Measuring the Cost of a Tariff War:
A Sufficient Statistics Approach

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Abstract

Tariff wars have attracted renewed interest, but measuring the *prospective* cost of a full-fledged tariff war remains a computationally burdensome task. Consequently, most studies on this topic are confined to a small set of countries and overlook input-output linkages. This paper develops a new methodology that measures the prospective cost of a full-fledged tariff war in one simple step, as a function of (a) *observable trade values*, (b) *input-output shares*, (c) *industry-level trade elasticities*, and (d) *markup wedges*. Applying this methodology to data on 44 economies and 56 industries, I find that (i) the prospective cost of a tariff war to global GDP has more-than-doubled over the past fifteen years, with small downstream economies being the most vulnerable. Moreover, (ii) many countries can lose significantly from a US-China tariff war even if they are not directly involved.

1 Introduction

The global economy is entering a new era of tariffs, with many economic leaders warning against the eminent threat of a global tariff war. Just recently, Christine Lagarde, head of the International Monetary Fund, labeled the escalating US-China tariff war as "the biggest risk to global economic growth."[1]

Concurrent with these real-world developments, there has been a growing academic interest in measuring the cost of a tariff war. One natural approach is the "ex-post" approach adopted by Amiti et al. (2019) and Fajgelbaum et al.

[1]Source: https://www.bloomberg.com/news/articles/2019-06-09/lagarde-says-u-s-china-trade-war-looms-large-over-global-growth

(2019). This approach, uses data on observed tariff hikes; employs economic theory to estimate the passthrough of tariffs onto consumer prices; and measures the welfare cost of these *already-applied* tariffs.

For all its merits, the "ex-post" approach does not address an important question facing policy-makers: *what is the prospective cost of a full-fledged tariff war?* To answer such "what if" questions, we first need to determine what Nash tariff levels will prevail under a full-fledged tariff war. The "ex-ante" approach developed by Perroni and Whalley (2000) and Ossa (2014) accomplishes this exact task.[2] They use economic theory to estimate the Nash tariff levels that will prevail and the welfare cost that will result from a hypothetical (but now imminent) tariff war that has fully escalated.

The "ex-ante" approach has been quite influential, and recent methodological advances have made it more accessible to researchers. However, existing techniques are still plagued with the curse of dimensionality when applied to many countries and industries. The current state-of-the-art technique computes the Nash tariffs using an iterative process where each iteration performs a country-by-country constrained optimization given the output of the previous iterations.[3] As the number of countries or industries grows, the computational burden underlying this approach can raise significantly. This is perhaps why the current implementations of the "ex-ante" approach are often limited to a smaller set of countries and do not account for input-output linkages.[4]

In this paper, I present a simple sufficient statistics methodology to measure the prospective cost of a full-fledged global tariff war.[5] The proposed methodology (i) circumvents the computational challenges facing existing "ex-ante" methodologies, and (ii) sheds fresh light on the degree to which input-output linkages amplify the burden of a tariff war. Moreover, due to its computational simplicity, the methodology can be readily applied to data from multiple years and many small, emerging economies. Doing so indicates that the prospective

[2] See Balistreri and Hillberry (2018) for a recent application of the ex-ante approach to the current US-China tariff war.

[3] See Ossa (2016) for a comprehensive review of the iterative global optimization technique. Advances that have made this technique more efficient include (i) reformulating the problem using the exact hat-algebra technique; (ii) parallelizing the country-by-country optimizations; and (iii) providing analytical derivatives for the optimization algorithm.

[4] In the existing literature, the set of countries is typically reduced by aggregating many countries into "the rest of the world" and treating them as one taxing authority.

[5] The sufficient statistics methodology developed here is akin to the Arkolakis et al. (2012) methodology, and exhibits key differences with the sufficient statistics approach popularized by Chetty (2009) in the public finance literature. See Chapter 7 in Costinot and Rodríguez-Clare (2014) for more discussion on these differences.

cost of a global tariff war has risen dramatically over the past two decades, with small downstream economies being –by far– the most vulnerable.

My methodology relies on the analytical characterization of Nash tariffs in an important class of quantitative trade models that accommodate multiple industries, input-output linkages, market distortions, and political economy pressures. Here, Nash tariffs refer to tariff levels that will (in theory) prevail in the event of a tariff war. In characterizing the Nash tariffs, I extend prior characterizations (e.g., Johnson (1953); Gros (1987); Felbermayr et al. (2013)) to a general equilibrium setup with arbitrarily many countries and industries.[6] The analytic formulas that result, describe the Nash tariffs for each country as a function of only structural parameters and observables.

Using my analytic formulas and the exact hat-algebra methodology, popularized by Dekle et al. (2007), I can compute the Nash tariffs and their welfare effects in one simple step, by solving a system of equations that depends on only (i) *observable trade volumes*, (ii) *input-output shares*, (iii) *industry-level trade elasticities*, and (iv) *industry-level markup wedges*. This method is remarkably fast and reliable for two main reasons. First, it does not involve any iterations or any constrained global optimizations. Second, the analytic formulas indicate that Nash tariffs are uniform along certain dimensions, which itself reduces dimensionality to a remarkable degree.

I apply my methodology to the World Input-Output Database (WIOD, Timmer et al. (2012)) from 2000 to 2014, covering 43 major countries and 56 industries. For each country in the WIOD sample in a given year, I compute the prospective cost of a full-fledged global tariff war as well as a two-way US-China tariff war. I first perform my analysis using a baseline multi-industry Eaton and Kortum (2002) model. Subsequently, I introduce (a) market distortions and political pressures, as well (b) input-output linkages into my baseline analysis to determine how these additional factors contribute to the cost of a tariff war. May analysis delivers four basic insights:

i. The prospective cost of a full-fledged tariff war is immense. Especially, when we account for the dependence of countries on global value chains and the fact that a trade war will exacerbate market distortions. In 2014, for instance, the prospective cost of a full-fledged tariff war was \$1.5 tril-

[6] My characterization of Nash tariffs also builds on and extends the results in Beshkar and Lashkaripour (2019) and Lashkaripour and Lugovskyy (2019). The aforementioned studies derive formulas from optimal *unilateral* trade taxes in a two-country general equilibrium trade model. This paper, in comparison, considers cases where arbitrarily many countries strategically impose tariffs against each other.

lion in terms of global GDP, which is the equivalent of erasing South Korea from the global economy.

ii. The prospective cost of a global tariff war has more-than-doubled from 2000 to 2014. The rising cost is driven by two distinct forces. First, the rise of global market power, which prompts countries to impose more-targeted (i.e., more-distortionary) Nash tariffs in the event of a tariff war. Second, the increasing dependence of emerging economies on the global value chain since 2000.

iii. Small downstream economies are the main casualties of a global tariff war. Take Estonia, for example, where imported intermediates account for 30% of the national output, inclusive of the service sector. Due to its strong dependence on imported intermediates, 10% of Estonia's real GDP will be wiped out by a global tariff war. Similar losses will be incurred by other small, downstream economies like Bulgaria, Latvia, and Luxembourg.

iv. A two-way US-China tariff war can shave off \$34 billion from the global economy, the equivalent of Paraguay's GDP. The US economy is the biggest loser, but many other countries can incur losses without even being directly involved in the tariff war. To give specific examples, Australia's economy can lose \$58 million or Ireland's economy can lose \$26 million from a US-China tariff war. These are losses incurred without the US-China tariffs directly targeting either country.

Aside from its methodological contributions, the present paper makes three conceptual contributions to the literature. First, my analytic formulas for Nash tariffs highlight a perviously overlooked contributor to the cost of tariff wars. I show that Nash tariffs (in all countries) are targeted at high-markup industries. As a result, they shrink global output in high-markup industries below their already sub-optimal level. These developments exacerbate pre-exciting market distortions and inflict an efficiency loss that is distinct from the standard trade-loss emphasized in the prior literature (e.g., Gros (1987)).

Second, this paper highlight a previously-overlooked determinant of tariff war outcomes. Dating back to Johnson (1953), a rich body of literature has emphasized how country size determines the winners and losers from a tariff war (Kennan and Riezman (2013)). My analysis shows that a country's degree of "upstream-ness" in the global value chain is an equally-determining factor. For instance, based on my analysis, Norway that is an upstream economy (due its commodity exports) can gain from a tariff war despite being small. These gains obviously come at the expense of small downstream economies incurring immense losses.

Third, my approach highlights the pitfalls of data aggregation, which is common-place in the tariff war literature. To elaborate, existing analyses of tariff wars often restrict attention to a small set of countries and aggregate the "rest of the world" into one taxing authority. Such aggregation schemes allow researchers to handle the computational complexities inherent to tariff war analysis. Capitalizing on the computational efficiency of my sufficient statistics approach, I can measure the cost of a tariff war *with* and *without* such aggregation schemes. Comparing the outcomes indicates that standard aggregation schemes overstate the loss from a tariff war quite considerably. Simply, because they artificially assign significant market power to the "rest of the world."

Finally, at a broader level, the approach developed here can be viewed as a sufficient statistics methodology to quantify the gains from global trade agreements. In that regard, it contributes to Arkolakis et al. (2012), Costinot and Rodríguez-Clare (2014), and Arkolakis et al. (2015) who propose sufficient statistics methodologies that quantify the gains from trade relative to autarky in an important class of trade models. Like the aforementioned studies, my proposed methodology quantifies the gains from trade, but it does so relative to a world without trade agreements as opposed to autarky.

This paper is organized as follows. Section 2 presents the theoretical model, based on which a sufficient statistics approach is developed to measure the prospective cost of a tariff war. Section 3 presents a quantitative implementation using actual trade data. Section 4 concludes.

2 Theoretical Framework

The present methodology applies to a range of quantitative trade models. In the interest of exposition, I begin my analysis with a *baseline* multi-industry, multi-country Ricardian model that nests the Eaton and Kortum (2002) and Armington models as a special case. I subsequently extend the baseline model to account for (a) political economy pressures and profit-shifting effects à la Ossa (2014), and (b) intermediate input trade à la Caliendo and Parro (2015).

Throughout my analysis, I consider a global economy consisting of $i = 1, ..., N$ countries and $k = 1, .., K$ industries, with $\mathbb{C}$ and $\mathbb{K}$ respectively denoting the set of countries and industries. Labor is the only *primary* factor of production. Each country i is populated with L_i workers, each supplying one unit of labor inelastically. Workers are perfectly mobile across industries but immobile across countries.

Demand. In the baseline Ricardian model, all varieties in industry k are differentiated by country of origin, with the triplet ji, k denoting a variety supplied by country j, to market i, in industry k— from the perspective of the Eaton and Kortum (2002) model, national product differentiation of this kind can be interpreted as the outcome of Ricardian specialization within industries. The representative consumer in Country i maximizes a general utility function, which yields an indirect utility function as follows:

$$V_i(Y_i, \tilde{\boldsymbol{P}}_i) = \max_{\boldsymbol{Q}_i} \quad U(\boldsymbol{Q}_i)$$
$$s.t. \quad \sum_k \sum_j \tilde{P}_{ji,k} Q_{ji,k} = Y_i. \tag{1}$$

In the above problem, Y_i denotes total income; $\boldsymbol{Q}_i = \{Q_{ji,k}\}$ denotes the vector of composite consumption quantities, and $\tilde{\boldsymbol{P}}_i = \{\tilde{P}_{ji,k}\}$ denotes the corresponding vector of *"consumer"* price indexes. I should emphasize here that, as a choice of notation, I use *tilde* to distinguish between "consumer" and "producer" prices throughout this paper. The above problem yields the following national-level Marshallian demand function,

$$Q_{ji,k} = \mathcal{D}_{ji,k}\left(Y_i, \tilde{\boldsymbol{P}}_i\right), \tag{2}$$

which can be summarized by a set of reduced-form demand elasticities facing each composite variety ji, k. Namely, the own-price elasticity of demand,

$$\varepsilon_{ji,k} \equiv \partial \ln \mathcal{D}_{ji,k}(.)/\partial \ln \tilde{P}_{ji,k}, \tag{3}$$

and the cross-price elasticity of demand between varieties ji, k and $ji, g \neq ji, k$,

$$\varepsilon_{ji,k}^{ji,g} \equiv \partial \ln \mathcal{D}_{ji,k}(.)/\partial \ln \tilde{P}_{ji,g}.$$

I assume that the aggregate demand functions are well-behaved such that $\varepsilon_{ji,k} < -1$ and $\varepsilon_{ji,k}^{ji,g} \geq 0$. The income elasticity of demand plays a less prominent role in my analysis, so I relegate its definition to the appendix.

Production. In the Ricardian model, production only employs labor and the average unit labor cost of production and transportation is also invariant to policy. Correspondingly, the "producer" price of composite variety ji, k can be expressed as a function of the labor wage rate in country j, w_j, times the constant unit labor cost of production and transportation, $\bar{a}_{ji,k}$:

$$P_{ji,k} = \bar{a}_{ji,k} w_j. \tag{4}$$

The "consumer" price, by definition, equals the "producer" price times the tariff applied by country i on variety ji, k, namely, $t_{ji,k}$:

$$\tilde{P}_{ji,k} = (1 + t_{ji,k}) P_{ji,k}. \tag{5}$$

Keep in mind that the assumption that $\bar{a}_{ji,k}$ is invariant to policy, corresponds to a flat export supply curve. In other words, it implies that the passthrough of taxes on to consumer prices is complete (once we net of general equilibrium wage effects). This assumption is consistent with ex-post studies of the recent tariff war, like Amiti et al. (2019) and Fajgelbaum et al. (2019).

Equilibrium. For any given vector of tariffs, $\boldsymbol{t} = \{t_{ji,k}\}$, equilibrium is a vector of wages, $\boldsymbol{w} = \{w_j\}$; a vector of "producer" and "consumer" price indexes, $\boldsymbol{P}_i = \{P_{ji,k}\}$ and $\tilde{\boldsymbol{P}}_i = \{\tilde{P}_{ji,k}\}$, that are described by Equations 4 and 5; as well as consumption quantities, $\boldsymbol{Q}_i$, given by 2 subject to total income in each country equaling the wage bill, $w_i L_i$, plus tax revenues:

$$Y_i = w_i L_i + \sum_j \sum_k t_{ji,k} P_{ji,k} Q_{ji,k}.$$

The above equation along with the representative consumer's budget constraint, ensure that trade is balanced between countries, i.e., $\sum_{j \neq i} \sum_k P_{ji,k} Q_{ji,k} = \sum_{j \neq i} \sum_k P_{ij,k} Q_{ij,k}$ for all $i \in \mathbb{C}$. For the reader's convenience, Table 1 reports a summary of the key variables and parameters. Also, provided that equilibrium is unique, all variables can be uniquely characterized as a function of global tariff rates, $\boldsymbol{t}$, and wages, $\boldsymbol{w}$, with the latter implicitly depending on $\boldsymbol{t}$. Total welfare in Country i can, accordingly, be expressed as $W_i(\boldsymbol{t}; \boldsymbol{w}) = V_i(Y_i(\boldsymbol{t}; \boldsymbol{w}); \boldsymbol{P}_i(\boldsymbol{t}; \boldsymbol{w}))$. Assigning labor in country j as the numeraire, the effect of imposing a tariff $t_{ji,k}$

on country i's welfare can be stated as follows:

$$\frac{\mathrm{d}W_i(.)}{\mathrm{d}(1+t_{ji,k})} = \frac{\partial W_i(.)}{\partial(1+t_{ji,k})} + \sum_{\jmath \neq j} \frac{\partial W_i(.)}{\partial w_\jmath} \frac{\mathrm{d}w_\jmath}{\mathrm{d}(1+t_{ji,k})}.$$

As shown in Appendix A.1, $\frac{\partial W_i(.)}{\partial w_\jmath} \frac{\mathrm{d}w_\jmath}{\mathrm{d}(1+t_{ji,k})} \propto \lambda_{\jmath i} r_{\jmath i,k} \lambda_{ji,k}$, where $r_{\jmath i,k}$ denotes the share of economy $\jmath$'s wage revenue collected from sales of good $\jmath i,k$. Based on actual trade data, $\lambda_{\jmath i} r_{\jmath i,k} / \lambda_{ii} r_{ii,k} \approx 0$ for $\jmath \neq i$.[7] So, changes in welfare can be approximated as

$$\frac{\mathrm{d}W_i(.)}{\mathrm{d}(1+t_{ji,k})} \approx \frac{\partial W_i(.)}{\partial(1+t_{ji,k})} + \frac{\partial W_i(.)}{\partial w_i} \frac{\mathrm{d}w_i}{\mathrm{d}(1+t_{ji,k})}. \tag{6}$$

In the following, I use the above approximation to derive sufficient statistics formulas for Nash tariffs. Appendix E derives sufficient statistics formulas for Nash tariffs without the above approximation. Computing Nash tariffs using the approximation-free formulas will be computationally more involved, but the computed tariff levels will be indistinguishable from the baseline levels.

Nash Tariffs. In the event of a tariff war, each country i chooses their vector of non-cooperative optimal tariffs $\boldsymbol{t}_i^* = \{t_{ji,k}^*\}$, given the tariffs applied by all other countries, which I denote by $\boldsymbol{t}_{-i}$. In other words, country i's best non-cooperative tariff response solves the following problem:

$$\boldsymbol{t}_i^*(\boldsymbol{t}_{-i}) = \arg\max \; V_i\left(Y_i(\boldsymbol{t}_i; \boldsymbol{t}_{-i}), \tilde{\boldsymbol{P}}_i(\boldsymbol{t}_i; \boldsymbol{t}_{-i})\right). \tag{7}$$

In what follows, I will analytically characterize the solution to this problem. But before doing that, let me briefly outline why calculating the Nash tariffs (that prevail under a tariff war) is plagued by the curse of dimensionality. By definition, the Nash tariffs solve the following system

$$\begin{cases} \boldsymbol{t}_1 = \boldsymbol{t}_1^*(\boldsymbol{t}_{-1}) \\ \vdots \\ \boldsymbol{t}_N = \boldsymbol{t}_N^*(\boldsymbol{t}_{-N}) \end{cases},$$

[7] Trade and production data from 44 major countries and 56 industries indicates that $\text{avg}_{\jmath \neq i,k}(\frac{r_{\jmath i,k}\lambda_{\jmath i}}{r_{ii,k}\lambda_{ii}}) = 5 \times 10^{-5}$ —see Section 3 for a full description of the data.

Table 1: Summary of Key Variables

Variable	Description
$\tilde{P}_{ji,k}$	Consumer price index of variety ji,k (exporter j–importer i–industry k)
$P_{ji,k}$	Producer price index of variety ji,k (exporter j–importer i–industry k)
$Q_{ji,k}$	Consumption quantity/Output of variety ji,k
$X_{ji,k}$	F.O.B. export value: $X_{ji,k} = P_{ji,k}Q_{ji,k}$
Y_i	Total income in country i
w_iL_i	Wage income in country i (wage$\times$population size)
Π_i	Total profits in country i
$t^*_{ji,k}$	Nash/Optimal tariff imposed by country i on variety ji,k
$\bar{t}_{ji,k}$	Applied (status-quo) tariff on variety ji,k
$\beta_{i,k}$	Country i's expenditure share on industry k
$\lambda_{ji,k}$	Expenditure share on variety ji,k: $\lambda_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}/\beta_{i,k}Y_i$
$r_{ji,k}$	Wage revenue share from variety ji,k: $r_{ji,k} = P_{ji,k}Q_{ji,k}/w_iL_i$
$\varepsilon_{ji,k}$	Own-price elasticity of demand: $\varepsilon_{ji,k} = \partial \ln Q_{ji,k}/\partial \ln \tilde{P}_{ji,k}$
$\varepsilon^{ji,g}_{ji,k}$	Cross-price elasticity of demand: $\varepsilon_{ji,k} = \partial \ln Q_{ji,k}/\partial \ln \tilde{P}_{ji,g}$
$\epsilon_k - 1$	Constant trade elasticity under the CES parmaterization
μ_k	Industry-level markup wedge
$\alpha_{j,k}(\ell, g)$	Share of input variety $\ell j, g$ in variety ji,k's output, $\forall i$
$\gamma_{j,k}$	Share of labor in variety ji,k's output, $\forall i$
$\tilde{\gamma}_{j,k}(\ell)$	Share of country ℓ's labor in *country j-industry* k's output in the reformulated IO model

where $\boldsymbol{t}^*_i(\boldsymbol{t}_{-i})$ can be obtained by solving Problem 7 separately for each country i. The curse of dimensionality here is driven by two factors. First, the above system involves $N(N-1)K$ tariff rates—a number than can grow rapidly with sample size. Second, to solve the above system numerically, one has to solve $\boldsymbol{t}^*_i = \boldsymbol{t}^*_i(\boldsymbol{t}_{-i})$ iteratively for all N countries. That is, the optimal tariffs are first computed for each country by conducting N constrained global optimizations, assuming zero tariffs in the rest of the world. Then, the optimal tariffs are updated by performing another N constrained global optimizations that condition on the optimal tariff levels obtained in the first step. This procedure is repeated iteratively until we converge to the unique solution of the above system.[8]

We can circumvent these issues, by obtaining an analytical characterization for $t_i^*(.)$. The following proposition takes an important step in this direction, which ultimately reduces the computational task highlighted above to that of solving *one* system of $3N$ equations and unknowns, which depend solely on structural elasticities and observable trade values, $X_{ji,k} \equiv P_{ji,k}Q_{ji,k}$.

