Demand Risk and Diversification through International Trade

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Abstract

I develop a theory of risk diversification through geography. In a general equilibrium trade model with monopolistic competition and stochastic demand, risk-averse entrepreneurs exploit the spatial correlation of demand across countries to lower the variance of their global sales. The model implies that both the probability of entry and trade flows to a country are increasing in the "Diversification Index", which depends on the multilateral covariance of the country's demand with all other markets. The risk diversification behavior can lead to higher welfare gains from trade than the ones predicted by trade models with risk neutrality. Using a panel of international sales of Portuguese firms, I estimate "risk-augmented" gravity regressions, which show that the Diversification Index significantly affects trade patterns at the extensive and intensive margins. In general equilibrium, the risk diversification channel increases welfare gains from trade by 17% relative to trade models with risk neutrality.

1 Introduction

Recent empirical evidence has shown that demand shocks explain a large fraction of the variation in firm sales across countries (see e.g. Eaton et al. (2011), Di Giovanni et al. (2014), Munch and Nguyen (2014) and Hottman et al. (2015)). When selling to a market, firms may not be able to perfectly insure against unexpected demand fluctuations. The role of demand uncertainty is particularly important in the case of costly irreversible investments, such as producing a new good or selling to a new destination (see Handley and Limao (2015)). According to a survey among 350 leading companies across the world, dealing with demand risk is the most important business challenge for global firms.[1] Therefore, it is crucial to understand how demand risk affects firms behavior across markets, and evaluate its economic implications.

I provide a theoretical characterization and an empirical assessment of the importance of demand risk for firms behavior on global markets. I argue that exporting to foreign countries is an opportunity to diversify demand risk. Selling to destinations with imperfectly correlated demand can hedge firms against idiosyncratic shocks hitting sales, in the spirit of classical portfolio theory (Markowitz (1952) and Sharpe (1964)). While this is an intuitive mechanism, it has not been fully explored by the macro and international trade literature. In a multi-country general equilibrium model of trade with spatially correlated demand, I characterize how firms' risk diversification behavior affects trade patterns and study its general equilibrium implications. I quantify that the risk diversification channel explains 15% of observed trade patterns across countries, and increases welfare gains from trade by 17% relative to trade models with risk neutrality.

In the first tier of my analysis, I develop a general equilibrium trade model with monopolistic competition and heterogeneous firms, along the lines of Melitz (2003). The model is characterized by two novel elements. First, the demand for a differentiated variety is subject to country-variety shocks, which are imperfectly correlated across countries. This is the only source of uncertainty in the economy, and it can reflect shocks to tastes, consumers confidence, regulation, firm reputation, etc. Second, firms are owned by risk-averse entrepreneurs. This assumption is motivated by the evidence that the volatility of cash-flows is a primary concern for many companies across the globe, especially if the managers' compensation is tied to the performance of the firm (see Ross (2004) and Panousi and Papanikolaou (2012)). I assume that financial markets are absent, thus the entrepreneurs cannot diversify away their idiosyncratic risk. In the limit case of no risk

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[1]This survey was conducted in 2011 and 2012 by the consulting firm Capgemini: https://www.capgemini.com/wp-content/uploads/2017/07/The_2012_Global_Supply_Chain_Agenda.pdf

aversion, the model is isomorphic to standard gravity models with monopolistic competition and Pareto distributed firms' productivities, as in Arkolakis et al. (2008) and Chaney (2008).

The production problem consists of two stages. In the first, firms know only the distribution of the demand shocks, but not their realization. Under uncertainty, they choose in which markets to operate, and perform costly marketing and distributional activities. In the second stage, firms learn the consumers' demand and produce, but they cannot change their set of destinations.[2]

The spatial correlation of demand across countries implies that, in the investment stage, entrepreneurs face a combinatorial problem, since both the extensive and the intensive margin decisions are interdependent across markets. I overcome this challenge by assuming that firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction n of the consumers in each location, in the spirit of Arkolakis (2010). This implies that the firm's choice variable is continuous rather than discrete, and firms simultaneously choose where to sell (depending on whether n is optimally zero or positive) and how much to sell (firms can choose to sell only to a fraction of the consumers). Therefore, the firm's extensive and intensive margin decisions are not taken independently for each market, as in standard trade models, but instead performing a global diversification strategy, which trades off the expected global profits with their variance.

I characterize the model-consistent exogenous measure of risk, which I name "Diversification Index." This variable measures the diversification benefits that a market provides to all firms exporting there, and it depends on the entire pattern of spatial covariance of demand across countries. I show that the probability of entering a market and the intensity of trade flows are increasing in the market's Diversification Index. If demand in a country is relatively stable and negatively/mildly correlated with demand in the other countries, then entrepreneurs optimally choose, *ceteribus paribus*, to export more there to hedge their business risk. This implies a fundamental trade-off: selling to a more remote destination may require higher trade and marketing costs, but it may also hedge firms against domestic fluctuations in demand.

In a two-country version of the model, I provide an analytical characterization of the welfare gains from trade. When the covariance of demand between the two countries is sufficiently low, firms use more intensively international trade to hedge their domestic

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[2]This assumption captures the fact that marketing activities present irreversibilities that make reallocation across countries too costly. For a similar assumption, but in dynamic settings, see Ramondo et al. (2013) and Conconi et al. (2016).

demand risk. This implies a stronger competitive pressure among firms, which in general equilibrium generates a pro-competitive effect which can lead to higher welfare gains from trade than in standard trade models with risk neutrality, such as the class of models considered in Arkolakis et al. (2012) (ACR henceforth).[3]

To assess the quantitative relevance of the risk diversification benefits of international trade, and test the model's predictions, I rely on a panel dataset of Portuguese manufacturing firms' domestic and international sales, from 1995 to 2005. Portugal exports to a wide range of destinations, and therefore is a good laboratory to study the implications of risk diversification for international trade.

First, I structurally recover the unobserved demand shocks from yearly firm-destination sales. I identify the demand shocks as innovations from the growth rate of domestic and international sales, similarly to Di Giovanni et al. (2014). The empirical methodology controls for unobserved firm and destination characteristics, and for firm-destination observables, which account for the firms' endogenous response to foreign demand shocks documented by a recent literature (Mayer et al. (2016) and Friedrich et al. (2018). Under the assumption of log-normally distributed demand, routinely maintained by the literature (see e.g. Cochrane (2009) and Eaton et al. (2011)), I estimate the country-level means and covariance matrix of the demand shocks, and compute the Diversification Index. Using the Portuguese firm-level data, I estimate "risk-augmented" gravity regressions at the firm and country level, which document that both the extensive and intensive margins of trade, even after controlling for standard gravity variables, are significantly affected by the Diversification Index, as predicted by my model.

In the second part of the analysis, I use the structural model to quantify the risk diversification benefits of international trade. First, I calibrate the remaining parameters of the model. Using the firm's first order conditions, I estimate the entrepreneurs's risk aversion in the cross-section of Portuguese firms. The implied "risk premium" is large, lending support to the assumption that entrepreneurs are risk averse. The structural calibration suggests that, overall, the risk diversification motive explains 15% of the observed variation in trade flows. Second, I follow Costinot and Rodriguez-Clare (2013) and compute the welfare gains of going from autarky, i.e. a world where trade costs are infinitely high, to the observed trade equilibrium in 2005. The results illustrate that welfare gains from trade in my model with risk-averse firms are typically larger than the gains predicted by risk neutral models. In relative terms, welfare gains in my model are, for the median country, 17% higher than in ACR. Therefore, the effect of risk diversification on

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[3]These include the models in Krugman (1980), Eaton and Kortum (2002), Melitz (2003), and Chaney (2008).

welfare gains is quantitatively relevant.

A strand of literature (Helpman and Razin (1978), Anderson (1981), Newbery and Stiglitz (1984), Helpman (1988)) has analyzed the effect of financial markets incompleteness on international trade under production uncertainty. While this old literature (recently revived by the works of Koren (2003), Kucheryavyy (2014), Ghironi and Wolfe (2018) and Heiland (2019)) has focused on the relationship between asset markets and international trade under uncertainty, I study the diametrically opposite case where financial markets are absent and analyze the role of international trade as a tool to diversify the demand uncertainty faced by uninsured exporters.[4]

A more recent quantitative literature has studied the role of uncertainty for international trade and welfare, such as Rob and Vettas (2003), Impullitti et al. (2013), Fillat and Garetto (2015), Allen and Atkin (2016). My contribution relative to this literature is twofold. First, while it typically focuses either on the volatility as measure of risk (see e.g. Vannoorenberghe et al. (2016), Kramarz et al. (2020) and De Sousa et al. (2020)), or on the covariance between the exporter' stock returns and the importer's aggregate demand (see e.g. Fillat and Garetto (2015), Barrot et al. (2019), Heiland (2019)), or on the covariance between domestic and foreign shocks (see e.g. Riaño (2011) and Ramondo et al. (2013)), my paper highlights the importance of the *multilateral* covariance of demand across countries, captured by the Diversification Index, in shaping international trade flows and welfare. Second, I provide a novel characterization of the welfare gains from trade as a function of demand risk, and determine, after a structural estimation, its quantitative importance in general equilibrium. Thus the paper innovates also with respect to the literature that examines the determinants of the welfare gains from trade, see e.g. Arkolakis et al. (2012) and Melitz and Redding (2015).

A vast literature, across several fields, has proposed different measures of uncertainty and studied their impact on the economy. These include proxies based on, among others, stock prices (Bloom (2009)), newspaper coverage (Baker et al. (2016)), tariffs (Pierce and Schott (2016)), GDP volatility (Koren and Tenreyro (2007)), consumption volatility (Boguth and Kuehn (2013)). The Diversification Index proposed in this paper captures a distinct aspect of uncertainty compared to existing measures, as it takes into account for the entire pattern of spatial covariance of demand across countries.

This paper also relates to the broad literature that studies the determinants of trade

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[4]While this paper studies the effect of demand risk on international trade and welfare gains from trade, a recent literature has highlighted the importance of other sources of risk that foreign markets can entail or exacerbate, such as sunk costs in dynamic settings (e.g. Fillat and Garetto (2015)), productivity shocks (e.g. Ramondo et al. (2013)), supply chain shocks (e.g. Blaum et al. (2020)), and policy uncertainty (e.g. Handley and Limão (2017) and Bianconi et al. (2021)).

patterns. Theoretically, previous models of firms' export decision have typically assumed that exporters make independent entry decisions for each destination market - see Melitz (2003), Chaney (2008) and Helpman et al. (2008a) among others. In contrast, in my model entry in a market depends on the global diversification strategy of the firm, which I characterize despite its inherent complexity. Other existing works, such as Ahn and McQuoid (2017), Lind and Ramondo (2018) and Morales et al. (2019), feature interdependence of exporting decisions across markets, but arising from supply-side forces, rather than from demand linkages. Empirically, while the existing literature has highlighted the importance of firm-destination shocks for the cross-sectional variation of sales (see e.g. Di Giovanni et al. (2014), Hottman et al. (2015), and Eaton et al. (2011)), a distinct contribution of this paper is to document that the spatial correlation of such shocks has important consequences for trade patterns.

Lastly, the paper complements the strand of literature that studies the effect of international trade on macroeconomic volatility. Di Giovanni et al. (2014) and di Giovanni et al. (2018) investigate the role of individual firms in international business cycle comovement and aggregate volatility. di Giovanni and Levchenko (2009) and Caselli et al. (2020) study the effect of trade openness on aggregate output volatility. My paper, in contrast, sheds light on the other direction, i.e. how demand risk affects international trade patterns through the firms' risk diversification behavior.

The remainder of the paper is organized as follows. Section 2 presents the general equilibrium model with risk aversion. In Section 3 I estimate the Diversification Index and test its role in shaping trade flows. In Section 4 I calibrate the model and perform the counterfactual exercise, while Section 5 concludes.

2 A trade model with risk-averse entrepreneurs

In this section, I propose a multi-country general equilibrium trade model featuring stochastic demand and risk-averse agents. Within the general framework, I show that both the extensive and intensive margins of trade depend on a model-consistent measure of demand risk. I then focus on the more tractable case of two symmetric countries, and show how the spatial correlation of demand affects the welfare gains from international trade.

2.1 Environment

Throughout the paper, I use bold variables to denote stacked vectors of country variables,

$z \equiv \{z_i\}_i$, and bar bold variables to denote matrices associated with origin country i and destination country j, $\bar{z} \equiv [z_{ij}]_{i,j}$. Each country j is populated by a continuum of agents of measure L_j, who maximize the following indirect utility

$$V_j(v) = E\left(\frac{y_j(v)}{P_j}\right) - \frac{\gamma}{2} Var\left(\frac{y_j(v)}{P_j}\right), \tag{1}$$

where $y_j(v)$ is agent v's income and $\gamma > 0$. The mean-variance specification above can be derived assuming that the agents maximize a CARA utility in real income, where γ is the coefficient of absolute risk aversion.[5] While the mean-variance utility has been widely used to study portfolio allocation problems (see e.g. Markowitz (1952), Sharpe (1964) and Ingersoll (1987)), it constitutes a departure from standard trade models typically used in the literature.[6]

Each agent chooses to become either a worker or an entrepreneur. If the agent chooses to become a worker, she earns a non-stochastic and homogeneous wage w_j. Thus the indirect utility of any worker simply equals the real wage, $V_j(v) = w_j/P_j$. If the agent chooses to become an entrepreneur, she earns the stochastic profits obtained from operating the firm. Thus the indirect utility of an entrepreneur depends on both the expected value and the variance of the profits, as indicated in equation (1).

Each agent v in country j chooses her consumption bundle by maximizing a CES aggregator of a continuum of varieties:

$$C_j(v) = \left(\sum_i \int_{\Omega_{ij}} \alpha_j(\omega)^{\frac{1}{\sigma}} q_{ij}(\omega, v)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where $\sigma > 1$ is the elasticity of substitution across varieties, and Ω_{ij} is the endogenous set of available varieties. The term $\alpha_j(\omega)$ reflects an exogenous demand shock specific to variety ω in market j. It is the only source of uncertainty in the economy, and it can reflect shocks to tastes, climatic conditions, consumers confidence, regulation, firm reputation, etc. Define $\boldsymbol{\alpha}(\omega) \equiv \alpha_1(\omega), ...\alpha_N(\omega)$ to be the vector of realizations of the demand shock for variety ω. I assume that:

Assumption 2a $\boldsymbol{\alpha}(\omega) \sim H(\boldsymbol{\mu}, \bar{\boldsymbol{\Sigma}})$, *i.i.d. across ω*

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[5]A second-order Taylor approximation of the expectation of a CARA utility leads to the expression in (1) (see Ingersoll (1987) and De Sousa et al. (2015)).

[6]Maloney and Azevedo (1995) and Allen and Atkin (2016) also assume risk aversion in the context of a trade model. Gervais (2018) proposes a model with risk averse managers, but studies the firm's sourcing problem.

Assumption 2a states that the demand shocks are drawn, independently across varieties, from a multivariate distribution characterized by a N-dimensional vector of positive means $\boldsymbol{\mu}$ and a $N \times N$ variance-covariance matrix $\bar{\boldsymbol{\Sigma}}$, where N is the number of countries.

Assumption 2a implies that the demand shocks are destination-variety specific, ruling out aggregate shocks that would affect the demand for all varieties in a given destination. This feature is appealing on both theoretical and empirical grounds. From a theoretical standpoint, I impose this assumption because, given the continuum of varieties, the demand shocks average out by the Law of Large Numbers and thus aggregate variables, such as wages and price indices, are non-stochastic.[7] In addition, recent empirical evidence suggests that firm-destination specific shocks account for the overwhelming majority of the variation in firms' sales across countries (see e.g. Di Giovanni et al. (2014) and Hottman et al. (2015)).

The CES aggregator in (2) implies the following optimal demand:

$$q_{ij}(\omega, v) = \alpha_j(\omega) \frac{p_{ij}(\omega)^{-\sigma}}{P_j^{1-\sigma}} y_j(v), \tag{3}$$

where P_j is the Dixit-Stiglitz price index.

2.2 The entrepreneurs problem

Once an agent becomes an entrepreneur, she owns a non-transferable technology to produce a differentiated variety ω under monopolistic competition, as in Melitz (2003), using only labor with a productivity z drawn from a known distribution, as highlighted in Assumption 2b:

Assumption 2b *A firm producing variety ω draws a productivity z from a known distribution $G(\cdot)$, independently from other firms and from the demand shocks vector $\boldsymbol{\alpha}(\omega)$.*

Since each firm with productivity z produces a unique variety ω, to simplify notation I will use z to identify both. Entrepreneurs choose how to operate their firm in country i by maximizing the indirect CARA utility in equation (1). The production problem consists of two stages. In the first, firms know only the distribution of the demand shocks, but not their realization. Under uncertainty, firms choose in which countries to operate, and in these markets perform costly marketing and distributional activities. In the second stage,

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[7]This restriction has been typically imposed by the literature that incorporates demand shocks into trade models, such as Eaton et al. (2011), Crozet et al. (2012) and Nguyen (2012).

firms learn the realized demand of the markets in which they have entered. Firms cannot change their set of destinations, but can adjust the quantity produced for each market depending on the magnitude of the observed shock.[8] I assume that financial markets are absent, thus firms cannot diversify away their idiosyncratic risk. This allows to focus on international trade as the only tool firms can use to diversify their demand risk.

