General Equilibrium Effects in Space: Theory and Measurement*

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Abstract

We document that estimates of the differential effect of the China shock measured by Autor et al. (2013) are much larger than those implied by existing quantitative frameworks. We develop a reduced-form representation of spatial models in which changes in regional outcomes combine: (i) each region's "shift-share" exposure to shocks in its excess labor demand, and (ii) the reduced-form effect that this exposure has directly on that region, and indirectly on other regions through spatial links. We use this representation to uncover the roots of the disconnect. The estimated employment losses are larger and more dispersed than those implied by existing quantitative frameworks.

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1 Introduction

International trade shocks do not affect all regions of a country in the same way. A recent wave of empirical work in international economics has exploited variation in regional exposure to trade shocks to evaluate their differential impact on regional outcomes – see Topalova (2010), Kovak (2013), Autor et al. (2013) and, for reviews, Autor et al. (2016) and Muendler (2017). This type of empirical strategy has become a popular tool to uncover causal evidence about how labor markets adjust to trade shocks. However, it suffers from the so-called "missing intercept" problem: it may not recover the aggregate, general equilibrium impact of the shock if regions are spatially connected – for example, when there are demand spillovers or upstream and downstream relationships across regions.[1] In this paper, we propose a reduced-form representation of a class of spatial models that incorporates general equilibrium links between regions, and use it to estimate the differential and aggregate impact of the "China shock" on U.S. regional markets.

We start by documenting that estimates of the differential effect of the China shock measured by Autor et al. (2013) (henceforth, ADH) are an order of magnitude larger than those implied by existing spatial frameworks used to quantify the shock's aggregate impact. This disconnect is more substantial when we extend ADH's specification to also include spatial linkages in the form of the import competition exposure of nearby regions and the regional shock exposure in term of (final and intermediate) consumption.

To address this disconnect, we first develop a reduced-form representation of a class of spatial models. Our representation establishes that changes in regional labor market outcomes combine (i) each region's "shift-share" exposure to shocks in its excess labor demand, and (ii) the reduced-form effect that this exposure has directly on that region, and indirectly on other regions through spatial links. We use our representation to uncover the roots of the disconnect, tracing it into how various economic mechanisms affect the magnitude of the model's reduced-form elasticities and, consequently, of its predicted differential effects. The same reduced-form elasticities are sufficient to aggregate the exposure of different regions to compute the shock's general equilibrium impact in the model, provided that the spatial linkages terms are included. This indicates that the credibility of the model's aggregate impact is severely undermined if it yields reduced-form elasticities that are inconsistent with their empirical counterparts.

An important empirical question is how to estimate these elasticities. We propose to use a specification derived from our model's reduced-form representation. It is a generalization of popular "shift-share" empirical specifications that incorporates spatial links in general equilibrium through direct and indirect effects. Identification of such effects relies on two assumptions. First, we assume that observed trade "shifters" across sectors and foreign markets are randomly assigned. Second, we impose parametric restrictions in the spatial model to characterize the (direct and indirect) reduced-form elasticities as a function of observed variables and a small set of unknown parameters. We use this specification to estimate

[1] This is related to the problem that difference-in-difference empirical strategies do not recover the general equilibrium effect of the "treated" on "non-treated" (Heckman et al., 1998). Muendler (2017) and Chodorow-Reich (2020) discuss this problem for specifications based on cross-regional variation in shock exposure. Moretti (2011), Nakamura and Steinsson (2018), and Chodorow-Reich provide comprehensive reviews of the literature.

the effects of the China shock: we find large direct effects and reinforcing indirect effects that, given our parametric restrictions, arise from strong agglomeration forces, strong (weak) employment sensitivity to wages (prices), and regional production and trade links. Our estimates imply employment losses from the China shock that are larger and more dispersed than those implied by existing quantitative frameworks.

In Section 2, with exactly the same data and sample used by ADH, we document three facts about how different mechanisms shaped the responses of U.S. Commuting Zones (CZs) to the China shock. First, spatial links propagated negative shocks in labor demand across regions: employment and wage growth were weaker in CZs geographically close to a CZ facing higher import competition. Second, stronger import growth in (final and intermediate) goods consumed in a CZ did not generate relative gains in employment and wages. Third, we do not find evidence that population responded to any measure of regional shock exposure. We attest that these findings are robust to alternative specifications, such as inference procedures, weighting schemes, control sets, exposure measures, and sectoral shifters.

This evidence leads us to document a substantial disconnect between the large empirical estimates and the small quantitative predictions in the existing literature for the shock's differential impact across regions. In particular, the cross-regional variation in ADH's estimates of the shock's impact on employment rates is several times higher than that of the predicted effects of quantitative spatial models in the literature. Such a disconnect could potentially arise from the fact that these frameworks account for spatial and production links, while the specification in ADH does not. However, this possibility is inconsistent with the three facts that we document. Instead, we obtain even larger differential treatment effects with our specification that intuitively approximates for regional exposure through such links.

Section 3 then proposes a reduced-form representation of a class of spatial models that allows us to both understand the roots of the disconnect above and derive an empirical strategy to estimate the differential and aggregate impacts of trade shocks in a unified way. We consider a multi-sector gravity trade model (as in Costinot et al. (2011)) extended to feature local agglomeration forces in production, as well as spatially immobile individuals who choose whether to work or not (similar to Kim and Vogel (2021)). We show that, up to a first-order approximation, log-changes in labor outcomes of regional market i, $\hat{Y}_i$, following shocks in the fundamentals of the global economy $\hat{\boldsymbol{\tau}}$ (e.g., trade costs and productivity) combine direct effects and spatial indirect effects:

$$\hat{Y}_i = \underbrace{\beta_{ii}(\boldsymbol{\theta})\hat{\eta}_i(\hat{\boldsymbol{\tau}})}_{\text{Direct effect}} + \underbrace{\sum_{j\neq i}\beta_{ij}(\boldsymbol{\theta})\hat{\eta}_j(\hat{\boldsymbol{\tau}})}_{\text{Indirect spatial effect}}, \tag{1}$$

where $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ is the shock-induced shift in each market's excess labor demand, and $\beta_{ij}(\boldsymbol{\theta})$ is the reduced-form elasticity of market i's outcome to the shift in excess labor demand of market j. In our model, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ captures the market's "revenue shock exposure," i.e. how much its revenue responds to the shock (holding constant wages and employment). It takes a shift-share form, as it sums the shocks in $\hat{\boldsymbol{\tau}}$ interacted with pre-shock regional exposure shares. The reduced-form elasticities $\beta_{ij}(\boldsymbol{\theta})$ capture how much the shock exposure of a market directly affects its own outcomes, and indirectly percolates to other markets. They are determined by the wage elasticity matrix of regional excess labor demand, which depends on both pre-shock spatial links and parameters in $\boldsymbol{\theta}$.

This structural relationship yields the general equilibrium impact of trade shocks from the aggregation of direct and indirect effects across regions. For any $\hat{\boldsymbol{\tau}}$, we measure the shock exposure of each market using outcomes observed prior to the shock. We then exploit the fact that the reduced-form elasticities, governed by the parameter vector $\boldsymbol{\theta}$ for each given model, are sufficient to measure the shock's general equilibrium impact in the model: we aggregate the exposure of different markets using estimates of the direct and indirect reduced-form elasticities that determine the shock's differential effects across regions.[2] The flip side of this result is that a disconnect between the (direct and indirect) reduced-form elasticities predicted by a given model and their empirical counterparts constitutes an empirical rejection of the model's reduced-form elasticities and, therefore, of its prediction for the shock's general equilibrium impact.

To rationalize the facts discussed above, we leverage this theoretical result and the fact that the shift-share exposure measure in ADH resembles a negative shock to revenue in our model. The theory yields three insights. First, indirect reduced-form elasticities are increasing in bilateral trade links and are positive when such links are strong enough. Thus, exposure to a negative revenue shock in a market endogenously reduces labor demand in nearby regions with which trade links are stronger. Second, while a higher pre-shock spending on imported goods that became cheaper directly affects the cost of living, it does not have any impact on employment and wages when import prices do not affect non-employment payoffs and production costs. Finally, reduced-form elasticities are increasing in the strength of both agglomeration and labor supply responses. We show that these two forces stand in for a slew of other micro-foundations of determinants of the slope of regional excess labor demand (see also the related discussion in Allen and Arkolakis (2022)). The fact that Ricardian quantitative spatial models used in the literature feature no agglomeration forces (i.e. a small response of labor demand to the trade shocks) and weak or no labor supply responses can help to explain the disconnect highlighted above.

This structural relationship also forms the basis for a specification to estimate the general equilibrium impact of observed trade shocks. In our model, observed changes in regional outcomes are the sum of the predicted response to the observed shock, given by (1), plus a constant and a residual solely determined by other unobserved shocks. If the observed shock is mean-independent from all unobserved shocks, this structural relationship yields a specification for the estimation of both the direct and the indirect reduced-form elasticities, $\beta_{ij}(\boldsymbol{\theta})$. We leverage our reduced-form representation under a specific parametrization of the model to reduce the dimensionality of $\boldsymbol{\theta}$ to a number of parameters that is feasible to estimate in practice. We follow this approach because a type of dimensionality curse prevents the non-parametric estimation of the reduced-form elasticities (that is, setting $\boldsymbol{\theta} \equiv \{\beta_{ij}\}$), as we only observe

[2] We further show how our formulas can be easily integrated to recover the exact impact of the shock. We use this integration algorithm to extend our empirical specification to account for non-linearities in the impact of the shock, and show that the first-order approximation performs well in our empirical application.

outcomes for I markets, but there are I^2 reduced-form elasticities.[3]

Our empirical specification is a generalization of the type of shift-share empirical strategy used by ADH. It not only contains the model-consistent measures of region shock exposure, but it also accounts for the direct and indirect effects that arise in general equilibrium (as specified by our parametric assumptions on the model). We argue that our reduced-form specification has three advantages. First, it transparently connects the shock's aggregate impact to the magnitude and sign of estimates of the direct and indirect reduced-form elasticities. Second, it yields the most efficient estimator of $\boldsymbol{\theta}$ since it leverages both the direct and the indirect effects of the observed shock. Third, it can be also used to evaluate whether the model's predicted differential effects are consistent with their empirical counterparts, when implemented for outcomes that are not used for estimation.[4]

In Section 4, we generalize this framework to account for other channels highlighted in recent quantitative trade and spatial models – for reviews, see Costinot and Rodríguez-Clare (2014) and Redding and Rossi-Hansberg (2017). We allow for trade in final and intermediate goods, as well as labor supply to depend on migration choices and import prices. These additional mechanisms yield an extended version of (1) and, thus, an analogous empirical specification. The general model entails three new theoretical insights. First, trade in intermediate goods introduces upstream production relationships into the measure of "revenue shock exposure." Second, higher usage of intermediates plays a similar role to stronger agglomeration forces in amplifying the (direct and indirect) reduced-form elasticities to revenue shock exposure. Third, the shift in excess labor demand now also incorporates two measures of "consumption shock exposure:" one accounting for the downstream effect of import cost shocks on sales, and another for the effect of import price shocks on labor supply.

In light of these results, Section 5 revisits the problem of estimating the impact of the China shock on U.S. CZs. We rely on Chinese productivity shocks in manufacturing industries that we recover from Chinese import growth in other high-income countries using the gravity structure of our model. We estimate large reduced-form elasticities, direct and indirect, to revenue shock exposure. In our parametrization of the model, such large reduced-form elasticities are a result of strong agglomeration forces, high use of intermediate goods in manufacturing, and high sensitivity of employment to wages. We also find that the two channels of consumption exposure create relatively weak employment responses to import price shocks.

Our estimated specification yields predictions that are consistent with the observed differential

[3] Note that this requires the structural spatial model used to parametrize $\beta_{ij}(\boldsymbol{\theta})$ to be well-specified. If it is not, the unobserved residual will contain part of the true impact of the observed shock and, thus, the moment conditions used for estimation will no longer be valid. This is similar to the typical requirement that parametric assumptions imposed in empirical models are valid.

[4] Compared to indirect inference procedures that match arbitrarily chosen moments, the empirical strategy based on (1) has the advantage of leveraging a structural equation in the model and, as such, it provides estimates of the (direct and indirect) reduced-form elasticities that are sufficient for aggregation. Indirect inference could lead to biased predictions if the targeted moments are not closely related to estimates of the model's reduced-form elasticities. For instance, by targeting the estimated coefficient of the direct effect reported in ADH, one does not guarantee that the model's indirect effects are consistent with their empirical analogs. In contrast, our approach implies that responses in the data discipline the magnitude of the reduced-form elasticities that determine the model's predicted impact of the shock.

responses in both outcomes used in estimation (i.e., wage and employment rates across CZs), as well as other outcomes not used in estimation (i.e., manufacturing employment share across CZs, and U.S. exports and imports across sectors). However, this does not hold for a calibration of our model motivated by the existing quantitative literature.[5] The alternative calibration predicts much smaller differential effects as a result of the assumptions of no agglomeration forces, weak employment sensitivity to wages, and strong employment sensitivity to import prices.

We conclude by using our estimated specification to measure the general equilibrium impact of the China shock on U.S. CZs. We find a large variation in employment responses across CZs. On aggregate, the China shock eliminated around 3 million jobs between 1990 and 2007. Due to our larger estimates of the (direct and indirect) reduced-form elasticities to revenue exposure, we obtain differential and aggregate losses in employment that are an order of magnitude larger than those predicted by existing quantitative frameworks recently used to study the China shock – e.g., Caliendo et al. (2019) and Galle et al. (2021). When we account for the compensating impact of the shock on the cost of living, we obtain only a small change in the median real wage in the U.S. However, in contrast to the existing quantitative literature, we estimate a large spatial dispersion in real wage responses, with declines for a large fraction of the CZs.

Our paper is related to the extensive literature summarized by Redding and Rossi-Hansberg (2017) that relies on models with rich calibrated spatial links to quantify the aggregate impact of trade shocks. Compared to this literature, the distinctive feature of our paper is the use of the model's reduced-form representation to accomplish two goals. Theoretically, to characterize measures of regional exposure to trade shocks, and analyze the determinants of the magnitude and sign of the reduced-form elasticities of regional outcomes to these exposure measures.[6] Empirically, to obtain a specification for the estimation of the model's reduced-form elasticities that yield the shock's general equilibrium impact from the aggregation of the shock exposure of all regions. Our approach provides a connection between the differential effects implied by the theory and their empirical counterparts in the context of the China shock. This leads to larger estimates of the shock's general equilibrium impact on U.S. CZs, both differentially and on aggregate.[7]

We also relate to a growing literature estimating how regional markets respond to trade shocks – see e.g. Topalova (2010), Autor et al. (2013), Kovak (2013), Pierce and Schott (2020). We show that a wide class of models yields a specification that generalizes typical shift-share empirical strategies. Our

[5] Under this alternative calibration, our model's predicted responses for the employment rate have the same dispersion as those reported in Caliendo et al. (2019), and their correlation is 0.5.

[6] Our reduced-form representation exploits an intuitive excess labor demand characterization of a class of spatial models, similar in spirit to those in Allen et al. (2020) and Bartelme (2018). In contemporaneous work, Baqaee and Farhi (2019) provide a first-order approximation for the impact of productivity shocks on wages and welfare in open economies linked through final and intermediate trade, without agglomeration forces and employment responses. Our work is also related to the literature on sufficient statistics in international trade, such as Arkolakis et al. (2012), Bartelme et al. (2020), and Kleinman et al. (2020).

[7] Recent evidence by Autor et al. (2021b) suggests that the impact of the China shock is highly persistent: differential effects remain large and stable over a decade after Chinese import penetration stopped growing. Similar findings are reported by Dix-Carneiro and Kovak (2017) in the context of the Brazilian trade liberalization. Our model is consistent with these findings as the responses to shocks in (1) are permanent. In contrast, the disconnect we uncover is unlikely to be driven by mechanisms that reduce employment only temporarily along a transition path, such as nominal rigidities or reallocation frictions.

specification is close in spirit to that in Kovak (2013), but accounts for a rich structure of spatial and production linkages. In the presence of such links, it can be used for estimating regional differential responses to economic shocks in general equilibrium and aggregated in a model-consistent way. Our empirical specification also complements structural estimation strategies based on equilibrium relationships between endogenous outcomes in spatial models: it provides additional moments that can be used both to estimate the model's reduced-form elasticities and to evaluate whether the model's differential predictions are consistent with those observed in the data.[8]

Finally, our paper is related to recent macroeconomic frameworks in which regional outcomes depend on the region's exposure to aggregate shocks and a "missing intercept" containing the common, general equilibrium impact of the shock on all regions – e.g. Nakamura and Steinsson (2014); Mian and Sufi (2014); Beraja et al. (2019). As Chodorow-Reich (2020) points out, with this approach the measurement of the shock's aggregate impact "depends heavily, and sometimes non-transparently, on the ingredients in the model as well as the particular parametrization." Chodorow-Reich (2020) also argues that identification with this approach relies on restrictive modeling assumptions that yield the Stable Unit Treatment Value Assumption (SUTVA). In our environment, this is equivalent to the shock's indirect effect being identical in all regions – that is, $\beta_{ij}(\boldsymbol{\theta}) = \beta_j(\boldsymbol{\theta})$ for all i in (1) –, which not only requires symmetry in spatial links (e.g., frictionless trade), but is also inconsistent with the type of heterogeneous spatial spillovers that we document for the China shock. We instead exploit the heterogeneous exposure of regions to exogenous trade shocks to identify the parameters regulating both the direct and indirect reduced-form elasticities.[9]

2 Adjustment of U.S. Regional Markets to Trade Shocks: Three Stylized Facts

We begin by extending the specification in ADH to establish three stylized facts. They indicate that spatial links in goods and labor markets did not offset, but rather amplified, the negative differential impact of Chinese import competition on U.S. CZs documented in ADH. Our findings point to a striking disconnect between the large estimates of the differential impact of the China shock (in ADH and in our extension of it) and their much smaller counterparts predicted by existing quantitative spatial models.

[8] This includes the so-called "market access" approach (see e.g. Redding and Venables (2004); Donaldson and Hornbeck (2016); Alder et al. (2015); Bartelme (2018)), based on the equilibrium relationship between endogenous regional outcomes and the endogenous market access. Notice also that our empirical specification remains valid under a flexible structure of spatial links and arbitrary unobserved shocks, while the measurement of market access requires restricting spatial links and accurately observing all trade costs (before and after the shock).

[9] See also Donaldson (2015) for a similar discussion for the literature on trade and growth. SUTVA also rules out heterogeneity in the direct "treatment" effect of regional shocks, which also arises from spatial links as shown by Monte et al. (2018). Two recent papers also document heterogeneous spatial indirect effects of regional shocks: Burchardi et al. (2020) for the impact of immigration shocks on innovation, and Hornbeck and Moretti (2018) for the impact of productivity gains on domestic migration.

2.1 Empirical Specification

Our empirical analysis evaluates the differential effect of the China shock across U.S. CZs on three labor market outcomes: log of average weekly wage, log of employment rate, and log of working-age population. We extend the empirical specification in ADH by introducing two new measures of shock exposure, in addition to the ADH employment exposure of CZ i to import competition at period t (IC_i^t). In particular, we also consider the impact of a geographic gravity-based measure of region i's indirect exposure to the rise in import competition faced by nearby CZs (GC_i^t), as well as the impact of a measure of CZ i's expenditure exposure to Chinese import growth (IE_i^t). Using these measures, we estimate the following specification:

$$\Delta Y_i^t = \alpha^t + \beta^{IC} IC_i^t + \beta^{GC} GC_i^t + \beta^{IE} IE_i^t + X_i^t \lambda + \epsilon_i^t \tag{2}$$

where Y_i^t is a labor market outcome, α^t is a time fixed-effect, and X_i^t is a set of regional controls. Our sample and outcome definitions are identical to those used in ADH for 722 CZs in mainland U.S. over 1990-2000 and 2000-2007.

We now define the exposure measures used in equation (2). The next sections show how they arise from a first-order approximation of various model specifications. As in ADH, CZ i's employment exposure to import competition is

$$IC_i^t \equiv \sum_s \ell_{i,s}^{t_0} \Delta M_{\text{China},s}^t, \tag{3}$$

where $\Delta M_{\text{China},s}^t$ is the change in imports from China in the 4-digit SIC sector s for a set of high-income countries divided by the U.S. initial employment in sector s, and $\ell_{i,s}^{t_0}$ is CZ i's employment share in sector s in the pre-shock period t_0.[10] Our definition of IC_i^t is identical to the shift-share instrumental variable (IV) in ADH. Thus, β^{IC} is the *direct* differential impact on the CZ's labor market outcomes of higher employment exposure to the growth of Chinese imports in other developed economies.

Our gravity-based measure of indirect exposure to the import competition faced by other CZs is

$$GC_i^t \equiv \sum_{j \neq i} \frac{D_{ij}^{-\delta}}{\sum_{k \neq i} D_{ik}^{-\delta}} IC_j^t, \tag{4}$$

where D_{ij} is the bilateral distance between the population centroids of CZs i and j. Our specification has a "gravity" structure: GC_i^t is higher if i is near CZs with higher import competition exposure. The parameter δ controls how much indirect exposure declines with distance – in our baseline, we use typical estimates of the trade elasticity and set $\delta = 5$. Accordingly, conditional on i's import competition exposure, β^{GC} is the spatial *indirect* differential effect of the shock exposure of nearby regions on i's labor market outcomes. It intuitively captures the net effect of different sources of spatial shock percolation

[10] We follow ADH by using 10-year equivalent changes in imports of eight high-income countries with trade data covering the sample period (Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland), and ten-year lagged employment shares (1980 for 1990-2000 and 1990 for 2000-2007).

in general equilibrium. These could be, for instance, labor demand spillovers from lower domestic sales to nearby regions or labor supply spillovers due to in-migration from more negatively exposed CZs.[11]

Finally, our measure of the CZ's expenditure exposure to Chinese import growth is

$$IE_i^t \equiv \sum_s e_{i,s}^{t_0} \Delta M_{\text{China},s}^t, \tag{5}$$

where $e_{i,s}^{t_0}$ is the pre-shock share of sector s in the total gross spending of CZ i. IE_i^t captures the notion that the expenditure shock in CZ i is stronger if i has a higher spending share on a sector s, $e_{i,s}^{t_0}$, in which China expanded more the world output supply, as measured by $\Delta M_{\text{China},s}^t$. Thus, β^{IE} is the differential effect of higher expenditure exposure to the shift in world output supply caused by the China shock. Such an impact can be positive if either labor supply or labor demand rises when there is a positive shock in the supply of goods used for final or intermediate consumption. Alternatively, the impact can be negative if higher availability of Chinese imports in a sector induces firms in the region to strongly substitute local labor for imported inputs.[12] Notice that IE_i^t, while arising from our model below, is also closely related to the expenditure exposure measure proposed by Hummels et al. (2014), but it is defined across regions instead of firms.

2.2 Data

To maintain our analysis close to that in ADH, our main data source is ADH's online replication package for all variables, except IE_i^t. To compute this variable, we follow Gervais and Jensen (2019) by measuring CZ i's share of gross spending in sector s as $e_{i,s}^{t_0} \equiv \frac{\xi_s^{t_0} + \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0}}{1 + \sum_k a_k^{t_0} \ell_{i,k}^{t_0}}$, where ξ_{sk}^{M,t_0} is the share of sector s in input spending of sector k, $a_k^{t_0}$ is the ratio of input-to-labor spending in sector k, and $\xi_s^{t_0}$ is the share of sector s in final consumption. We compute ξ_{sk}^{M,t_0} and $\xi_s^{t_0}$ from the BEA input-output table, and $a_k^{t_0}$ from the NBER manufacturing database for manufacturing sectors and from the WIOD database for non-manufacturing.[13]

Table B.1 and Figure B.1 in Appendix B.1 present moments of the main variables used in our empirical application. Our two new exposure measures vary considerably across CZs, but their standard deviations are around half of that of ADH's employment exposure to import competition. Despite being

[11]We use the gravity structure in (4) to approximate (and formalize in our model below) these two main sources of cross-regional links, highlighted in recent spatial gravity models – e.g., Allen and Arkolakis (2014) and Donaldson and Hornbeck (2016). Appendix B.1 shows that our results are robust to alternative specifications for GC_i^t.

[12]For example, import supply shocks can have a positive impact on labor supply because cheaper imports increase the opportunity cost of leisure. The ambiguous effect of input prices on labor demand arises from the productivity and substitution effects of higher foreign input supply – e.g., as in Feenstra and Hanson (1999), and Grossman and Rossi-Hansberg (2008). Our model below clarifies how these mechanisms affect regional exposure to trade shocks.

[13]Our procedure imposes that input and final spending shares are the same in all CZs, and trade is balanced. In Appendix C.1.2, we evaluate our procedure to construct $e_{i,s}^{t_0}$ by running a regression of gross spending shares implied by shipment inflows in the Commodity Flow Survey (CFS) on our measured spending shares when aggregated for states and CFS commodity groups. We obtain a coefficient close to 1 and an R^2 of 0.95.

Table 1: Differential Impact of the China Shock on U.S. CZs

	Change in average weekly log-wage		Change in log of employment rate		Change in log of working-age population	
	(1)	(2)	(3)	(4)	(5)	(6)
IC_i^t	-0.471***	-0.383***	-0.519***	-0.369***	0.273	0.127
	(0.127)	(0.113)	(0.089)	(0.079)	(0.180)	(0.155)
GC_i^t		-0.606***		-0.691***		0.348
		(0.156)		(0.155)		(0.212)
IE_i^t		0.077		-0.154		0.418
		(0.164)		(0.143)		(0.294)
Differential treatment effect (percentage points):						
	-1.78	-3.52	-1.97	-4.16	1.03	2.44

Notes: Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All endogenous variables are multiplied by 100. All specifications include the following two sets of controls. Regional controls in ADH: period and census division dummies, manufacturing employment share in 1990, college-educated population share in 1990, foreign-born population share in 1990, employment share of women in 1990, employment share in routine occupations in 1990, and average offshorability in 1990. Additional controls: CZ's share of spending in manufacturing in 1990 ($\sum_s e_{i,s}^{t_0}$), and CZ's indirect exposure to manufacturing employment share in 1990 ($\sum_{j\neq i} z_{ij} \sum_s l_{js}^{t_0}$, with $z_{ij} \equiv D_{ij}^{-5}/\sum_k D_{ik}^{-5}$). Differential treatment effect: difference between the estimated treatment effects of CZs in the 75^{th} and 25^{th} percentiles of the empirical distribution of the estimated treatment effects. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

constructed with the same sector-level shifters, the different exposure shares used to compute each measure imply that regions are not equally exposed to them. The correlation across CZs is 0.53 between IC_i^t and GC_i^t, but it is only 0.16 between IC_i^t and IE_i^t.

2.3 Results

Table 1 reports estimates of our baseline specification, which includes ADH's largest control set (described in Table 1's note), as well as two extra pre-shock controls: the share of gross spending on manufacturing, and the gravity-based measure of indirect exposure to the manufacturing employment share of nearby CZs. They control for potential confounding effects of exposure, through our two additional channels, to the secular manufacturing decline in the period.

In columns (1), (3) and (5), we first estimate the regression in (2) using only IC_i^t to replicate ADH's findings. The estimates indicate a relative decline in both the average wage and the employment rate of CZs with higher employment in industries experiencing stronger growth in Chinese import competition. Compared to the CZ in the 25^{th} percentile of the distribution of IC_i^t, the CZ in the 75^{th} percentile of the distribution experienced changes in the average wage and employment rate that were 1.8 p.p. and 2.0 p.p. lower, respectively. These are large differential effects when we consider that the standard deviation across CZs of changes in the average wage and the employment rate were 6.5 p.p. and 6.4 p.p., respectively. As in ADH, we find a non-significant impact of higher exposure to Chinese import competition on the CZ's population, but this estimated impact is also relatively imprecise with a 95% confidence interval between -0.09 and 0.63. Notice however that we reject substantial negative responses in local population.

