(t_i^{j*}\right)$ is significantly negatively correlated with KC_i^j but positively correlated with $\log\left(E_i^j\right)$. This indicates that optimal spatial carbon taxes balance the benefits of carbon reduction against the losses in production efficiency.

We also relate optimal spatial carbon taxes to emission intensity $\frac{E_i^j}{\sum_{n=1}^N X_{in}^j}$, a simple measure capturing the trade-off between promoting production and reducing emissions. Column (2) of Table 2 reveals that emission intensity is significantly positively correlated with optimal spatial carbon taxes, suggesting that the central government should impose higher t_i^j on (i, j) pairs with higher initial emission intensities. However, the R-squared of this regression is notably lower than that in Column (1), indicating that emission intensity is not as strong a predictor for optimal spatial carbon taxes as the model-guided variables KC_i^j and $\log\left(E_i^j\right)$.

Figure 2 depicts spatial reallocation of carbon emissions due to optimal spatial carbon taxes, indicating that these taxes shift emissions from high-emission provinces at the periphery of domestic production networks, such as Xinjiang and Inner Mongolia, to production hubs with higher KC_i^j. The regression coefficient for emission reallocation on Katz centrality in the observed inter-provincial trade networks is 10.9 with a standard error of 2.75.

It is often computationally demanding to solve for precise values of optimal spatial carbon taxes, as in our practice. Many policy practices require easily computable, albeit less accurate, indicators to guide policy design. The emission allocative efficiency (EAE),

Table 2: Optimal Spatial Carbon Taxes vs. Region-Sector Characteristics

Dependent Variable:	Optimal Carbon Taxes t_i^{j*}	
	(1)	(2)
KC_i^j	-2.049^{***}	
	(0.307)	
$\log\left(E_i^j\right)$	0.0184^{***}	
	(0.0036)	
$\log\left(\frac{E_i^j}{\sum_{n=1}^N X_{in}^j}\right)$		0.0198^{**}
		(0.00461)
r^2	0.114	0.009
N	450	450

Standard errors in parentheses. Standard errors are clustered at the sector-level.
* ($p < 0.10$), ** ($p < 0.05$), *** $p < 0.01$

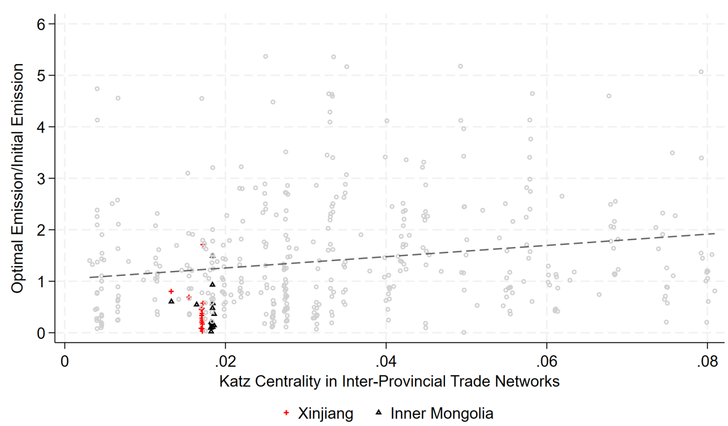


Figure 2: Emission Relocation Led by Optimal Carbon Taxes

(Notes: The dash line is the linear fit line. The regression coefficient of emission reallocation on Katz centrality is 10.9, with a standard error 2.75.)

$M_i^j(\mathbf{t})$ in Equation (16), offers two key advantages. First, it can be computed through the linear system in Equation (21), requiring significantly less computational time (3 minutes in our case) to solve for $\left(M_i^j(\mathbf{t})\right)$ in the initial economy. Second, as shown in Proposition 2, EAE predicts the ratio of optimal carbon taxes to initial carbon taxes, t_i^{j*}/t_i^j. In summary, EAE provides valuable qualitative guidance for designing spatial carbon taxes without requiring fully optimal policy calculations.

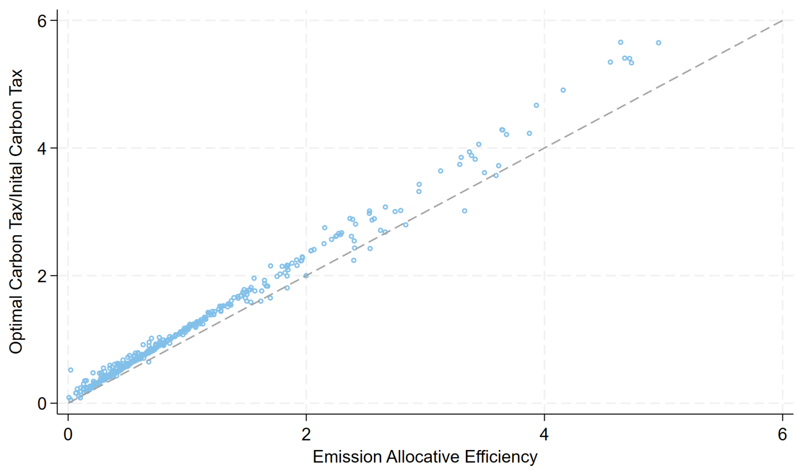


Figure 3: Emission Allocative Efficiency $M_i^j(\mathbf{t})$ as a Predictor to Optimal Carbon Taxes

(Notes: The dash line is the 45-degree line.)