Proposition 1. *Country i's optimal non-cooperative import tariff is uniform and can be characterized as*

$$1 + t_i^* = \frac{\sum_{j \neq i} \sum_k \sum_g \chi_{ij,k} \varepsilon_{ij,k}^{ij,g}}{1 + \sum_{j \neq i} \sum_k \sum_g \chi_{ij,k} \varepsilon_{ij,k}^{ij,g}},$$

in terms of only (i) reduced-form demand elasticities, $\varepsilon_{ij,k}^{ij,g}$; and (ii) observable export revenue shares, $\chi_{ij,k} \equiv X_{in,k} / \sum_{\ell \neq i} \sum_k X_{i\ell,k}$.

A formal proof for the above proposition is provided in Appendix A.1, but let me provide a brief intuition for this result. The uniformity of tariffs across industries arises from the unit labor cost being invariant to policy.[9] The uniformity of tariffs across suppliers, meanwhile, derives from the fact that (to a first-order approximation) the welfare effect of country i's tariff policy is driven by a change in w_i relative to wages in the rest of the world.[10] Since the optimal import tariff is uniform, it is akin to a uniform export tax or a markup applied to w_i when a good is exported.[11] This interpretation elucidates the optimal tariff formula specified by Proposition 1, which corresponds to an optimal monopoly markup on w_i applied to all exported goods. Alternatively, the optimal tariff formula can be reformulated as an equivalent optimal markdown on the wage embedded in Country i's imports.

[8] Ossa (2016) highlights an alternative approach, whereby the constrained global optimization is converted to a set of first-order and complementary slackness conditions. Under this approach, one can compute the Nash tariffs by solving a system of $2N + N(N-1)K$ equations. However, Ossa (2016) argues that given the given the sheer size of the problem, this approach is even less efficient than the iterative approach.

[9] That optimal non-cooperative tariffs are uniform in a *two-country* Ricardian model was first established by Opp (2010) and subsequently extended by Costinot et al. (2015). Beshkar and Lashkaripour (2019) show that a version of uniformity also extends to *multi-country* models.

[10] Appendix E derives an approximation-free sufficient statistics formula for Nash tariffs. When taken to data, however, the approximation-free formula predicts tariff rates that are indistinguishable from those predicted by my baseline formula.

[11] The equivalence between uniform import and export taxes is a manifestation of the Lerner symmetry. The aforementioned symmetry is often articulated in the context of a two-country model. But the same arguments apply to a multi-country setup subject to the welfare approximation in 6. Relatedly, we can re-formulate the optimal tariff specified by Proposition 1, so that is corresponds to the optimal mark-down of a multi-product monopsonist. Such a reformulation simply involves using the wage in country i as the numeraire.

Measuring the Cost of a Tariff War. We can employ Proposition 1 to measure the prospective cost of a full-fledged tariff war. But to do so, we first need to impose additional structure on the utility function, $U_i(.)$. One commonly-used specification in the quantitative trade literature is the Cobb-Douglas-CES specification, displayed below:

$$U_i(\mathbf{Q}_i) = \prod_k \left(\sum_i \varsigma_{ji,k} Q_{ji,k}^{\rho_k} \right)^{\beta_{i,k}/\rho_k}, \tag{8}$$

where $\varsigma_{ji,k}$ is a structural demand shifter. Adopting the above specification, the bilateral trade shares ($\lambda_{ji,k} \equiv \tilde{P}_{ji,k} Q_{ji,k} / \beta_{i,k} Y_i$) assume the following formulation:

$$\lambda_{ji,k} = \varsigma_{ji,k} \tilde{P}_{ji,k}^{-\epsilon_k} / \sum_\ell \left(\varsigma_{\ell i,k} \tilde{P}_{\ell i,k}^{-\epsilon_k} \right), \tag{9}$$

where $\epsilon_k \equiv \rho_k / (\rho_k - 1)$ denotes the *industry-level trade elasticity*. Moreover, under this specification, the cross-price elasticities of demand between varieties from different industries collapse to zero, while the own-price elasticity of demand reduces to

$$\varepsilon_{ji,k} = -1 - \epsilon_k \left(1 - \lambda_{ji,k}\right). \tag{10}$$

Using Equations 9 and 10 as well as Proposition 1; and employing the exact hat-algebra notation ($\hat{x} \equiv x'/x$); we can solve for Nash tariffs and their welfare effects in one simple step. The following proposition outlines this claim.

Proposition 2. *If preferences are described by functional form 8, the Nash tariffs, $\{t_i^*\}$, and their effect on wages, $\{\hat{w}_i\}$, and total income, $\{\hat{Y}_i\}$, can be solved as a solution to the following system:*

$$\begin{cases} 1 + t_i^* = \frac{1 + \sum_{j \neq i} \sum_k \left[\hat{\chi}_{ij,k} \chi_{ij,k} \epsilon_k (1 - \hat{\lambda}_{ij,k} \lambda_{ji,k}) \right]}{\sum_{j \neq i} \sum_k \left[\hat{\chi}_{ij,k} \chi_{ij,k} \epsilon_k (1 - \hat{\lambda}_{ij,k} \lambda_{ij,k}) \right]} \\ \hat{\chi}_{ij,k} \chi_{ij,k} = \frac{\hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / (1 + t_j^*)}{\sum_{n \neq i} \hat{\lambda}_{in,k} \lambda_{in,k} \beta_{n,k} \hat{Y}_n Y_n / (1 + t_n^*)} \\ \hat{\lambda}_{ji,k} = \left(\frac{1 + t_i^*}{1 + \bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \hat{\tilde{P}}_{i,k}^{\epsilon_k} \\ \hat{\tilde{P}}_{i,k}^{-\epsilon_k} = \sum_j \left[\left(\frac{1 + t_i^*}{1 + \bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \lambda_{ji,k} \right] \\ \hat{w}_i w_i L_i = \sum_k \sum_j \left[\hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / \left(1 + t_j^*\right) \right] \\ \hat{Y}_i Y_i = \hat{w}_i w_i L_i + \sum_k \sum_j \left(\frac{t_i^*}{1 + t_i^*} \hat{\lambda}_{ji,k} \lambda_{ji,k} \beta_{i,k} \hat{Y}_i Y_i \right) \end{cases},$$

which depends on structural industry-level trade elasticities, $\{\epsilon_k\}$; as well as three sets of observables: (i) applied tariffs, $\bar{t}_{ji,k}$, (ii) expenditure shares, $\lambda_{ji,k}$ and $\beta_{i,k}$, and (iii)

total expenditure and wage income, Y_i and $w_i L_i$.

Let me briefly elaborate on the significance of Proposition 2. The system specified by the above proposition involves $3N$ independent equations and unknowns: N Nash tariff rates, $\{t_i^*\}$; N wage changes, $\{\hat{w}_i\}$; and N income changes, $\{\hat{Y}_i\}$. Solving this system requires a set of sufficient statistics that are either observable or estimable. Namely, (a) data on observable applied tariffs, expenditure shares and total income in each country (namely, $\bar{t}_{ji,k}$, $\lambda_{ji,k}$, $\beta_{i,k}$, and Y_i); as well as (b) trade elasticities, ϵ_k.[12]

It is useful to compare the procedure outlined by Proposition 2 to the standard approach, whereby Nash tariffs are solved using an iterative global optimization procedure. Recall that in the standard approach, each iteration alone performs N constrained global optimizations over $2N + (N-1)K$ state variables. Using Proposition 2 we can not only bypass the need to iterate but also the need to perform a full-blown global optimization. In fact, we only need to solve a system of $3N$ equations and unknowns only once.

The solution to the system specified by Proposition 2 immediately pins down the prospective cost of a tariff war for each country i as

$$\%\Delta \text{Real GDP}_i = \hat{Y}_i \cdot \prod_k \left(\hat{\tilde{P}}_{i,k}^{-\beta_{i,k}} \right).$$

In the following subsections, I discuss how the above methodology easily extends to richer frameworks that accommodate political pressures, profit-shifting effects, and intermediate input trade. Later, in Section 3, I use Proposition 2 and these subsequent propositions to quantify the cost of a global tariff war.

2.1 Accounting for Market Distortions and Political Pressures

In the Ricardian model, the market equilibrium is efficient and Nash tariffs only internalize the terms-of-trade gains resulting from improving one's wage relative to the rest of the world. Ideally, we should also account for market distortions, which give rise to profit-shifting motives behind tariff imposition and

[12] Note that with data on $\bar{t}_{ji,k}$, $\lambda_{ji,k}$, $\beta_{i,k}$, and Y_i, we can immediately pin down $w_i L_i$ as

$$w_i L_i = \sum_k \sum_j \left[\lambda_{ij,k} \beta_{j,k} Y_j / (1 + \bar{t}_{ij,k}) \right].$$

political economy pressures. To introduce these two channels, I consider a generalized multi-industry Krugman (1980) model with restricted entry that nests Ossa (2014) as a special case. In this extension, firms enjoy market power and collect profits. As a result, tariffs can induce a profit-shifting externality that was absent in the baseline model. Moreover, as in Grossman and Helpman (1995), governments can assign different weights to profits collected in different industries in response to political pressures. For the sake of exposition, though, I start with the case where governments assign the same political weight to all industries. Subsequently, I show how introducing political pressures modifies the baseline results.

The model that follows extends the Ricardian model in two key dimensions. First, on the demand side, each composite country-level variety aggregates over differentiated firm-level varieties indexed by ω,

$$Q_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} q_{ji,k}(\omega)^{\varrho_k} d\omega \right)^{1/\varrho_k},$$

where $\varrho_k > 1$, while $\Omega_{j,k}$ denotes the set of firms serving industry k from country j. Noting the above specification, the Ricardian model can be viewed as a special case of the generalized Krugman model where $\varrho_k \to 1$.

Second, on the supply side, industry k in country j hosts a fixed number of firms, $\bar{M}_{j,k}$, that compete under monopolistic competition and charge a constant markup, $1 + \mu_k \equiv \rho_k$, over marginal cost. Each firm employs labor as the sole factor of production, with $a_{ji,k}(\omega)$ denoting the constant unit labor cost of production and transportation for goods sold by firm ω in Market i. Note that since firms incur no fixed marketing costs, the heterogeneity in $a_{ji,k}(\omega)$'s is inconsequential to my analysis.[13]

Combining these features, the "producer" price index of composite variety ji,k can be expressed as a function the labor wage rate in country j, w_j; the average unit labor cost of production and transportation, $\bar{a}_{ji,k}$;[14] the number of firms located in country j, $\bar{M}_{j,k}$; and the constant markup wedge, μ_k. In particular,

$$P_{ji,k} = (1 + \mu_k)\bar{a}_{ji,k}\bar{M}_{j,k}^{-\mu_k} w_j.$$

[13] As I will discuss later in Section 2.3, the present framework is isomorphic to one where $a_{ji,k}(\omega)$'s have a Pareto distribution and the fixed marketing costs is paid in terms of labor in the destination country.

[14] Stated formally, $\bar{a}_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} a_{ji,k}(\omega)^{\varrho_k/(\varrho_k - 1)} d\omega \right)^{(\varrho_k - 1)/\varrho_k}$.

Correspondingly, the "consumer" price index is given by $\tilde{P}_{ji,k} = (1 + t_{ji,k})P_{ji,k}$. Also, as in the Ricardian model, the pass-through of tariffs on to consumer prices is complete, once we net out general equilibrium wage effects.[15]

Equilibrium in the generalized Krugman model has a similar definition as the Ricardian model, except that total income in each country equals the wage bill, $w_i L_i$, plus total profits, $\Pi_i = \sum_k \sum_j (\mu_k/1+\mu_k) P_{ij,k} Q_{ij,k}$, and tariff revenues:

$$Y_i = w_i L_i + \Pi_i + \sum_j \sum_k t_{ji,k} P_{ji,k} Q_{ji,k}.$$

In the above setup, country i's tariffs have two distinctive effects on welfare. First, as in the Ricardian model, tariffs can alter country i's wage relative to the rest of the world. Second, tariffs can increase country i's profits, Π_i, by restricting imports and promoting domestic output in high-markup (high-μ) industries. Both of these effects also inflict a negative externality on the rest of the world. Despite this added layer of complexity, the Nash tariffs can still be analytically characterized in terms of reduced-form demand elasticities and observable shares, as outlined by the following proposition.[16]

Proposition 3. *Country i's optimal import tariff can be solved alongside a uniform shifter, τ_i^*, using the following system:*

$$\left[\frac{1+\bar{\tau}_i^*}{1+t_{ji,k}^*}\right]_{j\neq i} = \boldsymbol{\mathcal{E}}_{i,k}^{-1}\left(\mathbf{1}_{(N-1)\times 1} + \left[\frac{\lambda_{ii,k}\varepsilon_{ii,k}^{ji,k}}{\lambda_{ji,k}(1+\mu_k)}\right]_{j\neq i}\right), \quad \forall k$$

$$1+\bar{\tau}_i^* = \frac{\sum_{j\neq i}\sum_k \left[\chi_{ij,k}\varepsilon_{ij,k} - (t_{ji,k}^* - \bar{\tau}_i^*)\chi'_{ji,k}\varepsilon_{ji,k}^{ii,k}\right]}{1+\sum_{j\neq i}\sum_k \chi_{ij,k}\varepsilon_{ij,k}},$$

which features only (i) reduced-form demand elasticities that are partially contained in

[15] Under free entry, the pass-through of tariffs on to consumer prices will no longer be complete, at least for an excessively large economy. Lashkaripour and Lugovskyy (2019) formally analyze optimal tariffs in such a case. If country i is, however, sufficiently small relative to the rest of the world, the passthrough would remain complete even under free entry.

[16] As before (and in the absence of political pressures) optimal non-cooperative tariffs maximize welfare given applied tariffs in the rest of the world. In particular,

$$\boldsymbol{t}_i^*(\boldsymbol{t}_{-i}) = \arg\max \; V_i\left(Y_i(\boldsymbol{t}_i;\boldsymbol{t}_{-i}), \tilde{\boldsymbol{P}}_i(\boldsymbol{t}_i;\boldsymbol{t}_{-i})\right).$$

Also, it should be noted that the formula specified by Proposition 3 assumes zero cross-substitutability between industries, which is the relevant case for my quantitative analysis. But as shown in Appendix A.2, the formula easily extends to cases where there are arbitrary patterns of cross-substitutability between industries.

the $(N-1)\times(N-1)$ *matrix,* $\boldsymbol{\mathcal{E}}_{i,k} \equiv \left[\varepsilon_{ji,k}^{ji,k}\right]_{j,j\neq i}$*; (ii) constant markup wedges,* μ_k*; as well as* (iii) *observable export and import revenue shares,* $\chi_{ij,k} = / \sum_{\ell\neq i}\sum_k X_{i\ell,k}$ *and* $\chi'_{ji,k} = X_{ji,k}/\sum_{\ell\neq i}\sum_k X_{\ell i,k}$.

As with the baseline model, the above proposition can be used to measure the cost of a tariff war provided that we impose additional structure on preferences. Specifically, assuming that preferences have a Cobb-Douglas-CES parameterization (as in Equation 8), Proposition 3 implies that country i's Nash tariff is uniform across exporters and given by[17]

$$1+t_{i,k}^{*} = \left[\frac{\sum_{j\neq i}\sum_g X_{ij,g}\left[1+\epsilon_g\left(1-\lambda_{ij,g}\right)\right]}{\sum_{j\neq i}\sum_k\left[X_{ij,g}\epsilon_g\left(1-\lambda_{ij,g}\right)+\mathcal{X}_{ji,k}\epsilon_k\lambda_{ii,s}\right]}\right]\frac{\left(1+\mu_k\right)\left(1+\epsilon_k\lambda_{ii,k}\right)}{1+\mu_k+\epsilon_k\lambda_{ii,k}}. \tag{11}$$

where $\mathcal{X}_{ji,k} \equiv \sum_g\left[\frac{\mu_g\epsilon_g\lambda_{ii,g}}{1+\mu_g+\epsilon_g\lambda_{ii,g}}\right]X_{ji,k}$. To provide a brief intuition, the first term in the bracket reflects the uniform optimal markup over w_i, which is applied to all exported goods. This term was also present in the Ricardian model. The second term, which is industry-specific, reflects country i's incentive to protect and promote high-profit (high-μ) industries. This second term imposes a profit-shifting externality on the rest of the world that was absent in the baseline Ricardian model.[18]

Importantly, when all countries simultaneously protect their high-μ industries, global output in these industries shrinks below its already sub-optimal level. As a result, a full-fledged tariff war exacerbates misallocation in the global economy in a way that was absent in the baseline model. Later, when I map the model to data, it will become apparent that the cost of exacerbated misallocation is comparable to pure of cost of trade reduction in the event of a full-fledged tariff war.

Moving forward, we can appeal to Equation 11 in order to compute the Nash tariffs and the welfare cost associated with them in one simple step as a function of only observable shares and structural elasticities. The following proposition formally outlines this point.

[17] In the above equation, the uniform term is stated in terms of export levels (namely, $X_{ji,k}$) instead of export shares. Nonetheless, by dividing the numerator and denominator of the uniform term by $\sum_{j\neq i}\sum_k X_{ji,k} = \sum_{j\neq i}\sum_k X_{ij,k}$ we can express the same equation in terms of shares as in Proposition 3.

[18] As noted in Campolmi et al. (2018) and Lashkaripour and Lugovskyy (2019), the industry-specific term is an artifact of governments not having access to domestic subsidies. As a result, they resort to tariffs as a second-best policy for enhancing allocative efficiency in their local economy.

Proposition 4. *If preferences are described by functional form 8, the Nash tariffs, $\{t^*_{i,k}\}$, and their effect on wages, $\{\hat{w}_i\}$, and total income, $\{\hat{Y}_i\}$, can be solved as a solution to the following system:*

$$\begin{cases} 1+t^*_{i,k} = (1+\bar{\tau}^*_i)\left[\frac{1+\mu_k-\epsilon_k\hat{\lambda}_{ii,k}\lambda_{ii,k}}{(1+\mu_k)(1-\epsilon_k\hat{\lambda}_{ii,k}\lambda_{ii,k})}\right] \\ 1+\bar{\tau}^*_i = \frac{\sum_{j\neq i}\sum_k \hat{X}_{ij,k}X_{ij,k}\left[1+\epsilon_k(1-\hat{\lambda}_{ij,k}\lambda_{ji,k})\right]}{\sum_{j\neq i}\sum_k\left[\hat{X}_{ij,k}X_{ij,k}\epsilon_k(1-\hat{\lambda}_{ij,k}\lambda_{ij,k})+\hat{\mathcal{X}}_{ji,k}\mathcal{X}_{ji,k}\epsilon_k\hat{\lambda}_{ii,s}\lambda_{ii,s}\right]} \\ \hat{X}_{ij,k}X_{ij,k} = \hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{j,k}) \\ \hat{\mathcal{X}}_{ji,k}\mathcal{X}_{ji,k} = \sum_g\left(\frac{\mu_g\epsilon_g\hat{\lambda}_{ii,g}\lambda_{ii,g}}{1+\mu_g+\epsilon_g\hat{\lambda}_{ii,g}\lambda_{ii,g}}\right)\hat{X}_{ji,k}X_{ji,k} \\ \hat{\lambda}_{ji,k} = \left(\frac{1+t^*_{i,k}}{1+\bar{t}_{ji,k}}\hat{w}_j\right)^{-\epsilon_k}\hat{\tilde{P}}^{\epsilon_k}_{i,k} \\ \hat{\tilde{P}}^{-\epsilon_k}_{i,k} = \sum_j\left[\left(\frac{1+t^*_{i,k}}{1+\bar{t}_{ji,k}}\hat{w}_j\right)^{-\epsilon_k}\lambda_{ji,k}\right] \\ \hat{w}_iw_iL_i = \sum_k\sum_j\left[\hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{j,k})(1+\mu_k)\right] \\ \hat{\Pi}_i\Pi_i = \sum_k\sum_j\left[\mu_k\hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{j,k})(1+\mu_k)\right] \\ \hat{Y}_iY_i = \hat{w}_iw_iL_i+\hat{\Pi}_i\Pi_i+\sum_k\sum_j\left(\frac{t^*_{i,k}}{1+t^*_{i,k}}\hat{\lambda}_{ji,k}\lambda_{ji,k}\beta_{i,k}\hat{Y}_iY_i\right) \end{cases},$$

which depends on industry-level trade elasticities, $\{\epsilon_k\}$, and markup wedges, $\{\mu_k\}$; as well as three sets of observables: (i) applied tariffs, $\bar{t}_{ji,k}$, (ii) expenditure shares, $\lambda_{ji,k}$ and $\beta_{i,k}$, and (iii) total expenditure and wage income, Y_i and w_iL_i.

