

Geography, Value-Added and Gains From Trade: Theory and Empirics

Patrick D. Alexander*

Queen's University

January 16, 2015

JOB MARKET PAPER

Abstract

Standard new trade models depict firms as heterogeneous in total factor productivity. In this paper, I first extend the Eaton and Kortum (2002) and Melitz (2003) models of international trade to incorporate tradable intermediate inputs and firm heterogeneity in *value-added* productivity. In equilibrium, this yields a positive relationship between the response of international trade flows to changes in trade costs, the “trade elasticity”, and the intermediate inputs share. This relationship is absent from the standard models and driven by the extensive margin of trade. I then use sectoral data from the 1980s and 2000s to estimate the trade elasticity. Over both periods, I find empirical support for the positive relationship between the trade elasticity and the intermediate inputs share and for the importance of the extensive margin. I find that the gains from manufacturing trade are, on average, larger by 34% in the early-1980s and 57% in the early-2000s under the *value-added* framework relative to the standard models. I apply these results to provide explanations for the “international elasticity puzzle” and the “distance puzzle” from the empirical trade literature.

JEL classification codes: F11, F12, F14

Keywords: gravity models, value-added exports, trade elasticity, intermediate inputs, gains from trade, distance puzzle, international elasticity puzzle, extensive margin

*I would like to thank participants at the 2014 CESifo Conference on Gravity Models in Munich, the 2013 CIREQ Ph.D students conference in Montreal, the 2012 CEA meetings in Calgary, and the Queen's and Ryerson Ph.D students seminar series for all their input. Most of all, I would like to thank my advisers at Queen's, Beverly Lapham and Ian Keay, for invaluable guidance over the years. All mistakes are my own.

1 Introduction

In this paper, I first develop a theoretical model which incorporates intermediate goods and firm-level heterogeneity in *value-added* productivity. I examine the implications of this set-up for (i) measures of the response of trade volumes to changes in trade costs (the “trade elasticity”), (ii) measures of sectoral productivity dispersion and (iii) measures of the gains from trade. I then use data for nine countries and twenty sectors for the 1980s and 2000s to measure each of these items in accordance with the theory. I find empirical support for the importance of intermediate inputs, significantly different values for the items listed above, and evidence which offers an explanation for several recent puzzles in the empirical trade literature.

My work is motivated by a number of empirical observations. From the 1980s to the 2000s, trade as a share of total world GDP more than doubled.¹ This growth, moreover, came during a period of relatively modest changes in observed trade costs. From the late 1980s to the mid-2000s, average global tariffs declined by only a few percentage points and shipping cost margins experienced similarly small changes.² To match growth in world trade with such small changes in trade costs, the international trade elasticity needs to be in excess of 10 according to standard trade models. In trade models with heterogeneous firms, the trade elasticity is determined by the extensive margin (the number of exporting firms), and is equivalent to a parameter of inter-firm productivity dispersion. Most estimates of this dispersion parameter using firm-level data find it to be somewhere in the neighborhood of 5.³ Thus, it is difficult to reconcile the trade elasticity required to explain growth in world trade with the microfoundations of this elasticity based on standard heterogeneous firms trade models. This is referred to as the “international elasticity puzzle”.

Also, the impact of distance on bilateral trade appears to have remained fairly constant from the 1980s to the 2000s. Distance is widely used as a proxy variable for unobserved trade costs like information and communications. As these costs have declined over time due to technological change, we might expect that the impact of distance as a trade friction would have become weaker. However, according to most of the empirical

¹The share of total world exports in goods and services to world GDP (2005 USD) went from roughly 12.4% in the early 1980s to 27.5% in the mid-2000s. These figures are calculated using the International Trade (MEI) database and World Bank data for exports and GDP respectively.

²The simple mean of the applied tariff rate across OECD countries fell from 4.88% in 1989 to 3.36% in 2005 according to World Bank data. For non-OECD countries, tariffs fell by more, although the trend is still fairly modest. Hummels (2007) documents that ocean shipping, which constitutes the majority of world trade by value, experienced a decline in transport prices of only a few percentage points of relative to export value from the 1980s to the mid-2000s.

³For example, Eaton, Kortum and Kramarz (2011) estimate this parameter to be 4.87 using French firm data and results from di Giovanni, Levchenko and Rancière (2011) suggest similar estimates.

trade literature, distance has remained a stable and significant hindrance to international trade over time. This is often referred to as the “distance puzzle”.⁴

In the following, I examine the role that intermediate input goods play in reconciling these puzzles. Consider Figure 1 which plots the share of value-added in manufacturing exports from the 1980s to 2000s for several OECD countries.⁵ In all cases, this share is significantly less than 0.5, indicating that more than half of export value is produced from intermediate inputs. These inputs come from either domestic or foreign producers. Over time, the share of foreign-produced inputs has risen in place of falling domestic inputs across most sectors and countries. Overall, the value-added share fell over the two periods for each country; on average, from roughly 35% to 32%. That is, the share of intermediate inputs used in production was generally higher in the 2000s than in the 1980s.

Figure 1: Value-Added Share of Exports

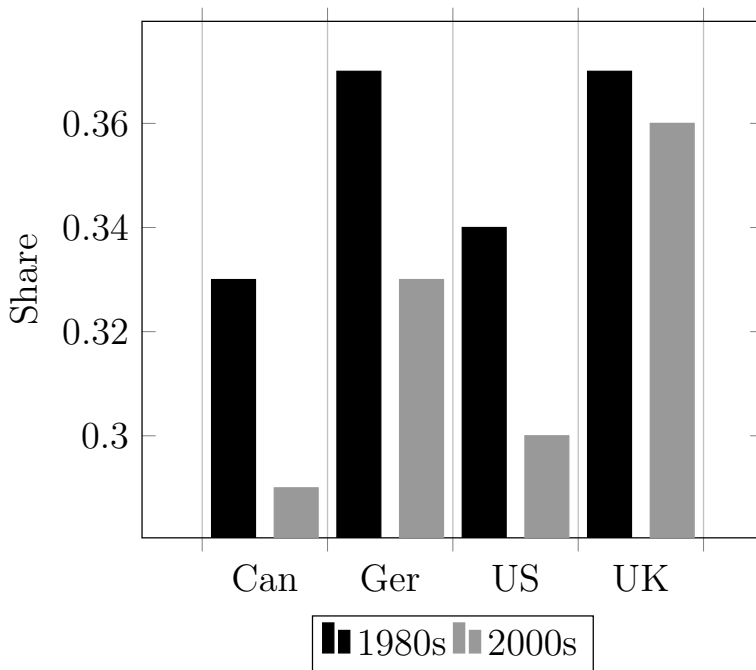


Figure 2 compares the average distance of value-added weighted exports to that of

⁴In a meta-analysis of this puzzle, Disdier and Head (2008) consider 1,467 estimates of the gravity equation from 103 papers. They find that the mean coefficient on distance remained fairly stable from the 1970s to the late 2000s.

⁵I report values for Canada, Germany, the United States and the United Kingdom. To calculate these shares, I took total output by the manufacturing sector and subtracted the share of intermediate inputs used in production. I then weighted sectoral exports by this value-added share and summed across sectors to find total value-added exports for the country. I made these calculations using OECD input-output data for each country from the early 1980s and early 2000s.

gross exports for the same group of four countries.⁶ In each case, the difference between these distances is greater than zero, implying that the geography of value-added exports is less regionalized than exports which include intermediate input goods. This pattern suggests that trade costs affect goods that include intermediate inputs more significantly than those that include only value-added.

In this paper, I extend the two leading heterogeneous firms models of international trade, Eaton and Kortum (2002) and Melitz (2003), to a framework with many countries, many sectors and heterogeneity in *value-added* (VA) productivity within sectors. The standard models have heterogeneity in total factor productivity (TFP) within sectors. In my framework, intermediate inputs are sourced from both at home and abroad, and firms differ with respect to their efficiency in adding value to these inputs. In equilibrium, this yields a closed-form gravity equation relating sectoral bilateral exports to market size, trade costs and the sectoral trade elasticity. The trade elasticity is a function of sectoral productivity dispersion (or “sectoral dispersion”) and an additional factor which is absent from previous models: the share of intermediate inputs in production. My model predicts that the sensitivity of trade flows to changes in trade costs is *higher* in sectors with a *higher* share of intermediate inputs. Under standard models, the trade elasticity is governed entirely by sectoral dispersion. Under my model, the trade elasticity is also driven by the share of intermediates and, as a result, over twice as large as sectoral dispersion for most sectors. Thus, trade responds significantly to changes in trade costs even in sectors where the dispersion parameter is low.

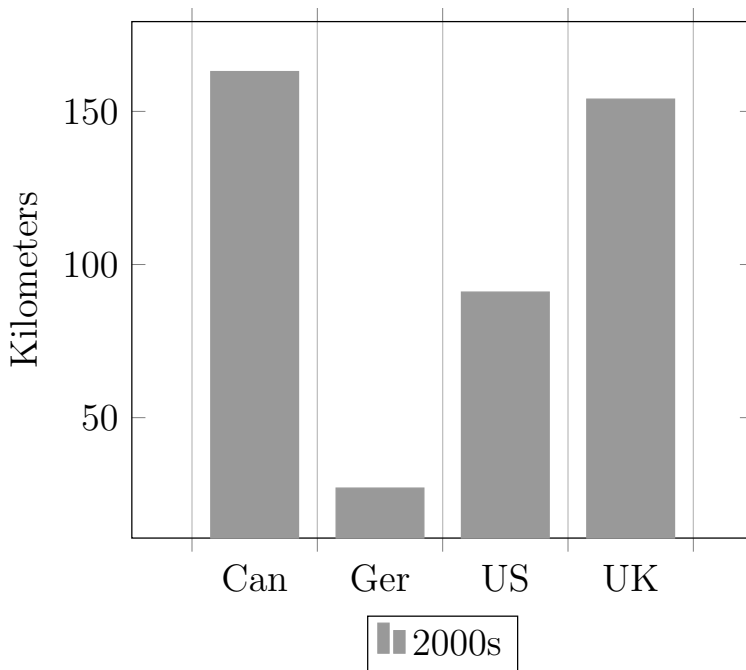
My framework also yields a new closed-form expression for the economic gains from international trade. Relative to standard models, the magnitude of the gains from trade is theoretically ambiguous, but can be calculated empirically using available data.

I then combine data on bilateral exports, input-output tables and trade costs into a cross-section with 9 countries and 20 manufacturing sectors for the early 1980s and the early 2000s.⁷ I estimate the trade elasticity for both periods. I find a positive and statistically significant relationship between the trade elasticity and the share of intermediate inputs (whether produced domestically or imported) as predicted by the model. This result holds across several different measures of trade costs, including bilateral distance. I also find evidence, consistent with the distance puzzle, that estimates of the elasticity of trade with respect to distance are stable over time. I argue, however, that this is not

⁶The average distance of exports for country n is calculated as $\sum_{i=1}^N \sum_{j=1}^J \left(\frac{X_{ni}^j}{X_n^j} \right) d_{ni}$ where i j and d_{ij} denote importer, sector and geographic distance between them respectively. The average distance of value-added weighted exports is calculated by weighting sectoral exports X_{ni}^j and X_n^j in this expression by the corresponding share of value-added in production.

⁷As indicators of trade costs, I include bilateral distance and dummy variables for regional trade agreement, common currency and common border.

Figure 2: Distance of Value-Added Exports minus Distance of Total Exports



puzzling in light of the impact that intermediate inputs have on these estimates. Once adjusting for variation in the intermediate inputs share across time, sectors and countries, I find evidence that the residual response of international trade to variation in bilateral distance was 13 percent lower (in absolute value) in the 2000s than in the 1980s.

Using tariff data, I also estimate parameters for sectoral dispersion according to my structural trade elasticity equation. I find that, compared with my estimates, previous estimates significantly *understate* the degree of dispersion within sectors: on average, by approximately a factor of 3. This translates to *larger* gains from trade under my specification. I calculate the gains from trade for 9 countries and 20 sectors from the 1980s and 2000s. Compared to standard models, I find that the gains from manufacturing trade are generally higher when measured according to my model: on average, gains from trade rise by 34% in the early 1980s and 57% in the early 2000s relative to the standard framework.

My framework also distinguishes between the intensive and extensive margins of trade. According to my theory, the positive relationship between the trade elasticity and the intermediate inputs share is driven entirely by the extensive margin. To identify this margin empirically, I link disaggregated trade data for 768 product varieties to the 20 sectors in my data and compute the count of goods exported between countries. Empirically, I find evidence that this relationship is particularly strong when using my constructed measure of the extensive margin.

Overall, my theoretical and empirical findings contribute to the literature in several

ways. The empirical distinction between intermediate inputs and value-added in exports using input-output analysis has been explored in many papers. For examples, see Hummels, Ishii and Yi (2001), Antras et al (2012), Johnson and Noguera (2012a, 2012b, 2012c) and recent papers by Koopman et al (2012, 2014) and Timmer et al (2014). These papers draw a particular distinction between imported intermediates and domestic value-added in exports. In my analysis, the main distinction is between intermediate inputs (domestic- or foreign-produced) and firm value-added. Empirically, I find a qualitatively similar pattern for domestic- and foreign-produced intermediates in relation to the trade elasticity.

My emphasis on theory-consistent estimation contributes to a substantial literature that addresses potential mis-specifications in empirical gravity models. For examples of representative firm models, see Anderson and Van Wincoop (2003) and Baldwin and Taglioni (2011). My theoretical framework is based on the gravity models with firm/product heterogeneity developed in Eaton and Kortum (2002) and Chaney (2008), combined with a production setting similar to Yi (2003, 2010).⁸ There is also an existing literature that aims to provide theoretical or empirical refinements in the estimation of trade elasticities. For examples, see Ruhl (2005) and Simonovska and Waugh (2014).

The welfare analysis in my paper adds to the discussion relating the gains from trade to recent micro-founded international trade models. For other contributions, see Arkolakis et al (2012), Caliendo and Parro (2012), Ossa (2012), Levchenko and Zhang (2014) and Costinot and Rodriguez-Clare (2013). The relationship between the gains from trade and intermediate inputs was recently explored in Melitz and Redding (2014).

In exploring the distance puzzle, my paper also adds to an extensive empirical literature. For recent examples, see Bhavnani et al (2002), Buch, Kleinert and Toubal (2004), Berthelon and Freund (2008), Disdier and Head (2008), Lin and Sim (2012), and Yotov (2012). For an example of a theoretical paper on the distance puzzle, see Chaney (2013).

Both the theoretical and the empirical sections of my paper consider differences across sectors. This contributes to a growing literature, including Caliendo and Parro (2012), Shikher (2012), Levchenko and Zhang (2014).

The importance of the extensive margin is also emphasized in other recent findings. These include Chaney (2008), Helpman, Melitz and Rubenstein (2008), Hummels and Hilberry (2008) and Crozet and Koenig (2008).

The remainder of the paper is organized as follows. Section 2 describes the theoretical framework, while Section 3 describes the data. Section 4 provides empirical results for the gains from trade and Section 5 provides empirical results for trade elasticities. Section 6

⁸Yi (2003) aims to explain growth in world trade by endogenous growth in imported intermediate inputs share. I expand his approach to include domestic intermediates, finding these inputs are also significant both qualitatively and quantitatively.

concludes. An appendix follows.

2 Model

In the following, I illustrate a framework with tradable intermediate and final goods, trade costs and heterogeneity in *value-added* productivity across firms/products. In this environment, the share of intermediate inputs used in production alters the relationship between international trade, trade costs and the gains from trade.

The results are derived under the settings of both perfect and monopolistic competition.