Figure 3 shows that EAE, $\left(M_i^j(\mathbf{t})\right)$, in our initial economy is a strong predictor of the ratio of observed to initial carbon taxes. Consequently, obtaining the easily computable EAE is sufficient for many policy practices.

The quantitative results in this subsection suggest that strategic spatial emission allocation is essential for balancing carbon reduction with production promotion. Our quantitative framework offers a valuable tool for designing practical carbon policies.

4.3 Extension: Decentralized Carbon Reduction

It is often impractical for the central government to formulate carbon policies tailored to specific regions and sectors. Consequently, emission constraints are typically assigned to regions, with local governments setting carbon taxes for their industries. In this decentralized process, local governments prioritize regional benefits and do not internalize the effects on emissions and production in other areas. What are the equilibrium carbon policies under decentralized allocation, and how do they differ from optimal policies? These questions are crucial for understanding the value of regional coordination in designing effective carbon reduction strategies.

In this section, we apply our framework to quantify the non-cooperative carbon taxes set by local governments. We extend our emission allocative efficiency measure in Equation (16) to characterize unilaterally optimal carbon taxes in each region and develop an algorithm to compute Nash equilibrium carbon taxes. We then apply this framework to the Chinese economy in 2017.

For any region k, we consider the problem where it chooses its local carbon taxes, $\left(t_k^j\right)_{j=1}^J$ to maximize its real income while adhering to its emission constraint $\bar{E}_k$, taking carbon taxes in all other regions as given:

$$\begin{aligned} &\max_{\left(t_k^j; w_i, L_i, P_n^j, X_n^j\right)} W_k \equiv \frac{Y_k/L_k}{P_k} \\ \text{s.t. } &\sum_{j=1}^{J} \frac{\xi_j}{t_1^j} \sum_{n=1}^{N} \lambda_{1n}^j X_n^j \leq \bar{E}_k \\ &\left(w_i, L_i, P_n^j, X_n^j\right) \text{ satisfy Equation (4), (5),(9), and (7)} \\ &\text{Take } \left(t_i^j\right) \text{ for } i \neq k \text{ as given} \end{aligned} \tag{26}$$

Solving the problem above for all regions, we obtain Nash carbon taxes in which each region maximizes its real income subject to a regional emission constraint, taking carbon taxes in other regions as given. In this Nash equilibrium, a region will not internalize the impacts of its carbon taxes on other regions' production, emission, and real income.

To solve for unilaterally optimal carbon taxes for each region, we propose a decentralized emission allocative efficiency (DEAE) measure analogous to EAE in Equation (16).

From this, we develop an iterative algorithm to solve for Nash carbon taxes across regions. The details of DEAE and the computation algorithm are presented in Appendix A.4.

We proceed by solving for Nash carbon taxes for 30 provinces in China. We consider two sets of regional emission constraints, $(\bar{E}_i)$: (i) the initial regional emissions, and (ii) the regional emissions under optimal carbon taxes, $\left(E_i^*\right)$. Using a personal computer, it takes about 4 hours to compute a single set of Nash equilibrium carbon taxes.

We find that if we impose initial emissions as regional emission constraints, Nash carbon taxes tend to reduce China's overall real income by 0.09%. If we impose $\left(E_i^*\right)$ as regional emission constraints, Nash carbon taxes tend to increase China's real income by 0.81%. As a comparison, optimal carbon taxes would increase the Chinese real income by 1.37%. Both results suggest that lack of regional carbon tax coordination could lead to considerable distortions and thereby welfare losses.

Why do these non-cooperative carbon taxes result in welfare losses? To address this question, we regress the ratio of Nash carbon taxes under optimal regional emission constraints to optimal taxes on the upstreamness measure developed by Antras, Chor, Fally, and Hillberry (2012). Specifically, let $\mathbf{Y}^p \equiv \left[\sum_{i,n} X_{in}^j\right]$ be the vector of sectoral production values and $\mathbf{\Gamma} \equiv \left[\gamma_{js}\right]$ be the input-output matrix. Then the vector of upstreamness $\mathbf{up} \equiv \left[up_j\right]$ satisfies $\mathbf{up} \cdot \mathbf{Y}^p = [\mathbf{I} - \mathbf{\Gamma}]^{-1} \mathbf{Y}^p$, where $\cdot$ refers to element-wise matrix multiplication.

Column (1) of Table 3 presents the regression results, indicating that Nash carbon taxes, under optimal regional emission constraints, are significantly higher in sectors with greater upstreamness compared to optimal carbon taxes. This discrepancy occurs because local governments, when setting carbon taxes non-cooperatively, do not account for the effects of their upstream carbon taxes on the production costs of downstream sectors in other regions. These excessively high carbon taxes in upstream sectors distort production and result in significant welfare losses compared to the optimal spatial carbon taxes.