The fact that demand is correlated across countries implies that, in the first stage, entrepreneurs face a combinatorial problem. Indeed, both the extensive margin (whether to export to a market) and the intensive margin (how much to invest in marketing) decisions are intertwined across markets: any decision taken in a market affects the outcome in the others. Then, for a given number of potential countries N, the choice set includes 2^N elements, and computing the indirect utility function corresponding to each of its elements would be computationally unfeasible.

I deal with such computational challenge by assuming that, in the first stage, firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction $n_{ij}(z)$ of consumers in location j (bounded between 0 and 1). This implies that the firm's choice variable is continuous rather than discrete, and firms simultaneously choose *where to sell* (if $n_{ij}(z)$ is optimally zero, firm z does not sell in country j) and *how much to sell* (firms can choose to sell to some or all consumers).

The first stage problem consists of choosing $n_{ij}(z)$ to maximize the following objective function:

$$\max_{\{n_{ij}(z)\}} \sum_j E\left(\frac{\pi_{ij}(z)}{P_i}\right) - \frac{\gamma}{2} \sum_j \sum_s Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i}\right) \tag{4}$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0 \tag{5}$$

where $\pi_{ij}(z)$ are net profits from destination j:

$$\pi_{ij}(z) = q_{ij}(z)p_{ij}(z) - q_{ij}(z)\frac{\tau_{ij}w_i}{z} - f_{ij}(z), \tag{6}$$

and $\tau_{ij} \geq 1$ are iceberg trade costs and $f_{ij}(z)$ are marketing costs. Since there is a continuum measure of agents, the total demand for variety z in country j is:

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[8]This assumption captures the idea that marketing and distributional activities present irreversibilities that make reallocation across countries too costly. For a similar assumption, but in different settings, see Ramondo et al. (2013), Albornoz et al. (2012) and Conconi et al. (2016).

$$q_{ij}(z) = \alpha_j(z)\frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}}n_{ij}(z)Y_j. \tag{7}$$

I assume that there is a non-stochastic cost, $f_j > 0$, to reach each consumer in country j, paid in foreign labor, such that $f_{ij}(z) = w_j f_j L_j n_{ij}(z)$.

Although there is no analytical solution to the first stage problem, because of the presence of $2N$ inequality constraints, it is instructive to look at the firm's interior first order condition:

$$\underbrace{r_{ij}(z)\bar{\alpha}_j - \gamma r_{ij}(z)\sum_s n_{is}(z)r_{is}(z)Cov(\alpha_j,\alpha_s)}_{\text{marginal benefit}} = \underbrace{\frac{1}{P_i}w_j f_j L_j}_{\text{marginal cost}}, \tag{8}$$

where $r_{ij}(z) \equiv \frac{1}{P_i}\frac{Y_j p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}}\left(p_{ij}(z) - \frac{\tau_{ij}w_i}{z}\right)$ are real gross profits. Equation (8) equates the real marginal benefit of adding one consumer to its real marginal cost. While the marginal cost is constant, the marginal benefit is decreasing in $n_{ij}(z)$. In particular, it is equal to the marginal revenues minus a "penalty" for risk, given by the sum of the profits covariances that destination j has with all other countries (including itself). The higher this covariance, the smaller the diversification benefits the market provides to a firm exporting there.[9]

To find the general solution of the firm problem, I make the following assumption:

Assumption 2c *The coefficient of risk aversion and the determinant of the covariance matrix are strictly positive, i.e. $\gamma > 0$ and $det(\bar{\Sigma}) > 0$.*

As shown in Appendix A.1, Assumption 2c is necessary and sufficient to have a unique optimal solution. For firm z from origin country i, define $\chi_i(z)$ to be the vector of Lagrange multipliers associated with the upper bound, and $\lambda_i(z)$ to be the vector of Lagrange multipliers associated with the lower bound. The optimal solution is as follows:

Proposition 1. *Under Assumptions 2a-c, for firm z from origin country i the unique vector of*

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[9]Note the difference in the optimality condition with Arkolakis (2010). In that paper, the marginal benefit of reaching an additional consumer is constant, while the marginal penetration cost is increasing in $n_{ij}(z)$. In my setting, instead, the marginal benefit of adding a consumer is decreasing in $n_{ij}(z)$, due to the concavity of the utility function of the entrepreneur, while the marginal cost is constant.

optimal $\mathbf{n}_i(z)$ satisfies:

$$\mathbf{n}_i(z) = \frac{1}{\gamma} \left(\bar{\bar{\mathbf{\Sigma}}}_i(z) \right)^{-1} \left[\boldsymbol{\pi}_i(z) - \boldsymbol{\chi}_i(z) + \boldsymbol{\lambda}_i(z) \right], \tag{9}$$

where $\bar{\bar{\mathbf{\Sigma}}}_i(z)$ is a NxN matrix , whose $k - j$ element equals $\left\{ \bar{\bar{\mathbf{\Sigma}}}_i(z) \right\}_{kj} = r_{ij}(z) r_{ik}(z) Cov(\alpha_j, \alpha_k)$, and $\boldsymbol{\pi}_i(z)$ is the vector of expected real profits, whose j element is $\{\boldsymbol{\pi}_i(z)\}_j = r_{ij}(z)\mu_j - w_j f_j L_j / P_i$.

Moreover, the optimal price charged in destination j is:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z}. \tag{10}$$

Proposition 1 resembles the solution of the well-known mean-variance portfolio problem (see e.g. Ingersoll (1987) and Campbell and Viceira (2002)), which dictates that the fraction of wealth allocated to an asset is proportional to the inverse of the covariance matrix times the vector of expected excess returns. The result implies that entrepreneurs, rather than solving a maximization problem country by country, as in traditional trade models, perform a *global* diversification strategy: they trade off the expected global profits with their variance, the slope being governed by the absolute degree of risk aversion $\gamma > 0$.

This implies that the firm's entry decision in a market (that is, whether $n_{ij}(z) > 0$ for some j) does not depend on a market-specific productivity cutoff and, upon entry, firms may optimally choose to reach only a fraction of consumers, instead of the entire market. This feature stands in contrast with traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008), in which firms, upon entry, serve all consumers. Interestingly, if the risk aversion is set to zero in the objective function (4), the optimal solution is isomorphic to the one in Melitz (2003) and Chaney (2008).[10]

Lastly, since the pricing decision is made after the uncertainty is resolved, and conditional on the number of consumers chosen in the first stage, the optimal price follows a standard constant markup rule over the marginal cost, shown in equation (10). Any realization of the shock in market j only shifts the demand curve without changing its slope, thus changing only the quantity produced.[11]

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[10]In Online Appendix B.2 I show that when $\gamma = 0$ a firm sells to country j only if its productivity exceeds the entry cutoff $(\bar{z}_{ij})^{\sigma-1} = \frac{w_j f_j L_j P_j^{1-\sigma} \sigma}{\mu_j \left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma}Y_j}$, and that, whenever $z \geq \bar{z}_{ij}$, it sells to all consumers, i.e. $n_{ij}(z) = 1$.

[11]A demand shock shifts up or down the quantity demanded, as shown in equation (3). This in turn

2.3 International Trade Flows

Proposition 1 implies that the sales of firm z to country j are given by:

$$x_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z). \tag{11}$$

Equation (11) suggests that the unobserved demand shocks $\alpha_j(z)$ can be structurally recovered from the observed firm-level trade flows, upon controlling for the other determinants of trade flows that appear in equation (11). In Section 3, I will use this feature of the model to back out the demand shocks and estimate their moments.

I now investigate how trade flows are affected by demand risk. To this end, I define the following *ex-ante* measure of risk:

Definition 1. *Given a multivariate distribution of demand shocks* $H(\boldsymbol{\mu}, \bar{\boldsymbol{\Sigma}})$, *the Diversification Index is defined as the vector*

$$\boldsymbol{D} \equiv \bar{\boldsymbol{\Sigma}}^{-1} \boldsymbol{\mu}. \tag{12}$$

The Diversification Index is an ex-ante measure of risk at the country-level. Consider for instance the case of two symmetric countries. For each country j, the Diversification Index simply equals:

$$D_j = \frac{\mu_j}{\sigma_j^2 (1 + \rho)}, \tag{13}$$

where σ_j^2 and μ_j denote the variance and the mean of the demand shocks, respectively, and ρ is the cross-country correlation. Equation (13) shows that the Diversification Index is increasing in the mean, decreasing in the variance, and decreasing in the correlation of demand with the other country. In the general case with N countries (as in equation (12)), which is the one I will consider in the empirical application, the Diversification Index for each country depends on the entire covariance matrix of demand and on the vector of means. It is easy to verify that D_j is decreasing in the covariance of j's demand with all other countries, and increasing in its mean. Therefore, the Diversification Index summarizes the *ex-ante* diversification benefits that a country provides to (domestic or foreign)

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affects the demand for labor: $L_i(z) = \frac{1}{z} \sum_j \mathbb{I}_{n_{ij}(z) > 0} \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z) Y_j$, but not the price charged. In Online Appendix B.1, I consider an alternative production setting, in which also the pricing decision is made under uncertainty, and show that the aggregate implications of the model are unchanged.

firms selling there, since it is inversely related to the overall riskiness of its demand.[12] Note that the Diversification Index is the same for all firms (irrespective of their productivity and their country of origin).

At this stage, it is useful to define the auxiliary matrix $\bar{A}$:

Definition 2. *For a given covariance matrix $\bar{\Sigma}$, the $k - j$ element of the associated cofactor matrix equals $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of $\bar{\Sigma}$. Define $\bar{A}$ the matrix whose $i - j$ element equals $A_{ij} \equiv - \sum_{k \neq i} C_{ik} \Sigma_{kj}$ for $i \neq j$, and $A_{ij} = 1$ for $i = j$.*

Recall also the definition of an M-matrix (see Berman and Plemmons (1994)):

Definition 3. *A matrix $\bar{A}$ is an M-matrix if and only if: i) the off-diagonal entries are less than or equal to zero, ii) $\bar{A}$ is nonsingular, iii) $\bar{A}^{-1}$ is nonnegative.*

In the following Proposition, I characterize how international trade patterns depend on the Diversification Index.

Proposition 2. *If $\bar{A}$ is a M-matrix, then $\frac{\partial n_{ij}(z)}{\partial D_j} \geq 0$ for all i and j. In other words, the probability of exporting and the amount exported to a given market j are increasing in its Diversification Index.*

Proposition 2 suggests that, conditional on aggregate variables and on marketing and variable trade costs, firms are more likely to enter in country j the higher the Diversification Index of j. Since sales are proportional to $n_{ij}(z)$, conditional on entering (i.e. $n_{ij}(z) > 0$) sales are also increasing in D_j. The sufficient condition to have a positive effect of the Diversification Index on $n_{ij}(z)$ is $\bar{A}$ to be a M-matrix, i.e. all its off-diagonal elements must be negative. It is easy to verify that this occurs whenever *some* demand covariances are negative.[13] Intuitively, the pattern of demand covariances across countries has to give enough diversification benefits in order for firms to engage in international trade. This will turn out to be important also in shaping the welfare gains from trade, as discussed in Proposition 4.

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[12]The Diversification Index can be seen as a generalization of the Sharpe Ratio typically used in finance to rank assets by their riskiness (see Sharpe (1966)). In fact, in the limit case in which all demand correlations are zero, the Diversification Index equals the simple ratio between mean and variance, similarly to the Sharpe Ratio, which is computed as an asset's "excess average return" divided by its standard deviation.

[13]This can be seen, for example, for the case $N = 4$, where a typical element of the matrix A is $A_{21} = \rho_{12}\sigma_1^3\sigma_2\sigma_3^2\sigma_4^2(1 - \rho_{13}^2 - \rho_{14}^2 - \rho_{34}^2 + 2\rho_{13}\rho_{14}\rho_{34})$, where ρ_{ij} is the demand correlation between j and i, and σ_i is the standard deviation of i. Then, to have $A_{21} < 0$, at least one correlation must be negative.

2.4 General equilibrium

I now describe the equations that define the trade equilibrium of the model. Following Chaney (2008), I assume that the productivities are drawn, independently across firms and countries, from a Pareto distribution with density:

$$g(z) = \theta z^{-\theta-1}, \ z \geq \underline{z}, \tag{14}$$

where $\underline{z} > 0$. The price index is:

$$P_i^{1-\sigma} = \sum_j M_j \int_{\underline{z}}^{\infty} \mu_i n_{ji}(z) p_{ji}(z)^{1-\sigma} g(z) dz, \tag{15}$$

where $n_{ji}(z)$ and $p_{ji}(z)$ are shown in Proposition 1. In equilibrium, to have a positive measure of both workers ($\tilde{L}_i$) and entrepreneurs (M_i), I impose that the expected utility from being an entrepreneur, $\int_{\underline{z}}^{\infty} E[V_i] \theta z^{-\theta-1} dz$, is the same as the expected utility of being a worker:

$$\int_{\underline{z}}^{\infty} E\left(\frac{\pi_i(z)}{P_i}\right) \theta z^{-\theta-1} dz - \frac{\gamma}{2} \int_{\underline{z}}^{\infty} Var\left(\frac{\pi_i(z)}{P_i}\right) \theta z^{-\theta-1} dz = \frac{w_i}{P_i}. \tag{16}$$

This condition determines the measure of agents that choose to become entrepreneurs. It also imposes, intuitively, that the expected profits have to be larger than the real wage, the difference being the "risk premium" that the entrepreneurs have to be compensated with.

I impose a balanced current account, such that the total expenditures in each country must equal to labor income plus business profits:

$$Y_i = w_i \tilde{L}_i + \Pi_i, \tag{17}$$

where profits are:

$$\Pi_i = M_i \sum_j \left(\frac{1}{\sigma} \int_{\underline{z}}^{\infty} \mu_j p_{ij}(z)^{1-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z) g(z) dz - \int_{\underline{z}}^{\infty} f_{ij}(z) g(z) dz\right). \tag{18}$$

The labor market clearing condition states that in each country the supply of labor must equal the amount of labor used for production and marketing:

$$M_i \sum_j \int_{\underline{z}}^{\infty} \frac{\tau_{ij}}{z} \mu_j p_{ij}(z)^{-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z) g(z) dz + \sum_j M_j \int_{\underline{z}}^{\infty} f_i L_i n_{ji}(z) g(z) dz = \tilde{L}_i. \quad (19)$$

The trade equilibrium in this economy is characterized by a vector of wages $\{w_i\}$, price indexes $\{P_i\}$, number of firms $\{M_i\}$, and income $\{Y_i\}$ that solve the system of equations (15), (17), (18) and (19), where $n_{ij}(z)$ is given by equation (9) and where $\tilde{L}_i = L_i - M_i$.

Lastly, since in equilibrium the expected utility of being an entrepreneur is equal to the utility of being a worker, welfare is equal to the real wage for all agents:

$$W_i = \tilde{L}_i \frac{w_i}{P_i} + M_i \int_{\underline{z}}^{\infty} E\left[V_i(z)\right] \theta z^{-\theta-1} dz = L_i \frac{w_i}{P_i}. \quad (20)$$

2.5 Two symmetric countries

To illustrate some properties of the model and obtain a closed-form expression for the welfare gains from trade, I study the special case of two symmetric countries. I consider two opposite equilibria: one in which there is autarky, and one in which there is free trade.[14] Under autarky, the Diversification Index is simply the ratio between the mean and the variance of the demand shocks, $D_j^A = \frac{\mu}{Var(\alpha)}$, while, under free trade, $D_j^{FT} = \frac{\mu}{Var(\alpha)(1+\rho)}$, where ρ is the correlation of demand between the two countries.