2.1 Perfect Competition

The following is a multi-sectoral Eaton-Kortum (2002) model of trade with intermediate inputs.⁹

2.1.1 Environment

Consider a world with N countries and J sectors. Country n has labor endowment L_n . Labor is the only factor of production and consumers in each country derive utility from consuming goods from each of the J sectors. Consumers in n buy C_n^j units of the final composite good from sector j to maximize the following CES utility function:

$$U_n = \prod_{j=1}^J C_n^j \alpha_n^j \quad (1)$$

where $\sum_{j=1}^J \alpha_n^j = 1$. The budget constraint for consumers in n is given by:

$$\sum_{j=1}^J P_n^j C_n^j = w_n L_n \quad (2)$$

where w_n denotes the wage rate and P_n^j denotes the aggregate price index in sector j of country n (described below).

Each sector is made up of a continuum of goods indexed by $\omega \in [0, 1]$ and labor is freely mobile within countries. Producers of good ω in sector j of country n draw *value-added* productivity $z_n^j(\omega)$ from a Fréchet distribution of the following form:

$$F_n^j(z_n^j) = \exp \left\{ -T_n^j z_n^j^{-\theta^j} \right\} \quad (3)$$

⁹The basic Eaton and Kortum (2002) model does not have intermediate inputs, although the authors provide an extension with intermediates in the second half of their original paper. Other multi-sectoral versions of the Eaton and Kortum (2002) model can be found in Shikher (2012), Caliendo and Parro (2012), Levchenko and Zhang (2014).

This distribution varies across both countries and sectors. A higher T_n^j implies higher average productivity for the country-sector pair, while a higher θ^j implies lower dispersion of *value-added* productivity draws within sector j .¹⁰

The corresponding production function for good ω is:

$$q_n^j(\omega) = [z_n^j(\omega)l_n^j(\omega)]^{1-\beta^j} \left[\prod_{k=1}^J M_n^{k,j}(\omega)^{\gamma^{k,j}} \right]^{\beta^j} \quad (4)$$

where z_n^j , l_n^j and $M_n^{k,j}$ denote labor productivity, labor inputs and intermediate input for the composite intermediate good in sector k respectively. The parameter $\gamma^{k,j}$ denotes the share of intermediate inputs from sector k used by producers in sector j , with $\sum_{k=1}^J \gamma^{k,j} = 1$. Equation 4 includes an important departure from the conventional Eaton and Kortum (2002) model. The parameter $z_n^j(\omega)$ does not enter here as total factor productivity (TFP) but as *value-added* (VA) productivity. As shown below, this difference is non-trivial: it provides for an additional role for intermediate inputs in the trade elasticity, and a lowering of the gains from trade. Composite goods Q_n^j are produced using the following CES production technology:

$$Q_n^j = \left(\int q_n^j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad (5)$$

where $\sigma > 1$ denotes elasticity of substitution across varieties. The composite goods from j are demanded by both consumers as final goods C_n^j and by producers as intermediate goods $M_n^{j,k}$ across all k sectors.

Total demand in n for good ω exported from i in sector j follows the CES demand function:

$$x_{ni}^j(\omega) = \left[\frac{p_{ni}^k(\omega)}{P_n^j} \right]^{1-\sigma} X_n^j \quad (6)$$

where X_n^j denotes total expenditure and

$$P_n^j = \left[\int p_n^j(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (7)$$

As mentioned, total expenditure on differentiated goods X_n^j consists of spending by both

¹⁰The original Eaton and Kortum (2002) model has a single sector, so T_n depicts a parameter of country-level average productivity while θ provides dispersion across productivity draws and, hence, a basis for gains from trade. In the present model, variance in T_n^j across sectors provides an additional basis for gains from trade due to comparative advantage in the traditional Ricardian sense. For more on this insight, see Levchenko and Zhang (2014).

consumers and producers. Given (1) and (4), this can be expressed as the following:

$$X_n^j = \alpha_n^j w_n L_n + \sum_{k=1}^J \gamma^{j,k} \beta^j Y_n^j \quad (8)$$

where Y_n^j denote gross production in sector j of country n . To clear the goods market for this sector:

$$Y_n^j = \sum_{i=1}^n X_{in}^j \quad (9)$$

Substituting this into total expenditure yields the following:

$$X_n^j = \sum_{k=1}^J \gamma^{j,k} \beta^j \left(\sum_{i=1}^N X_{in}^k \right) + \alpha_n^j w_n L_n \quad (10)$$

Price Index

As in Eaton and Kortum (2002), consumers and producers in n buy goods from the lowest cost producer. Producers are perfectly competitive, setting prices at marginal cost. Exports from i to n are subject to an additional iceberg trade cost of the form $\kappa_{ni} > \kappa_{ii} = 1$ where κ_{ni} units of a given variety need to be exported from i for each unit that arrives in n . As a result, the price of good ω exported from i to n takes the following form:

$$p_{ni}^j(z_i^j(\omega)) = \frac{c_i^j \kappa_{ni}^j}{(z_i^j(\omega))^{1-\beta^j}} \quad (11)$$

where

$$c_i^j = \Psi_i^j w_i^{1-\beta^j} \left[\prod_{k=1}^J P_i^k \gamma^{k,j} \right]^{\beta^j} \quad (12)$$

denotes unit cost of production and Ψ_i^j is a constant ¹¹.

Note that (11) is different here than it is in the standard Eaton and Kortum (2002) model with TFP heterogeneity. In that setting, the analogous expression is the following:

$$p_{ni}^j(z_i^j(\omega))^{EK} = \frac{c_i^j \kappa_{ni}^j}{(z_i^j(\omega))} \quad (13)$$

Expression (7) can be simplified by making use of some convenient properties of the Fréchet distribution. Let $F_{ni}^j(p)$ denote the probability that the price at which country i can supply a given variety in sector j to country n is lower than or equal to p . Rearranging

¹¹Specifically, $\Psi_i^j = \prod_{k=1}^J (\gamma^{k,j})^{-\gamma^{k,j}(1-\beta^j)} (\beta^j)^{-\beta^j} (1-\beta^j)^{\beta^j-1}$

(11) in terms of z_i^j and the using the distribution expression in (3), we find that:

$$F_{ni}^j(p) = 1 - F_i^j(z_i^j(\omega)) = 1 - F_i^j\left(\left(\frac{c_i^j \kappa_{ni}^j}{p}\right)^{\frac{1}{1-\beta^j}}\right) \quad (14)$$

Again, this probability is different from the standard Eaton and Kortum (2002) model, where the expression is the following:

$$F_{ni}^j(p)^{EK} = 1 - F_i^j(z_i^j(\omega)) = 1 - F_i^j\left(\frac{c_i^j \kappa_{ni}^j}{p}\right) \quad (15)$$

Let $p_n^j(\omega) \equiv \min\{p_{n1}^j(\omega), p_{n2}^j(\omega), \dots, p_{nN}^j(\omega)\}$ denote the lowest price of variety ω offered to country n for a particular sector. Then $p_n^j(\omega)$, the price which is actually paid for ω in n , is distributed according to the following function:

$$F_n^j(p) = 1 - \exp\left\{-\phi_n^j p^{\frac{\theta^j}{1-\beta^j}}\right\} \quad (16)$$

where

$$\phi_n^j = \sum_{i=1}^N T_i^j [c_i^j \kappa_{ni}^j]^{\frac{-\theta^j}{1-\beta^j}} \quad (17)$$

See the appendix for a proof of (16).

Substituting (11) into (7) yields the following closed-form solution for the aggregate price index for sector j in n :

$$P_n^j = A^j \left[\sum_{i=1}^N T_i^j [c_i^j \kappa_{ni}^j]^{\frac{-\theta^j}{1-\beta^j}} \right]^{\frac{1-\beta^j}{-\theta^j}} = A^j [\phi_n^j]^{\frac{1-\beta^j}{-\theta^j}} \quad (18)$$

where A^j is a constant ¹². See the appendix for a proof of (18).

2.1.2 Equilibrium

The international trade equilibrium satisfies goods market clearing for all sectors and countries and labor market clearing for all countries, optimization by all consumers and producers and balanced trade for all countries.

Total Bilateral Exports: A Gravity Equation

¹²In particular, $A^j = \Gamma\left(\frac{\theta^j + (1-\sigma)(1-\beta^j)}{\theta^j}\right)^{\frac{1}{(1-\sigma)(1-\beta^j)}}$ and Γ is the Gamma function.

We denote the share of expenditure in n on goods exported from i in sector j as $\pi_{ni}^j = X_{ni}^j/X_n^j$. Again, using some convenient properties of the Fréchet distribution, this share can be represented by the following:

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j [c_i^k \kappa_{ni}^j]^{-\frac{\theta^j}{1-\beta^j}}}{\phi_n^j} \quad (19)$$

See the appendix for a proof of equation (19). Rearranging (19) in terms of X_{ni}^j and substituting this into the goods market clearing equation in (9) yields:

$$Y_i^j = T_i^j (c_i^k)^{-\frac{\theta^j}{1-\beta^j}} \sum_{n=1}^N \frac{(\kappa_{ni}^j X_n^j)^{-\frac{\theta^j}{1-\beta^j}}}{\phi_n^j} \quad (20)$$

Solving this expressing for $T_i^j (c_i^k)^{-\frac{\theta^j}{1-\beta^j}}$ and substituting into (19) yields the following gravity equation:

$$X_{ni}^j = X_n^j Y_i^j \frac{(\kappa_{ni}^j/P_n^j)^{-\frac{\theta^j}{1-\beta^j}}}{\sum_{n=1}^N (\kappa_{ni}^j/P_n^j)^{-\frac{\theta^j}{1-\beta^j}}} \quad (21)$$

Equation (21) is different from standard multi-sectoral Eaton and Kortum gravity equation (e.g. Caliendo and Parro (2012)). In the standard setting with TFP heterogeneity, the gravity equation is the following:

$$X_{ni}^{j\text{ EK}} = X_n^j Y_i^j \frac{(\kappa_{ni}^j/P_n^j)^{-\theta^j}}{\sum_{n=1}^N (\kappa_{ni}^j/P_n^j)^{-\theta^j}} \quad (22)$$

Clearly, the main difference between these expressions relates to the $1 - \beta^j$ term in the exponent of (21). Denoting the trade elasticity with respect to variable trade costs κ_{ni}^j as $\eta_{X,\kappa}^j$, we can derive the following simple expression (controlling for X_n^j , Y_n^j and P_n^j):

$$\eta_{X,\kappa}^j = \frac{-\theta^j}{1 - \beta^j} \quad (23)$$

In contrast, the trade elasticity according to (22) is:

$$\eta_{X,\kappa}^{j\text{ EK}} = -\theta^j \quad (24)$$

In my model with heterogeneity in value-added productivity, sectors that use a higher share of intermediate inputs have a higher elasticity of trade with respect to trade costs. In the model with TFP productivity, this mechanism is absent.

2.2 Monopolistic Competition

The following is a multi-sectoral Melitz (2003) model of international trade based on Chaney (2008).¹³ As in the perfect competition model in Section 2.1, this model yields a closed-form gravity equation in equilibrium. It also provides closed-form distinctions between the intensive and extensive margins of trade. This is useful in identifying an important finding of this paper: that the intermediate inputs share specifically affects the extensive margin.

2.2.1 Environment

Consider a world with N countries and $J+1$ sectors. Country n has labor endowment L_n . Labor is the only factor of production. Consumers in each country derive utility from consuming goods in each of the $J+1$ sectors: the first sector, o , is made up of a single homogeneous good. The other J sectors are each made up of a sector-specific final composite good of differentiated varieties. Consumers in n buy c_n^o units of the homogenous good and C_n^j units of a composite final good in sector j in accordance with the following utility function:

$$U = c_n^o \alpha_n^o \prod_{j=1}^J C_n^j \alpha_n^j \quad (25)$$

where $\alpha_n^o + \sum_{j=1}^J \alpha_n^j = 1$. The budget constraint for consumers in n is given by:

$$\sum_{j=1}^J P_n^j C_n^j + p_n^o c_n^o = I_n \quad (26)$$

where P_n^j denotes the aggregate price index in sector j (described below) and p_n^o denotes the price of the homogenous good in country n .

Labor is freely mobile within a given country n . The homogenous good is produced according to the following constant returns to scale technology:

$$q_n^o = l_n^o \quad (27)$$

where l_n^o denotes the labor input for this sector.

For each of the differentiated sectors, the final composite good consists of a continuum of differentiated varieties indexed by $\omega \in \Omega^j$ where Ω^j is determined in equilibrium. Variety ω in sector j of country n is produced according to the following production

¹³In the original Chaney (2008) model labor is the only input.

function:

$$q_n^j(\omega) = [\varphi_n^j(\omega) l_n^j(\omega)]^{1-\beta^j} \left[\prod_{k=1}^J M_n^{k,j}(\omega) \gamma^{k,j} \right]^{\beta^j} \quad (28)$$

where φ_n^j , l_n^j and $M_n^{k,j}$ denote labor productivity, labor input and intermediate input for the composite intermediate good from sector k respectively. The parameter $\gamma^{k,j}$ denotes the share of intermediate inputs from sector k used in production of sector j , with $\sum_{k=1}^J \gamma^{k,j} = 1$. As with the perfect competition model from the previous section, the productivity parameter $\varphi_n^j(\omega)$ enters here as *value-added* productivity, not total factor productivity. Composite goods Q_n^j are produced using the following CES production technology:

$$Q_n^j = \left(\int_{\Omega^j} q_n^j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad (29)$$

where $\sigma > 1$ denotes elasticity of substitution across varieties. The composite goods are demanded by both consumers, as final goods C_n^j , and producers in sector k , as intermediate goods $M_n^{j,k}$.

Before deciding whether or not to produce, firms in sector j randomly draw φ from the following Pareto distribution:

$$G_j(\varphi) = 1 - \varphi^{-\gamma^j} \quad (30)$$

with $\gamma^j > \sigma - 1$, $dG_j(\varphi) = -\gamma^j \varphi^{-\gamma^j-1}$ and $\varphi \in [1, +\infty)$. The parameter γ^j can be thought of as an inverse dispersion parameters (analogous to θ^j in the previous section). A sector with higher γ^j is more homogenous in terms of productivity draws within the sector.

Goods in the homogenous sector o are traded freely both at home and abroad. Since this sector is perfectly competitive, firms set wages equal to marginal cost. As a result, wages in all countries are equivalent and equal to one: $w_n = w = 1$ for all n .

Goods in the differentiated goods sectors are subject to two sector-specific bilateral trade costs. The first is a variable iceberg cost $\kappa_{ni}^j > \kappa_{ii}^j = 1$ where κ_{ni}^j units of a given variety need to be exported from i for each unit that arrives in n . The second is a fixed cost where f_{ni}^j units of the numeraire good need to be spent before any units of the differentiated goods can be exported from i to n . I assume that $f_{ni}^j > f_{ii}^j = 0$ for all i and j . These bilateral fixed costs lead to country-pair specific increasing returns-to-scale for each differentiated sector j . Let c_i^j denote the unit cost of production in sector j of

country i . This can be represented as:

$$c_i^j = \Psi_i^j w_i^{1-\beta^j} \left[\prod_{k=1}^J P_i^k \gamma^{k,j} \right]^{\beta^j} \quad (31)$$

where Ψ_i^j is a constant ¹⁴. The cost of exporting q units of differentiated variety ω in sector j from country i to country n is:

$$c_{ni}^j(q, \omega) = \frac{c_i^j \kappa_{ni}}{(\varphi)^{1-\beta^j}} q + f_{ni}^j \quad (32)$$

As in the previous section, total demand in n for good ω in sector j follows the CES demand function:

$$x_{ni}^j(\omega) = \left[\frac{p_{ni}^k(\omega)}{P_n^j} \right]^{1-\sigma} X_n^j \quad (33)$$

where X_n^j denotes total expenditure and

$$P_n^j = \left[\int p_n^j(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (34)$$

denotes the aggregate price index in sector j of country n .

