Comparative Advantage and International Risk Sharing: Together at Last*

Konstantin Kucheryavyy†

November 14, 2014

Abstract

The overwhelming consensus in the theoretical literature is that access to international risk sharing in the presence of uninsured total factor productivity (TFP) shocks induces a country to specialize more in its comparative advantage industries. This paper demonstrates that the effect of financial integration on production patterns depends on preferences and on the structure of the variance-covariance matrix of TFP shocks present in the economy. Using a variant of the standard 2×2 Ricardian model with TFP shocks by Helpman and Razin (1978), I show that if TFP shocks affect each industry in all countries the same way, the standard assumption, then financial integration indeed leads to a more specialized production structure. However, if shocks are not correlated across countries and affect all industries in a country the same way, then the effect of financial integration is ambiguous. I also show that in the absence of international risk sharing the Helpman-Razin model generally has (discrete) multiple equilibria — an overlooked phenomenon. I build a framework with a continuum of goods in the spirit of Eaton-Kortum and show how the multiple equilibria can be numerically bounded. Using this framework I show that the gains from financial integration can very large — up to several hundred percent in expected welfare. This is in sharp contrast with the small gains from financial integration traditionally obtained in the literature on international risk sharing. This paper can be seen as a first attempt to bring together the discussion of international risk sharing and trade in the context of quantitative trade models of comparative advantage.

JEL: F15, F40, F65

*I am especially grateful to Jonathan Eaton and Stephen Yeaple for their continued guidance and support. I thank Barry Ickes who's ideas about the potential welfare benefits from international risk sharing through the feedback on the production structure led to the creation of this paper. The help of Felix Tintelnot on the early stages of this work was invaluable. I thank Michael Fabinger for helping me to formulate the numerical algorithm presented in this paper. I greatly appreciate Kenneth Judd's guidance and encouragement regarding the numerical part of this work. I thank participants of various seminars at Penn State University, trade lunch talk at Yale University, Midwest International Trade Fall 2014 Meetings, and Graduate Student Conference at University of Washington in St. Louis. I also thank Costas Arkolakis, Andrew Atkeson, Saroj Bhattarai, Russell Cooper, Bulat Gafarov, Edward Green, David Jinkins, Kala Krishna, Ahmad Lashkaripour, Gary Lyn, Kalina Manova, Andrés Rodríguez-Clare, James Tybout, Venky Venkateswaran, and Neil Wallace for many useful comments and suggestions. Financial support from the Center for Research on International Finance and Energy Security at Penn State is greatly appreciated.

†Department of Economics, The Pennsylvania State University, 605 Kern Building, University Park, PA 16802, USA (e-mail: kzk145@psu.edu).

1 Introduction

This paper analyses the interaction between comparative advantage and financial integration (i.e., access to international risk sharing) in the presence of uninsured total factor productivity (TFP) shocks. The central question of this paper is whether financial integration leads to more specialization in production. The first attempt to understand the link between financial integration and production dates back to the work of Helpman and Razin (1978a). Since then, a consensus has formed that financial integration unambiguously induces specialization of production.[1] The logic behind this view is simple: in the absence of access to international risk sharing countries hedge their risks through diversification of production. If perfect international risk sharing can be achieved, countries will insure their risks through financial markets and specialize more in their comparative advantage industries.

This paper demonstrates that, contrary to the conventional view, financial integration does not necessarily lead to more specialization in production. The outcome of financial integration depends on preferences and the structure of TFP shocks prevailing in the economy. A common assumption about the structure of TFP shocks made in the previous literature is that shocks are industry-specific, i.e., shocks affect one industry in all countries the same way.[2] I show that financial integration indeed leads to more specialized production under this assumption. However, if shocks are country-specific, i.e., if shocks affect all industries in one country the same way, then financial integration can lead to a more diversified production structure.

The assumption that shocks are industry-specific might not reflect well the observed shocks in the data. Recent evidence by Koren and Tenreyro (2007) shows that country-specific shocks — i.e., shocks that affect all industries in a country the same way — are as important as industry-specific shocks in explaining overall output volatility across countries.

To study the effects of financial integration, I use the standard 2×2 Ricardian model of comparative advantage with TFP shocks, a variant of the model by Helpman and Razin (1978a) often cited in the literature as unambiguously predicting that financial integration induces specialization. In this model consumers are risk-averse and factors of production are allocated before TFP shocks are realized. Expected comparative advantage is defined through comparison of the ratios of expected productivities for different goods across countries. International risk sharing can either be unrestricted or absent. A very important feature of this model is that production structure is endogenous. This is different from traditional assumptions in the international finance literature, which usually considers either endowment economies or economies where each country produces a unique tradeable good.

The main mechanism that drives the outcomes with country-specific shocks is insurance through prices of goods.[3] To understand this mechanism, consider an example with two countries, the U.S. and Brazil, producing two goods, planes and coffee, with the U.S. having comparative advantage in production of planes. Imagine that the U.S. has no shocks to productivity, while Brazil can

[1] For example, in Imbs (2004) "...financial integration may decrease (or increase) synchronization [of business cycles], but will also unambiguously induce specialization." (Section I, p. 723). And in Kalemli-Ozcan *et al.* (2003) "The theoretical foundations for the effect of risk sharing on industrial specialization are well established..." (Section I, p. 904)

[2] Helpman and Razin (1978a) and Koren (2003) are examples of papers that study the effects of financial integration and that assume industry-specific shocks.

[3] Newbery and Stiglitz (1981) studied this mechanism in the context of partial equilibrium models of trade. Cole and Obstfeld (1991) and Heathcote and Perri (2013) are examples of papers from the international finance literature that studied insurance through prices in the context of models with exogenous production structure.

have either a low or a high productivity shock affecting production of coffee and planes the same way. Both countries allocate factors (labor) to domestic production before the shock to Brazil is realized. After the shock is realized, the countries can freely trade.

If international risk sharing is not allowed, Brazil is an exporter of coffee, because the shock to Brazil does not change Brazil's comparative advantage. Then, since Brazil is the world supplier of coffee, when output of coffee in Brazil is low, the world price of coffee relative to the price of planes is high. In that sense Brazil is insured from the low productivity in its comparative advantage good through prices. At the same time, the fall in the relative price of planes acts as counter-insurance for the production of planes in Brazil. Since prices insure the production of coffee in Brazil and do the opposite for the production of planes, Brazil has an incentive to allocate more labor to the production of its insured good — coffee. At the same time, there is a similar and competing effect creating an incentive for Brazil to allocate more labor to the production of planes: when productivity in Brazil is high, Brazil might be producing too much coffee and so the relative price of coffee will be low. In this state, the world wants more planes and that is reflected in higher relative prices for planes. Under financial autarky, consumers get income only from their domestic production. Since consumers are risk-averse, they care more about what happens in the bad state of the world. Hence, Brazil will tend to put more labor into the production of coffee to exploit insurance of income through prices. After financial integration, the U.S. might provide insurance to Brazil from the low productivity outcome. In this case insurance through prices becomes less important for Brazil, and Brazil will put more resources into the production of planes, so that there is no "oversupply" of coffee when productivity in Brazil is high. As a result, we can see Brazil diversifying its production structure after financial integration.

Financial integration might also make a less volatile country more diversified — the U.S. in our example. Without international risk sharing, the U.S. earns income only from the domestic production. This income (measured in terms of coffee) is volatile only because the price of planes relative to the price of coffee is volatile. After financial integration, the U.S. invests in the production of coffee in Brazil, because this investment offers a high mean return. But this investment might also make the U.S. income more volatile, if the volatility of pre-integration prices was small relative to the volatility of shocks to Brazil. The U.S. might be willing to reduce the increased volatility of its income by producing more coffee. When output of coffee in Brazil is low, its price is high, while its output in the U.S. is stable, and, so, the U.S. earns more if it is committed to produce coffee instead of planes. Of course, the situation is the opposite when output of coffee in Brazil is high: its price is low and the U.S. earns less by producing coffee instead of planes. Hence, by moving at least some labor into the production of coffee, the U.S. reduces its income volatility. Alternatively, we can interpret the reallocation of labor in the U.S. into the production of coffee as the U.S. getting income insurance through the price of coffee.

This paper also shows that under the assumption of no international risk sharing and country-specific shocks the Helpman-Razin model generally has multiple equilibria — an earlier overlooked phenomenon. Again, insurance through prices plays an important role in generating multiple equilibria. To deal with the multiplicity, I bring the Helpman-Razin model into the framework of Eaton and Kortum (2002) with two countries, multiple industries with a continuum of goods in each industry, and free trade. Similar to the model with two goods, the resulting framework with a continuum of goods has two versions corresponding to financial markets structure: unrestricted international risk sharing or no risk sharing.

I show how any solution of the version of the model with a continuum of goods and no international risk sharing can be characterized using only aggregate quantities (aggregate labor allocations, trade shares, etc). In this version of the model each good is either produced by only one country in any equilibrium or is a disputed good. There is a continuum of equilibria each of which corresponds to some split of the set of disputed goods between the countries. The set of disputed goods can be bounded quantitatively. In this sense the multiple equilibria from the original Helpman-Razin model become manageable.

The second version of the model — with a continuum of goods and unrestricted international risk sharing — has a unique equilibrium, but it is very difficult to characterize the aggregate outcomes in this model for a general set of parameters. The particular values of parameters determine the set of aggregate expressions that characterize equilibrium. To solve the model I propose a solution approach that is of independent value and that can be applied in a much broader set of economic models.

I show that the model with a continuum of goods preserves the effects of financial integration from the model with two goods. The welfare analysis of the model with a continuum of goods reveals that the gains from international risk-sharing can be very large — up to several hundred percent in terms of expected welfare — if countries are subject to country-specific shocks. This is in contrast with the literature on international risk-sharing, which usually finds tiny gains from financial integration.[4] The reason for the large gains in my framework is that I explicitly model an endogenous feedback of production structure on the changes in the financial markets structure. Intuitively, under no international risk-sharing countries rely on insurance through prices and make very inefficient production choices comparing to the situation with unrestricted access to international risk-sharing: countries might become more specialized in the risky industries under no international risk-sharing. At the same time, the fact that countries might be more diversified under financial integration means that the countries have an option to produce a wide range of goods if they happen to be productive in all of them. When one country receives a positive country-wide productivity shock, the whole world can benefit a lot if this country produces a wide range of goods.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model in the general case with a continuum of goods and describes the solution approach. Section 4 provides analysis of the Helpman-Razin model and Section 5 provides analysis of the model with a continuum of goods. Section 6 concludes.

2 Literature Review

The role of productivity uncertainty in the trade models is a classic topic in the international trade literature. Pomery (1984) provides a good overview of challenges and theoretical results in that literature. The current paper is most closely related to Turnovsky (1974), Helpman and Razin (1978a), Eaton (1979), and Grossman and Razin (1985). These papers address whether countries specialize according to their comparative advantage in the presence of uncertainty of productivity. It was recognized in that literature that Ricardian comparative advantage ceases to be the sole determinant of specialization patterns: risk-aversion of agents and the structure of productivity shocks play an important role as well. Naturally, the literature devoted a lot of attention to seeking a set of assumptions that guarantee specialization according to comparative advantage. For example, Helpman and Razin (1978b) show that, if shocks to industries affect all countries in the same way, and if international trade in firm shares is allowed, then countries will specialize according to comparative advantage. So international trade in securities could make trade flow in

[4]See, for example, Lucas (1987) and Cole and Obstfeld (1991).

the “right” direction. However, as Grossman and Razin (1985) demonstrated, if country-specific shocks are allowed, then expected trade flows might go in the direction opposite to that predicted by expected comparative advantage. In the current paper, I start off by recognizing that both production and trade patterns can be changed in a non-trivial way, if we introduce uncertainty. The assumption that each industry has a continuum of goods allows me to get away from the problems associated with the knife-edge predictions of directions of trade flows and patterns of specialization.

A classical example in the international trade literature studying the link between financial integration and specialization patterns is the book by Helpman and Razin (1978a) mentioned in the previous paragraph. In their work, Helpman and Razin (1978a) not only show what set of assumptions is needed to make countries specialize according to their comparative advantage, but they also compare the outcomes of the models with and without financial integration. The authors show theoretically that, no matter what is the production structure in the absence of international risk sharing, if production risk can be insured through financial markets, a country would specialize in its comparative advantage industry. The critical assumption which drives this result is that shocks are industry-specific. Three other classical examples coming from the macroeconomics literature are papers by Saint-Paul (1992), Obstfeld (1994), and Acemoglu and Zilibotti (1997), in which the authors explicitly built in a trade off between diversification of risks through production versus diversification through financial markets. As a logical outcome, all of these papers find that better financial markets are associated with more specialization in production. The most recent and closely related to the current paper are the papers by Koren (2003) and Islamaj (2014) which present Ricardian models with uncertainty of production and varying degrees of financial markets incompleteness. Both of these papers study the relationship between financial openness and industrial specialization, and both of these papers come to the same conclusion that financial integration leads to a more specialized production structure. Koren (2003) assumes only industry-specific shocks, which drives his results. At the same time, even though Islamaj (2014) assumes that industry shocks within countries can be correlated, he never analyses the cases with strong correlation between the shocks within a country and, as a consequence, he only gets that financial integration induces specialization.

The current paper is also distantly related to another strand of the “old” literature on trade and uncertainty represented by Newbery and Stiglitz (1984), Eaton and Grossman (1986), Dixit (1987), Dixit (1989b), and Dixit (1989a). In these papers the authors were mostly concerned with the question of optimality of trade in the presence of uncertainty. Newbery and Stiglitz (1984) and Eaton and Grossman (1986) are classical papers showing the trade in the presence of uncertainty might not be optimal and, thus, some form of government intervention (e.g., tariffs) might be justified. The main assumption driving such an outcome was that insurance markets were not functioning properly in the analyzed economies. The above cited papers by Avinash Dixit came as a response to Newbery and Stiglitz (1984) and Eaton and Grossman (1986). Dixit showed that for many conceivable reasons of failures of insurance markets, if these failures are modeled explicitly, then free trade is constrained-Pareto optimal. Hence, there is no role for the government intervention. In the model presented here, multiple equilibria in the case of financial autarky can be ordered in terms of welfare from the point of view of a particular country. Then, the government in this country might be willing to purse some policies which move this country in a better equilibrium. On top of that, some of the equilibria can be Pareto-dominated for both countries. So, both countries can potentially purse a joint policy which will make them both better off.

Besides the papers by Koren (2003) and Islamaj (2014) mentioned above, the most recent papers directly descending from the "old" literature on trade and uncertainty also include the papers by Di Giovanni and Levchenko (2010) and Caselli *et al.* (2014). Di Giovanni and Levchenko (2010) analyze what determines how risky is the structure of country exports. The paper by Caselli *et al.* (2014) has a motivation similar to the current work. They show that — contrary to the common view — openness to trade can lead to less volatile output.

The role of prices as an insurance mechanism is a recognized phenomenon in both international trade and international finance literature. Among the first works that analyzed the insurance role of prices in the context of international trade was the book by Newbery and Stiglitz (1981). A classical example in the international finance literature is the paper by Cole and Obstfeld (1991), which shows how insurance through financial markets can be substituted with insurance through prices. The international finance literature traditionally abstracts away from comparative advantage in production and considers either endowment economies, as in Cole and Obstfeld (1991), or economies where each country produces a unique tradeable good, as in Heathcote and Perri (2013), who also talk about the effect of insurance through prices. As the analysis of the current paper reveals, by bringing the price insurance mechanism into an economy with an endogenous production structure, we can get a variety of interesting effects, which might help us in analyzing the consequences of financial integration in the real world.

The international finance literature traditionally finds small gains from international risk sharing. For example, Cole and Obstfeld (1991) find that the gains from financial integration in the model calibrated to the moments of the U.S. and Japanese data are about 0.2 percent of output per year. Another example is Lucas (1987), who also finds small gains from international risk sharing. In these studies gains come from consumption smoothing only, while a potential feedback of production structure on welfare is ignored. Also, in Cole and Obstfeld (1991) prices act as an insurance mechanism and reduce the role of financial markets, which results in small gains from international risk-sharing.

There are several studies which take into account the effect of international risk sharing on production structure. Among them are Obstfeld (1994) and Acemoglu and Zilibotti (1997), who find that gains from international risk-sharing can be substantial. These papers explicitly model the trade off between investing in high return but risky activities versus investing in low return but safer activities. In such environment countries use financial markets to insure their risks and, thus, are able to specialize in risky production technologies. However, these studies ignore the effect of insurance through prices, which — in view of the paper by Cole and Obstfeld (1991) — rises a natural question about whether their results survive if we take into account the price effects. As I show in the current paper, the feedback of production structure on welfare can be even larger than what Obstfeld (1994) and Acemoglu and Zilibotti (1997) obtain.

The empirical assessment of the relationship between financial integration and degree of specialization was performed by Koren (2003), Kalemli-Ozcan *et al.* (2003), Imbs (2004), Basile and Girardi (2010), and Bos *et al.* (2011). In all of these papers authors run regressions of various measures of risk sharing on indices of industrial specialization and find a positive relationship between these quantities. These results suggest that out of all effects of financial integration the ones which lead to more specialization might be the strongest in data. Nevertheless, as I discussed it in the introduction, there are several different reasons why financial integration can induce specialization. The evidence from these papers does not tell us which of the effects is most likely to be in action and what is the contribution of other effects in the data. So, there is still a lot of room for an empirical work in this area.

3 The Framework

The economy consists of $N = 2$ countries. Country 1 is labeled as "Home" and country 2 is labeled as "Foreign". Labor is the only factor of production. Country n is endowed with L_n units of labor, which are inelastically supplied. Labor is immobile across counties, but perfectly mobile across industries of one country at the stage when employment decisions are made. Each country can potentially produce a continuum of goods in each of the G industries, where G is a finite number. Goods are indexed by $\omega \in \Omega \equiv [0, 1]$ and industries are indexed by $g = 1, \ldots, G$.

Uncertainty The only sources of uncertainty in the economy are aggregate supply shocks: shocks which proportionally affect all efficiencies of production on the country-industry level. The state space of the international economy is denoted by $\mathcal{S}$, which is assumed to be discrete. The probability of state of the world $s \in \mathcal{S}$ is given by $h(s) > 0$, such that $\sum_{s\in\mathcal{S}} h(s) = 1$.

Each state $s \in \mathcal{S}$ of the world is described by the set of country-industry specific productivity shocks $A_{n,g}(s)$ for $g = 1, \ldots, G$ and $n = H, F$. When uncertainty is resolved, the realized state of the world is common knowledge.