In Appendix A.3, I show that in both equilibria the firm's optimal solution is:

Proposition 3. *Assume that f is sufficiently high. Under both autarky and free-trade, the unique optimal $n_{ij}(z)$ satisfies:*

$$n_{ij}(z) = 0 \qquad \text{if } z \leq z^*$$

$$n_{ij}(z) = \frac{D_j}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r_i(z)} < 1 \text{ if } z > z^*$$

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[14]To obtain a closed-form solution for $n(z)$, throughout this sub-section I assume that $f > \tilde{f}$ (where $\tilde{f}$ depends only on parameters), so that $n_{ij}(z) < 1$ for all z. For more details, see the proof of Proposition 3 in Appendix A.3.

where $r_j(z)$ are real gross profits, and the entry cutoff is:

$$z^* = \left(\left(\frac{\sigma}{\sigma - 1} \right)^{\sigma - 1} \frac{f P^{1 - \sigma} \sigma}{\mu Y} \right)^{\frac{1}{\sigma - 1}}.$$ (21)

In both equilibria, the Diversification Index is a sufficient statistics for risk in the entrepreneurs' optimal decision.[15] Under free trade, the more correlated is demand with the foreign country (and thus the lower D_j), the riskier is the global economy and the lower the number of consumers reached.[16]

I now characterize the effect of free trade on welfare. In order to directly compare the welfare gains from trade in my model to the ones predicted by standard trade models with risk neutrality, I express the welfare gains in terms of the log-change in the domestic trade share, $\ln \left(\hat{\lambda}_{jj} \right)$, where $\hat{\lambda}_{jj} = \frac{\lambda'_{jj}}{\lambda_{jj}}$ denotes the proportional change between the initial and counterfactual equilibria.

Proposition 4. *Welfare gains of going from autarky to free trade are given by:*

$$\ln \hat{W}_i = \underbrace{-\frac{1}{\theta} \ln \left(\hat{\lambda}_{jj} \right)}_{\text{standard gains}} \cdot \underbrace{\frac{\theta}{\theta + 1}}_{\text{variance effect}} - \underbrace{\frac{1}{\theta + 1} \ln \left(1 + \rho \right)}_{\text{diversification effect}} \cdot$$ (22)

Proposition 4 shows that free trade has three distinct effects on welfare.[17] The first is given by the reduction in the price index due to more competition from foreign firms, the standard channel present in trade models with risk neutrality, i.e. the class of models considered in ACR (see also Costinot and Rodriguez-Clare (2013)).[18] The second channel,

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[15]The perfect symmetry and the absence of trade costs imply that any firm will choose the same $n(z)$ in both the domestic and foreign market. This feature is the reason why perfect symmetry and free trade is the only case in which I can derive an analytical expression for $n(z)$. If there were trade costs $\tau_{ij} > 1$, the optimal $n(z)$ would still depend on the Lagrange multiplier of the other destination.

[16]The existence of a single entry cutoff means that there is strict sorting of firms into markets, as in Melitz (2003). However, that happens only because of the perfect symmetry between the two countries, which implies that $n(z)$ is not affected by the Lagrange multipliers of the other location. In the general case of asymmetric countries, firms do not strictly sort into foreign markets, as explained in the previous section.

[17]Note that both the risk aversion and the mean and variance of the shocks do not affect the welfare gains from trade because of the symmetry assumption and because $f > \tilde{f}$ (see footnote 14). In the general case, they do affect the welfare gains, as shown in Section 4.

[18]See Online Appendix B.2 for a proof that the model with $\gamma = 0$ delivers welfare gains from trade equal to $-\frac{1}{\theta} \ln \left(\hat{\lambda}_{jj} \right)$.

given by $\frac{\theta}{\theta+1}$, dampens the competition effect of trade on welfare gains. Lower prices induced by free trade increase the expected real profits, but also their variance, generating a feedback effect that lowers the number of consumers reached by the firms, weakening the competition among firms and increasing the price index.

The third effect, given by $\frac{1}{\theta+1}\ln(1+\rho)$, implies that gains from trade are larger the lower the correlation of demand with the foreign country. Intuitively, the lower such correlation, the more firms export, under free trade, in order to hedge their domestic demand risk, according to Proposition 3. This implies tougher competition among firms, which leads to lower prices and higher welfare gains. The combination of these general equilibrium effects implies that my model predicts larger welfare gains from trade than models with risk neutral firms as long as the correlation of demand is sufficiently low. For example, setting $\theta = 5$, a typical value for this parameter, welfare gains in my model are higher than in ACR, conditional on the same change in domestic trade share, as long as $\rho > -0.13$.

To sum up, the theoretical results discussed in this section highlight the importance of the sign and magnitude of the cross-country covariance of demand in shaping i) the direction of trade flows, ii) the risk diversification benefits of international trade, and iii) the welfare gains from trade. Therefore, the estimation of the covariance matrix of demand is crucial for the quantification of the benefits of international trade on risk diversification and welfare. To this end, in the following section I develop a methodology to estimate the first and second order moments of the demand shocks.

3   Estimation of the Diversification Index

The first step of the empirical analysis is the estimation of the destination-level measure of risk, the Diversification Index, which requires estimating the means and covariance matrix of the demand shocks. To this end, I rely on a panel dataset from Statistics Portugal on domestic and international sales of Portuguese firms (see Online Appendix B.5 for details). I consider the 10,934 manufacturing firms that, between 1995 to 2005, were selling domestically and exporting to at least one of the top 34 destinations served by Portugal.[19] Trade flows to these countries accounted for 90.56% of total manufacturing

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[19]This is the same set of countries considered in Section 4 for the quantitative analysis (see the list of countries in Table A.2 in Appendix A.5). I first select the top 45 destinations from Portugal by value of exports, and then I keep the countries for which there is data on manufacturing production, in order to construct bilateral trade flows.

exports from Portugal in 2005.[20] Portuguese manufacturing exporters are mostly small firms, that serve on average 10 destinations, with an average export share of 30%.

The estimation methodology follows a long tradition in empirical finance of using historical data to identify the moments of risky assets (see e.g. Cochrane (2009)), and a recent empirical literature that identifies demand shocks as innovations from the growth rate of sales (see e.g. Di Giovanni et al. (2014)). The methodology consists of two steps. First, I use the firm-destination level equation for sales to recover the unobserved demand shocks in each year of the sample. Then I use these shocks to compute the means and covariance matrix of demand across countries. For brevity, I define the variables $\tilde{x} \equiv \ln(x)$ and $\Delta x \equiv x_t - x_{t-1}$.

3.1 Identification of demand shocks

I assume that the structural model in Section 2 is the Data Generating Process:

Assumption 3a *In every year t of the sample period, the world economy is generated by the model of Section 2, and all parameters are constant throughout the sample period.*

This assumption and equation (11) imply that, in every year t, the log-sales of Portuguese firm s to country j (including Portugal itself) can be written as:

$$\tilde{x}_{js}^t = \delta^t + \delta_s^t + \delta_j^t + \varepsilon_{js}^t \tag{23}$$

where $\delta^t \equiv (1-\sigma)\ln\left(\frac{\sigma}{\sigma-1}w^t\right)$, $\delta_s^t \equiv (\sigma-1)\ln(z_s^t)$, $\delta_j^t \equiv (1-\sigma)\ln\left(\tau_j^t\right) + \ln\left(\frac{Y_j^t}{P_j^{t1-\sigma}}\right)$, and where the structural residual equals:

$$\varepsilon_{js}^t = \tilde{\alpha}_{js}^t + \tilde{n}_{js}^t. \tag{24}$$

Under Assumption 3a, and the fact that $\tilde{n}_{js}^t$ does not depend on the realization of the shocks (as shown in Proposition 1), taking the first-difference across time eliminates $\tilde{n}_{js}^t$, i.e. $\Delta\varepsilon_{js}^t = \Delta\tilde{\alpha}_{js}^t$. Assumption 2b, i.e. the orthogonality between firm-level productivity and demand shocks, implies that I can estimate the structural equation above with OLS:

$$\Delta\tilde{x}_{js}^t = \delta^t + \delta_s^t + \delta_j^t + \mathbf{Z}_{js}^t\boldsymbol{\beta} + \eta_{js}^t \tag{25}$$

where $\mathbf{Z}_{js}^t$ is a set of firm-destination specific controls, and $\eta_{js}^t = \Delta\tilde{\alpha}_{js}^t$. Therefore, by

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[20]I exclude from the analysis foreign firms' affiliates, i.e. firms operating in Portugal but owned by foreign owners, since their exporting decision is most likely affected by their parent's optimal strategy.

recovering the structural residual η_{js}^t one recovers $\Delta\tilde{\alpha}_{js}^t$, the *log-change* in demand shocks. The structural specification in (25) controls for firm-specific productivity shocks that are common to all destinations, by means of the firm fixed effect, and by destination-specific characteristics, such as trade costs and real income, by means of the destination fixed effect.

The vector $\mathbf{Z}_{js}^t$ includes log changes in firm-year-specific investment, capital intensity, and productivity (proxied by revenues per worker), all interacted with country-specific dummies.[21] Including the firms' investment rates and capital intensity controls for the possibility that firms endogenously respond to demand shocks in a market by changing their capital structure, as highlighted in Friedrich et al. (2018), thus affecting $\Delta\tilde{x}_{js}^t$. Controlling for the firm productivity interacted with country dummies accounts for the evidence, shown in Mayer et al. (2016), that foreign demand shocks may induce changes in firms' productivity, thus affecting sales.

Lastly, note that the specification in (25) controls for some features of the firms' behavior that the model does not capture explicitly. In fact, the firm and destination fixed effects control for endogenous markups (see De Loecker and Warzynski (2012)), while the first differencing absorbs the time-invariant component of pricing-to-market, which is firm-destination specific, and any time-invariant trade costs that are firm-destination specific.

3.2 Estimation of μ and $\bar{\Sigma}$

Once the demand shocks are structurally recovered, in order to estimate μ and $\bar{\Sigma}$ I make the following identifying assumption.

Assumption 3b *Demand shocks are independently and identically distributed across firms and time.*

The i.i.d assumption is useful because it allows to exploit both time-series and cross-sectional variation in the residuals. Since firms independently draw the shocks from the same distribution, to compute the covariance matrix of the *log-changes* of the shocks, for every pair of destinations j and k, I stack the residuals $\hat{\eta}_{js}^t$ and $\hat{\eta}_{ks}^t$ for each Portuguese firm s that was selling to both markets j and k in year t. Effectively, each firm-year pair is a vector of (at most) N correlated demand shocks draws. I compute an unbiased estimate of the covariance between country j and k as:

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[21]These are the variables that have the best coverage across the firms' balance sheets from the Central de Balancos dataset. Given the dimensionality of the data, replacing $\mathbf{Z}_{tjs}$ with an interaction of fixed effects would not be computationally feasible.

$$Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right) = \frac{1}{S_{jk}-1} \sum_{s=1}^{S_{jk}} \left(\hat{\eta}_{js} - \bar{\eta}_j\right)\left(\hat{\eta}_{ks} - \bar{\eta}_k\right) \tag{26}$$

where $\hat{\eta}_{ks}$ and $\hat{\eta}_{js}$ are the residuals from equation (23), S_{jk} is the number of observations (total number of firms-year pairs that sell to both markets j and k) and $\bar{\eta}_i \equiv \frac{1}{S_{jk}} \sum_{s=1}^{S_{jk}} \hat{\eta}_{is}$ for all i.

Note that, since the expectation and the covariance are linear operators, an alternative approach would be to compute the covariances for each Portuguese exporter using only time-series variation, and then, for each bilateral pair, compute the average covariance across firms (see Online Appendix B.6 for a proof). In the robustness exercise discussed below, I show that the resulting estimates are indeed very similar.

To compute the covariance matrix of the log shocks, I use the assumption that the shocks are i.i.d. across time to obtain (see Online Appendix B.7 for the proof):

$$\tilde{\Sigma}_{jk} \equiv Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) = Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right)/2. \tag{27}$$

Finally, in order to recover the covariance matrix from the covariance of the log shocks, $\tilde{\Sigma}_{jk}$, I make a parametric assumption on the distribution of the demand shocks:[22]

Assumption 3c *Demand shocks are log-normally distributed, with*

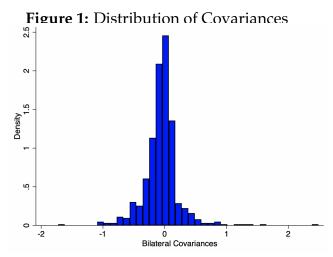$$log\alpha(z) \sim N\left(\mathbf{0}, \tilde{\Sigma}\right). \tag{28}$$

The log-normality assumption has been traditionally used in empirical asset pricing to model asset returns (see e.g. Cochrane (2009)), but recently also in the quantitative trade literature to model demand shocks (see Eaton et al. (2011), Nguyen (2012) and Crozet et al. (2012)). Using the properties of the normal distribution, I obtain the covariance of the *level* of the shocks as:

$$\Sigma_{jk} \equiv Cov\left(\alpha_j, \alpha_k\right) = e^{\frac{1}{2}\left(Var(\tilde{\alpha}_j) + Var(\tilde{\alpha}_k)\right)} \left(e^{Cov(\tilde{\alpha}_j, \tilde{\alpha}_k)} - 1\right) \tag{29}$$

and the expected value as:

$$\mu_k \equiv E\left[\alpha_k\right] = e^{\frac{1}{2}Var(\tilde{\alpha}_k)}. \tag{30}$$

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[22]The need to make a functional form assumption on the distribution of the shocks arises from the fact that I recover the growth rates of the demand shocks, and not directly their levels. If I were to estimate the structural equation for sales in levels, I could not identify the demand shocks separately from $\tilde{n}_{js}^t$.

Figure 1: Distribution of Covariances

Notes: Histogram of estimated bilateral covariances. Sample of 1,190 pairs of countries (35 countries excluding domestic pairs).

3.3 Results

Table A.1 in Appendix A.5 reports some summary statistics on the estimated moments. The covariances range from -1.73 to 2.49, with a median of -0.05, while variances are typically larger. Figure 1 displays the distribution of the estimated covariances. The heterogeneity in the covariances across countries highlights the potential for risk diversification that international trade can offer. In addition, Table 1 documents that more remote destinations offer, *ceteribus paribus*, better risk diversification benefits, since bilateral demand covariances are significantly negatively correlated with bilateral distance. This highlights the trade-off that exporters face. On one hand, the traditional profit maximization motive gives firms incentives to sell to nearby or "similar" destinations, because of lower trade costs. On the other hand, the risk minimization motive gives firms incentives to sell to remote countries, which could hedge against domestic fluctuations in demand.

Using the estimated μ and $\bar{\Sigma}$, I compute the destination-level Diversification Index, using equation (12). Table A.2 in Appendix A.5 reports the Diversification Indexes for the countries in the sample, together with their bootstrapped standard errors.[23] Standard errors are relatively small, suggesting that the Diversification Indexes are precisely estimated.

Table 1: Demand covariances and geography

	Bilateral Covariance		
	(1)	(2)	(3)
Log of bilateral distance	-0.024***	-0.069***	-0.063***
	(0.009)	(0.015)	(0.017)
Observations	1,190	1,190	1,190
Country FE	N	Y	Y
Gravity controls	N	N	Y

Notes: Sample of 1,190 pairs of countries (35 countries excluding domestic pairs). Column 1 reports the estimate of a regression of the bilateral demand covariances on the bilateral log distance, Column 2 adds country fixed effects, Column 3 adds dummies for contiguity, common language, common legal origins, colonial relationship. Robust standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Lastly, in Online Appendix B.8 I investigate the robustness of the estimates. I document that the estimated Diversification Indexes do not change substantially if I: i) do not control for firm-destination supply shocks, ii) compute the covariances for each Portuguese exporter using only time-series variation, iii) allow for aggregate demand shocks that affect the sales of all exporters to a certain destination, iv) include only "established" firm-destination pairs, i.e. exporters selling to a certain country for at least 5 years.

3.4 Diversification and Trade Flows

Armed with the estimated Diversification Index, I investigate whether the predictions of the model about demand risk and international trade hold in the data. Under the null hypothesis of risk-neutrality, the Diversification Index of a given destination country should not significantly affect neither the likelihood that firms enter that country, nor the level of the trade flows upon entry. I test this hypothesis with "risk-augmented" gravity regressions at the firm and country level.

Firm-level specifications. I first estimate the following "extensive margin" gravity regression:

$$Pr\left(x_{sj} > 0\right) = \beta_0 + \beta_1 ln\left(D_j\right) + \mathbf{X}_j'\boldsymbol{\beta} + \delta_s + \varepsilon_{js}, \tag{31}$$

where x_{sj} are the sales of Portuguese firm s to foreign country j in 2005, δ_s is a firm fixed effect, X_j is a set of destination-specific controls and standard gravity variables.[24]

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[23]To compute the standard errors, I sample with replacement from the structurally recovered demand shocks and use the steps shown in the previous section to compute the Diversification Index. I repeat this procedure 1,000 times and compute the standard deviation.