Table 3: Nash Carbon Taxes and Upstreamness

Dependent Variable:	$\log\left(\frac{\text{Nash tax (optimal)}}{\text{Optimal tax}}\right)$	$\log\left(\frac{\text{Nash tax (initial)}}{\text{Initial tax}}\right)$
	(1)	(2)
Upstreamness (in log)	1.261^{***}	1.969^{***}
	(0.079)	(0.138)
Prov. FE	✓	✓
r^2	0.194	0.258
N	450	450

Standard errors in parentheses. Standard errors are clustered at the provincial-level. * ($p < 0.10$), ** ($p < 0.05$), *** $p < 0.01$
Upstreamness is the upstreamness measure of sectors developed by Antras et al. (2012).
Nash taxes under optimal regional emissions increase China's real income by 0.81%
Nash taxes under initial regional emissions decrease China's real income by 0.09%
Optimal taxes increase China's real income by 1.37%

Column (2) of Table 3 shows the results of regressing the ratio of Nash carbon taxes under initial regional emission constraints to initial taxes on the upstreamness measure. Similar to the optimal case, Nash carbon taxes under initial regional emission constraints are higher in sectors with greater upstreamness compared to initial carbon taxes. This lack of regional coordination results in a 0.09% decline in China's aggregate real income.

In summary, our quantitative analysis reveals that under a decentralized carbon allocation system, local governments fail to internalize the effects of their carbon taxes on production and emissions in other regions, particularly for upstream sectors, leading to inefficiencies. Therefore, regional coordination is essential for designing effective spatial carbon policies.

5 Conclusion

In this paper, we develop a multi-sector quantitative spatial model with carbon emissions and taxes to quantify optimal spatial carbon policies. We propose a novel emission allocative efficiency (EAE) measure gauging the extent to which emission relocation could increase real income. We derive sufficient statistics that link EAE with data on inter-regional trade, labor, and emissions, highlighting the importance of regional heterogeneity, externalities, and interdependence in shaping optimal spatial carbon taxes. Based on EAE, we develop an iterative algorithm to compute high-dimensional optimal spatial carbon taxes. Applying our framework to the Chinese economy in 2017, we find that optimal spatial carbon taxes lead to considerable real income gains without changing total emissions. Furthermore, we utilize our framework to examine Nash carbon taxes implemented by local governments, underscoring the need for regional coordination in designing carbon policies.

Our framework has broad policy applications. First, it can calculate optimal spatial carbon policies for major economies including the U.S. and EU. Second, it can be used to compute optimal policies in other networks such as industrial policies in trade and production networks and innovation policies in knowledge networks. Notably, we do not specify how the optimal spatial carbon taxes could be practically implemented. This implementation question remains open for future research.

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A Theory

A.1 National Carbon Market

Consider an endowment of carbon emission $\bar{E}$ that can be frictionlessly traded across regions and sectors. The price of carbon emission is therefore $P_E > 0$ for all region-sectors. The revenue of carbon emission sales is distributed proportional to the wage income. The equilibrium under carbon emission market consists of $\left(w_i, L_i, P_i^j, X_i^j; P_E\right)$ satisfying Equation (5), (9), (4), (7), and

$$P_E \bar{E} = \sum_{j=1}^{J} \sum_{i=1}^{N} \sum_{n=1}^{N} \xi_j \lambda_{in}^j X_n^j. \tag{A1}$$

According to Proposition 2 in Fajgelbaum and Gaubert (2020), when $s_i = \frac{L_i}{\bar{L}}$ and $\psi_j = \beta = 0$, the decentralized spatial equilibrium is efficient. Consequently, the national carbon market mirrors the emissions allocation that would result from optimal carbon taxation. Under these conditions, the central government sets the carbon tax uniformly across regions and sectors at $t_i^j = t^* = P_E$ for all i and j, achieving the first-best allocation. Therefore, Proposition 1 holds.

A.2 Emission Allocative Efficiency

The Lagrange function of Problem (13) is defined as

$$\mathcal{L}\left(\mathbf{t}; \mu\right) \equiv \frac{1}{\bar{L}^{\beta}} \left[\sum_{k=1}^{N} \left(B_k \frac{Y_k / L_k}{P_k} \right)^{\frac{1}{\beta}} \right]^{\beta} + \mu \left[\bar{E} - \sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\xi_j}{t_i^j} \sum_{n=1}^{N} \lambda_{in}^j X_n^j \right]. \tag{A2}$$