The economy has two stages: before and after uncertainty is resolved. In the first stage financial markets are active, while in the second stage physical goods markets are active. Firms hire labor in the first stage by paying wages, which are not state contingent and cannot be carried over to the second stage. To pay the wages, firms sell claims on their output in the financial market. Consumers in the first stage sell their labor endowment to firms and use wage income to buy claims on endowments of physical goods in different states of the world in the second stage. When the second stage comes, firms use exactly the amount of labor they hired at the first stage. Actual production, international trade, and consumption of goods happen at the second stage.

I consider two cases of financial markets: complete markets and financial autarky. In the complete markets case countries are financially integrated, and consumers buy state-contingent goods for all states of the world. In the financial autarky case consumers can only buy claims on their domestic firms' output (in other words, there is no international risk sharing in this case). In the rest of the paper, when I talk about the effects of *financial integration*, I formally mean the effects of switching the financial markets structure from financial autarky to complete markets.

Production technology and expected comparative advantage. Technology of production is linear and stochastic: one unit of labor in industry g of country n can produce $A_{n,g}(s)z^{\omega}_{n,g} > 0$ units of good ω in state s. Here $z^{\omega}_{n,g}$ is the deterministic component of technology. It is convenient to assume that $E_s [A_{n,g}(s)] = 1$, so that $z^{\omega}_{n,g}$ is the expected efficiency of country n in producing good ω from industry g. The deterministic parts of technology, $z^{\omega}_{n,g}$, are independently (across goods, industries, and countries) drawn from a Fréchet distribution $F_{n,g}(x)$ with parameters $T_{n,g} > 0$ and $\theta > 1$:

$$F_{n,g}(x) = e^{-T_{n,g}x^{-\theta}}.$$

In the certainty case (e.g., in the Eaton-Kortum model), $T_{n',g}/T_{n'',g}$ defines comparative advantage of country n' relative to country n'' in producing goods from industry g. There are at least four different ways to extend this definition to the uncertainty case. We can say that country n' has expected comparative advantage in production of goods from industry g' relative to industry g'', if

(i) $\dfrac{T_{n',g'}}{T_{n'',g'}} > \dfrac{T_{n',g''}}{T_{n'',g''}}$; or

(ii) $E_s\left[\frac{A_{n',g'}(s)T_{n',g'}}{A_{n'',g'}(s)T_{n'',g'}}\right] > E_s\left[\frac{A_{n',g''}(s)T_{n',g''}}{A_{n'',g''}(s)T_{n'',g''}}\right]$; or

(iv) $\frac{A_{n',g'}(s)T_{n',g'}}{A_{n'',g'}(s)T_{n'',g'}} > \frac{A_{n',g''}(s)T_{n',g''}}{A_{n'',g''}(s)T_{n'',g''}}$ for any state s; or

(iii) $\frac{A_{n',g''}(s)T_{n',g''}}{A_{n'',g''}(s)T_{n'',g''}}$ is a mean-preserving spread of $\frac{A_{n',g'}(s)T_{n',g'}}{A_{n'',g'}(s)T_{n'',g'}}$.

I follow the previous literature on trade and uncertainty and choose the first definition — in terms of the ratios of the means of productivities. Apart from staying as close as possible to the previous literature, the advantage of using this definition is that with this definition expected comparative advantage does not depend on the structure of shocks. So, we can talk about riskiness (i.e., volatility of shocks) and expected comparative advantage separately.

International trade. International trade is free.

Preferences. Denote by $c^{\omega}_{n,g}(s)$ country n representative consumer's consumption of good ω from industry g in state s. For any two countries n and ℓ, good ω from industry g produced by country n in state s is a perfect substitute of good ω from industry g produced by country ℓ in state s. Country n's consumer combines consumption of goods ω into an industry-level consumption aggregate, $C_{n,g}(s)$, by the CES function with elasticity of substitution $\sigma_g > 0$:

$$C_{n,g}(s) \equiv \left[\int_{\omega\in\Omega} c^{\omega}_{n,g}(s)^{\frac{\sigma_g-1}{\sigma_g}}\, d\omega\right]^{\frac{\sigma_g}{\sigma_g-1}}.$$

Industry-level consumption aggregates are combined into a final consumption aggregate by the CES function with elasticity of substitution $\eta > 0$:

$$C_n(s) \equiv \left[\sum_{g=1}^{G} \alpha_g^{\frac{1}{\eta}} C_{n,g}(s)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $\alpha_g > 0$. In terms of this consumption aggregate, country n's consumer utility is given by the CRRA utility function:

$$U\left[C_n(s)\right] \equiv \frac{1}{1-\rho} C_n(s)^{1-\rho},$$

where $\rho > 0$ is the coefficient of relative risk aversion.

Market structure. It is assumed that firms are perfectly competitive, which implies that their expected profits are zero. Since goods ω from industry g produced by any two countries in state s are perfect substitutes, and since trade is free, price in state s of good ω from industry g is the same in any country. Denote this price by $p^{\omega}_g(s)$. Next, denote by $\psi_n(s)$ the pricing kernel in country n. This pricing kernel is an unknown which is derived later.[5] Country-n-producer hires the amount of labor, $l^{\omega}_{n,g}$, which maximizes its expected profit:

$$\max_{l^{\omega}_{n,g}} \sum_{s=1}^{\mathcal{S}} \psi_n(s) p^{\omega}_g(s) A_{n,g}(s) z^{\omega}_{n,g} l^{\omega}_{n,g} - w_n l^{\omega}_{n,g}.$$

[5]Since I focus on only two extreme cases — complete markets and financial autarky — the pricing kernel is uniquely defined on a country level.

Labor is hired *ex ante* at the cost of unit wage w_n. Wages are paid before uncertainty is realized. The firm's problem results in the following complementary slackness condition:[6]

$$l_{n,g}^{\omega} \geq 0 \textbf{ complementary to } w_n - \sum_{s=1}^{\mathcal{S}} \psi_n(s) p_g^{\omega}(s) A_{n,g}(s) z_{n,g}^{\omega} \geq 0.$$

Denote by $I_n(s)$ country-n-consumer's second stage income, which comes from collecting all the dividends from the firm shares held by the country-n-consumer. Then the second-stage expenditure on individual goods is given by:

$$\begin{aligned} x_{n,g}^{\omega}(s) &= \left(\frac{p_g^{\omega}(s)}{P_g(s)}\right)^{1-\sigma_g} X_{n,g}(s), \\ X_{n,g}(s) &= \left(\frac{P_g(s)}{P(s)}\right)^{1-\eta} \alpha_g I_n(s), \\ P_g(s) &= \left[\int_{\Omega} p_g^{\omega}(s)^{1-\sigma_g} d\omega\right]^{\frac{1}{1-\sigma_g}}, \\ P(s) &= \left[\sum_{g=1}^{G} \alpha_g P_g(s)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \end{aligned}$$

The indirect utility function is given by:

$$V\left(I_n(s), P(s)\right) \equiv \frac{1}{1-\rho}\left(\frac{I_n(s)}{P(s)}\right)^{1-\rho}.$$

I continue this section with the formal description of the complete financial markets case. After that I formally describe the financial autarky case.

3.1 Complete Markets

The consumer in country n maximizes her expected utility by choosing state-contingent income:

$$\max_{I_n(s)} E_s\left[V\left(I_n(s), P(s)\right)\right]$$

$$\text{s.t.} \quad \sum_{s=1}^{\mathcal{S}} \psi_n(s) I_n(s) = w_n L_n.$$

In the case of complete financial markets the pricing kernel, $\psi_n(s)$, is the same in all countries and can be normalized to 1. Then the consumer's problem implies

$$\begin{aligned} I_n(s) &= h(s)^{\frac{1}{\rho}} \left(\frac{P(s)}{P}\right)^{\frac{\rho-1}{\rho}} w_n L_n, \\ P &\equiv \left(E_s\left\{\left[\frac{P(s)}{h(s)}\right]^{\frac{\rho-1}{\rho}}\right\}\right)^{\frac{\rho}{\rho-1}}. \end{aligned}$$

[6] Variable $a \geq 0$ is complementary to variable $b \geq 0$, if $ab = 0$.

It is also useful to have an expression of the expected utility:

$$E_s\left[V\left(I_n(s),P(s)\right)\right]=\frac{1}{1-\rho}\left(\frac{w_nL_n}{P}\right)^{1-\rho}.$$

In the case of logarithmic utility ($\rho=1$) we have:

$$\begin{aligned}&I_n(s)=h(s)w_nL_n,\\&P=\exp\left\{E_s\left[\ln\frac{P(s)}{h(s)}\right]\right\},\\&E_s\left[V\left(I_n(s),P(s)\right)\right]=\ln\left(\frac{w_nL_n}{P}\right).\end{aligned}$$

3.2 Financial Autarky

Denote by $\zeta_{n,g}^{\omega}(s)$ country-n-consumer's income in state s from the domestic firm producing good ω from industry g. In equilibrium, $\zeta_{n,g}^{\omega}(s)=p_g^{\omega}(s)A_{n,g}(s)z_{n,g}^{\omega}l_{n,g}^{\omega}$. The total state-$s$ income of country-n consumer is given by

$$I_n(s)\equiv\sum_{g=1}^{G}\int_{\Omega}\zeta_n^{\omega}(s)d\omega.$$

Consumer in country n maximizes her expected utility by choosing her incomes from different goods produced by domestic firms:

$$\begin{aligned}&\max_{I_n(s),\zeta_{n,g}^{\omega}(s)}\quad E_s\left[V\left(I_n(s),P(s)\right)\right]\\&\text{s.t.}\\&\sum_{s=1}^{\mathcal{S}}\psi_n(s)I_n(s)=w_nL_n,\\&I_n(s)=\sum_{g=1}^{G}\int_{\Omega}\zeta_{n,g}^{\omega}(s)d\omega.\end{aligned}$$

By normalizing the Lagrange multiplier of the first budget constraint to 1, the consumer problem gives the expression for the pricing kernel:

$$\psi_n(s)=\frac{h(s)}{I_n(s)}\left(\frac{I_n(s)}{P(s)}\right)^{1-\rho}.$$

3.3 Equilibrium System of Equations

Since the financial autarky and complete market cases have many similar model components, the systems of equations that we need to solve to find equilibria in these cases — *the equilibrium systems of equations* — share many equations. Therefore, it is instructive to write the equilibrium systems of equations for these cases simultaneously.

To solve the model, we need to find $p_g^\omega(s)$, $P_g(s)$, $P(s)$, $s_{ni,g}^\omega(s)$, $c_{n,g}^\omega(s)$, $X_{n,g}(s)$, $I_n(s)$, $l_{n,g}^\omega$, w_n, which satisfy the following system of equations:

$$\begin{aligned}
&\sum_{n=1}^{N} c_{n,g}^\omega(s) = \sum_{n=1}^{N} A_{n,g}(s) z_{n,g}^\omega l_{n,g}^\omega, \quad g=1,\dots,G, \quad \omega\in\Omega, \quad s\in\mathcal{S};\\
&l_{i,g}^\omega \geq 0 \ \textbf{complementary to} \ w_i - E_s\left[\psi_i(s) p_g^\omega(s) A_{i,g}(s) z_{i,g}^\omega\right] \geq 0, \\
&\qquad\qquad\qquad\qquad\qquad\qquad i=1,\dots,N, \quad g=1,\dots,G, \quad \omega\in\Omega;\\
&\sum_{g=1}^{G}\int_\Omega l_{i,g}^\omega d\omega = L_i, \quad i=1,\dots,N.
\end{aligned} \tag{1}$$

Demand for individual goods, $c_{n,g}^\omega(s)$, is given by

$$\begin{aligned}
c_{n,g}^\omega(s) &= \frac{p_g^\omega(s)^{-\sigma_g}}{P_g(s)^{1-\sigma_g}} X_{n,g}(s),\\
X_{n,g}(s) &= \left(\frac{P_g(s)}{P(s)}\right)^{1-\eta} \alpha_g I_n(s),\\
P_g(s) &= \left[\int_\Omega p_g^\omega(s)^{1-\sigma_g} d\omega\right]^{\frac{1}{1-\sigma_g}},\\
P(s) &= \left[\sum_{g=1}^{G} \alpha_g P_g(s)^{1-\eta}\right]^{\frac{1}{1-\eta}}.
\end{aligned}$$

In the case of complete markets:

$$\begin{aligned}
\psi_n(s) I_n(s) &= h(s)^{\frac{1}{\rho}} \left(\frac{\psi_n(s)P(s)}{P}\right)^{\frac{\rho-1}{\rho}} w_n L_n,\\
P &= \left(\sum_{s=1}^{\mathcal{S}} \left[\psi_n(s)P(s)\right]^{\frac{\rho-1}{\rho}} h(s)^{\frac{1}{\rho}}\right)^{\frac{\rho}{\rho-1}}.
\end{aligned}$$

In the case of financial autarky:

$$\begin{aligned}
\psi_n(s) &= \frac{h(s)}{I_n(s)}\left(\frac{I_n(s)}{P(s)}\right)^{1-\rho},\\
I_n(s) &= \sum_{g=1}^{G}\int_\Omega p_g^\omega(s) A_{n,g}(s) z_{n,g}^\omega l_{n,g}^\omega d\omega.
\end{aligned}$$

3.4 Assumptions on the Structure of Shocks and Overview of Theoretical Implications

The main theoretical result of this paper is that financial integration *can* lead to a more diversified production structure. This result is in contrast with the conventional view that financial integration necessarily induces specialization. In this paper I show that the effect of financial integration actually depends on the structure of the variance-covariance matrix of TFP shocks affecting the economy. All interesting effects of financial integration can be described by focusing on two types of shocks: industry-specific and country-specific shocks. We can formally define these shocks in terms of the variance-covariance matrix. Figure 1 shows the general variance-covariance matrix of TFP shocks for the economy with 2 industries. The elements of this matrix are $\nu_{n'g',n''g''} \equiv Cov\left[A_{n',g'}(s), A_{n'',g''}(s)\right]$. Figure 2 shows the variance-covariance matrices for industry-specific and country-specific shocks. In the case of *industry-specific shocks*, each shock affects one industry in each country the same way, and shocks to different industries are not correlated with each other. An example of this type of shock is a new banana virus hitting production of bananas in Africa and Latin America. Alternatively, industry-specific shocks can be thought of and modeled as world-wide demand shocks in industries. In the case of *country-specific shocks*, each shock affects all industries in one country the same way, and shocks to different countries are not correlated with each other. An example of this type of shock is a labor strike in Argentina.

$$\begin{array}{lll}
 & & \begin{array}{cccc} \multicolumn{2}{c}{\text{Home}} & \multicolumn{2}{c}{\text{Foreign}} \\ \text{Industry 1} & \text{Industry 2} & \text{Industry 1} & \text{Industry 2} \end{array} \\
\begin{array}{l} \text{Home} \\ \text{Foreign} \end{array} & \begin{array}{l} \text{Industry 1} \\ \text{Industry 2} \\ \text{Industry 1} \\ \text{Industry 2} \end{array} & \left(\begin{array}{cc:cc} \nu_{H1,H1} & \nu_{H1,H2} & \nu_{H1,F1} & \nu_{H1,F2} \\ \nu_{H2,H1} & \nu_{H2,H2} & \nu_{H2,F1} & \nu_{H2,F2} \\ \hdashline \nu_{F1,H1} & \nu_{F1,H2} & \nu_{F1,F1} & \nu_{F1,F2} \\ \nu_{F2,H1} & \nu_{F2,H2} & \nu_{F2,F1} & \nu_{F2,F2} \end{array}\right)
\end{array}$$

Figure 1: General variance-covariance matrix of TFP shocks, $\nu_{n'g',n''g''} \equiv Cov\left[A_{n',g'}(s), A_{n'',g''}(s)\right]$

$$\begin{array}{lll}
 & & \begin{array}{cccc} \multicolumn{2}{c}{\text{Home}} & \multicolumn{2}{c}{\text{Foreign}} \\ \text{Industry 1} & \text{Industry 2} & \text{Industry 1} & \text{Industry 2} \end{array} \\
\begin{array}{l} \text{Home} \\ \text{Foreign} \end{array} & \begin{array}{l} \text{Industry 1} \\ \text{Industry 2} \\ \text{Industry 1} \\ \text{Industry 2} \end{array} & \left(\begin{array}{cc:cc} \nu_1 & 0 & \nu_1 & 0 \\ 0 & \nu_2 & 0 & \nu_2 \\ \hdashline \nu_1 & 0 & \nu_1 & 0 \\ 0 & \nu_2 & 0 & \nu_2 \end{array}\right)
\end{array}$$

(a) Industry-specific shocks, ν_g is the variance of shocks in industry $g = 1, 2$

$$\begin{array}{lll}
 & & \begin{array}{cccc} \multicolumn{2}{c}{\text{Home}} & \multicolumn{2}{c}{\text{Foreign}} \\ \text{Industry 1} & \text{Industry 2} & \text{Industry 1} & \text{Industry 2} \end{array} \\
\begin{array}{l} \text{Home} \\ \text{Foreign} \end{array} & \begin{array}{l} \text{Industry 1} \\ \text{Industry 2} \\ \text{Industry 1} \\ \text{Industry 2} \end{array} & \left(\begin{array}{cc:cc} \nu_H & \nu_H & 0 & 0 \\ \nu_H & \nu_H & 0 & 0 \\ \hdashline 0 & 0 & \nu_F & \nu_F \\ 0 & 0 & \nu_F & \nu_F \end{array}\right)
\end{array}$$

(b) Country-specific shocks, ν_n is the variance of shocks in country $n = H, F$

Figure 2: Special cases of the variance-covariance matrix of TFP shocks

In the rest of this paper I assume that there are only either industry-specific or country-specific shocks. I show by way of numerical examples, that in the case of industry-specific shocks financial integration induces specialization, while in the case of country-specific shocks financial integration *can* lead to a more diversified production structure. All the effects of financial integration can be demonstrated using the standard 2-good Ricardian model of Helpman and Razin (1978a) — these effects remain the same in the full model with a continuum of goods. Introduction of a continuum of goods into the Helpman-Razin model allows us to get smoothness of the model outcomes in response to changes in parameters, which, in turn, makes it possible to bring the Helpman-Razin model to data. One can think about the Helpman-Razin model as a special case of the framework presented in this paper, where each industry has just one good instead of a continuum of goods. Helpman and Razin (1978a) originally assumed only industry-specific shocks, and, as a consequence, came to a conclusion that financial integration necessarily induces specialization. I devote Section 4 to the analysis of the model by Helpman and Razin (1978a) with both industry-specific and country-specific shocks.