Compared to the Ricardian model, the above system involves $N(K+2)$ unknowns, namely, NK Nash tariff rates, $\{t_{i,k}\}$; N wage changes, $\{\hat{w}_i\}$; and N income changes, $\{\hat{Y}_i\}$. Also, in addition to data on $\bar{t}_{ji,k}$, $\lambda_{ji,k}$, $\beta_{i,k}$, and Y_i; and estimates for ϵ_k, we need estimates for industry-level markup wedge, μ_k, in order to solve the above system.[19] Once the system is solved, the solution immediately pins down the prospective cost of a tariff war for each country as

$$\%\Delta\text{Real GDP}_i = \hat{Y}_i\cdot\prod_k\left(\hat{\tilde{P}}^{-\beta_{i,k}}_{i,k}\right).$$

Introducing Political Pressures. To introduce political pressures, I follow Ossa's (2014) adaptation of Grossman and Helpman (1995). His approach

[19]Note that with data on $\bar{t}_{ji,k}$, $\lambda_{ji,k}$, $\beta_{i,k}$, and Y_i, we can immediately pin down w_iL_i and Π_i as

$$\begin{cases} w_iL_i = \sum_k\sum_j\left[\lambda_{ij,k}\beta_{j,k}Y_j/(1+\bar{t}_{ij,k})(1+\mu_k)\right] \\ \Pi_i = \sum_k\sum_j\left[\lambda_{ij,k}\beta_{j,k}Y_j/(1+\bar{t}_{ij,k})(1+\mu_k)\right] \end{cases}.$$

builds on the fact that under the Cobb-Douglas-CES utility, social welfare in Country i can be expressed as $W_i \equiv V_i(.) = \sum_{k,j}\left(X_{ij,k}/\tilde{P}_i\right)$, where $\tilde{P}_i = \prod_k\left(\sum \tilde{P}_{ji,k}^{-\epsilon_k}\right)^{-\beta_{i,k}/\epsilon_k}$ is the aggregate consumer price index. Instead of the government in Country i maximizing the plain social welfare, he assumes that it maximizes a politically-weighted welfare function:

$$W_i = \sum_{k,j}\theta_{i,k}\frac{X_{ij,k}}{\tilde{P}_i},$$

where $\theta_{i,k}$ is the political economy weight assigned to industry k, with the weights normalized such that $\sum_k\left(\theta_{i,k}\right)/K \;=1$. As shown in Appendix D, Propositions 3 and 4 characterize the Nash tariffs and their effects in this setup with no further qualification except that μ_k in all the formulas be replaced with

$$\tilde{\mu}_{i,k} = \frac{\theta_{i,k}\mu_k}{1+\left(1-\theta_{i,k}\right)\mu_k}.$$

Considering this, when mapping the model to data, accounting for political pressures involves the extra step of determining the political weights, $\theta_{i,k}$.

At this point, it can be useful to discuss how political pleasures may moderate or magnify the cost of a tariff war. If political pressures favor high-μ industries, then the Nash tariffs will be targeted even more intensively towards high-μ industries. In that case, politically-motivated Nash tariffs will drag the global economy further away from its efficiency frontier, making the tariff war more costly than implied by the baseline (non-political) model. Conversely, if political pressures favor low-μ industries, Nash tariffs will be less distortionary and the cost of a tariff war will be lower than implied by the baseline model.

2.2 Accounting for Intermediate Input Trade

Now, I consider an extended version of the baseline Ricardian model that features input-output (IO) linkages with tariffs that are subject to "duty drawbacks." In the interest of convenience, I hereafter refer to this model as the *IO model*. The drawback condition in the IO model corresponds to tariffs being applied on imported goods net of their re-exported content. Duty drawbacks are currently prevalent in many countries. More importantly, they are typically adopted voluntarily by governments. In the US, for instance, duty drawbacks have been an integral part of the tariff scheme since 1789. So, it is safe to assume that if a tariff war breaks out, governments will maintain the voluntarily-adopted duty drawbacks, which essentially serve as an export subsidy.

To present the IO model, let me temporarily abstract from tariffs. Here, production in each country combines labor and intermediate input varieties sourced from various international suppliers using a Cobb-Douglas aggregator. Assuming that the *final* and *intermediate* composite varieties are the same, the price index of composite variety ji,k can, thus, be expressed as,

$$P_{ji,k} = \bar{a}_{ji,k} w_j^{\gamma_{j,k}} \prod_{\ell,g} P_{\ell j,g}^{\alpha_{j,k}(\ell,g)}, \tag{12}$$

where $\gamma_{j,k} = 1 - \sum_{\ell,g} \alpha_{j,k}(\ell,g)$, with $\alpha_{j,k}(\ell,g)$ denoting the constant share of *country ℓ–industry g* inputs in the production of *country j–industry k* output. It is straightforward to verify that (from a welfare point of view) the IO model is isomorphic to a reformulated model where (i) instead of intermediate inputs crossing the borders, production employs labor from various locations, and (ii) only final goods (denoted by $\mathcal{F}$) are traded across borders. In this *reformulated IO model*, the price index of a final good variety ji,k can be expressed as

$$P_{ji,k}^{\mathcal{F}} = \tilde{a}_{ji,k} \prod_{\ell} w_\ell^{\tilde{\gamma}_{j,k}(\ell)}, \tag{13}$$

where $\tilde{a}_{ji,k}$ is composed of the constant unit labor costs (namely, the $\bar{a}_{ji,k}$'s), which are invariant to policy. $\tilde{\gamma}_{j,k}(\ell)$, meanwhile, denotes the share country ℓ's labor in the production of *country j–industry k's* final good. The full $NK \times K$ matrix of labor shares, $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_{j,k}(\ell)]_{jk,\ell}$, can be easily derived in terms of the IO shares as follows,[20]

$$\tilde{\boldsymbol{\gamma}} = (\boldsymbol{I}_{NK} - \boldsymbol{\alpha})^{-1} \boldsymbol{\gamma} \boldsymbol{I}_K \tag{14}$$

where $\boldsymbol{\alpha} \equiv [\alpha_{j,k}(\ell,g)]_{jk,\ell g}$ is the $NK \times NK$ global IO matrix; while $\boldsymbol{\gamma} = [\gamma_{j,k}]_{j,k}$ is a $NK \times 1$ vector. Let me provide a brief intuition behind the price formulation specified by Equation 13. There are two equivalent ways to interpret variety ji,k's production process. One where production employs intermediate inputs produced with labor from various countries, indexed by ℓ. Another, where production directly employs labor from various countries, indexed ℓ. Equation 13 corresponds to this latter interpretation. It is also straightforward to check that $\sum_\ell \tilde{\gamma}_{j,k}(\ell) = 1$ for all j and k.

Now, suppose tariffs are applied with duty drawbacks. The drawback

[20] Equation 14 can be obtained by applying the Implicit Function Theorem to Equation 12.

scheme ensures that tariffs do not propagate due to input-output linkages. Or put differently, tariffs with drawbacks are akin to a tariff applied on the traded final goods in the *reformulated* IO model. Accordingly, in the reformulated IO model, the consumer price index of the traded final goods can be expressed as

$$\tilde{P}^{\mathcal{F}}_{ji,k} = (1 + t_{ji,k})\tilde{a}_{ji,k} \prod_{\ell} w_{\ell}^{\tilde{\gamma}_{j,k}(\ell)}. \tag{15}$$

Equilibrium in the reformulated IO model also assumes a definition that is analogous to that of the baseline Ricardian model. That is, for any given vector of tariffs, $\boldsymbol{t} = \{t_{ji,k}\}$, equilibrium is a vector of wages, $\boldsymbol{w} = \{w_i\}$; a vector of "producer" and "consumer" final good price indexes, $\boldsymbol{P}^{\mathcal{F}}_i = \{P^{\mathcal{F}}_{ji,k}\}$ and $\tilde{\boldsymbol{P}}^{\mathcal{F}}_i = \{\tilde{P}^{\mathcal{F}}_{ji,k}\}$, specified by Equations 13 and 15; and consumption quantities, $\boldsymbol{Q}^{\mathcal{F}}_i$, given by $Q^{\mathcal{F}}_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}^{\mathcal{F}}_i)$, where $\mathcal{D}_{ji,k}(.)$ is implied by the utility-maximization Problem 1 subject to total income equaling

$$Y_i = w_i L_i + \sum_j \sum_k t_{ji,k} P^{\mathcal{F}}_{ji,k} Q^{\mathcal{F}}_{ji,k}.$$

Before moving forward, let me summarize the *reformulated* IO model one last time. Production in each economy employs labor from various locations to produce traded final goods, indexed by $\mathcal{F}$. Trade in the final good is subject to regular tariffs. In terms of welfare implications, the reformulated IO model is isomorphic to our original IO model where production employs local labor plus intermediate inputs, but with tariffs applied subject to duty drawbacks. Note that if tariffs were not subjected to drawbacks, they will multiply due to IO linkages and the original and reformulated IO models will no longer be isomorphic.

In the above setup, we can first show that the optimal tariff is again uniform and a function of observable revenue shares, reduced form demand elasticities, and IO shares. The following proposition formally outlines this claim.

Proposition 5. *Country i's optimal import tariff is uniform and can be characterized as*

$$1 + t^*_i = \frac{\sum_{j \neq i} \sum_k \sum_g \phi_{ij,k} \varepsilon^{ij,g}_{ij,k}}{1 + \sum_{j \neq i} \sum_k \sum_g \phi_{ij,k} \varepsilon^{ij,g}_{ij,k}},$$

in terms of only (i) reduced-form demand elasticities, and (ii) observable "value-added" export shares, $\phi_{ij,k} = \tilde{\gamma}_{i,k}(i) X^{\mathcal{F}}_{ij,k} / \sum_{j \neq i} \sum_g \tilde{\gamma}_{i,g}(i) X^{\mathcal{F}}_{ij,g}$, where $\tilde{\gamma}_{i,k}(i)$'s given by the IO matrix per Equation 14.

The intuition behind the uniformity of tariffs is that due duty drawbacks, Country i's tariffs can only influence the terms-of-trade through their effect on the vector of economy-wide wages, $\boldsymbol{w}$.[21] Hence, to a first-order approximation, the effect of country i's tariff on $W_i(.)$ is driven by a change in w_i relative to wages in the rest of the world.

The key distinction between the IO model and the baseline Ricardian model is that Nash tariff levels vary with country i's dependence on imported intermediate inputs. Specifically, strong dependence on imported intermediates, which is reflected in a low $\tilde{\gamma}_{i,k}(i)$, leads to less export market power and lower Nash tariffs. I will elaborate more on this issue on Section 3, where I fit the model to actual data.

As before, using Proposition 5 and imposing further structure on preferences, we can solve the Nash tariffs and the corresponding losses in one simple step. More importantly, in doing so, we only need information on observable export revenue and IO shares, as well as industry-level trade elasticities. The following proposition outlines this result.

Proposition 6. *If preferences are described by functional form 8, the Nash tariffs, $\{t_i^*\}$, and their effect on wages, $\{\hat{w}_i\}$, and total income, $\{\hat{Y}_i\}$, can be solved as a solution to the following system:*

$$\begin{cases} 1+t_i^* = \frac{1+\sum_{j\neq i}\sum_k\left[\hat{\phi}_{ij,k}\phi_{ij,k}\epsilon_k(1-\hat{\lambda}^{\mathcal{F}}_{ij,k}\lambda^{\mathcal{F}}_{ij,k})\right]}{\sum_{j\neq i}\sum_k\left[\hat{\phi}_{ij,k}\phi_{ij,k}\epsilon_k(1-\hat{\lambda}^{\mathcal{F}}_{ij,k}\lambda^{\mathcal{F}}_{ij,k})\right]} \\ \hat{\phi}_{ij,k}\phi_{ij,k} = \frac{\tilde{\gamma}_{i,k}(i)\hat{\lambda}^{\mathcal{F}}_{ij,k}\lambda^{\mathcal{F}}_{ij,k}\beta^{\mathcal{F}}_{j,k}\hat{Y}_jY_j/(1+t_j^*)}{\sum_{n\neq i}\sum_k\tilde{\gamma}_{i,k}(i)\hat{\lambda}^{\mathcal{F}}_{in,k}\lambda^{\mathcal{F}}_{in,k}\beta^{\mathcal{F}}_{n,k}\hat{Y}_nY_n/(1+t_n^*)} \\ \hat{\lambda}^{\mathcal{F}}_{ji,k} = \left[\frac{1+t_i^*}{1+\bar{t}_{ji,k}}\prod_\ell \hat{w}_\ell^{\gamma_{j,k}(\ell)}\right]^{-\epsilon_k}\left(\hat{P}^{\mathcal{F}}_{i,k}\right)^{\epsilon_k} \\ \hat{P}^{\mathcal{F}}_{i,k} = \sum_j\left(\left[\frac{1+t_i^*}{1+\bar{t}_{ji,k}}\prod_\ell \hat{w}_\ell^{\gamma_{j,k}(\ell)}\right]^{-\epsilon_k}\lambda^{\mathcal{F}}_{ji,k}\right)^{-1/\epsilon_k} \\ \hat{w}_iw_iL_i = \sum_k\sum_j\left[\hat{\lambda}^{\mathcal{F}}_{ij,k}\lambda^{\mathcal{F}}_{ij,k}\beta^{\mathcal{F}}_{j,k}\hat{Y}_jY_j/\left(1+t_j^*\right)\right] \\ \hat{Y}_iY_i = \hat{w}_iw_iL_i + \sum_k\sum_j\left(\frac{t_i^*}{1+t_i^*}\hat{\lambda}^{\mathcal{F}}_{ji,k}\lambda^{\mathcal{F}}_{ji,k}\beta^{\mathcal{F}}_{i,k}\hat{Y}_iY_i\right) \end{cases},$$

which depends on structural industry-level trade elasticities, $\{\epsilon_k\}$; as well as four sets of observables: (i) applied tariffs, $\bar{t}_{ji,k}$, (ii) final good expenditure shares $\lambda^{\mathcal{F}}_{ji,k}$ and $\beta^{\mathcal{F}}_{i,k}$, (iii) total final good expenditure and wage income, Y_i and w_iL_i, and (iv) IO shares, $\alpha_{j,k}(\ell,g)$.

[21] Without duty drawbacks tariffs can propagate through input-output linkages. This propagation effect gives countries *extraterritorial* taxing power, which can lead to non-unifrom optimal tariffs—see Beshkar and Lashkaripour (2019).

The system specified above involves the same set of unknowns as the baseline Ricardian model. However, solving it requires data on "final" good trade and expenditure, $\lambda^{\mathcal{F}}_{ji,k}$, $\beta^{\mathcal{F}}_{i,k}$, and Y_i; as well as data on the IO table, $\boldsymbol{\alpha}$, with the latter determining the $\tilde{\gamma}_{i,k}(i)$'s through Equation 14.[22] Once we solve the above system, the cost of a tariff war can be immediately pinned down as $\%\Delta\text{Real GDP}_i = \hat{Y}_i \cdot \prod_k \left(\hat{\tilde{P}}^{\mathcal{F}}_{i,k}\right)^{-\beta_{i,k}}$.

2.3 Discussion

To take stock, I presented a new methodology to compute the cost of a full-fledged tariff war in one simple step as function of (i) observable trade values, (ii) applied tariffs,[23] (iii) input-output shares, (iv) industry-level trade elasticities, and (v) and industry-level markup wedges. As a novel byproduct, my theory also highlighted that the cost of a full-fledged tariff war is driven by

i. pure trade reduction, the effect of which depends on a country's position in the global value chain, and

ii. the exacerbation of pre-existing market distortions, due to the governments' tendency to use targeted Nash tariffs.

Now, some readers may share Krugman's (1997) skepticism that governments do not necessarily set Nash tariffs with the objective to non-cooperatively maximize national welfare. This type of skepticism, however, does not pose a problem for the present methodology. Instead, the methodology is flexible enough to accommodate arbitrary preferences towards protection. For instance, if we believe that governments arbitrarily assign a higher weight to the agricultural sector, the present methodology can easily account for that.

That being said, let me discuss a few possible concerns with the above methodology. Some of these concerns are easy to address, but some others are more consequential and actually apply to the broader literature on this topic.

A first concern is my assumption on restricted entry. This assumption was adopted in line with Ossa (2014), with the justification that it makes the model amenable to the introduction of political pressures. But what happens if we

[22] Y_i in this setup has a slightly different interpretation than national expenditure. More specifically, it denotes total spending on only final goods, which is still a readily observable variable. Moreover, solving the system specified by Proposition 6 requires information on total wage income, $w_i L_i$, which can be uniquely inferred from $\lambda^{\mathcal{F}}_{ji,k}$, $\beta^{\mathcal{F}}_{i,k}$, Y_i, and $\tilde{\gamma}_{i,k}(i)$.

[23] Note that the full matrix of trade values and applied tariffs fully determines Y_i and $w_i L_i$ in each country, and also the expenditure shares, $\lambda_{ji,k}$ and $\beta_{i,k}$.

replace the *restricted entry* assumption with *free entry*? It is easy to verify that the optimal tariff formulas will remain intact. But the predicted losses from a tariff war can be quite different, and presumably larger under free entry–see Lashkaripour and Lugovskyy (2019) for a similar discussion but in the context of unilateral trade taxes.

A second concern is my abstraction from firm-selection effects. This concern is misplaced if we believe that the firm-level productivity distribution is Pareto and that the fixed marketing cost is paid in terms of labor in the destination country. In this *very* particular case, the heterogeneous firm model with selection effects becomes isomorphic to the generalized Krugman model introduced in Section 2.1.[24] Beyond this particular case, the concern is not easy to address. Mostly, because producing analytic formulas for Nash tariffs becomes increasingly difficult under arbitrary selection effects.[25]

A third and perhaps more serious concern, is that my analysis overlooks dynamic adjustment costs. This concern applies to a broader literature that employs static trade models when analyzing tariff wars. For instance, by imposing balanced trade, my analysis inevitably overlooks the losses from adjustments to the trade balance. Recently, several papers in the international macroeconomic literature, including Balistreri et al. (2018), Barattieri et al. (2018), and Bellora and Fontagné (2019), have used dynamic models to quantify these adjustments costs. The general consensus arising from these studies is that dynamic adjustment costs are non-trivial.

3 Quantitative Implementation

In this section, I employ Propositions 2, 4, and 6 to compute the prospective cost of a tariff war for 43 major economies and to study how this prospective cost has evolved over time. To solve the system specified by Propositions 2 and 4, I need data on the full matrix of industry-level bilateral trade values, $X_{ji,k} \equiv P_{ji,k}Q_{ji,k}$ and applied tariffs, $\bar{t}_{ji,k}$. Knowing these values, I can determine total expenditure, $Y_i = \sum_j \sum_k X_{ji,k}$; wage revenues, $w_i L_i = \sum_j \sum_k X_{ij,k}/(1+\bar{t}_{ij,k})$; as well as expenditure shares, $\beta_{i,k} = \sum_j \left(X_{ji,k}\right)/Y_i$, and $\lambda_{ji,k} = X_{ji,k}/\beta_{i,k}Y_i$.[26] To

[24] Kucheryavyy et al. (2016) establish this isomorphism under free entry. But the same isomorphism argument applies readily to the case of restricted entry.

[25] Costinot et al. (2016) have taken a notable step in this direction, by characterizing the optimal micro-level policy in a two-country model with general firm-selection effects.

[26] In the case of Proposition 4, we also need information on markup wedges to determine wage revenues: $w_i L_i = \sum_j \sum_k X_{ij,k}/(1+\bar{t}_{ij,k})(1+\mu_k)$.

solve the system specified by Proposition 6, I also need data on "final" good trade and the global IO matrix, $\boldsymbol{\alpha}$. Below, I describe how the needed data is collected from different sources.