The first-order conditions indicate that the optimal carbon taxes $\left(t_i^{j*}\right)$ satisfy:

$$\frac{\partial \mathcal{L}\left(\mathbf{t}^*;\mu^*\right)}{\partial t_i^j}=0\Rightarrow\frac{\partial W}{\partial t_i^j}=\mu^*\left[-\frac{R_i^{j*}}{\left(t_i^{j*}\right)^2}+\sum_{s=1}^{J}\sum_{k=1}^{N}\frac{\partial R_k^s}{\partial t_i^j}\frac{1}{t_k^{s*}}\right], \tag{A3}$$

and

$$\sum_{j=1}^{J}\sum_{i=1}^{N}E_i^{j*}=\bar{E}. \tag{A4}$$

Proof to Proposition 2

Equation (A3) can be expressed as:

$$1=\frac{\mu^*}{W^*}\left[E_i^{j*}+\sum_{s=1}^{J}\sum_{k=1}^{N}\left(-\frac{\partial\log R_k^s}{\partial\log t_i^j}\right)E_k^{s*}\right]\left(-\frac{\partial\log W}{\partial\log t_i^j}\right)^{-1}. \tag{A5}$$

The RHS of Equation (A5) is, by construction, $M_i^j\left(\mathbf{t}^*\right)$. Therefore, we have $M_i^j\left(\mathbf{t}^*\right)=1$.

Suppose that $\mathbf{t}=\mathbf{t}^*$ except for t_i^j. Notice that $\mathcal{L}\left(\mathbf{t};\mu\right)$ is continuously differentiable w.r.t. $\mathbf{t}$. Then there exists $\delta>0$ such that for any $t_i^j\in\left(t_i^{j*}-\delta,t_i^{j*}+\delta\right)$ we have

$$\frac{\partial \mathcal{L}\left(\mathbf{t};\mu\right)}{\partial t_i^j}>0\Rightarrow t_i^j<t_i^{j*}\quad\text{and}\quad\frac{\partial \mathcal{L}\left(\mathbf{t};\mu\right)}{\partial t_i^j}<0\Rightarrow t_i^j>t_i^{j*}. \tag{A6}$$

Equivalently, for any $t_i^j\in\left(t_i^{j*}-\delta,t_i^{j*}+\delta\right)$, we have if $M_i^j(\mathbf{t})>1$ then $t_i^{j*}>t_i^j$ and if $M_i^j(\mathbf{t})<1$ then $t_i^{j*}<t_i^j$.

Again, since $\mathcal{L}\left(\mathbf{t};\mu\right)$ is continuously differentiable w.r.t. $\mathbf{t}$, there exists $\delta>0$ such that for any $\mathbf{t}$ satisfying $\sum_{i,j}\left[t_i^j-t_i^{j*}\right]^2\leq\delta$, we have if $M_i^j(\mathbf{t})>1$ then $t_i^{j*}>t_i^j$ and if $M_i^j(\mathbf{t})<1$ then $t_i^{j*}<t_i^j$.

Q.E.D.

A.3 Sufficient Statistics in the One-Sector Case

In the one-sector case, the central government solves for the following spatial emission problem:

$$\begin{aligned}
&\max_{(t_i;w_i,L_i,X_i)} W \equiv \frac{1}{\bar{L}^{\beta}}\left[\sum_{k=1}^{N}\left(B_k\frac{X_k/L_k}{P_k}\right)^{\frac{1}{\beta}}\right]^{\beta} \\
&\text{s.t. } \sum_{i=1}^{N}\frac{\xi}{t_i}\sum_{n=1}^{N}\lambda_{in}X_n \le \bar{E}, \\
&w_iL_i = \gamma^L\sum_{n=1}^{N}\lambda_{in}X_n \\
&X_i = w_iL_i + s_iR, \quad R\equiv \xi\sum_{n=1}^{N}X_n, \quad s_i = \frac{w_iL_i}{\sum_{k=1}^{N}w_kL_k} \\
&P_n = \left[\sum_{k=1}^{N}A_kL_k^{\theta\psi}\left(\tau_{kn}w_k^{\gamma^L}t_k^{\xi}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \\
&\frac{L_i}{\bar{L}} = \frac{\left(B_i\frac{X_i/L_i}{P_i}\right)^{\frac{1}{\beta}}}{\sum_{k=1}^{N}\left(B_k\frac{X_k/L_k}{P_k}\right)^{\frac{1}{\beta}}}
\end{aligned} \tag{A7}$$

Lagrange:

$$\mathcal{L} = W + \mu\left(\bar{E} - \sum_{k=1}^{N}\frac{\xi X_k}{t_k}\right). \tag{A8}$$

F.O.C.

$$\frac{\partial W}{\partial t_i} + \mu\left(\frac{\xi X_i}{t_i^2} - \sum_{k=1}^{N}\frac{\xi}{t_k}\frac{\partial X_k}{\partial t_i}\right) = 0. \tag{A9}$$

Then

$$-\frac{\partial \log W}{\partial \log t_i} = \frac{\mu}{W^*}\left(\frac{\xi X_i^*}{t_i^*} - \sum_{k=1}^{N}\frac{\xi X_k^*}{t_k^*}\frac{\partial \log X_k}{\partial \log t_i}\right). \tag{A10}$$