[24]The controls are log of GDP, log of population, trade openness. The gravity controls include the dis-

Table 2: Risk-augmented Gravity Regressions

	Extensive margin		Intensive margin			Country regression	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$ln(D_j)$	0.177***	0.494***	1.511***	0.851***	0.807***	0.112***	0.208**
	(0.001)	(0.131)	(0.015)	(0.107)	(0.194)	(0.026)	(0.089)
$ln(GDP_j)$	0.076***	0.449***	0.740***	0.428***	0.338***	0.895***	0.706***
	(0.001)	(0.062)	(0.008)	(0.087)	(0.117)	(0.012)	(0.041)
Distance	-0.464***	-0.846***	-1.952***	-0.484**	-0.680**	-1.023***	-0.676***
	(0.002)	(0.219)	(0.036)	(0.232)	(0.312)	(0.091)	(0.053)
Observations	124,113	124,113	16,491	17,626	17,626	1,122	1,122

Notes: In cols (1) and (2) the dependent variable is an indicator equal to 1 if a Portuguese firm enters a market in 2005, and equal to 0 otherwise. Col. (1) estimates a two-stage FGLS, col. (2) estimates a Probit model. In cols. (3) and (4) the dependent variable is the log of trade flows of Portuguese firms, while col. (5) uses the level of trade flows. Col. (3) estimates a two-stage FGLS, col. (4) estimates a two-stage Heckman specification, col. (5) estimates a PPML specification. In col. (6) the dependent variable is the log of bilateral trade flows between the 35 countries in the sample, while col. (7) uses the level of trade flows. Additional not reported controls are: trade openness, log of population, average tariff rate, dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement. Cols. (1) and (3) add firm fixed effects, cols. (6) and (7) add origin fixed effects. Regressions in cols. (1) and (3) are weighted by firm's total sales. Robust standard errors are shown in parenthesis and are clustered by destination country (*** p<0.01, ** p<0.05, * p<0.1).

Since I cannot directly control for destination fixed effects, given the presence of $ln(D_j)$, I follow Head and Mayer (2013) and Baker and Fortin (2001) and use a two-step methodology, in which I first regress $Pr\left(x_{sj} > 0\right)$ on firm and destination fixed effects, and then regress the estimated destination fixed effect on $ln(D_j)$ and the other controls using Feasible Generalized Least Squares (FGLS). Column (1) in Table 2 shows that, consistent with my model, Portuguese firms are more likely to enter in countries with a higher D_j, even after controlling for trade barriers and destination specific characteristics.[25] The coefficient β_1 is statistically different from 0 at the 1% significance level, thus rejecting the risk-neutral hypothesis, and it implies that one standard deviation increase in the (log of) Diversification Index raises the probability of exporting by 7%. Column (2) estimates instead a Probit model, and confirms the findings of column (1).

I next investigate whether demand risk affects international trade on the intensive margin. To this end, I estimate the following gravity regression:

$$ln\left(x_{sj}\right) = \beta_0 + \beta_1 ln\left(D_j\right) + \mathbf{X}_j'\boldsymbol{\beta} + \delta_s + \varepsilon_{js}, \tag{32}$$

where the controls are the same as for the extensive margin regression. Column (3) in

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tance from Portugal, average ad-valorem tariff rate (from UNCTAD), dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement.

[25]Standard errors are clustered at the country level, but are similar if clustered at the firm level. Firm-level regressions are weighted by firms' total sales in 2005, but results are similar with equal weights.

Table 2 estimates the above with the Head and Mayer (2013) two-step methodology, and confirms the prediction of the model: conditional on entry, firms export more to countries with higher Diversification Index, even after controlling for trade barriers and destination specific characteristics. When I control in column (4) for selection of firms into exporting with a two-stage Heckman approach similarly to Helpman et al. (2008b), the coefficient is reduced but is still largely positive and significant, with an elasticity of trade flows to risk of 0.85.[26] Column (5) implements a Poisson Pseudo-Maximum Likelihood estimation to control for heteroskedasticity as in Silva and Tenreyro (2006), with similar results.

Country-level specifications. Lastly, I further estimate a gravity regression at the country level:

$$ln\left(X_{ij}\right) = \beta_0 + \beta_1 ln\left(D_j\right) + \mathbf{X}_j'\boldsymbol{\beta} + \mathbf{X}_{ij}'\boldsymbol{\xi} + \delta_i + \varepsilon_{ij}, \tag{33}$$

where X_{ij} are manufacturing trade flows in 2005 from country i to j, with $i \neq j$, from UN Comtrade, δ_i is an exporter fixed effect, $\mathbf{X}_j$ is a set of destination-specific controls and $\mathbf{X}_{ij}$ is a set of bilateral gravity variables.[27] Column (6) in Table 2 implements the Head and Mayer (2013) two-step methodology, while column (7) estimates the specification with PPML.[28] Both columns display a positive and 1% significant coefficient for $ln\left(D_j\right)$, with an implied elasticity of trade to risk between 0.11-0.20, suggesting that the Diversification Index affects trade flows also at the aggregate level.

4   Quantitative Analysis

Having estimated the Diversification Index and established its empirical relevance in shaping international trade patterns, I now quantify the role of risk diversification in determining the welfare gains from trade. To this end, I first calibrate the remaining parameters exploiting again the richness of the firm-level data, and then perform a counterfactual exercise.

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[26] As in Helpman et al. (2008b), I use the number of procedures to start a business, from Djankov et al. (2002), as the variable that satisfies the exclusion restriction, i.e. it affects selection into foreign markets but does not affect the second stage regression.

[27] The destination-specific controls are log of GDP, log of population, trade openness, while the bilateral controls include distance, log of bilateral average tariffs from UNCTAD, dummies for common language, common legal origins, contiguity, colonial links, common currency, WTO membership.

[28] Following Head and Mayer (2013) and Baker and Fortin (2001), I additionally control for $\bar{\mathbf{X}}_j \equiv \frac{1}{N}\sum_i \mathbf{X}_{ij}$, the average across exporters of each export-importer variable. As they recommend, standard errors are clustered at the importer level.

4.1 Calibration

I calibrate the model for the year 2005. The implicit assumption is that firms, when choosing their optimal risk diversification strategy in 2005, take as given the demand risk that each potential destination entails, which is summarized by the Diversification Index. Note that, since D_j is destination specific, exporters from different origin countries selling to a given market face the same Diversification Index, *ceteribus paribus*.

I augment the baseline model with a non-tradeable good produced, under perfect competition, with labor. Consumers spend a constant share ξ_i of their income on the manufacturing tradeable goods, and a share $1 - \xi_i$ on the non-tradeable good. The demand for the non-tradeable good is non-stochastic. I set the elasticity of substitution to $\sigma = 4$, consistent with estimates of an average mark-up of 33% in the manufacturing sector (see Christopoulou and Vermeulen (2012) and Broda and Weinstein (2006)). I set the technology parameter θ to 5, which is around the value found by a large empirical literature (see Head and Mayer (2013) and Simonovska and Waugh (2014)). L_j is measured as the total working-age population from the World Bank. I follow Arkolakis (2010) and assume that the cost to reach a certain number of consumers is lower in markets with a larger population, so the per-consumer cost is $f_j = L_j^{\chi-1}$, with $\chi = 0.42$ as in Arkolakis (2010). I obtain ξ_i as the share of final consumption on tradeable goods from WIOD.

To estimate the risk aversion parameter, I follow Allen and Atkin (2016) and directly use the firms' first order conditions. Intuitively, the risk aversion regulates the slope of the relationship between the average and the variance of a firm's *marginal* profits. In Appendix A.5.2 I show how equation (8) can be rearranged and used to estimate γ with a fixed-effect OLS regression and data on gross profits of Portuguese firms.[29] Table A.3 shows that there is a positive and statistically significant relationship between the expected marginal profits and their variance, with a risk-aversion parameter of 0.415. This estimate implies an average "risk premium" of 74%, which suggests a high level of risk aversion.[30]

Finally, I calibrate the matrix of variable trade costs by matching the observed matrix of international trade shares in the tradeable sector.[31] I use trade data from UN Com-

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[29] These additional data is obtained from Central de Balanços, a repository of yearly balance sheet data for non financial firms in Portugal. See Online Appendix (B.5).

[30] The *risk premium* expresses the risk aversion in relation to the magnitude of the risk taken. In other words, it is the fraction of the potential loss in a gamble that an agent is willing to pay to avoid facing that gamble. For a CARA utility the risk premium can be approximated as $\phi \approx \frac{ln(0.5(e^{-\gamma g}+e^{\gamma g}))}{\gamma g}$ where γ is the risk aversion and g is the size of the gamble (see Babcock et al. (1993)). Following the empirical literature, I proxy g by the standard deviation of firm sales.

[31] I normalize domestic trade barriers to $\tau_{ii} = 1$, and I further assume $\tau_{ij} \leq \tau_{iv}\tau_{vj}$ for all i, j, v to exclude the possibility of transportation arbitrage.

trade assembled by CEPII to measure country-to-country trade flows in manufacturing as the empirical counterpart of bilateral trade in the model, complemented with data on manufacturing production from WIOD (see Dietzenbacher et al. (2013)). Appendix A.5.3 describes in detail the Simulated Method of Moments algorithm used, and Figure A.1 shows that the model correctly reproduces the heterogeneity in trade shares that we observe in the data in 2005.

Once I calibrate the model, I next test its ability to reproduce some salient features of the data that were not targeted in the calibration. Table B.1 and Figure B.2 in Online Appendix B.10 document that the model predicts well the sorting of Portuguese firms into foreign markets and the distribution of sales across firms to each destination, and outperforms the standard model with risk neutrality in matching these moments.

Lastly, using the calibrated model I compute the fraction of observed trade flows that is explained by risk diversification. To this end, I set the risk aversion to zero and, holding constant the other parameters, I compute the implied trade shares, λ_{ij}^N. Then, 1 minus the R^2 of a regression of the calibrated trade shares on λ_{ij}^N represents the fraction of variation in the trade flows explained by risk diversification. Figure 2 plots λ_{ij}^N against the calibrated trade shares, and the associated R^2 of 0.85 suggests that risk diversification explains 15% of the observed trade flows.
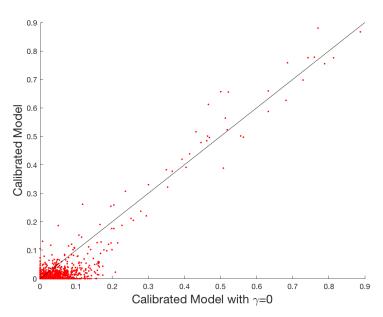
Figure 2: Risk and calibrated trade shares

Notes: The graph plots the trade shares of the calibrated model against the corresponding shares of the calibrated model with the risk aversion parameter set to 0. The figure also displays the 45-degree line.

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4.2 Welfare Gains from Trade

Following Costinot and Rodríguez-Clare (2014), I focus on an important counterfactual exercise: moving from autarky. Formally, starting from the calibrated trade equilibrium in 2005, I assume that variable trade costs in the new equilibrium are such that $\tau_{ij} = +\infty$ for all pair of countries $i \neq j$. All other structural parameters are the same as in the initial equilibrium. I then compute the welfare gains associated with moving from autarky to the observed calibrated equilibrium. In addition, I compute the welfare gains in the limit case of no risk aversion, $\gamma = 0$, which, conditional on the domestic trade shares in the initial calibrated equilibrium, is isomorphic to the gains predicted by the models considered in ACR, as discussed earlier.[32]
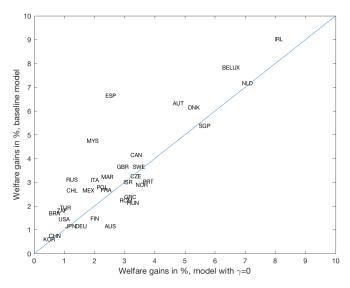
Figure 3 shows that welfare gains from trade in the baseline model with risk-averse firms are typically larger than the gains predicted by risk neutral models. Note that the welfare gains from trade are generally small because the model includes a large non-tradeable sector (on which consumers spend on average 73% of their income). In relative terms, welfare gains in my model are, for the median country, 17% higher than in ACR. Therefore, the effect of risk diversification on welfare gains is quantitatively relevant.

I next use the insights provided by the theory to investigate the determinants of the welfare gains from trade. In particular, I use equation (16) to decompose, in Figure 4, the percentage change in the real wage into the change in expected real profits (profit effect) and the change in the variance of real profits (variance effect).

All the countries experience an increase in their average real profits, which is the result of lower prices due to trade competition and higher profits due to expanded export opportunities. Countries with a higher increase in average profits tend to have larger welfare gains from trade. As for the variance effect, most countries experience a reduction in the variance of their real profits, a result of the risk diversification opportunities that international trade offers to firms. Interestingly, in general equilibrium some countries experience an increase in the variance of profits, but they still have an overall increase in their real wages.
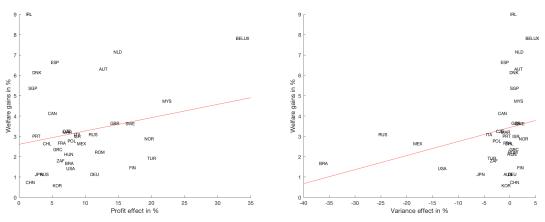
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[32]In the limit case of $\gamma = 0$, welfare gains can be computed using: the domestic trade shares in the initial calibrated equilibrium (since under autarky those are equal to 1), the share of total expenditures on the tradeable sector ξ_i, the share of total revenues from the tradeable sector, and the trade elasticity θ (see equation 23 in Costinot and Rodríguez-Clare (2014)). See also Esposito (2020) for a similar exercise.

Figure 3: Welfare gains from trade

Notes: The figure plots the percentage change in welfare predicted by the baseline model after moving from autarky to the calibrated equilibrium against the corresponding change predicted by the baseline model with $\gamma = 0$.

Figure 4: Welfare Gains from Trade - Decomposition

Notes: The figure on the left plots the welfare gains from trade against the percentage change of average real profits, while the figure on the right plots the welfare gains from trade against the percentage change of the variance of real profits. Both graphs also display the best fit line.

Robustness. Finally, in Online Appendix B.12 I investigate the robustness of the counterfactual predictions of the model along several dimensions. First, Figure B.3 plots the welfare gains from trade computed using means and demand covariances estimated using the alternative methodologies described in Online Appendix B.8. For all these alternative measures of risk, the resulting welfare gains are very similar to the baseline.

Second, Figure B.4 plots the median welfare gains across the countries in the sample computed using different values of the entrepreneurs' risk aversion. While the gains from trade are not substantially different from the baseline, they are a hump-shaped function of the risk aversion parameter γ. Intuitively, going from an economy with risk neutrality to one with risk aversion implies that firms use more intensively international trade to diversify their demand risk, increasing the welfare gains for most countries, as already shown in Figure 3. However, as the risk aversion increases, entrepreneurs optimally choose to be less exposed to foreign risk, leading to a weaker competitive pressure, and smaller welfare gains from trade.

Lastly, I repeat the counterfactual exercise assuming that the agents maximize a CRRA utility function, which features a decreasing absolute risk aversion (see Online Appendix B.11 for details). Figure B.5 documents that welfare gains with CRRA are highly correlated with the ones predicted by the baseline model, although they tend to be on average larger.[33]

5   Conclusions

In this paper I characterize the link between demand risk, firms' exporting decisions, and welfare gains from trade. The proposed framework is sufficiently tractable to deliver testable implications and to be calibrated using firm-level data. Theoretically, I stress the importance of the cross-country multilateral covariance of demand in amplifying the impact of a change in trade costs through a novel "pro-competitive" effect. Empirically, I show that the Diversification Index, the country-level measure of demand risk, significantly affects trade patterns in a gravity framework. Quantitatively, an important message emerges from the analysis: a trade liberalization affects the risk-return trade-off that firms face on global markets, implying general equilibrium effects that may increase welfare gains from trade relative to standard trade models with risk neutrality.

Several avenues for future research emerge from my study. For example, it would be

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[33]For this exercise, I calibrate the coefficient of relative risk aversion such that the average absolute risk aversion of Portuguese firms in the model is equal to the absolute risk aversion in the calibrated baseline model, which is $\gamma = 0.415$. The implied coefficient of relative risk aversion is 1.14.

interesting to introduce the possibility of product diversification as a tool to reduce profits volatility, as opposed to, or together with, geographical diversification, which has been the focus of this paper. In addition, one could enrich the model with dynamic learning, for example allowing firms to invest to reduce the degree of uncertainty over time.

Different measures of aggregate uncertainty have been used by the macro and trade literature. These include, among others: stock market volatility (Bloom (2009)), newspaper-based measures (Baker et al. (2016)), policy uncertainty measures (Pierce and Schott (2016)), GDP volatility (Koren and Tenreyro (2007)). The Diversification Index proposed in this paper differs from the existing measures, as it takes into account for the entire pattern of spatial correlation of demand across countries. Therefore, it could be used to control for demand risk in cross-country regressions.