Data on Trade Values and IO Shares. Data on bilateral trade values are taken from the 2016 release of the World Input-Output Database (WIOD, see Timmer et al. (2012)). The dataset spans years 2000 to 2014, covering 43 countries (plus an aggregate of the rest of the world) and 56 industries. The 43 countries featured in the WIOD are listed in the first column of Table 2. Following Costinot and Rodríguez-Clare (2014), I group the industries into 16 industrial categories, assuming that industries belonging to the same category are governed by the same trade elasticity parameter—the details of this categorization plus the list of industries is provided in Table 4 of Appendix C.

Solving the system specified by Propositions 6 requires two additional data points. First, I need the full matrix of "final good" trade values, $\{X^{\mathcal{F}}_{ji,k}\}$. This information is also readily available in each version of the WIOD. Second, I need data on international IO shares in order to construct the labor share matrix, $\tilde{\gamma}$, using Equation 14. For each country, the WIOD reports IO shares at the industry-level. With this information, I can construct the variety-level IO shares, $\alpha_{j,k}(\ell, g)$, as the variety-level import share, $\lambda_{ji,k}$, times the reported industry-level input share. Subsequently, the total wage bill and final good expenditure in Country i can be respectively determined as $w_i L_i = \sum_j \sum_n \sum_k \gamma_{j,k}(i) X^{\mathcal{F}}_{jn,k}$ and $Y_i = \sum_i \sum_k X^{\mathcal{F}}_{ji,k}$. With information on Y_i, I can also compute the final good expenditure shares as $\beta^{\mathcal{F}}_{i,k} = \sum_j (X^{\mathcal{F}}_{ji,k})/Y_i$ and $\lambda^{\mathcal{F}}_{ji,k} = X^{\mathcal{F}}_{ji,k}/\beta^{\mathcal{F}}_{i,k} Y_i$.

Importantly, to make the WIOD data compatible with theory, I also need to purge it from trade imbalances. To elaborate, Propositions 2, 4, and 6 implicitly assume that trade is balanced. So, applying these propositions to imbalanced trade data would identify the sum of the (i) actual cost of a tariff war, plus (ii) the cost associated with balancing global trade. In order to compute the pure cost of a tariff war, I purge the data from trade imbalances, closely following the methodology in Dekle et al. (2007).

Data on Applied Tariffs. To evaluate Propositions 2, 4, and 6, I also need information on applied tariffs for each of the countries and industries in the WIOD sample. For this purpose, I use data on applied tariffs from the United Nations Statistical Division, Trade Analysis and Information System

(UNCTAD-TRAINS). The UNCTAD-TRAINS for 2014 covers 31 two-digit (in ISIC rev.3) sectors, 185 importers, and 243 export partners. In line with Caliendo and Parro (2015), I use the *simple tariff line average* of the *effectively applied tariff* (AHS) to measure each of the $\bar{t}_{ji,k}$'s. When tariff data are missing in a given year, I use tariff data for the nearest available year, giving priority to earlier years. To aggregate the UNCTAD-TRAINS data into individual WIOD industries, I closely follow the methodology outlined in Kucheryavyy et al. (2016). Finally, I have to deal with the fact that individual European Union (EU) member countries are not represented in the UNCTAD-TRAINS data during the 2000-2014 period. To deal with this issue, I rely on the fact that the EU itself is featured as a reporter; and the fact that intra-EU trade is subject to zero tariffs while all EU members impose a common external tariff on non-members.

Industry-Level Trade Elasticities. I estimate the industry-level trade elasticities, $\{\epsilon_k\}$, using data on aggregate trade flows, $\{X_{ji,k}\}$, and applied tariff rates, $\bar{t}_{ji,k}$. To this end, I choose 2014 as the baseline year and employ the triple-difference methodology developed by Caliendo and Parro (2015) to estimate a trade elasticity for each of the WIOD industry categories in my analysis. Further details regarding the estimation procedure are provided in Appendix C. The estimated trade elasticities are also reported in Table 4 of the same appendix.[27]

In the case of the generalized Krugman model, I need mutually-consistent estimates for the constant industry-level markup wedges and the trade elasticities. Attaining such estimates requires micro-level data, and is not possible with the macro-level data reported by the WIOD. Considering this, I borrow the estimated μ_k and ϵ_k's from Lashkaripour and Lugovskyy (2019) for each of the WIOD industries in my analysis. These adopted values are reported in Table 5 of Appendix C. To maintain transparency, I also assume equal political economy weights for all industries, which is motivated by Ossa's (2016) point that *"average optimal tariffs and their average welfare effects are quite similar with and without political economy pressures."* The reason behind this apparent insignificance is that *"political economy pressures are more about the intranational rather than the international redistribution of rents."*[28]

[27] I normalize the trade elasticity for the service sector to 10, which is in between the two normalizations proposed by Costinot and Rodríguez-Clare (2014).

[28] As noted earlier, there are specific cases where political economy pressures magnify the efficiency loss resulting from a tariff war. One example is when governments assign higher political economy weights to high-profit (high-μ) industries, which leads to more distortionary

3.1 The Cost of a Tariff War Across Different Nations

Table 2 reports (i) the computed Nash tariff levels, as well as (ii) the per-cent loss in real GDP as a result of the tariff war for various countries and under various modeling assumptions. Recall that in the baseline Ricardian model, tariffs are targeted solely at improving a country's wage relative to the rest of the world. The Nash tariffs are, as a result, uniform and stand around 40% for the average economy. The cross-national variation in Nash tariffs, here, is driven primarily by the average trade elasticity underlying a country's exports. For instance, the Nash tariffs are significantly lower in Australia, Norway, and Russia who export predominantly in high-ϵ (or primary) industries.

From the perspective of the baseline Ricardian model, the average country loses 2.5% of its real GDP in the event of a tariff war. These losses are driven by pure trade reduction. Moreover, even though the losses are quite heterogeneous, all countries lose without exception. The losses are more pronounced for (i) smaller economies as well as (ii) economies with a relatively low export market power (i.e, with a relatively high export share in high-ϵ industries).

Once we account for market distortions, Nash tariffs are longer non-uniform and include two components: a *terms-of-trade-driven* component as well as a *profit-shifting-driven* component. The Nash tariffs are, as a result, higher, averaging around 44% across all countries and industries. The predicted losses from a tariff war are also magnified, averaging around 2.9% of the real GDP.

As noted earlier, there is a simple intuition for why the presence of market distortions amplifies the cost of a tariff war. In the presence of market distortions, a tariff war inflicts two types of inefficiency on the global economy: (i) an efficiency loss that is driven purely by trade reduction, and (ii) an efficiency loss due to the exacerbation of exiting market distortions. Let re-elaborate on this latter effect: Output in high-markup industries is already sub-optimal prior to the tariff war. As the tariff war escalates, countries impose tariffs that are more-targeted towards high-markup industries, thereby lowering global output in these industries and dragging the global economy further away from its efficiency frontier. Now, all countries lose from these developments; but economies like Korea and Taiwan that are net exporters in high-markup industries experience the greatest efficiency loss.[29]

Nash tariffs.

[29] It should be noted that using tariffs as a profit-shifting device is an artifact of domestic taxes being unavailable to the governments—see Lashkaripour and Lugovskyy (2019) for a more detailed discussion.

IO linkages too, magnify the Nash tariffs and the corresponding losses. Moreover, once we account for IO linkages, the implied losses are significantly more heterogeneous across countries. Somewhat surprisingly, some countries like Brazil, Norway, and Australia even gain –though modestly– from a tariff war. These gains, however, come at an immense cost to other economies like Malta, Slovakia, or Romania. More surprisingly, these supposed winners are not the largest economies by any account. Instead, they are economies that are positioned further upstream in the global value chain. On the flip side, the major losers are also small, downstream economies that depend heavily on imported intermediates. I will elaborate more on these patterns in Subsection 3.3.

Before going forward, let me briefly address an outstanding question: *How believable are these numbers?* To get a "rough" answer, we can contrast the present numbers with those following the only documented full-fledged tariff war in history. Namely, the tariff war triggered by the Smoot-Hawley Tariff Act of 1930. The tariffs that were imposed during this documented tariff war averaged around 50%, a number strikingly close to the numbers reported in Table 2.[30] Despite this stark resemblance, one should still keep in mind that the models considered here overlook many relevant cost channels. So, the present results should be interpreted with great caution nonetheless.

3.2 The Cost of a Tariff War Over Time

A key advantage of the approach developed here is its remarkable computational speed. To give some perspective, the baseline results reported in Table 2, were computed in 3-4 seconds on my personal laptop. Building on this advantage, I employ my method to compute the cost of a full-fledged tariff war under different modeling specifications and for many years, so far as data availability permits—that would be from 2000 to 2014 in the case of the WIOD data.

Figure 1 displays the final results. For every year, the cost of a tariff war to the global economy is calculated as the change in real global GDP. To calculate this change, I use yearly data on real GDP from the Penn World Tables. I multiply and add the per-cent loss in GDP for each country by its real GDP level in that year. This task is performed using not only the baseline Ricardian model but also the extended models that allow for pre-existing market distortions and IO linkages.

[30] See Bagwell and Staiger (2004) for more details regarding the tariff war that followed the Smoot-Hawley Tariff Act.

Table 2: *The welfare cost of a tariff war*

	Baseline Model		Model w/ Distortions		Model w/ IO Linkages	
Country	Nash Tariff	%Δ Real GDP	Nash Tariff	%Δ Real GDP	Nash Tariff	%Δ Real GDP
AUS	13.6%	-1.45%	31.3%	-1.59%	84.6%	-0.35%
AUT	45.7%	-2.95%	38.4%	-3.74%	56.8%	-1.49%
BEL	56.6%	-3.56%	32.9%	-4.78%	75.1%	-2.16%
BGR	38.1%	-2.57%	34.6%	-2.73%	39.0%	-4.70%
BRA	103.5%	-0.39%	39.9%	-0.83%	86.8%	0.04%
CAN	22.9%	-2.74%	33.9%	-2.66%	49.7%	-2.97%
CHE	51.9%	-2.22%	28.8%	-3.02%	67.7%	0.01%
CHN	41.0%	-0.24%	42.1%	-0.42%	42.7%	-0.22%
CYP	12.6%	-3.74%	22.5%	-3.74%	17.6%	-7.72%
CZE	49.4%	-3.05%	44.0%	-3.80%	54.3%	-3.17%
DEU	59.1%	-1.09%	51.5%	-0.95%	66.1%	0.71%
DNK	56.6%	-2.39%	28.5%	-3.84%	80.9%	-1.07%
ESP	61.2%	-1.64%	42.7%	-1.64%	77.8%	-0.82%
EST	28.6%	-4.39%	25.6%	-6.59%	42.0%	-4.86%
FIN	31.5%	-1.90%	47.4%	-1.98%	38.4%	-1.65%
FRA	52.2%	-1.84%	34.7%	-2.12%	65.0%	-1.32%
GBR	27.9%	-2.14%	31.8%	-1.52%	46.6%	-2.60%
GRC	12.5%	-2.88%	31.1%	-2.33%	15.1%	-5.79%
HRV	37.0%	-3.15%	29.2%	-3.74%	59.2%	-2.90%
HUN	52.6%	-4.36%	38.6%	-5.84%	57.1%	-2.17%
IDN	51.7%	-0.90%	39.8%	-1.43%	45.6%	0.15%
IND	47.5%	-0.53%	39.2%	-0.48%	50.5%	-0.02%
IRL	117.3%	-1.40%	24.8%	-6.60%	97.3%	-0.76%
ITA	49.6%	-0.89%	51.3%	-0.87%	57.5%	-0.05%
JPN	44.3%	-0.85%	44.1%	-1.04%	46.4%	-0.73%
KOR	43.6%	-1.29%	42.1%	-1.97%	46.7%	0.34%
LTU	31.6%	-4.51%	32.8%	-5.35%	64.4%	-2.41%
LUX	12.0%	-6.32%	19.9%	-6.03%	15.0%	-20.95%
LVA	26.0%	-3.33%	25.8%	-4.18%	47.6%	-6.66%
MEX	38.6%	-2.41%	38.8%	-2.52%	54.5%	-1.24%
MLT	12.4%	-5.68%	23.4%	-5.15%	18.2%	-18.79%
NLD	37.3%	-4.51%	27.9%	-5.32%	77.6%	1.46%
NOR	17.1%	-2.23%	34.5%	-2.24%	91.7%	2.46%
POL	46.1%	-2.83%	37.4%	-2.80%	59.4%	-2.54%
PRT	35.0%	-3.01%	35.6%	-2.44%	49.7%	-3.15%
ROU	32.7%	-2.70%	32.9%	-2.23%	40.2%	-3.67%
RUS	12.1%	-2.24%	33.4%	-1.96%	32.5%	-1.37%
SVK	41.7%	-4.69%	39.7%	-4.42%	46.1%	-4.78%
SVN	46.1%	-3.39%	36.5%	-4.11%	56.0%	-2.85%
SWE	38.4%	-2.06%	38.4%	-2.26%	54.3%	-0.18%
TUR	46.7%	-1.51%	43.7%	-1.81%	56.3%	-0.86%
TWN	35.4%	-2.60%	32.0%	-4.06%	35.7%	1.66%
USA	39.2%	-0.87%	36.8%	-0.64%	45.9%	-1.10%
Average	40.6%	-2.52%	35.4%	-2.94%	53.3%	-2.63%

Evidently, the prospective cost of a tariff war has risen rather dramatically from 2000 to 2014. Especially so, if we account for the global input-output structure and the exacerbation of market distortions by a tariff war. To provide numbers, if we account for exacerbation of market distortions, the prospective cost has nearly doubled from $707 billion in 2000 to around $1,395 billion in 2014. If we account for IO linkages, the prospective cost has more-than-doubled from $624 billion to $1,542 billion.

These numbers raise a basic question: *Why has the prospective cost of a tariff war risen so much?* The rise is driven by three independent factors:

i. The increased openness of small economies to foreign trade. The importance of this factor is evident from the fact the even from the perspective of the baseline Ricardian model, the cost of a tariff war has multiplied over time.

ii. The increased specialization of small, developing countries in high-μ industries. In light of this development, these countries will apply *more-targeted* Nash tariffs in the event of a tariff war. The more targeted the Nash tariffs, the more they exacerbate pre-existing market distortions. This factor perhaps explains the divergence between the losses predicted with and without accounting for market distortions.

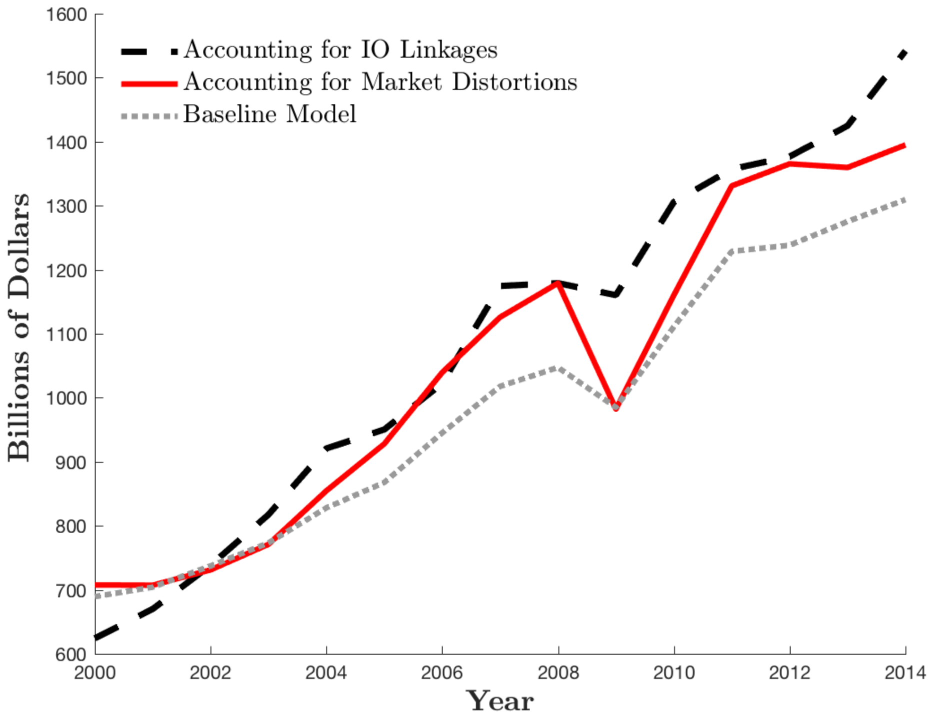
iii. The increased dependence of individual economies on the global value chain. Again, this factor explains why the prospective loss predicted by the model with IO linkages has risen rather dramatically relative to the cost predicted by the baseline model.

In any case, the present analysis indicates that given the current state of the global economy, the prospective cost of a full-fledged tariff war is immense. Take for instance the year 2014, where the prospective cost of a tariff war was $1,542 billion once we account for the global IO structure. Such a loss is the equivalent of erasing South Korea from the global economy.

3.3 Position in the Global Value Chain

The present analysis provides a novel glimpse into how global value chains have exposed some countries more than ever to a tariff war. To make this point formally, let me fix ideas by using the baseline Ricardian model as a conceptual benchmark. In this baseline, a country's market power is driven by its

Figure 1: *The prospective cost of a tariff war over time*



monopoly over differentiated varieties produced with local labor. Now, add global value chains into the mix. In that case, local labor will account for a smaller fraction of a country's differentiated output the more it specializes in downstream industries. Downstream-ness will, therefore, diminish a country's market power relative to the rest of the world. The intuition is that a downstream economy's tariffs have a relatively small effect on its terms-of-trade, as measured by its wage relative to the rest of the world. On the flip side, the market power of upstream economies will be multiplied by global value chains.

To demonstrate this point from the lens of the calibrated model, Figure 2 plots the national-level cost of a full-fledged tariff war against national-level dependence on intermediate inputs. The dependence index (assigned to the x-axis) is measured as one minus a trade-weighted average of $\gamma_{i,k}(i)$'s. Roughly speaking, this index tells us what percentage of a country's output is comprised of local labor content.

It is evident from Figure 2 that small downstream economies like Malta and Luxembourg, which depend more heavily on imported intermediates, experience the greatest losses from a tariff war. This outcome is aligned with my above assertion that IO linkages diminish market power for downstream economies. By contrast, a country like Norway that exports predominantly in upstream industries (like crude oil) can even gain from a full-fledged tariff war due to its upstream position in the global value chain.

On a broader level, the above arguments qualify an old belief that large countries can win a tariff war, whereas small countries always lose (Johnson (1953)). My analysis indicates that a country's degree of "upstream-ness" in the global value chain is as important of a factor as its size. consider again the case of Norway, which gains around 2.5% in the event of a tariff war once we account for IO linkages. By every account, Norway is a small economy. However, it exports primarily in upstream industries like Oil. Based on Johnson's (1953) theory, Norway should lose from a tariff war; and the baseline Ricardian model, which neglects global value chains, confirms this view. But this prediction is overturned, as soon as we account for the global IO structure.

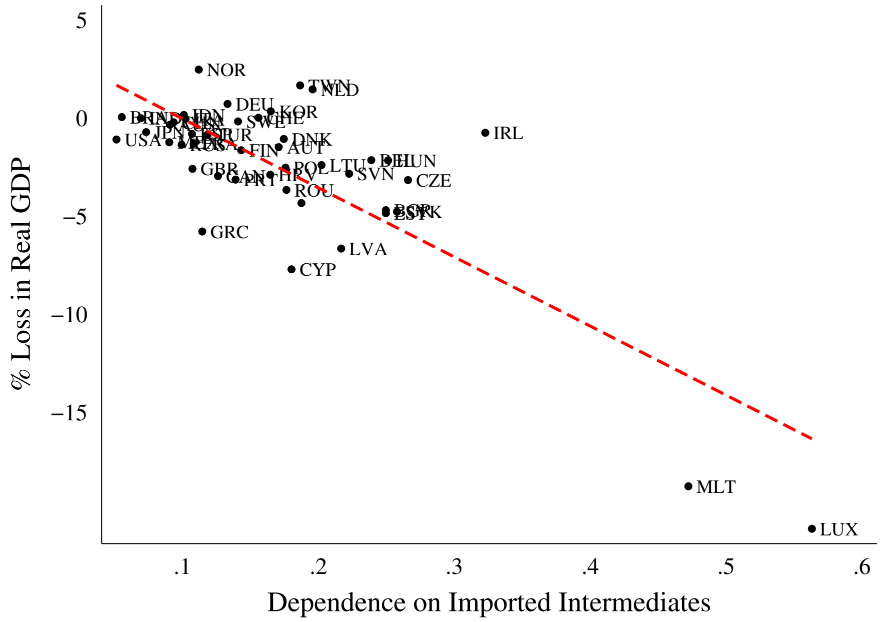
It should be noted once again that these results hinge on countries providing duty drawbacks in the event of a tariff war. As noted earlier, duty drawbacks are voluntarily adopted by many countries, and there is no reason they will be disposed of if a tariff war escalates.[31] Nonetheless, absent duty drawbacks, a country's market power also depends on its *extraterritorial taxing power* (see Beshkar and Lashkaripour (2019)). That is, a small downstream economy that re-exports a significant fraction of its imports can levy taxes on goods that are produced and eventually consumed outside its borders. As a result, it can raise revenue from foreign entities while imposing minimal distortion on its local economy. Duty drawbacks render extraterritorial taxing power obsolete, but so does retaliation. So, even without duty drawbacks, downstream economies will still experience immense losses, but the gains experienced by upstream economies will probably also diminish.