In the case of financial autarky with country-specific shocks the model presented here has multiple equilibria. The multiplicity of equilibria does not arise because we introduced a continuum of goods. Even the original framework by Helpman and Razin (1978a) has multiple equilibria, which was overlooked in the previous literature. In Section 4 I give an example with multiple equilibria in the Helpman-Razin model and provide intuition behind the multiplicity. Since the Helpman-Razin model has just 2 goods, it has a finite number of equilibria, and it is generally hard to find all the equilibria in this model. This is one reason why the multiplicity of equilibria in the Helpman-Razin model was not discovered up to this point. Once we introduce a continuum of goods into each industry, we get a continuum of equilibria. In Section 5 I show that these equilibria can be elegantly characterized and become "manageable" in the framework of this paper, which is another advantage — apart from "smoothness" — of introducing the continuum assumption.

3.5 Solution Approach

As I mentioned above, in Section 4 I analyze the Helpman-Razin model for both financial autarky and complete market cases with both industry- and country-specific shocks. In Section 5 I look at the model with a continuum of goods. I use different approaches to solve different variants of the model, because none of the approaches is suitable for all situations. Apart from the fact the model generally has multiple equilibria, the main complication with solving the model is that it is very difficult to find explicit expressions for prices of individual goods in different states of the world. The reason for that is that, contrary to the certainty case, state-specific prices are not equal to state-specific marginal costs:

$$\psi_n(s)p_g^{\omega}(s) \neq \frac{w_n}{A_n(s)z_{n,g}^{\omega}}.$$

We only have equivalence in expectation:

$$E_s\left[\psi_n(s)p_g^{\omega}(s)A_{n,g}(s)\right] = \frac{w_n}{z_{n,g}^{\omega}},$$

if good ω from industry g is produced in country n. That means that we have to explicitly include the complimentary slackness conditions (1):

$$l_{i,g}^{\omega} \geq 0 \text{ complementary to } w_i - E_s\left[\psi_i(s)p_g^{\omega}(s)A_{i,g}(s)z_{i,g}^{\omega}\right] \geq 0,$$

into the equilibrium system of equations. Solving such systems of equations is a challenging task.

To solve the Helpman-Razin model for the cases which have a unique equilibrium, I use the complementary slackness solver PATH.[7] To solve the Helpman-Razin model in the cases where it can potentially have multiple equilibria — the case of financial autarky with country-specific shocks, — I use a grid search on labor allocations to find all possible equilibria.

Next, in some cases with a continuum of goods it is possible to collapse the equilibrium system into a nonlinear system, which involves only aggregate quantities (aggregate labor allocations) and no complimentary slackness conditions. These are the cases of financial autarky with either industry- or country-specific shocks and the case of complete markets with industry-specific shocks. In these cases we can solve the model by the fixed point iteration on the aggregate quantities.

Unfortunately, a similar aggregation is very difficult — although possible in theory — in the case of complete financial markets with country-specific shocks. The difficulty with aggregating the model in this case is that we get a different set of expressions depending on particular values of model parameters. I explain this in detail in Section 5. To solve the model in this case, I use a completely different approach. I modify the model by artificially making it an Armington economy.[8] Namely, I introduce an artificial requirement that each country n buys each good ω from all countries. Varieties of good ω produced by different countries are combined by the CES utility function with elasticity σ'. The equilibrium system of the modified model does not involve complementary slackness conditions. As elasticity σ' goes to infinity, the artificial requirement disappears, and the modified model converges to the original model. Presumably, the solution of the modified model also converges to the solution of the original model. I assume that it is true. This solution approach can be applied in a broader context of trade models of comparative advantage and it is described in detail in Appendix A.

4 Effects of Financial Integration and Increased Riskiness in the Helpman-Razin Model

In this section I analyze numerical examples with the Helpman-Razin model for the cases of industry- and country-specific shocks. The purpose of these examples is to demonstrate the effects of financial integration and increased riskiness (i.e., volatility of shocks) as well as to show multiplicity of equilibria in the simple environment of the Helpman-Razin model. Understanding the intuition behind these effects in the standard 2-good Ricardian model will lay the foundation for understanding the effects in the model with a continuum of goods.

The Helpman-Razin model can thought of as a special case of the framework presented in this paper with 2 industries and one good in each industry instead of a continuum, i.e., with $G = 2$ and $\Omega = 1$. Since both industries have just one good, for the purposes of the current section we can use the words "industry" and "good" interchangeably to refer to an industry. Since there is only one good in each industry, the elasticity of substitution between goods from an industry, σ_g, does not play any role. So, we can forget about this parameter of the purposes of the current section.

[7] See Ferris and Munson (1999) for a description of the PATH solver. The corresponding programs — written in AMPL — are available on my web page, `http://www.personal.psu.edu/kzk145/`. One reason of why the Helpman-Razin model has not been carefully analyzed until now is that there were no tools to solve this model in the general case. PATH is one of the first nonlinear complementary slackness solvers which is able to solve the equilibrium systems of equations arising in the Helpman-Razin model. To the best of my knowledge, this paper is one of the first in the economics literature in general and in the international trade literature in particular to apply PATH to solve an economic model.

[8] I thank Michael Fabinger for giving me this idea.

Also, let us now interpret $T_{n,g}$ — the parameter of the Fréchet distribution from the model with a continuum of goods — as expected productivity, i.e., as the expected amount of good g which can be produced in country n with one unit of labor. For all examples below, each country is assumed to be endowed with one unit of labor, i.e., $L_n = 1$ for $n = H, F$.

	Home	Foreign
Good 1	**2**	**3**
Good 2	**1**	**3**

Table 1: Expected productivities for Sections 4.1 and 4.2, $T_{n,g}$

In Section 4.1 below I analyze the case of industry-specific shocks, and Section 4.2 I analyze the case of country-specific shocks. In both sections I use the same parametrization for expected productivities, $T_{n,g}$, which is given in Table 1. With this parametrization, Home has comparative advantage in good 1 and completely specializes in this good in the certainty case, while Foreign has absolute advantage in both goods and incompletely specializes in good 2 in the certainty case. This parametrization is chosen such that there is always a unique equilibrium for different values of preference parameters and different volatilities of shocks at which I look in Sections 4.1 and 4.2. At the same time, with this parametrization we can see all interesting effects of financial integration and increased riskiness. I look at the case with multiple equilibria in Section 4.3.

4.1 Industry-Specific Shocks

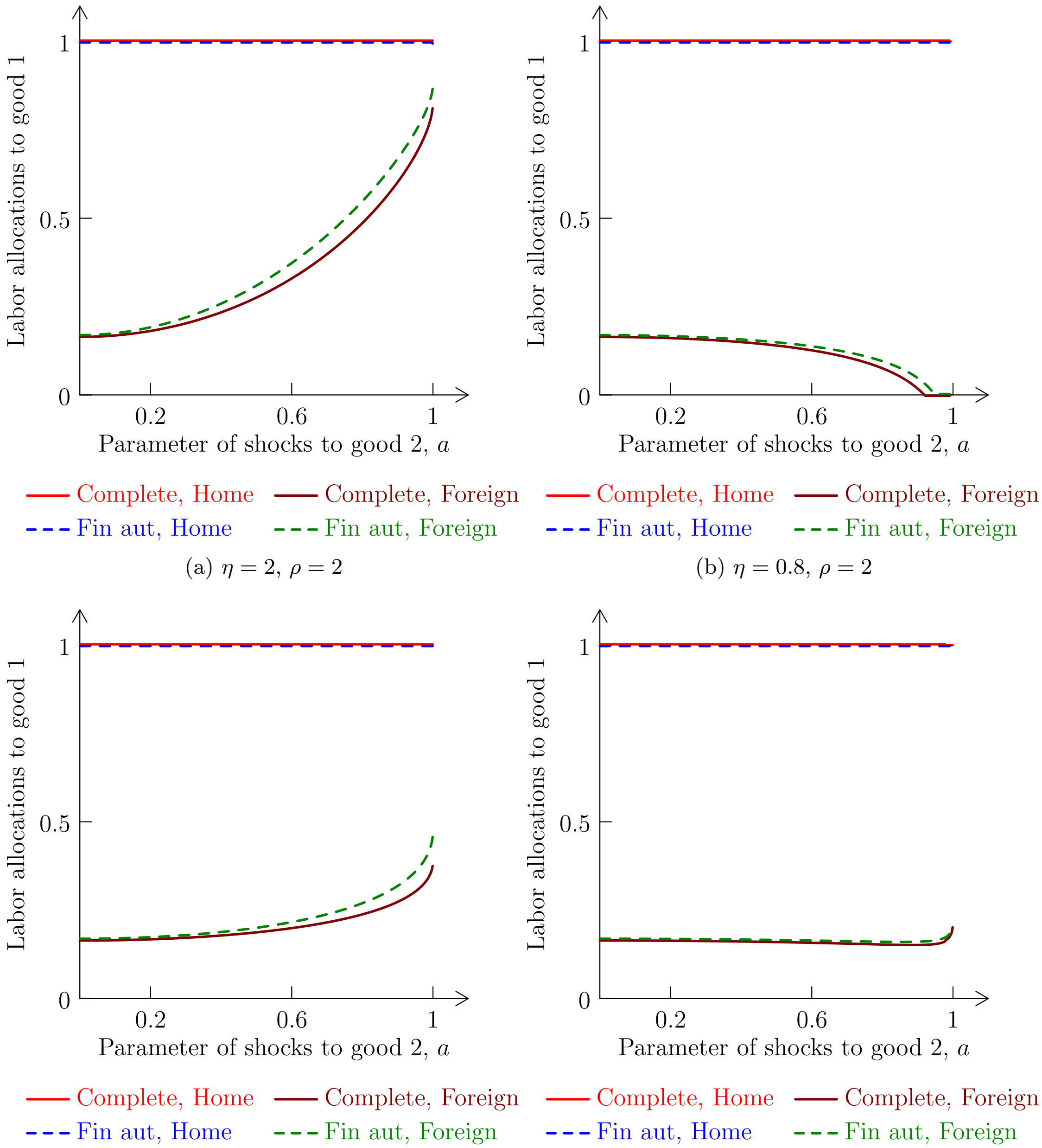
Let good 1 be a "safe" good, i.e., not subject to any shocks. Let good 2 be subject to a shock that can take a high or a low value with equal probabilities:

$$A_2(s) = \begin{cases} 1 - a, & \text{if } s = 1, \\ 1 + a, & \text{if } s = 2, \end{cases}$$

where $a \in [0, 1)$. The volatility of shock to good 2 is equal to a^2. So, the higher is a, the larger is the volatility of shock to good 2, and the riskier is production of this good.

Figure 3 depicts labor allocations to good 1 as functions of parameter a for different values of parameters η and ρ. Each of the graphs in Figure 3 captures two important aspects of the framework. The first aspect concerns the effects of changing the financial market structure: the graphs show what happens to labor allocations as we go from financial autarky to complete financial markets. The second aspect concerns the effects of increasing volatility of shock to good 2 (i.e., riskiness of production of good 2). Particular values of η and ρ are not of as much interest as their relation to 1: there are qualitative changes in effects of increased riskiness depending on whether η and/or ρ are smaller or greater than 1.

As it is clear from Figure 3, the effect of financial integration is qualitatively the same for all combinations of η and ρ. As we go from financial autarky to complete financial markets, countries become more specialized: the solid curves are either outside of the dashed curves or coincide with the dashed curves on all graphs. This is exactly the effect predicted by the conventional view about the role of financial integration. The intuition behind the effect of financial integration in this



(a) $\eta = 2$, $\rho = 2$

(b) $\eta = 0.8$, $\rho = 2$

(c) $\eta = 2$, $\rho = 0.4$

(d) $\eta = 0.8$, $\rho = 0.4$

Figure 3: Labor allocations to good 1 under industry-specific shocks

case is standard. *If countries are shut out from international risk-sharing, they hedge their risks by diversifying production. Once international risk-sharing is allowed, countries hedge their risks through financial markets and specialize more in their comparative advantage industries.*

Let us now discuss the effects of increased riskiness in production of good 2. Let us start with Figure 3a, which shows labor allocation to good 1 when $\eta > 1$ and $\rho > 1$. In this case goods 1 and 2 are substitutes and risk-aversion is "high". As we increase volatility of production of good 2, Foreign diverts labor from production of this good to production of good 1. Intuitively, as good 2 becomes riskier, it makes sense to substitute it with the safer good 1. Next, as Figure 3c shows, when risk-aversion is "low" and goods are substitutes ($\rho < 1$ and $\eta > 1$), the effect of increased riskiness is the same as in the case of "high" risk-aversion, although it is less pronounced.

When goods are complements, i.e., when $\eta < 1$, increased riskiness in good 2 can have the opposite effect on labor allocations compared to the case with $\eta > 1$. With $\eta < 1$, we have two competing effects. On the one hand, since good 2 is important for consuming good 1, as good 2 becomes riskier, it is important to invest even more into this good, to make sure there is enough output in bad times. On the other hand, since production of good 2 is high in good times, there no need to devote much labor to this good. Then, depending on the level of risk-aversion, one effect can dominate the other. When risk-aversion is "high", consumers care about bad outcomes much more than about good outcomes. So, as Figure 3b shows, as we increase volatility of shock to good 2, Foreign devotes even more labor to the production of this good. However, when risk-aversion is "low", consumers do not care as much about bad outcomes. Then, as Figure 3d shows, the second effect can start dominating the first effect at some point. We see from Figure 3d that, up to some level of volatility, labor allocation to good 1 in Foreign falls only slightly, and then it starts increasing with increase in riskiness in good 2.

It is also important to mention that when preferences over goods are Cobb-Douglas, i.e., when $\eta = 1$, increased riskiness has no impact on labor allocations. In this case labor allocations are the same as in the certainty case under both complete financial markets and financial autarky for any distribution of shocks to goods 1 and 2. With Cobb-Douglas preferences and industry-specific shocks, the price ratio of goods 1 and 2 is inversely proportional to the ratio shocks to goods 1 and 2. Also, the expenditure share on each good does not depend on the state of the world. Therefore, prices just adjust in each state of the world, leaving the spending shares unchanged and, so, labor allocations are the same as in the certainty case.

The outcomes of labor allocations described in the current subsection are not entirely new and can mostly be found in other works. This subsection can be seen as the first comprehensive overview of the effects of financial integration and increased riskiness in the Helpman-Razin model. The rest of the paper is devoted to the results which, to the best of my knowledge, cannot be found elsewhere. Therefore, before we proceed, it is instructive to pause and briefly talk about how the above results relate to the existing literature.

Rothschild and Stiglitz (1971) were the first who coherently analyzed effects of increased riskiness in a set of standard economic examples: investing into risky versus safe assets; saving more for tomorrow versus consuming more today; choosing the amount of capital today to complement or substitute labor tomorrow; and several other examples. All of these examples have a common theme: the effect of increased riskiness can switch from one direction to the other depending on preference parameters. In particular, Rothschild and Stiglitz (1971) showed that the effect of increased riskiness on portfolio allocation between safe and risky assets depends on whether the coefficient of relative risk aversion is smaller or greater than 1. In the same manner, Rothschild and Stiglitz (1971) showed that in a situation, where a firm faces uncertainty in productivity and needs to employ capital *ex ante* and labor *ex post*, the effect of increase in riskiness depends on whether capital and labor are complements or substitutes. Even though none of the examples from Rothschild and Stiglitz (1971) can directly be related to the Helpman-Razin model, the effects we observed in the current subsection are similar to the effects described in Rothschild and Stiglitz (1971).

Next, the effects of financial liberalization on labor allocations in a Ricardian model with uncertainty can be found in Helpman and Razin (1978a), Koren (2003), and Islamaj (2014). All of these papers have a conclusion that financial liberalization results in more specialized production structure. This is not surprising, because the assumptions made in these papers do not deviate enough from the assumption that countries are subject to only industry-specific shocks. For example, Helpman and Razin (1978a) and Koren (2003) literally work with only industry-specific shocks. Similarly, despite the fact that Islamaj (2014) introduces correlations between industry shocks, he only looks at examples where all industries in all countries are affected by shocks with the same or very similar volatilities. These assumptions are not enough to get more diversified production structure in response to financial liberalization. As we shall see in Section 4.2, such an outcome is possible under the assumption that shocks are country-specific and the volatility of shocks in one country is higher than in the other.

4.2 Country-Specific Shocks

Let us now look at examples of labor allocations similar to the ones we saw in the case of industry-specific shocks with the difference that now only Foreign is subject to a country-specific shock, while Home is a safe country. The shock to Foreign country can take one of two values — low or high — with equal probability:

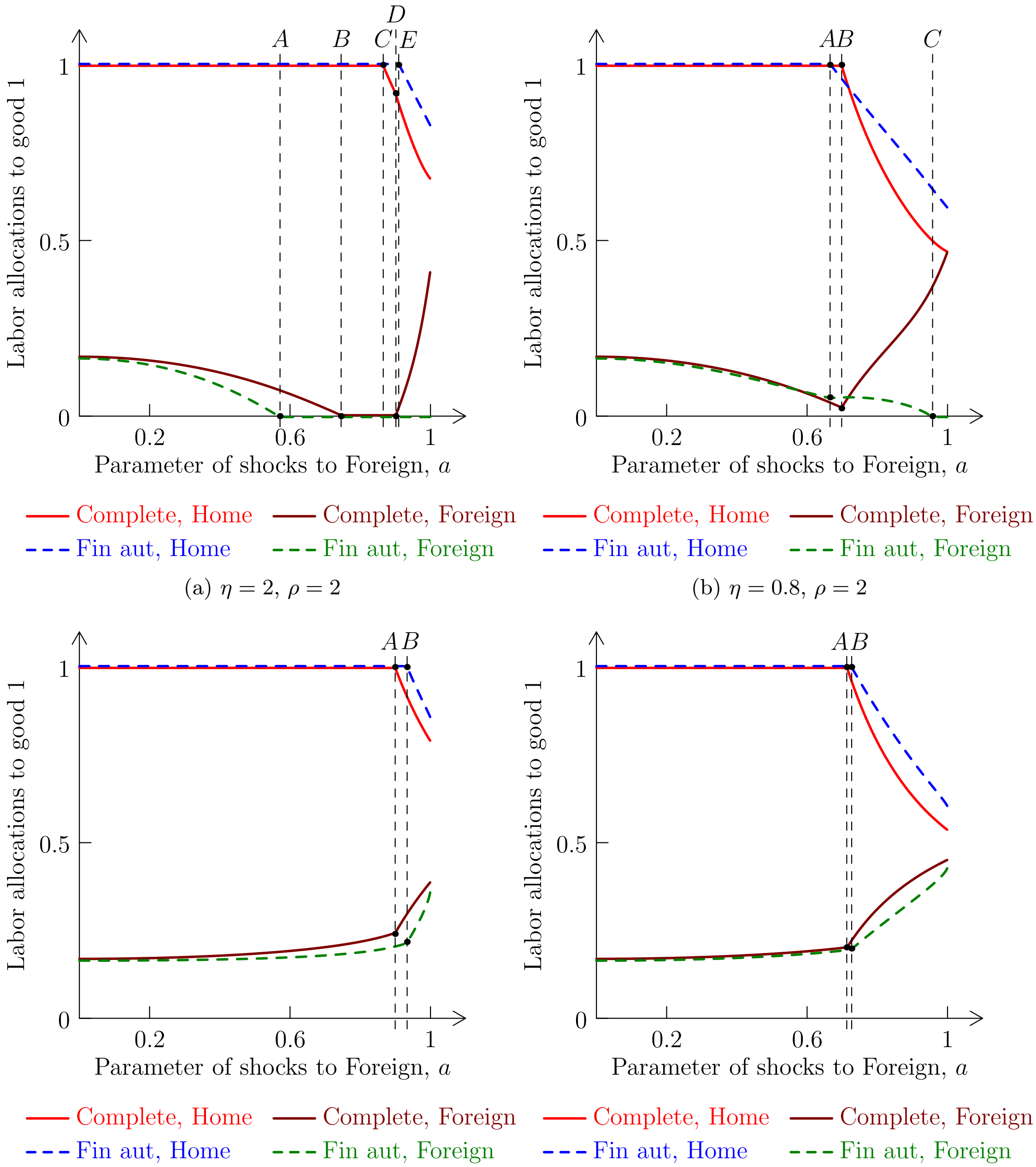
$$A_F(s) = \begin{cases} 1 - a, & \text{if } s = 1, \\ 1 + a, & \text{if } s = 2, \end{cases}$$

where $a \in [0, 1)$. The larger is a, the larger is the volatility of shock to Foreign, the riskier Foreign is.