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A Appendix

A.1 Proof of Proposition 1

Since the firm sets the optimal price after the realization of the shock, in the first stage it chooses the optimal fraction of consumers to reach in each market based on the expectation of what the price will be in the second stage. I solve the optimal problem of the firm by backward induction, starting from the second stage. At this stage, there is no uncertainty and thus the firm chooses the optimal pricing policy that maximizes profits, *given* the optimal $n_{ij}(z, E[p_{ij}(z)])$ chosen in the first stage:

$$\max_{\{p_{ij}\}} \sum_j \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z, E[p_{ij}(z)]) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right).$$

noting that the firm has already paid the marketing costs in the first stage. It is easy to see that this leads to the standard constant markup over marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z}. \tag{34}$$

Notice that, given the linearity of profits in $n_{ij}(z, E[p_{ij}(z)])$ and $\alpha_j(z)$, due to the assumptions of CES demand and constant returns to scale in labor, the optimal price does not depend on neither $n_{ij}(z, E[p_{ij}(z)])$ nor $\alpha_j(z)$. The optimal quantity produced is:

$$q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} \frac{n_{ij}(z, E[p_{ij}(z)]) Y_j}{P_j^{1-\sigma}}. \tag{35}$$

I now solve the firm problem in the first stage, when there is uncertainty on the realization of the shocks. By backward induction, in the first stage the firm takes as given the pricing rule in (34) and the quantity produced in (35). The maximization problem of firm z is:

$$\max_{\{n_{ij}\}} U_i(z) = \sum_j \mu_j n_{ij}(z) r_{ij}(z) - \frac{\gamma}{2} \sum_j \sum_s n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s) - \sum_j w_j n_{ij}(z) f_j L_j$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0$$

where $r_{ij}(z) \equiv \frac{1}{P_i} \frac{p_{ij}(z)^{-\sigma} Y_j}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right)$. Given the optimal price in (34), this simplifies to:

$$r_{ij}(z) = \frac{1}{P_i} \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma} \sigma}$$

The Lagrangian is (omitting the z to simplify notation):

$$\Gamma_i = \sum_j \mu_j n_{ij} r_{ij} - \frac{\gamma}{2} \sum_j \sum_s n_{ij} r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - \sum_j w_j n_{ij}(z) f_j L_j / P_i - \sum_j \chi_{ij} g(n_{ij})$$

where $g(n_{ij}) = n_{ij} - 1$. The necessary Karush–Kuhn–Tucker conditions are:

$$\frac{\partial \Gamma_i}{\partial n_{ij}} = \frac{\partial U_i}{\partial n_{ij}} - \chi_{ij} \frac{\partial g(n_{ij})}{\partial n_{ij}} \leq 0 \quad \frac{\partial \Gamma_i}{\partial n_{ij}} n_{ij} = 0$$

$$\frac{\partial \Gamma_i}{\partial \chi_{ij}} \geq 0 \quad \frac{\partial \Gamma_i}{\partial \chi_{ij}} \chi_{ij} = 0$$

A more compact way of writing the above conditions is to introduce the auxiliary variable λ_{ij}, which is such that

$$\frac{\partial U_i}{\partial n_{ij}} - \chi_{ij} \frac{\partial g(n_{ij})}{\partial n_{ij}} + \lambda_{ij} = 0$$

and thus $\lambda_{ij} = 0$ if $n_{ij} > 0$, while $\lambda_{ij} > 0$ if $n_{ij} = 0$. Then the first order condition for n_{ij} becomes:

$$\mu_j r_{ij} - \gamma \sum_s r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - w_j f_j L_j / P_i - \chi_{ij} + \lambda_{ij} = 0$$

I can write the solution for $n_{ij}(z)$ in matrix form as:

$$\mathbf{n}_i = \frac{1}{\gamma} \left(\bar{\bar{\Sigma}}_i \right)^{-1} \mathbf{r}_i, \tag{36}$$

where each element of the $N-$dimensional vector $\mathbf{r}_i$ equals:

$$r_i^j \equiv r_{ij} \mu_j - w_j f_j L_j / P_i - \chi_{ij} + \lambda_{ij}, \tag{37}$$

and $\bar{\bar{\Sigma}}_i$ is a NxN covariance matrix, whose k, j element is:

$$\tilde{\Sigma}_{i,kj} = r_{ij} r_{ik}(z) Cov(\alpha_j, \alpha_k).$$

The inverse of $\bar{\bar{\Sigma}}_i$ is, by the Cramer's rule:

$$\left(\bar{\tilde{\Sigma}}_i\right)^{-1} = \frac{\bar{r}_i \bar{C}_i \bar{r}_i}{det\left(\bar{\Sigma}\right)}, \tag{38}$$

where $\bar{r}_i$ is the inverse of a diagonal matrix whose j element is r_{ij}, and $\bar{C}_i$ is the (symmetric) matrix of cofactors of $\bar{\Sigma}$.[34] Replacing equations (38) and (37) into (36), the optimal n_{ij} is:

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{det(\bar{\Sigma})r_{ik}}\left(r_{ik}\mu_k - w_k f_k L_k / P_i - \chi_{ik} + \lambda_{ik}\right)}{\gamma r_{ij}},$$

where C_{jk} is the j, k cofactor of $\bar{\Sigma}$. Finally, the solution above is a global maximum if i) the constraints are quasi-convex and ii) the objective function is concave. The constraints are obviously quasi-convex since their are linear. The Hessian matrix of the objective function is:

$$H_i = \begin{bmatrix} \frac{\partial^2 U_i}{\partial^2 n_{ij}} & \cdot & \cdot & \frac{\partial^2 U_i}{\partial n_{ij} \partial n_{iN}} \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ \frac{\partial^2 U_i}{\partial n_{iN} \partial n_{ij}} & & & \frac{\partial^2 U_i}{\partial^2 n_{iN}} \end{bmatrix},$$

where, for all pairs j, k:

$$\frac{\partial^2 U_i}{\partial n_{ij} \partial n_{ik}} = \frac{\partial^2 U_i}{\partial n_{ik} \partial n_{ij}} = -\gamma r_{ij} r_{ik} Cov(\alpha_j, \alpha_k)$$

If $\gamma > 0$, then $\frac{\partial^2 U_i}{\partial^2 n_{ij}} < 0$ (since $Var(\alpha_j) > 0$), thus all the diagonal elements of the Hessian are positive. Therefore the Hessian is negative semi-definite if and only if its determinant is positive. It is easy to see that the determinant of the Hessian can be written as:

$$det\left(H_i\right) = \gamma det\left(\bar{\Sigma}\right) \prod_{j=1}^{N} r_{ij}^2, \tag{39}$$

which is positive if and only if $det\left(\bar{\Sigma}\right) > 0$ and $\gamma > 0$. Therefore under Assumption 2c the function is concave and the solution is a global maximum.∎

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[34]The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of $\bar{\Sigma}$. The minor of a matrix is the determinant of the sub-matrix formed by deleting the k-th row and j-th column.

A.2 Proof of Proposition 2

From Proposition 1, the optimal solution can be written as (omitting the z to simplify notation):

$$n_{ij} = \frac{D_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} (w_k f_k L_k / P_i)}{\gamma r_{ij}} + \frac{\sum_k \frac{C_{jk}}{r_{ik}} (\lambda_{ik} - \chi_{ik})}{\gamma r_{ij}} \tag{40}$$

where $D_j = \sum_k C_{jk} \mu_k$ is the Diversification Index of destination j. In the case of an interior solution, we have that:

$$n_{ij}(z) = \frac{D_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} (w_k f_k L_k / P_i)}{\gamma r_{ij}} \tag{41}$$

and therefore both the probability of entering j (i.e. the probability that $n_{ij}(z) > 0$) and the level of exports to j,

$$x_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) \tag{42}$$

are increasing in D_j. When instead there is at least one binding constraint (either the firm sets $n_{ik}(z) = 0$ or $n_{ik}(z) = 1$ for at least one k), then the corresponding Lagrange multiplier will be positive. Therefore:

$$\frac{\partial n_{ij}(z)}{\partial D_j} = \underbrace{\frac{1}{\gamma r_{ij}}}_{\text{direct effect}} + \underbrace{\frac{1}{\gamma r_{ij}} \left[\sum_{k \neq j} \frac{C_{jk}}{r_{ik}} \frac{\partial \lambda_{ik}}{\partial D_j} - \sum_{k \neq j} \frac{C_{jk}}{r_{ik}} \frac{\partial \chi_{ik}}{\partial D_j} \right]}_{\text{indirect effect}} \tag{43}$$

Note that λ_{ik} is zero if $n_{ik}(z) > 0$, otherwise it equals:

$$\lambda_{ik} = -\mu_k r_{ik} + \gamma r_{ik} \sum_{s \neq j} n_{is} r_{is} Cov(\alpha_k, \alpha_s) + w_k f_k L_k / P_i$$

and therefore

$$\frac{\partial \lambda_{ik}}{\partial D_j} = \gamma r_{ik} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial D_j} r_{is} Cov(\alpha_k, \alpha_s) \tag{44}$$

Similarly for the other Lagrange multiplier:

$$\chi_{ik} = \mu_k r_{ik} - \gamma r_{ik} \sum_{s \neq j} n_{is} r_{is} Cov(\alpha_k, \alpha_s) - \gamma r_{ik}^2 Var(\alpha_k) - w_k f_k L_k / P_i$$

and thus:

$$\frac{\partial \chi_{ik}}{\partial D_j} = -\gamma r_{ik} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial D_j} r_{is} Cov(\alpha_k, \alpha_s) = -\frac{\partial \lambda_{ik}}{\partial D_j} \tag{45}$$

Now notice that either $\chi_{ik} > 0$ and $\lambda_{ik} = 0$, or $\lambda_{ik} > 0$ and $\chi_{ik} = 0$. Combining this fact with equations (44) and (45), equation (43) becomes:

$$\frac{\partial n_{ij}(z)}{\partial D_j} = \frac{1}{\gamma r_{ij}} \left[1 + \gamma \sum_{k \neq j} C_{jk} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial D_j} r_{is} Cov(\alpha_k, \alpha_s) \right] \tag{46}$$

Define $x_j \equiv \frac{\partial n_{ij}(z)}{\partial D_j} \gamma r_{ij}$. Since $\gamma r_{ij} > 0$, the sign of x_j equals the sign of the derivative of interest, $\frac{\partial n_{ij}(z)}{\partial D_j}$. Then the above can be written as:

$$x_j = 1 + \sum_{k \neq j} C_{jk} \sum_{s \neq j} x_s Cov(\alpha_k, \alpha_s)$$

This is a linear system of N equations in N unknowns, x_j. We can rewrite it as $AX = B$, where A is the following NxN matrix:

$$A = \begin{bmatrix} 1 & -\sum_{k \neq 1} C_{1k} Cov(\alpha_k, \alpha_2) & \dots & -\sum_{k \neq 1} C_{1k} Cov(\alpha_k, \alpha_N) \\ -\sum_{k \neq 2} C_{2k} Cov(\alpha_k, \alpha_1) & 1 & \dots & -\sum_{k \neq 2} C_{2k} Cov(\alpha_k, \alpha_N) \\ \dots & \dots & \dots & \dots \\ -\sum_{k \neq N} C_{Nk} Cov(\alpha_k, \alpha_1) & -\sum_{k \neq N} C_{2k} Cov(\alpha_k, \alpha_2) & \dots & 1 \end{bmatrix},$$

that is

$$A_{ij} = \begin{cases} -\sum_{k \neq i} C_{ik} Cov(\alpha_k, \alpha_j), & i \neq j \\ 1 & , & i = j \end{cases}.$$

and B is a $Nx1$ vector of ones. It follows that

$$X = A^{-1} B.$$

Since B is a positive vector, in order to have X positive, it is sufficient that A^{-1} is a non-negative matrix. By Theorem 2.3. in chapter 6 of Berman and Plemmons (1994) (see also Pena (1995)), a necessary and sufficient condition for A^{-1} to be non-negative is A being a M-matrix, i.e. all off-diagonal elements are negative. ∎

A.3 Proof of Proposition 3

In Lemma 1 in Online Appendix B.3, I solve for the firm problem under autarky. The Lemma states that, if $f > \frac{P_A}{L} \frac{\mu^2}{4Var(\alpha)\gamma}$, then the optimal solution to the firm problem is:

- $n(z) = 0$ if $z \leq z^*$
- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{D_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{fP_A^{1-\sigma}\sigma}{\mu Y_A}\right)^{\frac{1}{\sigma-1}}$$

and

$$r(z) = \frac{1}{P_A}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma} \frac{Y_A}{P_A^{1-\sigma}\sigma}.$$

Plugging in the general equilibrium solution for P_A given in equation (B.73) in Online Appendix B.3, and inverting, the lower bound on f becomes:

$$f > \left(\frac{1}{2}\frac{\mu}{Var(\alpha)\gamma}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\sigma\right)^{\frac{\theta}{1-\sigma}}\mu\left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]\right)^{\frac{1-\sigma}{(1+\theta)(\sigma-1)-\theta}}\left(\frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)}L\right)^{\frac{-\theta}{(1+\theta)(\sigma-1)-\theta}}\left(\frac{\mu^2}{4LVar(\alpha)\gamma}\right)^{\frac{(1+\theta)(\sigma-1)}{(1+\theta)(\sigma-1)-\theta}}$$

In Lemma 2 in Online Appendix B.4, I solve for the firm problem under free-trade. The Lemma states that, if $f > \frac{P_{FT}}{L} \frac{\mu^2}{4Var(\alpha)(1+\rho)\gamma}$, then the optimal solution to the firm problem is:

- $n_{ij} = 0$ if $z \leq z^*$
- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{D_{FT}}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{fLP_{FT}^{1-\sigma}\sigma}{\mu Y_{FT}}\right)^{\frac{1}{\sigma-1}}$$

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and

$$r(z) = \frac{1}{P_{FT}} \left(\frac{\sigma}{\sigma - 1} \frac{1}{z} \right)^{1-\sigma} \frac{Y_{FT}}{P_{FT}^{1-\sigma} \sigma}.$$

Plugging in the general equilibrium solution for P_{FT} given in equation (B.82) in Online Appendix B.4, and inverting, the lower bound on f becomes:

$$f > \left(\frac{1}{2} \frac{\mu}{Var(\alpha)(1+\rho)\gamma} \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \sigma \right)^{\frac{\theta}{1-\sigma}} \mu \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right] \right)^{\frac{1-\sigma}{(1+\theta)(\sigma-1)-\theta}} \left(\frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) - \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)} L \right)^{\frac{-\theta}{(1+\theta)(\sigma-1)-\theta}} \left(\frac{\mu^2}{4LVar(\alpha)(1+\rho)\gamma} \right)^{\frac{(1+\theta)(\sigma-1)}{(1+\theta)(\sigma-1)-\theta}}$$

I combine the two equations above to state that if

$$f > \left(\frac{\mu^2}{2Var(\alpha)\gamma} \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \sigma \right)^{\frac{\theta}{1-\sigma}} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right] \right)^{\frac{1-\sigma}{(1+\theta)(\sigma-1)-\theta}} \left(\frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) L}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) - \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)} \right)^{\frac{-\theta}{(1+\theta)(\sigma-1)-\theta}} \left(\frac{\mu^2}{4LVar(\alpha)\gamma} \right)^{\frac{(1+\theta)(\sigma-1)}{(1+\theta)(\sigma-1)-\theta}} \left(max\left\{ \frac{1}{1+\rho}, 1 \right\} \right)^{\frac{(\sigma-1)\theta}{(1+\theta)(\sigma-1)-\theta}}$$

then

$$n_{ij}(z) = 0 \qquad if\ z \le z^*$$

$$n_{ij}(z) = \frac{D_j}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right)}{r_i(z)} < 1\ if\ z > z^* \quad \blacksquare$$

A.4 Proof of Proposition 4

To compare the welfare gains from trade in my model to ACR, I first write welfare as a function of domestic trade shares. These equal:

$$\lambda_{jj} = \frac{M_j \mu \int_{z^*}^{\infty} q_{jj}(z) p_{jj}(z) \theta z^{-\theta-1} dz}{w_j \tilde{L}_j + \Pi_j} = \delta_j P_j^{1+\theta} Y_j^{\frac{\theta-\sigma+1}{\sigma-1}} \tag{47}$$

where $\delta_j = M_j \mu \frac{D}{\gamma} \sigma \frac{\sigma-1}{\theta+\sigma-1} (x)^{\frac{\theta}{1-\sigma}}$ and $x \equiv \left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \frac{\sigma f L}{\mu}$. Substituting for Y_j from equation (B.72) in Online Appendix B.3 (which is the same as equation (B.81) in Online Appendix B.4) and rearranging:

$$P_j = \left(\frac{\lambda_{jj}}{\delta_j (\tilde{\chi} L)^{\frac{\theta-\sigma+1}{\sigma-1}}} \right)^{\frac{1}{\theta+1}} \tag{48}$$

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where $\tilde{\chi} \equiv \frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)}$. The welfare gains from trade are:

$$\hat{W}_j = \frac{W_{FT}}{W_A} = \frac{\frac{w_{FT}}{P_{FT}}\left(\tilde{L}+M_{FT}\right)}{\frac{w_A}{P_A}\left(\tilde{L}+M_A\right)} = \frac{P_A}{P_{FT}}$$

where in the last equality I have used the fact that wages are normalized to 1 and that $M_A = M_{FT}$ from equations (B.74) in Online Appendix B.3 and (B.83) in Online Appendix B.4. Using equation (48)

$$\hat{W}_j = \left(\hat{\lambda}_{jj}\right)^{-\frac{1}{\theta+1}}\left(\frac{M_{FT}\mu\frac{D_{FT}}{\gamma}\sigma\frac{\sigma-1}{\theta+\sigma-1}(x)^{\frac{\theta}{1-\sigma}}}{M_A\mu\frac{D_A}{\gamma}\sigma\frac{\sigma-1}{\theta+\sigma-1}(x)^{\frac{\theta}{1-\sigma}}}\right)^{\frac{1}{\theta+1}} =$$

$$= \left(\hat{\lambda}_{jj}\right)^{-\frac{1}{\theta+1}}\left(\frac{D_{FT}}{D_A}\right)^{\frac{1}{\theta+1}} =$$

$$= \left(\hat{\lambda}_{jj}\right)^{-\frac{1}{\theta+1}}\left(1+\rho\right)^{-\frac{1}{\theta+1}}$$

Taking logs

$$ln\left(\hat{W}_j\right) = \underbrace{-\frac{1}{\theta}\ln\left(\hat{\lambda}_{jj}\right)}_{\text{standard gains}} \cdot \underbrace{\frac{\theta}{\theta+1}}_{\text{variance effect}} - \underbrace{\frac{1}{\theta+1}\ln\left(1+\rho\right)}_{\text{diversification effect}} \quad \blacksquare$$

A.5 Estimation Results

A.5.1 Diversification Index

Table A.1: Summary statistics

	Covariance	Variance	Mean
Median	-0.04	3.35	1.55
St. dev.	0.3	4.48	0.22
Min	-1.73	1.01	1.27
Max	2.49	24.85	2.35

Notes: The first column reports summary statistics about the estimated covariances (sample of 1,190 pairs of countries), the second column about the variances (sample of 35 countries), the third column about the means (sample of 35 countries).