Given (25) and (28), total combined expenditure by consumers and producers in n of goods in sector j can be expressed as the following:

$$X_n^j = \alpha_n^j I_n + \sum_{k=1}^J \gamma^{j,k} \beta^j Y_n^j \quad (35)$$

where Y_n^j denote gross production in sector j of country n . To clear the goods market for this sector:

$$Y_n^j = \sum_{i=1}^n X_{in}^j \quad (36)$$

Substituting this into total expenditure yields the following:

$$X_n^j = \alpha_n^j I_n + \sum_{k=1}^J \gamma^{j,k} \beta^j \left(\sum_{i=1}^N X_{in}^k \right) \quad (37)$$

Consumers in n have two sources of income. The first is from wages w_n received in exchange for labor. The second is from dividends paid out by a global mutual fund. Some firms earn profits in equilibrium; these profits go to the fund which pays out dividends

¹⁴As with the perfect competition model, $\Psi_i^j = \prod_{k=1}^J (\gamma^{k,j})^{-\gamma^{k,j}(1-\beta^j)} (\beta^j)^{-\beta^j} (1-\beta^j)^{\beta^j-1}$

to shareholders. I denote global profits as the following:

$$\Pi_W = \sum_{j=1}^J \sum_{i,n=1}^N \int_{\Omega_j} \varpi_{ni}^j(\omega) d\omega$$

where $\varpi_{ni}^j(\omega)$ denotes profits that firm ω in sector j of country i produces from exporting to n . I assume that consumers across the world hold a share in the fund equal to their share of global labor income L_n/L_W , where $L_W = \sum_{i=1}^N L^i$. Total consumer income in n is the sum of labor income and income from the mutual fund: $I_n = (1 + \Pi_W/L_W)L_n$ ¹⁵.

Firms in the differentiated sectors choose prices to maximize profits. In this case, the profit-maximizing price is equal to a constant mark-up over marginal cost:

$$p_{ni}^j(\varphi) = p_i^j(\varphi)\kappa_{ni} = \frac{\lambda_i^j c_i^j \kappa_{ni}}{(\varphi)^{1-\beta^j}} \quad (38)$$

where $\lambda_i^j = \sigma(\sigma - 1)^{-1}$.

Note that prices in this setting are different from those found in Chaney's original model. In Chaney (2008), the price equation is equal to:

$$p_{ni}^j(\varphi)^{CH} = p_i^j(\varphi)\kappa_{ni} = \frac{\lambda_i^j c_i^j \kappa_{ni}}{\varphi} \quad (39)$$

The difference here is due to the form of productivity heterogeneity, which is value-added in my model but total factor productivity in the original Chaney (2008) setting.

Zero Profits Cut-Off

Firms from sector j in i will only export to n if the profits from doing so are positive. We can determine the threshold firm ω^* , characterized by productivity φ_{ni}^{*j} , by solving the zero profits condition, where $\varpi_{ni}(\omega^*) = x_{ni}^j(\omega^*) - c_{ni}^j(q_{ni}^j(\omega^*), \omega^*) = 0$. Substituting (32) and (33) into this equation yields the following:

$$\varpi_{ni}(\omega^*) = \left(\frac{\lambda_i^j \kappa_{ni}^j c_i^j}{P_n^j} (\varphi_{ni}^{*j})^{\beta^j - 1} \right)^{1-\sigma} \frac{X_n^j}{\sigma} - f_{ni}^j = 0 \quad (40)$$

Solving this expression in terms of φ_{ni}^{*j} yields:

$$\varphi_{ni}^{*j} = \left[\left(\frac{f_{ni}^j \sigma}{X_n^j} \right)^{\frac{1}{\sigma-1}} \frac{\lambda_i^j \kappa_{ni}^j c_i^j}{P_n^j} \right]^{\frac{1}{1-\beta^j}} \quad (41)$$

¹⁵The global mutual fund set up is taken from Chaney (2008).

In Chaney (2008), this cut-off is different, equal to the following:

$$\varphi_{ni}^{*j CH} = \left(\frac{f_{ni}^j \sigma}{X_n^j} \right)^{\frac{1}{\sigma-1}} \frac{\lambda_i^j \kappa_{ni}^j c_i^j}{P_n^j} \quad (42)$$

Since $\beta^j > 0$, it is clear that the cut-off is higher, meaning fewer firms export, in my model than in the Chaney (2008) model.

Aggregation

I denote the mass of firms that export from i to n in sector j as F_{ni}^j and restate the expression for the aggregate price index for sector j in n as the following:

$$P_n^j = \left(\sum_{i=1}^N \int_{\varphi_{ni}^{*j}}^{\infty} (p_{ni}^j(\omega))^{1-\sigma} F_{ni}^j \mu_{ni}^j(\omega) d\omega \right)^{\frac{1}{1-\sigma}} \quad (43)$$

where $\mu_{ni}^j(\omega)$ denotes the conditional distribution of $G_j(\omega)$ on $[\varphi_{ni}^{*j}, \infty)$ and can be represented as the following:

$$\begin{aligned} \mu_{ni}^j(\omega) &= \frac{G_j(\omega)}{(1 - G_j(\varphi_{ni}^{*j}))} && \text{if } \varphi > \varphi_{ni}^{*j} \\ &= 0 && \text{if } \varphi < \varphi_{ni}^{*j} \end{aligned} \quad (44)$$

I denote aggregate expenditure in country n and global profits as:

$$X_n^j = \sum_{i=1}^N X_{ni}^j = \sum_{i=1}^N \int_{\varphi_{ni}^{*j}}^{\infty} (x_{ni}^j(\omega)) F_{ni}^j \mu_{ni}^j(\omega) d\omega \quad (45)$$

$$\Pi_W = \sum_{j=1}^J \sum_{n,i=1}^N \int_{\varphi_{ni}^{*j}}^{\infty} (\varpi_{ni}^j(\omega)) F_{ni}^j \mu_{ni}^j(\omega) d\omega \quad (46)$$

2.2.2 Equilibrium

The international trade equilibrium satisfies the goods market clearing condition for all sectors and countries and the labor market clearing condition for all countries, optimization by all consumers and producers and balanced trade for all countries.

The entry and goods market clearing conditions provide for the equilibrium values for F_{ni}^j , X_{ni}^j , X_n^j and P_n^j for all n, i and j .

Entry

I assume that the total mass of potential entrants in any given country i is proportional

to L_i . As a result, the equilibrium mass $F_{ni}^j = L_i(1 - G_j(\varphi_{ni}^{*j}))$ ¹⁶. Since $G_j(\omega)$ is Pareto, $G_j(\varphi_{ni}^{*j}) = (\varphi_{ni}^{*j})^{-\gamma^j}$. Substituting (41) into this expression for F_{ni}^j yields the following expression for the mass of firms exporting from i to n in sector j :

$$F_{ni}^j = L_i \left(\frac{f_{ni}^j \sigma}{X_n^j} \right)^{\frac{-\gamma^j}{(\sigma-1)(1-\beta^j)}} \left(\frac{\lambda_p^j \kappa_{ni}^j \mathcal{C}_i^j}{P_n^j} \right)^{\frac{-\gamma^j}{1-\beta^j}} \quad (47)$$

In the original Chaney (2008) context, we would substitute 42 into F_{ni}^j to find the following expression for this mass:

$$F_{ni}^{j\ CH} = L_i \left(\frac{f_{ni}^j \sigma}{X_n^j} \right)^{\frac{-\gamma^j}{(\sigma-1)}} \left(\frac{\lambda_p^j \kappa_{ni}^j \mathcal{C}_i^j}{P_n^j} \right)^{-\gamma^j} \quad (48)$$

Price Index

Taking the expression for P_n^j in (43) and substituting in (44) and (38) for $\mu_{ni}^j(\omega)$ and p_{ni}^j respectively yields the following:

$$P_n^{j\ 1-\sigma} = \sum_{i=1}^N \left[[\lambda_i^j \kappa_{ni}^j \mathcal{C}_i^j]^{1-\sigma} \frac{F_{ni}^j}{(1 - G_j(\varphi_{ni}^{*j}))} \int_{\varphi_{ni}^{*j}}^{\infty} \varphi^{(1-\beta^j)(\sigma-1)} dG_j(\omega) \right] \quad (49)$$

Since $F_{ni}^j = L_i(1 - G_j(\varphi_{ni}^{*j}))$, the expression outside of the integral in the index simplifies to L_i^j . From the Pareto distribution, it is fairly simple to show that:

$$\int_{\varphi_{ni}^{*j}}^{\infty} \varphi^{(1-\beta^j)(\sigma-1)} dG_j(\omega) = \frac{\gamma^j}{\gamma^j - (\sigma - 1)(1 - \beta^j)} (\varphi_{ni}^{*j})^{(1-\beta^j)(\sigma-1) - \gamma^j} \quad (50)$$

Substituting (41) for φ_{ni}^{*j} into this expression and then plugging back into (49) yields the following closed-form solution for the price index for sector j in country n :

$$P_n^j = A^j \left[\sum_{i=1}^N L_i^j [\kappa_{ni}^j \mathcal{C}_i^j]^{\frac{-\gamma^j}{(1-\beta^j)}} \left(\frac{f_{ni}^j}{X_n^j} \right)^{1 - \frac{\gamma^j}{(\sigma-1)(1-\beta^j)}} \right]^{\frac{(1-\beta^j)}{-\gamma^j}} = A^j [\phi_n^j]^{\frac{1-\beta^j}{-\gamma^j}} \quad (51)$$

where A^j is a constant¹⁷. Notice that P_n^j is fairly similar to the price index in the perfect competition model. The main difference is the absence of sector-specific productivity

¹⁶This entry assumption is taken from Chaney (2008). Note that $(1 - G_j(\varphi_{ni}^{*j}))$ is the probability that a firm will draw φ above the threshold φ_{ni}^{*j} . This is also equal to the proportion of potential firms that are above this threshold.

¹⁷Specifically, $A^j = \lambda_i^j \left[\frac{\gamma^j}{\gamma^j - (\sigma-1)(1-\beta^j)} \right]^{\frac{1}{1-\sigma}} \sigma^{\frac{(\sigma-1)(1-\beta^j) - \gamma^j}{-\gamma^j(\sigma-1)}}$

parameters, the presence of the fixed cost parameters f_{ni}^j and the presence of total sectoral expenditure X_n^j and total labor input L_i for the importer and exporter respectively. Intuitively, the price index is lower, *ceterus paribus*, when partner specific trade costs are lower, or when the sectoral scale of production and/or consumption are higher.

Total Bilateral Exports: A Gravity Equation

Total exports from country i to n in sector j can be expressed as:

$$X_{ni}^j = \int_{\varphi_{ni}^{*j}}^{\infty} (x_{ni}^j(\omega)) F_{ni}^j \mu_{ni}^j(\omega) d\omega$$

Substituting (47) for F_{ni}^j into this expression and incorporating (33), we can restate total bilateral exports as:

$$X_{ni}^j = \left(\frac{\lambda_p^j \kappa_{ni}^j c_i^j}{P_n^j} \right)^{1-\sigma} X_n^j \frac{F_{ni}^j}{(1 - G_j(\varphi_{ni}^{*j}))} \int_{\varphi_{ni}^{*j}}^{\infty} (\varphi_{ni})^{(\sigma-1)(1-\beta^j)} G_j(\omega) d\omega \quad (52)$$

Again, since $F_{ni}^j = L_i(1 - G_j(\varphi_{ni}^{*j}))$, the expression in front of the integral simplifies to L_i . Substituting (41) for φ_{ni}^{*j} into expression (50) to simplify the integral in (52). This yields the following trade share equation:

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{\lambda_x^j L_i [\kappa_{ni}^j c_i^j]^{\frac{-\gamma^j}{(1-\beta^j)}} f_{ni}^j 1^{-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}}}{\phi_n^j} \quad (53)$$

where λ_x^j denotes a constant¹⁸. To find a gravity equation, I rearrange (53) in terms of X_{ni}^j and substitute this into the goods market clearing condition in (36). This yields the following:

$$Y_i^j = \lambda_x^j L_i (c_i^j)^{\frac{-\gamma^j}{(1-\beta^j)}} \sum_{n=1}^N \frac{\kappa_{ni}^j \frac{-\gamma^j}{(1-\beta^j)} f_{ni}^j 1^{-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}} X_n^j}{\phi_n^j} \quad (54)$$

Solving this expression in terms of $\lambda_x^j L_i (c_i^j)^{\frac{-\gamma^j}{(1-\beta^j)}}$ and substituting back into (53) yields the following gravity equation¹⁹:

$$X_{ni}^j = X_n^j Y_i^j \frac{(\kappa_{ni}^j / P_n^j)^{\frac{-\gamma^j}{1-\beta^j}} f_{ni}^j 1^{-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}}}{\sum_{n=1}^N (\kappa_{ni}^j / P_n^j)^{\frac{-\gamma^j}{1-\beta^j}} f_{ni}^j 1^{-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}}} \quad (55)$$

¹⁸Specifically, $\lambda_x^j = \left[\frac{\gamma^j}{\gamma^j - (\sigma-1)(1-\beta^j)} \right]^{1-\frac{\gamma^j}{\gamma^j(1-\sigma)}} \sigma^{\sigma-1}$

¹⁹See the appendix for a derivation of global profits Π_W , which is a constant in equilibrium.

Equation (55) is fairly similar to (21) in Section 2.1. The main difference is the presence of the fixed costs f_{ni}^j in this model, which are absent from the perfect competition model. Moreover, the main difference between the gravity model in (55) and previous gravity models with heterogeneous firms and increasing returns comes through the $1 - \beta^j$ term in the exponent on trade costs. In the Chaney (2008) setting, the following gravity equation can be derived (analogous to (55)):

$$X_{ni}^j{}^{CH} = X_n^j Y_i^j \frac{(\kappa_{ni}^j / P_n^j)^{-\gamma^j} f_{ni}^j{}^{1 - \frac{\gamma^j}{\sigma - 1}}}{\sum_{n=1}^N (\kappa_{ni}^j / P_n^j)^{-\gamma^j} f_{ni}^j{}^{1 - \frac{\gamma^j}{\sigma - 1}}} \quad (56)$$

I denote the trade elasticity with respect to variable trade costs κ_{ni}^j and fixed trade costs f_{ni}^j for sector j as $\eta_{X,\kappa}^j$, and $\eta_{X,f}^j$ respectively. Based on (55), I derive the following simple expressions (controlling for X_n^j , Y_n^j and P_n^j):

$$\eta_{X,\kappa}^j = \frac{-\gamma_j}{1 - \beta^j}, \quad \eta_{X,f}^j = 1 - \frac{-\gamma_j}{(1 - \beta^j)(\sigma - 1)} \quad (57)$$

Chaney (2008) derives a similar expression based on (56) the following:

$$\eta_{X,\kappa}^j{}^{CH} = -\gamma_j, \quad \eta_{X,f}^j{}^{CH} = 1 - \frac{-\gamma_j}{(\sigma - 1)} \quad (58)$$

Like for the perfect competition model, sectors in this model that use a higher share of intermediate inputs have a higher trade elasticity.

2.3 Extensive and Intensive Margins

Expression (47) denotes the mass of exporting firms from i to n in sector j . This provides for an exclusive identification of the extensive margin in the monopolistically competitive model. Denoting the elasticity of F_{ni}^j with respect to variable trade costs κ_{ni}^j and fixed trade costs f_{ni}^j for sector j as $\eta_{M,\kappa}^j$ and $\eta_{M,f}^j$ respectively, we can derive the following identical expression to (58) for the extensive margin:

$$\eta_{M,\kappa}^j = \frac{-\gamma_j}{1 - \beta^j}, \quad \eta_{M,f}^j = 1 - \frac{-\gamma_j}{(1 - \beta^j)(\sigma - 1)} \quad (59)$$

That is, the impact of the the intermediate inputs share β^j on the trade elasticity occurs at the extensive margin.