Figure 4 depicts labor allocations to good 1 as functions of parameter a for different values of parameters η and ρ. A quick look at Figure 4 should convince us that the effect of financial integration on production patterns is ambiguous. On graphs (a), (c), and (d) of Figure 4, the solid curves are always either inside of the dashed curves or coincide with them. This means that in these particular cases financial integration leads to a weakly more diversified production structure. At the same time, as it can be seen from graph (b), corresponding to $\eta < 1$ and $\rho > 1$, financial integration can also lead to more specialization even under country-specific shocks: there is a region on graph (b), where the solid curves are outside of the dashed curves. However, as I discuss it below, the reason for more specialization in this case is totally different from the case of industry-specific shocks, and the logic of the conventional view about the role of financial markets does not apply in this case.

Overall, there are two separate effects which can lead to more diversification under financial integration. One effect concerns production in Home, and the other effect concerns production in Foreign. In essence, these effects can be described as follows. *First, after financial integration, Home becomes exposed to the Foreign risk by investing into production in Foreign. As a result, Home might be willing to reduce this risk by diversifying domestic production. Second, in the good state of the world Foreign can have so strong an absolute advantage in its comparative disadvantage good, that it might make sense for Foreign to allocate labor to production of this good. However, under financial autarky, Foreign's terms of trade insure the risk in production of its comparative advantage good and act as a counter-insurance for its comparative disadvantage good. This counter-acts Foreign's willingness to produce its comparative disadvantage good. Under financial integration, production of the comparative disadvantage good in Foreign is insured through financial markets, and so Foreign might be willing to produce more of this good.*

It is best first to talk about the effects of increased riskiness, before we turn to the detailed discussion of the effects of financial integration.



(a) $\eta = 2$, $\rho = 2$

(b) $\eta = 0.8$, $\rho = 2$

(c) $\eta = 2$, $\rho = 0.4$

(d) $\eta = 0.8$, $\rho = 0.4$

Figure 4: Labor allocations to good 1 under country-specific shocks

Effects of increased riskiness. Let us start with the extreme case in which the volatility of output in Foreign is very high. To have a particular image in mind, look at the parts of the graphs in Figure 4 close to $a = 1$. In the good state of the world Foreign is very productive in both goods — it turns into a "big" economy which can potentially supply the world with both goods. But if the bad state of the world realizes, Foreign turns into a "small" economy. In this state Home dominates the world. What is the best labor allocation in this uncertain world under financial integration? The best labor allocation should somewhat resemble the trade autarky outcome! Indeed, with such production structure, Foreign will supply the world with both goods if it happens to be very productive, and Home will be just a "small" part of this world. Otherwise, Home will supply the world with both goods, and Foreign will be a "small" part of this world. The role of financial integration here is very straightforward: Foreign gets insurance in the bad state of the world and pays for that insurance in the good state of the world. Of course, Home and Foreign can still trade and neither of them is truly a small open economy. So, their labor allocations would not go the full way to the trade autarky outcome. Nevertheless, both Home and Foreign production structure under financial integration would be diversified. This is what we observe in all graphs (a)-(d) in Figure 4: the solid curves for Home and Foreign are close to each other and to the 0.5 level on the vertical axis when a is close to 1.

Can a similar labor allocation pattern happen under financial autarky? Surely it can if, for example, agents are risk-neutral. In this extreme case there will be no difference at all between labor allocations under financial autarky and complete financial markets. Obviously then, if agents are not very risk averse, the financial autarky outcomes will still resemble the complete financial markets outcomes. This is what we observe on graphs (c) and (d) in Figure 4: on these graphs the dashed curves have a similar pattern to the solid curves when a is close to 1 (and, actually, for other values of a as well).

However, the picture is different, if agents are very risk-averse. Looking at the dashed curves on graphs (a) and (b) in Figure 4, we see that Home and Foreign differ in the way they response to increased risk in financial autarky for high values of a. Home diversifies its production structure, while Foreign specializes in its comparative advantage good 2. To understand the difference in the outcomes, we need to understand three things, which follow one from another. First, productivity shocks to Foreign do not change its comparative advantage, and under financial autarky each country exports its comparative advantage good and imports the other good in any state of the world. At the same time, this two-way trade does not necessarily happen in each state of the world under financial integration. As we discussed it above, when volatility in Foreign is high, Home can be exporting both goods in the state with low output in Foreign and importing both goods in the other state under financial integration. Second, in our example — with no shocks to Home — the low output in Foreign hurts both countries. In other words, the bad state for Foreign is also bad for Home. Indeed, when Foreign's output is low, Foreign is obviously worse off relative to the state with high output. But Home is worse off as well, because it faces a higher price of the importing good relative to the state with high output in Foreign. Finally, in the bad state of the world Home and Foreign are in an unequal situation. Home faces adverse terms of trade, while Foreign faces favorable terms of trade.[9] Indeed, when output of good 2 in Foreign is low, the price of good 2 relative to the price of good 1 is high, because Foreign is the exporter of good 2. So, the price of good 2 relative to price of good 1 insures production of good 2 and acts as a counter-insurance for production of good 1 in both countries.

Then, if agents from the two countries are very risk-averse, they care more about the bad state of the world. To mitigate the adverse effect of the terms of trade in the bad state, Home diversifies production. At the same time, the terms of trade insure Foreign's production of good 2, and so Foreign puts even more resources into this good.

Up to this point we were discussing the effects of increased riskiness in extreme cases with very high volatility in Foreign. Let us now look at the cases with low volatility in Foreign, i.e., at the cases with parameter a being close to 0. For low values of output volatility Foreign remains the exporter of good 2 in any state of the world under both financial autarky and complete markets. Therefore, similar to the situation with high volatility of shocks, Foreign is insured by the terms of trade

[9] Terms of trade of country n are defined as the price of its export good relative to the price of its import good.

from low productivity. This force pushes Foreign to put more resources into production of good 2. However, there is another force, pushing Foreign to put more resources into production of good 1. In the good state of the world Foreign has absolute advantage in good 1. And the higher is the volatility of output in Foreign, the stronger the absolute advantage in the good state of the world is. Therefore, Foreign can potentially benefit more from putting more resources into the production of good 1. Then, if Foreign is very risk-averse, the first force dominates and labor allocation to good 1 decreases in response to increased riskiness. This effect can be seen in Figures 4a and 4b for dashed curves before vertical line A under financial autarky and for solid curves before vertical line B under complete markets. If, on the other hand, Foreign is not so risk-averse, the second force dominates and labor allocation to good 1 increases in response to increased riskiness. This effect can be seen in Figures 4c and 4d for dashed curves before vertical line B under financial autarky and for solid curves before vertical line A under complete markets.

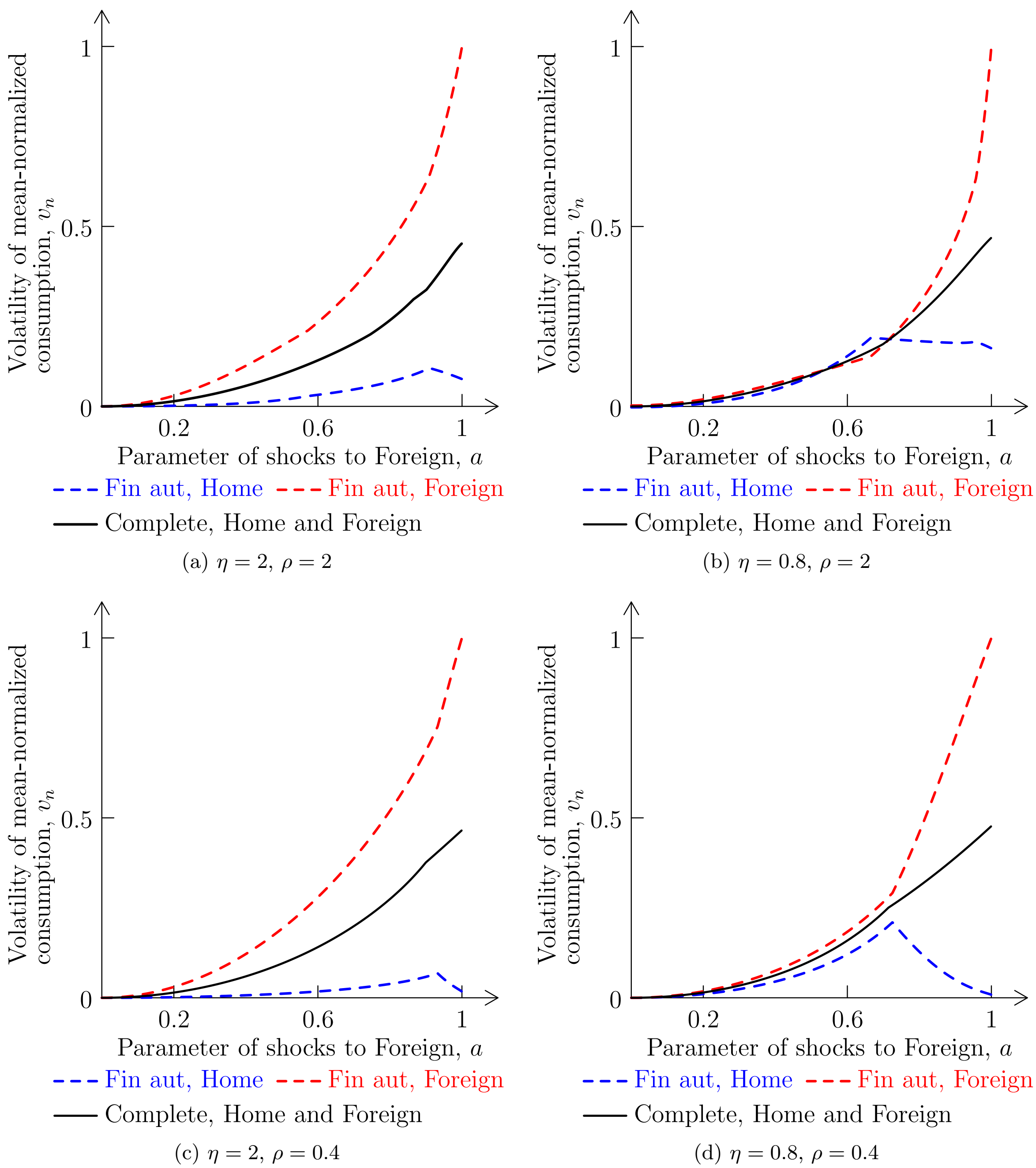
For intermediate values of shocks to Foreign, as a increases, we first see the same effect as with low shocks, and after some point we see the same effect as with extremely high shocks. For example, on Figure 4a the effect with low shocks explains the parts of dashed curves (corresponding to financial autarky) before the vertical line E, and the effect with high shocks starts kicking in after that. Similarly, on the same figure the effect with low shocks applies to the parts of solid curves (corresponding to complete markets) before the vertical line C, and the effect with high shocks starts being relevant after that.

Effects of financial integration. Now we are ready to proceed with a detailed discussion of the effects of financial integration using our examples in Figure 4. In order to understand the effects of financial integration in the case of country-specific shocks, it helps to start with a trivial observation that the aggregate risk in our economy is always present under any structure of financial markets. Indeed, financial markets do not magically eliminate shocks. The role of financial markets is to allow countries to share risks. This means that one country can insure the other country from bad outcomes. In our example Home is not affected by any shocks, while Foreign can have a low or a high productivity. Nevertheless, as we discussed it above, low productivity in Foreign hurts both countries. And it is actually not obvious, which country is in a worse situation in the bad state of the world: Home or Foreign. As a matter of fact, Foreign can potentially be in a relatively (to Home) better position, when Foreign's output is low, because Foreign faces favorable terms of trade, which insure Foreign to at least some extent. This argument brings us to an important conclusion that Foreign might be providing insurance to Home in the bad state under financial integration, even though Foreign experiences low productivity in this state. Of course, the opposite might happen as well: Home might be insuring Foreign from the bad state of the world.

Knowing which country provides insurance is the key to understanding in which direction the production structure of both countries changes after financial integration. When we observe the behavior of countries under financial integration, we can easily check which country provides insurance by looking at trade balance in the bad state of the world. The country with trade surplus in the bad state of the world (i.e., the country for which the value of exports exceeds the value of imports) provides insurance. There is also a straightforward way to check which country is going to provide insurance under financial integration by looking at the volatility of mean-normalized consumption under financial autarky, where the volatility of mean-normalized consumption in country n is defined as

$$v_n \equiv Var\left[\frac{C_n(s)}{E\left[C_n(s)\right]}\right].$$

The volatility of mean-normalized consumption represents the amount of risk a country is exposed to. The country with a lower v_n under financial autarky is in a relatively better position, and it is going to provide insurance to the other country under financial integration. Indeed, under complete markets, $v_H = v_F$. Since the aggregate risk is present under any structure of financial markets, as we go from financial autarky to complete markets, the aggregate risk does not disappear but is "reallocated" between the two countries. One country is going to "take" risk — by providing insurance — to the other country. Clearly, the country with higher v_n is in more need for insurance, and it is going to get it from the other country. As a result, v_n of the insured country is going to fall and v_n of the other country is going to rise after financial integration.



(a) $\eta = 2$, $\rho = 2$

(b) $\eta = 0.8$, $\rho = 2$

(c) $\eta = 2$, $\rho = 0.4$

(d) $\eta = 0.8$, $\rho = 0.4$

Figure 5: Volatility of mean-normalized consumption under country-specific shocks

Figure 5 depicts the volatilities of mean-normalized consumption for the four combinations of parameters we consider in this section. As we can see from Figure 5, in cases (a), (c), and (d) Foreign is riskier (in the sense of having higher v_n) than Home for all values of parameter a. And after financial integration Home "takes" some of the Foreign's risk in these cases. In case (b) there is a region around $a = 0.6$ where Home is actually riskier than Foreign. In that region Foreign provides insurance to Home after financial integration. The reason for Home being riskier in case (b) is that Home faces unfavorable terms of trade in the bad state of the world, and Home cannot substitute away consumption of good 2 with consumption of good 1, because $\eta < 1$ and so goods 1 and 2 are complements.

Having figured out the exact role of financial markets in our environment, we should now easily be able to understand the effects of financial integration on production structure. In cases (a), (c), and (d) from Figure 5 Home "takes" risk from Foreign after financial integration. As a result, Home becomes weakly more diversified: to establish this, we apply exactly the same logic which we used to understand the effects of increased riskiness on Home's production structure. At the same time, Foreign "off-loads" some of its risk to Home and becomes a less riskier country. Therefore, Foreign can exploit more the absolute advantage in good 1 in the good state of the world by devoting more resources to production of this good. As a consequence, Foreign also becomes more diversified in these cases. Again, this is exactly the same logic as we used earlier to understand the effects of increased riskiness on Foreign's production structure — with the only difference that now we apply this logic in the direction of reducing riskiness.

In case (b) from Figure 5 Foreign "takes" Home's risk in the region of values of a around 0.6. In that region both countries become more specialized after financial integration (see the corresponding region in Figure 4b). The logic behind this outcome is totally different from the standard logic behind the conventional view of financial integration. The conventional view might mistakenly attribute increased specialization in Foreign to Home insuring Foreign so that Foreign can focus on production of its comparative advantage good. As we now understand, the effect is actually quite the opposite. Foreign specializes more in its comparative advantage good, because Foreign insures Home and needs to absorb Home's risk by relying on the insurance through the terms of trade, which pushes Foreign to produce more of its insured good 2.

Let us close this subsection with some thoughts on the generality of the above results. Although I do not provide any mathematical proofs, some of the results above seem to be general. For example, when volatility of shocks to Foreign is high enough, financial integration shall always lead to a more diversified production structure, as long as there is a unique equilibrium under financial autarky. On the other hand, for low shocks the effect is certainly ambiguous for $\eta < 1$ and any value of ρ (it is just happened so that in case (d) from Figure 4 financial integration induces diversification for all values of a; it is possible to construct examples where financial integration induces specialization for low enough values of a and $\rho < 1$ and $\eta < 1$). But even if the effect of financial integration is ambiguous and the conventional view can sometimes make correct predictions, there is a certain value in the analysis we have done: as we saw, the conventional view might be making its correct predictions for the wrong reasons.

4.3 Multiple Equilibria in the Case of Financial Autarky with Country-Specific Shocks

To the best of my knowledge, this paper is the first one to point out that the Helpman-Razin model can have multiple equilibria in the case of financial autarky. The analysis of Section 4.2 should convince us that multiple equilibria do not appear for every set of parameters. As we shall see in the following examples, the multiplicity occurs when the comparative advantage of countries is not "strong", where the notion of "strong" depends on parameters (preferences, productivities, shocks) of the model.

Example 1. Let us consider an example with preference parameters $\eta = 2$ and $\rho = 2$. As in the previous subsection imagine that Foreign is subject to a low or a high shock with equal probabilities. Expected and state-specific productivities are provided in Table 2. The low shock is equal to 0.1, while the high shock is equal to 1.9 ($a = 0.9$ in terms of the parametrization from Section 4.2). With this parametrization Home has comparative advantage in good 1, but this comparative advantage is "weak".