Table A.2: Diversification Index

Australia	Czech Rep.	Ireland	Netherlands	South Africa
0.99 (0.05)	0.57 (0.03)	0.83 (0.05)	0.77 (0.04)	0.68 (0.04)
Austria	Denmark	Israel	Norway	South Korea
0.68 (0.04)	0.55 (0.04)	0.70 (0.04)	0.83 (0.05)	0.47 (0.03)
Benelux	Finland	Italy	Poland	Spain
0.87 (0.05)	0.93 (0.06)	0.53 (0.04)	0.55 (0.03)	0.84 (0.05)
Brazil	France	Japan	Portugal	Sweden
0.46 (0.03)	1.04 (0.06)	0.60 (0.03)	0.76 (0.05)	0.76 (0.04)
Canada	Germany	Malaysia	Romania	Turkey
0.64 (0.04)	0.83 (0.05)	0.21 (0.01)	0.46 (0.03)	0.38 (0.02)
Chile	Greece	Mexico	Russia	United King.
1.13 (0.04)	0.94 (0.05)	0.52 (0.03)	0.49 (0.04)	0.91 (0.05)
China	Hungary	Morocco	Singapore	United States
0.19 (0.02)	0.82 (0.03)	1.12 (0.05)	0.52 (0.03)	1.06 (0.06)

Notes: The table reports the estimated Diversification Index for each of the 35 countries in the sample. Bootstrapped standard errors are reported in parenthesis.

A.5.2 Estimation of γ

To estimate the risk aversion parameter γ, I directly take to the data the firms' first order condition. Assuming for simplicity that marketing costs are sufficiently high so that there is no Portuguese firm selling to the totality of consumers in any country (and thus $\mu_j(z) = 0$ for all j and z), for each destination j where firm z is selling to (and thus for which $\lambda_j(z) = 0$), the FOC can be written as:

$$\mu_j r_j(z) = w_j f_j L_j / P_j + \gamma \sum_s r_j(z) n_s(z) r_s(z) Cov(\alpha_j, \alpha_s) \tag{49}$$

where the left hand side is the *marginal* expected benefit of adding one additional consumer in country j, while the right hand side is its marginal cost, which is the sum of the marketing costs plus the *marginal* variance. Intuitively, the higher γ, the more entrepreneurs want to be compensated for taking additional risk (i.e. selling to an additional consumer in country j), and thus higher marginal variance of profits must be associated with higher marginal expected profits.

Both the marginal expected profits and their marginal variance are not observable in the data. However, I do observe gross profits from each destination, since they are a share σ of observed revenues, due to the CES assumption. The ratio between marginal expected gross profits, $MEGP_{js}$, and observed gross profits, GP_{js}, for firm s and market j, is given

by:

$$\frac{MEGP_{js}}{GP_{js}} = \frac{\mu_j r_{js}}{\alpha_{js} n_{js} r_{js}} = \frac{\mu_j}{\alpha_{js} n_{js}} \tag{50}$$

Taking logs and rearranging:

$$ln\left(MEGP_{js}\right) = ln\left(GP_{js}\right) + ln\left(\mu_j\right) - ln\left(\alpha_{js}\right) - ln\left(n_{js}\right) \tag{51}$$

Note that, for the estimation of the Diversification Index, I have already estimated $ln\left(\mu_j\right)$ and backed out $ln\left(\alpha_{js}\right) - ln\left(n_{js}\right)$, which is just the residual from equation (25). Since I directly observe $ln\left(GP_{js}\right)$ in the data, I immediately back out $MEGP_{js}$ for all firms and destinations in 2005.

To compute the marginal variance, note that the total covariance of gross profits in country j, i.e. the variance in j plus the covariances with all other countries, equals:

$$CGP_{js} = \sum_k Cov(GP_{js}, GP_{ks}) = \sum_k r_{js} r_{ks} n_{js} n_{ks} Cov(\alpha_j, \alpha_k) \tag{52}$$

I use data on observed gross profits from 1995 to 2004, for each firm-destination pair, to compute the left hand side of equation (52):

$$CGP_{js} = \sum_k \frac{1}{T} \sum_{t=1}^T \left(GP_{kst} - E[GP_{ks}]\right)\left(GP_{jst} - E[GP_{js}]\right) \tag{53}$$

I then compute r_{js} in equation (52) as $\frac{MEGP_{js}}{\mu_j}$ for all s and j, and since I have already estimated $Cov(\alpha_j, \alpha_k)$, I only need to solve for the vector n_{sj}. Thus, for each firm s selling to G markets, I solve a system of G equations (52) in G unknowns, i.e. $n_{gs}(z)$, for $g = 1, ...G$.

Finally, I compute the marginal variance of gross profits as

$$MVGP_{js} = \sum_k r_{js} n_{ks} r_{ks} Cov(\alpha_j, \alpha_k) \tag{54}$$

With the marginal variance of gross profits and the marginal expected gross profits for each pair of Portuguese firm and destination, I can implement the FOC above as a simple OLS regression:

$$MEGP_{js} = d_j + \gamma \cdot MVGP_{js} + \varepsilon_{js} \tag{55}$$

Table A.3: Risk aversion

| | Marginal expected gross profits | | |
	(1)	(2)	(3)
Marginal variance of gross profits	0.415***	0.282***	0.413***
	(0.106)	(0.052)	(0.065)
Observations	16,017	12,546	11,570
R^2	0.413	0.467	0.489
Destination FE	Y	Y	Y

Notes: Sample of 4,821 Portuguese firms. Col. (1) uses the baseline sample, col. (2) excludes sales to Portugal, col. (3) includes only established exporters. All regressions are weighted by firms' total sales in 2005. Robust standard errors, clustered at the destination level, are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

where $d_j \equiv w_j f_j L_j / P_j$ is a destination fixed effect that controls for country-specific real marketing costs, and ε_{js} is simple econometric error.

Table A.3 reports the results of the regression. Column (1) shows that there is a positive and statistically significant relationship between the expected marginal profits and their variance, with a risk-aversion parameter of 0.415. The regression is weighted by firms' total sales in 2005, both to minimize the influence of outliers and to estimate the appropriate average effect for the counterfactual analysis. The point estimate is a bit lower, 0.282, if I exclude domestic sales in column (2), and it remains the same if the sample includes only established exporters in column (3).

A.5.3 Calibration of trade costs
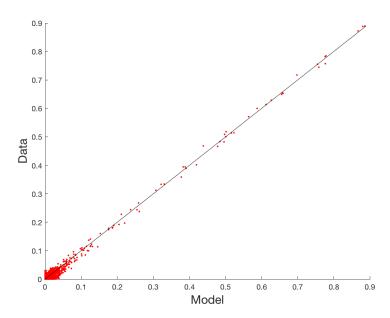
To calibrate trade costs, I implement the following Simulated Method of Moments algorithm:

1) Guess a matrix of trade costs τ_{ij} (normalizing domestic trade costs to 1). Stack them into the vector Θ.

2) Solve the trade equilibrium using the system of equations (15)-(19). To solve the general equilibrium model, I create a grid composed by 10,000 firms, each with a given productivity z, and compute the optimal $n_{ij}(z)$ for all firms and countries. By Walras' Law, one equation is redundant. Therefore, I normalize world GDP to a constant.

3) Compute manufacturing trade shares, $\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}}$, for $i \neq j$, where X_{ij} are total trade flows from i to j. I stack these trade shares in a $N(N-1)$-element vector $\hat{m}(\Theta)$ and compute the analogous moment in the data, m^{data}, using manufacturing international trade data for 2005.

4) Stack the differences between observed and simulated moments into a vector of length 1,190, $y(\Theta) \equiv m^{data} - \hat{m}(\Theta)$. I update Θ as $\Theta^{new} = \Theta + \varepsilon y(\Theta)$, where ε is arbitrarily small.

5) Iterate over 1-4 until $\max \{y(\Theta)\} < tol$, for tol sufficiently small.

The results from the Simulated Method of Moments are displayed in Figure A.1.

Figure A.1: Calibration of trade shares

Notes: The graph plots the trade shares of the calibrated model against the corresponding shares observed in the data in 2005.

B Online Appendix for "Demand Risk and Diversification through International Trade"

B.1 Alternative production structure

In this section I solve the problem of the firm under the assumption that the firm makes also the production decision under uncertainty, i.e. it pre-commits to production in the first stage. In such case, after the demand shocks are realized, the firm adjusts the price in each location such that the amount of goods already produced for each location exactly equal the realized demand. Specifically, there are three stages:

i. The firm maximizes the expected utility by choosing $n_{ij}(E[p_{ij}(z)], z)$, where $E[p_{ij}(z)]$ is the expectation of the price it will charge after the shock is realized. This stage is solved exactly in the same way as in Proposition 1.

ii. Given the set of destinations and the number of consumers chosen in the first stage, $n_{ij}(E[p_{ij}(z)], z)$, the firm hires workers to produce the corresponding quantity, still under uncertainty. However, since the firm has already decided the locations in which to sell to, the firm simply has to maximize expected gross profits in each destination separately:

$$\max_{\{p_{ij}\}} \sum_j \mu_j \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right)$$

which leads to

$$\tilde{p}_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \tag{B.56}$$

Thus the quantity produced at this stage is:

$$q_{ij}^{produced} = \mu_j \frac{\tilde{p}_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j \tag{B.57}$$

iii. The firm learns the demand shocks α_j in the countries it entered, and adjusts the price so that

$$q_{ij}^{produced} = q_{ij}^{realized} \tag{B.58}$$

in each destination j. Thus, the firm cannot reallocate resources across locations after the shocks, because it has already produced and paid workers, but it will only change the price.

Given equation (B.57), the above becomes

$$\mu_j \frac{\tilde{p}_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j = \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j$$

where $n_{ij}(E[p_{ij}(z)], z)$ is chosen in the first stage and cannot be changed. Then we have:

$$\mu_j \tilde{p}_{ij}(z)^{-\sigma} = \alpha_j(z) p_{ij}(z)^{-\sigma}$$

This implies the following final price

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \left(\frac{\alpha_j(z)}{\mu_j} \right)^{\frac{1}{\sigma}} \tag{B.59}$$

Intuitively, the higher the realized demand shock relative to the expected shock, the higher is the effective price charged, in order to extract more revenues from the higher demand.

By backward induction, in the first stage the firm takes the expectation over the price it will charge in the third stage, which is:

$$E[p_{ij}(z)] = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \tilde{\alpha}_j$$

where $\tilde{\alpha}_j \equiv E\left[(\alpha_j(z))^{\frac{1}{\sigma}} \right] (\mu_j)^{-\frac{1}{\sigma}}$. I assume that $E\left[(\alpha_j(z))^{\frac{1}{\sigma}} \right]$ is a finite moment. Therefore we have that:

$$r_{ij}(z) = \frac{1}{P_i} \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \tilde{\alpha}_j \frac{Y_j}{P_j^{1-\sigma} \sigma}$$

and $n_{ij}(z)$ is still given by:

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(r_{ik} \mu_k - w_k f_k L_k / P_i - \chi_{ik} + \lambda_{ik} \right)}{\gamma r_{ij}}.$$

B.2 Model with risk neutrality

With risk neutrality, I can set $\gamma = 0$ in the indirect utility in equation (1), and thus the entrepreneur's objective function becomes simply:

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$$\max_{\{n_{ij}\}} \sum_j \mu_j n_{ij}(z) r_{ij}(z) - \sum_j w_j n_{ij}(z) f_j L_j / P_i$$

Notice that the above is linear in $n_{ij}(z)$, and therefore it is always optimal, upon entry, to set $n_{ij}(z) = 1$. Therefore the firm's problem becomes a standard entry decision, as in Melitz (2003), where a firm with productivity z enters market j if and only if $z > \bar{z}_{ij}$, where the cutoff satisfies:

$$(\bar{z}_{ij})^{\sigma-1} = \frac{w_j f_j L_j P_j^{1-\sigma} \sigma}{\mu_j \left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i\right)^{1-\sigma} Y_j}. \tag{B.60}$$

To find the welfare gains from trade, I first write the equation for trade shares

$$\lambda_{ij} = \frac{M_i \int_{\bar{z}_{ij}}^\infty \mu_j p_{ij}(z) q_{ij}(z) g_i(z) dz}{w_j L_j} = \frac{M_i \int_{\bar{z}_{ij}}^\infty \mu_j p_{ij}(z)^{1-\sigma} g_i(z) dz}{P_j^{1-\sigma}} \tag{B.61}$$

Inverting the above and writing it for $i = j$, we get:

$$P_j^{1-\sigma} = \frac{M_j \phi_1(w_j)^{1-\sigma} (\bar{z}_{jj})^{\sigma-\theta-1}}{\lambda_{jj}}. \tag{B.62}$$

where ϕ_1 is a constant. Substituting for the cutoff, and using the fact that M_j is proportional to L_j under free-entry and Pareto (as in Arkolakis et al. (2008) and Arkolakis and Esposito (2014)), we obtain

$$P_j^{1-\sigma} = \frac{L_j \phi_2(w_j)^{1-\sigma} \left(\frac{w_j f_j L_j P_j^{1-\sigma} \sigma}{\mu_j \left(\frac{\sigma}{\sigma-1} w_j\right)^{1-\sigma} Y_j}\right)^{\frac{\sigma-\theta-1}{\sigma-1}}}{\lambda_{jj}} \tag{B.63}$$

where ϕ_2 is a constant. The free-entry condition and current account balance imply that $Y_j = L_j w_j$, thus we can rearrange the above as:

$$\lambda_{jj} = \left(\frac{w_j}{P_j}\right)^{-\theta} L_j \phi_2 \left(\frac{f_j \sigma}{\mu_j \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}\right)^{\frac{\sigma-\theta-1}{\sigma-1}} \tag{B.64}$$

which implies

$$\left(\frac{w_j}{P_j} \right) = \vartheta \lambda_{jj}^{-\frac{1}{\theta}}, \qquad (B.65)$$

where ϑ is a constant. Therefore welfare gains from trade are

$$\ln \left(\hat{W}_j \right) = -\frac{1}{\theta} \ln \left(\hat{\lambda}_{jj} \right).$$

B.3   Model with autarky

In this section, I assume that the home country cannot trade with the other country. I first solve the firm problem (Lemma 1), and then I analytically solve for all the endogenous variables in general equilibrium.