To illustrate, I reproduce the following firm-level bilateral exports equation from (33):

$$x_{ni}^j(\omega) = \left[\frac{\lambda c_i^j \kappa_{ni}^j (z_i^j)^{\beta^j - 1}}{P_n^j} \right]^{1-\sigma} X_n^j \quad (60)$$

Note that I have substituted in the price equation from (34). Clearly, the elasticity of trade with respect to variable trade costs for individual exporting firms is $1 - \sigma$. When trade costs fall, this firm-specific margin is exactly canceled by the compositional effect due to other firms in i that enter the export market to n . This is an artifact of the Pareto distribution. As for the trade elasticity with respect to fixed costs, there is no intensive or composition margin in the model, only an extensive margin. In the end, the extensive margin describes the entire trade elasticity at both margins in equilibrium.²⁰

The same point applies in the perfectly competitive model in Section 2.1, although the margins are not as clearly delineated as in the monopolistic competition model. At the firm-level, equation (6) depicts a similar equation for exports as (51); as such, the trade elasticity is again $1 - \sigma$. As with the Pareto distribution, the compositional effect with the Fréchet distribution is $\sigma - 1$ which fully cancels out the firm-specific margin. Overall, the trade elasticity in both models is entirely driven by the extensive margin or the number of firms exporting between two given trade partners.

2.4 Discussion

The main theoretical novelty in this framework relates the trade elasticity positively to the intermediate inputs share. This mechanism is not present in the standard Eaton and Kortum (2002) or Chaney's (2008) Melitz model.

This relationship relies on three components. First, production must include intermediate inputs. Second, there must be heterogeneity in firm productivity. Third, firm productivity must enhance value-added, not total factor productivity. A model missing any one of these elements will not produce this relationship.

The explanation for this mechanism is fairly intuitive. When intermediates are used in production, firms must carry an additional production cost. In the standard models, firms draw productivity that enhances all factors equally. As a result, this intermediate production cost includes additional productivity as well. In my framework, firms draw productivity that only enhances the value-added share of production. When firms export, they must pay an additional iceberg trade cost (or fixed trade cost) above domestic

²⁰This insight, described in Chaney (2008), explains why representative firm models like Krugman and Venables (1995), which have a similar production function as (33), do not yield a similar role for intermediate inputs in the trade elasticity as this model.

production costs. This additional trade cost affects total output, including intermediate inputs. Since firms pay a trade cost on total output but only benefit from productivity in value-added, the mass of firms that can export competitively is smaller when intermediate inputs are used in production.

When firms add a small share of value to an existing intermediate good, firm productivity must be significantly higher in order to compete internationally. In a world with global production chains where firms sometimes contribute a small piece along the chain, it is intuitive that trade barriers exact a significant influence on the extensive margin so that only very productive firms can operate. My framework is meant to capture this detail.

As in most of the previous literature, I use the Fréchet and Pareto distributions to model firm heterogeneity mainly because these distributions yield clean analytical solutions. However, Luttmer (2007) offers encouraging evidence, using number of employees to proxy for firm size, that the Pareto distribution provides a good approximation for the distribution of exporting firms in the United States. Since the number of employees is closely associated with value-added (as opposed to total output), I consider this evidence to be fairly supportive of using a Pareto distribution to model firm heterogeneity in value-added productivity.

The standard production frameworks in Eaton and Kortum (2002) and Melitz (2003) provide no relationship between the intermediate inputs share and the trade elasticity. In Section 5 of this paper, I find empirical evidence that the trade elasticity is higher for sectors and countries that use intermediate inputs in production. This pattern is consistent across various measures of trade costs and consistent with my theoretical findings.

2.5 Gains From Trade

To illustrate the welfare impact of international trade in this framework, I consider a simplified model where $\gamma^{j,j} = 1$ and $\gamma^{j,k} = 0$ for all $k \neq j$. That is, sector j uses only intermediate goods from its own sector in production.²¹ Welfare per capita in country n for this case is equal to that country's real wage, depicted as the following:

$$W_n = \frac{w_n}{P_n^c} \quad (61)$$

where $P_n^c = \prod_{j=1}^J P_n^j \alpha_n^j$ denotes the aggregate price index for consumers in n .²² Note that (61) applies to both the perfect and monopolistic competition models.

For the perfect competition model, from equation (19) we can rearrange to find the following expression for P_n^j :

$$P_n^j = \left(\frac{T_n^j}{\pi_{nn}^j} \right)^{\frac{1-\beta^j}{-\theta^j}} c_n^j \quad (62)$$

where κ_{nn}^j is assumed to be 1. Note that, given the simplified input-output assumption, the unit cost is $c_n^j = \Psi_n^j w_n^{1-\beta^j} P_n^j \beta^j$ from equation (12). Substituting this expression into (62) and solving for the price index P_n^j yields the following:

$$P_n^j = \left(\frac{T_n^j}{\pi_{nn}^j} \right)^{\frac{1}{-\theta^j}} \Psi_n^j w_n \quad (63)$$

Finally, substituting this expression into (61) yields the following expression for welfare per capita in n :

$$W_n = \prod_{j=1}^J \left(\frac{\lambda_{wP}^j}{\pi_{nn}^j} \right)^{\frac{\alpha_n^j}{\theta^j}} \quad (64)$$

where $\lambda_{wP}^j = (T_n^j)^{\frac{1}{\theta^j}} \Psi_n^j$ is a constant.

For the monopolistic competition model, from equation (53) we can rearrange to find

²¹When $\gamma^{j,j} = 1$ the gains from trade reduce to an simple analytical solution. This is convenient the purposes of illustrating the mechanisms of my model. Calculating the welfare impact in the model with sectoral linkages requires a more sophisticated quantitative model with sectoral data across all countries in the sample. While such data is available for more recent years, it is more difficult to find for the 1980s (see Caliendo and Parro (2012) for a breakdown of the channels and data requirements for this task). Levchenko and Zhang (2012) find that the model with $\gamma^{j,j} = 1$ provides an upper bound for the true gains from trade using data for 40 different countries from 2005.

²²In the monopolistic competition model in Section 2.2., $p_o = w = 1$ for all n .

the following expression for P_n^j :

$$P_n^j = \left(\frac{\lambda_x^j L_n}{\pi_{nn}^j} \right)^{\frac{1-\beta^j}{-\theta^j}} c_n^j \quad (65)$$

where κ_{nn}^j and f_{nn}^j are assumed to be 1. The unit cost expression (31), given the input-output assumption, is equal to $c_n^j = \Psi_n^j w_n^{1-\beta^j} P_n^{j\beta^j}$ as in the perfect competition model. Substituting this expression into (65) and solving for the price index P_n^j yields the following:

$$P_n^j = \left(\frac{\lambda_x^j L_n}{\pi_{nn}^j} \right)^{\frac{1}{-\theta^j}} \Psi_n^j w_n \quad (66)$$

Finally, substituting this expression into (61) yields the following expression for welfare per capita in n :

$$W_n = \prod_{j=1}^J \left(\frac{\lambda_{wM}^j}{\pi_{nn}^j} \right)^{\frac{\alpha_n^j}{\theta^j}} \quad (67)$$

where $\lambda_{wM}^j = (\lambda_x^j L_n)^{\frac{1}{\theta^j}} \Psi_n^j$ is a constant.

Notice that (64) and (67) are remarkably similar. The only difference between them relates to the constant terms λ_{wC}^j and λ_{wM}^j . I therefore denote the following generalization for welfare in either model:

$$W_n = \prod_{j=1}^J \left(\frac{\lambda_w^j}{\pi_{nn}^j} \right)^{\frac{\alpha_n^j}{\theta^j}} \quad (68)$$

where $\lambda_w^j = \lambda_{wC}^j$ in the perfect competition and $\lambda_w^j = \lambda_{wM}^j$ in the monopolistic competition model.

To find the gains from trade, I take the logarithm of (68) and consider comparative statics of going from autarky, where $\pi_{nn}^j = 1$ for all j , to the status quo where $\pi_{nn}^j = \tilde{\pi}_{nn}^j \leq 1$ for all j . The gains can be denoted as:

$$GFT_n = d \ln(W_n) = - \sum_{j=1}^J \frac{\alpha_n^j}{\theta^j} d \ln(\pi_{nn}^j) \quad (69)$$

To calculate the gains from trade in n , all that one needs is data on 3 variables: sectoral spending on final goods (α_n^j) for all j in n , the share of sectoral home consumption (π_{nn}^j) for all j in n , and sectoral dispersion parameters (θ^j).

Equation (68) is different here than it would be for the case with heterogeneity in TFP (e.g the standard the Eaton and Kortum (2002) model with intermediate inputs).

In that environment, welfare simplifies to the following:

$$W_n^{TFP} = \prod_{j=1}^J \left(\frac{\lambda_w^j}{\pi_{nn}^j} \right)^{\frac{\alpha_n^j}{\theta^j (1-\beta^j)}} \quad (70)$$

Note that in the TFP environment, consumers benefit more, *ceteris paribus*, from consuming goods from sectors produced with a larger share of intermediate inputs, as indicated by the $1 - \beta^j$ term in the exponent of (70).

The gains from trade with TFP heterogeneity are the following

$$GFT_n^{TFP} = d\ln(W_n^{TFP}) = - \sum_{j=1}^J \frac{\alpha_n^j}{\theta^j (1 - \beta^j)} d\ln(\pi_{nn}^j) \quad (71)$$

Since $\beta^j \in (0, 1)$ for all j , it is clear that the gains from trade are higher in (71) than (69) for a common set of π_{nn}^j , α_i^j and θ^j across the two models. In both models, intermediate goods are used to produce both intermediate and final goods. In the TFP model, this input-output loop for intermediate goods leads to an amplification effect in the gains from trade. As a result, the larger the share of intermediate inputs, the higher the gains from trade. In contrast, when productivity enhances value-added, as in my model, this amplification effect disappears. This reveals that the amplification is not due to the input-output loop *per se*, but depends of the form of the productivity parameter in the production function. In the standard TFP model, firm productivity enhances both value-added and intermediate inputs by the same factor, creating a compounding effect for productivity through the input-output loop. This mechanism is absent from my value-added framework.²³

This is not to say, however, that estimates of gains from trade will necessarily be higher using the TFP model. Equations (69) and (71) each depend on dispersion parameters θ_i^j which should be estimated with the model in mind. As I demonstrate in the Section 4, when these parameters are estimated using an empirical gravity equation, the estimates depend on the trade elasticity which differs across the two models.

²³Melitz and Redding (2014) reveal that the gains from trade can become arbitrarily large in framework with sequential production and TFP heterogeneity. This point can be equally demonstrated by setting β_i^j close to zero in the TFP model illustrated here.

3 Data

To compute the gains from trade under my specification in (69), data are needed for sectoral dispersion parameters (θ^j), sectoral home consumption (π_{nn}^j) and the sectoral consumption shares (α_n^j). To compare with the gains from trade under the standard TFP heterogeneity model according to (71), data for sectoral intermediate inputs shares in production (β^j) are also needed.

3.1 Sectoral Dispersion

To ascertain parameters for θ_i^j , it is common in the literature to estimate a gravity equation based on the theoretical model. In my model with value-added heterogeneity, this equation is represented by (21). In the standard TFP heterogeneity framework, this equation is given by something similar to the following:

$$X_{ni}^j = X_n^j Y_i^j \frac{(\kappa_{ni}^j / P_n^j)^{-\theta^j}}{\sum_{n=1}^N (\kappa_{ni}^j / P_n^j)^{-\theta^j}} \quad (72)$$

Caliendo and Parro (2012) provides a prominent recent example of sectoral estimates for θ_i^j under the TFP specification. The authors develop a multi-sectoral Eaton and Kortum (2002) model similar to the model in Section 2.1. They derive the following trade share equation for exports from n to i in sector j :

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j [c_i^k \kappa_{ni}^j]^{-\theta^j}}{\sum_{i=1}^N T_i^j [c_i^j \kappa_{ni}^j]^{-\theta^j}} \quad (73)$$

This equation is analogous to (19) in the value-added heterogeneity model. To estimate $-\theta^j$, they consider the following tetradic ratio for trade between n , i and h in sector j , based on (73):

$$\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} = \left(\frac{\kappa_{ni}^j \kappa_{ih}^j \kappa_{hn}^j}{\kappa_{in}^j \kappa_{hi}^j \kappa_{nh}^j} \right)^{-\theta^j} \quad (74)$$

This ratio conveniently eliminates everything in (73) except for bilateral trade costs and the dispersion parameter to be estimated. Note that any symmetric components of trade costs also cancel out in this expression. In fact, any country fixed effects cancel as well. To estimate (74), the authors gather asymmetric tariff data from UNCTAD-TRAINS from 1989 to 1993 across 16 economies and 20 sectors (18 manufacturing and 2 non-manufacturing).²⁴ Denoting bilateral tariffs on imports from i to n in sector j as τ_{ni}^j ,

²⁴The economies included are Argentina, Australia, Canada, Chile, China, the European Union, India,

they specify the following estimation equation based on the logarithm of (74):

$$\ln \left(\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} \right) = -\theta^j \ln \left(\frac{\tau_{ni}^j \tau_{ih}^j \tau_{hn}^j}{\tau_{in}^j \tau_{hi}^j \tau_{nh}^j} \right) + \epsilon^j \quad (75)$$

where ϵ^j denotes an i.i.d. error term. Caliendo and Parro estimate (75) using OLS with heteroskedasticity-robust standard errors, dropping observations with zero flows. In the first two columns of Table 1, I report the estimates and standard errors from their baseline full sample estimation.²⁵

Table 1: Dispersion Parameters for ISIC Rev. 2 Groups

Isic Rev. 2 group	θ_{EK}	Se.	θ_{VA}	Se.	Obs
Food, Beverages and Tobacco	2.545	(0.612)	0.607	(0.159)	495
Textiles, Apparel and Leather	5.561	(1.145)	1.864	(0.373)	437
Wood prod. and Furniture	10.833	(2.531)	3.699	(0.783)	315
Paper, Paper prod. and Printing	9.065	(1.693)	3.138	(0.641)	507
Industrial chemicals	4.750	(1.768)	1.628	(0.581)	430
Rubber and Plastic products	1.665	(1.409)	0.541	(0.447)	376
Non-metallic mineral products	2.765	(1.436)	1.088	(0.577)	342
Iron and Steel	7.986	(2.526)	2.478	(0.802)	388
Metal products	4.296	(2.154)	1.207	(0.681)	404
Non-electrical machinery	1.516	(1.806)	0.425	(0.591)	397
Office and Computing mach.	12.794	(2.140)	4.128	(0.666)	306
Electrical apparatus, nec	10.599	(1.376)	3.703	(0.471)	343
Radio, TV and Comm. equipment	7.075	(1.718)	2.033	(0.560)	312
Medical	8.981	(1.253)	2.871	(0.407)	383
Motor vehicles	1.015	(0.799)	0.331	(0.209)	237
Transport	0.370	(1.079)	0.065	(0.301)	245
Other manufacturing	5.002	(0.924)	1.764	(0.328)	412
Average	5.11		1.67		

According to my model with value-added heterogeneity described in Section 2.2., I derive the following analog to (74) based on (19):

$$\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} = \left(\frac{\kappa_{ni}^j \kappa_{ih}^j \kappa_{hn}^j}{\kappa_{in}^j \kappa_{hi}^j \kappa_{nh}^j} \right)^{\frac{-\theta^j}{1-\beta^j}} \quad (76)$$

Notice that the right-hand side of (76) is equal Caliendo and Parro's expression, to the exponent of $1/(1 - \beta^j)$. In order to find θ^j according to the value-added model, I adjust

Indonesia, Japan, Korea, New Zealand, Norway, Switzerland, Thailand and the United States.