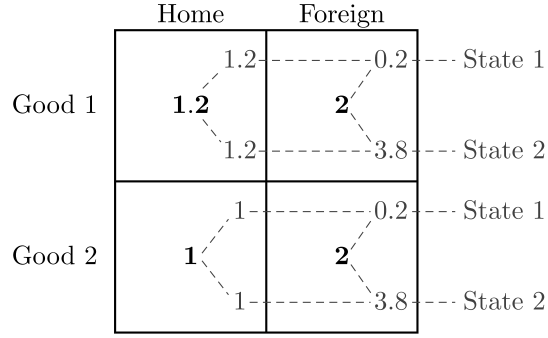


Table 2: Expected and state-specific productivities in Example 1 from Section 4.3, $T_{n,g}$ and $A_{n,g}(s)T_{n,g}$

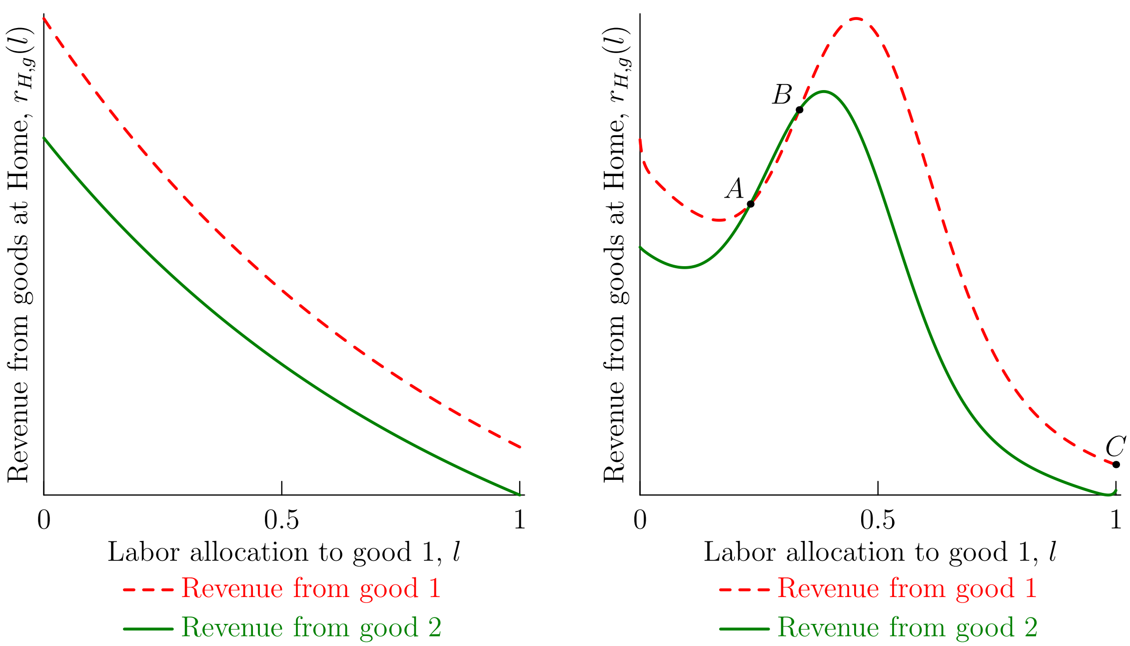
		Labor allocations			
		Financial autarky			Complete markets
		Equilibrium 1	Equilibrium 2	Equilibrium 3	
Home	Good 1	0.85	0.39	0.26	0.60
	Good 2	0.15	0.61	0.74	0.40
Foreign	Good 1	0.0	0.77	1.0	0.36
	Good 2	1.0	0.23	0.0	0.64

Table 3: Labor allocations in different equilibria in Example 1 from Section 4.3

There are three equilibria under financial autarky in this economy and a unique equilibrium under complete markets. Labor allocations in all equilibria are provided in Table 3. Let us first look at equilibria under financial autarky. Equilibrium 1 is the "standard" equilibrium — similar to the ones we saw in Section 4.2, in which Foreign takes advantage of the terms of trade insurance by specializing in its comparative advantage good 2. Equilibrium 3 is similar to equilibrium 1 in the sense that in equilibrium 3 Foreign also exploits the terms of trade insurance. The difference is that in equilibrium 3 Foreign captures production of its comparative disadvantage good 1 and leaves little place for Home to produce good 1. In equilibrium 3 Foreign will not move more resources into production of good 2 because its terms of trade act as a counter-insurance for this good, and Home will not move more resources into production of good 1, because the relative price of this good is very low when Foreign's productivity is high. Equilibrium 2 is an "intermediate" equilibrium, where both countries rely on diversification of production instead of terms of trade to insure themselves from low productivity in Foreign. In this equilibrium countries almost do not trade with each other in the bad state of the world and rely on their domestic production.

Let us now briefly discuss the complete markets equilibrium. In this equilibrium both countries diversify their production more than in all financial autarky equilibria. Nevertheless, we can directly compare only financial autarky equilibrium 1 with the complete markets equilibrium. In both of these equilbria countries incompletely specialize in their comparative advantage goods. At the same time, in financial autarky equilibria 2 and 3 countries incompletely specialize in their comparative disadvantage goods, and it is actually possible to find examples where in these kinds of equilibria under financial autarky countries diversify their production more than under financial integration.

Example 2. The multiplicity of equilibria under financial autarky arises because firms' expected revenues from production of different goods are non-convex functions of labor allocations, if consumers are risk-averse. The simplest way to see the non-convexity is to look at an example with symmetric countries which are subject to i.i.d. shocks with the same variance.



(a) $a = 0$ (b) $a = 0.9$

Figure 6: Expected revenues from production of goods 1 and 2

Suppose that preferences over goods are Cobb-Douglas with equal expenditure shares (i.e., $\eta = 1$ and $\alpha_1 = \alpha_2 = 1/2$). Expected productivities are given by $T_{H,1} = T_{F,2} = T$ and $T_{H,2} = T_{F,1} = 1$. There are two independent shocks: one affects production of both goods in Home only, and the other affects Foreign only. Each of the shocks can take a low or a high value with equal probability. The low value is equal to $1 - a$ and the high value is equal to $1 + a$ for both shocks, where $a \in [0, 1)$. Since shocks are independent, there are four equally probable states of the world corresponding to the four combinations of values of Home and Foreign shocks:

$$(A_H(s), A_F(s)) = \begin{cases} (1-a, 1-a), & \text{if } s = 1, \\ (1-a, 1+a), & \text{if } s = 2, \\ (1+a, 1-a), & \text{if } s = 3, \\ (1+a, 1+a), & \text{if } s = 4. \end{cases}$$

Let us search for symmetric equilibria only, i.e., for equilibria in which Home labor allocation to good 1 is equal to Foreign labor allocation to good 2, $l_{H,1} = l_{F,2}$. Consider any symmetric labor allocation in Home and Foreign such that $l_{H,1} = l_{F,2} = l$ and $l_{H,2} = l_{F,1} = 1 - l$, where $l \in [0, 1]$ (assuming that each country is endowed with one unit of labor). The goods market clearing conditions imply that prices as functions of l are given by:

$$\frac{p_1(s;l)}{p_2(s;l)} = \frac{A_H(s)(1-l) + A_F(s)Tl}{A_H(s)Tl + A_F(s)(1-l)}.$$

Income in state s in Home as a function of l is given by

$$I_H(s;l) = A_H(s)p_1(s;l)Tl + A_H(s)p_2(s;l)(1-l),$$

and the expected revenues from production of goods 1 and 2 in Home as functions of l are given by

$$r_{H,1}(l) = \sum_s h(s)A_H(s)I_H(s;l)^{-\rho}P(s;l)^{\rho-1}Tp_1(s;l),$$
$$r_{H,2}(l) = \sum_s h(s)A_H(s)I_H(s;l)^{-\rho}P(s;l)^{\rho-1}p_2(s;l),$$

where $P(s;l)$ is the Cobb-Douglas price index.

Suppose that $\rho = 5$ and $T = 1.2$. Figure 6 depicts expected revenues $r_{H,1}(l)$ and $r_{H,2}(l)$ as functions of l for two parametrizations of shocks: $a = 0$ and $a = 0.9$. The case with $a = 0$, depicted in Figure 6a, corresponds to the certainty case. In this case the revenue functions are convex, and the function for good 1 is always larger than the function for good 2. There is a unique equilibrium in this case with $l = 1$: since revenue from good 1 is always larger than the revenue from good 2, Home allocates all labor to production of good 1. In the case with $a = 0.9$, depicted in Figure 6b, the revenue functions are non-convex and intersect two times, resulting in three equilibria denoted by points A, B, and C. The first two equilibria occur at the points of intersection of curves $r_{H,g1}(l)$ and $r_{H,g1}(l)$. The third equilibrium is the same as in the certainty case with Home completely specializing in good 1.

5 Theoretical Predictions for the Model with a Continuum of Goods

In this section I analyze the model with a continuum of goods. As I discussed it earlier, in the case of financial autarky with country-specific shocks the model presented here has a continuum of equilibria. I first use the variant of the model with only one industry to show how the continuum of equilibria arises and how it can be characterized. I also analyze of the variant of the model with one industry for the case of complete financial markets and provide welfare comparisons of gains from financial integration on top of gains from trade. After that I move to the analysis of the model with two industries.

Before we move on, it is useful to have the following Claim which applies to any structure of shocks, either case of financial markets, and which is true regardless of how many equilibria exist in the model:

Claim 1. *Consider any country $i = H, F$ and any industry g. Suppose that goods $\omega', \omega'' \in \Omega$ from industry g are produced by country i only. Then, in both cases of financial markets and for an arbitrary structure of shocks,*

$$\frac{p_g^{\omega'}(s)}{p_g^{\omega''}(s)} = \left(\frac{z_{i,g}^{\omega'}}{z_{i,g}^{\omega''}}\right)^{-1} \quad \text{and} \quad \frac{l_{i,g}^{\omega'}}{l_{i,g}^{\omega''}} = \left(\frac{z_{i,g}^{\omega'}}{z_{i,g}^{\omega''}}\right)^{\sigma_g - 1},$$

where s is any state of the world.

For proof see the Algebra Appendix. This claim says that for any goods, in which countries completely specialize, all prices and labor allocations are the same up to a good-specific normalization. This property is the same as in the certainty case, and it is going to be the key property which allows us to find expressions for aggregate quantities.

5.1 One Industry and Country-Specific Shocks

For the purposes of this subsection we can drop industry index g to keep notation simple. Then shocks to countries are denoted by $A_H(s)$ and $A_F(s)$. Also, as usual, it is convenient to assume that goods within each industry are ordered so that comparative advantage of Home falls with good index ω, i.e., $z_F^{\omega}/z_H^{\omega}$ is an increasing function of index ω.

5.1.1 Financial Autarky

We can guess and verify that there exists a class of equilibria where goods are split into those produced by Home only and those produced by Foreign only (see the Algebra Appendix for all derivations). We focus only on these equilbira. Generally this framework results in equilibria where the chain of comparative advantage in the goods produced by one country breaks. I do not consider these equilibria. Let B be the threshold dividing goods produced by Home and Foreign, so that goods ω, such that $z_F^{\omega}/z_H^{\omega} < B$ are produced by Home. We need to find the conditions which determine this threshold.

It follows from Claim 1 that prices are given by

$$p^{\omega}(s) = \begin{cases} \Lambda_H(s)\,[z_H^{\omega}]^{-1}, & \text{if } \omega \text{ is produced by Home;} \\ \Lambda_F(s)\,[z_F^{\omega}]^{-1}, & \text{if } \omega \text{ is produced by Foreign;} \end{cases}$$

where $\Lambda_H(s)$ and $\Lambda_F(s)$ are some unknowns, which are part of the equilibrium solution. The analogs of $\Lambda_H(s)$ and $\Lambda_F(s)$ in the certainty case are Home and Foreign wages. It is important to note here that the above expressions for prices do not give us "out of equilibrium" prices: if a country does not produce some good ω in equilibrium, these expressions do not tell us what is the price of this good that would make this country willing to produce good ω.

The ratio $\Lambda_F(s)/\Lambda_H(s)$ defines the state-specific competitiveness of Foreign relative to Home in the same manner as how in the certainty case the ratio of wages w_F/w_H defines Foreign's relative competitiveness. We can show that for any two states s' and s'':

$$\frac{\Lambda_F(s')/\Lambda_H(s')}{\Lambda_F(s'')/\Lambda_H(s'')} = \left(\frac{A_F(s')/A_H(s'')}{A_F(s')/A_H(s'')}\right)^{-\frac{1}{\sigma}}.$$

Hence, the Foreign's relative competitiveness changes from state to state. In the certainty case Foreign's competitiveness level is the supply side of the condition which determines the threshold between goods produces by Home and Foreign. Since in the uncertainty case Foreign's competitiveness changes across states, while the threshold B is determined *ex ante*, the link between Foreign's competitiveness and the threshold B is indirect. It can be expressed through the following quantity:

$$\lambda\left(s\right) \equiv \frac{\Lambda_H(s)B}{\Lambda_F(s)}.$$

In the certainty case the corresponding quantity is just 1.

Next, in the certainty case, the equilibrium on the goods market is the demand side of the condition which determines the threshold between goods produced by Home and Foreign. In the uncertainty case, the equilibrium on the goods market pins down the link $\lambda(s)$ between competitiveness level and the threshold:

$$\lambda(s) = \left(\frac{A_H(s)L_H T_H^{-1}}{A_F(s)L_F T_F^{-1}}B^{-\theta-1}\right)^{-\frac{1}{\sigma}}, \tag{2}$$

while the threshold itself remains undetermined. This gives rise to the indeterminacy of equilibria. The link $\lambda(s)$ is directly related to Home's terms of trade:

$$tot_H\left(s\right) \equiv \frac{P_{HH}(s)}{P_{FF}(s)} = \lambda\left(s\right)\left(\frac{T_H}{B^{-\theta}T_F}\right)^{\frac{1}{1-\sigma}}, \tag{3}$$

where $P_{HH}(s)$ and $P_{FF}(s)$ are the price indices of goods produced by Home and Foreign. By the definition of threshold B, the higher is B, the more goods are produced by Home. Hence, as we can see from equations (2) and (3), the measure of goods produced by Home influences Home's terms of trade. The multiplicity of equilibria arises because, by capturing some measure of goods on the margin for production, Home gets insurance for these goods through the terms of trade, leaving no place for Foreign to produce these goods. This is exactly the same channel that gives rise to multiplicity of equilibria in the case with two goods.

Using the Home and Foreign firms' problems, we can find the conditions which put bounds on the threshold B. Recall that the Home and Foreign firms' problems result in the following complementary slackness condition (1):

$$l_n^\omega \geq 0 \text{ complementary to } w_n - \sum_{s=1}^{\mathcal{S}} \psi_n(s)p^\omega(s)A_n(s)z_n^\omega \geq 0.$$

Let us look at any good ω which Home does not produce in equilibrium, i.e., $l_H^\omega = 0$. The Home's firm complementary slackness condition implies that

$$\sum_{s=1}^{\mathcal{S}} \psi_H(s)p^\omega(s)A_H(s)z_H^\omega \leq w_H.$$

Since good ω is produced by Foreign, its price is $p^{\omega}(s) = \Lambda_F(s)\,[z_F^{\omega}]^{-1}$. Substituting this price into the above inequality, we get:

$$\frac{z_H^{\omega}}{z_F^{\omega}} \sum_{s=1}^{\mathcal{S}} \psi_H(s) A_H(s) \Lambda_F(s) \leq w_H.$$

Similarly, by looking at any good ω which Foreign does not produce in equilibrium, we get

$$\frac{z_F^{\omega}}{z_H^{\omega}} \sum_{s=1}^{\mathcal{S}} \psi_F(s) A_F(s) \Lambda_H(s) \leq w_F.$$

Since $z_F^{\omega}/z_H^{\omega}$ is a continuous function of index ω, the above two inequalities for Home and Foreign hold for the threshold good ω_B as well. Since $B = z_F^{\omega_B}\big/z_H^{\omega_B}$, we can combine the inequalities for Home and Foreign to get the conditions for the threshold:

$$\frac{\sum_{s=1}^{\mathcal{S}} \psi_H(s) A_H(s) \Lambda_F(s)}{w_H} \leq B \leq \frac{w_F}{\sum_{s=1}^{\mathcal{S}} \psi_F(s) A_F(s) \Lambda_H(s)}.$$

The conditions for the threshold B are complex expressions, which involve endogenous quantities. We can derive that

$$\frac{\sum_{s=1}^{\mathcal{S}} \psi_H(s) A_H(s) \Lambda_F(s)}{w_H} = \frac{L_H}{L_F} \cdot \frac{\sum_{s'} h(s') \pi_{FF}(s') \dfrac{A_H(s')}{A_F(s')} A_H(s')^{1-\rho} \left(1 + \dfrac{I_F(s')}{I_H(s')}\right)^{\rho} [\Xi_H(s')]^{1-\rho}}{\sum_{s'} h(s') \pi_{HH}(s') [A_H(s')]^{1-\rho} \left(1 + \dfrac{I_F(s')}{I_H(s')}\right)^{\rho} [\Xi_H(s')]^{1-\rho}},$$

$$\frac{w_F}{\sum_{s=1}^{\mathcal{S}} \psi_F(s) A_F(s) \Lambda_H(s)} = \frac{L_H}{L_F} \cdot \frac{\sum_{s'} h(s') \pi_{FF}(s') A_H(s')^{1-\rho} \left(1 + \dfrac{I_H(s')}{I_F(s')}\right)^{\rho} \Xi_H(s')^{1-\rho}}{\sum_{s'} h(s') \pi_{HH}(s') \dfrac{A_F(s')}{A_H(s')} A_H(s')^{1-\rho} \left(1 + \dfrac{I_H(s')}{I_F(s')}\right)^{\rho} \Xi_H(s')^{1-\rho}},$$

where

$$\Xi_H(s) \equiv L_H T_H^{\frac{1}{\theta}} \pi_{HH}(s)^{-\frac{\sigma}{\sigma-1}} m_H^{-\frac{1}{\theta}+\frac{1}{\sigma-1}}$$

is a short-cut introduced for convenience, $\pi_{HH}(s)$ is the expenditure share of Home on Home's goods, and m_H is the measure of goods produced by Home. Contrary to the certainty case, $\pi_{HH}(s) \neq m_H$, as can be seen from the corresponding expressions:

$$m_H = \frac{T_H}{T_H + T_F B^{-\theta}},$$
$$m_F = \frac{T_F B^{-\theta}}{T_H + T_F B^{-\theta}},$$

and

$$\pi_{HH}(s) = \pi_{FH}(s) = \frac{\lambda(s)^{1-\sigma} T_H}{\lambda(s)^{1-\sigma} T_H + T_F B^{-\theta}},$$
$$\pi_{HF}(s) = \pi_{FF}(s) = \frac{T_F B^{-\theta}}{\lambda(s)^{1-\sigma} T_H + T_F B^{-\theta}}.$$

Intuitively, since relative competitiveness of countries varies across states, the trade shares are different, while the share of goods produced by each country is determined *ex ante*.