We then turn to the full specification in (2) that also includes our two additional measures of exposure to the China shock, GC_i^t and IE_i^t. In the second row of Table 1, we report the differential impact of being close to CZs with higher exposure to import competition. Columns (2) and (4) show that the negative impact of local shock exposure propagates to nearby regions: a CZ whose neighbors are more exposed to Chinese import competition experienced relative declines in its average wage and employment rate. The simultaneous reduction of wages and employment suggests that general equilibrium links spatially spread the decline in regional labor demand and reinforce the effect of the China shock.[14] In column (6), we also estimate an indirect impact on population that is non-significant.

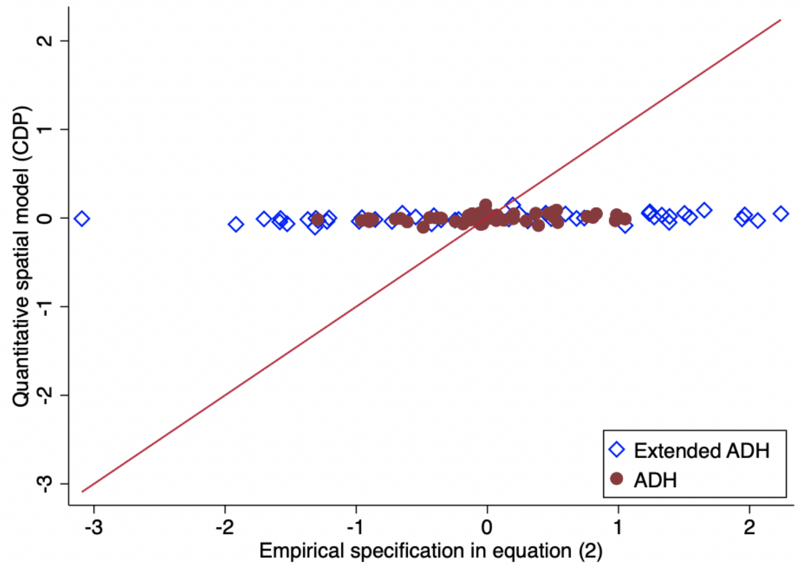
The third row of Table 1 reports the differential impact of higher spending exposure to the China shock, IE_i^t. For all outcomes, we find that the coefficients are not statistically different from zero. Importantly, this is driven by lower point estimates with standard errors whose magnitude are similar to those of the spatial indirect effects. Since IE_i^t is based on gross expenditure shares, our findings are consistent with weak differential responses in labor market outcomes to higher exposure to the input supply expansion caused by the China shock. This is similar to the evidence in Pierce and Schott (2016a) and Acemoglu et al. (2016a) of no differential growth in the national employment of industries more intensive in inputs of sectors in which Chinese imports grew more.

To summarize, our empirical analysis yields three novel stylized facts. First, spatial links amplify the negative impact of local exposure to import competition by generating relative reductions in the labor demand of other nearby regions. Second, we find no evidence of attenuating responses on employment and wages in regions more exposed to the positive shock in the supply of imported goods for (final and intermediate) consumption. Third, we find no evidence of population responses to the CZ's indirect exposure to the shock in nearby CZs, in addition to the lack of population responses to the CZ's own employment exposure documented in ADH. Hence, the spatial links embedded in our two new adjustment margins (namely, GC_i^t and IE_i^t) do not offset the differential negative impact of the China shock documented by ADH. Instead, the gravity-based measure of indirect exposure implies even larger differential effects on employment and wage rates across CZs, but neither margin induces significant differential responses in regional population.

Our estimates are at odds with the small differential impact of the China shock across regions implied by quantitative spatial frameworks in the literature. For instance, Caliendo et al. (2019) (henceforth CDP) find that the China shock had a small impact on employment, both on aggregate and differentially across U.S. states. In Figure 1, we compare the cross-state variation in the predicted employment rate changes in CDP to those implied by the estimates in columns (3) and (4) of Table 1. The figure shows a striking disconnect between the empirical estimates and the quantitative predictions for the shock's differential impact: those in CDP have a standard deviation of 0.05, while those implied by ADH

[14] As we discuss below, Table B.3 shows that estimates are essentially unchanged when we follow ADH in their choices of control set, data, sample, and weighting scheme, so that the only difference between our specification and theirs is the inclusion of the gravity-based measure of indirect exposure to import competition. However, if we follow Autor et al. (2021a) and restrict the sample to the period of 2000-2007, all estimates of direct and indirect effects become imprecise, with the exception of the direct effect of IC_i^t on the manufacturing employment share.

Figure 1: Differential Impact of the China Shock on Log Employment Rate



Notes: The figure compares the differential impact of the China shock on the log employment rate across U.S. states (multiplied by 100) between 2000 and 2007 that are predicted by the quantitative spatial model in CDP (vertical axis) and the estimates of the specification in equation (2) (horizontal axis). The red dots correspond to the state average of the predicted effects implied by the specification in column (3) of Table 1, and the blue hollow squares correspond to their conterparts implied by column (4) of Table 1. The red line is the 45-degree line. We obtain the predicted responses of CDP from their replication files. All variables are normalized to have mean zero.

(column (3) in Table 1) have a ten-times larger standard deviation of 0.54. The disconnect can also be seen from the comparison between the estimated coefficient obtained from the ADH specification across U.S. states when we set the dependent variable to be either the log-change in employment rates observed in the data or those predicted by CDP. The coefficient is -1.07 when we use the observed changes in the employment rate of U.S. states, but it is only -0.01 when we use the predicted changes from CDP.[15] Such a disconnect could potentially arise from the fact that the general equilibrium model in CDP accounts for spatial linkages, while the specification in ADH does not. However, this possibility is at odds with the even larger differential treatment effects implied by our extended empirical specification that intuitively approximates for gravity-based spatial links, as shown by the blue squares in Figure 1.

A similar disconnect from the response patterns observed in the data also arises in other recent quantitative models that predict small differential impacts of the China shock across U.S. CZs. For instance, the model extension in Galle et al. (2021) with endogenous employment responses yields a standard deviation of the predicted log-changes in employment rates across CZs of 0.08, which is again much smaller than that implied by the estimates in ADH and in Table 1. As Table A.3 in Galle et al. (2021) shows, these predicted differential effects are much smaller than those implied by the specification in ADH: when regressing their predicted effects on the ADH exposure measure, one obtains a coefficient

[15] Note that, compared to the estimates in Table 1, the estimated differential effect of the shock on the employment rate is even larger when we use U.S. states instead of U.S. CZs. This implies that the disconnect is not a consequence of the fact that CDP generate predictions at the state-level while ADH estimate their model across CZs.

of -0.04 for the log employment rate and -0.08 for the log average wage. These coefficients are much lower than the estimates reported in Table 1.

The disconnect we document is problematic and to some extent surprising because the analysis of the spatial quantitative literature on the China shock is motivated exactly by the need to complement the evidence of the differential effect in ADH with the general equilibrium channels of adjustment that may affect the aggregate impact of the shock. However, existing models predict differential effects that are an order of magnitude smaller than their empirical counterparts. This is true for both the original specification in ADH and our extension of it that approximates for spatial links among markets.

2.4 Robustness and Additional Results

We now discuss the robustness of our baseline results. Appendix B.1 displays all the tables.

Employment Outcomes in ADH. In Table B.2, we follow ADH in the choice of the dependent variables and focus on the change in the share of working-age population in manufacturing, non-manufacturing, unemployed, and out of the labor force. For all outcomes, we estimate statistically significant direct and indirect impacts of higher exposure to import competition. In Table B.3, we report similar estimates when we exactly follow ADH's specification, with the addition of GC_i^t.

Alternative Empirical Specifications. Table B.4 shows that estimates are similar when we consider only subsets of our exposure measures. We also document the absence of attenuating effects from indirect exposure to import expenditure shocks in nearby CZs. Column (2) of Table B.5 indicates that our estimates of the employment and wages responses to the CZ's direct and indirect shock exposure remain statistically significant at usual levels when we use the shift-share inference of Adão et al. (2019). Columns (3)–(4) of Table B.5 report similar results when we control for state fixed-effects and lagged population growth (as in Greenland et al. (2019)) to account for state-wide and persistent amenity shocks. Column (5) of Table B.5 controls for the CZ's initial manufacturing shares interacted with period dummies, which absorbs period-specific manufacturing shocks. This reduces the estimated impact of import competition on wages and employment, but only the direct effect on wages is not significant at 10%. Column (6) of Table B.5 reports similar results when we weigh CZs by their population.

Alternative Shock Exposure Measures. In Table B.6, we document the same reinforcing pattern of indirect responses to the shock exposure of nearby regions when we compute the gravity-based measure in (4) while setting the distance decay to one or eight (columns (2)-(3)), adjusting for the size of nearby CZs (column (4)), and excluding out-of-state CZs (column (5)).

Table B.7 considers alternative definitions of expenditure shock exposure. In column (2), we consider two separate exposure measures of the form in (5) built with sectoral spending shares out of final and intermediate expenditure (respectively, IEF_i^t and IEI_i^t).[16] We find that employment and wages do not differentially respond in CZs with higher shock exposure in terms of either final or intermediate expenditure. Column (3) reports similar estimates when we exclude input spending on the own sector in the computation of the intermediate spending shares. Lastly, column (4) reports estimates when we approximate for cross-industry supply links using the ''Leontief expenditure shares'' in Acemoglu et al. (2016a). In this case, we find that higher exposure to cheaper inputs from China causes a relative decline in the CZ's employment rate.

Table B.8 considers alternative measures of the China shock in each sector. This addresses concerns related to ADH's specification of the shifters in terms of import growth in other countries, which may be affected by productivity shocks in U.S. CZs or demand shocks in importing countries. In Panel A, we use China's exporter fixed-effect in each sector that we obtain from a gravity regression of log changes in bilateral trade shares on sector-origin and sector-destination fixed-effects. In Panel B, we construct exposure measures using the same sector-level NTR gaps used in Pierce and Schott (2016a). In both cases, we find similar qualitative patterns of responses to higher (direct or indirect) exposure to Chinese import competition.

Additional Migration Outcomes. Table B.9 investigates the impact of the China shock on gross migration flows across U.S. CZs. All measures of exposure to the China shock did not have statistically significant impacts on either the inflow or the outflow of migrants across CZs, but some of these estimates are imprecise and cannot rule out a wide range of responses.

3 Theory of General Equilibrium Effects in Space

Motivated by the evidence above, we now propose a simple spatial model that we use to show that the effect of a trade shock on each region can be expressed in its reduced-form: in terms of the regional shock exposure and the effects that each region's exposure creates directly on its own outcomes and indirectly on other regions through spatial links. Based on this characterization, we develop an empirical specification that allows us to estimate both the direct and the indirect elasticities to regional shock exposure, as well as connect the observed differential effects of a trade shock on regional outcomes to that shock's aggregate effect on the economy. While our modeling choices are guided by the stylized facts documented above, the next section shows how to extend our results to incorporate additional mechanisms present in a wide class of quantitative spatial models.

[16] As in the baseline, we construct intermediate spending shares using the national input-output table and the CZ's sectoral employment shares: the share of intermediate spending on sector s is $ei_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_k a_k^{t_0} \ell_{i,k}^{t_0}$. The share of final spending on sector s in CZ i, $ef_{i,s}^{t_0}$, is the share of average household expenditure in i's state across 3-digit SIC manufacturing sectors (constructed from the Consumer Expenditure Survey – see Appendix C.2.1).

3.1 Environment

We consider a multi-sector gravity trade model with I segmented markets grouped into countries. Each market comprises a product and labor market with a set of consumers and workers that face the same product and labor prices.[17] Let $i \in \mathcal{I}_c$ denote a market in country c. In sector s of market i, a representative competitive firm uses labor to produce a differentiated good with an endogenous production cost of $p_{i,s}$, and faces exogenous iceberg trade costs for selling to different destinations j of $\tau_{ij,s}$. Each market is endowed with a mass of heterogeneous individuals, $\bar{N}_i$, that endogenously decide whether or not to work by comparing the market's wage rate w_i to a government non-employment transfer b_i. Residents of market i face an income tax rate of v_i.

Gravity Trade Demand. All individuals in market j maximize the same nested Constant Elasticity of Substitution (CES) preferences. We consider a Cobb-Douglas aggregator of sector-specific composite goods where $\xi_{j,s}$ is the constant spending share on sector s. The sectoral composite good is a CES aggregator over the differentiated sector-specific products from different origins, with $\sigma > 1$ denoting the elasticity of substitution across origins.[18] Since markets are competitive, the price of market i's sector s differentiated good in market j is $\tau_{ij,s} p_{i,s}$. Thus, utility maximization implies that the bilateral sales in sector s from i to j are

$$X_{ij,s} = x_{ij,s} \xi_{j,s} E_j = \frac{(\tau_{ij,s} p_{i,s})^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s})^{1-\sigma}} \xi_{j,s} E_j, \tag{6}$$

where E_j is j's total expenditure. The associated consumption price index in i is

$$P_i = \prod_s (P_{i,s})^{\xi_{i,s}}, \quad \text{with} \quad P_{i,s} = \left[\sum_o (\tau_{oi,s} p_{o,s})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{7}$$

This demand structure implies that market i's revenue is the sum of sectoral sales to different destinations, $R_i = \sum_{j,s} X_{ij,s}$. These sales, in turn, are a function of bilateral trade costs, $\tau_{ij,s}$, and the trade elasticity, $1-\sigma$. To the extent that $\tau_{ij,s}$ depends on distance, we show below that our model features the type of spatial percolation in regional labor demand shocks that we documented in Section 2. This multi-sector gravity-based demand has become a standard way of modeling spatial links in the trade literature – see e.g. Anderson (1979); Eaton and Kortum (2002); Costinot et al. (2010); Arkolakis

[17] We define a product market as a set of consumers with access to the same products and prices, a common approach in industrial organization (e.g., Berry and Haile (2014)). Similarly, we define labor markets as sets of producers that face the same labor cost, as in neoclassical and gravity trade models (e.g., Dixit and Norman (1980), Costinot and Rodríguez-Clare (2014)). We incorporate wage differences across sectors when markets are groups of sectors within a region – for instance, when each region has two distinct markets, one for the set of manufacturing industries and another for the set of non-manufacturing industries. We return to this point in Section 4.

[18] This demand specification greatly simplifies exposition, but we show below that our insights do not rely on assumptions of either nested CES preferences or a single elasticity of substitution for all sectors.

et al. (2012) and, for a review, Costinot and Rodríguez-Clare (2014).

Labor Supply. Individuals are heterogeneous and choose whether to be employed or not. If employed, individual ι supplies $l(\iota)$ efficiency units, obtaining an after-tax labor income of $(1-v_i)w_i l(\iota)$. If non-employed, individual ι's income is $(1-v_i)b_i u(\iota)$, with $u(\iota)$ denoting ι's non-employment income potential. The pair $(l(\iota),u(\iota))$ is drawn independently from a Frechet distribution with shape parameter $\phi>1$ and scale 1, so that the employment rate in market i is

$$n_i=\Pr\left[(1-v_i)\frac{w_i}{P_i}l(\iota)\geq(1-v_i)\frac{b_i}{P_i}u(\iota)\right]=\frac{w_i^\phi}{w_i^\phi+b_i^\phi}. \tag{8}$$

Up to a first order approximation, the log-change in the share of employed residents in market i is $\Delta\ln n_i=\phi(1-n_i)\Delta\ln(w_i/b_i)$ and, therefore, is proportional to the change in the ratio of the market's wage rate to the return of the non-employment outside option, with a sensitivity controlled by ϕ. Under this specification, a reduction in market's labor demand leads to a decline in both wages and employment rates, in line with the evidence in Section 2.[19] This structure of selection of heterogeneous individuals into employment is a standard way of modeling changes in the extensive margin of labor supply – e.g., see Heckman and Sedlacek (1985), Rogerson (1988), Mulligan and Rubinstein (2008), and Chetty et al. (2013a). It is also consistent with the evidence in Autor et al. (2013) and Pierce and Schott (2020) that the number of recipients of different types of government transfers increases in regions more exposed to the China shock.

The presence of heterogeneous individuals allows us to incorporate in our analysis a salient feature of the data: individuals with lower initial income are more likely to become non-employed when exposed to higher Chinese import competition (see Autor et al. (2014)). This is true in our model because individuals differ in their efficiency, implying that the wage rate w_i is not identical to the observable average log of labor earning, $\overline{\ln w_i}$, used to document the wage responses in Section 2.[20] Instead, our model yields the following equation:

$$\Delta\overline{\ln w_i}=\Delta\ln w_i-\frac{1}{\phi}\Delta\ln n_i. \tag{9}$$

The employment rate in our model also depends on the reservation wage, b_i. In our baseline specification, we take the simplest approach of assuming that, in every market i, non-employment

[19] Kim and Vogel (2021) impose similar assumptions to model the choice of labor force participation of heterogeneous workers. The same employment rate expression arises if we relax the Frechet assumption as in Adão (2016), but that would introduce an additional parameter to control selection forces in wages. All our results are identical if $u(\iota)$ is a private benefit of not working rather than an income potential. Appendix A.5 shows that an expression for the change in the employment rate in terms of changes in w_i/b_i also arises in a competitive search environment in which firms post vacancies with a given wage and workers decide whether to search for a job. In this case, the employment rate elasticity also depends on the efficiency parameter of the matching function (as in Kim and Vogel (2021)).

[20] Several models used to study the impact of trade shocks on regional economies cannot account for this fact since they miss either non-employment or heterogeneity in worker efficiency – e.g., Burstein et al. (2019); Caliendo et al. (2019). Adão (2016) and Kim and Vogel (2021) constitute recent exceptions.

benefits do not respond to trade shocks and are exogenously set in terms of a common numeraire function of wages: $b_i = \bar{b}_i \Omega(\{w_j\}_j)$, where $\Omega(.)$ is homogeneous of degree one and $\omega_i \equiv \frac{\partial \ln \Omega(\{w_j\}_j)}{\partial \ln w_i}$. Thus, through the proper specification of $\Omega(\{w_j\}_j)$, our model can replicate the evidence in Chodorow-Reich and Karabarbounis (2016) that changes in the aggregate non-employment payoff (i.e., average changes in $\bar{b}_i \Omega(\{w_j\}_j)/P_i$) are positively, but only partially, correlated with changes in the aggregate real wage (i.e., average changes in w_i/P_i). In Section 5, we use their evidence to specify $\Omega(\{w_j\}_j)$ so that the non-employment payoff, $b_i/P_i = \bar{b}_i \Omega(\{w_j\}_j)/P_i$, is a function of real income in different markets.[21]

Production Technology. We start with a simple structure of production where, in each sector s of market i, output is proportional to the representative firm's endogenous employment choice, $L_{i,s}$, as well as to a term capturing economies of scale that are external to the firm and increasing in the market's employment rate. Specifically, the production function is $Q_{i,s} = (\frac{n_i}{1-n_i})^{\psi} L_{i,s}$ and, thus, the unit production cost is

$$p_{i,s} = w_i^{1-\psi\phi} b_i^{\psi\phi}. \tag{10}$$

Agglomeration forces may arise from a variety of economic mechanisms such as entry externalities (e.g., Krugman (1991)), Marshallian production externalities (e.g., Ethier (1982) and Kucheryavyy et al. (2016)), and search frictions (see Appendix A.5). The importance of this mechanism to analyze regional responses to local shocks in labor demand has been emphasized by several recent papers – e.g., Greenstone et al. (2010), Kline and Moretti (2014), Dix-Carneiro and Kovak (2017), and Peters (2019).[22] Our specification captures the combination of these economic forces in a reduced-form way through the combined strength of agglomeration and labor supply forces in $\psi\phi$ and, thus, our functional form choice is guided by its convenient implication that the pass-through from wages to prices is the constant $1-\psi\phi$. As shown below, this connects the combined strength of agglomeration and labor supply forces in $\psi\phi$ to the curvature of the regional labor demand function. In Section 4, we substantially generalize the structure of production by introducing intermediate inputs. Such an extension implies that the pass-through from wages to prices decreases with the intermediate input share in production.

Equilibrium. To analyze the equilibrium, we characterize the labor demand in market i. Since labor is the only factor of production, this is simply given by the sum of sectoral revenues in equation (6)

[21] In Section 4, we specify $b_i = \bar{b}_i P_i^{\lambda} (\Omega(\{w_j\}_j))^{1-\lambda}$ and show that the impact of import expenditure exposure on labor market outcomes is increasing in λ. Given the evidence in Section 2, our estimated λ is close to zero in Section 5, which roughly corresponds to our baseline specification of b_i. Thus, our estimates and the evidence in Chodorow-Reich and Karabarbounis (2016) reject that the non-employment payoff is invariant to shocks (i.e., that b_i/P_i is constant as imposed in Caliendo et al. (2019) and Galle et al. (2021), in which case λ would be one).

[22] This channel is absent in recent quantitative spatial frameworks based on the Ricardian model of Eaton and Kortum (2002) used to quantify the impact of trade shocks on regional economies – e.g. Caliendo et al. (2019), Lyon and Waugh (2019), Galle et al. (2021), and Kim and Vogel (2021).

(after substituting for the production cost in (10)):

$$R_i = \sum_s \sum_j \frac{\tau_{ij,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\kappa-\sigma+1}}{\sum_o \tau_{oj,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\kappa-\sigma+1}} \xi_{j,s} E_j, \tag{11}$$

where $\kappa \equiv (\sigma-1)(1-\psi\phi)$ is a parameter determining the sensitivity of labor demand to changes in the wage rate of different markets (conditional on total spending). As such, κ is a key determinant of the differential responses in wages and employment to shocks in economic fundamentals. In our model, the labor demand elasticity is lower if the trade elasticity, $(\sigma-1)$, is lower, or the combined strength of the agglomeration and labor supply elasticities, $\psi\phi$, is higher.

To solve for the equilibrium and simplify our analysis, we impose that the local income tax v_i is set such that the benefit payments equal the tax revenues in equilibrium: $v_i(W_i+B_i)=B_i$, with W_i and B_i denoting total wage and benefit payments in market i, respectively. The market level spending is thus $E_i=W_i$.[23] Given our labor supply structure, total income in market i is given by $W_i = w_i^{\phi}(w_i^{\phi}+b_i^{\phi})^{\frac{1-\phi}{\phi}} \bar{N}_i \varrho$ where $\varrho \equiv \Gamma(1-1/\phi)$ and $\Gamma(.)$ is the gamma function. This indicates that, in our model, ϕ determines the elasticity of both employment and spending in each market to changes in the local wage rate. For this reason, ϕ is also key to determine how labor market outcomes respond to shocks in economic fundamentals.

We then define the equilibrium as a wage vector that yields an excess labor demand of zero in every market. Formally, consider a wage vector $\boldsymbol{w} \equiv \{w_o\}_o$ with $w_m \equiv 1$ for an arbitrary numeraire market m. It is an equilibrium if $D_i(\boldsymbol{w}|\boldsymbol{\tau})=0$ for all i, such that

$$D_i(\boldsymbol{w}|\boldsymbol{\tau}) \equiv \sum_j \left(\sum_s \frac{\tau_{ij,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\kappa-\sigma+1}}{\sum_o \tau_{oj,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\kappa-\sigma+1}} \xi_{j,s} - \mathbb{I}_{i=j} \right) w_j^{\phi} \left(w_j^{\phi} + \bar{b}_j^{\phi} (\Omega(\boldsymbol{w}))^{\phi} \right)^{\frac{1-\phi}{\phi}} \bar{N}_j \varrho, \tag{12}$$

where $\boldsymbol{\tau} \equiv \{\tau_{id,s}\}_{ids}$ is a vector of bilateral trade costs, and $\mathbb{I}_{i=j}$ is an indicator function that equals one if, and only if, $i=j$. Note that when $\psi=0$ and $\phi \to 1$, equation (12) is isomorphic to the excess demand function of a multi-sector gravity trade model with a fixed labor supply (see e.g. Costinot et al. (2010)).

3.2 General Equilibrium Effects of Trade Shocks in Space

We now study how exogenous changes in trade costs $\tau_{ij,s}$ affect different markets. Given the definition of $\tau_{ij,s}$, our analysis applies also to productivity shocks when changes in trade costs are the same for all destinations. We use 0 superscripts to denote variables in the initial equilibrium, z_j^0; hats to denote log changes in variables between the initial and new equilibria, $\hat{z}_j \equiv \ln(z_j/z_j^0)$; bold variables to denote stacked vectors of market outcomes, $\boldsymbol{z} \equiv \{z_i\}_i$; and bar bold variables to denote matrices with bilateral variables associated with origin i and destination j, $\bar{\boldsymbol{z}} \equiv \{z_{ij}\}_{i,j}$.

[23] This assumption is not important for our results. In Section 4, we show that an arbitrary structure of (endogenous and exogenous) transfers across markets only determines how E_i depends on wages in different markets. In addition, our empirical findings below are similar when fiscal transfers are endogenous (as specified in Appendix A.2.6).

The response of the wage rate in each market to changes in trade costs follows directly from the total differentiation of the equilibrium definition in terms of excess labor demand. This yields the two key objects in our analysis. The first is the partial equilibrium shift in the excess labor demand caused by the shock (holding wages constant),

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \equiv (\bar{\boldsymbol{R}}^0)^{-1}\big(\nabla_{\ln\boldsymbol{\tau}}\boldsymbol{D}\big(\boldsymbol{w}^0|\boldsymbol{\tau}^0\big)\big)\hat{\boldsymbol{\tau}}, \tag{13}$$

where $\bar{\boldsymbol{R}}^0$ is the diagonal matrix of initial revenues. The second is the "spatial links" matrix,

$$\bar{\boldsymbol{\gamma}}^0 \equiv -(\bar{\boldsymbol{R}}^0)^{-1}\big(\nabla_{\ln w}\boldsymbol{D}\big(\boldsymbol{w}^0|\boldsymbol{\tau}^0\big)\big), \tag{14}$$

which captures the elasticity of a market's excess labor demand to wages in different markets. Written as such, our analysis is a traditional comparative statics exercise in general equilibrium, as in Arrow and Hahn (1971)and Mas-Colell et al. (1995).

We can express the wage response (in terms of the economy's numeraire) to trade shocks as

$$\bar{\boldsymbol{\gamma}}^0\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}). \tag{15}$$

In the rest of this section, we first establish that the excess demand shift in each market, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$, takes the form of a shift-share variable based on the sum of trade shocks interacted with market-specific exposure shares. We then characterize the sources of spatial links embedded in $\bar{\boldsymbol{\gamma}}^0$. We finally invert expression (15) to characterize the reduced-form elasticities that are sufficient to compute the general equilibrium impact of the shock exposure vector, $\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$, on market-level outcomes.

3.2.1 A Shift-Share Measure for Shocks in Excess Labor Demand

A log-linearization of equation (12) implies that $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ takes the form of a shift-share variable:

$$\hat{\eta}_i(\hat{\boldsymbol{\tau}}) = (1-\sigma)\sum_s \ell^0_{i,s}\mu_{i,s}(\hat{\boldsymbol{\tau}}), \tag{16}$$

where $\ell^0_{i,s}$ is the initial share of labor in market i employed in sector s, and $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ is the shift in the demand for i's goods in sector s,

$$\mu_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_j r^0_{ij,s}\left(\hat{\tau}_{ij,s} - \sum_o x^0_{oj,s}\hat{\tau}_{oj,s}\right), \tag{17}$$

with $r^0_{ij,s} \equiv X^0_{ij,s}/\sum_d X^0_{id,s}$ denoting the initial share of market j in market i's sales in sector s. $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ is the market's "revenue shock exposure" since it is the sum across sectors of the shock to the demand for i's goods in each sector, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$, weighted by the sector's initial share in i's employment $\ell^0_{i,s}$. The sector-level demand shock $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ itself is the sum across destinations j of the impact of market i's own trade shock on the demand for its goods minus the demand shift caused by competitors' trade shocks in that sector, weighted by the revenue importance of each destination $r^0_{ij,s}$. Note that all components of $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ can be computed with measures of the bilateral trade shocks and information on initial bilateral trade flows.[24]

The excess labor demand shift in (16) is closely related to shift-share measures of exposure to sectoral shocks used in the literature (such as that used in Section 2). To see this, consider a foreign shock with an identical impact on the sectoral demand of all destinations: formally, $\hat{\tau}_{oj,s}=0$ for all $o\neq \mathrm{F}$ and $\hat{\zeta}_{F,s}\equiv(1-\sigma)x^0_{Fj,s}\hat{\tau}_{Fj,s}$ for all j. Then, $\hat{\zeta}_{F,s}$ is the common impact, the "shift", that the shock in the foreign country has on the sectoral demand of every other market, and thus

$$\hat{\eta}_i = -\sum_s \ell^0_{i,s}\hat{\zeta}_{\mathrm{F},s}. \tag{18}$$

If the foreign country becomes more productive in sector s ($\hat{\zeta}_{\mathrm{F},s}>0$), then every other market suffers a negative shift in its excess labor demand, $\hat{\eta}_i<0$ for $i\neq \mathrm{F}$. The magnitude of this impact is proportional to the initial share of sector s in i's labor demand, as measured by the "share" $\ell^0_{i,s}$. In Section 5, we use the common component of the growth in sectoral Chinese imports across destinations to link the movement in a region's excess labor demand to its shift-share exposure to import competition (as defined in ADH and in Section 2).