3.4 The Cost of a US-China Tariff War on the Global Economy

The recent tariff face-off between the US and China has been the source of revived academic interest in tariff wars. Currently, a feared scenario is one where the US and China engage in a two-way tariff war without necessarily raising tariffs on other, non-involved trading partners. The methodology developed here can be equally applied to compute the cost associated with such a two-

[31] From a pure social welfare perspective, duty drawbacks act as an export subsidy that harms a country's terms-of-trade. But, their voluntary adoption can be easily rationalized in two ways: (a) they are motivated by political economy considerations, or (ii) due to administrative constraints, they are adopted as a *second-best* substitute for domestic subsidies.

Figure 2: *Cost of a tariff war vs. Dependence on imported intermediates*



way tariff war.

In Appendix B, I show that an analog of Propositions 2, 4, and 6 can be produced for the case where *only* two (or any subset of) countries engage in a two-way tariff war. Using these analog propositions, we can compute the cost of a two-way tariff war in one step, with knowledge of observable as well as the structural markup wedges and trade elasticities (μ_k and ϵ_k).

I apply this method to analyze a two-way tariff war involving the US and China, using data from 2014. The results indicate that the two-way Nash tariffs adopted by China and the US are slightly lower than those adopted in the event of a full-fledged global tariff war. Perhaps encouragingly, the baseline Ricardian model predicts that the US would impose a 25% China-specific Nash tariff—a number that is remarkably close to what the US authorities have been pointing to in light of their recent face-off with China. The model that accounts for market distortions, however, predicts higher bilateral Nash tariffs that are closer to 50%.

My analysis indicates that a US-China tariff war would shave $34 billion off global GDP, which is the equivalent of Paraguay's economy.[32] The US-China

[32]These are numbers implied by the generalized Krugman model. The losses implied by the Ricardian and IO models are somewhat smaller. In the generalized Krugman model targeted (non-uniform) tariffs inflict an additional efficiency loss to the global economy, by lowering output in high-profit industries below its already sub-optimal level.

Table 3: *The main winners and Losers from a US-China tariff war*

Main Losers		Main Winners	
Country	ΔReal GDP (millions of dollars)	Country	ΔReal GDP (millions of dollars)
United States	-\$24,680	Mexico	\$2,028
China	-\$15,774	India	\$678
Australia	-\$58	Japan	\$581
Ireland	-\$26	Canada	\$460

tariff war also creates winners and losers. Table 3 lists the countries that experience the largest negative effects as well as those that experience the largest positive effects. Expectedly the US and China are the main losers, respectively losing \$25 and \$16 billion worth of real GDP.

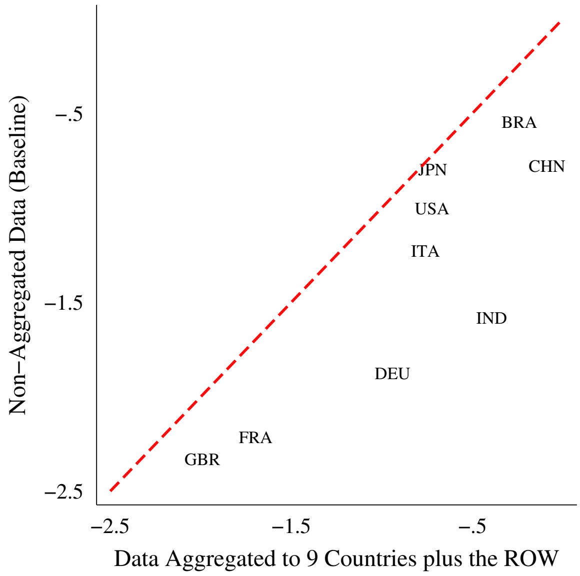
These numbers resonate with those estimated by Amiti et al. (2019), who use the "ex-post" approach described in the Introduction. Specifically, they estimate a \$16.8 billion loss for the US economy, implied by already-applied US tariff rates ranging between 10% and 50% and retaliatory Chinese tariffs averaging 16%. The \$25 billion loss predicted here corresponds to a full-fledged version of the US tariff war, which involves Nash tariffs averaging around 45% for the US and 52% for China.

An interesting finding here is that many countries can lose from a US-China tariff war, even without being directly involved. Australia and Ireland, for instance, lose from trade destruction and diversion. Specifically, as total income in the US and China drops, these two economies lose part of their international demand. Moreover, some markets divert their imports from Australia and Ireland to the US and China to benefit from the reduced wages in the latter two economies. On the flip side, Mexico, Canada, India, and Japan experience significant gains from trade diversion. Simply, because the two-way tariff war induces the US and China to divert imports from each other to the likes of Mexico, India, Canada, and Japan.

3.5 Data Aggregation Distorts the Estimated Cost

As noted in the Introduction, existing analyses of tariff wars often restrict their attention to a limited sample of countries. This is done by aggregating smaller countries into a single taxing authority that is labeled the rest of the wold (ROW). Also, recall that this aggregation scheme is adopted to overcome the

Figure 3: *% Loss from a Tariff War: with and without aggregation*



computational complexities inherent to tariff war analysis[33].

Capitalizing on the computational efficiency of my sufficient statistics approach, I can test if such aggregation schemes pose a problem. To this end, I re-do my analysis with aggregated data, which is restricted to Brazil, China, Germany, Great Britain, France, Italy, India, Japan, and the United States. The remaining 34 countries (in the aggregated data) are lumped with the ROW and treated as one taxing authority.

Table 3 compares the welfare losses computed using the *non-aggregated* sample to those computed using the *aggregated* sample. Evidently, aggregating the data overstates the cost of a tariff war. There is a simple intuition behind this outcome. Aggregating many countries into the ROW, gives the ROW an artificially high degree of market power. As a result, the ROW imposes artificially high Nash tariffs that inflict a large welfare loss on other (non-aggregated) economies. By adopting the sufficient statistics approach developed here, researchers can avoid such data aggregation and the bias that accompanies it.

[33]See Ossa (2016) for an overview of this literature. To give specific examples, Perroni and Whalley (2000) and Ossa (2014) aggregate the data into 6 economies and an aggregate of the ROW. Note, however, that they aggregate EU member countries into one taxing authority and the ROW only includes non-EU countries.

4 Concluding Remarks

Building on recent advances in quantitative trade theory, I developed a simple, sufficient statistics methodology to compute the prospective cost of a full-fledged global tariff war. My proposed methodology has two basic advantages. First, by relying on analytic formulas, it is incredibly fast, delivering a more than 100-fold increase in computational speed relative to alternatives. Second, it can account for input-output linkages, which are usually a missing ingredient in existing *ex-ante* analyses of tariff wars.

Applying my methodology to data across many countries, industries, and years, and by accounting for the global input-structure and market distortions, my analysis shed fresh light on the consequences of a full-fledged global tariff war. Among other results, I showed that (i) a significant fraction of the cost associated with a full-fledged tariff war is due to the exacerbation of already-existing market distortions; (ii) the prospective cost of a tariff war to the global economy has more-than-doubled over the past 15 years; (iii) that small downstream economies are the main losers; and (iv) that countries can incur significant losses from a US-China tariff war even if they are not directly involved.

Moving forward, a natural next step is to apply the proposed methodology to an even broader set of countries and industries using richer, confidential data. Previously, such applications were partially impeded by computational burden; but practitioners can employ the present methodology to circumvent this particular obstacle. Another avenue for future research is to extend the methodology, itself, by incorporating multiple factors of production and other short-run adjustment costs.

References

Amiti, M., S. J. Redding, and D. Weinstein (2019). The impact of the 2018 trade war on us prices and welfare. Technical report, National Bureau of Economic Research.

Arkolakis, C., A. Costinot, D. Donaldson, and A. Rodríguez-Clare (2015). The elusive pro-competitive effects of trade. Technical report, National Bureau of Economic Research.

Arkolakis, C., A. Costinot, and A. Rodriguez-Clare (2012). New trade models, same old gains? *American Economic Review 102*(1), 94–130.

Bagwell, K. and R. W. Staiger (2004). *The economics of the world trading system*. MIT press.

Balistreri, E. J., C. Böhringer, and T. Rutherford (2018). Quantifying disruptive trade policies.

Balistreri, E. J. and R. Hillberry (2018). 21st century trade wars.

Barattieri, A., M. Cacciatore, and F. Ghironi (2018). Protectionism and the business cycle. Technical report, National Bureau of Economic Research.

Bellora, C. and L. Fontagné (2019). Shooting oneself in the foot? trade war and global value chains.

Beshkar, M. and A. Lashkaripour (2019). Interdependence of Trade Policies in General Equilibrium.

Caliendo, L., R. C. Feenstra, J. Romalis, and A. M. Taylor (2015). Tariff Reductions, Entry, and Welfare: Theory and Evidence for the Last Two Decades. Technical report, National Bureau of Economic Research.

Caliendo, L. and F. Parro (2015). Estimates of the Trade and Welfare Effects of NAFTA. *Review of Economic Studies 82*(1), 1–44.

Campolmi, A., H. Fadinger, and C. Forlati (2018). Trade and domestic policies in models with monopolistic competition.

Chetty, R. (2009). Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annu. Rev. Econ. 1*(1), 451–488.

Costinot, A., D. Donaldson, J. Vogel, and I. Werning (2015). Comparative Advantage and Optimal Trade Policy. *The Quarterly Journal of Economics 130*(2), 659–702.

Costinot, A. and A. Rodríguez-Clare (2014). Trade Theory with Numbers: Quantifying the Consequences of Globalization. *Handbook of International Economics 4*, 197.

Costinot, A., A. Rodríguez-Clare, and I. Werning (2016). Micro to Macro: Optimal Trade Policy with Firm Heterogeneity. Technical report, National Bureau of Economic Research.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced Trade. Technical report, National Bureau of Economic Research.

Eaton, J. and S. Kortum (2002). Technology, Geography, and Trade. *Econometrica 70*(5), 1741–1779.

Fajgelbaum, P. D., P. K. Goldberg, P. J. Kennedy, and A. K. Khandelwal (2019). The return to protectionism. Technical report, National Bureau of Economic Research.

Felbermayr, G., B. Jung, and M. Larch (2013). Optimal tariffs, retaliation, and the welfare loss from tariff wars in the melitz model. *Journal of International Economics 89*(1), 13–25.

Gros, D. (1987). A note on the optimal tariff, retaliation and the welfare loss from tariff wars in a framework with intra-industry trade. *Journal of International Economics 23*(3-4), 357–367.

Grossman, G. M. and E. Helpman (1995). Trade wars and trade talks. *Journal of political Economy 103*(4), 675–708.

Johnson, H. G. (1953). Optimum tariffs and retaliation. *Review of Economic Studies 21*(2), 142–153.

Kennan, J. and R. Riezman (2013). Do big countries win tariff wars? In *International Trade Agreements and Political Economy*, pp. 45–51. World Scientific.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review 70*(5), 950–959.

Krugman, P. (1997). What should trade negotiators negotiate about? *Journal of Economic Literature 35*(1), 113–120.

Kucheryavyy, K., G. Lyn, and A. Rodríguez-Clare (2016). Grounded by Gravity: A Well-Behaved Trade Model with Industry-Level Economies of Scale. NBER Working Paper 22484.

Lashkaripour, A. and V. Lugovskyy (2019). Scale economies and the structure of trade and industrial policy.

Opp, M. M. (2010). Tariff wars in the ricardian model with a continuum of goods. *Journal of International Economics 80*(2), 212–225.

Ossa, R. (2014). Trade Wars and Trade Talks with Data. *The American Economic Review 104*(12), 4104–46.

Ossa, R. (2016). Quantitative Models of Commercial Policy. In *Handbook of Commercial Policy*, Volume 1, pp. 207–259. Elsevier.

Perroni, C. and J. Whalley (2000). The new regionalism: trade liberalization or insurance? *Canadian Journal of Economics/Revue canadienne d'économique 33*(1), 1–24.

Timmer, M., A. A. Erumban, R. Gouma, B. Los, U. Temurshoev, G. J. de Vries, I.-a. Arto, V. A. A. Genty, F. Neuwahl, J. Francois, et al. (2012). The World Input-Output Database (WIOD): Contents, Sources and Methods. Technical report, Institute for International and Development Economics.

A Proofs and Derivations

In the proofs that follow, I will express all equilibrium outcomes as a function of feasible tariff-wage combinations, i.e., $(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})$. For instance, welfare in Country i can be expressed as $W_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})$, where $\boldsymbol{w}$ is the vector of equilibrium wages that implicitly depend on the applied tariffs. This combination is feasible in that the tariff and wage rates satisfy the equilibrium conditions outlined in Section 2.

A.1 Proof of Proposition 1

The first order condition for a tariff on imported variety ji, k can be expressed as follows

$$\begin{aligned}\frac{\mathrm{d}\ln W_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\mathrm{d}\ln(1+t_{ji,k})} =& \frac{\partial \ln V(.)}{\partial \ln Y_i}\frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1+t_{ji,k})} + \frac{\partial \ln V(.)}{\partial \ln \tilde{P}_{ji,k}}\frac{\partial \ln \tilde{P}_{ji,k}(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1+t_{ji,k})} \\ &+ \sum_{\ell}\left[\frac{\partial \ln V(.)}{\partial \ln Y_i}\frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln w_\ell}\frac{\mathrm{d}\ln w_\ell}{\mathrm{d}\ln(1+t_{ji,k})}\right] \\ &+ \sum_{\ell}\sum_{g}\left[\frac{\partial \ln V(.)}{\partial \ln \tilde{P}_{\ell i,g}}\frac{\partial \ln \tilde{P}_{\ell i,g}(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln w_\ell}\frac{\mathrm{d}\ln w_\ell}{\mathrm{d}\ln(1+t_{ji,k})}\right] = 0. \end{aligned} \tag{16}$$

The first term in the F.O.C. accounts for the effect of tariffs on income through tax revenues. Noting that (i) $Y_i = w_i L_i + \sum_j \sum_k t_{ji,k} P_{ji,k} Q_{ji,k}$, and (ii) $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}_i)$, this term can be expressed as

$$\frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1+t_{ji,k})} = Y_i^{-1}\left\{\tilde{P}_{ji,k}Q_{ji,k} + \sum_g \sum_j \left(t_{ji,g}P_{ji,g}Q_{ji,g}\left[\frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln \tilde{P}_{ji,k}}\frac{\partial \ln \tilde{P}_{ji,k}(.)}{\partial \ln(1+t_{ji,k})} + \frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln Y_i}\frac{\partial \ln Y_i(.)}{\partial \ln(1+t_{ji,k})}\right]\right)\right\} \tag{17}$$

The second term in the F.O.C. accounts for the effect of tariffs on consumer prices. Given Roy's identity, $\frac{\partial V(.)/\partial P_{ji,k}}{\partial V(.)/Y_i} = -Q_{ji,k}$, and the complete passthrough of tariffs on to consumer prices, $\partial \ln \tilde{P}_{ji,k}(.)/\partial \ln(1+t_{ji,k}) = 1$, this term can be expressed as

$$\frac{\partial \ln V(.)}{\partial \ln \tilde{P}_{ji,k}}\frac{\partial \ln \tilde{P}_{ji,k}(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1+t_{ji,k})} = -\frac{\partial \ln V(.)}{\partial \ln Y_i}\frac{\tilde{P}_{ji,k}Q_{ji,k}}{Y_i}. \tag{18}$$

I, hereafter, assign w_j as the numeraire and use the fact that, in Equation 16, $\frac{\partial W_i}{\partial \ln w_\ell}\frac{\mathrm{d}\ln w_\ell}{\mathrm{d}\ln(1+t_{ji,k})} \approx 0$ for all $\ell \neq i$. That is, to a first-order approximation, the effect of $t_{ji,k}$ on W_i is driven by a change in w_i relative to wages in the rest of the world (see Appendix A.4). Considering this; using our earlier definition for the demand elasticity, $\varepsilon_{ji,k} \equiv \partial \ln \mathcal{D}_{ji,k}(.)/\partial \ln \tilde{P}_{ji,k}$; letting $\eta_{ji,k} \equiv \partial \ln \mathcal{D}_{ji,k}(.)/\partial \ln Y_i$ denotes the income elasticity; noting that $\lambda_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}/Y_i$; and plugging Equations 17 and 18 back into 16, implies the following optimality condition:

$$\lambda_{ji,k}+\frac{t_{ji,k}}{1+t_{ji,k}}\lambda_{ji,k}\varepsilon_{ji,k}+\sum_{J}\sum_{g}\left[\frac{t_{Ji,g}}{1+t_{Ji,g}}\lambda_{Ji,g}\eta_{Ji,g}\right]\frac{\partial\ln Y_i(.)}{\partial\ln(1+t_{ji,k})}$$
$$-\lambda_{ji,k}+\left(\frac{\partial\ln Y_i(.)}{\partial\ln w_i}+\sum_{g}\frac{\partial\ln V(.)/\partial\ln\tilde{P}_{ii,g}}{\partial\ln V(.)/\partial\ln Y_i}\frac{\partial\ln\tilde{P}_{ii,g}(.)}{\partial\ln w_i}\right)\frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln(1+t_{ji,k})}=0$$

To evaluate the above condition, we can apply the Implicit Function Theorem to the balanced trade condition, $\sum_{j\neq i}\sum_k P_{ji,k}Q_{ji,k}=\sum_{j\neq i}\sum_k P_{ij,k}Q_{ij,k}$, to characterize $d\ln w_i/d\ln(1+t_{ji,k})$ as follows:

$$\begin{aligned}\frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln(1+t_{ji,k})}&=-\frac{\frac{\partial}{\partial\ln(1+t_{ji,k})}\left[\sum_j\sum_g P_{ji,g}Q_{ji,g}-\sum_j\sum_g P_{ij,g}Q_{ij,g}\right]}{\frac{\partial}{\partial\ln w_i}\left[\sum_j\sum_g P_{ji,g}Q_{ji,g}-\sum_j\sum_g P_{ij,g}Q_{ij,g}\right]}\\
&=-\frac{\sum_J\sum_g\left(P_{Ji,g}Q_{Ji,g}\left[\frac{\partial\ln\mathcal{D}_{Ji,g}(.)}{\partial\ln\tilde{P}_{ji,k}}\frac{\partial\ln\tilde{P}_{ji,k}(.)}{\partial\ln(1+t_{ji,k})}+\frac{\partial\ln\mathcal{D}_{Ji,g}(.)}{\partial\ln Y_i}\frac{\partial\ln Y_i(.)}{\partial\ln(1+t_{ji,k})}\right]\right)}{\frac{\partial}{\partial\ln w_i}\left[\sum_j\sum_g P_{ji,g}Q_{ji,g}-\sum_j\sum_g P_{ij,g}Q_{ij,g}\right]}\\
&=-\frac{\sum_J\sum_g\left(\frac{\lambda_{Ji,g}}{1+t_{Ji,g}}\left[\varepsilon^{ji,k}_{Ji,g}+\eta_{Ji,g}\frac{\partial\ln Y_i(.)}{\partial\ln(1+t_{ji,k})}\right]\right)Y_i}{\frac{\partial}{\partial\ln w_i}\left[\sum_j\sum_g P_{ji,g}Q_{ji,g}-\sum_j\sum_g P_{ij,g}Q_{ij,g}\right]}.\end{aligned}$$

Defining,

$$\bar{\tau}_i\equiv\frac{\frac{\partial Y_i(.)}{\partial\ln w_i}+\sum_g\frac{\partial V(.)/\partial\tilde{P}_{ii,g}}{\partial V(.)/\partial Y_i}\frac{\partial\tilde{P}_{ii,g}(.)}{\partial\ln w_i}}{\frac{\partial}{\partial\ln w_i}\left[\sum_j\sum_g P_{ji,g}Q_{ji,g}-\sum_j\sum_g P_{ij,g}Q_{ij,g}\right]},\tag{19}$$