It is convenient to introduce notation for bounds on B:

$$b_H \equiv \frac{\sum_{s=1}^{\mathcal{S}} \psi_H(s) A_H(s) \Lambda_F(s)}{w_H},$$
$$b_F \equiv \frac{w_F}{\sum_{s=1}^{\mathcal{S}} \psi_F(s) A_F(s) \Lambda_H(s)}.$$

Expressions for b_H and b_F are relatively simple when $\eta = 1$, $\sigma = 1$, and $\rho = 1$:

$$b_H = \frac{L_F}{L_H} \cdot \frac{T_H}{T_F} \cdot B^\theta \cdot \left(\sum_{s'} h(s') \frac{A_H(s')}{A_F(s')} \right)^{-1},$$
$$b_F = \frac{L_F}{L_H} \cdot \frac{T_H}{T_F} \cdot B^\theta \cdot \sum_{s'} h(s') \frac{A_F(s')}{A_H(s')}.$$

Note that, when there is no uncertainty, we get the familiar expression $b_H = b_F = \frac{L_F}{L_H} \cdot \frac{T_H}{T_F} \cdot B^\theta$.

As it should be clear from the above discussion, b_H and b_F are endogenous quantities and they are different across different equilibria. The smallest value for b_H is achieved in an equilibrium with $B = b_H$, while the largest value for b_F is achieved in an equilibrium with $B = b_F$. Let us denote the corresponding values by $\bar{b}_H$ and $\bar{b}_F$. Then, in any equilibrium, goods ω such that $z_F^\omega / z_H^\omega < \bar{b}_H$ are produced by Home, and goods ω such that $z_F^\omega / z_H^\omega > \bar{b}_F$ are produced by Foreign. The set of goods between $\bar{b}_H$ and $\bar{b}_F$ can be labeled as a "set of disputed goods". Each particular value of B determines the split in the set of disputed goods between Home and Foreign: goods ω such that $\bar{b}_H \leq z_F^\omega / z_H^\omega \leq B$ are produced by Home, and the rest is produced by Foreign. Each equilibrium from the continuum can be associated with the percentage of disputed goods produced by Home given by $\kappa \equiv \left(B - \bar{b}_H\right) / \left(\bar{b}_F - \bar{b}_H\right)$. Having this notation for κ gives us a way to compare equilibria for different values of volatility of shocks corresponding to the same value of κ. In what follows, I will call **κ-equilibrium** an equilibrium corresponding to a particular value of κ.

Let us now look at gains from trade. In any equilibrium they are given by:

$$GT_n(s) = \pi_{nn}(s)^{-\frac{1}{\sigma-1}} m_n^{-\frac{1}{\theta}+\frac{1}{\sigma-1}}.$$

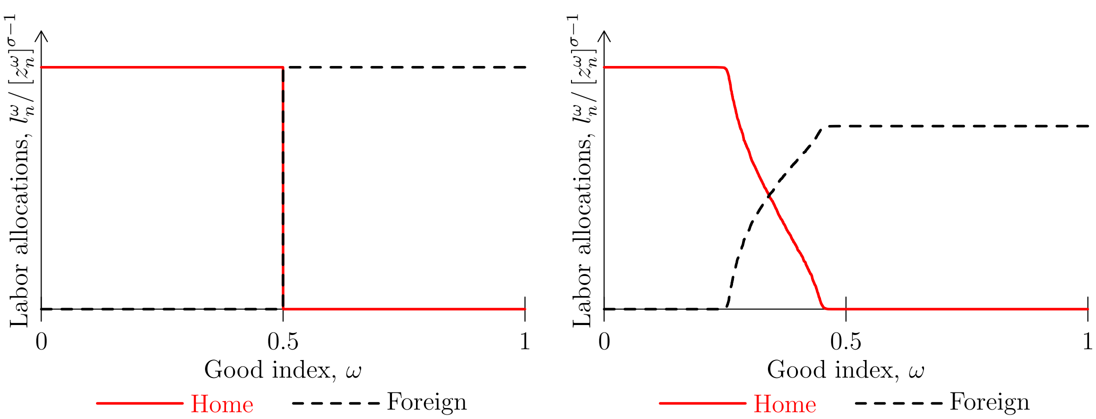
There is an interesting economic intuition behind this expression.[10] Let us focus on some particular equilibrium of our economy. Let us label the economy with this equilibrium by O ("original"). Economy O is described by the threshold B dividing goods produced by Home and Foreign, by trade shares $\pi_{nn}(s)$, and by the measure of goods produced by country n, m_n. Now imagine an Armington economy, where each country is forced to produce their measures m_H and m_F of goods. In other words, in this economy each country is exogenously assigned to produce exactly the goods they produce in the economy O. Let us label this Armington economy by A. By the definition of economy A, in the trade autarky of economy A countries produce their exogenously assigned measures of goods m_H and m_F. Consequently, when in the economy A countries open up to trade, the measures of goods they produce do not change, but the measures of goods they consume expand to the full set of goods of the economy O. Hence, the gains from trade in economy A come from the love for variety and are given by $\pi_{nn}(s)^{-\frac{1}{\sigma-1}}$.

[10] I thank Andrés Rodríguez-Clare for sharing with me this intuition.

Next, let us look at the welfare gains from going from the trade autarky of economy O to the trade autarky of economy A. These welfare gains come from two components. On the one hand, as we go from the trade autarky in O to the trade autarky in A, we shrink the set of the consumed goods, and, therefore, suffer a welfare loss of $m_n^{\frac{1}{\sigma-1}}$, which, again, comes from the love for variety. On the other hand, there is now more labor available to produce each good. This gives as a welfare gain of $m_n^{-\frac{1}{\theta}}$. Therefore, the overall welfare gain from going from the trade autarky of economy O to the trade autarky of economy A, is given by $m_n^{-\frac{1}{\theta}+\frac{1}{\sigma-1}}$.

Finally, the welfare gains from going from the trade autarky of economy O to the free trade in economy O, can be decomposed into (i) welfare gains from going from the trade autarky of economy O to the trade autarky of economy A; (ii) welfare gains from going from the trade autarky of economy A to the free trade in economy A (which is the same as the free trade in economy O). Hence, the overall gains from trade in economy O are given by $\pi_{nn}(s)^{-\frac{1}{\sigma-1}} m_n^{-\frac{1}{\theta}+\frac{1}{\sigma-1}}$.

5.1.2 Complete Financial Markets



(a) Certainty and financial autarky cases

(b) Complete financial markets

Figure 7: Labor allocations across individual goods

In the case of complete financial markets, there are regions of incomplete specialization even if trade is free. Figure 7a shows how labor is allocated to individual goods when there is no uncertainty or in the case of financial autarky. In these cases production of goods is split between Home and Foreign. Figure 7b shows labor allocations to individual goods in the case of complete financial markets. There is a region of incomplete specialization in the middle. Countries export (or import) different sets of goods within that region in different states of the world. As the volatility of country-specific shocks increases, that region becomes wider. The intuition behind why this region of incomplete specialization occurs comes directly from the case of two goods. First, as countries financially integrate, they take some part of each others risk, and need to reduce that risk by diversifying their production. Second, after financial integration countries do not have to rely on the terms of trade insurance. Hence, they can exploit their absolute advantage in the marginal goods in the states when their productivity is high and rely on financial markets to insure from the states with low productivity.

Because of the regions of incomplete specialization, it is hard to get analytic expressions (price indices, the share of goods produced by each country, welfare, etc.) in the case of complete financial markets. These expressions have to be derived for each value of θ separately, and they have very different forms for different values of θ. In the regions of incomplete specialization price of any good ω is a function of both Home and Foreign efficiencies, z_H^ω and z_F^ω. For example, if there are no shocks to Home and the shock to Foreign can take only two values, prices in states 1 and 2 are given by:

$$p^\omega(1) = \left(A_F(2)\frac{w_H}{z_H^\omega} - \frac{w_F}{z_F^\omega}\right)(A_F(2) - A_F(1))^{-1},$$
$$p^\omega(2) = \left(\frac{w_F}{z_F^\omega} - A_F(1)\frac{w_H}{z_H^\omega}\right)(A_F(2) - A_F(1))^{-1}.$$

Expressions for price indices of such prices involve taking integrals of rational polynomial functions having θ in the powers of polynomials. While it is possible to find closed-form expressions for such integrals, these expressions have very different forms depending on the value of θ. Therefore, to solve the complete financial markets case in the most general form, I use the algorithm described in Appendix A.

5.1.3 A Numerical Example

Parameter	Value
σ	4
ρ	2
θ	5
T_n	1

Table 4: Parameter values for the example with one industry

Let us now look at a numerical example which illustrates the effect of financial integration and increased riskiness on the measure of goods produced in Home and Foreign as well as on welfare. The elasticity of substitution between goods from different industries, η, is irrelevant for this section, because we have only one industry. The values of other parameters are provided in Table 4. The uncertainty in the world is described by one shock, which applies to Foreign only, while Home has no shocks. The shock to Foreign can take either a high or a low value with equal probabilities and is parametrized by the number $a \in [0, 1)$ in the following way:

$$A_F(s) = \begin{cases} 1 - a, & \text{if } s = 1 \\ 1 + a, & \text{if } s = 2. \end{cases}$$

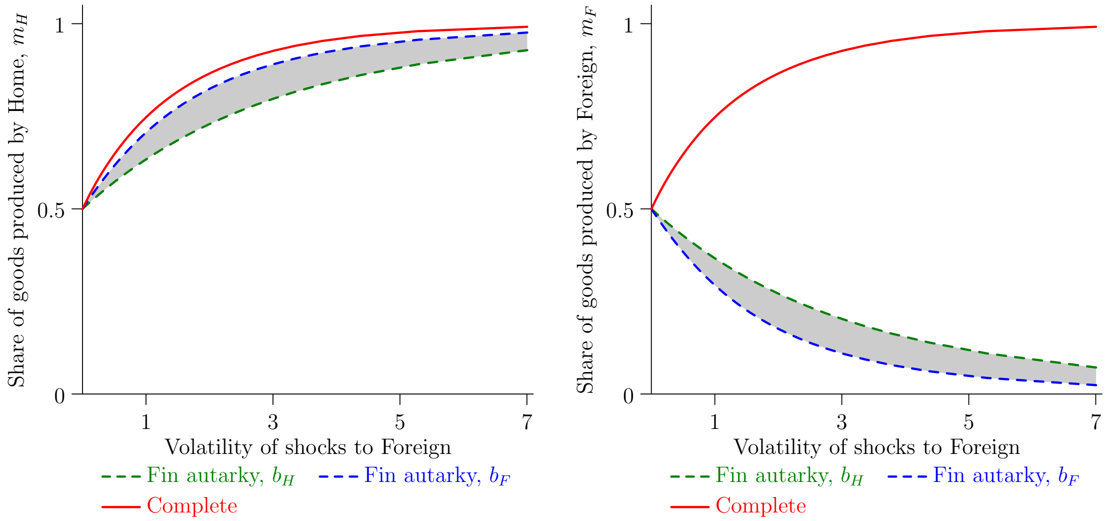
Below I show the measure of goods produced by each country and welfare as functions of the volatility of shock to Foreign. It is more revealing to look at the measures of goods and welfare as functions of log-volatility. I calculate this volatility as the variance of log deviation from the mean:

$$V\left[\hat{A}_F(s)\right] = E\left[\left(\hat{A}_F(s) - E\left[\hat{A}_F(s)\right]\right)^2\right],$$

where

$$\hat{A}_F(s) \equiv \log A_F(s) - \log E\left[A_F(s)\right].$$

Such way of calculating volatilities in the static framework presented here corresponds to the volatility of growth rates (i.e., volatility of log deviations from a trend) in a truly dynamic framework.



(a) Share of goods produced by Home, m_H

(b) Share of goods produced by Foreign, m_F

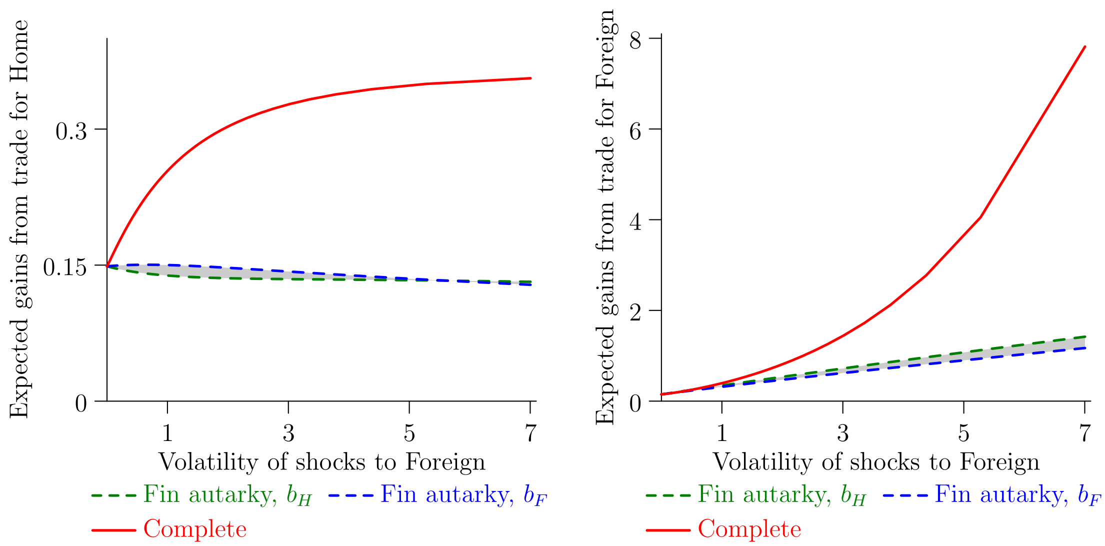
Figure 8: Shares of goods produced by Home and Foreign

Given the parametrization described above, there is a continuum of equilibria in the case of financial autarky. Each of these equilibria is characterized by the threshold B and the corresponding value of κ. Each value of the threshold B defines the share of goods produced by Home and Foreign, m_H and m_F. These shares are depicted in Figure 8 in the gray area between the dashed lines, which correspond to the bounds b_H and b_F within which the threshold B varies.

In the case of complete financial markets there is a unique equilibrium for each value of volatility in Foreign. The solid red lines in Figure 8 depict the shares of goods produced by Home and Foreign. These shares are calculated as part of the solution of the problem, and they include *all* goods produced by Home and Foreign — not only the goods in which Home and Foreign completely specialize. For example, in Figure 7b, the share of goods produced by Home is equal to the percent of goods from $[0, 1]$ for which the solid red line is above level zero. Among those goods, there is a large share of goods in which Home incompletely specializes.

Not surprisingly, Figure 8 reveals that the measures of goods produced by Home and Foreign in the model with a continuum of goods follow the same pattern as the amount of labor allocated to goods in the framework with two goods (compare Figure 8 to Figure 4a). In the case of financial autarky, increasing volatility in Foreign results in a smaller share of goods produced in Foreign and a bigger share of goods produced in Home in any κ-equilibrium. Intuitively, Foreign moves resources into production of its best goods because these goods are insured by the terms of trade. In the case of complete financial markets, increasing volatility in Foreign results in a bigger share of goods produced in both countries. In this case both countries can benefit from the state with high aggregate productivity in Foreign, if Foreign produces a wider range of goods. And both countries rely on production in Home when aggregate productivity in Foreign is low.

Also, as Figure 8 demonstrates, for any κ-equilibrium, financial integration leads to more diversified production in Home and Foreign in terms of the share of goods produced by the countries: the solid red lines are above the gray regions in Figures 8a and 8b. In this case the terms of trade insurance becomes not so important for Foreign, and Foreign exploits the comparative advantage in a wide range of goods when its productivity is high. At the same time, Home insures Foreign from the state with low productivity and, so, Home needs to absorb that additional risk by producing a wider range of goods.



(a) Expected gains from trade for Home

(b) Expected gains from trade for Foreign

Figure 9: Expected gains from trade for Home and Foreign

Figure 9 shows the gains from trade under financial autarky and complete financial markets. These gains are calculated in the following way:

$$GT_n \equiv \begin{cases} \dfrac{E\left[U\left(C_n^A(s)\right)\right]}{E\left[U\left(C_n(s)\right)\right]} - 1, & \text{if } \rho > 1, \\ \dfrac{E\left[U\left(C_n(s)\right)\right]}{E\left[U\left(C_n^A(s)\right)\right]} - 1, & \text{if } \rho \leq 1, \end{cases} \tag{4}$$

where $C_n^A(s)$ is country n's aggregate consumption in the trade autarky and $C_n(s)$ is country n's aggregate consumption in the case of free trade with either complete markets or financial autarky.[11] So, the expected gains from trade measure the expected welfare increase resulting from going from trade autarky to free trade.

Figure 9 demonstrates several things. First, there are always gains from trade: for any value of volatility in Foreign and under both complete markets and financial autarky. Second, gains from trade under complete markets are much larger than gains from trade under financial autarky for both countries. Third, Foreign gains much more from trade than Home. Gains from trade under complete markets are especially large for Foreign: in the extreme case, when the level of volatility in Foreign is high, Foreign can gain as much as 800% from trade. And even for moderate values of volatility in Foreign these gains are still large. Such large differences between gains from trade under complete financial markets relative to financial autarky come from the fact that both countries can exploit Foreign's comparative advantage in production of a wide range of goods when aggregate

[11] Recall that if $\rho > 1$, then $U\left(C_n(s)\right) = \frac{1}{1-\rho} C_n(s)^{1-\rho} < 0$, and so the larger absolute value of $U\left(C_n(s)\right)$ means the lower utility in this case.

productivity in Foreign is high, and can rely on Home's production when aggregate productivity in Foreign is low.