Then

$$t_i^* = \frac{\mu\xi}{W^*}\left[X_i^* + \sum_{k=1}^{N}\frac{t_i^*}{t_k^*}\left(-\frac{\partial \log X_k}{\partial \log t_i}\right)X_k^*\right]\left(-\frac{\partial \log W}{\partial \log t_i}\right)^{-1}. \tag{A11}$$

A.3.1 Proof to Proposition 3

Notice that the aggregate consumption expenditure is equal to the aggregate production value, *i.e.* $\sum_{i=1}^{N} X_i = \sum_{i=1}^{N} \sum_{n=1}^{N} \lambda_{in} X_n$. Also $\gamma^L + \xi = 1$. Therefore, we have $R = \frac{\xi}{1-\xi} \sum_{i=1}^{N} w_i L_i$ and $X_i = \frac{1}{1-\xi} w_i L_i$. Then the equilibrium system can be expressed in terms of $(w_i, L_i, P_i; W)$:

$$\begin{aligned} w_i L_i &= \sum_{n=1}^{N} A_i L_i^{\theta\psi} \left(\tau_{in} w_i^{1-\xi} t_i^{\xi}\right)^{-\theta} P_n^{\theta} w_n L_n \\ P_i^{-\theta} &= \sum_{n=1}^{N} A_n L_n^{\theta\psi} \left(\tau_{ni} w_n^{1-\xi} t_n^{\xi}\right)^{-\theta} \\ L_i &= \left(\frac{1}{1-\xi}\right)^{\frac{1}{\beta}} W^{-\frac{1}{\beta}} \left(B_i \frac{w_i}{P_i}\right)^{\frac{1}{\beta}} \\ \sum_{i=1}^{N} L_i &= \bar{L}. \end{aligned} \tag{A12}$$

Then we have

$$A_i^{-1} w_i^{1+\theta(1-\xi)} L_i^{1-\theta\psi} t_i^{\theta\xi} = \left(\frac{1}{1-\xi}\right)^{\theta} W^{-\theta} \sum_{n=1}^{N} \tau_{in}^{-\theta} B_n^{\theta} w_n^{1+\theta} L_n^{1-\theta\beta}, \tag{A13}$$

and

$$B_i^{-\theta} w_i^{-\theta} L_i^{\theta\beta} = \left(\frac{1}{1-\xi}\right)^{\theta} W^{-\theta} \sum_{n=1}^{N} \tau_{ni}^{-\theta} A_n L_n^{\theta\psi} \left(w_n^{1-\xi} t_n^{\xi}\right)^{-\theta}. \tag{A14}$$

Then we have

$$A_i^{-1} w_i^{1+\theta(1-\xi)} L_i^{1-\theta\psi} t_i^{\theta\xi} = \phi B_i^{-\theta} w_i^{-\theta} L_i^{\theta\beta}, \tag{A15}$$

where $\phi > 0$ is some scalar.

Therefore,

$$w_i = \phi^{\frac{1}{1+\theta(2-\xi)}} \left(A_i B_i^{-\theta}\right)^{\frac{1}{1+\theta(2-\xi)}} L_i^{\frac{\theta(\beta+\psi)-1}{1+\theta(2-\xi)}} t_i^{-\frac{\theta\xi}{1+\theta(2-\xi)}}. \tag{A16}$$

Then

$$
\begin{aligned}
&A_i^{-\frac{\theta}{1+\theta(2-\xi)}} B_i^{-\frac{\theta[1+\theta(1-\xi)]}{1+\theta(2-\xi)}} L_i^{\theta\frac{1+[1+\theta(1-\xi)]\beta-\theta\psi}{1+\theta(2-\xi)}} t_i^{\frac{\theta^2\xi}{1+\theta(2-\xi)}} \\
&= \phi^{\frac{1+\theta}{1+\theta(1-\xi)}} \left(\frac{1}{1-\xi}\right)^{\theta} W^{-\theta} \sum_{n=1}^{N} \tau_{in}^{-\theta} A_n^{\frac{1+\theta}{1+\theta(2-\xi)}} B_n^{\frac{\theta^2(1-\xi)}{1+\theta(2-\xi)}} L_n^{\theta\frac{(1-\xi)-\theta(1-\xi)\beta+(1+\theta)\psi}{1+\theta(2-\xi)}} t_n^{-\theta\xi\frac{1+\theta}{1+\theta(2-\xi)}}.
\end{aligned}
\tag{A17}
$$

Let $\breve{\theta} = \frac{\theta}{1+\theta(2-\xi)}$, $\delta_1 = 1 + [1+\theta(1-\xi)]\,\beta - \theta\psi$, and $\delta_2 = (1-\xi) - \theta(1-\xi)\beta + (1+\theta)\psi$. Then

$$
L_i^{\breve{\theta}\delta_1} t_i^{\breve{\theta}\theta\xi} = \phi^{\frac{1+\theta}{1+\theta(1-\xi)}} \left(\frac{1}{1-\xi}\right)^{\theta} W^{-\theta} \sum_{n=1}^{N} K_{in} L_n^{\breve{\theta}\delta_2} t_n^{-\breve{\theta}(1+\theta)\xi}, \tag{A18}
$$

where $K_{in} = \tau_{in}^{-\theta} A_n^{\breve{\theta}\left(1+\frac{1}{\theta}\right)} B_n^{\breve{\theta}\theta(1-\xi)} A_i^{\breve{\theta}} B_i^{\breve{\theta}[1+\theta(1-\xi)]}$.