3.2.2 Spatial Links in General Equilibrium

We proceed with the characterization of the spatial links in the economy: $\bar{\boldsymbol{\gamma}}^0$ in (14). This matrix summarizes the spatial percolation of shocks in our model as it regulates how much wage changes in one market affect excess labor demand in other markets. By defining $\phi^0_i\equiv\phi-(\phi-1)n^0_i$, Appendix A.1 shows that

$$\gamma^0_{ij} = \left(\phi^0_i+\kappa\right)\mathbb{I}_{[i=j]}-\rho^0_{ij} \quad \text{where} \quad \rho^0_{ij}\equiv r^0_{ij}\phi^0_j+\kappa\sum_{s,d}\ell^0_{i,s}r^0_{id,s}x^0_{jd,s}+\omega^0_j\sum_d r^0_{id}(\phi^0_i-\phi^0_d). \tag{19}$$

The first component of this expression is the own-elasticity of i's excess labor demand to its wage, which corresponds to the sum of the labor demand and labor supply elasticities, regulated by κ and ϕ^0_i, respectively. Following the usual logic in supply-demand frameworks, a lower value of $\phi^0_i+\kappa$ implies stronger wage responses to the same shock.

The second component ρ^0_{ij} is the cross-wage elasticity of excess labor demand. A higher ρ^0_{ij} creates a stronger dependence of outcomes in i to labor demand shocks in j. Such a dependence arises from three sources. The term $r^0_{ij}\phi^0_j$ captures the positive impact that an increase on j's wage has on its total expenditure (proportional to ϕ^0_j) and, consequently, on the sales of i (proportional to the share of j in i's

[24] Note that, with a single sector, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ is the partial equilibrium (i.e. holding wages constant in all markets) change in the firm market access. The concept of firm market access introduced in Anderson and Van Wincoop (2003) and Redding and Venables (2004) is widely used to measure the revenue potential of a location in the literature (e.g., Redding and Sturm (2008), Donaldson and Hornbeck (2016), Bartelme (2018)).

revenue, r_{ij}^0). The next term captures endogenous changes in excess labor demand arising from demand substitution across suppliers due to changes in j's labor cost. It is proportional to the sensitivity of demand to wages κ and, importantly, to the covariance between i's sales $\ell_{i,s}^0 r_{id,s}^0$ and j's market share $x_{jd,s}^0$ across sectors and destinations. The last term is the impact on excess labor demand of changes in labor supply due to the non-employment benefit's numeraire and arises because of the heterogeneity in the labor supply elasticity across markets – in fact, it is zero if $n_i^0 = n^0$ and, thus, $\phi_i^0 = \phi^0$ for all i.

3.2.3 General Equilibrium Effects in Space and their Determinants

We now characterize the reduced-form elasticity of wages to trade shocks in general equilibrium. This is a "sufficient statistics" characterization: it yields responses in terms of market-level measures of shock exposure (determined by $\hat{\eta}_i$ in (16)) and market-to-market reduced-form elasticities to these measures (determined by γ_{ij} in (19)). Both components are functions of variables observed in the initial equilibrium, as well as parameters controlling the elasticities in the model. Appendix A.1 contains the proofs of the results in this section.

Throughout our analysis, we impose sufficient conditions for equilibrium uniqueness given any $\boldsymbol{\tau}$. This guarantees that our counterfactual analysis yields unambiguous predictions for the impact of shocks in economic fundamentals. Following Arrow and Hahn (1971) T.9.12 (p. 234), we assume that the excess demand system satisfies diagonal dominance: there exists $\{h_i\}_{i \neq m} \gg 0$ such that, for all $i \neq m$,[25]

$$h_i \gamma_{ii}^0 > \sum_{j \neq m,i} h_j |\gamma_{ij}^0|. \tag{20}$$

Theorem 1. *(Sufficient Statistics for Reduced-Form Responses) Consider any shock to bilateral shifters $\hat{\boldsymbol{\tau}}$. If condition (20) holds, then (up to a first-order approximation)*

$$\hat{w}_i = \underbrace{\beta_{ii}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_i(\hat{\boldsymbol{\tau}})}_{\textit{Direct effect}} + \underbrace{\sum_{j \neq i} \beta_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j(\hat{\boldsymbol{\tau}})}_{\textit{Spatial indirect effect}}, \quad \textit{with} \quad \beta_{ij} = \frac{1}{\phi_j^0 + \kappa}\left(\mathbb{I}_{[i=j]} + \tilde{\gamma}_{ij} + \sum_{d=2}^{\infty} \tilde{\gamma}_{ij}^{(d)}\right), \tag{21}$$

where $\tilde{\gamma}_{ij}^{(d)}$ is the i-j entry of $(\bar{\tilde{\boldsymbol{\gamma}}})^d$ such that $\tilde{\gamma}_{ij} \equiv (\phi_i^0 + \kappa)^{-1} \rho_{ij}^0 \mathbb{I}_{[i,j \neq m]}$, $\boldsymbol{\theta} \equiv (\phi, \kappa)$ is a parameter vector, and $\boldsymbol{\mathcal{W}}^0 \equiv \{n_i^0, \omega_i^0, \{X_{ij,s}^0\}_{j,s}\}_i$ is a matrix of initial conditions.

Theorem 1 yields a set of sufficient statistics for counterfactual analysis in general equilibrium: the vector of excess labor demand shifts (i.e, $\hat{\eta}_i$ in (16)), as well as the reduced-form elasticities to such measures (i.e., β_{ij} in (21)). The formula for wage changes (in terms of the economy's numeraire) in (21) *aggregates* the direct effect of the market's own shock exposure and the spatial indirect effect of the shock exposure of

[25]This assumption is weaker than the gross substitution property (i.e., $\gamma_{ii} > 0$ and $\gamma_{ij} < 0$ for all $i \neq j$) that yields uniqueness of one-sector gravity trade models with exogenous labor supply (Alvarez and Lucas, 2007).

all other markets, weighted by the reduced-form elasticities β_{ii} and β_{ij}, respectively. The aggregation formula thus maps measures of shock exposure in partial equilibrium for all markets (i.e., the shifts in excess labor demand) into general equilibrium responses of wages in each market. As a special case, it provides a closed-form characterization (up to a first-order approximation) for the solution of the non-linear system of equations for counterfactuals in gravity trade models (see e.g. Proposition 2 in Arkolakis et al. (2012)).

The reduced-form elasticity β_{ij} is a series expansion of the spatial links matrix $\bar{\boldsymbol{\gamma}}^0$. Thus, spatial spillovers are stronger between markets with tighter ties in terms of bilateral sales or competition, as captured by ρ^0_{ij}, and in terms of third-market connections in the network, as captured by the power series term. Intuitively, any wage change necessary to restore market clearing in market j following an exogenous shock to its labor demand will endogenously shift labor demand in all other markets i through changes in both j's demand for i products and j's market share in other markets served by i. These endogenous shifts in the labor demand of other markets must also be corrected in general equilibrium, triggering the multiple rounds of adjustment summarized in the higher-order terms of the power series. This generates a pattern of spatial percolation of regional shocks that is similar to that of the percolation of shocks across production networks (Acemoglu et al. (2016b) and Carvalho and Tahbaz-Salehi (2019)).

The representation in (21) links our model to the evidence in Section 2: for foreign shocks in which $\hat{\eta}_i$ takes the shift-share form in (18), the direct effect, $\beta_{ii}\hat{\eta}_i$, is related to the direct impact of the market's employment exposure to import competition IC^t_i, while the spatial indirect effect, $\sum_{j\neq i}\beta_{ij}\hat{\eta}_j$, is related to the impact of the gravity-based measure of exposure to shocks in other markets, GC^t_i. This link between the model's predictions and the empirical evidence emphasizes the importance of measuring the magnitude of both the direct and indirect elasticities in order to correctly quantify the aggregate effect of trade shocks and, thus, also highlights the importance of solving the disconnect discussed in the previous section.

We can also use this characterization to rationalize our empirical findings. We first show that trade links generate the type of reinforcing spatial indirect effects documented in Section 2 – that is, direct and indirect reduced-form elasticities that have the same sign.

Corollary 1. *If $\kappa > 0$ and $\max_{i,j}|n^0_i - n^0_j|$ is low enough, then $\tilde{\gamma}_{ij} \geq 0$ and $\beta_{ij} \geq 0$ $\forall i,j$.*

Consider again the same foreign productivity gain introduced in Section 3.2.1 ($\hat{\zeta}_{\mathrm{F},s} > 0$), while setting the foreign wage to be the economy's numeraire ($\beta_{iF} = 0$ for all i). This leads to a negative shift in i's excess demand ($\hat{\eta}_i < 0$, for all $i \neq F$), which then has a negative effect not only on the labor demand in that market, but also on all other markets ($\beta_{ji} \geq 0$). Intuitively, the negative demand shift pushes down i's wage (relative to the foreign country) and, consequently, also the trade demand in all other markets j through losses in both their sales to i (captured by $r^0_{ji}\phi^0_i$) and their market share in all destinations (captured by $\sum_{s,d}\ell^0_{j,s}r^0_{jd,s}x^0_{id,s}$). In this case, spatial links between regions reinforce the negative direct effect of an import competition shock. The bound on the dispersion of n^0_i guarantees that these demand channels are not overturned by labor supply changes due to the impact of wages on the non-employment payoff.

Second, we investigate the determinants of the size of the reduced-form elasticities to understand the drivers of the large differential effects estimated in Section 2. To do so, it is useful to focus on the special case in which the spatial indirect effects are identical, which arises when labor supply elasticities and trade links are the same in all markets.

Corollary 2. *Assume that markets have the same labor supply elasticity ($\phi_j^0 = \phi^0$) and trade links ($\xi_{j,s} = \xi_s$, $x_{ij,s}^0 = x_{i,s}^0$, and $\frac{\sum_s \xi_s x_{i,s}^0 x_{j,s}^0}{\sum_s \xi_s x_{i,s}^0} = \chi_j$). Then,*

$$\hat{w}_i \quad = \quad \tfrac{1}{\kappa+\phi^0}\hat{\eta}_i(\hat{\boldsymbol{\tau}})+\bar{\eta} \quad \textit{such that} \quad \bar{\eta} \equiv \textstyle\sum_j \tfrac{\beta_j}{\kappa+\phi^0}\hat{\eta}_j(\hat{\boldsymbol{\tau}}) \ . \tag{22}$$

The direct reduced-form elasticity $(\kappa+\phi^0)^{-1}$ is positive, increasing in $\psi\phi$ and decreasing in σ, and the indirect reduced-form elasticity β_j is positive and increasing in j's size.

The differential direct impact of shock exposure on wages, $(\kappa+\phi^0)^{-1}$, is decreasing in the labor demand elasticity, κ. In fact, our estimates below indicate that the disconnect documented in Section 2.3 arises in part from the high value for the labor demand elasticity implied by Ricardian spatial models (in which $\kappa = \sigma - 1$ due to the lack of agglomeration forces, $\psi = 0$). The corollary also indicates that market j's (symmetric) impact on other markets is proportional to its size.

In addition, the symmetry in spatial links gives rise to an "endogenous" fixed-effect, $\bar{\eta}$, comprising all the spatial indirect effects of the shock in general equilibrium. Hence, Corollary 2 establishes sufficient conditions for wage changes in a market to be a linear combination of its shift-share shock exposure plus a common fixed-effect. This special case thus yields a tight connection between our characterization and empirical shift-share specifications that followed Bartik (1991). The frameworks proposed in Nakamura and Steinsson (2014) and Beraja et al. (2019), given the absence of trade costs, are akin the case of identical spatial linkages across markets considered in Corollary 2.

Lastly, we characterize the importance of the expenditure shock exposure. While it does not matter for responses in wages and employment, it does affect changes in the price index.

Corollary 3. *Consider any shock to bilateral shifters $\hat{\boldsymbol{\tau}}$. If condition (20) holds, then (up to a first-order approximation)*

$$\hat{P}_i = \sum_j \beta_{ij}^C \hat{\eta}_j(\hat{\boldsymbol{\tau}}) + \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) \quad \textit{where} \tag{23}$$

$$\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \sum_{s,o} \xi_{i,s} x_{oi,s}^0 \hat{\tau}_{oi,s}, \quad \textit{and} \quad \beta_{ij}^C \equiv \sum_o \left(x_{oi}^0 \frac{\kappa}{\sigma-1} + \left(1 - \frac{\kappa}{\sigma-1}\right)\omega_o^0 \right) \beta_{oj}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0). \tag{24}$$

The price index change combines two effects. The first term, $\sum_j \beta_{ij}^C \hat{\eta}_j(\hat{\boldsymbol{\tau}})$, measures the impact of the shock on the market's consumption cost through the endogenous changes in production costs arising from the wage responses in Theorem 1. The second term, $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$, measures the shock's impact on the exogenous component of consumption costs. It is the average change in bilateral trade shifters of a destination market, weighted by its final spending share across sectors and origins. To gain intuition for this term, consider again the foreign sectoral shock introduced in Section 3.2.1 for which $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ is a shift-share variable based on sectoral spending shares, $\hat{\eta}_i^C \propto -\sum_s \xi_{i,s} \hat{\zeta}_{\mathrm{F},s}$. In this case, the price index falls more in markets with a higher initial spending share on sectors in which the foreign country experienced stronger productivity growth. In the absence of intermediate goods, final and gross spending shares are equal, implying that $\hat{\eta}_i^C$ is proportional to the import expenditure exposure IE_i used in Section 2.

Two comments are useful at this point. First, in this simple model, consumption cost exposure does not affect wages and employment across markets. While this is consistent with the evidence in Section 2, Section 4 shows that the sensitivity of labor supply to the consumption price index controls how much $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ affects labor market outcomes. Second, changes in the real wage, w_i/P_i, combine the direct impact of the shock on consumption costs, measured by $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$, with the terms-of-trade effects implied by the shock, measured by $\sum_j (\beta_{ij} - \beta_{ij}^C) \hat{\eta}_j(\hat{\boldsymbol{\tau}})$.[26]

3.3 From Theory to an Empirical Specification

In this section, we use Theorem 1 to derive an empirical specification that yields the general equilibrium impact of observed trade shocks on regional outcomes. We consider two observed equilibria that differ because of the realization of random shocks, $\hat{\tau}_{ij,s}$, and assume that we observe a component of these shocks, $\hat{\tau}_{ij,s}^{\mathrm{obs}}$. Without loss of generality, we can define the unobserved component of the shocks as $\hat{\boldsymbol{\tau}}^{\mathrm{unbs}} = \hat{\boldsymbol{\tau}} - \hat{\boldsymbol{\tau}}^{\mathrm{obs}}$, so that

$$\hat{\eta}_i(\hat{\boldsymbol{\tau}}) = \sum_s \ell_{i,s}^0 \hat{z}_{i,s}^{\mathrm{obs}} + \hat{\eta}_i(\hat{\boldsymbol{\tau}}^{\mathrm{unbs}}), \tag{25}$$

where $\hat{z}_{i,s}^{\mathrm{obs}} \equiv (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}^{\mathrm{obs}})$ is the impact of $\hat{\boldsymbol{\tau}}^{\mathrm{obs}}$ on market i's sector s demand (defined in (17)).

We show in Appendix A.1.6 that by combining the decomposition in (25), the wage response in (21), and the supply relationships in (8)–(9), we obtain a structural relationship between changes in observed labor market outcomes and market exposure to observed and unobserved shocks:

$$\begin{bmatrix} \Delta \overline{\ln w_i} \\ \Delta \ln n_i \end{bmatrix} = \begin{bmatrix} \alpha^w \\ \alpha^n \end{bmatrix} + \sum_j \begin{bmatrix} \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \\ \beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \end{bmatrix} \left(\sum_s \ell_{j,s}^0 \hat{z}_{j,s}^{\mathrm{obs}} \right) + \begin{bmatrix} \nu_i^w \\ \nu_i^n \end{bmatrix}, \tag{26}$$

where $\beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \equiv (n_i^0 \beta_{ij} + (1-n_i^0)\sum_d \omega_d^0 \beta_{dj})$ and $\beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \equiv \phi(1-n_i^0)(\beta_{ij} - \sum_d \omega_d^0 \beta_{dj})$, with $\beta_{ij} = \beta_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ given by (21). In this expression, α^w and ν_i^w are, respectively, the average and idiosyncratic changes in wages generated by the unobserved component of trade shocks $\hat{\boldsymbol{\tau}}^{\mathrm{unbs}}$.[27] α^n and ν_i^n are similarly defined for changes in the employment rate.

Through the lens of our model, both the residuals (ν_i^w, ν_i^n) and the constants (α^w, α^n) are not functions

[26] Note that in our framework the welfare of an individual corresponds to real wage (if working), or to real benefit from non-employment (if not working). Even in a setting with a representative agent with endogenous labor supply, it is easy to show that the equivalent variation associated with a trade shock is proportional to the change in the real wage (see Appendix B.3.1 of the old version of our paper, Adao et al. (2020a)).

[27] Formally, $\alpha^w \equiv I^{-1} \sum_{i,j} \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) E[\hat{\eta}_j(\hat{\boldsymbol{\tau}}^{\mathrm{unbs}})]$ and $\nu_i^w \equiv \sum_j \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \hat{\eta}_j(\hat{\boldsymbol{\tau}}^{\mathrm{unbs}}) - \alpha^w$.

of the observed shocks in $\hat{z}^{\text{obs}}_{i,s}$. Because of this property, knowledge of the reduced-form elasticities $\beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ and $\beta^n_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ is sufficient to compute *both* the differential *and* the aggregate impacts in general equilibrium of the observed shock exposure of markets on employment and wages. Thus, estimates of the reduced-form elasticities based on equation (26) can be aggregated in order to obtain the general equilibrium impact of the observed shock. The flip side of this observation is that a disconnect between the model's reduced-form elasticities and their empirical counterparts constitutes a rejection of both the differential and aggregate predictions of the model.

To take equation (26) to the data, we impose the following assumption.

Assumption 1. *For all markets and sectors, (i) the observed shock has the same expectation,* $E[\hat{\tau}^{obs}_{ij,s}|\boldsymbol{\mathcal{W}}^0]=\tau^{obs}$, *and (ii) observed and unobserved shocks are uncorrelated,*

$$Cov(\hat{\tau}^{obs}_{ij,s},\hat{\tau}^{unbs}_{od,k}|\boldsymbol{\mathcal{W}}^0)=0. \tag{27}$$

This assumption guarantees the causal interpretation of estimates in the literature on the impact of trade cost shocks on trade flows, firms, industries, or regions (see e.g. Autor et al. (2013), Kovak (2013) and Pierce and Schott (2016a)). It is equivalent to the quasi-random assignment of shocks that yields identification of shift-share specifications – see Adão et al. (2019). Since this assumption is not testable, how reasonable it is must be evaluated in each particular application. We return to this point below in the context of the China shock.[28]

As shown in Appendix A.1.7, the orthogonality assumption in (27) implies that the unobserved residuals in (26) are orthogonal to measures of market-level exposure to the observed shocks:

$$E\left[\nu^w_i\sum_j h^w_{ij}Z_j\right]=E\left[\nu^n_i\sum_j h^n_{ij}Z_j\right]=0 \quad \text{for any real matrices} \quad \{h^w_{ij},h^n_{ij}\}_j, \tag{28}$$

where $Z_j\equiv\sum_s\ell^0_{j,s}(\hat{z}^{\text{obs}}_{j,s}-\bar{z}^{\text{obs}}_{j,s})$ is market j's exposure to the de-meaned shock, with $\bar{z}^{\text{obs}}_{j,s}\equiv(1-\sigma)\mu_{j,s}(\tau^{\text{obs}})$ computed by setting all observed shocks to their expected value. The use of de-meaned shifters avoids identification threats arising, even under (27), from markets being more exposed to all types of shocks.

We now discuss a number of advantages of using (26) and (28) for empirical analyses of the aggregate and differential effects of observed trade shocks. First, our specification links in a transparent way the shock's impact in general equilibrium to exposure measures and reduced-form effects (direct and indirect). Equations (26) and (28) then connect such an impact to moments in the data associated with the elasticity of market-level outcomes to the shock exposure of different markets. The empirical content of (26) and (28) is a significant departure from the common approach of computing the shock's general equilibrium impact using calibrated spatial models – either in quantitative frameworks with rich calibrated spatial

[28] It is easy to allow for shocks in $\bar{b}_i$ (akin to labor supply or amenities shocks) to affect outcomes through the definitions of ν^w_i and ν^n_i. In this case, in addition to condition (27), we must assume that $Cov(\hat{\tau}^{\text{obs}}_{ij,s},\hat{\bar{b}}_o|\boldsymbol{\mathcal{W}}^0)=0$.

links (as in Redding and Rossi-Hansberg (2017)), or in frameworks combining an empirical strategy of the form in (22) and a calibrated spatial model to quantify the common "missing intercept" (as in Kovak (2013); Nakamura and Steinsson (2014); Mian and Sufi (2014); Beraja et al. (2019)).

Second, (26) and (28) can be used to estimate the parameter vector $\boldsymbol{\theta}$ and, therefore, $\beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ and $\beta^n_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$. Intuitively, identification comes from how market-level outcomes directly and indirectly respond to the shock exposure of markets with stronger (bilateral and higher-order) cross-market links in γ_{ij} (as defined in (19)). Formally, it follows from applying the usual rank condition for non-linear moment conditions in Newey and McFadden (1994) and Chen et al. (2014) to the specification in (26) that is non-linear in $\boldsymbol{\theta}$.[29] In addition, we show in Appendix A.1.8 that the estimation of $\boldsymbol{\theta}$ with (26)–(28) is more efficient than using "intuitive" instrumental variables for structural relationships between endogenous outcomes. Formally, we build on Chamberlain (1987) to derive the optimal moment conditions in the context of our general equilibrium model: that is, we characterize the weights, $\{h^w_{ij}, h^n_{ij}\}_j$, that minimize the variance of the GMM estimator of $\boldsymbol{\theta}$ based on (26)–(28). The efficiency gains arise because the optimal weights $\{h^w_{ij}, h^n_{ij}\}_j$ rely on both market i's own shock exposure associated with $\boldsymbol{\theta}$ as well as its indirect exposure to other markets in general equilibrium.

Third, (26) and (28) can be used to evaluate whether spatial models generate predictions that are consistent with the actual regional responses to observed shocks. Specifically, for a given $\boldsymbol{\theta}$, the predicted response in any labor market outcome Y_i to the observed shock can be written as $\hat{Y}^M_i(\boldsymbol{Z}|\boldsymbol{\theta},\boldsymbol{W}^0) \equiv \sum_j \beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) Z_j$ and, therefore, the observed log-change in the outcome, $\hat{Y}_i$, is

$$\hat{Y}_i = \alpha^Y + \rho^Y \hat{Y}^M_i(\boldsymbol{Z}|\boldsymbol{\theta},\boldsymbol{W}^0) + \nu^Y_i, \qquad E[\nu^Y_i \hat{Y}^M_i(\boldsymbol{Z}|\boldsymbol{\theta},\boldsymbol{W}^0)] = 0. \tag{29}$$

Under the null hypothesis that the model is well specified, the pass-through coefficient from predicted to actual changes in any outcome is one (i.e., $\rho^Y = 1$). Intuitively, if the orthogonality condition in (27) holds, an estimated coefficient larger than one means that the predicted responses in the model must be re-scaled by a large coefficient to match the differential impact of the observed shock across markets and, therefore, are too small. The opposite holds if the estimated fit coefficient is small and/or non-significant.[30]

Importantly, this additional moment has the advantage of relying exactly on the reduced-form

[29] Leveraging the facts that (26) is additive in the residual and that $\boldsymbol{\theta}$ only enters (26) through functions that are multiplicative on the random variables $\hat{z}^{\text{obs}}_{j,s}$, identification of $\boldsymbol{\theta}$ follows from the rank of $\sum_{i,j,d} \big(h^w_{ij} \nabla_{\boldsymbol{\theta}} \beta^w_{id}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}), h^n_{ij} \nabla_{\boldsymbol{\theta}} \beta^n_{id}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\big) E[Z_j Z_d|\boldsymbol{\mathcal{W}}^0]$ being equal to $\dim(\boldsymbol{\theta})$. Notice that, since $E[Z_j Z_d|\boldsymbol{\mathcal{W}}^0] \neq 0$ for some j and d is a weak condition (as it includes $j = d$), identification essentially relies on all entries of $\boldsymbol{\theta}$ being associated with heterogeneous (direct and indirect) reduced-form effects across markets. In other words, we cannot identify parameters that are *only* associated with a common component of the reduced-form effect on all markets i. This condition is weaker than the Stable Unit Treatment Value Assumption (SUTVA) that yields identification of the direct reduced-form elasticity to local shock exposure in structural models with a common "missing intercept" – see result 2 of Chodorow-Reich (2020). SUTVA rules out that shock exposure of a region differentially affects outcomes in other regions, as we documented to be the case for the China shock in Section 2.

[30] This type of "slope" test for evaluating the model fit has a long tradition in international economics – e.g., see Davis and Weinstein (2001) and Costinot and Donaldson (2012). Recently, Kovak (2013) and Adao et al. (2020b) use a version of it to evaluate model predictions for how factor prices respond to trade shocks.

elasticities that are sufficient for the computation of the model's counterfactual predictions in general equilibrium. Therefore, a fit coefficient very different from one constitutes a rejection of the predicted differential responses of the model and, given the discussion above, also undermines the credibility of the model's aggregate predictions. Note that this is possible even when $\boldsymbol{\theta}$ is structurally estimated, as predicted reduced-form responses to observed shocks may not be consistent with their estimated counterparts – for example, see the discussion in Section 2.3. Moreover, in contrast to the type of statistical decompositions proposed by Kehoe et al. (2017), the estimation of the fit coefficient does not depend on how much of the cross-market variation in the outcome of interest is driven by other shocks, because the orthogonality condition in (27) guarantees the identification of the impact of the observed shock while holding other unobserved shocks constant.

Fourth, it is worth mentioning that (26)–(28) remain valid under a flexible structure of spatial links and arbitrary unobserved shocks. Such a flexibility is in contrast with the "market access" approach in Donaldson and Hornbeck (2016). In such a setting, market access is an endogenous variable obtained from solving the general equilibrium model under restrictive assumptions on the economy's spatial links – specifically, symmetric trade costs that are fully observed before and after the shock.[31] Even under these assumptions, one cannot simply aggregate the empirical specification to compute the general equilibrium impact of changes in market access as it also involves an endogenous common component that is not separately identified from the constant.

Finally, our empirical strategy is distinct from an indirect inference procedure that calibrates parameters to match arbitrarily chosen moments generated in the model with simulated shocks. Such a procedure may yield biased estimates of the (direct and indirect) reduced-form elasticities if the chosen moments are not closely related to the model-implied relationship in (26), or the model replicates the chosen moments mostly through other unobserved shocks instead of the observed shock of interest. In contrast, our strategy is not subject to these concerns because we derived it from the model's predictions for the impact of the observed shock.