²⁵Caliendo and Parro present estimates according to 20 ISIC revision 3 industries. The values in Table 1 are converted into ISIC revision 2 classification using the correspondence in the appendix.

Caliendo and Parro's tetradic tariff ratio data to be consistent with the specification in (76). That is, I adjust the regressors from Caliendo and Parro's data to the following:

$$\ln \left(\frac{\tau_{ni}^j \tau_{ih}^j \tau_{hn}^j}{\tau_{in}^j \tau_{hi}^j \tau_{nh}^j} \right)^* = \left(\frac{1}{1 - \tilde{\beta}_i^j} \right) * \ln \left(\frac{\tau_{ni}^j \tau_{ih}^j \tau_{hn}^j}{\tau_{in}^j \tau_{hi}^j \tau_{nh}^j} \right) \quad (77)$$

where $\tilde{\beta}_i^j$ is the mean of intermediate inputs share across the three countries n , i and h for the given sector j , calculated using input-output data.²⁶ I then re-estimate θ^j under the following value-added heterogeneity specification:

$$\ln \left(\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} \right) = -\theta^j \ln \left(\frac{\tau_{ni}^j \tau_{ih}^j \tau_{hn}^j}{\tau_{in}^j \tau_{hi}^j \tau_{nh}^j} \right)^* + \epsilon^j \quad (78)$$

In the fourth and fifth columns of Table 1, I report my estimates from the full sample. Not surprisingly, the adjusted estimates of θ^j are significantly lower than the original estimates due to the impact of dividing the original regressors by $(1 - \tilde{\beta}_i^j)$. This translates into a higher degree of dispersion within sectors. Moreover, variation in the TFP estimates is considerably higher than that in the value-added estimates. Based on the TFP estimates, we would conclude that dispersion varies significantly across sectors; that technological competitiveness is much higher in, for example, the Transport sector where $\tilde{\theta}_j = 0.370$ than in the Office, Computing and Machinery sector where $\tilde{\theta}_j = 12.79$. By comparison, the estimates from the value-added framework suggest far lower variation in dispersion across sectors, meaning that technological competitiveness is relatively similar across sectors.

The gains from trade are higher when sectoral dispersion parameters are low. This is true for both the value-added and TFP frameworks, as indicated by equations (71) and (69). Note, however, that the TFP specification in (71) has a $(1 - \beta^j)$ term which (69) is missing. This raises the gains from trade under TFP heterogeneity. In the end, the lower estimates of θ^j from the value-added specification counterbalance the welfare-reducing impact of the missing $(1 - \beta^j)$, resulting in an ambiguous overall difference in the gains from trade between the two theoretical models. Empirically, whether the gains from trade are higher under the value-added versus TFP specification will come down to quantitative differences in the dispersion parameters and the β^j shares.

²⁶Due to difficulty in finding input-output data for Thailand, I simply set that countries values of β^j to the overall average across all other countries.

3.2 Intermediate Inputs Shares

To capture sectoral intermediate inputs shares (β^j), I use data from OECD input-output tables. Because this share varies across countries, for the empirical analysis I allow β^j to vary across i . Previous literature specifically emphasizes the relationship between the trade elasticity and the imported intermediate share (Johnson and Noguera (2012b)). However, the model I present in Section 2 suggests that trade elasticity should have a negative relationship with intermediate inputs share regardless of whether the inputs are produced domestically or abroad. To explore this, I distinguish between β_{ih}^j for domestic- and β_{if}^j for foreign-produced intermediates inputs share.

For each country, data for 22 manufacturing ISIC Revision 3 sectors are available from the OECD for 2000 and 2005. To avoid the impact of primary resources, which could be governed by forces outside of the purview of the models in Section 2, I exclude two manufacturing sectors: i) Coke, refined petroleum products and nuclear fuel, and ii) Non-ferrous metals. A list of the sectors included is provided in the appendix. For the years prior to 1995, tables are available under ISIC Revision 2 classification. For consistency, I converted the later years to Revision 2 using a self-constructed correspondence table provided in the appendix. Since I am interested in comparing recent periods to the 1980s, I am restricted to nine exporting countries. The specific years included and availability of the data by year for each exporting country is reported in the appendix.

Table 10 in the appendix reports average shares of domestic- and foreign-sourced intermediate inputs by sector over time across these nine countries. As we see, β_{ih}^j is consistently higher than β_{if}^j across all sectors. However, if we compare over time, the domestic share has generally decreased while the foreign share increased from the 1980s to 2000s across nearly every sector. On average, the share of intermediate inputs used in production has risen a few percentage points over time.

3.3 Exports

For exports (X_{ni}^j), I use bilateral export data from UN comtrade. To consider the extensive margin, I initially obtained data disaggregated to the 4-digit SITC Revision 2 level.²⁷ I then aggregated the data to correspond with the 20 ISIC Revision 2 sectors, and kept count of the number of varieties F_{ni}^j in each sector.²⁸ As a result, I can define the following expression:

$$X_{ni}^j = F_{ni}^j \times \bar{X}_{ni}^j$$

²⁷This level of aggregation includes 768 product varieties. The same level of aggregation is used in Berthelon and Freund (2008) to examine this time period.

²⁸To link SITC Revision 2 to ISIC Revision 2 groups, I used correspondence tables downloaded from the United Nations Statistical Division website.

where \overline{X}_{ni}^j denotes the average bilateral exports between i and n in sector j . This provides for distinction between the extensive (F_{ni}^j) and compositional/intensive (\overline{X}_{ni}^j) margins separately in the empirical analysis. For all three dependent variables, I include data from the nine exporting countries to 151 recipient countries.²⁹

3.4 Trade Costs

To estimate the trade elasticity, I consider several different measures of trade costs (κ_{ni}). These include bilateral distance (d_{ni}) and dummy variables for bilateral regional trade agreement (rta_{ni}), common border (b_{ni}) and currency union (cu_{ni}).³⁰ All of these data came from CEPII.³¹

3.5 Other Input-Output Parameters

To calculate the gains from trade, we need measures of home consumption shares π_{nn}^j and sectoral consumption shares α_n^j across countries and sectors. I find both using the set of input-output tables listed in the appendix.

Home exports are calculated by sector as:

$$\tilde{\pi}_{nn}^j = 1 - \frac{(\text{total imports})_n^j}{(\text{total expenditure})_n^j} \quad (79)$$

Total expenditure is calculated as the summation of sectoral intermediate consumption plus final consumption expenditure by households, non-profit organizations and government, gross fixed capital formation and changes in inventories.

One issue that arises in this analysis is the change in these consumption shares over time. In the early 1980s, the manufacturing share in total consumption was on average 36.4% across the countries in my sample; by the mid-2000s, this share dropped to 26.7%. As a result, supposing that these shares remain unchanged in the autarky counter-factual, the gains from manufacturing trade are significantly higher in the 1980s despite the fact that home consumption shares fell on average over the two periods. To address this issue, I take the mean of the consumption shares across the two periods for each sector and set this equal to the share for both periods. That is, I focus on changes in the gains from

²⁹A list of recipient countries is provided in the appendix.

³⁰I also explored using common language and colonial linkage dummy variables. These measures, however, had little explanatory power.

³¹Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) provides several different measures of bilateral distance. I use the population-weighted measure of agglomeration-by-agglomeration distance created in Head and Mayer (2008). This is calculated as: $d_{ni} = (\sum_{k \in n} (pop_k / pop_n) \sum_{l \in i} (pop_l / pop_i) d_{kl})$ where pop_k denotes population in agglomeration k inside country n .

trade due to changes in intermediate inputs shares and home consumption, rather than changes in the share of manufacturing in total consumption.³²

To address potential measurement biases in year-specific data, I took averages for each variable across the early-1980s and early-2000s. Summary statistics are provided in Table 2.

Table 2: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
$\ln(X_{ni}^j)$	13.445	3.348	0.69	24.518	39045
$\ln(F_{ni}^j)$	2.347	0.981	0.693	4.585	39104
$\ln(\bar{X}_{ni}^j)$	6.668	3.015	0	27.245	39045
β_i^j	0.546	0.097	0.297	0.871	39104
β_{fi}^j	0.185	0.115	0.009	0.776	39104
β_{hi}^j	0.36	0.127	0.028	0.798	39104
$\ln(d_{ni})$	8.729	0.778	5.938	9.847	39104
b_{ni}	0.016	0.127	0	1	39104
cu_{ni}	0.009	0.096	0	1	39104
rta_{ni}	0.124	0.33	0	1	39104
θ_{VA}^j	1.747	1.259	4.128	0.065	39104
θ_{EK}^j	5.354	3.718	12.794	0.37	39104

³²A comprehensive analysis of the overall gains from trade would need to consider why consumption shares have changed over time and include the gains from non-manufacturing trade which have surely increased over time. My analysis is more focused on exploring the impact my value-added specification on the gains from trade in the presence of intermediate inputs than on calculating the overall economy-wide gains from trade. As a result, I ignore these additional channels of welfare analysis.

4 Gains from Trade: Empirical Results

I calculate the gains from manufacturing trade using data for sectoral spending on final goods (α_n^j), the share of sectoral home consumption (π_{nn}^j), and my sectoral dispersion parameters estimates (θ_{VA}^j), as described in Section 3.

These are calculated according to my value-added heterogeneity specification based on the following equation (derived in Section 2.5):

$$GFT_n = d\ln(W_n) = - \sum_{j=1}^J \frac{\alpha_n^j}{\theta^j} d\ln(\pi_{nn}^j) \quad (80)$$

I also calculate the gains from trade under the total factor productivity specification. These were calculated using Caliendo and Parro's estimates of θ_{EK}^j according to the following standard gains from trade equation:

$$GFT_n^{TFP} = d\ln(W_n^{TFP}) = - \sum_{j=1}^J \frac{\alpha_n^j}{\theta^j (1 - \beta^j)} d\ln(\pi_{nn}^j) \quad (81)$$

I report the results under the value-added specification in columns 1 and 2 of Table 3 for all 9 countries for the 1980s and 2000s. The results based on the total factor productivity specification are reported in columns 3 and 4.

Table 3: Gains from Trade

Country	<i>Value-Added</i>		<i>TFP</i>	
	1980s	2000s	1980s	2000s
Australia	15.5%	17.9%	9.7%	13.3%
Canada	55.3%	85.3%	52.0%	64.7%
Denmark	23.9%	87.4%	15.2%	47.8%
France	12.8%	24.5%	10.2%	18.2%
Germany	11.2%	43.0%	8.1%	27.1%
Italy	9.3%	16.5%	6.7%	11.7%
Japan	2.8%	6.6%	1.6%	4.4%
United Kingdom	28.8%	64.4%	15.5%	31.6%
United States	4.8%	9.1%	3.7%	6.8%
Average	18.3%	39.4%	13.6%	25.1%

The gains from manufacturing trade are, in all cases, larger under the value-added specification for both the 1980s and 2000s. On average, the gains rise by 34% in the 1980s and 57% in the 2000s in moving from the TFP to the value-added framework. This difference is driven by the lower dispersion parameters estimated under the value-

added framework, reported in Table 1. Although the TFP specification might yield higher gains through the intermediate inputs channel (which is absent under the value-added model), this effect is less significant quantitatively than the difference in productivity dispersion estimates. Among the nine countries listed, the gains from trade are highest in Canada and Denmark in the 2000s. These two countries were particularly open to trade during this period.

Figure 3: Percentage Growth in Gains from Trade, 1980s to 2000s

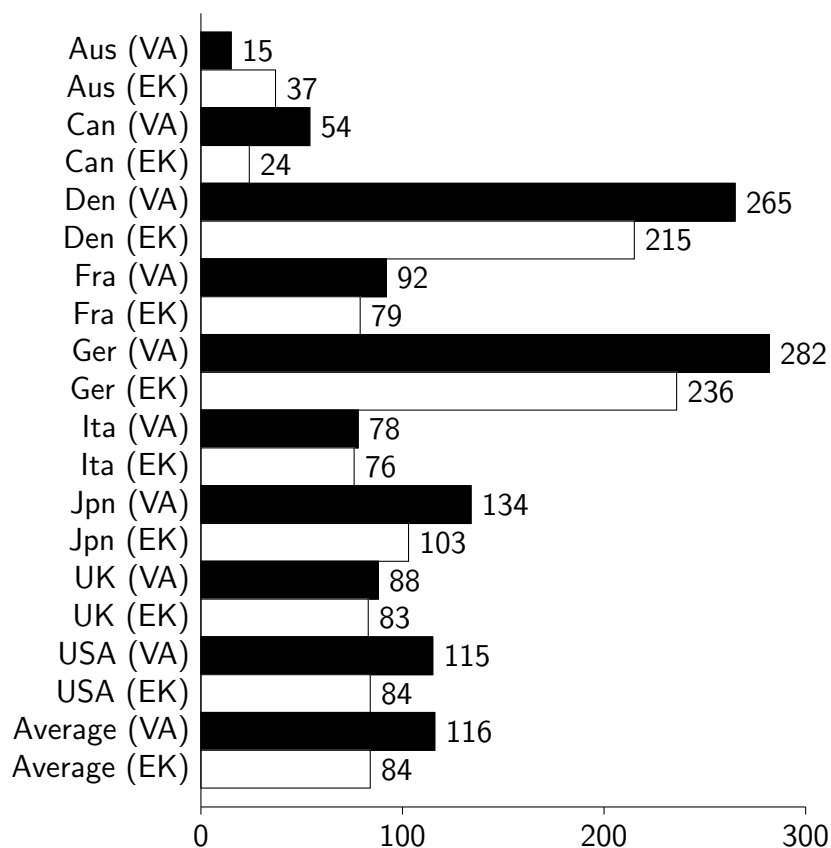


Figure 3 depicts the growth in gains from trade over the two periods for each country. In all cases, the gains from trade grew substantially from the 1980s to 2000s. Growth was generally higher under the value-added specification. On average, the gains grew by 116% and 84% under the value-added and TFP specifications respectively. This growth was highest in Germany and Denmark. This is not surprising since the European economy became much more integrated over between these two periods.

Overall, these findings suggest that the distinction between TFP and value-added frameworks is quite significant quantitatively and should be considered when calculating the welfare gains from international trade.

5 The Trade Elasticity: Empirical Results

5.1 Empirical Specification

The gravity models in (21) and (55) have two distinct features that are different from the previous gravity equations in the literature. The first is that the elasticity of trade with respect to trade costs (the “trade elasticity”) is positively related to the share of intermediate inputs used in production (β^j). The second is that this relationship is driven by the extensive margin; the number of goods exported from i to n is more sensitive to changes in trade costs when intermediates inputs are used in production.