Lemma 1. *Assume that the home country is under autarky. Assume that $f > \frac{P}{L} \frac{\mu^2}{4 Var(\alpha) \gamma}$. Then the optimal solution to the firm problem is:*

- $n(z) = 0$ if $z \leq z^*$

- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{D_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \frac{f P_A^{1-\sigma} \sigma}{\mu Y_A} \right)^{\frac{1}{\sigma-1}}$$

and

$$r(z) = \frac{1}{P_A} \left(\frac{\sigma}{\sigma-1} \frac{1}{z} \right)^{1-\sigma} \frac{Y_A}{P_A^{1-\sigma} \sigma}.$$

    Proof. I omit the autarky subscript for simplicity. As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1} \frac{1}{z}$$

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and thus real gross profits are:

$$r(z) = \frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$$

The Lagrangian is:

$$\mathcal{L}_i(z) = \mu n(z)r(z) - \frac{\gamma}{2}Var(\alpha)n^2(z)r^2(z) - n(z)fL/P + \lambda n(z) + \chi(1-n(z))$$

and the FOCs are:

$$\mu r(z) - fL/P - \gamma n(z)r^2(z)Var(\alpha) + \lambda - \chi = 0$$

Note that n can be either 0, 1 or $0 < n < 1$. Consider first the interior solution. Setting $\lambda = \chi = 0$, it holds that

$$n(z) = \frac{\mu r(z) - fL/P}{r^2(z)Var(\alpha)\gamma}$$

If $n = 0$, then and $\chi = 0$, and

$$\lambda = -\mu r(z) + fL/P > 0$$

and thus it must hold that

$$fL/P > \mu r(z)$$

We can find the productivity cutoff such that this is zero:

$$fL/P = \mu\frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z^*}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$$

and thus

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fLP^{1-\sigma}\sigma}{\mu Y}\right)^{\frac{1}{\sigma-1}}$$

All $z < z^*$ will optimally set $n = 0$, while all $z \geq z^*$ will have $n > 0$. Consider the case $n = 1$, then $\lambda = 0$ and

$$\mu r(z) - fL/P - \gamma r^2(z)Var(\alpha) = \chi$$

We can find the productivity cutoff such that this is zero:

$$\mu\kappa(z)^{\sigma-1} - fL/P - \gamma Var(\alpha)\kappa^2(z)^{2(\sigma-1)} = 0 \tag{B.66}$$

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where $\kappa \equiv \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. The above equation is a quadratic polynomial in $z^{\sigma-1}$. For tractability (i.e. in order to solve for the entire general equilibrium model), I impose a restriction on marketing costs f such that it is always optimal to choose $n(z) < 1$. When the optimal solution is $n = 0$, then this holds trivially. If instead $n > 0$, and thus $\lambda = 0$, then it must hold that:

$$n(z) = \frac{\mu r(z) - fL/P}{r^2(z)Var(\alpha)\gamma} < 1$$

Rearranging:

$$f > \frac{b(z)}{L} \left(\mu - \frac{1}{P}b(z)Var(\alpha)\gamma \right) \tag{B.67}$$

where $b(z) = \left(\frac{\sigma}{\sigma-1} \frac{1}{z} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max} = \left(\frac{\mu}{2Var(\alpha)\tilde{u}\gamma} \right)^{\frac{1}{\sigma-1}} \tag{B.68}$$

where $\tilde{u} = \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have (B.67) is:

$$f > \frac{b(z_{max})}{L} \left(\mu - \frac{1}{P}b(z_{max})Var(\alpha)\gamma \right)$$

or

$$f > \frac{P}{L} \frac{\mu^2}{4Var(\alpha)\gamma} \tag{B.69}$$

If this holds, then any firm will always choose to set $n_{ij}(z) < 1$. Then, the FOC becomes:

$$\mu r(z) - fL/P - \gamma n(z)r^2(z)Var(\alpha) + \lambda = 0$$

I now guess and verify that the optimal $n(z)$ is such that: if $z > z^*$ then $n(z) > 0$, otherwise $n(z) = 0$. First I find such cutoff by solving $n(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \frac{fLP^{1-\sigma}\sigma}{\mu Y} \right)^{\frac{1}{\sigma-1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{\mu}{\gamma Var(\alpha)} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:
$$\mu r(z) - fL + \lambda = 0$$

and so the multiplier is:

$$\lambda = fL - \mu r(z)$$

which is positive only if $fL > \mu r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written more compactly as:

$$n(z) = \frac{D_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where $D_A \equiv \frac{\mu}{Var(\alpha)}$ is the Diversification Index.∎

Equilibrium. Given the solution for $n(z)$ from Lemma 1, I now solve for all the endogenous variables that constitute the general equilibrium. Throughout this section I assume that $\theta > \sigma - 1 > 0$, and normalize the wage to 1. First, the indifference condition between the expected utility from being an entrepreneur and the expected utility of being a worker (equation 16), becomes

$$\frac{\mu D_A}{\gamma} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{fLP_A^{1-\sigma}\sigma}{\mu Y_A} \right)^{\frac{-\theta}{\sigma-1}} \left[\left(\frac{\sigma-1}{\theta+\sigma-1} - \left(\frac{\theta}{\theta+\sigma-1} - \frac{\theta}{\theta+2\sigma-2} \right) \right) - \frac{1}{2} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma} \right) \right] = \frac{1}{P_A}$$

Rearranging to find the price index:

$$P_A = (\zeta)^{-\frac{1}{1+\theta}} Y_A^{\frac{\theta}{(1-\sigma)(1+\theta)}} \tag{B.70}$$

where $\zeta \equiv \frac{D_A}{2\gamma} (x)^{\frac{\theta}{1-\sigma}} \mu \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma fL}{\mu}$. The current account balance is

$$Y_A = \tilde{L} + M_A \kappa_1 P_A^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}} \tag{B.71}$$

where $\kappa_1 \equiv \frac{D_A}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \mu \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right]$.
The price index equation is:

$$P_A^{1-\sigma} = \mu M_A \int_{z^*}^{\infty} n(z)p(z)^{1-\sigma}\theta z^{-\theta-1}dz =$$

$$= Y_A^{\frac{-\theta-1+\sigma}{1-\sigma}} P_A^{2-\sigma+\theta} M_A \kappa_2$$

where $\kappa_2 \equiv \mu \frac{D_A \sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1} \right)$, which can be rearranged as

$$P_A^{1+\theta} = Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} / (M_A \kappa_2)$$

Combining the above equations, some algebra give that

$$Y_A = \tilde{\chi}L \tag{B.72}$$

where $\tilde{\chi} \equiv \frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)}$. The price index is

$$P_A = (\zeta)^{-\frac{1}{1+\theta}} (\tilde{\chi}L)^{\frac{\theta}{(1-\sigma)(1+\theta)}} \tag{B.73}$$

and the equilibrium number of entrepreneurs is

$$M_A = \phi L \tag{B.74}$$

where $\phi = \frac{\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)} \leq 1$.

B.4 Two-country model with free trade

In this section, I assume that the home country can trade with the other symmetric country, and the trade costs are set to 1. I first solve the firm problem (Lemma 2), and then I analytically solve for all the endogenous variables in general equilibrium.

Lemma 2. *Assume countries are perfectly symmetric and there is free trade. Assume that $f > \frac{P_{FT}}{L} \frac{\mu^2}{4Var(\alpha)(1+\rho)\gamma}$. Then the optimal solution of the firm problem is:*
- $n_{ij} = 0$ if $z \leq z^*$
- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{D_{FT}}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \frac{fLP_{FT}^{1-\sigma}\sigma}{\mu Y_{FT}} \right)^{\frac{1}{\sigma-1}}$$

and

$$r(z) = \frac{1}{P_{FT}} \left(\frac{\sigma}{\sigma-1} \frac{1}{z} \right)^{1-\sigma} \frac{Y_{FT}}{P_{FT}^{1-\sigma}\sigma}.$$

Proof: I omit the free-trade subscript for simplicity. As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1} \frac{1}{z}$$

and thus real gross profits are:

$$r_{ij}(z) = \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \frac{1}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}\sigma}$$

In the first stage, the FOCs are:

$$\mu_j r_{ij}(z) - \gamma \sum_s r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s) - w_j f_j L_j / P_i + \lambda_{ij} - \chi_{ij} = 0$$

Imposing symmetry, and that $w = 1$:

$$\mu r(z) - \gamma r(z)^2 n(z) Var(\alpha) (1+\rho) - fL/P + \lambda - \chi = 0$$

Note that n can be either 0, 1 or $0 < n < 1$. Consider first the interior solution. Setting $\lambda = \chi = 0$, it holds that

$$n(z) = \frac{\mu r(z) - fL/P}{r^2(z) Var(\alpha) (1+\rho) \gamma}.$$

If $n = 0$, then $\chi = 0$, and

$$\lambda = -\mu r(z) + fL/P > 0$$

and thus it must hold that

$$fL/P > \mu r(z)$$

We can find the productivity cutoff such that this is zero:

$$fL/P = \mu \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \frac{1}{z^*} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$$

and thus

$$z^* = \left(\left(\frac{\sigma}{\sigma - 1} \right)^{\sigma - 1} \frac{fLP^{1-\sigma}\sigma}{\mu Y} \right)^{\frac{1}{\sigma - 1}}$$

All $z < z^*$ will optimally set $n = 0$, while all $z \geq z^*$ will have $n > 0$.

Consider the case $n = 1$, then $\lambda = 0$ and

$$\mu r(z) - fL/P - \gamma r^2(z) Var(\alpha) (1 + \rho) = \chi$$

We can find the productivity cutoff such that this is zero:

$$\mu \kappa (z)^{\sigma - 1} - fL/P - \gamma Var(\alpha) (1 + \rho) \kappa^2 (z)^{2(\sigma - 1)} = 0 \qquad \text{(B.75)}$$

where $\kappa \equiv \frac{1}{P} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. The above equation is quadratic in $z^{\sigma - 1}$, which means that it admits two solutions, z_1^* and z_2^*. To find an analytical solution for the full problem, I impose a restriction on marketing costs f such that it is always optimal to choose $n(z) < 1$. When the optimal solution is $n = 0$, then this holds trivially. If instead $n > 0$, and thus $\lambda = 0$, then it must hold that:

$$n(z) = \frac{\mu r(z) - fL/P}{r^2(z) Var(\alpha) (1 + \rho) \gamma} < 1$$

Rearranging:

$$f > \frac{b(z)}{L} \left(\mu - \frac{1}{P} b(z) Var(\alpha) (1 + \rho) \gamma \right) \qquad \text{(B.76)}$$

where $b(z) = \left(\frac{\sigma}{\sigma - 1} \frac{1}{z} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max} = \left(\frac{\mu}{2 Var(\alpha) (1 + \rho) \tilde{u} \gamma} \right)^{\frac{1}{\sigma - 1}} \qquad \text{(B.77)}$$

where $\tilde{u} = \frac{1}{P} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have B.76 is:

$$f > \frac{b(z_{max})}{L} \left(\mu - \frac{1}{P} b(z_{max}) Var(\alpha) (1 + (N - 1)\rho) \gamma \right)$$

or

$$f > \frac{P}{L} \frac{\mu^2}{4 Var(\alpha) (1 + \rho) \gamma} \qquad \text{(B.78)}$$

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If this holds, then any firm will always choose to set $n_{ij}(z) < 1$. Then, the FOC becomes:

$$\mu r(z) - fL/P - \gamma n(z)r^2(z)Var(\alpha)(1+\rho) + \lambda = 0$$

I now guess and verify that the optimal $n(z)$ is such that: if $z > z^*$ then $n(z) > 0$, otherwise $n(z) = 0$. First I find such cutoff by solving $n(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma - 1} \right)^{\sigma - 1} \frac{fLP^{1-\sigma}\sigma}{\mu Y} \right)^{\frac{1}{\sigma - 1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{1}{\gamma} \frac{\mu}{Var(\alpha)(1+\rho)} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma - 1} \right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:

$$\mu r(z) - fL + \lambda = 0$$

and so the multiplier is:

$$\lambda = fL - \mu r(z)$$

which is positive only if $fL > \mu r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written more compactly as:

$$n(z) = \frac{D_{FT}}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z} \right)^{\sigma - 1} \right)}{r(z)}$$

where $D_{FT} \equiv \frac{\mu}{Var(\alpha)(1+\rho)}$ is the Diversification Index.∎

Equilibrium. Given the solution for $n(z)$ from Lemma 2, I now solve for all the endogenous variables that constitute the general equilibrium. Throughout this section I assume that $\theta > \sigma - 1 > 0$, and normalize the wage to 1. The indifference condition (16) can be expanded, imposing symmetry, as:

$$2 \left(\mu \int_{z^*}^{\infty} n(z)r(z)\theta z^{-\theta-1}dz - \int_{z^*}^{\infty} \frac{f}{P_{FT}} n(z)\theta z^{-\theta-1}dz \right) - \frac{\gamma}{2} \int_{z^*} \xi(z)\theta z^{-\theta-1}dz = \frac{1}{P_{FT}}$$

where $\xi(z) \equiv \left(Var(\alpha) \left(\frac{\pi_{HH}(z)}{P_{FT}} \right)^2 + Var(\alpha) \left(\frac{\pi_{HF}(z)}{P_{FT}} \right)^2 + 2\frac{\pi_{HF}(z)}{P_{FT}} \frac{\pi_{HH}(z)}{P_{FT}} Cov(\alpha_H, \alpha_F) \right)$. Some

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algebra gives

$$P_{FT} = (\zeta_2)^{-\frac{1}{1+\theta}} Y_{FT}^{\frac{\theta}{(1-\sigma)(1+\theta)}}$$

where $\zeta_2 \equiv \mu \frac{D_{FT}}{\gamma}(x)^{\frac{-\theta}{\sigma-1}}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)$. The price index equation becomes

$$Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}}/(M_{FT}\kappa_3) = P_{FT}^{1+\theta} \tag{B.79}$$

where $\kappa_3 \equiv \mu 2\frac{D_{FT}\sigma}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. The current account balance becomes

$$Y_{FT} = L - M_{FT} + M_{FT}\kappa_4 P_{FT}^{1+\theta} Y_{FT}^{\frac{\theta}{\sigma-1}} \tag{B.80}$$

where $\kappa_4 \equiv \frac{2D_{FT}}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\mu\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f L}{\mu}$. By combining the three equations above, it is easy to find that

$$Y_{FT} = \tilde{\chi}L \tag{B.81}$$

where $\tilde{\chi} \equiv \frac{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)}$. The price index is

$$P_{FT} = (\zeta_2)^{-\frac{1}{1+\theta}}(\tilde{\chi}L)^{\frac{\theta}{(1-\sigma)(1+\theta)}} \tag{B.82}$$

and the equilibrium number of entrepreneurs is

$$M_{FT} = \phi L \tag{B.83}$$

where $\phi = \frac{\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}}{2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right)} \leq 1$.

B.5   Data Appendix

Trade data. Statistics Portugal collects data on export and import transactions by firms that are located in Portugal on a monthly basis. These data include the value and quantity of internationally traded goods (i) between Portugal and other Member States of the EU (intra-EU trade) and (ii) by Portugal with non-EU countries (extra-EU trade). Data on extra-EU trade are collected from customs declarations, while data on intra-EU trade are collected through the Intrastat system, which, in 1993, replaced customs declarations as the source of trade statistics within the EU. The same information is used for official statistics and, besides small adjustments, the merchandise trade transactions in our dataset aggregate to the official total exports and imports of Portugal. Each transaction record includes, among other information, the firm's tax identifier, an eight-digit Com-

bined Nomenclature product code, the destination/origin country, the value of the transaction in euros, the quantity (in kilos and, in some case, additional product-specific measuring units) of transacted goods, and the relevant international commercial term (FOB, CIF, FAS, etc.). I use data on export transactions only, aggregated at the firm-destination-year level.

Data on firm characteristics. The second main data source, Quadros de Pessoal, is a longitudinal dataset matching virtually all firms and workers based in Portugal. Currently, the dataset collects data on about 350,000 firms and 3 million employees. As for the trade data, I was able to gain access to information from 1995 to 2005. The data is made available by the Ministry of Employment, drawing on a compulsory annual census of all firms in Portugal that employ at least one worker. Each year, every firm with wage earners is legally obliged to fill in a standardized questionnaire. Reported data cover the firm itself, each of its plants, and each of its workers. Variables available in the dataset include the firm's location, industry (at 5 digits of NACE rev. 1), total employment, sales, ownership structure (equity breakdown among domestic private, public or foreign), and legal setting. Each firm entering the database is assigned a unique, time-invariant identifying number which I use to follow it over time.

The two datasets are merged by means of the firm identifier. As in Mion and Opromolla (2014) and Cardoso and Portugal (2005), I account for sectoral and geographical specificities of Portugal by restricting the sample to include only firms based in continental Portugal while excluding agriculture and fishery (Nace rev.1, 2-digit industries 1, 2, and 5) as well as minor service activities and extra-territorial activities (Nace rev.1, 2-digit industries 95, 96, 97, and 99). The analysis focuses on manufacturing firms only (Nace rev.1 codes 15 to 37) because of the closer relationship between the export of goods and the industrial activity of the firm. The location of the firm is measured according to the NUTS 3 regional disaggregation.