So far, we have discussed the advantages of using equation (26) for empirical analysis. The use of this expression is, however, subject to two important caveats. The first is that the separability of the unobserved residuals (ν_i^w,ν_i^n), which is necessary for the derivation of the moment conditions above, follows from the log-linearization of the model around the initial equilibrium. This raises the concern that equation (26) may be a poor approximation for the model's predictions depending on the application. We propose, and implement below, two ways of addressing such a concern that rely on the exact solution for the model's predictions that we obtain with the integral of our formulas (as described in Appendix A.3.3). First, once the model has been estimated, we attest the quality of the linear approximation by

[31] Donaldson and Hornbeck (2016) point out that "the calculation of market access (via equation (9)) requires the measurement of all trade costs." This is true even if one extends their environment to obtain expressions in terms of changes in market access. In this case, knowledge of initial trade flows subsumes knowledge of initial trade costs, but it is still necessary to observe all components of bilateral trade shocks (in our notation, $\hat{\boldsymbol{\tau}}^{\text{unbs}}=0$). In gravity trade models, identifying $\hat{\boldsymbol{\tau}}$ typically requires assuming symmetric shocks, as in Head and Ries (2001).

showing that it yields predictions that are similar to the exact predicted impact of the observed shock. Second, to account for non-linear responses to the observed shock, we show that the estimates of $\boldsymbol{\theta}$ are similar when we extend equation (26) to use the integral of our first-order formulas for the impact of the observed shock. Thus, results are robust to removing any "approximation error" from the structural residual in (26). Appendix B.2.2 presents further details about the implementation of these procedures.

Lastly, one may also be concerned that we specify the reduced-form elasticities as parametric functions of the data in $\boldsymbol{\mathcal{W}}^0$ and the parameters in $\boldsymbol{\theta}$. We follow this approach because a type of dimensionality curse prevents the non-parametric estimation of the reduced-form elasticities in (26), as we only observe outcomes for I markets, but (34) has I^2 reduced-form elasticities.[32] Thus, as in any structural framework, the derivation of (26) requires the spatial model to be well specified. In case it is not, additional channels will be included in the residuals and the constant, which would lead to the violation of the exclusion restriction in equation (27) and the mis-measurement of the aggregate effects. To explore additional channels previously highlighted by the literature, we extend our methodology to a broader set of models in the next section.

4 Other Margins of General Equilibrium Effects in Space

We now extend the empirical specification in Section 3.3 for an economy with trade in intermediate goods as well as a labor supply that depends on migration choices and consumption prices.

4.1 Environment

Labor Supply with Endogenous Population. Each country c has a continuum $\bar{N}_c$ of workers. Individuals have heterogeneous preferences for the amenities of different markets and draw market-specific amenities $\{a_i(\iota)\}_{i\in\mathcal{I}_c}$ independently from a Frechet distribution with shape parameter ϑ and scale $\bar{\nu}_j$. As before, we assume that, conditional on residing in market i, individuals independently draw a realization of their income potentials $(l(\iota),u(\iota))$ from the same Frechet distribution used in Section 3. Thus, the employment rate is given by n_i in (8), and the average log wage by $\overline{\ln w_i}$ in (9). Worker ι chooses in which market $i\in\mathcal{I}_c$ to reside based on expected payoffs, $U_i(\iota)=a_i(\iota)\varrho w_i^{\phi}(w_i^{\phi}+b_i^{\phi})^{\frac{1-\phi}{\phi}}/P_i$. This implies a location choice similar to that of recent spatial frameworks (Allen and Arkolakis, 2014; Redding, 2016):

$$N_i=\frac{\bar{\nu}_i P_i^{-\vartheta} w_i^{\phi\vartheta}(w_i^{\phi}+b_i^{\phi})^{\vartheta\frac{1-\phi}{\phi}}}{\sum_{j\in\mathcal{I}_{c(i)}}\bar{\nu}_j P_j^{-\vartheta} w_j^{\phi\vartheta}(w_j^{\phi}+b_j^{\phi})^{\vartheta\frac{1-\phi}{\phi}}}\bar{N}_c. \tag{30}$$

[32] This procedure effectively projects the reduced-form elasticities onto observable variables regulating the strength of spatial links. It is similar to the common practice in demand estimation of specifying cross-price demand elasticities in terms of observable variables (Berry, 1994; Berry et al., 1995).

Population in market i (and consequently labor supply) is higher whenever the per-capita real income in i is higher relative to that of other markets in the country. ϑ controls the sensitivity of a market's population to changes in its relative per-capita real income and, as we formally show below, the type of responses in population to regional shock exposure studied in Section 2.

We further generalize the model by introducing a parameter that controls the sensitivity of the payoff of not working to local prices: $b_i = \bar{b}_i P_i^{\lambda}(\Omega(\{w_j\}_j))^{1-\lambda}$. When λ is higher, the same decline in import prices has a stronger positive impact on the relative payoff of working and, consequently, on labor supply. Thus, λ determines the magnitude of the responses of wages and employment to shocks in the supply of imported goods (such as those that we investigated in Section 2). Note that, in the limit case of $\lambda = 1$, labor supply becomes a function of the market's real wage.

Gravity Trade in Final and Intermediate Goods. We follow the gravity trade framework with intermediate inputs of Caliendo and Parro (2015) and Costinot and Rodríguez-Clare (2014). We maintain sectoral gravity trade links across markets: sector s of origin i has a representative competitive firm that produces a differentiated tradable good at a cost of $p_{i,s}$ and faces iceberg trade costs of $\tau_{ij,s}$ to sell to j. In each sector and destination, the differentiated products of all origins are combined to produce a composite non-tradable good, using a CES aggregator with elasticity σ. These sectoral composite goods are inputs for the production of the final consumption good and the tradable differentiated goods.

The production function of the final consumption good is a Cobb-Douglas aggregator of the sectoral non-tradable composite goods with shares $\xi_{i,s}$, so that the final good price is still given by (7). In addition, we assume that the production function of the differentiated good of sector s is Cobb-Douglas between labor and an intermediate input aggregator, with spending shares of $a_{i,s}^L$ and $a_{i,s}^M$, respectively. The intermediate input aggregator in sector s, $M_{i,s}$, is also a Cobb-Douglas function of the sectoral non-tradable composite goods, with $\xi_{i,ks}^M$ denoting the share of intermediate spending on sector k ($\xi_{i,ks}^M > 0$ and $\sum_k \xi_{i,ks}^M = 1$). We maintain the assumption of external economies of scale associated with the market's employment rate (as regulated by an elasticity ψ).[33] From cost minimization, the production cost in sector s of market i is

$$p_{i,s} = (w_i)^{1-\psi\phi - a_{i,s}^M} (P_{i,s}^M)^{a_{i,s}^M} (b_i)^{\psi\phi}, \quad \text{with} \quad P_{i,s}^M = \Pi_k (P_{i,k})^{\xi_{i,ks}^M}. \tag{31}$$

Notice that, relative to the model of Section 3, the pass-through of wages to production costs is now a function of the share of intermediate goods in production. Given the same value of $\psi\phi$, a higher $a_{i,s}^M$ will lower the sensitivity of labor demand to the local wage, since input prices also depend on the labor cost in other markets through input purchases. As we formally show below, this mechanism generates wage responses to a given shift in excess labor demand that are larger when the share of intermediate goods in production is higher.

[33] The general specification of the model in Appendix A.3 also features an elasticity of productivity to population. We set this elasticity to zero in this section because we cannot estimate it given the lack of population responses to the China shock documented in Section 2.

Finally, in Appendix A.2, we define the equilibrium wage vector in terms of an excess labor demand system: $D_i(\boldsymbol{w}|\boldsymbol{\tau})=0$ for all i. All the remaining derivations for this section are in the same Appendix.

4.2 An Extended Reduced-Form Representation

We now extend the empirical specification in Section 3.3, and characterize how the presence of intermediate goods changes our measures of shock exposure. Consider first the shift in market-level sales caused by the shock (holding constant all endogenous variables), the "revenue shock exposure" defined as $\eta_i^R(\hat{\boldsymbol{\tau}})\equiv\sum_{s,o,d}\frac{\partial \ln R_i}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s}$:

$$\eta_i^R(\hat{\boldsymbol{\tau}})=(1-\sigma)\sum_s \ell_{i,s}^0\big(\mu_{i,s}(\hat{\boldsymbol{\tau}})+\mu_{i,s}^U(\hat{\boldsymbol{\tau}})\big) \quad \text{where} \quad \mu_{i,s}^U(\hat{\boldsymbol{\tau}})\equiv\sum_{j,k} b_{is,jk}^U \mu_{j,k}(\hat{\boldsymbol{\tau}}), \quad \bar{\boldsymbol{b}}^U\equiv\sum_{d=1}^{\infty}\big(\bar{\boldsymbol{r}}^U\big)^d, \tag{32}$$

and $\bar{\boldsymbol{r}}^U\equiv[r_{is,jk}^U]_{is,jk}$ with $r_{is,jk}^U\equiv X_{ij,sk}^M/R_{i,s}$ denoting the share of revenue in sector s of market i, $R_{i,s}$, coming from its intermediate sales to sector k of market j, $X_{ij,sk}^M$. Here, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ is the same shock to the demand for goods of sector s of market i defined in (17). Since the demand shift for the products of a sector-market affects its input purchases, it also generates revenue shifts for upstream sectors and markets that we capture in the series expansion of the upstream matrix of revenue shares, $\bar{\boldsymbol{r}}^U$.

We also consider the shock's impact on input costs, $\eta_{i,s}^M(\hat{\boldsymbol{\tau}})\equiv\sum_{k,o,d}\frac{\partial \ln P_{i,s}^M}{\partial \ln \tau_{od,k}}\hat{\tau}_{od,k}$:

$$\eta_{i,s}^M(\hat{\boldsymbol{\tau}})=\mu_{i,s}^M(\hat{\boldsymbol{\tau}})+\sum_{j,k} b_{is,jk}^D \mu_{j,k}^M(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \mu_{i,s}^M(\hat{\boldsymbol{\tau}})\equiv\sum_{j,k} x_{ji,k}^0 \xi_{i,ks}^M \hat{\tau}_{ji,k}, \quad \bar{\boldsymbol{b}}^D\equiv\sum_{d=1}^{\infty}\big(\bar{\boldsymbol{x}}^D\big)^d, \tag{33}$$

and $\bar{\boldsymbol{x}}^D\equiv[x_{is,jk}^D]_{is,jk}$ with $x_{is,jk}^D\equiv a_{j,k}^M X_{ji,ks}^M/a_{i,s}^M R_{i,s}$ denoting the share of input expenditure in sector s from i that corresponds to input purchases from sector k of market j. This "input shock exposure" has again two terms. $\mu_{i,s}^M(\hat{\boldsymbol{\tau}})$ is the direct impact of the shock on the unit input cost of sector s from market i, which by Shepard's lemma is simply an average of the shocks across sectors and markets, weighted by the spending shares on them. In addition, cost shocks in other sectors and markets have a downstream impact on the cost of production in sector s from i through its intermediate input purchases, with weights given by the series expansion of the matrix of intermediate cost shares, $\bar{\boldsymbol{x}}^D$.

Our theoretical exposure measures are closely related to measures of upstreamness and downstreamness (in levels) for open economies suggested by Fally (2012). They are the open economy analogs of the Leontief matrices controlling shock percolation across sectors in a closed economy network model (see Acemoglu et al. (2016b) and Carvalho and Tahbaz-Salehi (2019)), and related to the forces highlighted in the open economy model of Baqaee and Farhi (2019).

Theorem 1 still holds in this general setting and, as in Section 3.3, the changes in any labor market outcome $\hat{Y}_i \in \{\Delta\overline{\ln w_i}, \Delta\ln n_i, \Delta\ln N_i\}$ have the following reduced-form representation:

$$\hat{Y}_i = \alpha^Y + \sum_j \beta_{ij}^{Y,R}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j^R(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_j \beta_{ij}^{Y,C}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j^C(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_{j,s} \beta_{ij,s}^{Y,M}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_{j,s}^M(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \nu_i^Y, \tag{34}$$

where we now define the parameter vector and the matrix of initial conditions as $\boldsymbol{\theta} \equiv (\phi,\psi,\lambda,\vartheta,\sigma)$ and $\boldsymbol{\mathcal{W}}^0 \equiv \{n_i^0, \omega_i^0, \{X_{ij,s}^0\}_{j,s}, \{\xi_{i,s}, a_{i,s}^L\}_s, \{\xi_{i,ks}^M\}_{k,s}\}_i$. Under Assumption 1,

$$E\left[\nu_i^Y \sum_j h_{ij}^{Y,R}\hat{\eta}_j^R(\ddot{\boldsymbol{\tau}}^{\text{obs}})\right] = E\left[\nu_i^Y \sum_j h_{ij}^{Y,C}\hat{\eta}_j^C(\ddot{\boldsymbol{\tau}}^{\text{obs}})\right] = E\left[\nu_i^Y \sum_{j,s} h_{ij,s}^{Y,M}\hat{\eta}_{j,s}^M(\ddot{\boldsymbol{\tau}}^{\text{obs}})\right] = 0 \tag{35}$$

for the de-meaned shock, $\ddot{\boldsymbol{\tau}}^{\text{obs}} \equiv \hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}}$, and any real matrices $\{h_{ij}^{Y,R}, h_{ij}^{Y,C}, \{h_{ij,s}^{Y,M}\}_s\}_j$.

Equations (34) and (35) generalize the empirical specification in Section 3.3. As such, (34)–(35) inherit all the properties outlined in Section 3.3. However, (34) yields three additional insights.

The first term in (34) is the analog for this general model of the reduced-form responses in (26) for the simpler model of Section 3. Not only trade in intermediate goods requires the measurement of upstream revenue exposure (i.e., $\sum_s \ell_{i,s}^0 \mu_{i,s}^U(\hat{\boldsymbol{\tau}})$ in (32)), but it also alters the reduced-form elasticities to revenue shock exposure (i.e., $\boldsymbol{\mathcal{W}}^0$ includes final and intermediate spending shares). In particular, higher intermediate input usage plays a similar role to stronger agglomeration forces in amplifying reduced-form elasticities by flattening the labor demand curve. We formalize this intuition in Appendix A.2.5 by showing that, in a symmetric economy, the labor demand elasticity is instead $\kappa = (\sigma-1)(1-\psi\phi-a^M)$ and wage responses are increasing in κ.

The second term indicates that shocks in the price of imported final goods, $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}^{\text{obs}})$, also affect labor market outcomes in this more general framework. This follows from the impact that such shocks have on both the non-employment payoff (as regulated by λ) and the allocation of individuals across markets (as regulated by ϑ). Formally, we can write $\beta_{ij}^{Y,C} = \lambda\tilde{\beta}_{ij}^{Y,C\lambda} + \vartheta\tilde{\beta}_{ij}^{Y,C\vartheta}$. Thus, when λ and ϑ are higher, the impact of consumption exposure on labor market outcomes is also stronger. In fact, $\beta_{ij}^{Y,C} = 0$ for the labor supply structure of Section 3 that entails $\lambda = \vartheta = 0$.

The last term captures how outcomes respond to shocks in the cost of imported inputs, $\hat{\eta}_{j,s}^M(\hat{\boldsymbol{\tau}}^{\text{obs}})$. Such responses arise from two channels. When input costs fall in a market, the market's labor demand increases due to market share gains in all destinations. Moreover, input cost shocks affect labor supply through changes in the consumption price index across markets.

Notice that the representation in (34) links our model to the evidence in Section 2. For the same foreign shock $\hat{\zeta}_{\text{F},s}$ of Section 3.2.1, the import expenditure exposure in (5) is a weighted average of the regional exposure to shocks in the cost of final and intermediate goods.[34] Thus, the evidence in Section 2 suggests that any potential gain in wages and employment created by declines in consumption costs, $\hat{\eta}_i^C$ and $\hat{\eta}_{i,s}^M$, are not strong enough to offset the negative impact caused by revenue losses due to import competition, $\hat{\eta}_i^R$.

[34] Formally, $\sum_s e_{i,s}^0 \hat{\zeta}_{F,s} \propto \sum_k \ell_{i,k}^0 a_{i,k}^0 \mu_{i,k}^M(\hat{\boldsymbol{\zeta}}) + (1 - \sum_k \ell_{i,k}^0 a_{i,k}^0)\eta_i^C(\hat{\boldsymbol{\zeta}})$ with $a_{i,k} = a_{i,k}^M / a_{i,k}^L$.

Generality of the Empirical Specification. In Appendix A.3.1, we show that our results hold for a general class of models encompassing most of the recent quantitative trade and spatial models reviewed by Costinot and Rodríguez-Clare (2014) and Redding and Rossi-Hansberg (2017). We outline general conditions that yield (34) with the same shift-share measures of exposure $\{\hat{\eta}_j^R, \hat{\eta}_j^C, \hat{\eta}_{j,s}^M\}$ that satisfy (35). This characterization follows three steps: (i) specifying the observed and unobserved trade shocks, (ii) solving for the first-order approximation of log-changes in observable outcomes, and (iii) defining the reduced-form elasticities as a function of initial conditions and elasticities in the model.

5 The General Equilibrium Effect of The China Shock

Our theoretical analysis has established that the general equilibrium impact of trade shocks on regions is intrinsically related to the reduced-form elasticities of regional outcomes to the shock exposure of different markets. We now use our characterization of these elasticities to empirically investigate how U.S. CZs were affected by the China shock.

5.1 Measuring the China Shock

We back out model-consistent sectoral demand shifts from ADH's measure of the China shock – that is, the per-worker growth in Chinese imports by eight developed countries between years t_0 and t, $\Delta M^t_{\text{China},s}$. Without loss of generality, we consider a decomposition of the shift in sectoral demand triggered by the China shock into a common component and destination-specific components: $(1-\sigma)x^{t_0}_{\text{China}j,s}\hat{\tau}^t_{\text{China}j,s} = \hat{\zeta}^t_{\text{China},s} + \hat{\varepsilon}^t_{\text{China}j,s}$ such that the size-weighted average of $\hat{\varepsilon}^t_{\text{China}j,s}$ is zero, $\sum_j E^{t_0}_{j,s}\hat{\varepsilon}^t_{\text{China}j,s}/\sum_j E^{t_0}_{j,s} = 0$. Our observed measure of the China shock is the common sectoral component $\hat{\zeta}^t_{\text{China},s}$, which we back out from $\Delta M^t_{\text{China},s}$ using the following relationship shown in Appendix A.4.1:

$$\Delta M^t_{\text{China},s} = \left(\frac{\sum_j E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}}\right)\hat{\zeta}^t_{\text{China},s} + \frac{\sum_j X^{t_0}_{\text{China}j,s}\Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}}, \tag{36}$$

where $\Lambda^t_{j,s}$ is the destination-sector fixed-effect in a sector-level gravity regression for log-changes in bilateral trade flows between years t_0 and t, and $L^{t_0}_{\text{US},s}$ is the U.S. employment in sector s at t_0.[35]

The structural relationship in (36) indicates that the sectoral shifter used in ADH combines two components. The first is proportional to the sectoral demand shift associated with shocks in China's trade costs, $(\sum_j E^{t_0}_{j,s}/L^{t_0}_{\text{US},s})\hat{\zeta}^t_{\text{China},s}$. The second is the average across destinations of changes in sectoral demand, $\sum_j X^{t_0}_{\text{China}j,s}\Lambda^t_{j,s}/L^{t_0}_{\text{US},s}$, which depends on changes in endogenous and exogenous variables in the world economy. As illustrated in Panel A of Figure B.2 in Appendix B.2.1, $\Delta M^t_{\text{China},s}$ and $(\sum_j E^{t_0}_{j,s}/L^{t_0}_{\text{US},s})\hat{\zeta}^t_{\text{China},s}$ have a correlation of 0.96 and, thus, China's productivity growth is the main

[35]We estimate $\Lambda^t_{j,s}$ with a gravity equation that also includes origin fixed-effects (but not a constant). We consider the same set of destinations used by ADH, and weigh observations by trade flows at t_0.

driver of the cross-sector variation in Chinese imports by developed countries.[36] This suggests that shocks other than Chinese productivity have little impact on the measure $\Delta M^t_{\text{China},s}$. Indeed, Panel C of Table B.8 in Appendix B.1 shows that the results in Table 1 are qualitatively similar when we compute the shift-share exposure variables in (3)–(5) using $\hat{\zeta}^t_{\text{China},s}$ instead of $\Delta M^t_{\text{China},s}$.

Our empirical specification requires $\hat{\zeta}^t_{\text{China},s}$ to satisfy Assumption 1. Given initial conditions, shocks to Chinese productivity must be uncorrelated with other unobserved shocks in the world economy. This is reasonable because the reduction in China's trade costs has been largely driven by China's transition to a market-oriented economy in this period and China's accession to the WTO in 2001 – for discussions, see Hsieh and Klenow (2009), Brandt et al. (2012) and Autor et al. (2013). In addition, Assumption 1 also requires observing the expected value of the shock, so that we can compute the de-meaned exposure measures used in (35). To maintain our analysis close to ADH, we follow the assumption implicit in their specification that the shock had the same mean across sectors and periods.[37] Thus, since all (de-meaned) exposure measures are a function of $x^{t_0}_{ij,s}\ddot{\hat{\tau}}^{\text{obs},t}_{ij,s}$, we compute them by setting $(1-\sigma)x^{t_0}_{\text{China}j,s}\ddot{\hat{\tau}}^{\text{obs},t}_{\text{China}j,s} = \hat{\zeta}^t_{\text{China},s} - (1/2S)\sum_{s',t'}\hat{\zeta}^{t'}_{\text{China},s'}$ for all j. This implies that the de-meaned revenue exposure (without intermediate production) in (28) is $Z_j = -\sum_s \ell^0_{j,s}(\hat{\zeta}^t_{\text{China},s} - (1/2S)\sum_{s',t'}\hat{\zeta}^{t'}_{\text{China},s'})$.

5.2 Measuring the Spatial Links

Next, we discuss the specification of the variables in $\boldsymbol{\mathcal{W}}^0$ necessary to compute the reduced-form elasticities for any given $\boldsymbol{\theta}$. We consider links between 722 U.S. CZs and 52 foreign countries in 152 SIC 3-digit manufacturing sectors, and one non-manufacturing sector. Appendix C presents details about the data construction procedure.

We first construct bilateral trade flows in each sector. We use trade data from UN Comtrade assembled by CEPII to measure country-to-country trade flows in each sector. We use the gravity structure of our model to impute domestic sales in each sector by combining bilateral trade flows and information on domestic sales in aggregate sectors obtained from Eora MRIO. Second, we distribute U.S. domestic and international trade flows across CZs using again the gravity structure of our model. Specifically, we first split U.S. Census data on imports and exports for each industry-country across CZs using measures of each CZ's share in that industry's national spending and production. We then impute bilateral trade shares across CZs using a gravity specification estimated with bilateral shipment data from the Commodity Flow Survey (CFS). Since our baseline model imposes trade balance, we

[36] Panel B of Figure B.2 shows that the correlation between $\Delta M^t_{\text{China},s}$ and $\hat{\zeta}^t_{\text{China},s}$ is lower, because $\hat{\zeta}^t_{\text{China},s}$ is not normalized by U.S. sectoral employment. However, some of the sectors in which China caused the strongest demand shifts are also those in which per-worker imports from China grew at the fastest pace, including toys, clothing, and furniture (see Table B.10 in Appendix B.2.1).

[37] Borusyak et al. (2018) argue that, since the realized growth in Chinese imports was stronger after 2000, this assumption raises the concern that estimates may capture confounding shocks to manufacturing that were also stronger in the second period. As a robustness, Appendix B.2.2 reports that our point estimates are similar but less precise when we implement their preferred specification that allows the shock's expected value to vary across the two periods.

adjust market sizes to balance trade flows given the bilateral trade shares.[38]

For U.S. CZs, we obtain final expenditure shares from the Public-use Micro-data from the Consumer Expenditure Surveys. For foreign countries, we use the final spending shares from the BEA input-output matrix. We also use the BEA input-output matrix to specify the sectoral intermediate cost shares for all markets. Finally, we set the share of intermediate inputs in total cost in each sector and market by assuming that $a^M_{j,k} = a_j a^M_k$ with a^M_k obtained from the NBER Manufacturing database, and selecting a_j to match observed value-added in each market.[39]

To specify the numeraire function of non-employment benefits $\Omega(\boldsymbol{w})$, we use the evidence in Chodorow-Reich and Karabarbounis (2016) that the non-employment payoff in the U.S. (the average change in b_i/P_i across U.S. CZs) varies substantially over the business cycle, exhibiting a correlation of 0.64 with annual changes in the U.S. per-capita real income (the average change in w_i/P_i across U.S. CZs).[40] We match this correlation by setting $\Omega(\boldsymbol{w})$ as the geometric average of income in the U.S. and the World, $\Omega(\boldsymbol{w}) = (W_{\text{US}}(\boldsymbol{w}))^{\bar{\omega}}(W_{\text{W}}(\boldsymbol{w}))^{1-\bar{\omega}}$. As shown in Appendix A.4.2, this implies that $\omega^{t_0}_j = \frac{\partial \ln \Omega(\boldsymbol{w}^{t_0})}{\partial \ln w_j} = \bar{\omega} W^{t_0}_j / W^{t_0}_{\text{US}} \mathbb{I}_{j \in \mathcal{I}_{\text{US}}} + (1-\bar{\omega}) W^{t_0}_j / W^{t_0}_{\text{W}}$ with $\bar{\omega} = 0.62$, where $W^{t_0}_j$ is j's GDP at t_0. In all other countries, we simplify our analysis by imposing that labor supply is exogenous.

5.3 Estimation of Reduced-Form Elasticities

Table 2 presents the estimates of $\boldsymbol{\theta}$ obtained with a GMM estimator based on (34)–(35) using the pooled sample of 722 U.S. CZs in 1990-2000 and 2000-2007. Because $\hat{\zeta}^t_{\text{China},s}$ already accounts for the trade elasticity, we do not estimate this parameter and set it to five (i.e., $\sigma - 1 = 5$), a typical value in the literature (see Costinot and Rodríguez-Clare (2014)).[41] In all specifications, we use the same control set in Table 1, and use the weights suggested by the optimal moment conditions in Appendix A.1.8:

$$h^{Y,R,t}_{ij} = \nabla_{\boldsymbol{\theta}} \beta^{Y,R}_{ij}(\boldsymbol{\theta}|\boldsymbol{W}^{t_0}), \quad h^{Y,C,t}_{ij} = \nabla_{\boldsymbol{\theta}} \beta^{Y,C}_{ij}(\boldsymbol{\theta}|\boldsymbol{W}^{t_0}), \quad h^{Y,M,t}_{ij,s} = \nabla_{\boldsymbol{\theta}} \beta^{Y,M}_{ij,s}(\boldsymbol{\theta}|\boldsymbol{W}^{t_0}). \tag{37}$$

In Panel A of Table 2, we consider the most general version of the model in Section 4. The first column reports an estimate of ϕ equal to 4.4. In our model, this parameter controls the (Marshallian)

[38] Table C.3 in Appendix C.1.2 reports validation tests using the CFS data. Regressions of actual on predicted trade flows across states and SCTGs yield coefficients close to 1 and R^2 of 0.48–0.83.

[39] We impose that final and intermediate spending shares are the same across countries because we are not aware of any comprehensive dataset that includes this information for all countries and 3-digit SIC sectors considered in our empirical application. Figure C.1 in Appendix C.2.2 shows that our calibration procedure almost exactly matches the observed shares of value added across U.S. CZs and foreign countries.

[40] Compared to annual fluctuations, changes over longer horizons in the non-employment payoff are even more volatile (e.g., the dispersion is three times higher for ten-year changes), but the correlation with changes in real income remains similar (e.g., it is 0.51 for ten-year changes).

[41] This is without loss of generality for the simple model in Section 3 as reduced-form elasticities only depend on the labor demand elasticity $\kappa \equiv (\sigma - 1)(1 - \psi\phi)$. The choice of σ affects the estimate of ψ, but does not alter the model predictions. In the more general model in Section 4, reduced-form elasticities depend separately on σ and ψ, but it is hard to separately identify these parameters in practice as they have similar effects on the model's predictions.