To examine these relationships in a reduced form, one must control for every item in these equations except for the bilateral trade cost expressions $\kappa_{ni}^j \frac{-\theta^j}{1-\beta^j}$, $\kappa_{ni}^j \frac{-\gamma^j}{1-\beta^j}$ and $f_{ni}^j 1 - \frac{\gamma^j}{(\sigma-1)(1-\beta^j)}$. One way to achieve this would be to divide these expressions by exporter and importer home consumption (π_{nn}^j and π_{ii}^j).³³ This approach, however, requires input-output tables for all importing and exporting countries. Since I only have these tables for 9 countries, the sample would be severely restricted. Instead, I follow a tetradic ratio approach developed by Head et al. (2010). Considering sectoral exports between n , i , a reference exporter l and a reference importer k in sector j , we can derive the following tetradic ratios that accord with (21) and (55) respectively:

$$\frac{X_{ni}^j X_{kl}^j}{X_{nl}^j X_{ki}^j} = \left(\frac{\kappa_{ni}^j \kappa_{kl}^j}{\kappa_{nl}^j \kappa_{ki}^j} \right)^{\frac{-\theta^j}{1-\beta^j}}, \quad (82)$$

$$\frac{X_{ni}^j X_{kl}^j}{X_{nl}^j X_{ki}^j} = \left(\frac{\kappa_{ni}^j \kappa_{kl}^j}{\kappa_{nl}^j \kappa_{ki}^j} \right)^{\frac{-\gamma^j}{1-\beta^j}} \left(\frac{f_{ni}^j f_{kl}^j}{f_{nl}^j f_{ki}^j} \right)^{1 - \frac{\gamma^j}{(\sigma-1)(1-\beta^j)}} \quad (83)$$

These ratios conveniently cancel out any exporter and importer sectoral fixed effects that are found in the theoretical gravity equations. Unlike (74) from Caliendo and Parro (2012), however, these ratios do not cancel out symmetric bilateral trade costs. Taking the logarithm of (82), I define the following log-linearized theoretically consistent empirical gravity specification for marginal trade costs:

$$\ln \left(\tilde{X}_{ni}^j \right) = \left(\frac{-\theta^j}{1 - \beta^j} \right) \ln \left(\tilde{\kappa}_{ni}^j \right) + \epsilon_{ni}^j \quad (84)$$

where $\tilde{X}_{ni}^k = \frac{X_{ni}^k X_{kl}^k}{X_{nl}^k X_{ki}^k}$, $\tilde{\kappa}_{ni}^k = \frac{\kappa_{ni}^k \kappa_{kl}^k}{\kappa_{nl}^k \kappa_{ki}^k}$ and ϵ_{ni}^j is an error term assumed to be i.i.d .

³³This approach is referred to in the literature as the Head-Ries Index. Note that I also cannot use country-sector fixed effects, since this would eliminate much of the variation in β_i^j that I wish to exploit.

As mentioned in the previous section, data on β^j varies across countries empirically. Allowing for this variation, I define $\tilde{\beta}_i^j$ as the mean of intermediate inputs shares between the exporter i and the reference exporter l in sector j .

In theory, $\tilde{\kappa}_{ni}^j$ consists of both observed and unobserved bilateral trade costs. To capture these, I include data on the log of bilateral distance (d_{ni}) and dummy variables for bilateral regional trade agreements (rta_{ni}), common borders (b_{ni}) and currency unions (cu_{ni}). I assume that any unobserved determinants of intermediate inputs shares and trade costs that are excluded are orthogonal to the error term ϵ_{ni}^j .

The tetradic reference country method raises the difficulty of choosing reference countries. Including reference countries inevitably restricts the sample of observations. Since I only have 9 exporting countries to begin with, I would like to choose a reference importer that is not among this group of 9 to maximize the number of observations. The reference importer should be a large economy that is relatively open to imports, again to provide as many observations as possible. I have chosen the United States and the Netherlands as the reference exporter and importer respectively.

I analyze the relationship between X_{ni}^j , κ_{ni}^j and β_i^j in (84) using two methodologies. In the first, I estimate the following equation based on the theoretical trade elasticity:

$$\ln\left(\tilde{X}_{ni}^j\right) = \lambda_0 + \lambda_1 \left(\frac{1}{1 - \lambda_2 \tilde{\beta}_i^j}\right) \ln\left(\tilde{\kappa}_{ni}^j\right) + \epsilon_{ni}^j \quad (85)$$

This equation is analogous to a typical gravity equation with fixed effects, which is usually specified as the following:

$$\ln\left(\tilde{X}_{ni}^j\right) = \lambda_0 + \lambda_1 \ln\left(\tilde{\kappa}_{ni}^j\right) + \epsilon_{ni}^j \quad (86)$$

Note that my specification differs from the standard approach due to the structure of the trade elasticity associated with the trade cost variable in (85). I am interested in whether or not λ_2 is positive, as well as its magnitude. In my theoretical framework, $\lambda_2 = 1$. I am also interested in the estimate of λ_1 , particularly when using $\ln(d_{ni})$ as a proxy variable for trade costs. As mentioned in the introduction, distance is widely viewed as a proxy for unobserved information and communication costs. As these costs have declined over time due to technological change, we should expect that estimates of λ_1 in (86) are higher (in absolute value) using data from the 1980s when compared with estimates using more recent data from the 2000s. In fact, most of the gravity literature finds that λ_1 has remained stable over time when κ_{ni} is proxied using the log of distance. This stability is known as the “distance puzzle”.

Given my theoretical findings in Section 2, it is not necessarily surprising that the

distance estimates have remained stable over time. The fact that intermediate inputs shares have, on average, increased over time suggests that the elasticity of trade with respect to trade costs in (85) should have also increased. Moreover, β_i^j varies across countries and sectors, which a reduced form estimate of the trade elasticity according to (86) fails to take into account. Under the specification in (85), I adjust for variation in β_i^j across countries, exporters and time. The remaining impact of distance, captured with the λ_1 parameter, should perhaps be falling over time (in absolute value) due to changes in communication and information costs.

I also examine the relationship between for the domestic and imported share of intermediate inputs implied by (85). That is, I run the following regression:

$$\ln(\tilde{X}_{ni}^j) = \lambda_0 + \lambda_1 \left(\frac{1}{1 - \lambda_2 \tilde{\beta}_{ih}^j - \lambda_3 \tilde{\beta}_{if}^j} \right) \ln(\tilde{\kappa}_{ni}^j) + \epsilon_{ni}^j \quad (87)$$

where h and f denote the home and foreign share of intermediates. Other authors have examined the relationship between the elasticity of trade with respect to distance and imported intermediates in particular (for examples, see Yi (2003, 2010) and Johnson and Noguera (2012b)). The relationship in my framework is not confined to imported intermediates, but also applies to domestic intermediates. In estimating (87), I am interested in the signs and magnitudes of λ_2 and λ_3 . According to the theory, they should *both* be positive and significant.

To explore the extensive margin and average exports per good, I also estimate (85) and (87) replacing X_{ni} by F_{ni} and \bar{X}_{ni} . I expect, based on my model, that the relationship for overall exports should be driven by the extensive margin.

5.2 Results

The first set of results from estimating (85) using total bilateral exports and distance as a proxy for trade costs are reported in columns 3 and 4 in Table 4. Columns 1 and 2 report results from estimating (86), which I will call the standard gravity equation. When we compare columns 1 and 2, we see that the estimate of the trade elasticity λ_1 is slightly higher in the 2000s than in the 1980s. This pattern is consistent with previous literature and indicative of the distance puzzle. The magnitude of these estimates are also similar to estimates from other studies (Berthelon and Freund (2008), Disdier and Head (2008)).

In columns 3 and 4, we see that the estimates of λ_2 from (85) are positive and significant at the 1% level for the both the 1980s and 2000s. The magnitude of the parameter is between 0.5 and 1 in both periods and rises over time. Although the theoretical model in

Table 4: Dependent Var: Total Exports, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s	1980s	2000s
Const.	0.674*** (0.0518)	0.836*** (0.0560)	0.674*** (0.0519)	0.843*** (0.0561)
λ_1	-1.034*** (0.0318)	-1.071*** (0.0348)	-0.713*** (0.0467)	-0.618*** (0.0353)
λ_2			0.597*** (0.0613)	0.784*** (0.0360)
Observations	15372	15956	15372	15956
Adjusted R^2	0.193	0.243	0.196	0.250

Note: Standard errors are reported in parentheses. All errors are clustered by country-pair.

Section 2 predicts that $\lambda_2 = 1$, I take this result to be fairly supportive of the mechanisms described in the model. Note that in the standard framework with TFP heterogeneity, λ_2 is set equal to 0, which is clearly rejected by the data. Remarkably, the estimate λ_1 in columns 3 and 4 falls (in absolute value) over time. That is, once we adjust for the impact of intermediate inputs in the trade elasticity, the impact of distance on international trade became weaker from the 1980s to the 2000s. This is in contrast to the opposite pattern in columns 1 and 2 where λ_2 is assumed to be 0. I will elaborate more on this below.

In Tables 12 to 14 in the appendix, I report estimates of (85) using a regional trade agreement, currency union and shared border dummies as proxy variables for trade costs. In each case, estimates of λ_2 are positive and significant as suggested by the theoretical predictions in Section 2. That is, the trade elasticity, when estimated using any of these indicators for trade costs, has a positive and significant relationship with the intermediate inputs share. In addition, all of these estimates of λ_2 fall in the range from 0.712 to 1.271.

In Table 5, I report estimates of specification (87) where I differentiate between domestic- and foreign-produced intermediate inputs. Estimates of λ_2 and λ_3 are positive and significant in the 1980s and 2000s. Johnson and Noguera (2012b) find a pattern that is qualitatively consistent with $\lambda_3 > 0$, suggesting that sectors which use imported intermediate inputs have a higher elasticity of trade with respect to distance. In Table 5, we see evidence that this relationship holds for domestically produced intermediates as well. This is consistent with the qualitative predictions from my model, where the source of intermediate goods plays no role in relation to their impact on the trade elasticity.

Again, λ_1 falls over time. In terms of magnitudes, there are significant changes in both $\tilde{\beta}_{hi}^j$ and $\tilde{\beta}_{fi}^j$ over time that are difficult to interpret. We know that $\tilde{\beta}_{fi}^j$ rose while $\tilde{\beta}_{hi}^j$ fell significantly from the 1980s to the 2000s. As such, part of these changes could be simply

Table 5: Dependent Var: Total Exports, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s
Const.	0.680*** (0.0518)	0.829*** (0.0564)
λ_1	-0.673*** (0.0441)	-0.662*** (0.0370)
λ_2	0.575*** (0.0672)	0.968*** (0.0438)
λ_3	1.022*** (0.174)	0.190** (0.0926)
Observations	15372	15956
Adjusted R^2	0.197	0.254

Note: Standard errors are reported in parentheses.
All errors are clustered at the country-pair level.

compositional since some of the intermediate goods that were domestically sourced in the 1980s become sourced from abroad by the 2000s. In Tables 17-19 in the appendix, I report results from estimates of (87) using regional trade agreement, common currency and common language as indicators of trade costs. For all three costs, the estimate on λ_2 is positive and significant in both years.³⁴ For the imported intermediate share, the estimate of λ_3 is either positive and insignificant or even negative and significant in some periods. In light of my theory, this pattern is unsettling. However, it could be explained by a selection effect whereby the industries that use imported intermediate inputs have lower trade costs, on average, than those that use domestic intermediates. Again, it is difficult to get clear evidence either in support or against my theoretical pattern when we differentiate between imported and domestic intermediates, especially since my theory does not suggest that there should be any difference here.

Overall, I interpret the results from specification (87) with caution. My main purpose in differentiating between imported and domestic intermediates was to verify that the negative relationship between intermediate goods and the trade elasticity was not being driven entirely by imported intermediates. The evidence appears to offer support for this argument since the relationship between domestic intermediates and the trade elasticity is positive and significant in all years and when using any one of our indicators for trade costs.

The evidence from Tables 4 and 5 offers an interesting reconciliation of the “distance puzzle”. Again, this puzzle refers to the common finding that, despite our assurance

³⁴In the case for common currency, this estimate is only significant at the 10% level for 1985

that trade-related costs like information and communication have fallen over the years, the elasticity of trade with respect to distance has remained stable and significant. In columns 1 and 2, we see that this elasticity has risen from the 1980s to the 2000s, which is consistent with this puzzle. However, we see in columns 3 and 4 that once we adjust in variation in the intermediate input share across time, sectors and countries, the remaining impact of distance falls over time. This suggests that variation in impact of intermediate inputs on the trade elasticity can help explain the puzzle.

5.2.1 Extensive Margin

My theory predicts that the negative relationship between the intermediate inputs share and the trade elasticity should be driven by the extensive margin rather than the intensive or compositional margins.

In this Section, I test this prediction by replacing total exports with the number of goods exported as the dependent variable in specification (85). In Table 6, I report these results using the log of distance to indicate trade costs. Again, in columns 1 and 2 I report estimates from the standard gravity specification from (86), which does not include the intermediate inputs share. Notice that, contrary to the pattern for total exports, the elasticity of trade with respect to distance has fallen from the 1980s to the 2000s; that is, there is not much of a distance puzzle at the extensive margin. When intermediates inputs are included (columns 3 and 4) as in (85), estimates of λ_3 are positive and significant in both the 1980s and 2000s. Moreover, these estimates are larger in magnitude and closer to 1 than the estimates from Table 4 using total exports as the dependent variable. This difference is particularly striking when compared to the estimate of λ_2 in row 2. The overall trade elasticity is less than 0.05 at the extensive margin, but the coefficient on the intermediate inputs share is roughly 1.³⁵

I also estimate specification (85) using the residual between the total exports and the number of goods exported, which can be interpreted as the average exports per good. According to my theory, there should not be any relationship between the intermediate inputs share and the trade elasticity at this margin. However, as we see in columns 3 and 4 of Table 7, the relationship is positive and significant at this margin as well. Although this appears to contradict my theory, it is important to note that this margin is not a very good measure of what, theoretically, should be the intensive and compositional margins combined. My empirical measure of the extensive margin is the count of products traded between countries at the SITC 4-digit classification level. This includes 768 product

³⁵I also considered estimates of (85) with the number of goods exported and using my other measures of trade costs. Unfortunately, γ_3 could not be identified with this data due to the coarseness of both this dependent variables and these trade cost indicators.

Table 6: Dependent Var: Number of Goods Exported, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s	1980s	2000s
Const.	-0.188*** (0.0248)	-0.132*** (0.0246)	-0.187*** (0.0246)	-0.129*** (0.0245)
λ_1	-0.102*** (0.0152)	-0.0747*** (0.0161)	-0.0409*** (0.00855)	-0.0352*** (0.0112)
λ_2			1.122*** (0.0488)	0.993*** (0.100)
Observations	15372	15956	15372	15956
Adjusted R^2	0.024	0.014	0.029	0.016

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

categories which, while a much lower level of aggregation than the 20 2-digit ISIC sectors, is still not sufficiently low to fully capture the extensive margin. Within each of these product codes, there are actually many firms and many products. As a result, my intensive/compositional margin actually contains a combination of the intensive, the compositional and some of the extensive margin.

Table 7: Dependent Var.: Average Exports per Good, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s	1980s	2000s
Const.	0.862*** (0.0426)	0.968*** (0.0442)	0.862*** (0.0427)	0.973*** (0.0443)
λ_1	-0.932*** (0.0260)	-0.997*** (0.0269)	-0.691*** (0.0475)	-0.597*** (0.0314)
λ_2			0.499*** (0.0788)	0.744*** (0.0387)
Observations	15372	15956	15372	15956
Adjusted R^2	0.179	0.246	0.180	0.252

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Therefore, we might be more interested how Tables 6 and 7 compare quantitatively in relation to λ_2 . It is clear that the estimate of λ_2 in Table 6 is larger and closer to 1 than the estimate in Table 7. This is especially true in comparison to the overall trade elasticity which, for extensive margin, is less than 0.05. In contrast, the estimates of λ_3 in columns 3 and 4 of Table 7 are between 0.5 and 0.75 while the estimates of overall trade elasticity are in the same range (in absolute value).

Overall, this evidence suggests that when we isolate the extensive margin, the relationship between the intermediate inputs share and the trade elasticity is particularly strong. This is consistent with the theoretical model, which predicts that this relationship is driven by the extensive margin of trade.

6 Conclusion

This paper makes several contributions. First, I extend the Eaton and Kortum (2002) and Melitz (2003) models of international trade to a framework with intermediate inputs and firm heterogeneity in *value-added* productivity. My framework reveals that using value-added productivity instead of TFP heterogeneity in this setting leads to a positive relationship between the trade elasticity and the share of intermediate inputs in production. I also show that the gains from trade are different under the value-added specification.