Next, Figure 9 demonstrates that, for each level of volatility in Foreign, the multiple equilibira existing under financial autarky can be ordered in terms of welfare from the point of view of a particular country. For example, the welfare in Foreign is the highest when it produces all disputed goods, i.e., when $B = b_H$, which correspond to the green dashed line in Figure 9b. Intuitively, on the one hand, when Foreign's aggregate productivity is low, the disputed goods produced by Foreign are insured through the terms of trade. On the other hand, when Foreign's aggregate productivity is high, Foreign has comparative advantage in the disputed goods, and, hence, Foreign benefits from the fact that it produces these goods with productivities which are higher relative to Home's productivities in these goods. Note that for any level of volatility in Foreign, Foreign is always better off from capturing production of all disputed goods. For Home the situation is different. When volatility in Foreign is low, Home benefits from producing all disputed goods: the dashed blue line is above the dashed green line in Figure 9a. In this case Home produces the disputed goods with a safe technology and does not face high prices on these goods in the bad state of the world (which happens if Foreign produces the disputed goods). However, if volatility in Foreign is high, Home benefits more if it does not produce the disputed goods: the dashed blue line is below the dashed green when volatility in Foreign is higher than 5. This happens because, by allowing Foreign to produce the disputed goods, Home benefits from high aggregate productivity in Foreign in the good state of the world, and this benefit offsets the losses from high prices on the disputed goods in the bad state of the world.

The welfare analysis we have just done reveals that countries might be willing to pursue industrial policies to move to a better equilibrium. Moreover, sometimes both countries can benefit from such industrial policies. Nevertheless, all these gains happen on the margin, and the biggest benefit is reaped when the countries open financially.

The potentially large gains from financial integration obtained in the framework of this paper are in sharp contrast with the small gains from financial integration traditionally found in the international finance literature. For example, in the classical paper by Cole and Obstfeld (1991) the gains from financial integration in the model calibrated to the moments of the U.S. and Japanese data are reported to be about 0.2 percent of output per year. The big difference in the gains from financial integration is due to the fact that the international finance literature usually focuses on consumption smoothing in endowment economies, and ignores the channels through which financial integration can interact with comparative advantage and impact the production structure.

5.2 Multiple Industries and Industry-Specific Shocks

Let us now consider the case with multiple industries and industry-specific shocks only.[12] This case is very different from what we have just seen. One important distinction is that under industry-specific shocks the equilibrium is unique for any structure of financial markets. The purpose of this section is to demonstrate that the main effects from the model with 2 goods are in action on the aggregate level in the model with a continuum of goods. So, we are interested to look at the effects of increased riskiness and financial integration on aggregate labor allocations. Also, we will look at welfare to compare the outcomes with the previously considered case of a single industry with country-specific shocks.

[12] See the Algebra Appendix for all derivations.

Since there are only industry-specific shocks, we can drop country index from the notation for shocks: $A_{n,g}(s) = A_g(s)$. One can show that in this case there is a unique equilibrium in which goods in each industry are split into those produced only by Home and those produced only by Foreign. Suppose that goods within each industry are ordered so that comparative advantage of Home falls with good index ω. Let B_g be a threshold dividing goods produced by Home and Foreign in industry g, so that goods ω, such that $z^{\omega}_{F,g} \big/ z^{\omega}_{H,g} < B_g$, are produced by Home. Then for both cases of international risk sharing

$$B_g = \left(\frac{T_{H,g}}{T_{F,g}} \cdot \frac{L_{F,g}}{L_{H,g}} \right)^{-\frac{1}{1+\theta}}.$$

The same expression is true for the certainty case as well. The difference between the certainty case, complete financial markets, and financial autarky will be in the amount of labor, $L_{n,g}$, devoted to each industry.

Let us now look at trade shares and labor allocations. Trade shares are the same for all states of the world:

$$\pi_{HH,g} = \pi_{FH,g} = \frac{T_{H,g}}{T_{H,g} + T_{F,g} B_g^{-\theta}},$$
$$\pi_{HF,g} = \pi_{FF,g} = \frac{T_{F,g} B_g^{-\theta}}{T_{H,g} + T_{F,g} B_g^{-\theta}}.$$

This is the same expression as in the certainty case. The difference is in the value of threshold, B_g. Allocation of Home labor for the case of complete financial markets is given by the expression:

$$\frac{L_{H,g'}}{L_{H,g''}} = \frac{\alpha_{g'}}{\alpha_{g''}} \left(\frac{T_{H,g'}}{T_{H,g''}} \right)^{\frac{\eta-1}{\theta}} \left(\frac{\pi_{HH,g'}}{\pi_{HH,g''}} \right)^{1-\frac{\eta-1}{\theta}} \left(\frac{\Gamma_{g'}}{\Gamma_{g''}} \right)^{\eta-1}$$
$$\times \left(\frac{\sum_{s'} h(s') A_{g'}(s')^{\frac{\eta-1}{\eta}} \Xi(s')^{\frac{1}{\eta}-\rho}}{\sum_{s'} h(s') A_{g''}(s')^{\frac{\eta-1}{\eta}} \Xi(s')^{\frac{1}{\eta}-\rho}} \right)^{\eta},$$

where

$$\Xi(s) \equiv \left[\sum_{g'} \alpha_{g'} \left(\alpha_{g'}^{-1} A_{g'}(s) T_{H,g'}^{\frac{1}{\theta}} \pi_{HH,g'}^{-\frac{1+\theta}{\theta}} L_{H,g'} \Gamma_{g'} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

Similarly, for the case of financial autarky:

$$\frac{L_{H,g'}}{L_{H,g''}} = \frac{\alpha_{g'}}{\alpha_{g''}} \left(\frac{T_{H,g'}}{T_{H,g''}} \right)^{\frac{\eta-1}{\theta}} \left(\frac{\pi_{HH,g'}}{\pi_{HH,g''}} \right)^{1-\frac{\eta-1}{\theta}} \left(\frac{\Gamma_{g'}}{\Gamma_{g''}} \right)^{\eta-1}$$
$$\times \left(\frac{\sum_{s'} h(s') \left(1 + \frac{I_F(s')}{I_H(s')}\right)^{\rho} A_{g'}(s')^{\frac{\eta-1}{\eta}} \Xi(s')^{\frac{1}{\eta}-\rho}}{\sum_{s'} h(s') \left(1 + \frac{I_F(s')}{I_H(s')}\right)^{\rho} A_{g''}(s')^{\frac{\eta-1}{\eta}} \Xi(s')^{\frac{1}{\eta}-\rho}} \right)^{\eta},$$

where $\Xi(s)$ is given by the same expression as in the case of complete financial markets. Note that the expressions for labor allocation in the case of complete financial markets and financial autarky differ only in the term $(1 + I_F(s')/I_H(s'))^{\rho}$, which enters the sums in the case of financial autarky. If consumers are risk-neutral, $\rho = 0$, then the financial markets structure has no impact on labor allocations across industries (but the allocations are different from the certainty case). If preferences over industries are Cobb-Douglas, $\eta = 1$, then in both cases of financial markets:

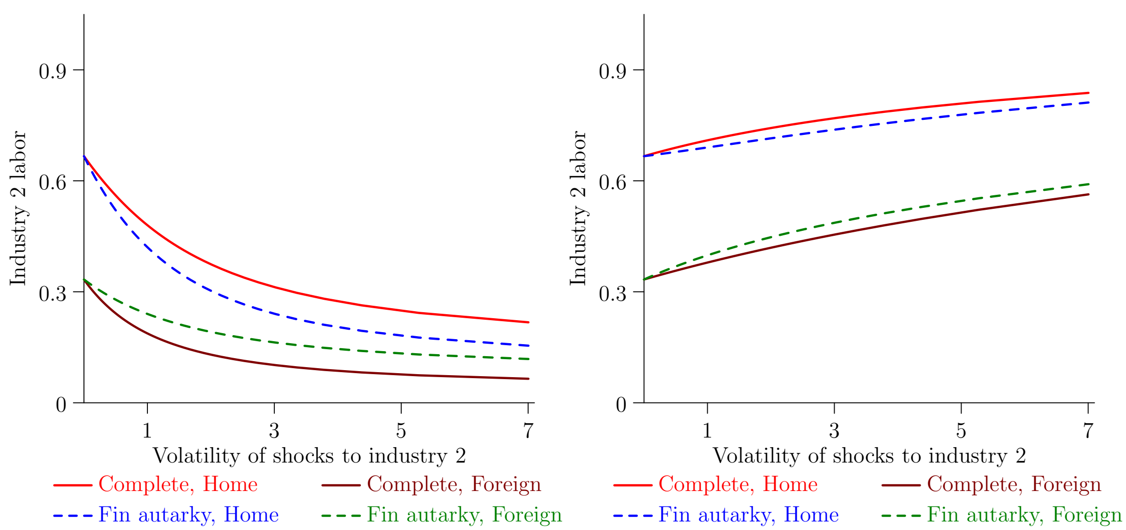
$$\frac{L_{H,g'}}{L_{H,g''}} = \frac{\alpha_{g'}}{\alpha_{g''}} \cdot \frac{\pi_{HH,g'}}{\pi_{HH,g''}},$$

which does not depend on the shocks and is the same as in the certainty case.

Let us now look at the examples of labor allocations for the case of two industries, which are mirror images of each other in terms of parameters T of Fréhet distribution: $T_{H,1} = T_{F,2} = 1$ and $T_{H,2} = T_{F,1} = 2$. With this parametrization Home has comparative advantage in industry 2, while Foreign has comparative advantage in industry 1. All other parameters are identical for the two countries: $\rho = 2$, $\theta = 5$, $\sigma_1 = \sigma_2 = 4$, $\eta = 0.8$ or 2. Let industry 1 be a safe industry, while industry 2 be subject to a shock which can take a high or a low value with equal probabilities:

$$A_2(s) = \begin{cases} 1 - a, & \text{if } s = 0 \\ 1 + a, & \text{if } s = 1, \end{cases}$$

where $a \in [0, 1)$.



(a) $\eta = 2$

(b) $\eta = 0.8$

Figure 10: Aggregate employment in the risky industry

Figure 10 demonstrates that the patterns of aggregate labor allocations in the model with a continuum of goods are similar to the patterns of labor allocation in the model with two goods (the only difference is that having continuum of goods allows us to achieve smoothness of the outcomes). First, financial integration makes both countries more specialized in their comparative advantage industries: the solid lines are outside of the dashed lines in both Figures 10a and 10b. Second, as volatility of shocks to industry 2 increases, both countries pull labor out of this industry if $\eta > 1$, and put even more labor into this industry if $\eta < 1$. Intuitively, similar to the case with 2 goods, if $\eta > 1$ and industry 2 becomes riskier, then it makes sense to substitute its goods with goods from the safer industry 1. On the other hand, if $\eta < 1$, industry 2 goods are important for consuming

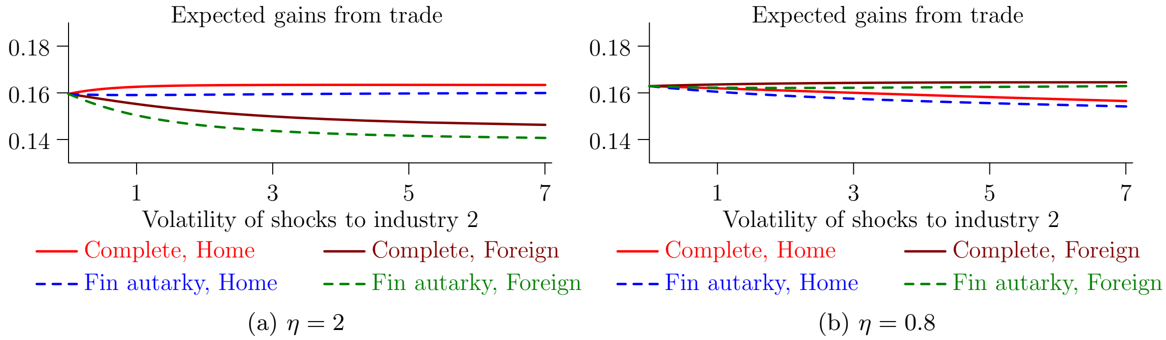


Figure 11: Expected gains from trade

goods from industry 1. And so, as industry 2 becomes riskier, it is important to invest even more into this industry, to make sure there is enough output in bad times.

Figure 11 shows the expected gains from trade calculated using formula (4). As we can see from Figure 11, the expected gains from trade do not change much as volatility of shocks to industry 2 increases. The gains from financial integration on top of the gains from trade (measured as increase in welfare resulting from allowing international risk sharing) are small in this case as well. These small gains are related to the fact that in case with $\eta = 1$ both cases of financial markets for any level of volatility in industry 2 result in the labor allocations and trade shares the same as in the certainty case. Hence, if $\eta = 1$, the expected gains from trade for any level of volatility and any structure of financial markets are just the same as in the certainty case. Then, as we vary η around 1, the gains from trade do not change much.

5.3 Multiple Industries and Country-Specific Shocks

In this section we focus on the country-specific shocks and shut down the industry-specific shocks: $A_{n,g}(s) = A_n(s)$.[13] Formal analysis of the economy with multiple industries is similar to analysis of the economy with a single industry, which we have already done. So, in this section we start right away with a numerical example.

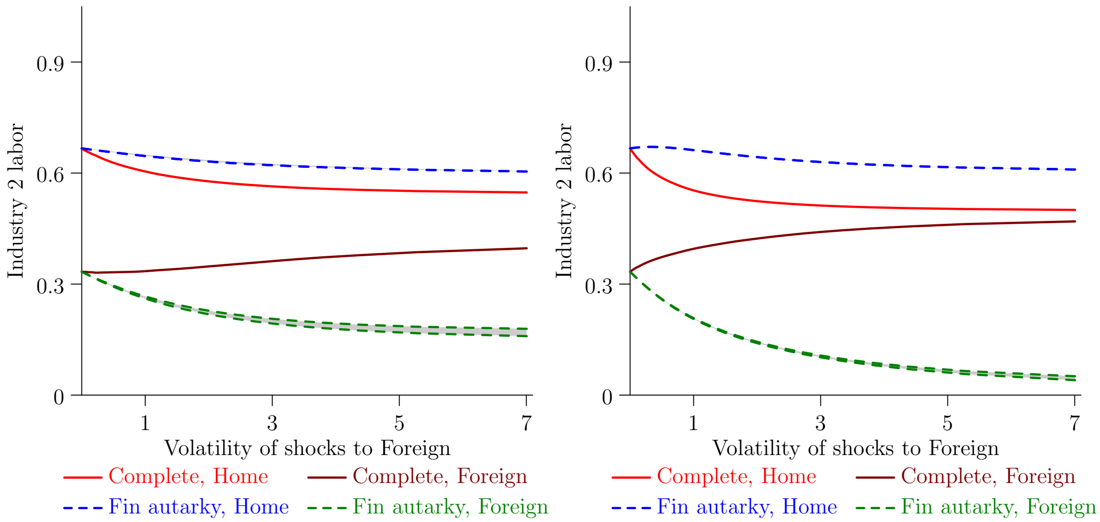
Let us look at aggregate labor allocations and gains from trade in an example similar to the one we analyzed in the case of industry-specific shocks. All parameter values are the same as in the example with industry-specific shocks. The only difference is that now only Foreign is subject to a country-specific shock, while Home is a safe country. The shock to Foreign country can take one of two values — low or high — with equal probabilities:

$$A_F(s) = \begin{cases} 1 - a, & \text{if } s = 0 \\ 1 + a, & \text{if } s = 1, \end{cases}$$

where $a \in [0, 1)$.

Figure 12 depicts aggregate labor allocations in industry 2 for $\eta = 2$ and $\eta = 0.8$. Let us first look at Figure 12a corresponding to $\eta = 2$. The two green dashed curves in Figure 12a bound labor allocations in Foreign for all possible κ-equilibria. As we can see, these curves are very close

[13] See the Algebra Appendix for all derivations.



(a) $\eta = 2$

(b) $\eta = 0.8$

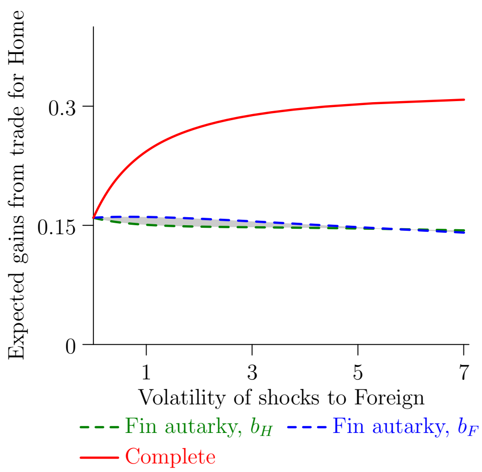
Figure 12: Aggregate employment in the risky industry

to each other, meaning that there is not much variation in aggregate labor allocations in Foreign across different κ-equilibria. The variation in aggregate labor allocations across κ-equilibria is even smaller (almost non-existent) for Home: the blue dashed curves in Figure 12a basically coincide with each other. The situation is similar for the case with $\eta = 0.8$ depicted in Figure 12b.

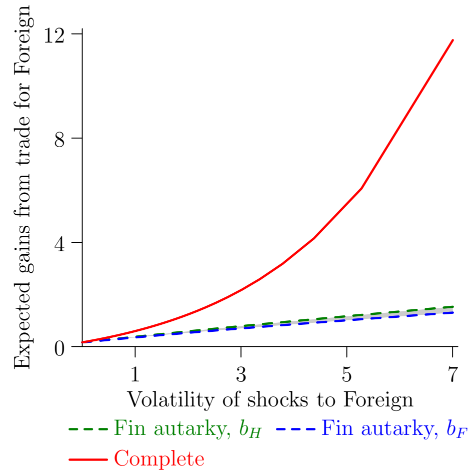
Similar to the case with two goods, we see from Figure 12 that financial integration leads to more diversified production structure in terms of aggregate labor allocations: the solid curves are outside of the dashed curves on both Figures 12a and 12b. An increase in riskiness makes Foreign devote more labor to its comparative advantage industry 1 under financial autarky, because this industry is insured by the terms of trade: the green dashed curves are decreasing on both Figures 12a and 12b. In the case of $\eta = 2$, Home also devotes more resources to industry 1, to diversify its risks of possible adverse terms of trade in the state with low aggregate productivity in Foreign. In the case of $\eta = 0.8$, the response of aggregate labor allocations in Home to increased riskiness is non-monotonic. For small levels of volatility in Foreign, Home puts more labor into industry 2 in response to increased riskiness. This happens because, as riskiness increases, Foreign produces less of goods from industry 2, but both countries need these goods because they complement consumption of goods from industry 1. So, Home starts producing more of goods from industry 2. At the same time, Home faces adverse terms of trade in industry 2. So, Home has incentives to move labor out of this industry. As riskiness in Foreign becomes too large, the terms of trade effect dominates, and the aggregate labor allocation in industry 2 in Home falls.