Table 2: Estimates of the Structural Parameters

ϕ	ψ	λ	ϑ
Panel A: Model with intermediate production in Section 4			
4.39	0.05	0.21	-0.19
(1.28)	(0.02)	(0.32)	(0.27)
Panel B: Model with intermediate production in Section 4			
4.16	0.05	0	0
(1.23)	(0.01)	-	-
Panel C: Model without intermediate production in Section 3			
2.53	0.35	0	0
(0.37)	(0.05)	-	-

Notes: Panels A and B report GMM estimates of $\boldsymbol{\theta}$ implied by the specification in (34) and (35), with the weight matrix in (37). Panel C reports GMM estimates of $\boldsymbol{\theta}$ implied by the specification in (26) and (28). Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications also include the baseline control vector used in Table 1. Standard errors in parentheses are clustered by state.

labor supply elasticity, which corresponds to $\phi(1-n_i^{t_0})$. Given that the median employment rate across CZs is 0.7, our estimate implies that the typical CZ has a labor supply elasticity of 1.3. This is similar to the labor supply elasticity implied by the ratio of the estimates of employment to wage responses to import competition reported in Table 1. Our point estimate of the Marshallian labor supply elasticity is closer to existing estimates based on market-level variation across regions and countries, but it is higher than estimates of the Hicksian elasticity based on micro-level responses of individuals – see Adão et al. (2019) and, for a review, see Chetty et al. (2013b).

The second column reports an estimate of ψ equal to 0.05, which implies strong agglomeration forces: the median elasticity of production costs to regional employment across CZs is $\psi/(1-n_i^{t_0})=0.17$, a value similar to that implied by the models in Krugman (1980) and Krugman (1991) given our trade elasticity of five. Our large agglomeration elasticity is consistent with evidence on regional responses to local demand shocks in the U.S. and Brazil (Kline and Moretti, 2014; Dix-Carneiro and Kovak, 2017) and regional labor supply shocks in Germany (Peters, 2019), and is in the upper range of the sectoral scale elasticities at the country-level in Bartelme et al. (2019).

In the third column, our estimate of λ is positive, but not statistically different from zero. The point estimate of 0.21 for λ implies that a decrease of 1% in the local price index is associated with a median increase in labor supply across U.S. CZs of $\lambda\phi(1-n_i)=0.28\%$. In line with the discussion in Section 4, such a small, non-significant estimate of λ follows from the evidence in Table 1 that higher expenditure exposure to the China shock had small, non-significant impacts on wages and employment across CZs. The fact that we reject $\lambda=1$ at usual levels indicates that our estimate is consistent with the evidence in Chodorow-Reich and Karabarbounis (2016) that the non-employment payoff in the U.S., b_i/P_i, responds to shocks in labor demand and supply.

Finally, the fourth column reports a negative and imprecise estimate of the elasticity of location choice to real wages, ϑ. Since ϑ is proportional to the reduced-form response of population to regional

Table 3: Reduced-form Elasticities and Shifts in Excess Labor Demand for U.S. CZs, 2000-2007

	Reduced-form Elasticity		Shift in Excess Labor Demand
	Direct	Indirect	
	β_{ii}	β_{ij}	$\hat{\eta}_i$
Percentiles of empirical distribution, Full Model of Section 4			
10^{th} percentile	0.288	0.000	-0.071
50^{th} percentile	0.314	0.003	-0.027
90^{th} percentile	0.395	0.039	-0.009
99^{th} percentile	1.138	0.494	-0.001

Notes: The table reports the percentiles of the empirical distribution for the 722 U.S. CZs of the reduced-form elasticities in 2000 and the shift in excess labor demand in 2000-2007 implied by the model in Section 4 for the estimates in Panel A of Table 2.

shock exposure (see Part C of Appendix A.2.4), our estimate of ϑ follows from the evidence in Table 1 that the differential impact of higher exposure to Chinese import competition on regional population is not statistically different from zero, with a positive point estimate. Our result is consistent with a growing body of literature documenting that recent shocks in regional labor demand in the U.S. triggered weak population responses over ten-year horizons – see Molloy et al. (2011), Autor et al. (2013), Cadena and Kovak (2016), Yagan (2019), and Benguria (2020).

Panels B and C of Table 2 present estimates of two restricted versions of the model. Panel B reports similar estimates of ϕ and ψ when we shut down the two additional margins of labor supply responses introduced in Section 4 (i.e., $\lambda = \vartheta = 0$). Panel C reports estimates of ϕ and ψ when we consider the simpler model in Section 3 that ignores intermediate production (i.e., $a^M_{i,s} = 0$). In this case, the estimate of ϕ is lower, but not statistically different from that in Panel A. However, the higher estimated ψ suggests a much stronger agglomeration force than that implied by the estimate in Panel A. This is a consequence of the fact that, as discussed in Section 4, a higher share of intermediates in production yields a flatter labor demand function for any given value of $\psi\phi$. Thus, accounting for intermediate inputs in production is essential for the model to simultaneously generate reasonable agglomeration forces and reduced-form elasticities that are consistent with those in the data. As discussed in the next section, the direct implication of this argument is that a Ricardian setting without intermediate inputs and agglomeration forces (like that used by Galle et al. (2021) and Kim and Vogel (2021)) yields reduced-form elasticities that are much smaller than our estimates.

Moreover, in Appendix B.2.2, we implement the two procedures described in Section 3.3 that use the integral of our formulas to show that the first-order approximation of the model's predictions performs well in our application.

Lastly, using the estimated parameters in Panel A of Table 2, we report in Table 3 the reduced-form elasticities, β_{ij}, and the shifts in excess labor demand caused by the China shock, $\hat{\eta}_i$. We focus only on the 722 U.S. CZs in the second period. The first column indicates that, for the median U.S. CZ, a 1% increase in its excess labor demand triggers an increase in the local wage of 0.31%. There is substantial heterogeneity in this direct reduced-form elasticity across CZs, as it can be seen from the value of the 99^{th} percentile, due to their distinct conditions before the shock (e.g., employment rate, openness and size). The second column shows that the indirect reduced-form elasticities are positive and, thus, imply a reinforcing spatial propagation of regional demand shocks. The median indirect elasticity of 0.003 is small, but the combined spatial indirect effect may be relatively large as there are 721 CZs indirectly affecting each region. A small subset of large or centrally-connected CZs create much stronger spatial indirect effects: the 99^{th} percentile of the indirect reduced-form elasticity is 0.49. Lastly, the third column of Table 3 reports the percentiles of the shift in excess labor demand across CZs. Although the China shock reduced the excess labor demand in most CZs, the magnitude of this reduction varied substantially across markets.

5.4 Evaluating the Fit of Different Specifications of Spatial Links

Our next goal is to evaluate which specifications of spatial links imply predicted responses to the China shock that are aligned with those observed across U.S. CZs. To do so, we estimate the fit coefficient in (29) for different outcomes and specifications.

We start in Table 4 with the predicted responses implied by the specification in Panel A of Table 2.[42] Columns (1) and (2) present the estimates for the two labor market outcomes used in the estimation of the reduced-form elasticities in Section 5.3: the changes in the average log wage and the log of the employment rate. It is thus reassuring, but not surprising, that we cannot reject that the fit coefficients are one for these two outcomes.

In columns (3) and (4), we present estimates of the fit coefficient for the predicted responses in the CZ's sectoral employment composition (as derived in Appendix A.1.9). Since these outcomes were not used in the estimation of $\boldsymbol{\theta}$, the fit coefficient of one is an over-identification restriction that we now use for testing our estimated model.[43] Results indicate that our estimated model generates differential responses in sectoral employment composition that are consistent with those observed following the China shock. Column (3) shows that the estimated fit coefficient is close to one for the change in the share of the CZ's working-age population employed in manufacturing (the main dependent variable in ADH). Finally, in column (4), we estimate the fit coefficient for the change in the share of manufacturing in the CZ's total employment. We again obtain a fit coefficient close to one, which indicates that the results in column (3) are not only driven by the change in the employment rate used in estimation.

In addition, Table B.14 in Appendix B.2.2 investigates our model's fit for changes in exports and

[42] Table B.13 in Appendix B.2.2 shows that results are similar for the different versions of the model in Table 2. We use standard errors clustered by state that impose independence of residuals across states. Because our model's predictions take a shift-share form, Panel D in Table B.13 also shows that standard errors are similar when we allow for arbitrary cross-market correlation in the residuals using the inference procedure in Adão et al. (2019). Because of the computational burden involved with manipulating the high-dimensional matrices for the full model, we only implement this inference procedure for the model without intermediate production.

[43] Note that there could be many reasons why our model may fail to match these non-targeted moments, as it does not feature search frictions (e.g. as in Helpman and Itskhoki (2010)), mobility costs and amenity preferences (e.g. as in Caliendo et al. (2019)), or sector-specific human capital (e.g. as in Burstein et al. (2019); Galle et al. (2021)).

Table 4: Fit of the Model for Labor Market Outcomes across U.S. CZs

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in working-age population	Share of Manufacturing in employed population
	(1)	(2)	(3)	(4)
Fit Coef. (ρ^Y)	1.16	1.07	0.86	0.79
	(0.48)	(0.20)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	73.9%	70.5%	42.6%	21.4%

Notes: Estimation of (29) in the pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. The regressor is the predicted impact of the (de-meaned) exposure to the China shock obtained from the model in Section 4 for the estimates in Panel A of Table 2. Robust standard errors in parentheses are clustered by state.

imports across sectors of the U.S. (aggregated across all CZs). We again estimate fit coefficients close to one. Hence, although U.S. trade outcomes were not used in estimation, our model's predicted responses for both U.S. exports and imports are consistent with those observed in the data.

Lastly, in Table 5, we investigate the fit of alternative model specifications. We setup as a benchmark a calibration of the model in Section 4 that is consistent with that used in CDP: a multi-sector Ricardian framework with input-output links and no agglomeration forces ($\psi = 0$), isomorphic employment and location choices ($\phi = \vartheta$), and non-employment payoff proportional to the price index ($\lambda = 1$). We set $\phi = 1.5$ so that the median labor supply elasticity is 0.5 across CZs.[44] In this benchmark calibration, our model's predicted changes in the employment rate for U.S. states in 2000-2007 are similar to the long-run predicted changes in CDP: they have a correlation of 0.5, and almost identical standard deviations of 0.05%. However, column (1) of Table 5 shows that it implies responses that are substantially smaller than those in the data: the fit coefficient is 4.5 for the employment rate and 1.9 for the average log wage. While the fit is also less precise than that for our estimated specifications, one can reject this alternative specification based on the fit for the employment rate at a 5% significance level.

The disconnect between the predictions of our alternative calibration and the observed responses across CZs may arise because our model is not dynamic and labor is not imperfectly mobile across sectors. These two features have been emphasized by recent papers studying the China shock, and are present in the calibrated model used by CDP. To investigate whether this is the case, Figure B.5 in Appendix B.2.2 plots the employment changes observed in the data against the impact of the China shock predicted by CDP (as reported in their online replication package). This is essentially a graphical representation of the estimation of equation in (29), where the regressor is the predicted response to the China shock in CDP. We obtain a fit coefficient that is much larger than one, again a consequence

[44] In CDP, ϕ and ϑ correspond to β/ν, which is estimated to be 0.2 or 0.5 at the quarterly or annual frequencies, respectively. They caution readers that this parameter should be higher at longer horizons. So, given that we implement our model for changes over ten years, we prefer to calibrate this parameter using the estimates for the long-run (Hicksian) elasticity of labor supply in Chetty et al. (2013b).

Table 5: Fit of the Model for Labor Market Outcomes across U.S. CZs – Alternative Specifications

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Change in log of employment rate						
Fit Coef. (ρ^Y)	4.48	1.93	1.43	1.82	3.70	6.63
	(1.42)	(0.35)	(0.28)	(0.33)	(0.75)	(1.67)
p-value of $H_0 : \rho^Y = 1$	1.5%	0.7%	12.1%	1.3%	0.0%	0.1%
Panel B: Change in log of average weekly wage						
Fit Coef. (ρ^Y)	1.89	1.88	2.08	2.07	1.46	2.34
	(0.96)	(0.84)	(0.65)	(0.86)	(0.64)	(0.70)
p-value of $H_0 : \rho^Y = 1$	35.2%	29.1%	9.7%	21.2%	47.8%	5.7%
Intermediate Production:	Yes	Yes	Yes	Yes	Yes	No
Parameter Calibration:						
ϕ	1.50	4.40	4.40	4.40	1.50	1.50
ψ	0.00	0.05	0.05	0.00	0.05	0.00
λ	1.00	0.21	1.00	0.21	0.21	1.00
ϑ	1.50	1.50	0.00	0.00	0.00	0.00

Notes: Estimation of (29) in the pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. The regressor is the predicted impact of the (de-meaned) exposure to the China shock obtained from the model with intermediate production in Section 4 in columns (1)-(5), and from the model without intermediate production in Section 3 in column (6). In each column, we use the set of parameters reported in the bottom of the table. Robust standard errors in parentheses are clustered by state.

from the fact that the model in CDP generates differential effects that are too small compared to their empirical counterparts (in line with Figure 1).

The remaining columns of Table 5 investigate why this alternative specification is rejected, while our estimated model is not. We estimate the fit coefficient by sequentially modifying our baseline estimates. For the model with intermediate production, we impose that the migration choice elasticity is 1.5 in column (2), the non-employment payoff is proportional to the price index in column (3), the agglomeration elasticity is zero in column (4), and the labor supply elasticity parameter is 1.5 in column (5).[45] For all these alternative specifications, the fit coefficients are higher than those in Table 4 and are statistically different from one for either wages or employment at a 10% significance level.

In column (6), we report the fit coefficient when we restrict the production side of the model to a Ricardian framework without intermediate goods and agglomeration forces, such as that used in Galle et al. (2021) and Kim and Vogel (2021). In this case, because labor demand is even more elastic, the responses are smaller and the fit coefficients are larger. Figure B.4 in Appendix B.2.2 shows that strong agglomeration forces are essential for the model to generate a fit coefficient of one for *both* employment and wages. In fact, without agglomeration forces (i.e., high κ), the labor supply elasticity that would yield a fit coefficient of one for employment is so high that wage responses would become too small and, thus, the fit coefficient for wages would become much larger than one. This happens because, given the same labor demand shock,

[45] In columns (3)–(6), we set the location choice elasticity to zero, as our negative point estimate is not statistically distinguishable from zero and a negative value would not be used in the literature.

a higher labor supply elasticity leads to stronger employment responses, but weaker wage responses.

To summarize, our results identify the roots of the disconnect, documented in Section 2, between the small differential effects implied by quantitative spatial models in the literature and their much larger estimated counterparts. Such a disconnect disappears when we consider (i) the combination of strong agglomeration forces and high sensitivity of employment to wages, and (ii) weak responses of employment to the price of imported consumption goods.

5.5 The Impact of the China Shock in General Equilibrium

We conclude by presenting the predictions of our estimated specification for the general equilibrium impact of the China shock on U.S. CZs. We focus our discussion on the aggregate and distributional effects of the shock on employment rates and real wages, which we summarize in Figure 2.[46]

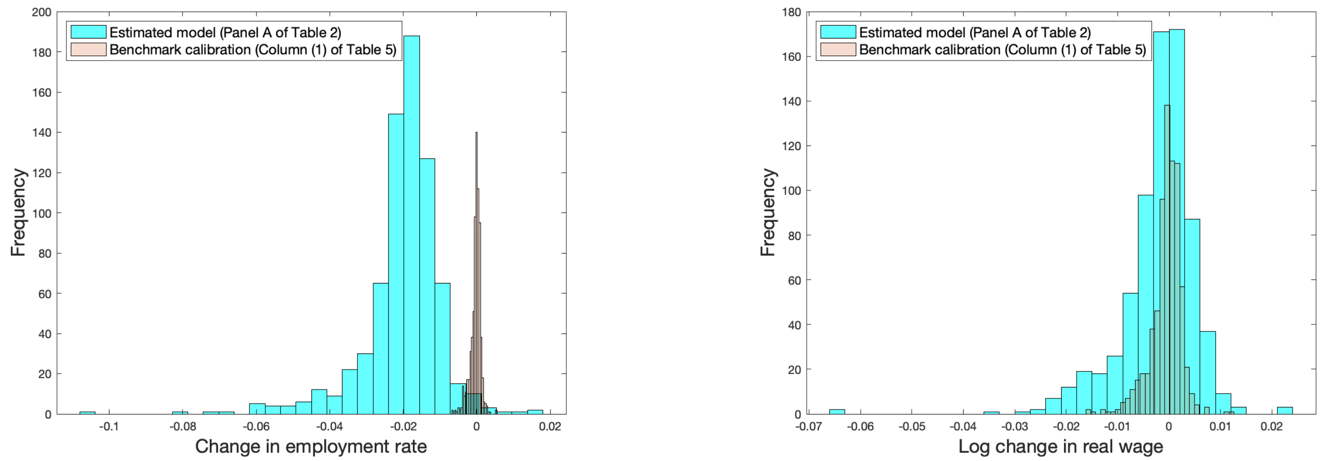
We find that the employment effects of the China shock are negative for the vast majority of the CZs. There is, however, a large spatial dispersion in these adverse effects, with a standard deviation of employment rate responses equal to 1 p.p. The Southeast and Midwest are the most negatively affected areas of the U.S. (see Figure B.6 in Appendix B.2.3), containing the ten CZs with the largest declines in employment rates. This is a consequence of both areas' higher exposure to the shock, due to their initial patterns of industry and consumption specialization, and the propagation of indirect effects across the nearby CZs within these areas. Overall, our estimates imply aggregate employment losses of approximately 3 million jobs between 1990 and 2007. In contrast, the benchmark specification that corresponds to the existing quantitative literature in column (1) of Table 5, as anticipated by the discussion in Section 5.4, implies employment effects that are close to zero with a much smaller standard deviation of 0.1 p.p.

The right panel of Figure 2 shows that the general equilibrium impact of the shock on real wages is close to zero for the median CZ, but again there is a large spatial dispersion in these responses, with regions in the Southeast bearing the largest losses (see Figure B.6 in Appendix B.2.3). The key reason for this more dampened average effect on real wages is the positive impact of the China shock on the price index through both the consumption exposure channel in Corollary 3 and the downstream cost channel in Section 4. Note that our predicted effects on real wages have a similar average as those implied by the quantitative benchmark calibration in column (1) of Table 5, but our estimated model yields much more dispersed real wage responses.

Lastly, Figure B.7 in Appendix B.2.3 evaluates whether the simple exposure measures considered in Section 2 are able to capture the predictions obtained with our estimates of the general equilibrium specification in (34). On the vertical axis, we plot the changes in the employment rate between 1990 and 2007 predicted by the specification in (34) with the parameter estimates in Panel A of Table 2. On the horizontal axis, we plot the fitted values implied by a regression of the model-implied employment changes on a constant, IC_t, IE_t and GC_t (computed with the sectoral shifter defined as

[46]Figure B.8 in Appendix B.2.3 shows that results are similar when we consider endogenous fiscal transfers across CZs.

Figure 2: Impact of China Shock on U.S. CZs in General Equilibrium



Notes: The histogram on the left displays the response of the employment rate to the China shock for each of the 722 CZs that we compute with the sum of the predicted effects in 1990-2000 and 2000-2007 implied by the specification in (34). The histogram on the right displays the analog for the predicted response of the log of the real wage. The light blue bars correspond to the predictions obtained with parameter estimates in Panel A of Table 2, while the light pink bars correspond to predictions obtained with the benchmark calibration of the model in column (1) of Table 5.

$\hat{\zeta}^t_{\text{China},s}$, instead of $\Delta M^t_{\text{China},s}$). The scatter plot shows that these simple exposure measures generate a pattern of cross-regional variation in employment responses that is similar to that implied by our general equilibrium model (the correlation between them is 0.56).

6 Conclusion

The use of cross-regional variation in shock exposure to study how labor markets adjust to economic shocks has become an important part of the toolkit of researchers in international, macro and urban economics. This approach has an important shortcoming, though: estimates of the differential responses of local outcomes to the market's shock exposure may not fully capture all the adjustment channels operating in general equilibrium. In this paper, we show how to recover the general equilibrium impact of economic shocks from the aggregation of an empirical specification based on cross-regional variation in shock exposure. Our specification relies on the reduced-form characterization of a general class of spatial models whose predictions can be expressed in terms of shifts in regional excess labor demand and reduced-form elasticities, direct and indirect, to these shifts. These reduced-form elasticities are sufficient for aggregating the exposure of different markets in order to compute the shock's general equilibrium impact. This reduced-form characterization yields an empirical specification – a generalization of shift-share empirical strategies – that can be used for either estimating the parameters of the model's reduced-form elasticities or testing the model's differential predictions.

Our approach provides a connection between the differential effects implied by spatial models and their empirical counterparts. A class of quantitative spatial models has emerged, motivated by the critique that empirical strategies exploiting cross-regional variation in shock exposure can recover the shock's differential effect but not its aggregate effect. The ultimate goal of these papers is to use instead their spatial frameworks to quantify the general equilibrium impact of economic shocks. Despite matching any cross-section of observed regional outcomes with free parameters, a priori these frameworks are not guaranteed to generate predicted responses of regional outcomes to observed shocks that are consistent with the shock's actual differential effect across markets – and indeed we show that they very often do not. We argue in this paper that quantitative spatial models should be held to the same standard as the empirical strategies that they are supposed to complement, by generating differential responses to economic shocks that are credibly supported by evidence. This is important because, as our theoretical results show, the model's differential predicted responses depend on the same reduced-form elasticities that determine the model's predicted aggregate impact.

Our methodology allows the evaluation of the empirical content of spatial general equilibrium models in terms of their implications for the differential impact of exogenous shocks across markets. It thus makes progress in achieving the standards set by Kehoe (2005): *"Such evaluations also help make applied GE analysis a scientific discipline in which there are well-defined puzzles with clear successes and failures for competing theories."* The advantage of our unified theoretical and empirical approach is also evinced by our findings that spatial models anchored to the reduced-form moments in the data imply a larger and more dispersed impact of the China shock on U.S. CZs when compared to alternative specifications whose differential predictions are not consistent with their empirical counterparts.

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A Appendix: Proofs and Additional Results (Not for publication)

A.1 Proofs for Section 3

A.1.1 Proof of Equation (14)

The definitions of γ_{ij} in (14) and $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (12) immediately imply that

$$\gamma_{ij} = \mathbb{I}_{i=j}(\phi-(\phi-1)n_i)-(\phi-1)(1-n_i)\omega_j^0 - \frac{1}{R_i^0}\frac{\partial R_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j}$$

where

$$\frac{1}{R_i^0}\frac{\partial R_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j} = \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 \left[\mathbb{I}_{d=j}(\phi-(\phi-1)n_d)-(\phi-1)(1-n_d)\omega_j^0-(\sigma-1)(1-\psi\phi)\left(\mathbb{I}_{i=j}-x_{jd,s}^0\right)\right].$$

Thus, using the definitions of $\kappa \equiv (\sigma-1)(1-\psi\phi)$, $\phi_i^0 \equiv \phi-(\phi-1)n_i$ and $r_{ij}^0 \equiv \sum_s \ell_{i,s}^0 r_{ij,s}^0$, we can re-write this expression as

$$\begin{aligned}
\gamma_{ij} &= \mathbb{I}_{i=j}\phi_i^0 + (1-\phi_i^0)\omega_j^0 - \textstyle\sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 \left[\mathbb{I}_{d=j}\phi_d^0 + (1-\phi_d^0)\omega_j^0 - \kappa\left(\mathbb{I}_{i=j}-x_{jd,s}^0\right)\right] \\
&= \mathbb{I}_{i=j}\left(\phi_i^0+\kappa\right) - \textstyle\sum_s \ell_{i,s}^0 r_{ij,s}^0 \phi_j^0 - \kappa \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0\left(\phi_i^0 - \sum_s\sum_d \ell_{i,s}^0 r_{id,s}^0 \phi_d^0\right) \\
&= \mathbb{I}_{i=j}\left(\phi_i^0+\kappa\right) - r_{ij}^0\phi_j^0 - \kappa \textstyle\sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0\left(\phi_i^0 - \sum_d r_{id}^0 \phi_d^0\right)
\end{aligned}$$

which is equivalent to (19) since $\sum_d r_{id}^0 = 1$.

A.1.2 Proof of Theorem 1

We re-define the system in (15) to set the change in the wage of market m to zero. Consider the matrix $\bar{\boldsymbol{M}}$ obtained by deleting the m-th row from the identity matrix with dimension equal to the number of markets. If $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is non-singular, then we can write

$$\bar{\boldsymbol{M}}\hat{\boldsymbol{w}} = \left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}},$$

which yields the representation in (21) when we define $\bar{\boldsymbol{\beta}} \equiv \bar{\boldsymbol{M}}'(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}')^{-1}\bar{\boldsymbol{M}}$.

In the rest of the proof, we first show that $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is non-singular and then establish that $\bar{\boldsymbol{\beta}}$ admits the series representation in (21). To simplify exposition, we abuse notation by defining

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}', \quad \hat{\boldsymbol{w}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{w}} \qquad \text{and} \qquad \hat{\boldsymbol{\eta}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}}.$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\lambda}}$ be the diagonal matrix defined by the vector of $\lambda_i^0 \equiv \kappa + \phi_i$, and $\tilde{\boldsymbol{\gamma}}$ be the matrix with entries $\tilde{\gamma}_{ij} \equiv \rho_{ij}/(\kappa+\phi_i)$, so that

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\boldsymbol{\gamma}}}\right).$$

Consider the vector $\{h_i\}_{i \neq m} \gg 0$ that guarantees the diagonal dominance of $\bar{\boldsymbol{\gamma}}$ in the initial equilibrium. Let

$\bar{\boldsymbol{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i. Thus, the system in (15) is equivalent to

$$\begin{aligned}
\bar{\boldsymbol{\lambda}}(\bar{\boldsymbol{I}}-\bar{\tilde{\gamma}})\left(\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^{-1}\right)\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}} \\
\bar{\boldsymbol{\lambda}}(\bar{\boldsymbol{h}}-\bar{\tilde{\gamma}}\bar{\boldsymbol{h}})\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}} \\
(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}})\left(\bar{\boldsymbol{I}}-\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}
\end{aligned}$$

which implies that

$$\hat{\boldsymbol{w}}=\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}}\right)^{-1}(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}})^{-1}\hat{\boldsymbol{\eta}}, \quad \bar{\bar{\tilde{\gamma}}}\equiv\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}. \tag{38}$$

Notice that, for all i, $\bar{\tilde{\tilde{\gamma}}}_{ij}=\tilde{\gamma}_{ij}h_j/h_i=\rho_{ij}h_j/(\kappa+\phi_i)h_i$.

First, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is non-singular, so that we can write the expression in (38). We proceed by contradiction. Suppose that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is singular, so $\mu=0$ is an eigenvalue of $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$. Take the eigenvector $\boldsymbol{x}$ associated with the zero eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})\boldsymbol{x}=0$, so that the i-row of this system is

$$1-\sum_{j\neq m}\bar{\tilde{\tilde{\gamma}}}_{ij}x_j=0 \quad\Longrightarrow\quad 1-\frac{\rho_{ii}}{\kappa+\phi_i}-\sum_{j\neq i,m}\frac{\rho_{ij}}{\kappa+\phi_i}\frac{h_j}{h_i}x_j=0$$

Thus, because $|x_j|\leq 1$ and $h_j>0$ for all j,

$$(\kappa+\phi_i-\rho_{ii})h_i=\sum_{j\neq i,m}\rho_{ij}h_jx_j\leq\sum_{j\neq i,m}|\rho_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\rho_{ij}||h_j|,$$

which contradicts (20).