We then log-linearize Equation (A18). For any variables $(Z_i)_{i=1}^{N}$ with $Z_i > 0$, we denote $\tilde{Z}_i \equiv d\log Z_i$ and the column vector $\tilde{\mathbf{Z}} \equiv \left(\tilde{Z}_1, \ldots, \tilde{Z}_N\right)'$. We also denote $\nabla_{\tilde{\mathbf{t}}}\tilde{\mathbf{Z}} \equiv \left[\frac{\partial \log Z_k}{\partial \log t_i}\right]$ as a matrix with k denoting the rows and i denoting the columns.

Notice that Equation (A18) is derived from (A13). Then we have

$$
\tilde{L}_i = -\frac{\theta}{\breve{\theta}\delta_1}\tilde{W} - \frac{\theta\xi}{\delta_1}\tilde{t}_i + \sum_{n=1}^{N} \chi_{in}\left[\frac{\delta_2}{\delta_1}\tilde{L}_n - \frac{(1+\theta)\xi}{\delta_1}\tilde{t}_n\right], \tag{A19}
$$

where $\chi_{in} = \frac{X_{in}}{\sum_{k=1}^{N} X_{ik}}$.

We then have the matrix expression as

$$
\tilde{\mathbf{L}} = -\frac{\theta}{\breve{\theta}\delta_1}\mathbf{e}\tilde{W} - \frac{\theta\xi}{\delta_1}\tilde{\mathbf{t}} + \frac{\delta_2}{\delta_1}\chi\tilde{\mathbf{L}} - \frac{(1+\theta)\xi}{\delta_1}\chi\tilde{\mathbf{t}}. \tag{A20}
$$

Therefore,

$$
\tilde{\mathbf{L}} = -\frac{\theta}{\breve{\theta}\delta_1}\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}\tilde{W} - \left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]\tilde{\mathbf{t}}. \tag{A21}
$$

We also have

$$\sum_{i=1}^{N} \iota_i \tilde{L}_i = 0. \tag{A22}$$

Therefore

$$0 = \boldsymbol{\iota}'\tilde{\mathbf{L}} = -\frac{\theta}{\breve{\theta}\delta_1}\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}\tilde{W} - \boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]\tilde{\mathbf{t}}. \tag{A23}$$

Then

$$\tilde{W} = -\frac{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]}{\frac{\theta}{\breve{\theta}\delta_1}\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}}\tilde{\mathbf{t}}. \tag{A24}$$

Then

$$\nabla_{\tilde{\mathbf{t}}}\tilde{W} = -\frac{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]}{\frac{\theta}{\breve{\theta}\delta_1}\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}}. \tag{A25}$$

The effects of carbon taxes on labor can be expressed as

$$\begin{aligned}\tilde{\mathbf{L}} = &\frac{1}{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}}\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]\tilde{\mathbf{t}} \\ &- \left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]\tilde{\mathbf{t}}.\end{aligned} \tag{A26}$$

Therefore,

$$\tilde{\mathbf{L}} = -\left[\mathbf{I} - \frac{1}{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}}\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}\boldsymbol{\iota}'\right]\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]\tilde{\mathbf{t}}. \tag{A27}$$

Then

$$\nabla_{\tilde{\mathbf{t}}}\tilde{\mathbf{L}} = -\left[\mathbf{I} - \frac{1}{\boldsymbol{\iota}'\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}}\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\mathbf{e}\boldsymbol{\iota}'\right]\left[\mathbf{I} - \frac{\delta_2}{\delta_1}\chi\right]^{-1}\left[\frac{\theta\xi}{\delta_1}\mathbf{I} + \frac{(1+\theta)\xi}{\delta_1}\chi\right]. \tag{A28}$$

Moreover,

$$\tilde{\mathbf{w}} = \breve{\theta}\left[(\beta+\psi) - \frac{1}{\theta}\right]\tilde{\mathbf{L}} - \breve{\theta}\xi\tilde{\mathbf{t}}. \tag{A29}$$

Since $\tilde{\mathbf{X}} = \tilde{\mathbf{w}} + \tilde{\mathbf{L}}$. Therefore,

$$\nabla_{\tilde{\mathbf{t}}}\tilde{\mathbf{X}} = \left[1 + \breve{\theta}\left(\beta + \psi - \frac{1}{\theta}\right)\right]\nabla_{\tilde{\mathbf{t}}}\tilde{\mathbf{L}} - \breve{\theta}\xi\mathbf{I}. \tag{A30}$$

Q.E.D.