The optimality condition specified by Equation 16, reduces to the following:

$$\sum_J\sum_g\left((1-\frac{1+\bar{\tau}_i}{1+t_{Ji,g}})\lambda_{Ji,g}\left[\varepsilon^{ji,k}_{Ji,g}+\eta_{Ji,g}\frac{\partial\ln Y_i(.)}{\partial\ln(1+t_{ji,k})}\right]\right)=0.$$

Stated in the matrix form, Country i's optimal tariffs thus solve the following system,

$$\boldsymbol{\Omega}_i\left[1-\frac{1+\bar{\tau}_i}{1+t_{Ji,g}}\right]_{j,k}=\mathbf{0},$$

where $\boldsymbol{\Omega}_i=\left[\lambda_{Ji,g}(\varepsilon^{ji,k}_{Ji,g}+\eta_{Ji,g}\frac{\partial\ln Y_i(.)}{\partial\ln(1+t_{ji,k})})\right]_{Jg,jk}$ is a $NK\times NK$ matrix. Provided that $\boldsymbol{\Omega}_i$ is non-singular, the unique solution to the above system is the trivial solution,

$$1+t^*_{ji,k}=1+\bar{\tau}_i,\quad\forall ji,k.$$

So, to determine the optimal tariff we simply need to characterize $\bar{\tau}_i$, supposing that $t^*_{ji,k}=\bar{\tau}_i$. Using my choice of notation for trade values ($X_{ji,k}=P_{ji,k}q_{ji,k}$), the aforementioned step yields the following:

$$\bar{\tau}_i \equiv \frac{\frac{\partial Y_i(.)}{\partial \ln w_i} + \sum_g \frac{\partial V(.)/\partial \tilde{P}_{ii,g}}{\partial V(.)/\partial Y_i} \frac{\partial \tilde{P}_{ii,g}(.)}{\partial \ln w_i}}{\frac{\partial}{\partial \ln w_i} \left[\sum_{j \neq i} \sum_g P_{ji,g} Q_{ji,g} - \sum_{j \neq i} \sum_g P_{ij,g} Q_{ij,g} \right]}$$

$$= \frac{w_i L_i + \sum_{j \neq i} \sum_g \left[\bar{\tau}_i X_{ji,g} \sum_s \left(\frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln \tilde{P}_{ii,s}} \frac{\partial \ln \tilde{P}_{ii,s}(.)}{\partial \ln w_i} \right) \right] - \sum_g X_{ii,g}}{\sum_{j \neq i} \sum_g \left[X_{ji,g} \sum_s \left(\frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln \tilde{P}_{ii,s}} \frac{\partial \ln \tilde{P}_{ii,s}(.)}{\partial \ln w_i} \right) \right] - \sum_{j \neq i} \sum_g \left[X_{ij,g} \left[\frac{\partial \ln \tilde{P}_{ij,g}(.)}{\partial \ln w_i} + \sum_s \left(\frac{\partial \ln \mathcal{D}_{ij,g}(.)}{\partial \ln \tilde{P}_{ij,s}} \frac{\partial \ln \tilde{P}_{ij,s}(.)}{\partial \ln w_i} \right) \right] \right]}$$

$$= \frac{\sum_{j \neq i} \sum_g X_{ij,g}}{- \sum_{j \neq i} \sum_g X_{ij,g} \left[1 + \sum_s \varepsilon_{ij,g}^{ij,s} \right]} = \frac{\sum_j \sum_g \sum_s \chi_{ij,g} \varepsilon_{ij,g}^{ij,s}}{1 + \sum_j \sum_g \sum_s \chi_{ij,g} \varepsilon_{ij,g}^{ij,s}} - 1$$

where the second line follows from Roy's identity that $\frac{\partial V(.)/\partial P_{ii,g}}{\partial V(.)/\partial Y_i} = -Q_{ii,k}$, and $\chi_{ij,g} \equiv X_{ij,g} / \sum_{j \neq i} \sum_g X_{ij,g}$.

A.2 Proof of Proposition 3

The first order condition for a tariff on imported variety ji, k can be expressed as follows

$$\frac{\mathrm{d} \ln W_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\mathrm{d} \ln(1 + t_{ji,k})} = \frac{\partial \ln V(.)}{\partial \ln Y_i} \frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1 + t_{ji,k})} + \frac{\partial \ln V(.)}{\partial \ln \tilde{P}_{ji,k}} \frac{\partial \ln \tilde{P}_{ji,k}(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1 + t_{ji,k})}$$
$$+ \sum_{\ell} \left[\frac{\partial \ln V(.)}{\partial \ln Y_i} \frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln w_\ell} \frac{\mathrm{d} \ln w_\ell}{\mathrm{d} \ln(1 + t_{ji,k})} \right]$$
$$+ \sum_{\ell} \sum_g \left[\frac{\partial \ln V(.)}{\partial \ln \tilde{P}_{\ell i,g}} \frac{\partial \ln \tilde{P}_{\ell i,g}(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln w_\ell} \frac{\mathrm{d} \ln w_\ell}{\mathrm{d} \ln(1 + t_{ji,k})} \right] = 0.$$

The above F.O.C. is similar to that analyzed earlier, with one basic difference. The first term in the F.O.C. that accounts for the effect of tariffs on total income now includes the effect on both tariff revenue and total profits. Specifically, given that $Y_i = w_i L_i + \Pi_i + \sum_j \sum_k t_{ji,k} P_{ji,k} Q_{ji,k}$, we can write this terms as follows:

$$\frac{\partial \ln Y_i(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w})}{\partial \ln(1 + t_{ji,k})} = Y_i^{-1} \left\{ \tilde{P}_{ji,k} Q_{ji,k} + \sum_g \sum_{j \neq i} \left(t_{ji,g} P_{ji,g} Q_{ji,g} \left[\frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln \tilde{P}_{ji,k}} \frac{\partial \ln \tilde{P}_{ji,k}(.)}{\partial \ln(1 + t_{ji,k})} + \frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln Y_i} \frac{\partial \ln Y_i(.)}{\partial \ln(1 + t_{ji,k})} \right] \right) \right.$$
$$\left. + \sum_g \left(\frac{\mu_g}{1 + \mu_g} P_{ii,g} Q_{ii,g} \left[\frac{\partial \ln \mathcal{D}_{ii,g}(.)}{\partial \ln \tilde{P}_{ji,k}} \frac{\partial \ln \tilde{P}_{ji,k}(.)}{\partial \ln(1 + t_{ji,k})} + \frac{\partial \ln \mathcal{D}_{ii,g}(.)}{\partial \ln Y_{i,k}} \frac{\partial \ln Y_i(.)}{\partial \ln(1 + t_{ji,k})} \right] \right) \right\}.$$

Noting that Equations still apply under the *generalized Krugman model*, they (along with the above equation) imply the following optimality condition:

$$\sum_g \sum_{j \neq i} \left[\left(t_{ji,g} - \bar{\tau}_i \right) \frac{\lambda_{ji,g}}{1 + t_{ji,g}} \varepsilon_{ji,g}^{ji,k} \right] + \sum_g \left(\frac{\mu_g}{1 + \mu_g} \lambda_{ii} \varepsilon_{ii,g}^{ji,k} \right) \tag{20}$$

$$+\sum_g \sum_{j\neq i} \left[\left(\left(t_{ji,g} - \bar{\tau}_i\right) \frac{\lambda_{ji,g}}{1+t_{ji,g}} \eta_{ji,g} \right) + \sum_g \left(\frac{\mu_g}{1+\mu_g} \lambda_{ii,g} \gamma_{ii,g} \right) \right] \frac{\partial \ln Y_i(.)}{\partial \ln(1+t_{ji,k})} = 0,$$

where $\bar{\tau}_i$ is defined as earlier. The second term in the above equation can be eliminated due to the redundancy in optimal tariffs. Specifically, since country i is a small open economy, multiplying wages in the rest of the world and dividing all of Country i's tariff rates by (i.e., $1+t_{ji,k}$'s) by $1+\delta > 0$, leads to the same exact equilibrium. Considering this, by choice of δ, there is always one optimal combination of tariffs and wages such that

$$\Delta(\boldsymbol{t}_i, \boldsymbol{t}_{-i}; \boldsymbol{w}) = \left[\sum_g \sum_{j\neq i} \left(\left(t_{ji,g} - \bar{\tau}_i\right) \frac{\lambda_{ji,g}}{1+t_{ji,g}} \gamma_{ji,g} \right) + \sum_g \left(\frac{\mu_g}{1+\mu_g} \lambda_{ii,g} \gamma_{ii,g} \right) \right] = 0. \quad (21)$$

Considering this; assuming zero cross-substitutability between industries; and noting that $\sum_j \lambda_{ji,k} \varepsilon_{ji,k}^{ji,k} = -\lambda_{ji,k}$, one of the multiple optimal tariffs combinations is given by

$$\left[\frac{1+\tilde{\tau}_i}{1+t^*_{ji,k}} \right]_{j\neq i} = \boldsymbol{\mathcal{E}}_{i,k}^{-1} \left(\mathbf{1}_{(N-1)\times 1} + \left[\frac{\lambda_{ii,k} \varepsilon_{ii,k}^{ji,k}}{\lambda_{ji,k}(1+\mu_k)} \right]_{j\neq i} \right) \quad, \forall k \quad (22)$$

where $\boldsymbol{\mathcal{E}}_{i,k} \equiv \left[\varepsilon_{ji,k}^{ji,k} \right]_{j,j}$ is an $(N-1)\times(N-1)$ matrix of demand elasticities, while $\tilde{\tau}_i$ is τ_i (as defined by 19) adjusted by the choice of δ that ensures 21. Note that the above solution is one of the multiple optimal tariff solutions. Depending on how wages are normalized in the rest of the world, Equation 22 with $\bar{\tau}_i$ replaced with $\tilde{\tau}_i$ specifies another solution. Considering this and following the same steps taken in Appendix A.1 to determine $\bar{\tau}_i$, we can show that country i's Nash tariffs solve the following system

$$\left[\frac{1+\bar{\tau}^*_i}{1+t^*_{ji,k}} \right]_{j\neq i} = \boldsymbol{\mathcal{E}}_{i,k}^{-1} \left(\mathbf{1}_{(N-1)\times 1} + \left[\frac{\lambda_{ii,k} \varepsilon_{ii,k}^{ji,k}}{\lambda_{ji,k}(1+\mu_k)} \right]_{j\neq i} \right), \quad \forall k$$

$$1+\bar{\tau}^*_i = \frac{\sum_{j\neq i} \sum_k \left[\chi_{ij,k} \varepsilon_{ij,k} - (t^*_{ji,k} - \bar{\tau}^*_i) \chi'_{ji,k} \varepsilon_{ji,k}^{ii,k} \right]}{1 + \sum_{j\neq i} \sum_k \chi_{ij,k} \varepsilon_{ij,k}},$$

which features $(N-1)\times K$ tariff rates, $\{1+t^*_{ji,k}\}$ and a uniform shifter, $1+\bar{\tau}^*_i$. In the above expression, $\chi_{ij,k} \equiv X_{ij,k} / \sum_{n\neq i} \sum_k X_{in,k}$ and $\chi'_{ji,k} \equiv X_{ji,k} / \sum_{n\neq i} \sum_k X_{ni,k}$, as defined in the main text. The extra term showing up in the numerator of the expression for $\bar{\tau}^*$, accounts for the cross-cost passthrough facing country i, when acting as a multi-product monopolist. To elaborate more, a uniform tariff is akin to a markup applied on w_i for all exported goods. Such a markup, however, affects country i's tax revenues which can be viewed as revenue from selling a second product (aside from the output of local labor). The term $\sum_{j\neq i} \sum_k \sum_s (t^*_{ji,k} - \bar{\tau}^*_i) \nu_{ji,k} \varepsilon_{ji,k}^{ii,s}$ accounts for these crosspassthrough effects.

The above formula can be greatly simplified if we further assume that preferences are given by Cobb-Douglas-CES parametrization in Equation 8. Then, $t^*_{ji,k}$ becomes uniform across countries, i.e., $t^*_{ji,k} = t^*_{i,k}$ for all j. Moreover, given that (i) $\varepsilon^{ji,k}_{ii,k} = \epsilon_k \lambda_{ji,k}$, (ii) $\varepsilon_{ji,k} = -1 - \epsilon_k(1-\lambda_{ji,k})$, and (iii) that $\sum_{j\neq i}\sum_k \chi_{ij,k} = 1$, it is straightforward to verify that

$$1+t^*_{i,k} = \left[\frac{\sum_{j\neq i}\sum_g X_{ij,g}\left[1+\epsilon_g\left(1-\lambda_{ij,g}\right)\right]}{\sum_{j\neq i}\sum_g\left[X_{ij,g}\epsilon_g\left(1-\lambda_{ij,g}\right)+\mathcal{X}_{ji,k}\epsilon_g\lambda_{ii,g}\right]}\right]\frac{\left(1+\mu_k\right)\left(1+\epsilon_k\lambda_{ii,k}\right)}{1+\mu_k+\epsilon_k\lambda_{ii,k}}.$$

where $\mathcal{X}_{ji,g} \equiv \sum_s\left(\frac{\mu_s\epsilon_s\lambda_{ii,s}}{1+\mu_s+\epsilon_s\lambda_{ii,s}}\right)X_{ji,g}$. Note that in the above expression, the uniform term is stated using export levels. But dividing both the numerator and denominator of the uniform term by $\sum_{j\neq i}\sum_k X_{ij,k} = \sum_{j\neq i}\sum_k X_{ji,k}$, we can also express it in terms of export shares $\chi'_{ji,k}$ and $\chi_{ij,k}$.

A.3 Proof of Proposition 5

The proof of Proposition 5 is very similar to that of Proposition 1, except that $\tilde{P}_{ji,k}(.)$ depends on the wage rate in every country. Specifically,

$$\tilde{P}_{ji,k}(\boldsymbol{t}_i,\boldsymbol{t}_{-i};\boldsymbol{w}) = \left(1+t_{ji,k}\right)\tilde{a}_{ji,k}\prod_{\ell} w_\ell^{\tilde{\gamma}_{j,k}(\ell)}.$$

Hence, whereas in the Ricardian model $\partial\ln\tilde{P}_{ji,k}(.)/\partial\ln w_j = 1$, in the IO model, $\partial\ln\tilde{P}_{ji,k}(.)/\partial\ln w_j = \gamma_{j,k}(j)$. Considering this and following the same exact steps as those conducted in Section A.1, we can show that

$$1+t^*_i = 1+\bar{\tau}_i, \quad \forall i\in\mathbb{C}$$

where $\bar{\tau}_i$ is given by Equation 19. So, to prove Proposition 5, we need to derive an expression for $\bar{\tau}_i$ subject to $\partial\ln\tilde{P}_{ji,k}(.)/\partial\ln w_j = \gamma_{j,k}(j)$. We can do so along the following lines (using $X^{\mathcal{F}} \equiv P^{\mathcal{F}}Q^{\mathcal{F}}$ to denote the value of "final good" trade in the reformulated IO model):

$$\bar{\tau}_i \equiv \frac{\frac{\partial Y_i(.)}{\partial\ln w_i}+\sum_g\frac{\partial V(.)/\partial\tilde{P}^{\mathcal{F}}_{ii,g}}{\partial V(.)/\partial Y_i}\frac{\partial\tilde{P}^{\mathcal{F}}_{ii,g}(.)}{\partial\ln w_i}}{\frac{\partial}{\partial\ln w_i}\left[\sum_{j\neq i}\sum_g P^{\mathcal{F}}_{ji,g}Q^{\mathcal{F}}_{ji,g}-\sum_{j\neq i}\sum_g P^{\mathcal{F}}_{ij,g}Q^{\mathcal{F}}_{ij,g}\right]}$$

$$=\frac{w_iL_i+\sum_{j\neq i}\sum_g\left[\bar{\tau}_iX^{\mathcal{F}}_{ji,g}\sum_s\left(\frac{\partial\ln\mathcal{D}_{ji,g}(.)}{\partial\ln\tilde{P}^{\mathcal{F}}_{ii,s}}\frac{\partial\ln\tilde{P}^{\mathcal{F}}_{ii,s}(.)}{\partial\ln w_i}\right)\right]-\sum_g X^{\mathcal{F}}_{ii,g}}{\sum_{j\neq i}\sum_g\left[X^{\mathcal{F}}_{ji,g}\sum_s\left(\frac{\partial\ln\mathcal{D}_{ji,g}(.)}{\partial\ln\tilde{P}^{\mathcal{F}}_{ii,s}}\frac{\partial\ln\tilde{P}^{\mathcal{F}}_{ii,s}(.)}{\partial\ln w_i}\right)\right]-\sum_{j\neq i}\sum_g\left[X^{\mathcal{F}}_{ij,g}\left[\frac{\partial\ln\tilde{P}^{\mathcal{F}}_{ij,g}(.)}{\partial\ln w_i}+\sum_s\left(\frac{\partial\ln\mathcal{D}_{ij,g}(.)}{\partial\ln\tilde{P}^{\mathcal{F}}_{ij,s}}\frac{\partial\ln\tilde{P}^{\mathcal{F}}_{ij,s}(.)}{\partial\ln w_i}\right)\right]\right]}$$

$$=\frac{\sum_{j\neq i}\sum_g \tilde{\gamma}_{i,g}(g)X^{\mathcal{F}}_{ij,g}}{-\sum_{j\neq i}\sum_g \tilde{\gamma}_{i,g}(g)X^{\mathcal{F}}_{ij,g}\left[1+\sum_s \varepsilon^{ij,s}_{ij,g}\right]}=\frac{\sum_{j\neq i}\sum_g\sum_s \phi_{ij,g}\varepsilon^{ij,s}_{ij,g}}{1+\sum_{j\neq i}\sum_g\sum_s \phi_{ij,g}\varepsilon^{ij,s}_{ij,g}}-1,$$

where $\phi_{ij,g}\equiv \tilde{\gamma}_{i,g}(g)X_{ij,g}/\sum_{j\neq i}\sum_g \tilde{\gamma}_{i,g}(g)X_{ij,g}$ denotes the value-added export share.

A.4 Welfare Approximation

This subsection shows that $\frac{\partial W_i}{\partial \ln w_j}\frac{dw_j}{d1+t_{ji,k}}\propto r_{ji,k}\lambda_{ji,k}\lambda_{ji,k}$, which justifies the welfare approximation presented under Equation 6. The term $\partial W_i/\partial \ln w_j$ can be characterized as follows

$$\begin{aligned}\frac{\partial W_i}{\partial \ln w_j}=&\frac{\partial V_i(.)}{\partial Y_i}\frac{\partial Y_i}{\partial \mathcal{R}_i}\frac{\partial \mathcal{R}_i}{\partial w_j}+\sum_k\frac{\partial V_i(.)}{\partial \tilde{P}_{ji,k}}\frac{\partial \tilde{P}_{ji,k}}{\partial w_j}\\=&\frac{\partial V_i(.)}{\partial Y_i}\left\{\sum_k\sum_\ell\left(t_{\ell i,k}P_{\ell i,k}Q_{\ell i,k}[\mathbb{1}_{j=J}+\varepsilon^{ji,k}_{ji,k}]\right)-\sum_k\tilde{P}_{ji,k}Q_{ji,k}\right\},\end{aligned}$$

where the second line uses Roy's identity, $\frac{\partial V_i(.)}{\partial \tilde{P}_{ji,k}}/\frac{\partial V_i(.)}{\partial Y_i}=-Q_{ji,k}$. Under the CES-Cobb-Douglas specification, the above expression immediately reduces to[34]

$$\frac{\partial W_i}{\partial \ln w_j}=Y_i\left[\sum_k\epsilon_k\left(\overline{\frac{t_{i,k}}{1+t_{i,k}}}-\frac{t_{ji,k}}{1+t_{ji,k}}\right)\frac{\beta_{i,k}\lambda_{ji,k}}{\lambda_{ji}}-1\right]\lambda_{ji},$$

where $\overline{\frac{t_{i,k}}{1+t_{i,k}}}\equiv\sum_\ell\frac{t_{\ell i,k}}{1+t_{\ell i,k}}\lambda_{\ell i,k}$. To determine $\mathrm{d}\ln w_j/\mathrm{d}\ln(1+t_{ji,k})$, we can apply the implicit function theorem to the system of labor market clearing conditions:

$$\begin{cases}R_1(\boldsymbol{t};\boldsymbol{w})\equiv w_1L_1-\sum_i P_{1i}(\boldsymbol{t};\boldsymbol{w})Q_{1i}(\boldsymbol{t};\boldsymbol{w})=0\\ \vdots\\ R_N(\boldsymbol{t};\boldsymbol{w})\equiv w_NL_N-\sum_i P_{Ni}(\boldsymbol{t};\boldsymbol{w})Q_{Ni}(\boldsymbol{t};\boldsymbol{w})=0\end{cases},$$

which implies that

$$\frac{\mathrm{d}\ln\boldsymbol{w}}{\mathrm{d}\ln(\boldsymbol{1}+\boldsymbol{t}_i)}=-\frac{\partial\boldsymbol{R}(.)}{\partial\ln\boldsymbol{w}}^{-1}\frac{\partial\boldsymbol{R}(.)}{\partial\ln(\boldsymbol{1}+\boldsymbol{t}_i)}.$$

Noting from actual trade data that $r_{ji,k}\lambda_{\ell i,k}\approx 0$ if $j,\ell\neq i$, the above expression implies that

$$\frac{\mathrm{d}\ln w_j}{\mathrm{d}\ln(1+t_{ji,k})}\approx\frac{\epsilon_k r_{ji,k}\lambda_{ji,k}}{1+\sum_\iota\sum_g\epsilon_g r_{j\iota,g}}.$$

[34]As similar conclusion can be reached without the CES-Cobb-Douglas specification, by using the fact that $\sum_\ell\sum_k\lambda_{\ell i,k}\varepsilon^{ji,k}_{\ell i,k}=-\lambda_{ji}$. The aforementioned result itself follows from the demand function being homogeneous of degree zero.