Second, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}$ admits the series representation in (21). This is true whenever the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\boldsymbol{x}$ associated with the largest eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $\mu\boldsymbol{x}=\bar{\bar{\tilde{\gamma}}}\boldsymbol{x}$ so that the i-row of this system is

$$1\leq\mu=\sum_{j\neq m}\frac{\rho_{ij}}{\kappa+\phi_i}\frac{h_j}{h_i}x_j.$$

Since $\kappa+\phi_i$ and h_i are positive, the same steps used above imply that

$$(\kappa+\phi_i-\rho_{ii})h_i\leq\sum_{j\neq i,m}|\rho_{ij}|h_j,$$

which contradicts the assumption of diagonal dominance. Thus, the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one, allowing us to write $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}=\sum_{d=0}^{\infty}(\bar{\bar{\tilde{\gamma}}})^d$. Substituting this series expansion into (38) yields

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we now show that $\bar{\boldsymbol{h}}(\bar{\bar{\tilde{\gamma}}})^d\bar{\boldsymbol{h}}^{-1}=(\bar{\tilde{\gamma}})^d$. We proceed by induction. For $d=1$, it is trivial to see that $\bar{\boldsymbol{h}}(\bar{\tilde{\tilde{\gamma}}})\bar{\boldsymbol{h}}^{-1}=\bar{\tilde{\gamma}}$. Assume that it holds for d, we now show that it also holds for $d+1$:

$$\bar{\boldsymbol{h}}\left(\bar{\tilde{\tilde{\gamma}}}\right)^{d+1}\bar{\boldsymbol{h}}^{-1}=\bar{\boldsymbol{h}}\left(\bar{\tilde{\tilde{\gamma}}}\right)^{d}\bar{\tilde{\tilde{\gamma}}}\bar{\boldsymbol{h}}^{-1}=\bar{\boldsymbol{h}}\left(\bar{\tilde{\tilde{\gamma}}}\right)^{d}\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}=\left(\bar{\boldsymbol{h}}\left(\bar{\tilde{\tilde{\gamma}}}\right)^{d}\bar{\boldsymbol{h}}^{-1}\right)\bar{\tilde{\gamma}}=(\bar{\tilde{\gamma}})^{d+1}.$$

Thus,

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}(\bar{\tilde{\gamma}})^{d}\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}},$$

which immediately implies the result.

A.1.3 Proof of Corollary 1

The series expansion representation of β_{ij} indicates that $\beta_{ij}>0$ if $\tilde{\gamma}_{ij}>0$ for all i and j. We now show that $\tilde{\gamma}_{ij}>0$ whenever $\max_{o,d}|n_o-n_d|$ is low enough. Since $\sum_d r_{id}^0(\phi_i^0-\phi_d^0)=(\phi-1)\sum_d r_{id}^0(n_d^0-n_i^0)>-(\phi-1)\max_{o,d}|n_o-n_d|$,

$$\tilde{\gamma}_{ij}>\frac{r_{ij}^0\phi_j^0+\kappa\sum_s\sum_d\ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0-\omega_j(\phi-1)\max_{o,d}|n_o-n_d|}{\phi_i^0+\kappa}$$

and, thus,

$$\max_{o,d}|n_o-n_d|<\frac{r_{ij}^0\phi_j^0+\kappa\sum_s\sum_d\ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0}{\omega_j(\phi-1)}\Rightarrow\tilde{\gamma}_{ij}>0.$$

Since the numerator is positive in our model, there exists $\max_{o,d}|n_o-n_d|\geq 0$ such that the condition above holds for all i and j.

A.1.4 Proof of Corollary 2

We establish this result in two steps.

Step 1. First we show that, if $\bar{\boldsymbol{\gamma}}\equiv\lambda^0\left(\bar{\boldsymbol{I}}-\mathbf{1}\boldsymbol{\rho}'\right)$ where $\mathbf{1}$ is a column vector of ones and $\boldsymbol{\rho}\equiv\{\rho_j\}_{j\neq m}$ is column vector, then $\bar{\boldsymbol{\gamma}}^{-1}=(\lambda^0)^{-1}\left(\bar{\boldsymbol{I}}+\rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'\right)$.

$$\begin{aligned}
\bar{\boldsymbol{\gamma}}^{-1}\bar{\boldsymbol{\gamma}} &= \bar{\boldsymbol{I}}+\rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'-\mathbf{1}\boldsymbol{\rho}'-\rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}}+\rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'-\mathbf{1}\boldsymbol{\rho}'-\left(\rho_m^{-1}\textstyle\sum_{j\neq m}\rho_j\right)\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}}+\left(\rho_m^{-1}(1-\textstyle\sum_{j\neq m}\rho_j)-1\right)\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}}
\end{aligned}$$

where the second equality follows from $\boldsymbol{\rho}'\mathbf{1}=\sum_{j\neq m}\rho_j$, and the fourth from $\sum_j\rho_j=1$ (since $\sum_j\gamma_{ij}=0$ for all i).

Step 2. We now establish conditions that allow us to write $\gamma_{ij}=\lambda^0(\mathbb{I}_{i=j}-\rho_j)$. Assume that $n_i^0=n^0$, so $\phi_i^0=\phi+(1-\phi)n^0$ and we can define $\lambda^0\equiv\kappa+\phi^0$. We now use the fact that $x_{ij,s}=x_{i,s}$ and $\xi_{j,s}=\xi_s$ so that

$$r_{ij}^0=\frac{\sum_s x_{ij,s}^0\xi_{j,s}E_j^0}{\sum_j\sum_s x_{ij,s}^0\xi_{j,s}E_j^0}=\frac{\sum_s x_{i,s}^0\xi_s E_j^0}{\sum_j\sum_s x_{i,s}^0\xi_s E_j^0}=\frac{E_j^0}{\sum_j E_j^0}=e_{Wj}^0.$$

In addition,

$$\kappa\sum_s\sum_d\ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0=\kappa\sum_s\sum_d\frac{x_{i,s}^0\xi_s E_d^0}{\sum_j\sum_s x_{i,s}^0\xi_s E_j^0}x_{j,s}^0=\kappa\frac{\sum_s\xi_s x_{i,s}^0 x_{j,s}^0}{\sum_s\xi_s x_{i,s}^0}=\kappa\chi_j$$

Since $n_i^0 = n^0$, then $\sum_d r_{id}^0(\phi_i^0 - \phi_d^0) = 0$. This implies that $\rho_{ij}^0 = e_{Wj}^0 \phi^0 + \kappa\chi_j$, which we can use to define $\rho_j \equiv (e_{Wj}^0 \phi_j^0 + \kappa\chi_j)/(\kappa + \phi^0)$ and $\beta_j \equiv \mathbb{I}_{j \neq m} \rho_j / \rho_m$.

A.1.5 Proof of Corollary 3

By Shepard's lemma, the price index expression in (7) implies that

$$\hat{P}_i = \sum_{s,o} \xi_{i,s} x_{oi,s}^0 (\hat{\tau}_{oi,s} + \hat{p}_o) = \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) + \sum_o x_{oi}^0 \hat{p}_o$$

where $x_{oi}^0 \equiv \sum_s \xi_{i,s} x_{oi,s}^0$ is the share of o in the total spending of i. Thus,

$$\begin{aligned} \hat{P}_i &= \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) + \textstyle\sum_o x_{oi}^0 \Big((1-\psi\phi)\hat{w}_o + \psi\phi\hat{\Omega} \Big) \\ &= \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) + \textstyle\sum_o \big(x_{oi}^0 (1-\psi\phi) + \psi\phi\omega_o^0 \big) \hat{w}_o, \\ &= \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) + \textstyle\sum_o \Big(x_{oi}^0 \frac{\kappa}{\sigma-1} + \Big(1 - \frac{\kappa}{\sigma-1}\Big)\omega_o^0 \Big) \hat{w}_o \end{aligned}$$

where the first expression follows from $\hat{p}_o$ in (10), the second from $\hat{b}_i = \hat{\Omega} = \sum_o \omega_o^0 \hat{w}_o$, and the last from the definition of $\kappa = (\sigma-1)(1-\psi\phi)$. The combination of this expression and the expression for $\hat{w}_o$ in (21) immediately implies (23).

A.1.6 Proof of Equation (26)

The labor supply equation in (8) with $\hat{b}_i = \sum_d \omega_d^0 \hat{w}_d$ implies that $\hat{n}_i = \phi(1-n_i^0)(\hat{w}_i - \sum_d \omega_d^0 \hat{w}_d)$. Using (21), we get that

$$\hat{n}_i = \phi(1-n_i^0) \sum_j \left(\beta_{ij} - \sum_d \omega_d^0 \beta_{dj} \right) \hat{\eta}_j, \tag{39}$$

which immediately implies the second expression in (26) when combined with (25).

The combination of (9), (21), and (39) implies that

$$\Delta \overline{\ln w_i} = \sum_j \beta_{ij} \hat{\eta}_j - (1-n_i^0) \sum_j \left(\beta_{ij} - \sum_d \omega_d^0 \beta_{dj} \right) \hat{\eta}_j,$$

and, thus,

$$\Delta \overline{\ln w_i} = \sum_j \left[n_i^0 \beta_{ij} + (1-n_i^0) \sum_d \omega_d^0 \beta_{dj} \right] \hat{\eta}_j.$$

This immediately implies the first expression in (26) when combined with (25).

A.1.7 Proof of Equation (28)

To establish condition (28) first notice that, by definition,

$$\begin{aligned} \nu_i^w &= (1-\sigma) \quad \textstyle\sum_j \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \sum_s \ell_{j,s}^0 \Big(\sum_d r_{jd,s}^0 \hat{\tau}_{jd,s}^{\text{unbs}} - \sum_o \sum_d r_{jd,s}^0 x_{od,s}^0 \hat{\tau}_{od,s}^{\text{unbs}} \Big) - \alpha^w \\ &= \textstyle\sum_{s,d,o} \Big[\sum_j (1-\sigma) \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \ell_{j,s}^0 r_{jd,s}^0 \Big(\mathbb{I}_{o=j} - x_{od,s}^0 \Big) \Big] \hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w. \end{aligned}$$

Using the definition $Z_j \equiv \sum_s \ell^0_{j,s}(\hat{z}^{obs}_{j,s} - \bar{z}^{obs}_{j,s})$, we have that

$$\begin{aligned} Z_j &= (1-\sigma) \quad \sum_s \ell^0_{j,s}\left(\sum_d r^0_{jd,s}\left(\hat{\tau}^{obs}_{jd,s} - \tau^{obs}\right) - \sum_o \sum_d r^0_{jd,s} x^0_{od,s}\left(\hat{\tau}^{obs}_{od,s} - \tau^{obs}\right)\right) \\ &= \sum_{s,d,o}\left[(1-\sigma)\ell^0_{j,s} r^0_{jd,s}\left(\mathbb{I}_{o=j} - x^0_{od,s}\right)\right]\left(\hat{\tau}^{obs}_{od,s} - \tau^{obs}\right). \end{aligned}$$

For arbitrary i and j,

$$E[\nu^w_i Z_j|\mathcal{W}^0] = E\left[\left(\sum_{s,d,o}\left[\sum_h \beta^w_{ih}(\boldsymbol{\theta}|\mathcal{W}^0)\ell^0_{h,s} r^0_{hd,s}\left(\mathbb{I}_{o=h} - x^0_{od,s}\right)\right]\hat{\tau}^{unbs}_{od,s} - \alpha^w\right)\left(\sum_{s',d',o'} \ell^0_{j,s'} r^0_{jd',s'}\left(\mathbb{I}_{o'=j} - x^0_{o'd',s'}\right)\left(\hat{\tau}^{obs}_{o'd',s'} - \tau^{obs}\right)\right)|\mathcal{W}^0\right]$$

and, thus,

$$\begin{aligned} E[\nu^w_i Z_j|\mathcal{W}^0] &= \sum_{s,d,o,s',d',o'}\left(\sum_h \beta^w_{ih}(\boldsymbol{\theta}|\mathcal{W}^0)\ell^0_{h,s} r^0_{hd,s}\left(\mathbb{I}_{o=h} - x^0_{od,s}\right)\right)\ell^0_{j,s'} r^0_{jd',s'}\left(\mathbb{I}_{o'=j} - x^0_{o'd',s'}\right)E\left[\hat{\tau}^{unbs}_{od,s}\left(\hat{\tau}^{obs}_{o'd',s'} - \tau^{obs}\right)|\mathcal{W}^0\right] \\ &- \alpha^w\left(\sum_{s',d',o'} \ell^0_{j,s'} r^0_{jd',s'}\left(\mathbb{I}_{o'=j} - x^0_{o'd',s'}\right)\right)E\left[\left(\hat{\tau}^{obs}_{o'd',s'} - \tau^{obs}\right)|\mathcal{W}^0\right]. \end{aligned}$$

Since $E\left[\left(\hat{\tau}^{obs}_{o'd',s'} - \tau^{obs}\right)|\mathcal{W}^0\right] = 0$,

$$E[\nu^w_i Z_j|\mathcal{W}^0] = \sum_{s,d,o}\sum_{s',d',o'}\left(\sum_h \beta^w_{ih}(\boldsymbol{\theta}|\mathcal{W}^0)\ell^0_{h,s} r^0_{hd,s}\left(\mathbb{I}_{o=h} - x^0_{od,s}\right)\right)\ell^0_{j,s'} r^0_{jd',s'}\left(\mathbb{I}_{o'=j} - x^0_{o'd',s'}\right)Cov\left(\hat{\tau}^{unbs}_{od,s}, \hat{\tau}^{obs}_{o'd',s'}|\mathcal{W}^0\right).$$

Thus, by the assumption in (27), $E\left[\nu^w_i Z_j|\mathcal{W}^0\right] = 0$ and, therefore, $E[\nu^w_i Z_j] = E\left[E\left[\nu^w_i Z_j|\mathcal{W}^0\right]\right] = 0$. This immediately establishes that, for any real matrix h^w_{ij}, $E\left[\nu^w_i \sum_j h^w_{ij} Z_j\right] = \sum_j h^w_{ij} E[\nu^w_i Z_j] = 0$. We can follow the same steps to show that $E\left[\nu^n_i \sum_j h^n_{ij} Z_j\right] = 0$.

A.1.8 Model-Implied Optimal Moment Condition

To simplify exposition without loss of generality, we assume that all variables are demeaned, so that (26) and (28) can be written in the following vector form:

$$v_i(\boldsymbol{\theta}) = Y_i - (\beta_i(\boldsymbol{\theta}|\boldsymbol{W}))'\boldsymbol{Z} \quad \text{such that} \quad E\left[v_i(\boldsymbol{\theta})\sum_j h_{ij} Z_j\right] = 0. \tag{40}$$

Define $\boldsymbol{H}_i = \left[\left(\boldsymbol{h}^k_i \boldsymbol{Z}\right)'\right]^{dim(\boldsymbol{\theta})}_{k=1}$ where $\boldsymbol{h}^k_i \boldsymbol{Z}$ has dimension $dim(v_i) \times 1$. Thus, for any $\boldsymbol{h}^k_i$, the condition above is equivalent to

$$E[\boldsymbol{H}_i v_i(\boldsymbol{\theta})] = 0,$$

which yields the following class of GMM estimators of $\boldsymbol{\theta}$,

$$\hat{\boldsymbol{\theta}}_H \equiv \arg min_{\boldsymbol{\theta}}\left[\sum_i \boldsymbol{H}_i v_i(\boldsymbol{\theta})\right]'\left[\sum_i \boldsymbol{H}_i v_i(\boldsymbol{\theta})\right].$$

Optimal Moment Conditions with Independent Residuals. We follow Chamberlain (1987) to derive the optimal moment conditions. We start with the assumption that v_i are independent across (clusters of) markets. We show below how the formula changes when we instead assume that the observed shocks are independent, while allowing residuals to have an arbitrary correlation.

When v_i is independent across markets, the usual optimal IV formula in Chamberlain (1987) holds, so that the optimal moment condition is

$$H_i^* \equiv \left(E\left[v_i(\boldsymbol{\theta})v_i(\boldsymbol{\theta})'|\boldsymbol{Z}\right]\right)^{-1}\nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta})$$

where, given (40),

$$\nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta}) = -\nabla_{\boldsymbol{\theta}} \beta_i(\boldsymbol{\theta}|\boldsymbol{W})\boldsymbol{Z}.$$

The term $(E[v_i(\boldsymbol{\theta})v_i(\boldsymbol{\theta})'|\boldsymbol{Z}])^{-1}$ adjusts the weight of each observation to the inverse of the variance of its residuals. It is the usual adjustment that arises in generalized least squares under heteroskedasticity. This term is irrelevant under homoskedasticity. For each observation, $\nabla_{\boldsymbol{\theta}} v_i(\boldsymbol{\theta})$ attributes a higher weight to the exposure of the markets whose bilateral reduced-form elasticities are more sensitive to changes in each parameter.

Intuition for Efficiency Gains. We now illustrate the source of the efficiency gains from the use of the optimal moment conditions in the context of our model. To fix ideas, we assume that the goal is to estimate ϕ when the labor demand elasticity κ is already known. In this case, the efficiency gains arise from the fact that the reduced-form expression for wage changes in (26) is the model-consistent first-stage specification for the estimation of ϕ.

To see this, notice that the labor supply structure in (8)–(9) implies that

$$\frac{n_i^0}{1-n_i^0}\Delta \ln n_i = \hat{\Omega} + \phi \Delta \overline{\ln w_i} + v_i^n \tag{41}$$

where $v_i^n \equiv -\Delta \ln \bar{b}_i$ and $\hat{\Omega}$ is the change in the numeraire of transfers.

Given that κ is assumed to be known, the reduced-form expression for wage changes in (26) yields the model-consistent first-stage expression:

$$\Delta \overline{\ln w_i} = \alpha^w + \sum_j \beta_{ij}^w(\phi)\hat{\eta}_j + v_i^w. \tag{42}$$

Notice that if we substitute (42) into (41), then we recover the reduced-form expression for employment in (26). Thus, equations (41)-(42) imply the same relationship between market-level shock exposure (as measured by $\hat{\eta}_j$) and employment and wage changes across markets.

Using the general equilibrium predictions of the model in (41)-(42), the optimal moment condition for the estimation of ϕ is

$$\left(E\left[v_i(\phi)v_i(\phi)'|\boldsymbol{Z}\right]\right)^{-1}\nabla_\phi v_i(\phi) = \begin{bmatrix} \sigma_n^2 & \sigma_{wn} \\ \sigma_{wn} & \sigma_w^2 \end{bmatrix}^{-1} \sum_j \begin{bmatrix} \beta_{ij}^w(\phi) + \phi \nabla_\phi \beta_{ij}^w(\phi) \\ \nabla_\phi \beta_{ij}^w(\phi) \end{bmatrix} \hat{\eta}_j, \tag{43}$$

where σ_n^2 and σ_w^2 denote the variance of v_i^n and v_i^w respectively and σ_{wn} denotes the covariance between v_i^w and v_i^n.

We now compare the optimal moment condition implied by the general equilibrium predictions of the model to the optimal moment condition that a researcher would obtain if she estimates ϕ with the equilibrium relationship in (41) but imposes an *arbitrary* first-stage relationship between the average log-wage in market i (i.e., the endogenous variable) and that market's exogenous shock exposure (i.e., the instrumental variable):

$$\Delta \overline{\ln w_i} = \alpha + \beta \hat{\eta}_i + u_i. \tag{44}$$

This approach yields a consistent estimator of ϕ under the same orthogonality conditions introduced in Section 3.3. However, it yields a less efficient estimator. To see this, we now apply the optimal moment condition to (41) and (44) (which is equivalent to the 2SLS estimator of ϕ when $\hat{\eta}_i$ is the IV):

$$\left(E\left[v_i(\phi)v_i(\phi)'|\boldsymbol{Z}\right]\right)^{-1}\nabla_{\phi}v_i(\phi)=\begin{bmatrix}\sigma_n^2 & 0\\ 0 & \sigma_u^2\end{bmatrix}^{-1}\begin{bmatrix}\beta\hat{\eta}_i\\ 0\end{bmatrix}=\begin{bmatrix}\frac{\beta}{\sigma_n^2}\hat{\eta}_i\\ 0\end{bmatrix}\propto\begin{bmatrix}\hat{\eta}_i\\ 0\end{bmatrix}, \tag{45}$$

where we assume that the researcher also imposed that the covariance between u_i and v_i is zero – as typically done in practice.

Under the assumption that the general equilibrium model is well specified, the fact that (43) and (45) are different indicates that the arbitrary first-stage assumption leads to a less efficiency estimator. The difference is that the derivation of (45) does not take into account the general equilibrium structure of the model, and, thus, ignores two channels through which shocks affect endogenous outcomes in general equilibrium: it ignores (i) information on shocks in other regions that may affect wages and employment in i (i.e., the arbitrary first-stage ignores spatial indirect effects), and (ii) the impact that the parameter ϕ has on wage responses (i.e., the arbitrary first-stage ignores that wage responses depend on ϕ).

Optimal Moment Conditions with Correlated Residuals. We now also derive the optimal moment conditions using the results in Borusyak and Hull (2020) that allow for arbitrary cross-market correlation in v_i but assume that the observed shocks are independent from each other. In this case, they show that

$$\boldsymbol{H}^*\equiv\left(E\left[\boldsymbol{v}(\boldsymbol{\theta})\boldsymbol{v}(\boldsymbol{\theta})'|\boldsymbol{\mathcal{W}}\right]\right)^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{v}(\boldsymbol{\theta}).$$

We then apply this formula to our reduced-form representation of the predictions of general equilibrium spatial models: using (40),

$$\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})=-\nabla_{\boldsymbol{\theta}}\beta_i(\boldsymbol{\theta}|\boldsymbol{W})\boldsymbol{Z}.$$

This formula is similar to the one above. The term $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ is identical in the two formulas. The only difference is the first term, which now accounts for the potential covariance between the residuals of different markets. In our model, such a correlation may arise if markets are exposed to similar unobserved shocks in economic fundamentals – a point recently raised by Adão et al. (2019) in the context of shift-share specifications. To see this, assume that $\hat{\tau}_{od,s}^{\text{unbs}}$ are independently drawn from an arbitrary distribution with mean zero and variance of σ_τ^2. As shown in Appendix A.1.7, the residual can be written in a general form, $v_i(\boldsymbol{\theta})=\sum_{s,d,o}\boldsymbol{\beta}_{i,ods}^v(\boldsymbol{\theta}|\boldsymbol{W})\hat{\tau}_{od,s}^{\text{unbs}}$ where $\beta_{i,ods}^v=\sum_j[\beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}),\beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}})]'\ell_{j,s}y_{jd,s}(\mathbb{I}_{o=j}-x_{od,s})$. Thus,

$$E\left[v_i(\boldsymbol{\theta})v_j(\boldsymbol{\theta})'|\boldsymbol{\mathcal{W}}\right]=\sigma_\tau^2\sum_{s,d,o}\left(\boldsymbol{\beta}_{i,ods}^v(\boldsymbol{\theta}|\boldsymbol{W})\right)\left(\boldsymbol{\beta}_{j,ods}^v(\boldsymbol{\theta}|\boldsymbol{W})\right)'.$$

This expression shows that the correlation between the residuals of markets i and j is higher if they have higher exposure to the same shocks. This is the case if the two markets are similar in terms of employment shares across sectors and/or within-sector revenue shares across destinations.

Implementation Comments. We conclude with two comments on implementation. First, while it is trivial to compute $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ because of our reduced-form characterization in (40), it is much harder to compute the variance adjustment term as it requires knowledge of the unobserved residuals. For this reason, it is common to ignore this adjustment term in practice by constructing moment conditions with $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$. This yields a consistent estimator of $\boldsymbol{\theta}$, but it is possible that implementing the variance correction term could further improve the estimator's efficiency. Second, $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ must be evaluated at the true value of $\boldsymbol{\theta}$. To simplify the estimator's implementation, one can adopt an asymptotically equivalent two-step GMM estimator of $\boldsymbol{\theta}$ where, in the first-stage, we obtain a consistent estimator $\hat{\boldsymbol{\theta}}_1$ using $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta}_0)$ computed with an arbitrary guess $\boldsymbol{\theta}_0$ and, in the second-stage, we estimate $\hat{\boldsymbol{\theta}}_2$ using $\nabla_{\boldsymbol{\theta}}v_i(\hat{\boldsymbol{\theta}}_1)$ computed with the first-stage estimate $\hat{\boldsymbol{\theta}}_1$.

A.1.9 Reduced-form Responses in Sectoral Employment Outcomes

To derive the change in sectoral employment, recall that, in our model, the share of employment in sector s is equal to the share of revenue in that sector, so that $\hat{\ell}_{i,s} = \hat{R}_{i,s} - \sum_k \ell^0_{i,k} \hat{R}_{i,k}$. The definitions $R_{i,s} \equiv \sum_j x_{ij,s} \xi_{j,s} E_j$, $r_{ij,s} \equiv x_{ij,s} \xi_{j,s} E_j / R_{i,s}$ and $\mu_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_j r^0_{ij,s}(\hat{\tau}_{ij,s} - \sum_o x^0_{oj,s} \hat{\tau}_{oj,s})$ imply

$$\hat{R}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) + (1-\sigma)\hat{p}_i + (\sigma-1)\sum_o \left(\sum_j r^0_{ij,s} x^0_{oj,s} \right) \hat{p}_o + \sum_j r^0_{ij,s} \hat{E}_j.$$

Thus, since $\hat{\eta}_i(\hat{\boldsymbol{\tau}}) \equiv (1-\sigma)\sum_k \ell_{i,k} \mu_{i,k}(\hat{\boldsymbol{\tau}})$,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + (\sigma-1)\sum_o \left(\sum_j r^0_{ij,s} x^0_{oj,s} - \sum_k \ell^0_{i,k} \sum_j r^0_{ij,k} x^0_{oj,k} \right) \hat{p}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right) \hat{E}_j.$$

Using the fact that $\hat{p}_o \equiv (1-\psi\phi)\hat{w}_o + \psi\phi\hat{\Omega}$, this expression is equivalent to

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \kappa\sum_o \left(\sum_j r^0_{ij,s} x^0_{oj,s} - \sum_k \ell^0_{i,k} \sum_j r^0_{ij,k} x^0_{oj,k} \right) \hat{w}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right) \hat{E}_j$$

and, therefore,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \kappa\sum_o \left(\chi^0_{io,s} - \chi^0_{io}\right) \hat{w}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right) \hat{E}_j$$

where $\chi^0_{io,s} \equiv \sum_d r^0_{id,s} x^0_{od,s}$ and $\chi^0_{io} \equiv \sum_s \ell^0_{i,s} \chi^0_{io,s}$.

Recall that $E_j = W_j = w_i (n_i)^{\frac{\phi-1}{\phi}} N_i \varrho$ and $\hat{E}_j = \phi^0_j \hat{w}_j + (1-\phi^0_j)\hat{\Omega}$. Thus,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \sum_j \left[\left(\chi^0_{ij,s} - \chi^0_{ij}\right)\kappa + \left(r^0_{ij,s} - r^0_{ij}\right)\phi^0_j\right] \hat{w}_j + \rho^0_{i,s} \hat{\Omega}$$

where $\rho^0_{i,s} \equiv -\sum_d (r^0_{id,s} - r^0_{id})\phi^0_d$. Since $\hat{\Omega} = \sum_j \omega^0_j \hat{w}_j$, this expression becomes

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \sum_j \pi^0_{ij,s} \hat{w}_j \tag{46}$$

where $\pi^0_{ij,s} \equiv \left(\chi^0_{ij,s} - \chi^0_{ij}\right)\kappa + \left(r^0_{ij,s} - r^0_{ij}\right)\phi^0_j + \rho^0_{i,s}\omega^0_j$. Notice that, up to a first-order approximation, the change in the share of employment in sector s is $\Delta\ell_{i,s} = \ell^0_{i,s}\hat{\ell}_{i,s}$ and share of population employed in sector s is $\Delta n_{i,s} = \ell^0_{i,s} n^0_i (\hat{\ell}_{i,s} + \hat{n}_i)$.

A.2 Proofs for Section 4

The equilibrium requires the gross revenue $\{R_{i,s}\}_{i,s}$ to solve

$$R_{i,s} = \sum_j \frac{\left(\tau_{ij,s} p_{i,s}\right)^{1-\sigma}}{\sum_o \left(\tau_{oj,s} p_{o,s}\right)^{1-\sigma}} \left(\xi_{j,s} E_j + \sum_k \xi^M_{j,sk} a^M_{j,k} R_{j,k} \right) \quad \text{for all} \quad (i,s), \tag{47}$$

where $E_i = W_i = \varrho w^\phi_i (w^\phi_i + b^\phi_i)^{\frac{1-\phi}{\phi}} N_i$.