Data on firms' profits. I obtain data on firms' net profits, investment rate, capital expenditures from Central de Balanços, a repository of yearly balance sheet data for non financial firms in Portugal.

B.6 Equivalence in the moments estimation

In this section I prove that, if the shocks are i.i.d. over time, computing the moments of the shocks using variation across time for each year firm and taking the average across firms is equivalent to stacking together the observations for all years and computing the moments once.

To save notation, define $X \equiv \Delta\tilde{\alpha}_x$ and $Y \equiv \Delta\tilde{\alpha}_y$, where x and y are any two des-

tinations. The covariance between X and Y, computed stacking together the structural residuals, is:

$$Cov(X, Y) = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} (y_k - \bar{y}) (x_k - \bar{x}) \tag{B.84}$$

where x_k (y_k) is the observed change in the log of the shock in destination $x(y)$ for k, where k is a pair of firm s and year t, S is the number of firms selling to both x and y, T is the number of years. Since $\bar{x} \equiv E[\Delta \tilde{\alpha}_x] = 0$ and $\bar{y} \equiv E[\Delta \tilde{\alpha}_p] = 0$, the above becomes:

$$Cov(X, Y) = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} y_k x_k \tag{B.85}$$

If instead I compute the covariance for each firm, it equals:

$$Cov(X, Y)_s = \frac{1}{T} \sum_{y=1}^{T} y_{ts} x_{ts} \tag{B.86}$$

where x_{ts} (y_{ts}) is the observed change in the log of the shock in destination x (y) in year t and for firm s. The average across firms of this covariance is simply:

$$\frac{1}{S} \sum_{s=1}^{S} Cov(X, Y)_s = \frac{1}{S} \sum_{s=1}^{S} \frac{1}{T} \sum_{t=1}^{T} y_{ts} x_{ts} =$$

$$= \frac{1}{T \cdot S} \sum_{s=1}^{S} \sum_{t=1}^{T} y_{ts} x_{ts} = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} y_k x_k \tag{B.87}$$

by the associative property. Therefore, equation (B.85) is equivalent to equation (B.87). A similar proof works for the expected value of the shocks.

B.7 Proof of equation (27)

Starting from the covariance of the log-change of the shocks:

$$Cov\left(\Delta \tilde{\alpha}_j, \Delta \tilde{\alpha}_k\right) = E\left[\Delta \tilde{\alpha}_j \Delta \tilde{\alpha}_k\right] - E\left[\Delta \tilde{\alpha}_j\right] E\left[\Delta \tilde{\alpha}_k\right] =$$

$$= E\left[\left(\tilde{\alpha}_{j,t} - \tilde{\alpha}_{j,t-1}\right)\left(\tilde{\alpha}_{k,t} - \tilde{\alpha}_{k,t-1}\right)\right] - E\left[\tilde{\alpha}_{j,t} - \tilde{\alpha}_{j,t-1}\right] E\left[\tilde{\alpha}_{k,t} - \tilde{\alpha}_{k,t-1}\right] =$$

$$= Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t}\right) - Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t-1}\right) - Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t}\right) + Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t-1}\right) =$$

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$$= 2Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) - 2Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t}\right) \tag{B.88}$$

where the last equality follows because the covariance is assumed constant over the estimation period, i.e. $Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t}\right) = Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t-1}\right) = Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right)$, and by symmetry of the covariance matrix. By Assumption 3b, $Cov\left(\tilde{\alpha}_{j,t-1}\tilde{\alpha}_{k,t}\right) = 0$ for all j and k. Therefore, the covariance of the log of the shocks is

$$Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) = Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right)/2.$$

B.8   Robustness on the estimation of D_j

I now examine the robustness of the estimates to some of the assumptions made for the empirical methodology just described, and show that they do not significantly affect the results.

No supply shocks. I first investigate how controlling for the firm-destination supply shocks affects the estimates. Since data on capital intensity and investment rates is missing for some firms in the sample, the baseline approach could generate a selection bias. To this end, I re-estimate equation (25) without the controls $\mathbf{Z}_{js}^t$, and compute the Diversification Index as explained above. Reassuringly, Figure B.1 documents that omitting the controls for supply shocks does not significantly alter the estimated Diversification Index.

Firm-level covariances. As discussed in the previous section, an alternative approach would be to compute the covariances for each Portuguese exporter using only time-series variation, and then, for each bilateral pair, compute the average covariance across firms. Figure B.1 highlights that such approach would produce a Diversification Index that is highly correlated with the baseline one.[35]

Aggregate shocks. The baseline empirical methodology rules out the presence of country-specific demand shocks, such as a monetary tightening or exchange rate fluctuations, that affect the sales of all exporters to a certain destination. To take into account for aggregate shocks, I assume that the log demand shocks can be decomposed into a macro component, common to all firms, and a firm-destination component:

$$\tilde{\alpha}_{js} = \tilde{\xi}_j + \tilde{\varsigma}_{js}$$

In the structural equation (23), the macro shock $\tilde{\xi}_j$ is absorbed by the destination fixed effect. In order to back it out, I use as before the assumption that the parameters stay

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[35]Using this approach, however, leads to higher standard errors, since each firm-level covariance is computed with fewer observations. This is the reason why I do not use these estimates as baseline.

constant between two consecutive years, and take the first difference of both the estimated destination fixed effect and the residual (controlling for supply shocks as in equation (25)):

$$\delta_{jt} - \delta_{j,t-1} + \varepsilon_{js,t} - \varepsilon_{js,t-1} = \Delta\tilde{\bar{\zeta}}_{j,t} + \Delta\tilde{\zeta}_{js,t} = \Delta\tilde{\alpha}_{js,t}$$

Using $\Delta\tilde{\alpha}_{js,t}$, I compute the moments of the demand shocks in exactly the same way I do in the baseline, thus assuming that $\tilde{\alpha}_{js}$ is normally distributed. Figure B.1 shows that the baseline Diversification Index and the one augmented with macro shocks are highly correlated. Therefore, while macro shocks are certainly an important component of the overall demand that exporters face, the baseline setting seems to be a good approximation of the demand risk faced by exporters.[36]

Learning. There is evidence that, in the short run, firms sequentially enter different markets to learn their demand behavior, and often exit very soon (see Albornoz et al. (2012), Ruhl and Willis (2014) and Berman et al. (2015) among others). This short-run learning behavior may contaminate the estimation of the moments. For this reason, I re-estimate the moments of the distribution considering only "established" firm-destination pairs, i.e. exporters selling to a certain market for at least 5 years, for which the learning process is most likely over. Figure B.1 shows that using only established firm-destination pairs does not significantly affect the estimates.[37]

B.9   Model with non-tradeable sector

Each agent v makes consumption choices by maximizing a Cobb-Douglas function of a non-tradeable good and a CES sub-aggregator of tradeable goods:

$$U_j(v) = \left(C_j(v) \right)^{\xi_j} \left(S_j(v) \right)^{1-\xi_j}$$

where

$$C_j(v) = \left(\sum_i \int_{\Omega_{ij}} \alpha_j(\omega)^{\frac{1}{\sigma}} q_j(\omega,v)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

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[36]This result is consistent with the recent empirical evidence that firm-destination specific shocks, rather than aggregate shocks, account for a large fraction of the variation in firms' sales across countries (see e.g. Di Giovanni et al. (2014) and Hottman et al. (2015)).

[37]On one hand, having at least 5 years of observations for each firm-destination pair allows to have more accurate estimates of the moments of the distribution. On the other hand, this approach selects firms that have most likely faced positive demand shocks, generating a selection bias. Sales by established exporters represent 89% of total exports volume throughout the sample period.

Figure B.1: Diversification Index: robustness

Notes: The figures plot the baseline Diversification Index against the Diversification Index computed, respectively: i) without firm-destination supply controls; ii) taking an average across each firm's covariances; iii) taking into account for macro shocks; iv) including only established firm-destination pairs. The displayed relationships are all 1

I assume that the demand for the non-tradeable good is non-stochastic. Maximizing the above function gives

$$S_j(v) = \frac{y(v)(1 - \tilde{\xi}_j)}{P_j^{NT}}$$

and

$$C_j(v) = \frac{y(v)\tilde{\xi}_j}{P_j}$$

where $y(v)$ is the income of agent v, P_j^{NT} is the price of the non-tradeable good, P_j is the price of the CES aggregator. The consumption aggregator equals

$$U_j(v) = \left(\frac{y(v)\tilde{\xi}_j}{P_j}\right)^{\tilde{\xi}_j} \left(\frac{y(v)(1 - \tilde{\xi}_j)}{P_j^{NT}}\right)^{1-\tilde{\xi}_j} = \frac{y(v)}{\tilde{P}_j}$$

where the aggregate price index is

$$\tilde{P}_j = \tilde{\xi} \left(P_j\right)^{\tilde{\xi}_j} \left(P_j^{NT}\right)^{1-\tilde{\xi}_j}$$

where $\tilde{\xi}_j \equiv \left(\tilde{\xi}_j\right)^{-\tilde{\xi}_j} \left(1 - \tilde{\xi}_j\right)^{\tilde{\xi}_j - 1}$.

I assume perfect competition in the non-traded sector, with production occurring with only labor and an unitary productivity. Assuming perfect mobility of workers between the two sectors, it holds that:

$$P_j^{NT} = w_j$$

The general equilibrium equations are the same as in the baseline model, except that the quantity demanded for each variety is

$$q_{ij}(z) = \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z) \xi_j Y_j$$

and the labor market condition becomes:

$$M_i \sum_j \int_{\underline{z}}^{\infty} \frac{\tau_{ij}}{z} \mu_j q_{ij}(z) g(z) dz + M_i \sum_j \int_{\underline{z}}^{\infty} f_j n_{ij}(z) L_j g(z) dz + \frac{Y_i(1-\xi_i)}{w_i} = L_i - M_i$$

Given the equalization of welfare between workers and entrepreneurs, welfare is simply equal to the real wage:

$$W_j = L_j \frac{w_j}{\tilde{P}_j}$$

Plugging in the aggregate price index in the expression for welfare, we have

$$W_j = \frac{L_j w_j}{\tilde{\xi}_j (P_j)^{\xi_j} \left(P_j^{NT}\right)^{1-\tilde{\xi}_j}} = \frac{L_j w_j}{\tilde{\xi}_j (P_j)^{\xi_j} (w_j)^{1-\tilde{\xi}_j}} = \frac{L_j}{\tilde{\xi}_j} \left(\frac{w_j}{P_j}\right)^{\xi_j}$$

B.10 Untargeted moments

Entry of firms. As discussed in the theoretical section, Proposition 1 implies that the firm's entry decision in a market (that is, whether $n_{ij}(z) > 0$) does not depend on a market-specific entry cutoff. Therefore, firms' sorting into exporting is not strictly hierarchical, as in traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008). In such models, as well as in my model with $\gamma = 0$, any firm selling to the $k+1$ most popular destination necessarily sells to the k most popular destination as well, since that has a lower entry cutoff.

Table B.1 reports, for each of the top destinations from Portugal, the fraction of exporters that are strictly sorted. Specifically, for each destination j, I consider all Portuguese exporters selling there, and compute the fraction of firms selling to all destinations more popular than j. For instance, the table suggests that only 23% of Portuguese firms that in 2005 were exporting to US, the 7th most popular destination, were also exporting to all

Table B.1: Sorting of Portuguese exporters

Destination	Rank	Fraction sorted, data	Fraction sorted, model
France	2	77%	94%
UK	3	66%	77%
Germany	4	50%	65%
Netherlands	5	46%	57%
Belgium	6	35%	47%
USA	7	23%	34%
Total		54%	68%

Notes: For each destination ranked k-th in terms of popularity, I consider all Portuguese firms exporting to that destination, and compute the fraction of exporters that sells to all markets with rank lower than k.

the top 6 destinations. Across the top 7 destinations, the fraction of strictly sorted firms in the data is 54%. While standard trade models with fixed costs and risk neutrality (e.g. Melitz (2003)) would predict that all exporters follow a strict sorting into exporting, my model with risk averse firms instead is able to predict fairly well the fraction of strictly sorted firms (68%).

Sales distribution. Propositions 1 and 2 suggest how firms, following their global diversification strategy, may optimally reach only few consumers in a given market, and thus export small amounts. I test this feature of the model by plotting, in Figure B.2, the distribution of sales, across all destinations, of Portuguese small firms. The graph suggests that the model with risk aversion is able to reproduce very well the left tail of the observed distribution. In contrast, the model with $\gamma = 0$, which corresponds to a standard fixed cost model such as Melitz (2003) and Chaney (2008), cannot predict the existence of small exporters, because of the presence of fixed costs of entry.
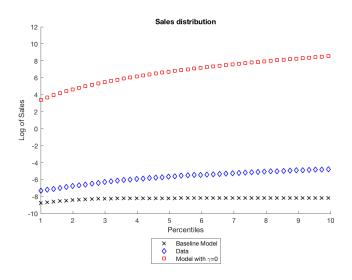
B.11 CRRA utility

I consider an extension of the model where the agents have a CRRA utility, and thus a decreasing absolute risk aversion. In particular, the entrepreneurs now maximize the following utility:

$$\max E\left[\frac{1}{1-\rho}\left(\frac{y_i(z)}{P_i}\right)^{1-\rho}\right] \tag{B.89}$$

where the coefficient of absolute risk aversion is given by

Figure B.2: Sales distribution

Notes: The figure plots the left tail of the distribution of log sales in the calibrated model and in the data, for 2005. I first compute the percentiles for each destination and then I take an average across destinations for each percentile.
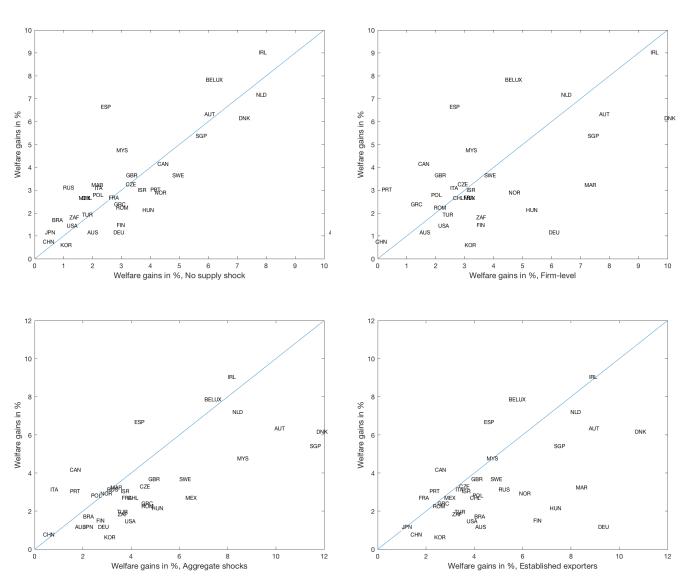
$$ARA = \rho \left(\frac{y_i(z)}{P_i} \right)^{-1} \tag{B.90}$$

and therefore is decreasing in the size of the firm, and the coefficient of relative risk aversion is simply $\rho > 0$. Taking a second-order expansion of $E\left[\frac{1}{1-\rho} z^{1-\rho} \right]$ around the expected value, this becomes:

$$\max \left(E\left[\frac{y_i(z)}{P_i} \right] \right)^{1-\rho} - \frac{\rho}{2} \left(E\left[\frac{y_i(z)}{P_i} \right] \right)^{-1-\rho} Var\left(\frac{y_i(z)}{P_i} \right). \tag{B.91}$$
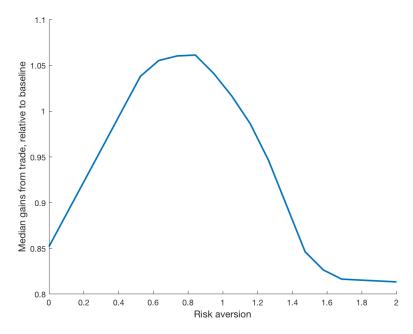
B.12   Welfare gains from trade, Robustness
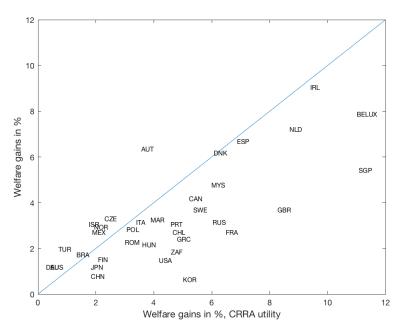
Figure B.3: Welfare Gains from Trade - Robustness

Notes: The figures plot the baseline welfare gains from trade against the gains computed using means and covariances estimated with the alternative methodologies described in Section B.8.

Figure B.4: Welfare Gains from Trade - Different risk aversion

Notes: The figure reports the welfare gains from trade for different values for the risk aversion, relative to the baseline welfare gains (computed with a risk aversion of 0.415).

Figure B.5: Welfare Gains from Trade - CRRA Utility

Notes: The figure plots the baseline welfare changes against the corresponding changes computed with a CRRA utility function.