Combining the expressions for $\frac{\partial W_i}{\partial \ln w_J}$ and $\frac{\mathrm{d} \ln w_J}{\mathrm{d} \ln(1+t_{ji,k})}$, we can arrive at the following expression:

$$\frac{\partial W_i}{\partial \ln w_J} \frac{\mathrm{dln} w_J}{\mathrm{d} \ln(1+t_{ji,k})} \approx Y_i \frac{\epsilon_k \left[\sum_g \epsilon_g \left(\frac{t_{i,g}}{1+t_{i,g}} - \frac{t_{ji,g}}{1+t_{ji,g}} \right) \frac{\beta_{i,g}\lambda_{ji,g}}{\lambda_{ji}} - 1 \right]}{1 + \sum_g \sum_\iota \epsilon_k r_{\jmath\iota,g}} \lambda_{\jmath i} r_{\jmath i,k} \lambda_{ji,k} \propto \lambda_{\jmath i} r_{\jmath i,k} \lambda_{ji,k}.$$

B Measuring the Cost of a Two-Way Tariff War

Now, I consider the case where only two countries, namely, i and ℓ, engage in a two-way tariff war. I consider the Generalized Krugman model, noting that similar arguments apply to other models. In this two-way war, countries i and ℓ adopt an optimal tariff in response to each-other's tariffs, without changing the applied tariff on other trading partners. Country i's optimal tariff on country ℓ's exports in industry k (namely, $t^*_{\ell i,k}$) will, therefore, satisfy the F.O.C. implied by Equation 20, setting $t_{ji,g} = 0$ for all $j \neq \ell$. In particular,

$$\sum_g \left(t^*_{\ell i,g} - \bar{\tau}_i \right) \frac{\lambda_{\ell i,g}}{1 + t_{\ell i,g}} \varepsilon^{\ell i,k}_{\ell i,g} + \sum_g \frac{\mu_g}{1+\mu_g} \lambda_{ii} \varepsilon^{\ell i,k}_{ii,g} = 0.$$

Setting cross-industry demand elasticities to zero, which is the relevant case for my quantitative analysis, the above condition implies

$$1 + t^*_{\ell i,g} = (1 + \bar{\tau}_i) \left[1 + \frac{\frac{\mu_g}{1+\mu_g} \lambda_{ii} \varepsilon^{\ell i,k}_{ii,g}}{\lambda_{\ell i,g} \varepsilon_{\ell i,g}} \right]^{-1},$$

where $\bar{\tau}_i$ is defined as 19. To determine $1 + \bar{\tau}_i$, we can follow the same steps as before using our notation for trade values ($X = PQ$):

$$\begin{aligned}
\bar{\tau}_i \equiv & \frac{\frac{\partial Y_i(.)}{\partial \ln w_i} + \sum_g \frac{\partial V(.)/\partial P_{ii,g}}{\partial V(.)/\partial Y_i} \frac{\partial P_{ii,g}(.)}{\partial \ln w_i}}{\frac{\partial}{\partial \ln w_i} \left[\sum_{j \neq i} \sum_g P_{ji,g} Q_{ji,g} - \sum_{j \neq i} \sum_g P_{ij,g} Q_{ij,g} \right]} \\
= & \frac{w_i L_i + \sum_g \left[t_{\ell i,g} X_{\ell i,g} \sum_s \left(\frac{\partial \ln \mathcal{D}_{\ell i,g}(.)}{\partial \ln \tilde{P}_{ii,s}} \frac{\partial \ln \tilde{P}_{ii,s}(.)}{\partial \ln w_i} \right) \right] - \sum_g X_{ii,g}}{\sum_{j \neq i} \sum_g \left[X_{ji,g} \sum_s \left(\frac{\partial \ln \mathcal{D}_{ji,g}(.)}{\partial \ln \tilde{P}_{ii,s}} \frac{\partial \ln \tilde{P}_{ii,s}(.)}{\partial \ln w_i} \right) \right] - \sum_{j \neq i} \sum_g \left[X_{ij,g} \left[\frac{\partial \ln \tilde{P}_{ij,g}(.)}{\partial \ln w_i} + \sum_s \left(\frac{\partial \ln \mathcal{D}_{ij,g}(.)}{\partial \ln \tilde{P}_{ij,s}} \frac{\partial \ln \tilde{P}_{ij,s}(.)}{\partial \ln w_i} \right) \right] \right]} \\
= & \frac{\sum_g \left(t_{\ell i,g} P_{\ell i,g} Q_{\ell i,g} \varepsilon^{ii,g}_{\ell i,g} \right) + \sum_{j \neq i} \sum_g X_{ij,g}}{- \sum_{j \neq i} \sum_g \left(X_{ij,g} \left[1 + \sum_s \varepsilon^{ij,s}_{ij,g} \right] - X_{ji,g} \sum_s \varepsilon^{ii,s}_{ji,g} \right)}.
\end{aligned}$$

As a final step, we can use the last line in the above expression to produce the following characterization:

$$1 + \bar{\tau}_i = \frac{-\sum_{j\neq i}\sum_g \left(X_{ij,g}\varepsilon_{ij,g}^{ii,g}\right)}{1 - \sum_{j\neq i}\sum_g \left(X_{ij,g}\varepsilon_{ij,g} + \mathcal{X}_{ji,g}\varepsilon_{ji,g}^{ii,g}\right)}$$

where $\mathcal{X}_{ji,g} = X_{ji,g}$ if $j \neq \ell$ and $\mathcal{X}_{ji,g} = \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii}\varepsilon_{ii,g}^{\ell i,k}}{\lambda_{\ell i,g}\varepsilon_{\ell i,g}}\left(1 + \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii}\varepsilon_{ii,g}^{\ell i,k}}{\lambda_{\ell i,g}\varepsilon_{\ell i,g}}\right)^{-1} X_{ji,g}$. Assuming a CES-Cobb-Douglas utility parametrization (as in Equation 8), the above equation reduces to the following:

$$1 + t^*_{\ell i,g} = \left(\frac{\sum_{j\neq i}\sum_g X_{ij,g}\left[1 + \epsilon_g\left(1 - \lambda_{ij,g}\right)\right]}{\sum_{j\neq i}\sum_g \left(X_{ij,g}\epsilon_g\left(1 - \lambda_{ij,g}\right) + \mathcal{X}_{ji,g}\epsilon_g\lambda_{ii,g}\right)}\right)\left[1 - \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii,g}\epsilon_g}{1 + \epsilon_g(1 - \lambda_{\ell i,g})}\right]^{-1}, \tag{23}$$

where $\mathcal{X}_{ji,g} = X_{ji,g}$ if $j \neq \ell$ and $\mathcal{X}_{ji,g} = \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii,g}\epsilon_g}{1+\epsilon_g(1-\lambda_{\ell i,g})}\left(1 - \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii,g}\epsilon_g}{1+\epsilon_g(1-\lambda_{\ell i,g})}\right)^{-1} X_{ji,g}$. A similar characterization applies to country ℓ's Nash tariff on country i, i.e., $t^*_{i\ell,k}$. Moreover, analogous to the above equation, the two-way Nash tariff in the Ricardian and IO models would be uniform (across industries), but with an extra term in the denominator compared to the formulas specified by Propositions 1 and 3. This extra term accounts for the cross-cost passthrough facing country i, when it acts as a multi-product monopolist. More specifically, country i sells multiple goods to different international markets. The tariff on goods sold to market ℓ, internalize how country i's wage change affects the demand for the goods sold to all other (non-ℓ) markets. In the baseline specification, the tariff was applied uniformly on all exported goods, so this extra term canceled out.

Finally, given Equation 23, we can produce the following analog of Proposition 4, to compute the cost of two-way tariff war:

$$\begin{cases} 1+t^*_{ji,k} = 1+\bar{t}_{ji,k}, \quad j \neq \ell \\ 1+t^*_{\ell i,k} = (1+\bar{\tau}^*_i)\left[1 - \frac{\frac{\mu_g}{1+\mu_g}\lambda_{ii,g}\epsilon_g}{1+\epsilon_g(1-\lambda_{\ell i,g})}\right]^{-1} \\ 1+\bar{\tau}^*_i = \frac{\sum_{j\neq i}\sum_k \hat{X}_{ij,k}X_{ij,k}\left[1+\epsilon_k(1-\hat{\lambda}_{ij,k}\lambda_{ji,k})\right]}{\sum_{j\neq i}\sum_k\left[\hat{X}_{ij,k}X_{ij,k}\epsilon_k(1-\hat{\lambda}_{ij,k}\lambda_{ij,k})+\hat{\mathcal{X}}_{ji,k}\mathcal{X}_{ji,k}\epsilon_k\hat{\lambda}_{ii,s}\lambda_{ii,s}\right]} \\ \hat{X}_{ij,k}X_{ij,k} = \hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{ij,k}) \\ \hat{\mathcal{X}}_{ji,k}\mathcal{X}_{ji,k} = \hat{X}_{ji,k}X_{ji,k}, \quad j\neq\ell \\ \hat{\mathcal{X}}_{\ell i,k}\mathcal{X}_{\ell i,k} = \frac{\frac{\mu_g}{1+\mu_g}\hat{\lambda}_{ii,g}\lambda_{ii,g}\epsilon_g}{1+\epsilon_g(1-\hat{\lambda}_{\ell i,g}\lambda_{\ell i,g})}\left(1-\frac{\frac{\mu_g}{1+\mu_g}\hat{\lambda}_{ii,g}\lambda_{ii,g}\epsilon_g}{1+\epsilon_g(1-\hat{\lambda}_{\ell i,g}\lambda_{\ell i,g})}\right)^{-1}\hat{X}_{\ell i,k}X_{\ell i,k} \\ \hat{\lambda}_{ji,k} = \left(\frac{1+t^*_{ji,k}}{1+\bar{t}_{ji,k}}\hat{w}_j\right)^{-\epsilon_k}\hat{\tilde{P}}^{\epsilon_k}_{i,k} \\ \hat{\tilde{P}}^{-\epsilon_k}_{i,k} = \sum_j\left[\left(\frac{1+t^*_{ji,k}}{1+\bar{t}_{ji,k}}\hat{w}_j\right)^{-\epsilon_k}\lambda_{ji,k}\right] \\ \hat{w}_iw_iL_i = \sum_k\sum_j\left[\hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{ij,k})(1+\mu_k)\right] \\ \hat{\Pi}_i\Pi_i = \sum_k\sum_j\left[\mu_k\hat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\hat{Y}_jY_j/(1+t^*_{ji,k})(1+\mu_k)\right] \\ \hat{Y}_iY_i = \hat{w}_iw_iL_i + \hat{\Pi}_i\Pi_i + \sum_k\sum_j\left(\frac{t^*_{ji,k}}{1+t^*_{ji,k}}\hat{\lambda}_{ji,k}\lambda_{ji,k}\beta_{i,k}\hat{Y}_iY_i\right) \end{cases}.$$

Note that we can follow the same steps to characterize Nash tariffs and the corresponding costs for any localized tariff war. For instance, we can characterize the cost of a tariff war involving any $M \geq 2$ out of the total N countries.

C Estimation of Trade Elasticities

In this appendix, I describe the estimation procedure used to attain the industry-level trade elasticities. Following the notation introduced in the main text, let $X_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}$ trade values, and let $\bar{t}_{ji,k}$ denote effectively applied tariffs. Following Caliendo and Parro (2015), the industry-level trade elasticity in the Ricardian model can be estimated using the following estimating equation that combines tariff and trade data for any triple set of countries j, i, and k:

$$\ln\frac{X_{ji,k}X_{in,k}X_{nj,k}}{X_{ij,k}X_{ni,k}X_{jn,k}} = -\hat{\epsilon}_k\ln\frac{(1+\bar{t}_{ji,k})(1+\bar{t}_{in,k})(1+\bar{t}_{nj,k})}{(1+\bar{t}_{ij,k})(1+\bar{t}_{ni,k})(1+\bar{t}_{jn,k})} + \varepsilon_{jin,k}.$$

The error term, $\varepsilon_{jin,k}$, is composed of (idiosyncratic) bilateral non-tariff trade barriers. Under the identifying assumption that bilateral non-tariff barriers are uncorrelated with bilateral tariffs, we can employ an OLS estimator to identify $\hat{\epsilon}_k$ for each industry k.

To perform the above estimation, I use the full sample of countries in the aggregated 2014 WIOD database, consisting of 44 economies and 16 industries. In line with

Caliendo and Parro (2015), I drop zeros from the sample. I also apply Caliendo and Parro's (2015) trim, whereby exporters with the lowest/highest 2.5% share in each industry are dropped from the sample.[35] Data on applied tariffs are from UNCTAD-TRAINS, as explained in Section . To repeat myself, the applied tariff is measured as the *simple tariff line average* of the *effectively applied tariff.*

The estimation results are reported in Table 4, the cross-industry variation in the trade elasticities broadly aligns with those in Caliendo and Parro (2015). Unfortunately for the "Mining" and "Metal" industries, my estimation did not render meaningful estimates for $\hat{\epsilon}_k$. Presumably, this is due to the main exporters in these two industries being WTO members in 2014, which leads to a lack of sufficient variation in discriminatory tariffs.[36] Considering this, I simply adopt Caliendo and Parro's (2015) for these two industries—the adopted values are highlighted in gray.

To measure the cost of a tariff war in the generalized Krugman model, I need mutually-consistent estimates for both ϵ_k and μ_k. Attaining estimates for these parameters is only possible with micro-level data. That is, I cannot use the macro-level WIOD data to discipline both of these parameters. As an alternative solution, I borrow the estimates from Lashkaripour and Lugovskyy (2019), who use transaction-level data from 251 exporting countries during 2007-2013 to estimate the ϵ_k and μ_k for each of the WIOD industries used in my analysis. These adopted estimates are reported in Table 5. For the service-related industries, the parameters are normalized to $\epsilon = 5$ and $\mu = 0$.

[35] The Caliendo and Parro (2015) estimation involves only 16 countries and uses data from 1993. In comparison, my estimation involves 44 countries, some of which of relatively small. To handle, extreme observation due to my larger sample size, I also drop observations with the highest/lowest 2.5% values for $\frac{X_{ji,k}X_{in,k}X_{nj,k}}{X_{ij,k}X_{ni,k}X_{jn,k}}$ and $\frac{\bar{t}_{ji,k}\bar{t}_{in,k}\bar{t}_{nj,k}}{\bar{t}_{ij,k}\bar{t}_{ni,k}\bar{t}_{jn,k}}$.

[36] Ossa (2016) also faced a similar issue when applying the Caliendo and Parro (2015) estimation methodology to more contemporary data. He attributed this to most countries in his sample being WTO members, which leads to a lack of variation in discriminatory tariffs. I am inclined to believe that the same caveat applies here.

Table 4: *List of industries and estimated trade elasticities.*

Number	Description	trade elasticity ϵ_k	std. err.	N
1	Crop and animal production, hunting Forestry and logging Fishing and aquaculture	0.69	0.12	11,440
2	Mining and Quarrying	13.53	3.67	...
3	Food, Beverages and Tobacco	0.47	0.13	11,440
4	Textiles, Wearing Apparel and Leather	3.33	0.53	11,480
5	Wood and Products of Wood and Cork	5.73	0.93	11,326
6	Paper and Paper Products Printing and Reproduction of Recorded Media	8.50	1.52	11,440
7	Coke, Refined Petroleum and Nuclear Fuel	14.94	2.05	8,798
8	Chemicals and Chemical Products Basic Pharmaceutical Products	0.92	0.96	11,440
9	Rubber and Plastics	1.69	0.78	11,480
10	Other Non-Metallic Mineral	1.47	0.89	11,440
11	Basic Metals Fabricated Metal Products	3.28	1.23	...
12	Computer, Electronic and Optical Products Electrical Equipment	3.44	1.07	11,480
13	Machinery and Equipment n.e.c	3.64	1.45	11,480
14	Motor Vehicles, Trailers and Semi-Trailers Other Transport Equipment	1.38	0.46	11,480
15	Furniture; other Manufacturing	1.64	0.60	11,480
16	All Service-Related Industries (WIOD Industry No. 23-56)	4	...	...

Table 5: *The structural parameters used in the generalized Krugman model.*

Number	**Description**	Trade Ealsticity ϵ_k	Markup Wedge μ_k
1	Crop and animal production, hunting Forestry and logging Fishing and aquaculture	6.212	0.14
2	Mining and Quarrying	6.212	0.141
3	Food, Beverages and Tobacco	3.333	0.265
4	Textiles, Wearing Apparel and Leather	3.413	0.207
5	Wood and Products of Wood and Cork	3.329	0.270
6	Paper and Paper Products Printing and Reproduction of Recorded Media	2.046	0.397
7	Coke, Refined Petroleum and Nuclear Fuel	0.397	1.758
8	Chemicals and Chemical Products Basic Pharmaceutical Products	4.320	0.212
9	Rubber and Plastics	3.599	0.162
10	Other Non-Metallic Mineral	4.561	0.186
11	Basic Metals and Fabricated Metal	2.959	0.189
12	Computer, Electronic and Optical Products Electrical Equipment	1.392	0.453
12	Machinery, Nec	8.682	0.100
14	Motor Vehicles, Trailers and Semi-Trailers Other Transport Equipment	2.173	0.133
15	Furniture; other Manufacturing	6.704	0.142
16	All Service-Related Industries (WIOD Industry No. 23-56)	4	0

A withstanding question here, is why the trade elasticities differ between the two models? A straightforward answer is that they are estimated using different datasets and different identification strategies. For instance, the correlation between non-tariff trade barriers and tariffs can bias the estimates implied by the Caliendo et al. (2015) methodology, but not those implied by the methodology in Lashkaripour and Lugovskyy (2019). A deeper answer, though, is that tariffs presumably trigger selection effects. In that case, the trade elasticity estimated in Lashkaripour and Lugovskyy (2019) has to be adjusted for selection effects. The exact adjustment, however, depends on whether tariffs are applied after or before markups are charged—see Footnote 30 in Costinot and Rodríguez-Clare (2014) for more details.