The equilibrium can be expressed as a wage vector in terms of an excess labor demand system: $D_i(\boldsymbol{w}|\boldsymbol{\tau})=0$ for all i, such that

$$D_i(\boldsymbol{w}|\boldsymbol{\tau})\equiv\sum_s a_{i,s}^L R_{i,s}(\boldsymbol{w}|\boldsymbol{\tau})-W_i(\boldsymbol{w}|\boldsymbol{\tau}). \tag{48}$$

A.2.1 Proof of Excess Labor Demand in (48)

Step 1. We now implicitly characterize $P_{j,s}(\boldsymbol{w})$ from the combination of $P_{j,s}$ in (7), $p_{i,s}$ in (31), n_i in (8), N_i in (30) with $b_i=\bar{b}_i(P_i)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}$. Thus, any $\{P_{j,s}\}_{j,s}\in\{P_{j,s}(\boldsymbol{w})\}_{j,s}$ solves

$$(P_{j,s})^{1-\sigma}=\sum_i(\tau_{ij,s})^{1-\sigma}\left(\frac{w_i}{\bar{b}_i\left(\Pi_k(P_{i,k})^{\xi_{i,k}}\right)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}}\right)^{\phi\psi(\sigma-1)}\left[(w_i)^{a_{i,s}^L}\left(\Pi_k(P_{i,k})^{\xi_{i,ks}^M}\right)^{a_{i,s}^M}\right]^{1-\sigma}.$$

Step 2. For any $\boldsymbol{w}$, take $\{P_{j,s}\}_{j,s}\in\{P_{j,s}(\boldsymbol{w})\}_{j,s}$ to compute $P_i(\boldsymbol{w}|\boldsymbol{\tau})=\Pi_k(P_{i,k})^{\xi_{i,k}}$, and $P_{i,s}^M(\boldsymbol{w}|\boldsymbol{\tau})=\Pi_k(P_{i,k})^{\xi_{i,ks}^M}$. We then obtain $p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau})$ from (31), $n_i(\boldsymbol{w}|\boldsymbol{\tau})$ from (8), $N_i(\boldsymbol{w}|\boldsymbol{\tau})$ from (30), and

$$E_j(\boldsymbol{w}|\boldsymbol{\tau})=W_j(\boldsymbol{w}|\boldsymbol{\tau})=w_i^\phi\left(w_i^\phi+\left(\bar{b}_i\left(\Pi_k(P_{i,k})^{\xi_{i,k}}\right)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}\right)^\phi\right)^{\frac{1-\phi}{\phi}}N_i(\boldsymbol{w}|\boldsymbol{\tau})\varrho.$$

Step 3. To define the revenue function, we solve for

$$R_{i,s}-\sum_k\sum_j\frac{(\tau_{ij,s}p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o(\tau_{oj,s}p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}\xi_{j,sk}^M a_{j,k}^M R_{j,k}=\sum_j\frac{(\tau_{ij,s}p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o(\tau_{oj,s}p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}\xi_{j,s}E_j(\boldsymbol{w}|\boldsymbol{\tau})$$

This system has a unique solution because, for every (j,k),

$$\sum_s\sum_i\frac{(\tau_{ij,s}p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o(\tau_{oj,s}p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}\xi_{j,sk}^M a_{j,k}^M=\sum_s\xi_{j,sk}^M a_{j,k}^M=a_{j,k}^M<1,$$

so that

$$[R_{i,s}(\boldsymbol{w}|\boldsymbol{\tau})]_{i,s}=\sum_{d=0}^{\infty}\left(\bar{\boldsymbol{A}}(\boldsymbol{w}|\boldsymbol{\tau})\right)^d\left[\sum_j\frac{(\tau_{ij,s}p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o(\tau_{oj,s}p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}\xi_{j,s}E_j(\boldsymbol{w}|\boldsymbol{\tau})\right]_{i,s}$$

where

$$\bar{\boldsymbol{A}}(\boldsymbol{w}|\boldsymbol{\tau})\equiv\left[\frac{(\tau_{ij,s}p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o(\tau_{oj,s}p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}\xi_{j,sk}^M a_{j,k}^M\right]_{is,jk}.$$

A.2.2 Proof of Equation (32)

In all the remaining proofs of this section, we simplify notation by omitting the superscript 0. The system in (47) implicitly defines $\{R_{i,s}\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{p_{i,s}\}_{i,s}$. We can then use the implicit function theorem to write

$$\frac{\partial\ln R_{i,k}}{\partial\ln\tau_{od,s}}=\mathbb{I}_{s=k}\frac{X_{id,s}}{R_{i,s}}(1-\sigma)(\mathbb{I}_{i=o}-x_{od,s})+\sum_{k'}\sum_j\frac{x_{ij,s}\xi_{j,sk'}^M a_{j,k'}^M R_{j,k'}}{R_{i,k}}\frac{\partial\ln R_{j,k'}}{\partial\ln\tau_{od,s}}$$

which implies that

$$\left[\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}}\right]_{ik} = (1-\sigma)\left(\bar{\boldsymbol{I}}+\bar{\boldsymbol{b}}^U\right)\left[\mathbb{I}_{s=k}\frac{X_{id,s}}{R_{i,s}}(\mathbb{I}_{i=o}-x_{od,s})\right]_{ik}$$

where we have used the definition $\bar{\boldsymbol{b}}^U \equiv \sum_{d=1}^{\infty}\left(\bar{\boldsymbol{r}}^U\right)^d$, where $\bar{\boldsymbol{r}}^U \equiv [r^U_{is,jk}]_{is,jk}$ with $r^U_{is,jk} \equiv X^M_{ij,sk}/R_{i,s}$.

Thus,

$$\begin{aligned}
\left[\textstyle\sum_{s,o,d}\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s}\right]_{ik} &= (1-\sigma)\quad \left(\bar{\boldsymbol{I}}+\bar{\boldsymbol{b}}^U\right)\left[\textstyle\sum_{o,d}\frac{X_{id,k}}{R_{i,k}}(\mathbb{I}_{i=o}-x_{od,k})\hat{\tau}_{od,k}\right]_{ik} \\
&= (1-\sigma)\quad \left(\bar{\boldsymbol{I}}+\bar{\boldsymbol{b}}^U\right)[\mu_{i,k}(\hat{\boldsymbol{\tau}})]_{ik} \\
&= (1-\sigma)\quad \left[\mu_{i,k}(\hat{\boldsymbol{\tau}})+\textstyle\sum_{j,k'}b^U_{ik,jk'}\mu_{j,k'}(\hat{\boldsymbol{\tau}})\right]_{ik} \\
&= (1-\sigma)\quad \left[\mu_{i,k}(\hat{\boldsymbol{\tau}})+\mu^U_{i,k}(\hat{\boldsymbol{\tau}})\right]_{ik}
\end{aligned}$$

This immediately implies (32) because $\hat{\eta}^R_i(\hat{\boldsymbol{\tau}}) \equiv \sum_{s,o,d}\frac{\partial \ln R_i}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s} = \sum_k \ell_{i,k}\sum_{s,o,d}\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s}$.

A.2.3 Proof of Equation (33)

Equations (7) and (31) define $\{P^M_{i,s}\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{w_i,L_i\}_i$ from the solution of

$$P^M_{i,s} = \Pi_k\left(\sum_o(\tau_{oi,k})^{1-\sigma}\left(\frac{w_o}{b_o}\right)^{\phi\psi(\sigma-1)} w_o^{(1-\sigma)a^L_{o,s}}\left(P^M_{o,k}\right)^{(1-\sigma)a^M_{o,s}}\right)^{\frac{\xi^M_{i,ks}}{1-\sigma}}.$$

Using the implicit function theorem, we have that

$$\frac{\partial \ln P^M_{i,s}}{\partial \ln \tau_{od,k}} = \mathbb{I}_{i=d}\xi^M_{i,ks}x_{oi,k} + \sum_{k'}\sum_j \xi^M_{i,k's}x_{ji,k'}a^M_{j,k'}\frac{\partial \ln P^M_{j,k'}}{\partial \ln \tau_{od,k}}$$

which, by defining $\bar{\boldsymbol{g}}^D \equiv (\bar{\boldsymbol{I}}+\bar{\boldsymbol{b}}^D)$ implies that

$$\left[\frac{\partial \ln P^M_{i,s}}{\partial \ln \tau_{od,k}}\right]_{is} = \bar{\boldsymbol{g}}^D\left[\mathbb{I}_{i=d}\xi^M_{i,ks}x_{oi,k}\right]_{is}.$$

Thus, $[\hat{\eta}^M_{i,s}(\hat{\boldsymbol{\tau}})]_{i,s} = \bar{\boldsymbol{g}}^D[\mu^M_{i,s}(\hat{\boldsymbol{\tau}})]_{is} = \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k}b^D_{is,jk}\mu^M_{j,k}(\hat{\boldsymbol{\tau}})$.

A.2.4 Proof of Equations (34)–(35)

There are four parts to the derivation of equations (34)–(35). The first part is to characterize the matrix of spatial links, $\bar{\boldsymbol{\gamma}}$, and the vector of shifts in excess labor demand, $\hat{\boldsymbol{\eta}}$ (as in Sections 3.2.1 and 3.2.2). The second part is to characterize the reduced-form response of wages to shifts in excess labor demand (as in Section 3.2.3). The third is the derivation of the reduced-form responses in labor market outcomes to shocks in bilateral productivity. The last part is the derivation of (34) for observed and unobserved shocks, and the associated moment conditions in (35) (as in Section 3.3).

Part A: Derivation of the matrix of spatial links, $\bar{\boldsymbol{\gamma}}$, and the vector of shifts in excess labor demand, $\hat{\boldsymbol{\eta}}$. We first establish that, by totally differentiating the equilibrium conditions, we can write them as

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}. \tag{49}$$

Step 1. We first derive changes in labor market outcomes as a function of $\hat{\boldsymbol{w}}$ and $\hat{\boldsymbol{P}}$. Expression (8) with $b_i = \bar{b}_i P_i^{\lambda}(\Omega(\boldsymbol{w}))^{1-\lambda}$ implies that $\hat{n}_i = \phi(1-n_i)(\hat{w}_i - \lambda \hat{P}_i - (1-\lambda)\sum_j \omega_j \hat{w}_j)$. So,

$$\hat{\boldsymbol{n}} = \phi \bar{\boldsymbol{\phi}}^{n,w} \hat{\boldsymbol{w}} + \phi \lambda \bar{\boldsymbol{\phi}}^{n,P} \hat{\boldsymbol{P}} \tag{50}$$

such that $\phi_{ij}^{n,w} \equiv (1-n_i)(\mathbb{I}_{i=j} - (1-\lambda)\omega_j)$ and $\phi_{ij}^{n,p} \equiv -(1-n_i)\mathbb{I}_{i=j}$.

Expression (30) with $b_i = \bar{b}_i P_i^{\lambda}(\Omega(\boldsymbol{w}))^{1-\lambda}$ implies

$$\begin{array}{rcl} \hat{N}_i & = & \vartheta \sum_j \left(\mathbb{I}_{i=j} - \frac{N_j}{N_{c(i)}} \mathbb{I}_{c(i)=c(j)} \right) \left(\hat{w}_j - \hat{P}_j + (\phi-1)(1-n_j) \left(\hat{w}_j - \lambda \hat{P}_j \right) \right) \\ + & & (\phi-1)(1-\lambda) \left(n_i - \sum_{j \in c(i)} \frac{N_j}{N_{c(j)}} n_j \right) \sum_d \omega_d \hat{w}_d. \end{array}$$

Thus,

$$\hat{\boldsymbol{N}} = \vartheta \bar{\boldsymbol{\phi}}^{N,w} \hat{\boldsymbol{w}} + \vartheta \bar{\boldsymbol{\phi}}^{N,P} \hat{\boldsymbol{P}}, \tag{51}$$

where $\tilde{\boldsymbol{N}} \equiv \left[\mathbb{I}_{i=j} - \mathbb{I}_{c(i)=c(j)} N_j / N_{c(j)} \right]_{i,j}$, $\bar{\boldsymbol{n}} \equiv [n_i \mathbb{I}_{i=j}]_{i,j}$,

$$\bar{\boldsymbol{\phi}}^{N,w} \equiv \tilde{\boldsymbol{N}} \left(\bar{\boldsymbol{I}} + (\phi-1) \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{n}} \right) \right) + (\phi-1)(1-\lambda) \tilde{\boldsymbol{N}} \boldsymbol{n} \boldsymbol{\omega}', \quad \text{and} \quad \bar{\boldsymbol{\phi}}^{N,P} \equiv -\tilde{\boldsymbol{N}} \left(\bar{\boldsymbol{I}} + \lambda(\phi-1) \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{n}} \right) \right).$$

Expressions (50) and (51) imply that $\hat{W}_i = \hat{w}_i + \hat{n}_i(\phi-1)/\phi + \hat{N}_i$ is given by

$$\hat{\boldsymbol{E}} = \hat{\boldsymbol{W}} = \bar{\boldsymbol{\phi}}^{W,w} \hat{\boldsymbol{w}} + \left((\phi-1)\lambda \bar{\boldsymbol{\phi}}^{n,P} + \vartheta \bar{\boldsymbol{\phi}}^{N,P} \right) \hat{\boldsymbol{P}}, \tag{52}$$

where $\bar{\boldsymbol{\phi}}^{W,w} \equiv \bar{\boldsymbol{I}} + (\phi-1) \bar{\boldsymbol{\phi}}^{n,w} + \vartheta \bar{\boldsymbol{\phi}}^{N,w}$.

Step 2. We now derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (31),

$$\hat{P}_{i,s}^M - \sum_{j,k} \xi_{i,ks}^M x_{ji,k} a_{j,k}^M \hat{P}_{j,k}^M = \mu_{i,s}^M(\hat{\boldsymbol{\tau}}) + \sum_{j,k} \xi_{i,ks}^M x_{ji,k} \left(a_{j,k}^L \hat{w}_j - \frac{\psi}{1-n_j} \hat{n}_j \right)$$

which, by defining $\bar{\boldsymbol{x}}_{NS \times NS}^D \equiv [a_{j,k}^M \xi_{i,ks}^M x_{ji,k}]_{is,jk}$, $\bar{\boldsymbol{x}}_{NS \times NS}^M \equiv [\xi_{i,ks}^M x_{ji,k}]_{is,jk}$, $\bar{\boldsymbol{a}}_{NS \times N}^L \equiv [a_{i,k}^L \mathbb{I}_{i=j}]_{ik,j}$, $\bar{\boldsymbol{v}}_{NS \times N}^n = [\mathbb{I}_{i=j}(1-n_i)^{-1}]_{ik,j}$, implies

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D \right) \hat{\boldsymbol{P}}^M = \boldsymbol{\mu}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^M \left(\bar{\boldsymbol{a}}^L \hat{\boldsymbol{w}} - \psi \bar{v}^n \hat{\boldsymbol{n}} \right)$$

and, therefore,

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^D \bar{\boldsymbol{x}}^M \left(\bar{\boldsymbol{a}}^L \hat{\boldsymbol{w}} - \psi \bar{v}^n \hat{\boldsymbol{n}} \right). \tag{53}$$

From Shepard's lemma, $\hat{P}_i = \sum_s \xi_{i,s} \sum_j x_{ji,s}(\hat{\tau}_{ji,s} + \hat{p}_{j,s})$, which implies that

$$\hat{P}_i = \sum_s \xi_{i,s} \sum_j x_{ji,s} \left(\hat{\tau}_{ji,s} + a_{j,s}^L \hat{w}_j + a_{j,s}^M \hat{P}_{j,s}^M - \frac{\psi}{1-n_j} \hat{n}_j \right)$$

which, by defining $\bar{\boldsymbol{x}}_{N \times NS}^C \equiv [\xi_{i,s} x_{ji,s}]_{i,js}$, $\bar{\boldsymbol{a}}_{NS \times NS}^M \equiv [a_{j,k}^M \mathbb{I}_{is=jk}]_{is,jk}$,

$$\hat{\boldsymbol{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \left(\bar{\boldsymbol{a}}^L \hat{\boldsymbol{w}} - \psi \bar{v}^n \hat{\boldsymbol{n}} \right) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{P}}^M.$$

Substituting (53) into this expression,

$$\hat{\boldsymbol{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{c}}^M \left(\bar{\boldsymbol{a}}^L \hat{\boldsymbol{w}} - \psi \bar{v}^n \hat{\boldsymbol{n}}\right),$$

where $\bar{\boldsymbol{c}}^M \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{a}}^M \bar{\boldsymbol{g}}^D \bar{\boldsymbol{x}}^M$.

By plugging (50) into this expression,

$$\begin{aligned} \hat{\boldsymbol{P}} &= \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{c}}^M \bar{\boldsymbol{a}}^L \hat{\boldsymbol{w}} \\ &- \psi \bar{\boldsymbol{x}}^C \bar{\boldsymbol{c}}^M \bar{v}^n \left(\phi \bar{\boldsymbol{\phi}}^{n,w} \hat{\boldsymbol{w}} + \phi \lambda \bar{\boldsymbol{\phi}}^{n,P} \hat{\boldsymbol{P}}\right) \end{aligned}$$

and, therefore,

$$\hat{\boldsymbol{P}} = \bar{\boldsymbol{\alpha}}^{P,w} \hat{\boldsymbol{w}} + \boldsymbol{\rho}^P \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right), \tag{54}$$

with $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \boldsymbol{\rho}^P \bar{\boldsymbol{x}}^C \bar{\boldsymbol{c}}^M \left(\bar{\boldsymbol{a}}^L - \phi \psi \bar{v}^n \bar{\boldsymbol{\phi}}^{n,w}\right)$ and $\boldsymbol{\rho}^P \equiv \left(\bar{\boldsymbol{I}} + \phi \psi \lambda \bar{\boldsymbol{x}}^C \bar{\boldsymbol{c}}^M \bar{v}^n \bar{\boldsymbol{\phi}}^{n,P}\right)^{-1}$.

Substituting $\hat{\boldsymbol{P}}$ in (54) into (50), we have that

$$\hat{\boldsymbol{n}} = \phi \bar{\boldsymbol{\alpha}}^{n,w} \hat{\boldsymbol{w}} + \phi \lambda \bar{\boldsymbol{\alpha}}^{n,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{55}$$

with $\bar{\boldsymbol{\alpha}}^{n,w} \equiv \bar{\boldsymbol{\phi}}^{n,w} + \lambda \bar{\boldsymbol{\phi}}^{n,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{n,P} \equiv \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^P$.

Substituting $\hat{\boldsymbol{P}}$ in (54) into (51), we have that

$$\hat{\boldsymbol{N}} = \vartheta \bar{\boldsymbol{\alpha}}^{N,w} \hat{\boldsymbol{w}} + \vartheta \bar{\boldsymbol{\alpha}}^{N,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{56}$$

with $\bar{\boldsymbol{\alpha}}^{N,w} \equiv \bar{\boldsymbol{\phi}}^{N,w} + \bar{\boldsymbol{\phi}}^{N,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{N,P} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^P$.

Substituting $\hat{\boldsymbol{P}}$ in (54) into (52), we have that

$$\hat{\boldsymbol{W}} = \bar{\boldsymbol{\alpha}}^{W,w} \hat{\boldsymbol{w}} + \left(\lambda \bar{\boldsymbol{\alpha}}_\lambda^{W,P} + \vartheta \bar{\boldsymbol{\alpha}}_\vartheta^{W,P}\right) \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{57}$$

with $\bar{\boldsymbol{\alpha}}^{W,w} \equiv \bar{\boldsymbol{\phi}}^{W,w} + \left((\phi-1)\lambda \bar{\boldsymbol{\phi}}^{n,P} + \vartheta \bar{\boldsymbol{\phi}}^{N,P}\right) \bar{\boldsymbol{\alpha}}^{P,w}$, $\bar{\boldsymbol{\alpha}}_\lambda^{W,P} \equiv (\phi-1) \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^P$, and $\bar{\boldsymbol{\alpha}}_\vartheta^{W,P} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^P$.

Finally, we can solve for the change in the input price index using (53):

$$\hat{\boldsymbol{P}}^M = \bar{\boldsymbol{\alpha}}^{M,w} \hat{\boldsymbol{w}} + \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) - \psi \phi \lambda \bar{\boldsymbol{\alpha}}_\lambda^{M,P} \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{58}$$

where $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\boldsymbol{g}}^D \bar{\boldsymbol{x}}^M \left(\bar{\boldsymbol{a}}^L - \phi \psi \bar{v}^n \bar{\boldsymbol{\alpha}}^{n,w}\right)$, and $\bar{\boldsymbol{\alpha}}_\lambda^{M,P} \equiv \bar{\boldsymbol{g}}^D \bar{\boldsymbol{x}}^M \bar{v}^n \bar{\boldsymbol{\alpha}}^{n,P}$.

Step 3. We now solve for the change in revenue of sector-market pairs. From (47), by defining $\bar{\boldsymbol{r}}^C_{NS\times N} \equiv [x_{ij,s} \xi_{j,s} E_j / R_{i,s}]_{is,j}$,

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^U\right) \hat{\boldsymbol{R}} = \left[\sum_d r_{id,s} \hat{x}_{id,s}\right]_{is} + \bar{\boldsymbol{r}}^C \hat{\boldsymbol{W}},$$

where, from (6),

$$\sum_d r_{id,s} \hat{x}_{id,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \sum_d r_{id,s} (\mathbb{I}_{i=j} - x_{jd,s}) \left(a_{j,s}^L \hat{w}_j + a_{j,s}^M \hat{P}_{j,s}^M - \frac{\psi}{1-n_j} \hat{n}_j\right).$$

By defining $\bar{\boldsymbol{\chi}}_{NS\times NS}\equiv[\mathbb{I}_{s=k}\sum_d r_{id,s}(\mathbb{I}_{i=j}-x_{oj,s})]_{is,jk}$ and $\bar{\boldsymbol{g}}^U\equiv\bar{\boldsymbol{I}}+\bar{\boldsymbol{b}}^U$, we can then write

$$\hat{\boldsymbol{R}} \quad = \quad (1-\sigma)\bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})+(1-\sigma)\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\Big(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}}-\psi\bar{v}^n\hat{\boldsymbol{n}}+\bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M\Big)+\bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^C\hat{\boldsymbol{W}}. \tag{59}$$

Step 4. We now characterize the system in (49). The equilibrium definition in (48) implies that $\hat{W}_i=\sum_s\ell_{i,s}\hat{R}_{i,s}$, which, by defining $\bar{\boldsymbol{\ell}}_{N\times NS}\equiv[\ell_{j,s}\mathbb{I}_{[i=j]}]_{i,js}$, and $\bar{\boldsymbol{v}}_{NS\times N}=[\mathbb{I}_{i=j}]_{ik,j}$, can be written as

$$\hat{\boldsymbol{W}}=\bar{\boldsymbol{\ell}}\hat{\boldsymbol{R}}.$$

Plugging (59) into this expression, we have that

$$\big(\bar{\boldsymbol{I}}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^C\big)\hat{\boldsymbol{W}} \quad = \quad (1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})+(1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\Big(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}}-\psi\bar{v}^n\hat{\boldsymbol{n}}+\bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M\Big)\ .$$

Using (55)–(57),

$$\bar{\boldsymbol{\gamma}}^w\hat{\boldsymbol{w}}=(1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})+(1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma\hat{\boldsymbol{P}}^M+\big(\lambda\bar{\boldsymbol{\gamma}}^P_\lambda+\vartheta\bar{\boldsymbol{\gamma}}^P_\vartheta\big)\Big(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\Big),$$

where

$$\bar{\boldsymbol{\gamma}}^w \quad \equiv \quad \big(\bar{\boldsymbol{I}}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\big)\bar{\boldsymbol{\alpha}}^{W,w}-(1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\big(\bar{\boldsymbol{a}}^L-\psi\phi\bar{v}^n\bar{\boldsymbol{\alpha}}^{n,w}\big),$$

$$\bar{\boldsymbol{\gamma}}^P_\lambda\equiv-\big(\bar{\boldsymbol{I}}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\big)\bar{\boldsymbol{\alpha}}^{W,P}_\lambda-(1-\sigma)\psi\phi\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\bar{v}^n\bar{\boldsymbol{\alpha}}^{n,P},$$

$$\bar{\boldsymbol{\gamma}}^P_\vartheta\equiv-\big(\bar{\boldsymbol{I}}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\big)\bar{\boldsymbol{\alpha}}^{W,P}_\vartheta,$$

$$\bar{\boldsymbol{\alpha}}^M_\sigma\equiv\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^M.$$

Substituting $\hat{\boldsymbol{P}}^M$ with (58),

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})+(1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})+\big(\lambda\bar{\boldsymbol{\alpha}}^P_\lambda+\vartheta\bar{\boldsymbol{\alpha}}^P_\vartheta\big)\Big(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\Big)$$

where

$$\bar{\boldsymbol{\gamma}}\equiv\bar{\boldsymbol{\gamma}}^w-(1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma\bar{\boldsymbol{\alpha}}^{M,w} \tag{60}$$

$$\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})\equiv(1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}),,\quad \bar{\boldsymbol{\alpha}}^P_\lambda\equiv\bar{\boldsymbol{\gamma}}^P_\lambda-(1-\sigma)\psi\phi\bar{\boldsymbol{\alpha}}^M_\sigma\bar{\boldsymbol{\alpha}}^{M,P}_\lambda,\quad \bar{\boldsymbol{\alpha}}^P_\vartheta\equiv\bar{\boldsymbol{\gamma}}^P_\vartheta.$$

Thus,

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$$

with

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})\equiv\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})+\big(\lambda\bar{\boldsymbol{\alpha}}^P_\lambda+\vartheta\bar{\boldsymbol{\alpha}}^P_\vartheta\big)\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})+\big((1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma+\lambda\bar{\boldsymbol{\alpha}}^M_\lambda+\vartheta\bar{\boldsymbol{\alpha}}^M_\vartheta\big)\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{61}$$

$$\bar{\boldsymbol{\alpha}}^M_\lambda\equiv\bar{\boldsymbol{\alpha}}^P_\lambda\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M,\quad\text{and}\quad\bar{\boldsymbol{\alpha}}^M_\vartheta\equiv\bar{\boldsymbol{\alpha}}^P_\vartheta\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M.$$

Part B: Derivation of the reduced-form response of wages to shifts in excess labor demand. We now establish that the system above, $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$, yields a reduced-form representation for wage changes, $\hat{\boldsymbol{w}}=\bar{\boldsymbol{\beta}}\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$, where $\bar{\boldsymbol{\beta}}$ has a series expansion representation. We impose the same diagonal dominance condition: In any equilibrium, there is a vector $\{h_i\}_{i\neq m}\gg0$ such that, for all $i\neq m$,

$$h_i\gamma_{ii}>\sum_{j\neq i,m}|\gamma_{ij}|h_j. \tag{62}$$

The proof now proceeds in the same way as the proof in Section A.1.2. We start by redefining the system to set the change in the wage of market m to zero. Using the same matrix $\bar{\boldsymbol{M}}$ defined in Section A.1.2, we show that, if $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is non-singular, then $\hat{\boldsymbol{w}}=\bar{\boldsymbol{\beta}}\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$ for $\bar{\boldsymbol{\beta}}\equiv\bar{\boldsymbol{M}}'(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}')^{-1}\bar{\boldsymbol{M}}$. In the rest of the proof, we first show that $\bar{\boldsymbol{\beta}}$ exists and then that it admits a series representation. To simplify exposition, we again abuse notation by defining

$$\bar{\boldsymbol{\gamma}}\equiv\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}',\quad \hat{\boldsymbol{w}}\equiv\bar{\boldsymbol{M}}\hat{\boldsymbol{w}}\qquad\text{and}\qquad \hat{\boldsymbol{\eta}}\equiv\bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}).$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m=0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\lambda}}$ be the diagonal matrix with the diagonal elements of $\bar{\boldsymbol{\gamma}}$: $\bar{\boldsymbol{\lambda}}$ s.t. $\lambda_{ii}=\gamma_{ii}$ and $\lambda_{ij}=0$ for $i\neq j$. Thus, we can write the system as

$$\bar{\boldsymbol{\gamma}}=\bar{\boldsymbol{\lambda}}\big(\bar{\boldsymbol{I}}-\bar{\tilde{\boldsymbol{\gamma}}}\big)\quad st\quad \bar{\tilde{\boldsymbol{\gamma}}}\equiv\bar{\boldsymbol{I}}-\bar{\boldsymbol{\lambda}}^{-1}\bar{\boldsymbol{\gamma}},$$

which implies that $\tilde{\gamma}_{ii}=0$ and $\tilde{\gamma}_{ij}=-\gamma_{ij}/\gamma_{ii}$. Let $\bar{\boldsymbol{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i. Thus, $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$ is equivalent to

$$\begin{aligned}
\bar{\boldsymbol{\lambda}}\big(\bar{\boldsymbol{I}}-\bar{\tilde{\boldsymbol{\gamma}}}\big)\Big(\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^{-1}\Big)\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\
\bar{\boldsymbol{\lambda}}\big(\bar{\boldsymbol{h}}-\bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}\big)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\
\big(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\big)\Big(\bar{\boldsymbol{I}}-\Big(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}\Big)\Big)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}},
\end{aligned}$$

which implies that

$$\hat{\boldsymbol{w}}=\bar{\boldsymbol{h}}\Big(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}}\Big)^{-1}\big(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\big)^{-1}\hat{\boldsymbol{\eta}},\quad \bar{\tilde{\tilde{\boldsymbol{\gamma}}}}\equiv\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}. \tag{63}$$

Notice that, for all i, $\tilde{\tilde{\gamma}}_{ii}=0$ and $\tilde{\tilde{\gamma}}_{ij}=-\gamma_{ij}h_j/\gamma_{ii}h_i$.