Under complete financial markets both countries diversify their aggregate labor allocations in response to increased riskiness: Home allocates more labor to industry 1 and Foreign allocates more labor to industry 2. Again, this is driven by the same effect, which we saw in the case of two goods: by diversifying production both countries can benefit from the state with high aggregate productivity in Foreign, and both countries rely on production in Home to hedge the risk of low aggregate productivity in Foreign.

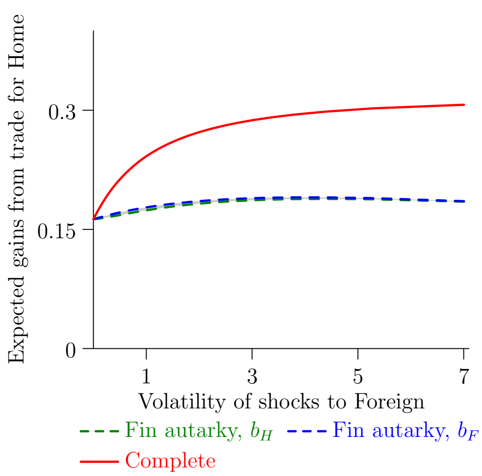
The patterns of gains from trade in the case with two industries — depicted in Figure 13 — are very similar to the corresponding patterns in the case with one industry. The only difference is



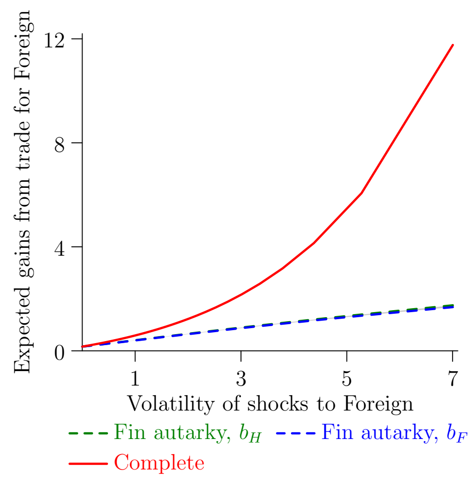
(a) Expected gains from trade for Home, $\eta = 2$



(b) Expected gains from trade for Foreign, $\eta = 2$



(c) Expected gains from trade for Home, $\eta = 0.8$



(d) Expected gains from trade for Foreign, $\eta = 0.8$

Figure 13: Expected gains from trade for Home and Foreign for $\eta = 2$ and $\eta = 0.8$

that now the gains from trade in the case of complete financial markets are even bigger for Foreign: they can go as high as 1200% in the case with two industries versus 800% in the case with one industry.

6 Conclusions

This papers provides careful analysis of the standard 2×2 Ricardian model with uncertainty — a variant of the Helpman-Razin model. This analysis reveals that — contrary to the prevailing view — the effect of financial integration on production structure of countries is ambiguous: this effect depends on the structure of TFP shocks and preferences. Under industry-specific shocks countries can become more specialized after financial integration, while under country-specific shocks countries can become more diversified after financial integration. The analysis also shows that the Helpman-Razin model generally has multiple equilibria, which was overlooked in the earlier literature.

Introduction of the continuum of goods into the Helpman-Razin model preserves all interesting effects and makes the multiple equilibria "manageable". Building the model with a continuum of goods is a critical step towards quantifying with data the effects of financial integration and increased riskiness as well as measuring the welfare gains from financial integration on top of the gains from trade. The international finance literature traditionally finds small gains from financial integration. Preliminary welfare analysis of the framework with a continuum of goods presented in this paper shows that the gains from financial integration can be significant: up to several hundred percent in terms of welfare. The next natural step in research on the topic of the current paper will be to bring the presented framework to data.

References

Acemoglu, D. and Zilibotti, F. (1997). Was prometheus unbound by chance? risk, diversification, and growths. *Journal of Political Economy*, **105** (4), 709–751. 2

Basile, R. and Girardi, A. (2010). Specialization and risk sharing in European regions. *Journal of Economic Geography*, **10** (5), 645–659. 2

Bos, J. W. B., Economidou, C. and Zhang, L. (2011). *Specialization in the Presence of Trade and Financial Integration: Explorations of the Integration-Specialization Nexus*. Tech. rep. 2

Caselli, F., Koren, M., Lisicky, M. and Tenreyro, S. (2014). Diversification through Trade. *Unpublished Draft*. 2

Cole, H. L. and Obstfeld, M. (1991). Commodity trade and international risk sharing : How much do financial markets matter? *Journal of Monetary Economics*, **28** (1), 3–24. 3, 4, 2, 5.1.3

Costinot, A., Donaldson, D. and Komunjer, I. (2012). What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas. *Review of Economic Studies*, **79** (2), 581–608. A.2

Di Giovanni, J. and Levchenko, A. A. (2010). *The Risk Content of Exports: A Portfolio View of International Trade*. NBER Working Papers 16005, National Bureau of Economic Research, Inc. 2

Dixit, A. (1987). Trade and insurance with moral hazard. *Journal of International Economics*, **23** (3-4), 201–220. 2

— (1989a). Trade and Insurance with Adverse Selection. *Review of Economic Studies*, **56** (2), 235–47. 2

— (1989b). Trade and Insurance with Imperfectly Observed Outcomes. *The Quarterly Journal of Economics*, **104** (1), 195–203. 2

Dornbusch, R., Fischer, S. and Samuelson, P. A. (1977). Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods. *American Economic Review*, **67** (5), 823–39. A.2

Eaton, J. (1979). The Allocation of Resources in an Open Economy with Uncertain Terms of Trade. *International Economic Review*, **20** (2), 391–403. 2

— and Grossman, G. M. (1986). Optimal trade and industrial policy under oligopoly. *The Quarterly Journal of Economics*, **101** (2), 383–406. 2

— and Kortum, S. (2002). Technology, Geography, and Trade. *Econometrica*, **70** (5), 1741–1779. 1, A.2

Ferris, M. C. and Munson, T. S. (1999). Interfaces to path 3.0: Design, implementation and usage. *Computational Optimization and Applications*, **12** (1-3), 207–227. 7, A.3

Grossman, G. M. and Razin, A. (1985). The pattern of trade in a ricardian model with country-specific uncertainty. *International Economic Review*, **26** (1), 193–202. 2

Heathcote, J. and Perri, F. (2013). The International Diversification Puzzle Is Not as Bad as You Think. *Journal of Political Economy*, **121** (6), 1108 – 1159. 3, 2

Helpman, E. and Razin, A. (1978a). *A Theory of International Trade under Uncertainty*. New York: Academic Press. 1, 2, 2, 3.4, 4.1

— and — (1978b). Uncertainty and international trade in the presence of stock markets. *The Review of Economic Studies*, **45** (2), 239–250. 2

Imbs, J. (2004). Trade, Finance, Specialization, and Synchronization. *The Review of Economics and Statistics*, **86** (3), 723–734. 1, 2

Islamaj, E. (2014). Industrial specialization, financial integration and international consumption risk sharing. *The B.E. Journal of Macroeconomics (forthcoming)*. 2, 4.1

Judd, K. L. (1998). *Numerical Methods in Economics*, *MIT Press Books*, vol. 1. The MIT Press. 15

Kalemli-Ozcan, S., Sorensen, B. E. and Yosha, O. (2003). Risk sharing and industrial specialization: Regional and international evidence. *The American Economic Review*, **93** (3), 903–918. 1, 2

Koren, M. (2003). *Financial Globalization, Portfolio Diversification, and the Pattern of International Trade*. IMF Working Papers 03/233, International Monetary Fund. 2, 2, 4.1

— and Tenreyro, S. (2007). Volatility and Development. *The Quarterly Journal of Economics*, **122** (1), 243–287. 1

Lucas, R. E. (1987). *Models of Business Cycles*. Oxford: Basil Blackwell. 4, 2

Newbery, D. M. G. and Stiglitz, J. E. (1981). *The theory of commodity price stabilization: a study in the economics of risk*. Clarendon Press. 3, 2

— and — (1984). Pareto Inferior Trade. *Review of Economic Studies*, **51** (1), 1–12. 2

Obstfeld, M. (1994). Risk-Taking, Global Diversification, and Growth. *American Economic Review*, **84** (5), 1310–29. 2

Pomery, J. (1984). Uncertainty in trade models. In R. W. Jones and P. B. Kenen (eds.), *Handbook of International Economics*, *Handbook of International Economics*, vol. 1, *9*, Elsevier, pp. 419–465. 2

Rothschild, M. and Stiglitz, J. E. (1971). Increasing risk II: Its economic consequences. *Journal of Economic Theory*, **3** (1), 66–84. 4.1

Saint-Paul, G. (1992). Technological choice, financial markets and economic development. *European Economic Review*, **36** (4), 763–781. 2

Turnovsky, S. J. (1974). Technological and price uncertainty in a ricardian model of international trade. *The Review of Economic Studies*, **41** (2), 201–217. 2

Appendix A: Solution Algorithm

A.1 System of Equations for the Modified Model

Let $p^{\omega}_{nn,g}(s)$ be the price of the good ω produced by country n in industry g in state s. We have to find $p^{\omega}_{nn,g}(s)$, $p^{\omega}_{g}(s)$, $P_g(s)$, $P(s)$, $x^{\omega}_{n,g}(s)$, $X_{n,g}(s)$, $I_n(s)$, $\Pi^{\omega}_{n,g}(s)$, $\Pi^{\omega}_{n,g}$, $l^{\omega}_{n,g}$, w_n, which solve the following system of equations:

$$
\begin{aligned}
p^{\omega}_{g}(s) &= \left[\sum_{i=1}^{N} p^{\omega}_{ii,g}(s)^{1-\sigma'}\right]^{\frac{1}{1-\sigma'}}, \\
P_g(s) &= \left[\int_{\omega\in\Omega} p^{\omega}_{g}(s)^{1-\sigma_g} d\omega\right]^{\frac{1}{1-\sigma_g}}, \\
P(s) &= \left[\sum_{g=1}^{G} \alpha_g P_g(s)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \\
x^{\omega}_{n,g}(s) &= \left(\frac{p^{\omega}_{g}(s)}{P_g(s)}\right)^{1-\sigma_g} X_{n,g}(s), \\
X_{n,g}(s) &= \left(\frac{P_g(s)}{P(s)}\right)^{1-\eta} \alpha_g I_n(\varsigma), \\
\Pi^{\omega}_{i,g}(s) &= \left[\frac{1}{A_{i,g}(s)} \sum_{n=1}^{N} p^{\omega}_{g}(s)^{\sigma'-1} x^{\omega}_{n,g}(s)\right]^{\frac{1}{\sigma'}}, \\
w_i &= \left(\frac{1}{L_i}\sum_{g=1}^{G}\int_{\omega'\in\Omega}\left[z^{\omega'}_{i,g}\right]^{\sigma'-1}\left[\Pi^{\omega'}_{i,g}\right]^{\sigma'} d\omega'\right)^{\frac{1}{\sigma'}}, \\
l^{\omega}_{i,g} &= \left[z^{\omega}_{i,g}\right]^{\sigma'-1}\left(\frac{\Pi^{\omega}_{i,g}}{w_i}\right)^{\sigma'}, \\
p^{\omega}_{ii,g}(s) &= \frac{\Pi^{\omega}_{i,g}(s)}{\Pi^{\omega}_{i,g}} \cdot \frac{w_i}{z^{\omega}_{i,g}}.
\end{aligned}
$$

For complete financial markets:

$$
\begin{aligned}
\Pi^{\omega}_{n,g} &= \sum_{s=1}^{\mathcal{S}} A_{n,g}(s)\Pi^{\omega}_{n,g}(s), \\
I_n(s) &= \frac{h(s)^{\frac{1}{\rho}} P(s)^{\frac{\rho-1}{\rho}}}{\sum_{s=1}^{\mathcal{S}} h(s)^{\frac{1}{\rho}} P(s)^{\frac{\rho-1}{\rho}}} w_n L_n.
\end{aligned}
$$

For financial autarky:

$$
\Pi^{\omega}_{n,g} = E_s\left[A_{n,g}(s)\Pi^{\omega}_{n,g}(s) I_n(s)^{-\rho} P(s)^{\rho-1}\right],
$$

$$
I_n(s) = \sum_{g=1}^{G} A_{n,g}(s)\int_{\omega\in\Omega} p^{\omega}_{nn,g}(s) z^{\omega}_{n,g} l^{\omega}_{n,g} d\omega.
$$

A.2 Discretization Scheme

The most straightforward way to solve the model in the general case is to discretize the continuum of goods, i.e., randomly draw a finite number of efficiencies $z_{n,g}^{\omega}$ from corresponding Fréchet distributions.[14] This is the strategy I follow in the current paper. However, the discretization brings two related problems, which we should be aware of.

First, the Fréchet distribution has infinite support and a fat tail. Any discretization scheme will necessarily truncate the support, throwing away some part of the tail. This introduces a truncation error into the solution, which can be substantial. To get a sense of this problem, imagine that we have a Dornbush-Fisher-Samuelson economy (Dornbusch *et al.* (1977)) with the underlying Fréchet distribution of efficiencies of production and totally identical countries. Suppose that we take 1000 random draws of efficiencies for each country. There is a big chance that one of the countries (say, Home) will get a draw much further down the tail than the other country (Foreign). If that happens, Home becomes a much more technologically advanced economy. As a result, the trade shares and wages will be significantly altered.

Second, as it is well known, the standard error of an integral computed by Monte Carlo integration is inversely proportional to $\sqrt{M}$, where M is the number of draws.[15] In other words, to get one more digit of accuracy in aggregate values, we need to take 100 times more draws. This exponential growth of the number of points needed to get accurate solutions can be a limiting factor in solving the model.

Clearly, the two problems mentioned above are not specific to the model presented here. Any model with a fat-tail distribution, which needs to be solved by discretization, suffers from these problems. A natural question to ask here is why would we want choose the Fréchet distribution in the current paper, if we solve the model numerically anyway? One reason for choosing the Fréchet distribution is to stay as close as possible to the existing trade models of comparative advantage (Eaton and Kortum (2002) and Costinot *et al.* (2012)). Another reason is that with the Fréchet distribution we can actually go a long way in charaterizing the solution in the special cases (free trade and only country- or industry-specific shocks).

To mitigate the discretization errors, I propose the following approach. Suppose we choose M discretization points. Then for every country-industry we first construct a non-random sample of efficiencies $\tilde{z}_{n,g}^{m}$ by taking M equidistant points on the interval $[0,1]$ and translating them into the Fréchet distribution:

$$\tilde{z}_{n,g}^{m} = \left(-\frac{1}{T_{n,g}}\ln u^{m}\right)^{-\frac{1}{\theta}},$$
$$u^{m} = \frac{m-0.5}{M}, \quad m = 1, \ldots, M.$$

After that we make the sample pseudo-random by shuffling indices of goods for each country-industry, i.e., for each country-industry, we take a random permutation $(\omega_1, \ldots, \omega_M)$ of indices $(1, \ldots, M)$ and reassign efficiencies:

$$z_{n,g}^{\omega_m} = \tilde{z}_{n,g}^{m}.$$

The sample obtained this way has two advantages over the regular Monte Carlo sampling. First, it gives stable solutions: it preserves the order of countries in terms of their levels of technology.

[14] An alternative approach can be to approximate price distributions with some basis functions.

[15] See, for example, Section 8.2 in Judd (1998).

Second, it reduces the variance of numerical errors. Thus, we need fewer points to achieve the same level accuracy comparing to the regular Monte Carlo sampling. In the current work, I was able to achieve much better results by using this discretization scheme over the regular Monte Carlo sampling.

A.3 Coping with Complementary Slackness Conditions

Solving the above equilibrium system of equations is a non-trivial task, because it involves complementary slackness conditions. A direct way to cope with them is to use a complementary slackness solver. As of today, the best complementary slackness solver is PATH described in Ferris and Munson (1999). When applied to the above problem with a small number of discrete points (e.g., $M = 500$), it gives great results. However, to get accurate aggregate solutions, we need at least $10,000$ points. As we increase the number points, we increase the number of complementary slackness conditions. With $10,000$ points the memory requirements for PATH and the time of solution become too restrictive to use PATH on, say, desktop computers, or laptops, or even on bigger machines.

In the current paper I use an indirect approach to cope with complementary slackness conditions. I modify the model by introducing an artificial requirement that each country n buys each good ω from all countries. Varieties of good ω produced by different countries are combined by the CES utility function with elasticity σ'. The equilibrium system of the modified model does not involve complementary slackness conditions. As elasticity σ' goes to infinity, the artificial requirement disappears, and the modified model converges to the original model. Presumably, the solution of the modified model also converges to the solution of the original model. I assume that it is true.

We can find an approximate solution of the original model by solving a sequence of modified models corresponding to an increasing sequence of elasticities $\{\sigma'_d\}_{d=1}^{D}$ and using the solution of the model with σ'_d as a starting point for the system of equations corresponding to σ'_{d+1}. The value of σ'_D should be large enough so that the change in the aggregate outcomes from σ'_{D-1} to σ'_D is small. In my calculations I always use $\sigma'_D = 2^{17} = 131,072$. In practice, most of the time of the algorithm is spend in computing powers of σ' and $1/\sigma'$. Since we are free to choose any increasing sequence of $\{\sigma'_d\}_{d=1}^{D}$, it is best to use powers of 2: $\sigma'_d = 2^d$, $d = 1, \ldots, D$. Taking any number to power σ' generally requires at least $\log_2 \sigma'$ multiplications, with exactly $\log_2 \sigma'$ multiplications when σ' is a power of 2. So, if we choose $\sigma'_d = 2^d$, we need to make the least theoretically possible number of multiplications each time when we take powers of σ'_d. The reduction in solving time can be significant: several times faster comparing to a random sequence of σ'.

The nonlinear system for the modified model can and must be solved by the fixed point iteration. The reason for this is that for large values of σ' the gradients of the equations are numerically unstable. So, the regular Newton's method cannot be applied here. When using the fixed point iteration, we also need to use damping.

There is one more crucial detail which is important to know when using the suggested approach with increasing elasticities. Solving the equilibrium system requires evaluating expressions of the following kind:

$$p = \left[\sum_{i=1}^{N} p_i^{1-\sigma'} \right]^{\frac{1}{1-\sigma'}} .$$

As σ' becomes large, it is likely that the direct evaluation of such expression will result in arithmetic overflow. Instead, one has to evaluate the following (theoretically equivalent) expression:

$$p = p_{\text{min}} \left[\sum_{i=1}^{N} \left(\frac{p_i}{p_{\text{min}}} \right)^{1-\sigma'} \right]^{\frac{1}{1-\sigma'}},$$

with

$$p_{min} = \min_i \{p_i\}.$$

Only with transformations of this kind we are able to use elasticities σ' as large as 2^{17}.