A.3.2 EAE under $s_i = \frac{L_i}{\bar{L}}$

Without loss of generality, we let $\bar{L} = 1$ and $\sum_{i=1}^{N} w_i L_i = 1$. Then $s_i = \frac{L_i}{\bar{L}}$ implies that

$$X_i = \left(w_i + \frac{\xi}{1-\xi}\right)L_i. \tag{A31}$$

Therefore, the equilibrium system can be expressed as

$$\begin{aligned}
w_i L_i &= (1-\xi)\sum_{n=1}^{N} A_i L_i^{\theta\psi}\left(\tau_{in} w_i^{1-\xi} t_i^{\xi}\right)^{-\theta} P_n^{\theta}\left(w_n + \frac{\xi}{1-\xi}\right)L_n \\
P_i^{-\theta} &= \sum_{n=1}^{N} A_n L_n^{\theta\psi}\left(\tau_{ni} w_n^{1-\xi} t_n^{\xi}\right)^{-\theta} \\
W &= L_i^{-\beta} B_i \frac{w_i + \frac{\xi}{1-\xi}}{P_i} \\
\sum_{i=1}^{N} L_i &= \bar{L}.
\end{aligned} \tag{A32}$$

Then we have

$$A_i^{-1} w_i^{1+\theta(1-\xi)} L_i^{1-\theta\psi} t_i^{\theta\xi} = (1-\xi)\, W^{-\theta} \sum_{n=1}^{N} \tau_{in}^{-\theta} B_n^{\theta} \left(w_n + \frac{\xi}{1-\xi} \right)^{1+\theta} L_n^{1-\theta\beta}, \tag{A33}$$

and

$$B_i^{-\theta} \left(w_i + \frac{\xi}{1-\xi} \right)^{-\theta} L_i^{\theta\beta} = W^{-\theta} \sum_{n=1}^{N} \tau_{ni}^{-\theta} A_n L_n^{\theta\psi} \left(w_n^{1-\xi} t_n^{\xi} \right)^{-\theta}. \tag{A34}$$

Log-linearization leads to

$$\begin{aligned}
[1+\theta(1-\xi)]\,\tilde{w}_i + (1-\theta\psi)\,\tilde{L}_i + (\theta\xi)\,\tilde{t}_i &= -\theta\tilde{W} + \sum_{n=1}^{N} \chi_{in} \left[(1+\theta)\,\delta_n^w \tilde{w}_n + (1-\theta\beta)\,\tilde{L}_n \right] \\
\delta_i^w \tilde{w}_i - \beta\tilde{L}_i &= \tilde{W} + \sum_{n=1}^{N} \lambda_{ni} \left[-\psi\tilde{L}_n + (1-\xi)\,\tilde{w}_n + \xi\tilde{t}_n \right]
\end{aligned} \tag{A35}$$

In matrix form:

$$\begin{aligned}
\left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right]\tilde{\mathbf{w}} + \left[(1-\theta\psi)\,\mathbf{I} - (1-\theta\beta)\,\boldsymbol{\chi}\right]\tilde{\mathbf{L}} &= -\theta\tilde{W}\mathbf{e} - \theta\xi\tilde{\mathbf{t}} \\
\left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]\tilde{\mathbf{w}} - \left[\beta\mathbf{I} - \psi\boldsymbol{\lambda}\right]\tilde{\mathbf{L}} &= \tilde{W}\mathbf{e} + \xi\boldsymbol{\lambda}\tilde{\mathbf{t}}
\end{aligned} \tag{A36}$$

where $\boldsymbol{\delta}^w$ is a diagonal matrix whose diagonal is $\left[\delta_1^w, \ldots, \delta_N^w\right]'$.

Then

$$\begin{aligned}
&\left\{ \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \left[\beta\mathbf{I} - \psi\boldsymbol{\lambda}\right] + \left[(1-\theta\psi)\,\mathbf{I} - (1-\theta\beta)\,\boldsymbol{\chi}\right] \right\} \tilde{\mathbf{L}} \\
&= -\tilde{W} \left\{ \theta\mathbf{I} + \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \right\} \mathbf{e} \\
&- \xi \left\{ \theta\mathbf{I} + \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \boldsymbol{\lambda} \right\} \tilde{\mathbf{t}}
\end{aligned} \tag{A37}$$

Let $\Omega \equiv \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \left[\beta\mathbf{I} - \psi\boldsymbol{\lambda}\right] + \left[(1-\theta\psi)\,\mathbf{I} - (1-\theta\beta)\,\boldsymbol{\chi}\right]$. Notice that $\boldsymbol{\iota}'\tilde{L} = 0$. Therefore, we have

$$\nabla_{\tilde{\mathbf{t}}}\tilde{W} = -\frac{\xi\boldsymbol{\iota}'\Omega^{-1} \left\{ \theta\mathbf{I} + \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \boldsymbol{\lambda} \right\}}{\boldsymbol{\iota}'\Omega^{-1} \left\{ \theta\mathbf{I} + \left[(1+\theta(1-\xi))\,\mathbf{I} - (1+\theta)\,\boldsymbol{\chi}\boldsymbol{\delta}^w\right] \left[\boldsymbol{\delta}^w - (1-\xi)\,\boldsymbol{\lambda}\right]^{-1} \right\} \mathbf{e}} \tag{A38}$$

A.4 Decentralized Carbon Allocation

To characterize region i's unilaterally optimal carbon taxes, we define the following decentralized emission allocative efficiency (DEAE):

$$DM_i^j(\mathbf{t}_i) \equiv \frac{\mu_i}{W_i}\left[E_i^j + \sum_{s=1}^{J}\left(-\frac{\partial \log R_i^s}{\partial \log t_i^j}\right)E_i^s\right]\left(-\frac{\partial \log W_i}{\partial \log t_i^j}\right)^{-1}, \tag{A39}$$

where μ_i is the Lagrange multiplier of the regional carbon emission constraint in Equation (26).