Second, I estimate the trade elasticity in accordance with the theoretical relationship provided by my model. I find evidence that the sensitivity of trade flows in response to changes in trade costs is positively related to the intermediate inputs share in production, which is consistent with my model’s prediction. Since the share of intermediate inputs varies across time, countries and sectors, the trade elasticity differs across these variables as well. I also estimate sectoral productivity dispersion parameters under my specification, finding that standard models *overestimate* the magnitude of these parameters (i.e. *underestimate* the degree of dispersion), on average by a factor of approximately 3. I then calculate the gains from trade under my specification using these estimates, finding that the gains are, on average, 34% higher in the early 1980s and 57% higher in the early 2000s according to my model relative to the standard framework.

These findings shed light on several empirical puzzles. From the 1980s to 2000s, trade in manufacturing goods grew significantly as a share of GDP despite apparently modest declines in tariffs and transportation costs. This suggests that the international trade elasticity is greater than 10 according to standard trade models. In trade models with firm heterogeneity, the trade elasticity is governed by the degree of productivity dispersion across firms. According to firm-level evidence, the degree of this dispersion is high which should translate to values of the trade elasticity in the region of 5. Under my specification, the trade elasticity is also affected by the intermediate inputs share. Given that most sectors use intermediate shares between 0.5 and 0.6, the trade elasticity is generally over twice as large as productivity dispersion alone would suggest. Thus, my model provides a mechanism for reconciling relatively large responses in trade to modest changes in trade costs amidst lower parameters of sectoral dispersion, which corresponds well with empirical evidence.

Over the same period, the elasticity of trade with respect to distance (or “distance elasticity”) remained fairly stable. Since distance is widely viewed as an indicator of trade-related frictions like information and communication, many find it surprising that this relationship has not weakened over time in the midst of significant technological

change. I also find evidence that the distance elasticity remained stable over this period. I argue, however, that this finding is not surprising given that the intermediate inputs share, which positively affects the distance elasticity according to my model, has grown over time and varies across countries and sectors. I separate the distance elasticity into two components: an intermediate inputs component and a residual component. I find that, after adjusting for the intermediate inputs component, the magnitude of the residual component falls by roughly 13% over time. This provides evidence that, while the distance elasticity has remained stable over time, this stability is driven by the impact of the intermediate inputs share and is not necessarily puzzling.

Overall, these findings indicate an important role for economic theory in shedding light on apparently puzzling empirical results in international trade.

References

- Anderson, James E., and Eric van Wincoop (2003) ‘Gravity with gravitas: A solution to the border puzzle.’ *American Economic Review* 93(1), 170–192
- Antras, Pol, and Davin Chor (2013) ‘Organizing the Global Value Chain.’ *Econometrica* 81(6), 2127–2204
- Antras, Pol, Davin Chor, Thibault Fally, and Russell Hillberry (2012) ‘Measuring the Upstreamness of Production and Trade Flows.’ *American Economic Review* 102(3), 412–16
- Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare (2012) ‘New Trade Models, Same Old Gains?’ *American Economic Review* 102(1), 94–130
- Baldwin, Richard, and Daria Taglioni (2011) ‘Gravity chains: Estimating bilateral trade flows when parts and components trade is important.’ NBER Working Papers 16672, National Bureau of Economic Research, Inc, January
- Berthelon, Matias, and Caroline Freund (2008) ‘On the conservation of distance in international trade.’ *Journal of International Economics* 75(2), 310–320
- Bhavnani, Rikhil, Natalia T. Tamirisa, Arvind Subramanian, and David T. Coe (2002) ‘The Missing Globalization Puzzle.’ IMF Working Papers 02/171, International Monetary Fund, October
- Broda, Christian, and David E. Weinstein (2006) ‘Globalization and the Gains from Variety.’ *The Quarterly Journal of Economics* 121(2), 541–585
- Buch, Claudia M., Jorn Kleinert, and Farid Toubal (2004) ‘The distance puzzle: on the interpretation of the distance coefficient in gravity equations.’ *Economics Letters* 83(3), 293–298
- Caliendo, Lorenzo, and Fernando Parro (2012) ‘Estimates of the Trade and Welfare Effects of NAFTA.’ NBER Working Papers 18508, National Bureau of Economic Research, Inc, November
- Chaney, Thomas (2008) ‘Distorted gravity: The intensive and extensive margins of international trade.’ *American Economic Review* 98(4), 1707–21
- Chaney, Thomas (2013) ‘The Gravity Equation in International Trade: An Explanation.’ CEPR Discussion Papers 9613, C.E.P.R. Discussion Papers, August

- Clausing, Kimberly A. (2001) ‘Trade creation and trade diversion in the Canada - United States Free Trade Agreement.’ *Canadian Journal of Economics* 34(3), 677–696
- Costinot, Arnaud, and Andres Rodriguez-Clare (2013) ‘Trade Theory with Numbers: Quantifying the Consequences of Globalization.’ NBER Working Papers 18896, National Bureau of Economic Research, Inc, March
- Crozet, Matthieu, and Pamina Koenig (2010) ‘Structural gravity equations with intensive and extensive margins.’ *Canadian Journal of Economics* 43(1), 41–62
- di Giovanni, Julian, Andrei A. Levchenko, and Romain Rancière (2011) ‘Power laws in firm size and openness to trade: Measurement and implications.’ *Journal of International Economics* 85(1), 42–52
- Disdier, Anne-Célia, and Keith Head (2008) ‘The puzzling persistence of the distance effect on bilateral trade.’ *The Review of Economics and Statistics* 90(1), 37–48
- Eaton, Jonathan, and Samuel Kortum (2002) ‘Technology, Geography, and Trade.’ *Econometrica* 70(5), 1741–1779
- Eaton, Jonathan, Samuel Kortum, and Francis Kramarz (2011) ‘An Anatomy of International Trade: Evidence From French Firms.’ *Econometrica* 79(5), 1453–1498
- Goldberg, Pinelopi Koujianou, Amit Kumar Khandelwal, Nina Pavcnik, and Petia Topalova (2010) ‘Imported Intermediate Inputs and Domestic Product Growth: Evidence from India.’ *The Quarterly Journal of Economics* 125(4), 1727–1767
- Head, Keith, and John Ries (2001) ‘Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade.’ *American Economic Review* 91(4), 858–876
- Head, Keith, and Thierry Mayer (2013) ‘Gravity Equations: Workhorse, Toolkit, and Cookbook.’ CEPR Discussion Papers 9322, C.E.P.R. Discussion Papers, January
- Head, Keith, Thierry Mayer, and John Ries (2010) ‘The erosion of colonial trade linkages after independence.’ *Journal of International Economics* 81(1), 1–14
- Helpman, Elhanan, Marc Melitz, and Yona Rubinstein ‘Estimating trade flows: Trading partners and trading volumes’
- Hillberry, Russell, and David Hummels (2008) ‘Trade responses to geographic frictions: A decomposition using micro-data.’ *European Economic Review* 52(3), 527–550

- Hummels, David (2007) ‘Transportation Costs and International Trade in the Second Era of Globalization.’ *Journal of Economic Perspectives* 21(3), 131–154
- (IMF), International Monetary Fund (2011) ‘Changing patterns of global trade.’ *IMF Policy Paper*
- Johnson, Robert C., and Guillermo Noguera (2012a) ‘Accounting for intermediates: Production sharing and trade in value added.’ *Journal of International Economics* 86(2), 224–236
- (2012b) ‘Fragmentation and Trade in Value Added over Four Decades.’ NBER Working Papers 18186, National Bureau of Economic Research, Inc, June
- (2012c) ‘Proximity and production fragmentation.’ *American Economic Review* 102(3), 407–11
- Kasahara, Hiroyuki, and Beverly Lapham (2013) ‘Productivity and the decision to import and export: Theory and evidence.’ *Journal of International Economics* 89(2), 297–316
- Kasahara, Hiroyuki, and Joel Rodrigue (2008) ‘Does the use of imported intermediates increase productivity? Plant-level evidence.’ *Journal of Development Economics* 87(1), 106–118
- Koopman, Robert, Zhi Wang, and Shang-Jin Wei (2012) ‘Estimating domestic content in exports when processing trade is pervasive.’ *Journal of Development Economics* 99(1), 178–189
- Koopman, Robert, Zhi Wang, and Shang-Jin Wei (2014) ‘Tracing Value-Added and Double Counting in Gross Exports.’ *American Economic Review* 104(2), 459–94
- Levchenko, Andrei A., and Jing Zhang (2014) ‘Ricardian productivity differences and the gains from trade.’ *European Economic Review* 65(C), 45–65
- Limao, Nuno, and Anthony J. Venables (1999) ‘Infrastructure, geographical disadvantage, and transport costs.’ Policy Research Working Paper Series 2257, The World Bank, December
- Lin, Faqin, and Nicholas C.S. Sim (2012) ‘Death of distance and the distance puzzle.’ *Economics Letters* 116(2), 225–228
- Luttmer, Erzo G. J. (2007) ‘Selection, Growth, and the Size Distribution of Firms.’ *The Quarterly Journal of Economics* 122(3), 1103–1144

- Maggi, Giovanni, and Pinelopi Koujianou Goldberg (1999) ‘Protection for Sale: An Empirical Investigation.’ *American Economic Review* 89(5), 1135–1155
- Melitz, Marc J. (2003) ‘The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity.’ *Econometrica* 71(6), 1695–1725
- Melitz, Marc J., and Stephen J. Redding (2014) ‘Missing Gains from Trade?’ *American Economic Review* 104(5), 317–21
- Ossa, Ralph (2012) ‘Why Trade Matters After All.’ NBER Working Papers 18113, National Bureau of Economic Research, Inc, November
- Ruhl, Kim J. (2008) ‘The International Elasticity Puzzle.’ Technical Report
- Shikher, Serge (2012) ‘Putting industries into the Eaton–Kortum model.’ *The Journal of International Trade & Economic Development* 21(6), 807–837
- Silva, J. M. C. Santos, and Silvana Tenreyro (2006) ‘The Log of Gravity.’ *The Review of Economics and Statistics* 88(4), 641–658
- Simonovska, Ina, and Michael E. Waugh (2014) ‘The elasticity of trade: Estimates and evidence.’ *Journal of International Economics* 92(1), 34–50
- Timmer, Marcel P., Abdul Azeez Erumban, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries (2014) ‘Slicing Up Global Value Chains.’ *Journal of Economic Perspectives* 28(2), 99–118
- Tinbergen, Jan (1962) ‘An analysis of world trade flows.’ New York, NY Twentieth Century Fund, Shaping the World Economy
- Yi, Kei-Mu (2003) ‘Can vertical specialization explain the growth of world trade?’ *Journal of Political Economy* 111(1), 52–102
- (2010) ‘Can Multistage Production Explain the Home Bias in Trade?’ *American Economic Review* 100(1), 364–93
- Yotov, Yoto V. (2012) ‘A simple solution to the distance puzzle in international trade.’ *Economics Letters* 117(3), 794–798

7 Appendix

In the following appendix, I include three sections. Section 1 includes theoretical proofs mentioned in Section 2 of the paper. Section 2 includes any tables that were referred to but not included in the body of the paper. Section 3 includes regression tables from Section 5 of the paper that were not included in the body of the paper.

7.1 Proofs

Proof of equation (16):

Incorporating (11) into (3), it is fairly simple to show that:

$$F_{ni}^j(p) = 1 - \exp \left\{ -T_i^j (c_i^j \kappa_{ni}^j)^{-\frac{\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}} \right\}$$

It follows that the probability of receiving a price in n below p for a given variety from *any* country is equal to $F_n^j(p) = \prod_{i=1}^N F_{ni}^j(p)$. Solving for this expression yields:

$$(\text{??}) F_n^j(p) = 1 - \prod_{i=1}^N \exp \left\{ -T_i^j (c_i^j \kappa_{ni}^j)^{-\frac{\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}} \right\} = 1 - \exp \left\{ \sum_{i=1}^N -T_i^j (c_i^j \kappa_{ni}^j)^{-\frac{\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}} \right\} \quad (88)$$

This is equivalent to the expression found in equation (16).

Proof of equation (18):

Expression (7) can be rearranged to $(P_{s,n}^j)^{1-\sigma} = \int_0^1 p_{s,n}^j (\omega^j)^{1-\sigma} d\omega = \int_0^\infty p^{1-\sigma} dF_n^j(p)$. Expanding $dF_n^j(p)$ using (16) and substituting this into the price index yields the following:

$$(P_n^j)^{1-\sigma} = \int_0^\infty p^{1-\sigma} \phi_n^j \left(\frac{\theta^j}{1-\beta^j} \right) p^{\frac{\theta^j}{1-\beta^j}-1} \exp \left\{ -\phi_n^j p^{\frac{\theta^j}{1-\beta^j}} \right\} dp \quad (89)$$

From here, I employ integration by substitution. Letting $x = \phi_n^j p^{\frac{\theta^j}{1-\beta^j}}$, it follows that $dx = \phi_n^j \left(\frac{\theta^j}{1-\beta^j} \right) p^{\frac{\theta^j}{1-\beta^j}-1} dp$ and $p = (x/\phi_n^j)^{\frac{1-\beta^j}{\theta^j}}$. Substituting these expressions into (89) yields the following:

$$\begin{aligned} (P_n^j)^{1-\sigma} &= \int_0^\infty \left(\frac{x}{\phi_n^j} \right)^{\frac{-\theta^j}{(1-\sigma)(1-\beta^j)}} \exp \{-x\} dx \\ &= (\phi_n^j)^{\frac{\theta^j}{(1-\sigma)(1-\beta^j)}} \int_0^\infty x^{\frac{(1-\sigma)(1-\beta^j)}{\theta^j}} \exp \{-x\} dx \end{aligned} \quad (90)$$

The second part of this expression can be simplified as

$$\int_0^{\infty} x^{\frac{-\theta^j}{(1-\sigma)(1-\beta^j)}} \exp\{-x\} dx = \Gamma\left(\frac{\theta^j + (1-\sigma)(1-\beta^j)}{\theta^j}\right)^{\frac{1}{(1-\sigma)(1-\beta^j)}} \quad (91)$$

where Γ denotes the Gamma function (a constant).³⁶ Substituting (91) into (90) and multiplying by the exponent of $1/(1-\sigma)$ yields the expression in (18):

$$P_n^j = (\phi_n^j)^{\frac{-\theta^j}{1-\beta^j}} \Gamma\left(\frac{\theta^j + (1-\sigma)(1-\beta^j)}{\theta^j}\right)^{\frac{1}{(1-\sigma)(1-\beta^j)}} \quad (92)$$

Proof of equation (11):

We can represent π_{ni}^j as $\pi_{ni}^j = Pr(p_{ni}^j(\omega^j) \leq \min\{p_{nk}^j(\omega^j); k \neq i\})$. Suppose that $p_{ni}^j(\omega^j) = p$; then, this probability can be represented as:

$$\begin{aligned} \prod_{k \neq i} Pr(p_{nk}^j(\omega^j) \geq p) &= \prod_{k \neq i} [1 - F_{nk}^j(p)] = \exp\left\{\sum_{k \neq i} -T_i^j(c_i^j \kappa_{ni}^j)^{\frac{-\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}}\right\} \\ &= \exp\left\{-\phi_{n \neq i}^j p^{\frac{\theta^j}{1-\beta^j}}\right\} \end{aligned} \quad (93)$$

where $\phi_{n \neq i}^j = \sum_{k \neq i} -T_i^j(c_i^j \kappa_{ni}^j)^{\frac{-\theta^j}{1-\beta^j}}$. To find π_{ni}^j , we integrate (14) over all possible p 's times the density $dF_i^j(p)$, which is itself equal to:

$$dF_i^j(p) = -T_i^j(c_i^j \kappa_{ni}^j)^{\frac{-\theta^j}{1-\beta^j}} \frac{\theta^j}{1-\beta^j} p^{\frac{\theta^j}{1-\beta^j}} \exp\left\{-T_i^j(c_i^j \kappa_{ni}^j)^{\frac{-\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}}\right\} dp \quad (94)$$

We can therefore solve for π_{ni}^j as:

$$\begin{aligned} &\phi_n^j \exp\left\{-\phi_n^j p^{\frac{\theta^j}{1-\beta^j}} p^{\frac{\theta^j}{1-\beta^j}} - 1\right\} \\ &= \left(\frac{T_i^j [c_i^j \kappa_{ni}^j]^{\frac{-\theta^j}{1-\beta^j}}}{\phi_n^j}\right) \int_0^{\infty} dF_n^j(p) dp \end{aligned} \quad (95)$$

Since $\int_0^{\infty} dF_n^j(p) dp = 1$, (19) has been proven.