D Accounting for Political Economy Weights

In this appendix, I demonstrate how the methodology developed here can easily accommodate political economy considerations. To this end, consider the multi-industry Krugman model introduced in Section 2.1, where preferences have a Cobb-Douglas-CES parametrization as in Equation 8. Also, following Ossa (2014), suppose that the policy maker in Country i maximizes a weighted welfare function,

$$W_i = \sum_k \theta_{i,k} \frac{X_{i,k}}{\tilde{P}_i}$$

where $\theta_{i,k}$ is the political economy weight assigned to industry k, $X_{i,k} = \sum_j P_{ij,k} Q_{ij,k}$ is total sales of industry k in country i, and $\tilde{P}_i$ is the Cobb-Douglas-CES consumer price index, $\tilde{P}_i = \prod_k \left(\sum \tilde{P}_{ji,k}^{-\epsilon_k} \right)^{-\beta_{i,k}/\epsilon_k}$. Also, suppose that $\theta_{i,k}$'s are normalized such that $\sum_k (\theta_{i,k}) / K = 1$. Following the same steps covered in Appendix A.2, we can easily show that (for every i and k) the Nash tariff is given by

$$1 + t_{i,k}^* = \left[\frac{\sum_{j \neq i} \sum_g X_{ij,g} \left[1 + \epsilon_g \left(1 - \lambda_{ij,g} \right) \right]}{\sum_{j \neq i} \sum_k \left[X_{ij,g} \epsilon_g \left(1 - \lambda_{ij,g} \right) + \mathcal{X}_{ji,k} \epsilon_k \lambda_{ii,s} \right]} \right] \frac{(1 + \mu_k)(1 + \epsilon_k \lambda_{ii,k})}{1 + \mu_k + \epsilon_k \lambda_{ii,k}},$$

where $\mathcal{X}_{ji,k} \equiv \sum_g \left[\frac{\mu_g \epsilon_g \lambda_{ii,g}}{1 + \mu_g + \epsilon_g \lambda_{ii,g}} \right] X_{ji,k}$, with

$$\tilde{\mu}_{i,k} = \frac{\theta_{i,k} \mu_k}{1 + (1 - \theta_{i,k}) \mu_k}. \tag{24}$$

Note that without political economy considerations, i.e., $\theta_{i,k} = 1$, the above equation simply implies that $\tilde{\mu}_{i,k} = \mu_k$. Beholding the above result, suppose we estimate the political economy weights using data on non-cooperative tariffs à la Ossa (2014). Then, we can simply compute the political economy-adjusted Nash tariffs and the welfare losses associated with them, using the following variation of Proposition 4.

Proposition 7. *If preferences are described by functional form 8 and $\{\theta_{i,k}\}$ describes the political economy weights in each country, then the Nash tariffs, $\{t_{i,k}\}$, and their effect on wages,*

$\{\hat{w}_i\}$, and total income, $\{Y_i\}$, can be solved as a solution to the following system:

$$\begin{cases} 1 + t^*_{i,k} = (1 + \bar{\tau}^*_i) \left[\frac{1+\tilde{\mu}_{i,k} - \epsilon_k \hat{\lambda}_{ii,k} \lambda_{ii,k}}{(1+\tilde{\mu}_{i,k})(1-\epsilon_k \hat{\lambda}_{ii,k} \lambda_{ii,k})} \right] \\ 1 + \bar{\tau}^*_i = \frac{\sum_{j \neq i} \sum_k \hat{X}_{ij,k} X_{ij,k} \left[1+\epsilon_k(1-\hat{\lambda}_{ij,k}\lambda_{ji,k})\right]}{\sum_{j \neq i} \sum_k \left[\hat{X}_{ij,k} X_{ij,k} \epsilon_k (1-\hat{\lambda}_{ij,k}\lambda_{ij,k}) + \hat{\mathcal{X}}_{ji,k} \mathcal{X}_{ji,k} \epsilon_k \hat{\lambda}_{ii,s} \lambda_{ii,s}\right]} \\ \hat{X}_{ij,k} X_{ij,k} = \hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / (1 + t^*_{j,k}) \\ \hat{\mathcal{X}}_{ji,k} \mathcal{X}_{ji,k} = \sum_g \left(\frac{\tilde{\mu}_g \epsilon_g \hat{\lambda}_{ii,g} \lambda_{ii,g}}{1+\tilde{\mu}_g + \epsilon_g \hat{\lambda}_{ii,g} \lambda_{ii,g}} \right) \hat{X}_{ji,k} X_{ji,k} \\ \hat{\lambda}_{ji,k} = \left(\frac{1+t^*_{i,k}}{1+\bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \hat{\tilde{P}}^{\epsilon_k}_{i,k} \\ \hat{\tilde{P}}^{-\epsilon_k}_{i,k} = \sum_j \left[\left(\frac{1+t^*_{i,k}}{1+\bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \lambda_{ji,k} \right] \\ \hat{w}_i w_i L_i = \sum_k \sum_j \left[\hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / (1 + t^*_{j,k})(1 + \mu_k) \right] \\ \hat{\Pi}_i \Pi_i = \sum_k \sum_j \left[\mu_k \hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / (1 + t^*_{j,k})(1 + \mu_k) \right] \\ \hat{Y}_i Y_i = \hat{w}_i w_i L_i + \hat{\Pi}_i \Pi_i + \sum_k \sum_j \left(\frac{t^*_{i,k}}{1+t^*_{i,k}} \hat{\lambda}_{ji,k} \lambda_{ji,k} \beta_{i,k} \hat{Y}_i Y_i \right) \end{cases},$$

which depends on only (i) observable expenditure shares and national output levels, $\lambda_{ji,k}$, $\beta_{i,k}$, and $Y_i = w_i L_i$; (ii) industry-level trade elasticities, ϵ_k; as well as (iii) $\tilde{\mu}_{i,k}$'s, which are given by the constant industry-level markup wedges, μ_k, and the estimated political economy weights per Equation 24.

With the above results in hand, let me elaborate on how or when political economy considerations may alter the conclusions reached in my main analysis. Without political economy considerations, countries will target Nash tariffs at high-μ industries. These targeted tariffs are costly, as they shrink output in high-μ industries below their already sub-optimal level. Now, suppose countries assign a greater political economy weight to high-μ industries, which is analog to

$$\partial \theta_k / \partial \mu_k > 0.$$

In that case, political economy considerations will push the global economy even further away from the efficiency frontier. Hence, the global cost of a tariff war would be greater with than without political economy considerations. To the contrary, suppose countries assign a lower political economy weight to high-μ industries, which is analog to

$$\partial \theta_k / \partial \mu_k < 0.$$

In that case, political economy considerations countervail the profit-shifting incentives that motivate targeted tariffs. As a result, in the event of a tariff war, Nash tariffs will be relatively less targeted towards high-μ industries. The global cost of tariff war would, therefore, be smaller with than without political economy weights. Presumably, in practice, high-profit industries are better positioned to lobby for protection. So, it is highly possible that we are dealing with the former case. If so, my main analysis provides a lower bound for the global cost of a full-fledged global tariff war.

E Computing Nash Tariffs without Approximation

This appendix derives sufficient statistics formulas for Nash tariffs without the approximation specified by Equation 6. First, I appeal to Proposition 1 in Beshkar and Lashkaripour (2019), which states that the country i's optimal (or Nash) tariff is uniform across industries, i.e., $t^*_{ji,k} = t^*_{ji,g}$ for all j, k, and g. This result reduces the task of solving the Nash tariffs from a problem involving $N(N-1)K$ tariffs rates to one that involves only $(N-1)N$ tariff rates. To economize on the notation, let $\delta^{ji}_{J} \equiv \mathrm{d}\ln w_J / \mathrm{d}\ln(1+t_{ji})$ denote the general equilibrium effect of t_{ji} on w_J. With this choice of notation and assigning labor in Country i as the numeraire (i.e., $w_i = 1$), the first-order condition with respect to t_{ji} can be expressed as[37]

$$\frac{\mathrm{d}W_i(.)}{\mathrm{d}\ln(1+t_{ji})} = \sum_{\ell \neq i} t_{\ell i} P_{\ell i} Q_{\ell i} \varepsilon^{ji}_{\ell i} + \sum_{\ell \neq i} \left[t_{\ell i} P_{\ell i} Q_{\ell i} \left(\delta^{ji}_{\ell} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) - \tilde{P}_{\ell i} Q_{\ell i} \delta^{ji}_{\ell} \right] = 0,$$

where $\varepsilon^{ji}_{\ell i} \equiv \sum_k (r_{\ell i,k}/r_{\ell i}) \varepsilon^{ji,k}_{\ell i,k}$. Rearranging the above equation and dividing by Y_i, yields the following expression:

$$\sum_{\ell \neq i} \left(1 - \frac{1}{1+t_{\ell i}}\right) \frac{\tilde{P}_{\ell i} Q_{\ell i}}{Y_i} \left(\varepsilon^{ji}_{\ell i} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) - \sum \frac{1}{1+t_{\ell i}} \frac{\tilde{P}_{\ell i} Q_{\ell i}}{Y_i} \delta^{ji}_{\ell}$$

$$\sum_{\ell \neq i} \lambda_{\ell i} \left(\varepsilon^{ji}_{\ell i} + \delta^{ji}_{\ell} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) \frac{1}{1+t_{\ell i}} = \sum_{\ell \neq i} \lambda_{\ell i} \left(\varepsilon^{ji}_{\ell i} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) = 0.$$

For each country i, the above system characterizes a vector of Nash tariffs. To simplify this system, we can rewrite it in matrix-form as follows:

$$\boldsymbol{A}_i \boldsymbol{\mathcal{T}}^*_i = \boldsymbol{b}_i \implies \boldsymbol{\mathcal{T}}^*_i = \boldsymbol{A}_i^{-1} \boldsymbol{b}_i, \quad \forall i \in \mathbb{C} \tag{25}$$

where $\boldsymbol{\mathcal{T}}^*_i \equiv [1/(1+t^*_{ji})]_{j \neq i}$ is an $(N-1) \times 1$ vector of (inverse) Nash tariffs; $\boldsymbol{A}_i \equiv \left[\left(\varepsilon^{ji}_{\ell i} + \delta^{ji}_{\ell} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) \lambda_{\ell i} \right]_{j \neq i, \ell \neq i}$ is an $(N-1) \times (N-1)$ matrix; and $\boldsymbol{b}_i \equiv \left[\sum_{\ell \neq i} \lambda_{\ell i} \left(\varepsilon^{ji}_{\ell i} + \sum_{J \neq i} \varepsilon^{ji}_{\ell i} \delta^{ji}_{J} \right) \right]_{j \neq i}$ is an $(N-1) \times 1$ vector. Importantly, $\boldsymbol{A}_i$ and $\boldsymbol{b}_i$ are composed of only observable expenditure shares, reduced-form demand elasticities, and δ^{ji}_{J}s. Next, I show that the matrix $\boldsymbol{\Delta}_i \equiv [\delta^{ji}_{J}]_{J,j}$ can be also calculated as a function of only

[37] As in the proof of Proposition 1 (in Appendix A.1), the above expression uses envelope conditions to eliminate the direct effect of tariffs on revenue and consumer prices.

observables and reduced-form demand elasticities. To this end, I apply the Implicit Function Theorem to the set of country-specific labor market clearing conditions:

$$\begin{cases} R_1(\boldsymbol{t};\boldsymbol{w}) \equiv w_1 L_1 - \sum_\iota P_{1\iota}(\boldsymbol{t};\boldsymbol{w}) Q_{1\iota}(\boldsymbol{t};\boldsymbol{w}) = 0 \\ \vdots \\ R_N(\boldsymbol{t};\boldsymbol{w}) \equiv w_N L_N - \sum_\iota P_{N\iota}(\boldsymbol{t};\boldsymbol{w}) Q_{N\iota}(\boldsymbol{t};\boldsymbol{w}) = 0 \end{cases}.$$

Doing so, fully determines the δ_j^{ji}'s in Equation 25 as a function of reduced-form demand elasticities and observable revenue shares:

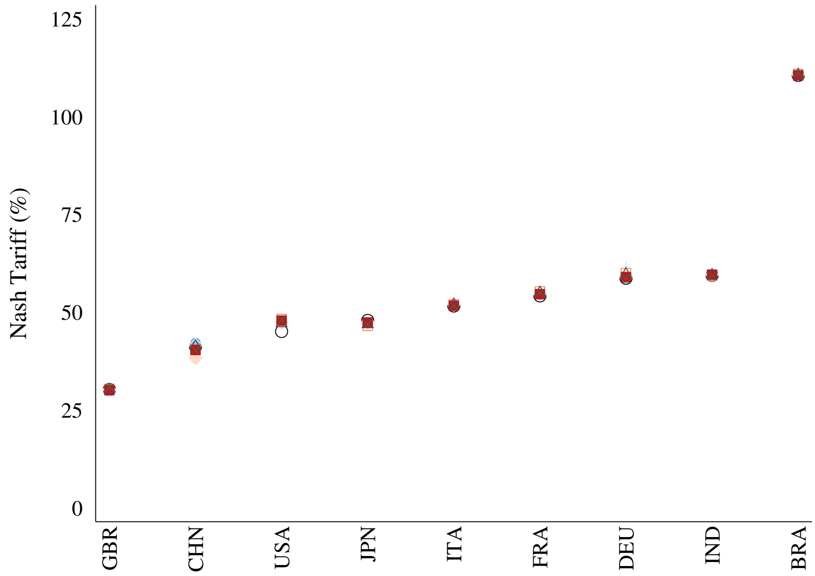
$$\begin{aligned} \boldsymbol{\Delta}_i &= -\frac{\partial \boldsymbol{R}(.)}{\partial \ln \boldsymbol{w}}^{-1} \frac{\partial \boldsymbol{R}(.)}{\partial \ln(\boldsymbol{1}+\boldsymbol{t}_i)} \\ &= -\left(\boldsymbol{I}_N - \left[\sum_\iota r_{j\iota} \left(\varepsilon_{j\iota}^{\ell\iota} + \mathbb{1}\{j=\ell\} \right) \right]_{j,\ell} \right)^{-1} \left[r_{ji} \varepsilon_{ji}^{\ell i} \right]_{j,\ell}. \end{aligned} \quad (26)$$

Together, Equations 25 and 26 provide a sufficient statistics characterization of Nash tariffs as a function of reduced-form demand elasticities; observable expenditure shares; and observable revenue shares. Assuming the Cobb-Douglas-CES preferences characterized by Equation 8, the reduced-form demand elasticities become $\varepsilon_{ji,k}^{\ell i,k} = -\mathbb{1}\{j=\ell\}(\epsilon_k+1) + \epsilon_k \lambda_{\ell i,k}$. So, as in the baseline case, we can use the exact hat-algebra notation to jointly solve (a) the Nash tariffs specified by Equation 25 and (b) the equilibrium conditions. Doing so involves solving the following system features $N(N-1)+2N$ independent equation and $N(N-1)+2N$ independent unknowns, namely, $\boldsymbol{t}^* \equiv \{t_{ji}^*\}$, $\hat{\boldsymbol{w}} \equiv \{\hat{w}_i\}$, and $\hat{\boldsymbol{Y}} \equiv \{\hat{Y}_i\}$:

$$\begin{cases} [1/(1+t_{ji}^*)]_{j\neq i} = \boldsymbol{A}_i^{-1} \boldsymbol{b}_i \\ \boldsymbol{A}_i = \left[\sum_k \left(-\mathbb{1}\{j=\ell\} + \epsilon_k \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} + \sum_\jmath \epsilon_k \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} \delta_\jmath^{ji} \right) \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} \right]_{j\neq i,\ell\neq i} \\ \boldsymbol{b}_i = \left[\sum_\ell \sum_k \left(-\mathbb{1}\{j=\ell\} + \left(\epsilon_k + \delta_\ell^{ji} \right) \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} + \sum_\jmath \epsilon_k \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} \delta_\jmath^{ji} \right) \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} \right]_{j\neq i} \\ \left[\delta_j^{\ell i} \right]_{j\neq i,\ell\neq i} = \left[\sum_\iota \sum_k \epsilon_k \lambda_{\ell \iota,k} \hat{\lambda}_{\ell i,k} \frac{\lambda_{j\iota,k} \hat{\lambda}_{j\iota,k}}{1+t_{j\iota}^*} \right]_{j\neq i,\ell\neq i}^{-1} \left[-\mathbb{1}_{j=\ell} (\epsilon_k+1) + \epsilon_k \lambda_{\ell i,k} \hat{\lambda}_{\ell i,k} \frac{\lambda_{ji,k} \hat{\lambda}_{ji,k}}{1+t_{ji}^*} \right]_{j\neq i,\ell\neq i} \\ \hat{\lambda}_{ji,k} = \left(\frac{1+t_i^*}{1+\bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \hat{\tilde{P}}_{i,k}^{\epsilon_k} \\ \hat{\tilde{P}}_{i,k}^{-\epsilon_k} = \sum_j \left[\left(\frac{1+t_i^*}{1+\bar{t}_{ji,k}} \hat{w}_j \right)^{-\epsilon_k} \lambda_{ji,k} \right] \\ \hat{w}_i w_i L_i = \sum_k \sum_j \left[\hat{\lambda}_{ij,k} \lambda_{ij,k} \beta_{j,k} \hat{Y}_j Y_j / \left(1+t_j^* \right) \right] \\ \hat{Y}_i Y_i = \hat{w}_i w_i L_i + \sum_k \sum_j \left(\frac{t_i^*}{1+t_i^*} \hat{\lambda}_{ji,k} \lambda_{ji,k} \beta_{i,k} \hat{Y}_i Y_i \right) \end{cases}.$$

Computing the Nash tariffs using the above system is more efficient than the standard iterative optimization procedure, but is more computationally involved than the base-

Figure 4: *Nash tariffs computed using the approximation-free formulas.*



Note: this graph compares the Nash tariff rates implied by the approximation-free formulas to the baseline rates. Each dot corresponds to a Nash tariff applied on an individual export partner, by a country featured on the x-axis. The countries feature on the x-axis are the largest economies in the 2014 WIOD sample, excluding EU member countries.

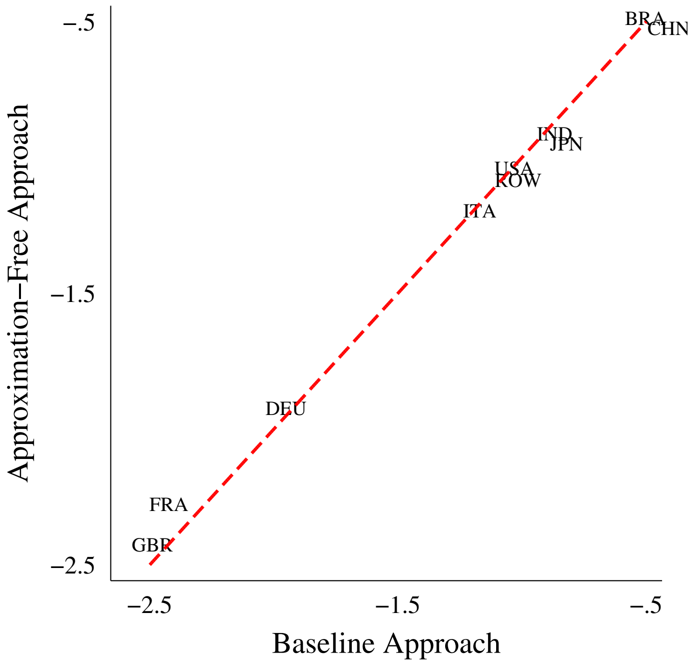
line approach presented in Section 2. My objective here is to compare my baseline results to the approximation-free results obtained from solving the above system of equations. Given this objective, I aggregate the 2014 WIOD sample into the 10 largest countries plus an aggregate of the rest of the world. By doing so, I am essentially focusing on the set of countries for which my welfare approximation is most suspect.

The computed Nash tariffs under the approximation-free approach are displayed in Figure 4. When interpreting this graph, note that in the Ricardian model, Nash tariffs are always uniform across industries but may vary across exporters if a country trades excessively with another partner. If my assumption that $\lambda_{ji}r_{ji,k}/\lambda_{ii}r_{ii,k} \approx 0$ for $j \neq i$ is credible, then the Nash tariffs should be approximately uniform across the board. Based on Figure 4 this is indeed the case.

Next, I compare the welfare losses implied by the baseline approach to those implied by the approximation-free approach. The comparison is displayed in Figure 5. Once again it is clear that the two approaches deliver indistinguishable predictions. Albeit, with different degrees of computational efficiency: on my personal computer, for instance, the baseline approach produced output around 50-times faster than the approximation-free approach, which itself converged more than 20-times faster than standard iterative approach.

Now, is perhaps a good time to reflect on the computational speed of the sufficient

Figure 5: *% Loss in real GDP from a tariff war*



statistics methodology relative to the standard iterative method. On the same computing device, my proposed methodology converges close to 1000-times faster than the standard methodology. Moreover, based on my experience, when smaller countries are included in the analysis, the standard methodology (based on the FMINCON solver in MATLAB) becomes increasingly sensitive to the choice of initial values. My purposed methodology, however, is not susceptible to this problem as it does not involve a global optimization and also imposes *theory-driven* uniformity constraints. Finally, another word caution is that when I implemented the standard methodology using the FMINCON solver in MATLAB, I obtained output that did not actually correspond to a global optimum in some instances. I noticed this by cross-checking the output from FMINCON with that implied by my analytic formulas and comparing the objective function's values. This is not a criticism of the standard iterative methodology per-se, but more so a word caution regarding the use of FMINCON.