Corollary 5 (Decentralized Emission Allocative Efficiency) *Consider region i, taking* $\left(t_k^j\right)$ *for all* $k \neq i$ *as given. Let* $\mathbf{t}_i^* \equiv \left(t_i^{j*}\right)$ *be the solution of Problem (26). Then there exists* $\delta > 0$ *such that for any* $\mathbf{t}_i$ *satisfying* $\sum_j \left[t_i^j - t_i^{j*}\right]^2 \leq \delta$, $DM_i^j(\mathbf{t}_i)$ *defined by Equation (A39) has the following properties:*

1. $DM_i^j(\mathbf{t}_i^*) = 1$ *for all* j.
2. *If* $DM_i^j(\mathbf{t}_i) > 1$, *then* $t_i^{j*} > t_i^j$.
3. *If* $DM_i^j(\mathbf{t}_i) < 1$, *then* $t_i^{j*} < t_i^j$.

To derive $DM_i^j(\mathbf{t}_i)$, we can still calculate $\left(\tilde{w}_i, \tilde{L}_i, \tilde{P}_i^j, \tilde{X}_i^j\right)$ by solving Equation (21). Notice that the first-order changes in real income can be expressed as

$$\tilde{W}_i = \tilde{Y}_i - \tilde{L}_i - \tilde{P}_i. \tag{A40}$$

Based on Corollary 5, we develop an algorithm to solve for Nash carbon taxes $\mathbf{t}_i^*$ which is analogous to Algorithm 4.

Algorithm 6 (Nash Carbon Taxes) *The Nash carbon taxes* $\mathbf{t}^*$ *can be calculated as follows:*

- *Guess* $\mathbf{t}^0 \in \mathbb{R}_{++}^{N\times J}$.

- *Given* $\left(t_k^{j0}\right)$ *for all* $k \neq i$, *derive* $\mathbf{t}_i^*$ *that solves the problem in Equation* (26):

  1. *Guess* $\mathbf{t}_i^* \in \mathbb{R}_{++}^J$.
  2. *Solve for* $\left(X_{in}^j, \iota_i\right)$ *under* $\mathbf{t}_i^*$ *by "exact-hat algebra" in Equation* (12).
  3. *Solve the linear system* (21).
  4. *Calculate* $DM_i^j\left(\mathbf{t}_i^*\right)$ *using Equation* (A40), (23), *and* (A39).
  5. *Update* t_i^{j*} *by* $t_i^{j*}DM_i^j(\mathbf{t}_i^*)$.
  6. *Repeat Step 1-5 until* $t_i^{j*} = t_i^{j*}DM_i^j(\mathbf{t}_i^*)$ *for all* j.
  7. *Adjust the level of* $\mathbf{t}_i^*$ *to bind the regional emission constraint in Equation* (26).

- *Iterate until* $\mathbf{t}^* = \mathbf{t}^0$.

Notably, our DEAE framework accelerates the computation of unilaterally optimal carbon taxes in the inner loop of Algorithm 6. Additionally, we can parallelize the computation of regional optimal carbon taxes, further speeding up the overall process.

Proof to Corollary 5

The Lagrange function of Problem (26) for region i is

$$\mathcal{L}_i\left(\mathbf{t}_i, \mu_i\right) = W_i + \mu_i\left[\bar{E}_i - \sum_{j=1}^{J}\frac{R_i^j}{t_i^j}\right]. \tag{A41}$$

The first-order condition with respect to t_i^j is

$$\frac{\partial W_i}{\partial t_i^j} = \mu_i^*\left[-\frac{R_i^{j*}}{\left(t_i^{j*}\right)^2} + \sum_{s=1}^{J}\frac{\partial R_i^s}{\partial t_i^j}\frac{1}{t_i^{s*}}\right]. \tag{A42}$$

Then we have

$$1 = \frac{\mu_i^*}{W_i}\left[E_i^{j*} + \sum_{s=1}^{J}\left(-\frac{\partial \log R_i^s}{\partial \log t_i^j}\right)E_i^{s*}\right]\left(-\frac{\partial \log W_i}{\partial \log t_i^j}\right)^{-1}. \tag{A43}$$

Therefore, $DM_i^j\left(\mathbf{t}_i^*\right) = 1$ for all j. The rest of the proof follows the arguments in Appendix A.2.

Q.E.D.