³⁶The general formula for the Gamma function is $\Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx$.

Derivation for Global Profits from Section 2.2:

Consider the aggregated expression for Π_W from (44) :

$$\Pi_W = \sum_{h=1}^j \sum_{n,i=1}^N \int_{\varphi_{ni}^{*j}}^{\infty} (\pi_{ni}^j(\omega)) F_{ni}^j \mu_{ni}^j(\omega) d\omega$$

Incorporating F_{ni}^j from (47) and expanding $\pi_{ni}^j(\omega)$, this expression can be depicted as:

$$\Pi_W = \sum_{j=1}^J \sum_{n,i=1}^N \left[\frac{L_i X_j^j}{\sigma} \left(\frac{\lambda_p^j \kappa_{ni}^j C_i^j}{P_n^j} \right)^{1-\sigma} \int_{\varphi_{ni}^{*j}}^{\infty} (\varphi_{ni})^{(\sigma-1)(1-\beta^j)} G_j(\omega) d\omega + L_i \int_{\varphi_{ni}^{*j}}^{\infty} f_{ni}^j \right]$$

The first expression in the square brackets is equivalent to X_{ni}^j/σ . The second is equal to $F_{ni}^j f_{ni}^j$, which can be expanded to:

$$F_{ni}^j f_{ni}^j = L_i \left(\frac{f_{ni}^j \sigma}{E_j^j} \right)^{1-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}} \left(\frac{\lambda_p^j \kappa_{ni}^j C_i^j}{P_n^j} \right)^{\frac{-\gamma^j}{1-\beta^j}}$$

We can simplify global profits as:

$$\Pi_W = \sum_{n=1}^j \sum_{n,i=1}^N \left[\lambda_j^X X_i^j \left(\frac{\kappa_{ni}^j C_i^j}{P_j^j} \right)^{\frac{-\gamma^j}{(1-\beta^j)}} f_{ni}^j 1^{-\frac{\gamma^j}{(\sigma-1)(1-\beta^j)}} \right] \left[\frac{X_j^j}{\sigma} + \lambda_j^f \right]$$

where $\lambda_j^f = \frac{\gamma^j - (\sigma-1)(1-\beta^j)}{\gamma^j}$. Substituting (89) and X_j^j into this expression and rearranging yields the following equilibrium expression for global profits:

$$\Pi_W = \frac{\sum_{j=1}^J \left(\alpha_j L^W + \frac{\beta^j}{1-\beta^j} L^W + \sigma \lambda_j^f \right)}{\sum_{j=1}^J (\sigma - \alpha_j)}$$

This expression is simply a function of constant parameters.

7.2 Additional Tables

The following is a table listing OECD input-output availability over the period examined in this paper.

Table 8: OECD input-output data availability

	Early 1980s	Mid 1980s	2000	2005
Australia	-	1986	✓	✓
Canada	1981	1986	✓	✓
Denmark	1980	1985	✓	✓
France	1980	1985	✓	✓
Germany	1978	1986	✓	✓
Italy	-	1985	✓	✓
Japan	1980	1985	✓	✓
United Kingdom	1979	1984	✓	✓
United States	1982	1985	✓	✓

The correspondence table for ISIC Revision 2 to 3 is provided below:

Table 9: Isic Revision 2 to 3 correspondence table

Isic Revision 2 group	Group code	Isic Revision 3 group	Group code
Food, beverages and tobacco	31	-	15+16
Textiles, apparel and leather	32	-	17+18+19
Wood prod. and furniture	33	Wood prod. and cork	20
Paper, paper prod. and printing	34	-+ pulp and publish	21+22
Industrial chemicals	35 ex. 3522	Chemicals	24 ex. 2423
Drugs and medicines	3522	Pharmaceuticals	2423
Rubber and plastic products	355+356	-	25
Non-metallic mineral products	36	-	26
Iron and steel	371	-	271
Non-ferrous metals	372	-	272
Metal products	381	-	28
Non-electrical machinery	382 ex. 3825	Machinery and equip., nec	29
Office and computing mach.	3825	Office, account. and compu. mach.	30
Electrical apparatus, nec	383 ex. 3832	Elec. mach. and appar., nec	31
Radio, TV and comm. equipment	3832	-	32
Shipbuilding and repairing	3841	Build./repair. of ships and boats	351
Other transport	3842	Rail. and trans. equip., nec	352A
Motor vehicles	3843	Motor veh. and trailers	34
Aircraft	3845	Aircraft and space-	353
Professional goods	385	Med., precision and optical instr.	33
Other manufacturing	39	Manuf. nec and recyc.	36+37

Note: (-) indicates that the sector has the same name for both revision groups.

The following table reports average shares of intermediate inputs used in production, differentiating across time, sector and origin of inputs (h versus f):

Table 10: Average Sectoral Intermediate Inputs Shares (β_i^j) across Countries and Time

Isic Rev. 2 group	β_{h80s}	β_{h00s}	β_{f80s}	β_{f00s}	β_{80s}	β_{00s}
Food, beverages and tobacco	0.593	↓ 0.521	0.098	↑ 0.117	0.691	↓ 0.638
Textiles, apparel and leather	0.396	↓ 0.315	0.186	↑ 0.240	0.582	↓ 0.555
Wood prod. and furniture	0.415	↓ 0.400	0.142	↑ 0.147	0.556	↓ 0.547
Paper, paper prod. and printing	0.384	↓ 0.315	0.128	↑ 0.140	0.512	↓ 0.455
Industrial chemicals	0.436	↓ 0.335	0.203	↑ 0.257	0.639	↓ 0.592
Drugs and medicines	0.320	↓ 0.244	0.139	↑ 0.197	0.459	↓ 0.441
Rubber and plastic products	0.363	↓ 0.314	0.181	↑ 0.227	0.543	↓ 0.541
Non-metallic mineral products	0.361	↓ 0.323	0.087	↑ 0.117	0.448	↓ 0.440
Iron and steel	0.513	↓ 0.388	0.151	↑ 0.273	0.664	↓ 0.661
Metal products	0.388	↓ 0.334	0.118	↑ 0.146	0.506	↓ 0.480
Non-electrical machinery	0.377	↓ 0.348	0.127	↑ 0.188	0.503	↑ 0.537
Office and computing mach.	0.293	↓ 0.203	0.219	↑ 0.438	0.512	↑ 0.641
Electrical apparatus, nec	0.372	↓ 0.332	0.129	↑ 0.229	0.501	↑ 0.561
Radio, TV and comm. Equipment	0.316	↓ 0.242	0.179	↑ 0.328	0.495	↑ 0.570
Shipbuilding and repairing	0.396	↓ 0.386	0.123	↑ 0.227	0.518	↑ 0.613
Other transport	0.402	↓ 0.397	0.116	↑ 0.189	0.518	↑ 0.586
Motor vehicles	0.473	↓ 0.430	0.170	↑ 0.281	0.643	↑ 0.710
Aircraft	0.291	↓ 0.199	0.193	↑ 0.323	0.484	↑ 0.522
Professional goods	0.290	↓ 0.231	0.127	↑ 0.202	0.417	↑ 0.433
Other manufacturing	0.334	↓ 0.341	0.173	↑ 0.190	0.508	↑ 0.532
Mean	0.379	0.330 ↓	0.173	0.242 ↑	0.552	↑ 0.573

The following table provides a list of countries included in the analysis. Note that, for time-series comparison, several of the countries were aggregated into a single group.³⁷

Table 11: List of import-receiving countries

Afghanistan	Egypt	Kenya	Rwanda
Albania	El Salvador	Kuwait	Saudi Arabia
Algeria	Eritrea	Kyrgyzstan	Senegal
Angola	Estonia	Lao People's Dem. Rep.	Serbia and Montenegro
Argentina	Ethiopia	Latvia	Sierra Leone
Armenia	Finland	Lebanon	Singapore
Australia	Fmr Arab Rep. of Yemen	Liberia	Slovakia
Austria	Fmr Dem. Yemen	Libya	Slovenia
Azerbaijan	Fmr Ethiopia	Lithuania	Somalia
Bangladesh	Fmr Fed. Rep. of Germany	Luxembourg	South Africa
Belarus	Fmr Sudan	Madagascar	Spain
	Fmr USSR	Malawi	Sri Lanka
Belgium-Luxembourg	Fmr Yugoslavia	Malaysia	Suriname
Belgium	France	Mauritania	Swaziland
Bosnia Herzegovina	Gabon	Mexico	Sweden
Botswana	Gambia	Mongolia	Switzerland
Brazil	Georgia	Morocco	Syria
Bulgaria	Germany	Mozambique	TFYR of Macedonia
Burkina Faso	Ghana	Namibia	Tajikistan
Cambodia	Greece	Nepal	Thailand
Cameroon	Grenada	Netherlands	Togo
Canada	Guatemala	New Zealand	Trinidad and Tobago
Central African Rep.	Guinea	Nicaragua	Tunisia
Chad	Guinea-Bissau	Niger	Turkey
Chile	Guyana	Nigeria	Turkmenistan
China	Haiti	Norway	USA
Colombia	Honduras	Oman	Uganda
Congo	Hungary	Pakistan	Ukraine
Costa Rica	Iceland	Panama	United Arab Emirates
Croatia	India	Papua New Guinea	United Kingdom
Cuba	Indonesia	Paraguay	United Rep. of Tanzania
Cyprus	Iran	Peru	Uruguay
Czech Rep.	Iraq	Philippines	Uzbekistan
Czechoslovakia	Ireland	Poland	Venezuela
Cote d'Ivoire	Israel	Portugal	Viet Nam
Dem. People's Rep. of Korea	Italy	Qatar	Western Sahara
Dem. Rep. of the Congo	Jamaica	Rep. of Korea	Yemen
Denmark	Japan	Rep. of Moldova	Zambia
Dominican Rep.	Jordan	Romania	Zimbabwe
Ecuador	Kazakhstan	Russian Federation	

³⁷The following countries were combined to make groups (in brackets): Belgium and Luxembourg (Belgium-Luxembourg); Czech Republic and Slovakia (Fmr. Czechoslovakia); Armenia, Azerbaijan, Belarus, Estonia, Georgia, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Rep. of Moldova, Russian Federation, Tajikistan, Turkmenistan, Ukraine and Uzbekistan (Fmr. USSR); Fmr. Arab Republic of Yemen, Fmr. Democratic Republic of Yemen (Yemen); Bosnia Herzegovina, Croatia, Serbia and Montenegro, Slovenia, TFYR of Macedonia (Fmr. Yugoslavia).

7.3 Regression Tables

The following tables report estimates from various regression specifications defined in the paper. The first two columns of Tables 12 to 15 report estimates for specification (86); columns 3 and 4 report estimates for (85). Tables 16 to 19 reported estimates for specification (87). I include different tables for different indicators of trade costs:

Table 12: Dependent Var.: Total Exports, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s	1980s	2000s
	1980s	2000s	1980s	2000s
Const.	0.674*** (0.0518)	0.836*** (0.0560)	0.674*** (0.0519)	0.843*** (0.0561)
λ_1	-1.034*** (0.0318)	-1.071*** (0.0348)	-0.713*** (0.0467)	-0.618*** (0.0353)
λ_2			0.597*** (0.0613)	0.784*** (0.0360)
Observations	15372	15956	15372	15956
Adjusted R^2	0.193	0.243	0.196	0.250

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 13: Dependent Var.: Total Exports, $\kappa_{ni} = \widetilde{rta}_{ni}$

	1980s	2000s	1980s	2000s
Const.	0.342*** (0.0643)	0.530*** (0.0659)	0.351*** (0.0643)	0.532*** (0.0659)
λ_1	1.540*** (0.105)	1.765*** (0.0988)	0.944*** (0.109)	0.954*** (0.0864)
λ_2			0.749*** (0.0727)	0.841*** (0.0469)
Observations	15372	15956	15372	15956
Adjusted R^2	0.064	0.111	0.067	0.116

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 14: Dependent Var.: Total Exports, $\kappa_{ni} = \tilde{c}u_{ni}$

	1980s	2000s	1980s	2000s
Const.	-0.515*** (0.0601)	-0.347*** (0.0699)	-0.515*** (0.0601)	-0.324*** (0.0680)
λ_1	1.868*** (0.346)	0.158 (0.139)	1.168** (0.472)	0.0564 (0.0371)
λ_2			0.712** (0.284)	1.271*** (0.0406)
Observations	15372	15956	15372	15956
Adjusted R^2	0.006	-0.000	0.006	0.002

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 15: Dependent Var.: Total Exports, $\kappa_{ni} = \tilde{b}_{ni}$

	1980s	2000s	1980s	2000s
Const.	-0.342*** (0.0592)	-0.192*** (0.0615)	-0.341*** (0.0592)	-0.191*** (0.0615)
λ_1	1.672*** (0.144)	1.500*** (0.150)	0.936*** (0.138)	0.814*** (0.134)
λ_2			0.841*** (0.0853)	0.850*** (0.0876)
Observations	15372	15956	15372	15956
Adjusted R^2	0.041	0.037	0.042	0.039

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 16: Dependent Var.: Total Exports, $\kappa_{ni} = \ln(\tilde{d}_{ni})$

	1980s	2000s
Const.	0.680*** (0.0518)	0.829*** (0.0564)
λ_1	-0.673*** (0.0441)	-0.662*** (0.0370)
λ_2	0.575*** (0.0672)	0.968*** (0.0438)
λ_3	1.022*** (0.174)	0.190** (0.0926)
Observations	15372	15956
Adjusted R^2	0.197	0.254

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 17: Dependent Var.: Total Exports, $\kappa_{ni} = \widetilde{rta}_{ni}$

	1980s	2000s
Const.	0.350*** (0.0643)	0.540*** (0.0658)
λ_1	0.883*** (0.108)	1.062*** (0.0930)
λ_2	0.720*** (0.0823)	1.067*** (0.0513)
λ_3	1.174*** (0.278)	0.0914 (0.120)
Observations	15372	15956
Adjusted R^2	0.067	0.120

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 18: Dependent Var.: Total Exports, $\kappa_{ni} = \widetilde{cu}_{ni}$

	1980s	2000s
Const.	-0.515*** (0.0601)	-0.324*** (0.0672)
λ_1	1.131*** (0.423)	0.0934* (0.0566)
λ_2	0.662* (0.357)	1.719*** (0.0903)
λ_3	1.082 (0.880)	-0.891 (0.544)
Observations	15372	15956
Adjusted R^2	0.006	0.003

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.

Table 19: Dependent Var.: Total Exports, $\kappa_{ni} = \widetilde{b}_{ni}$

	1980s	2000s
Const.	-0.340*** (0.0594)	-0.189*** (0.0616)
λ_1	0.963*** (0.151)	1.002*** (0.147)
λ_2	0.863*** (1.047)	1.376*** (0.377)
λ_3	0.570 (0.159)	-1.056*** (0.100)
Observations	15372	15956
Adjusted R^2	0.042	0.041

Note: Standard errors are reported in parentheses. All errors are clustered at the country-pair level.