First, we show that $(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$ is non-singular, so that we can write the expression in (63). We proceed by contradiction. Suppose that $(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$ is singular, so $\mu=0$ is an eigenvalue of $(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$. Take the eigenvector $\boldsymbol{x}$ associated with the zero eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}})\boldsymbol{x}=0$, so that the i-row of this system is

$$1+\sum_{j\neq i,m}-\tilde{\tilde{\gamma}}_{ij}x_j=0\quad\Longrightarrow\quad 1+\sum_{j\neq i,m}\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j=0.$$

Thus,

$$\gamma_{ii}h_i=-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq\Big|\sum_{j\neq i,m}\gamma_{ij}h_jx_j\Big|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$$

where the last inequality holds because $|x_j|\leq 1$ and $h_j>0$. Thus, $\gamma_{ii}h_i\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts (62).

Second, we show that $(\bar{\boldsymbol{I}}-\bar{\tilde{\tilde{\boldsymbol{\gamma}}}})^{-1}$ admits a series representation. This is true whenever the largest eigenvalue of $\bar{\tilde{\tilde{\boldsymbol{\gamma}}}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\boldsymbol{x}$ associated with the largest eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $\mu\boldsymbol{x}=\bar{\tilde{\tilde{\boldsymbol{\gamma}}}}\boldsymbol{x}$ so that the i-row of this system is

$$1\leq\mu=\sum_{j\neq i,m}-\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j$$

Since γ_{ii} and h_i are positive,

$$\gamma_{ii}h_i\leq-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq\Big|\sum_{j\neq i,m}\gamma_{ij}h_jx_j\Big|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|$$

Since $|x_j| \le 1$ and $h_j > 0$, $\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j| \le \sum_{j\neq i,m}|\gamma_{ij}|h_j$. Thus, $\gamma_{ii}h_i \le \sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts (62). Thus, the largest eigenvalue of $\bar{\tilde{\bar{\boldsymbol{\gamma}}}}$ is below one, allowing us to write $(\bar{\boldsymbol{I}} - \bar{\tilde{\bar{\boldsymbol{\gamma}}}})^{-1} = \sum_{d=0}^{\infty}(\bar{\tilde{\bar{\boldsymbol{\gamma}}}})^d$. Substituting this series expansion into (63) yields

$$\hat{\boldsymbol{w}} = \sum_{d=0}^{\infty}\left(\bar{\boldsymbol{h}}\left(\bar{\tilde{\bar{\boldsymbol{\gamma}}}}\right)^d \bar{\boldsymbol{h}}^{-1}\right)\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we can follow the same steps in Appendix A.1.2 to show that $\bar{\boldsymbol{h}}(\bar{\tilde{\bar{\boldsymbol{\gamma}}}})^d\bar{\boldsymbol{h}}^{-1} = (\bar{\tilde{\boldsymbol{\gamma}}})^d$. Thus,

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}\hat{\boldsymbol{\eta}} = \sum_{d=0}^{\infty}(\bar{\tilde{\boldsymbol{\gamma}}})^d\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}},$$

which immediately implies that

$$\hat{w}_i = \sum_j \beta_{ij}\hat{\eta}_j(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \beta_{ij} = \frac{1}{\gamma_{ii}}\left(\mathbb{I}_{[i=j]} - \frac{\gamma_{ij}}{\gamma_{jj}}\mathbb{I}_{[i\neq j]}\right) + \sum_{d=2}^{\infty}\frac{\tilde{\gamma}_{ij}^{(d)}}{\gamma_{jj}} \tag{64}$$

with $\tilde{\gamma}_{ij}^{(d)}$ denoting the i-j entry of $(\bar{\tilde{\boldsymbol{\gamma}}})^d$ defined as $\tilde{\gamma}_{ij} \equiv -\frac{\gamma_{ij}}{\gamma_{ii}}\mathbb{I}_{[i\neq j;\ i,j\neq m]}$.

Part C: Reduced-form representation for changes in labor market outcomes. We start by writing the reduced-form response in wages:

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}^R\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^C\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{65}$$

with

$$\bar{\boldsymbol{\beta}}^R = \bar{\boldsymbol{\beta}}, \quad \bar{\boldsymbol{\beta}}^C \equiv \bar{\boldsymbol{\beta}}\big(\lambda\bar{\boldsymbol{\alpha}}_\lambda^P + \vartheta\bar{\boldsymbol{\alpha}}_\vartheta^P\big), \quad \boldsymbol{\beta}^M \equiv \boldsymbol{\beta}\big((1-\sigma)\bar{\boldsymbol{\alpha}}_\sigma^M + \lambda\bar{\boldsymbol{\alpha}}_\lambda^M + \vartheta\bar{\boldsymbol{\alpha}}_\vartheta^M\big).$$

The combination of (65) and (55)–(56) implies

$$\hat{\boldsymbol{n}} = \bar{\boldsymbol{\beta}}^{n,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{66}$$

$$\hat{\boldsymbol{N}} = \bar{\boldsymbol{\beta}}^{n,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{67}$$

where

$$\bar{\boldsymbol{\beta}}^{n,R} \equiv \phi\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^R, \quad \bar{\boldsymbol{\beta}}^{n,C} \equiv \phi\Big(\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^C + \lambda\bar{\boldsymbol{\alpha}}^{n,P}\Big), \quad \bar{\boldsymbol{\beta}}^{n,M} \equiv \phi\Big(\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^M + \lambda\bar{\boldsymbol{\alpha}}^{n,P}\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\Big)$$

$$\bar{\boldsymbol{\beta}}^{N,R} \equiv \vartheta\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^R, \quad \bar{\boldsymbol{\beta}}^{n,C} \equiv \vartheta\Big(\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^C + \bar{\boldsymbol{\alpha}}^{N,P}\Big), \quad \bar{\boldsymbol{\beta}}^{N,M} \equiv \vartheta\Big(\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^M + \bar{\boldsymbol{\alpha}}^{N,P}\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\Big)$$

Finally, from the expression for $\Delta\overline{\ln w_i}$ in (9), $\Delta\overline{\ln\boldsymbol{w}} = \hat{\boldsymbol{w}} - (1/\phi)\hat{\boldsymbol{n}}$. By substituting (64) and (66),

$$\Delta\overline{\ln\boldsymbol{w}} = \bar{\boldsymbol{\beta}}^{w,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{w,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{w,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{68}$$

where

$$\bar{\boldsymbol{\beta}}^{w,R} \equiv \bar{\boldsymbol{\beta}}^R - (1/\phi)\bar{\boldsymbol{\beta}}^{n,R}, \quad \bar{\boldsymbol{\beta}}^{w,C} \equiv \bar{\boldsymbol{\beta}}^C - (1/\phi)\bar{\boldsymbol{\beta}}^{n,C}, \quad \bar{\boldsymbol{\beta}}^{w,M} \equiv \bar{\boldsymbol{\beta}}^M - (1/\phi)\bar{\boldsymbol{\beta}}^{n,M}.$$

Part D: Empirical specification in (34)–(35). For outcomes $\Delta\ln Y_i \in \{\Delta\ln n_i, \Delta\ln N_i, \Delta\overline{\ln\boldsymbol{w}}\}$, equations (66)–(68) imply that, up to a first-order approximation,

$$\Delta\ln\boldsymbol{Y} = \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}).$$

By definition, $\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})$, $\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})$ and $\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})$ are linear combinations of $\hat{\tau}_{ij,s}$. Thus, $\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{\tau}}^{\text{obs}} + \hat{\boldsymbol{\tau}}^{\text{unbs}}$ implies that

$$\Delta \ln \boldsymbol{Y} = \alpha^Y + \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \boldsymbol{\nu}^Y$$

where $\boldsymbol{\nu} \equiv \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^{\text{unbs}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^{\text{unbs}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}^{\text{unbs}})$, $\alpha^Y \equiv I^{-1}\sum_i E[\nu_i|\boldsymbol{\mathcal{W}}^0]$, and $\nu_i^Y \equiv \nu_i - \alpha^Y$.

We now establish that, if $Cov(\hat{\tau}^{\text{obs}}_{ij,s}, \hat{\tau}^{\text{unbs}}_{od,k}|\boldsymbol{\mathcal{W}}^0) = 0$, then $E[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}})] = 0$ for $E \in \{R, C, M\}$. The proof is analogous to that in Appendix A.1.7. First notice that the definitions of $\bar{\boldsymbol{\beta}}^{Y,E}$ and $\hat{\boldsymbol{\eta}}^E(\hat{\boldsymbol{\tau}})$ imply that we can write

$$\nu_i^Y = \sum_{s,d,o} \beta^{Y,\tau}_{i,sdo}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\tau}^{\text{unbs}}_{od,s} - \alpha^Y, \quad \text{and} \quad \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}}) = \sum_{s,d,o} H^{E,\tau}_{j,sdo}(\boldsymbol{\mathcal{W}}^0)\left(\hat{\tau}^{\text{obs}}_{od,s} - \tau^{\text{obs}}\right).$$

Thus,

$$\begin{aligned} E\left[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}})|\boldsymbol{\mathcal{W}}^0\right] &= \textstyle\sum_{s,d,o}\sum_{s',d',o'} \beta^{Y,\tau}_{i,sdo}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) H^{\tau}_{j,s'd'o'}(\boldsymbol{\mathcal{W}}^0) E\left[\hat{\tau}^{\text{unbs}}_{od,s}(\hat{\tau}^{\text{obs}}_{od,s} - \tau^{\text{obs}})|\boldsymbol{\mathcal{W}}^0\right] \\ &- \alpha^Y \textstyle\sum_{s,d,o} H^{\tau}_{j,sdo}(\boldsymbol{\mathcal{W}}^0) E\left[\hat{\tau}^{\text{obs}}_{od,s} - \tau^{\text{obs}}|\boldsymbol{\mathcal{W}}^0\right] \end{aligned}$$

Note that $E[\hat{\tau}^{\text{obs}}_{od,s} - \tau^{\text{obs}}|\boldsymbol{\mathcal{W}}^0] = 0$ and $E[\hat{\tau}^{\text{unbs}}_{od,s}(\hat{\tau}^{\text{obs}}_{od,s} - \tau^{\text{obs}})|\boldsymbol{\mathcal{W}}^0] = 0$ from $Cov(\hat{\tau}^{\text{obs}}_{ij,s}, \hat{\tau}^{\text{unbs}}_{od,k}|\boldsymbol{\mathcal{W}}^0) = 0$. Hence, $E[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}})] = E[E[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \tau^{\text{obs}})|\boldsymbol{\mathcal{W}}^0]] = 0$ for any i and j. This immediately implies that (35) holds for any real matrix h^E_{ij}.

A.2.5 Extension of Reduced-Form Response in (22) for a Symmetric Economy with Intermediate Inputs in Production

There are no trade costs, $\tau_{ij,s} = \tau_{i,s}$ for all j, the input spending shares are the same in all sectors and markets, $a^M_{i,s} = a^M$ and $\xi^M_{i,ks} = \xi^M_k$. We consider the same labor supply structure in Section 3 where $\lambda = \vartheta = 0$. Thus,

$$P^M_{i,s} = P^M = \Pi_k \left[\sum_o (\tau_{o,k} p_o)^{1-\sigma}\right]^{\frac{\xi_k}{1-\sigma}},$$

$$p_i = (w_i)^{1-\psi\phi-a^M}\left(P^M\right)^{a^M} \bar{b}_i^{\psi\phi}(\Omega(\boldsymbol{w}))^{\psi\phi}.$$

By defining $\kappa \equiv (\sigma-1)(1-\psi\phi-a^M)$, these expressions imply that

$$x_{ij,s} = x_{i,s} = \frac{(\tau_{i,s}p_i)^{1-\sigma}}{\sum_o(\tau_{o,s}p_o)^{1-\sigma}} = \frac{\tau_{i,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o \tau_{o,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\psi\phi(1-\sigma)}}.$$

In this case, labor market clearing requires that

$$\begin{aligned} W_i &= (1-a^M)\textstyle\sum_s\sum_j x_{i,s}\left(\xi_s W_j + \sum_k \xi^M_s a^M R_{j,k}\right) \\ W_i &= \textstyle\sum_s\sum_j x_{i,s}\left((1-a^M)\xi_s W_j + a^M \xi^M_s \sum_k W_{j,k}\right) \\ W_i &= \textstyle\sum_s\sum_j x_{i,s}\left((1-a^M)\xi_s + a^M\xi^M_s\right)W_j \\ W_i &= \textstyle\sum_s\sum_j x_{i,s} e_s W_j \end{aligned}$$

where $e_s \equiv (1-a^M)\xi_s + a^M\xi^M_s$ is the share of gross spending on sector s (common to all markets i).

Finally, the supply of labor efficiency units is

$$W_j = w_j^{\phi}\left(w_j^{\phi} + \bar{b}_j^{\phi}(\Omega(\boldsymbol{w}))^{\phi}\right)^{\frac{1-\phi}{\phi}} \bar{N}_j \varrho.$$

The combination of the equilibrium conditions above implies that the equilibrium wage vector solves $D_i(\boldsymbol{w}|\boldsymbol{\tau})=0$ for all i such that

$$D_i(\boldsymbol{w}|\boldsymbol{\tau})\equiv\sum_s\sum_j\left[\frac{\tau_{i,s}^{1-\sigma}w_i^{-\kappa}\bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o\tau_{o,s}^{1-\sigma}w_o^{-\kappa}\bar{b}_o^{(1-\sigma)\psi\phi}}e_s-\mathbb{I}_{i=j}\right]w_j^{\phi}\left(w_j^{\phi}+\bar{b}_j^{\phi}(\Omega(\boldsymbol{w}))^{\phi}\right)^{\frac{1-\phi}{\phi}}\bar{N}_j\varrho.$$

Given the alternative definition of the labor demand parameter κ, this excess labor demand system is isomorphic to that in (12) for the model of Section 3 in the special case of $\tau_{ij,s}=\tau_{i,s}$ and $\xi_{j,s}=e_s$ for all j. This implies that Corollary 2 holds for this economy, and wage responses are given by (22).

A.2.6 Adding Endogenous Transfers to the Model

We now extend our model to allow for endogenous transfers across markets to finance the non-employment benefits. We maintain the same assumptions of Section 4 for production and labor supply. In this case, it is useful to write the location choice in terms of per-capita spending in each market, so that

$$\frac{N_i}{N_j}=\frac{P_i^{-\vartheta}(E_i/N_i)^{\vartheta}}{P_j^{-\vartheta}(E_j/N_j)^{\vartheta}}$$

and, therefore,

$$N_i=\frac{(E_i/P_i)^{\frac{\vartheta}{1+\vartheta}}}{\sum_{j\in\mathcal{I}_{c(i)}}(E_j/P_j)^{\frac{\vartheta}{1+\vartheta}}}. \tag{69}$$

We further assume that a fraction ϖ of non-employment benefits is financed with local taxes and that the remaining balance is financed with a common national income tax. Hence,

$$E_i=W_i+B_i-v_i(W_i+B_i)-v_c(W_i+B_i)$$

such that, in equilibrium,

$$v_i(W_i+B_i)=\varpi B_i,\quad\text{and}\quad v_c\sum_{i\in\mathcal{I}_c}(W_i+B_i)=(1-\varpi)\sum_{i\in\mathcal{I}_c}B_i.$$

Using the fact that $\frac{W_i}{B_i}=\frac{n_i}{1-n_i}$, this expression is equivalent to

$$E_i=W_i+(1-\varpi)\left[1-n_i-\frac{\sum_{j\in\mathcal{I}_{c(i)}}(1-n_j)(W_j/n_j)}{\sum_{j\in\mathcal{I}_{c(i)}}(W_j/n_j)}\right]\frac{W_i}{n_i}. \tag{70}$$

Notice that, as in our baseline specification, $E_i=W_i$ if $\varpi=1$. For any $\varpi\in[0,1]$, equations (8), (69), and (70) constitute a system of equations that can be locally solved as a function of $\boldsymbol{w}$ and $\boldsymbol{P}$. Conditional on these expressions, we can follow exactly the same steps in Appendix A.2.4 to characterize the reduced-form responses in the model.

A.3 Proofs of General Version of the Model in Section 4

A.3.1 Environment

Suppose that each country c is populated by a continuum of individuals divided into multiple groups indexed by $g=1,...,G$. Workers of group g in market i receive a wage of w_{gi} for each efficiency unit supplied. As in the baseline, each market has a competitive representative firm that produces a differentiated tradable intermediate good in each sector s, whose endogenous production cost is $p_{i,s}$ and iceberg trade cost of selling in j is $\tau_{ij,s}$.[47] There is a representative firm that produces a single non-tradable final consumption good in each market, and sells it at price of P_i.

We now present the three central parts of the model that, up to a first-order approximation, yield log-linear equilibrium relationships that are sufficient to derive the reduced-form representation for the model's predictions.

Labor Supply. Assume that worker preferences and efficiency implicitly define all labor outcomes as a (local) function of the wage rate and the price of the consumption good across markets. That is, for any group g in market i, we have the following local representation for one outcome Y_{gi} out of the employment rate n_{gi}, population N_{gi}, employment L_{gi}, spending E_{gi}, wage bill W_{gi}, or log average wage $\Delta\overline{\ln w}_{gi}$:

$$Y_{gi} = \Phi^Y_{gi}(\{w_{fj}\}_{fj}, \{P_j\}_j).$$

Notice that the models in Sections 3 and 4 satisfy this general restriction. It allows for (endogenous or exogenous) transfers across markets (for example, as in Appendix A.2.6). The general representation also covers a generalized Roy model with arbitrary individual heterogeneity in market-specific efficiency and preferences (for example, as in Adão (2016)). In addition, it allows for a rich structure of preferences for leisure and home production (as specified in Appendix B of the old version of our paper, Adao et al. (2020a)), as well as competitive search environments (such as the one described in Appendix A.5).

The first-order approximation for changes in any outcome Y_{gi} is

$$\hat{Y}_{gi} = \sum_{f,j} \phi^{Y,w}_{gi,fj} \hat{w}_{fj} + \sum_j \phi^{Y,P}_{gi,j} \hat{P}_j, \tag{71}$$

where $\phi^{Y,w}_{gi,fj} \equiv \frac{\partial \ln \Phi^Y_{gi}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln w_{fj}}$ and $\phi^{Y,P}_{gi,j} \equiv \frac{\partial \ln \Phi^Y_{gi}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln P_j}$ are the labor supply elasticities with respect to wages and prices, respectively.

Final Consumption Good. Assume that the production function for the final good combines the differentiated good from all origins: $C_j = F^C_j(\{c_{ij,s}\}_{i,s})$ where $c_{ij,s}$ is the quantity of the differentiated good of sector s from i used to produce the final good in market j. Assume that F^C_j is continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. Thus, cost minimization and zero profit imply that

$$P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k}) \equiv \min_{\{c_{ij,s}\}_{i,s}} \sum_{o,k} \tau_{oj,k} p_{o,k} \quad \text{such that} \quad F^C_j(\{c_{ij,s}\}_{i,s}) = 1.$$

The first-order approximation for changes in the final good price and final spending shares are given by

$$\hat{P}_j = \sum_{i,s} x^C_{ij,s}(\hat{\tau}_{ij,s} + \hat{p}_{i,s}), \quad \text{and} \quad \hat{x}^C_{ij,s} = \sum_{o,k} \chi^C_{ij,s,ok}(\hat{\tau}_{oj,k} + \hat{p}_{o,k}), \tag{72}$$

where $x^C_{ij,s} \equiv \frac{\partial \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s})}$ is the share of good s from i in final spending of market j, and $\chi^C_{ij,s,ok} \equiv \frac{\partial^2 \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s}) \partial \ln(\tau_{oj,k} p_{i,k})}$ is the elasticity of $x^C_{ij,s}$ to changes in the cost of good k from o. This final good production structure allows for arbitrary final spending shares and cross-price elasticities in the initial equilibrium. It is

[47] It is straightforward to define markets as groups of sectors by assuming that, for a subset of sectors, $\tau_{ij,s} = \infty$ for all j (including i).

equivalent to allowing individuals to have arbitrary homothetic preferences for the differentiated products of different sectors and origins.

Differentiated Good. Assume that the production function for the differentiated good of sector s from market i is subject to external economies of scale and combines labor of different groups and inputs from different sectors and origins. That is, $Q_{i,s} = \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j}) F_{i,s}\Big(\{L_{gi,s}\}_g, F^M_{i,s}(\{M_{oi,ks}\}_{o,k})\Big)$, where $L_{gi,s}$ is the number of efficiency units employed in sector s of market i, $M_{oi,ks}$ is the quantity of the differentiated good of sector k from o used to produce good s in market i, and $\Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})$ is the endogenous productivity term (but external to the firm) in sector s of market i that depends on employment outcomes across groups and markets. Assume that $F_{i,s}$ and $F^M_{i,s}$ are continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. This production function allows us to solve the firm's cost minimization problem in two stages.

Consider first the cost minimization problem of selecting intermediate inputs:

$$P^M_{i,s}(\{\tau_{oi,k} p_{o,k}\}_{o,k}) \equiv \min_{\{M_{oi,ks}\}_{o,k}} \sum_{o,k} \tau_{oi,k} p_{o,k} M_{oi,ks} \quad \text{s.t.} \quad F^M_{i,s}(\{M_{oi,ks}\}_{o,k}) = 1,$$

which implies that

$$\hat{P}^M_{i,s} = \sum_{j,k} x^M_{ji,ks}(\hat{\tau}_{ji,k} + \hat{p}_{j,k}), \quad \text{and} \quad \hat{x}^M_{ji,ks} = \sum_{o,h} \chi^M_{ji,ks,oh}(\hat{\tau}_{oi,h} + \hat{p}_{o,h}), \tag{73}$$

where $x^M_{ji,ks} = \frac{\partial \ln P^M_{i,s}(\{\tau_{oi,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ji,k} p_{j,k})}$ is the share of spending on sector k from j in the total input spending of sector s in market i, and $\chi^M_{ji,ks,oh} \equiv \frac{\partial^2 \ln P^M_{i,s}(\{\tau_{oi,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ji,k} p_{j,k}) \partial \ln(\tau_{oi,h} p_{o,h})}$ is the elasticity of $x^M_{ji,ks}$ to changes in the cost of the good from sector h of market o.

We then solve the firm's optimal spending on labor and inputs:

$$c_{i,s}\big(\{w_{gi}\}_g, P^M_{i,s}\big) = \min_{\{L_{gi,s}\}_g, M_{i,s}} \sum_g w_{gi} L_{gi,s} + P^M_{i,s} M_{i,s} \quad \text{s.t.} \quad F_{i,s}(\{L_{gi,s}\}_g, M_{i,s}) = 1,$$

which implies that

$$\hat{c}_{i,s} = \sum_g a^L_{gi,s} \hat{w}_{gi} + a^M_{i,s} \hat{P}^M_{i,s}, \quad \hat{a}^L_{gi,s} = \sum_{g'} \epsilon^L_{gi,s,g'} \hat{w}_{g'i} + \epsilon^{LM}_{gi,s} \hat{P}^M_{i,s}, \quad \text{and} \quad \hat{a}^M_{i,s} = \sum_g \epsilon^{LM}_{gi,s} \hat{w}_{gi} + \epsilon^M_{i,s} \hat{P}^M_{i,s} \tag{74}$$

where $a^L_{gi,s} \equiv \frac{\partial \ln c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})}{\partial \ln w_{gi}}$ and $a^M_{i,s} = \frac{\partial \ln c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})}{\partial \ln P^M_{i,s}}$ are the shares of labor and inputs on the total cost of sector s from i, and $\epsilon^L_{gi,s,g'} \equiv \frac{\partial^2 \ln c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})}{\partial \ln w_{gi} \partial \ln w_{g'i}}$, $\epsilon^{LM}_{gi,s} \equiv \frac{\partial^2 \ln c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})}{\partial \ln w_{gi} \partial \ln P^M_{i,s}}$ and $\epsilon^M_{i,s} \equiv \frac{\partial \ln c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})}{\partial \ln P^M_{i,s} \partial \ln P^M_{i,s}}$ are the elasticities of labor and input cost shares with respect to changes in wages and input prices.

Expressions in (73)–(74) allow for a flexible structure of production. In the initial equilibrium, each sector-market pair can have arbitrary spending shares on labor of different groups and intermediate goods from different sectors and origins. In addition, we flexibly allow for a nested elasticity structure in the labor and input demand functions. We do not impose parametric restrictions on the cross-price elasticity matrix for intermediate spending, allowing for different substitution patterns across goods from different sectors and markets. This structure also yields an arbitrary demand substitution pattern for labor of different groups. Importantly, while changes in the unit input cost can differentially affect demand for different labor groups, the nested production function imposes that the cost of inputs of distinct sectors and origins only affect factor demand through a single unit input cost index.

Finally, production cost of s in i is $p_{i,s}=c_{i,s}(\{w_{gi}\}_g,P^M_{i,s})/\Psi_{i,s}(\{n_{gj},N_{gj}\}_{g,j})$, which implies that

$$\hat{p}_{i,s}=\sum_g a^L_{gi,s}\hat{w}_{gi}+a^M_{i,s}\hat{P}^M_{i,s}-\sum_{g,j}\psi^n_{is,gj}\hat{n}_{gj}-\sum_{g,j}\psi^N_{is,gj}\hat{N}_{gj}, \tag{75}$$

where $\psi^n_{is,gj}\equiv\frac{\partial\ln\Psi_{i,s}(\{n_{gj},N_{gj}\}_{g,j})}{\partial\ln n_{gj}}$ and $\psi^N_{is,gj}\equiv\frac{\partial\ln\Psi_{i,s}(\{n_{gj},N_{gj}\}_{g,j})}{\partial\ln N_{gj}}$. This general agglomeration elasticity matrix allows for technology diffusion between regions, as in Fujita et al. (1999) and Lucas and Rossi-Hansberg (2003), and differences across sectors in economies of scale – e.g., Krugman and Venables (1995), Balistreri et al. (2010), Kucheryavyy et al. (2016). In addition, the cross-market elasticity of labor productivity may also incorporate congestion forces implied by the re-allocation of other factors of production, like land and capital (see Appendix B of the old version of our paper, Adao et al. (2020a)).

Equilibrium. The equilibrium requires both goods and labor markets to clear. For every sector s and market i, the vector of gross revenues $\{R_{i,s}\}_{i,s}$ must solve

$$R_{i,s}=\sum_j x^C_{ij,s}E_j+\sum_j x^M_{ij,sk}a^M_{j,k}R_{j,k}. \tag{76}$$

For every group g and market i, the vector of wages must guarantee that

$$W_{gi}=\sum_s a^L_{gi,s}R_{i,s}. \tag{77}$$

A.3.2 Reduced-form Representation

The following proposition characterizes the reduced-form representation of the wage change for each group and market as a function of measures of market-level shock exposure.

Proposition 1. *For an arbitrary $\hat{\boldsymbol{\tau}}\equiv\{\hat{\tau}_{ij,s}\}_{ijs}$, the vector of wage change, $\hat{\boldsymbol{w}}\equiv\{\hat{w}_{gi}\}_{gi}$, solves $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$. Under the diagonal dominance condition in (62), $\hat{\boldsymbol{w}}$ has a representation of the form:*

$$\hat{\boldsymbol{w}}=\bar{\boldsymbol{\beta}}\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \quad \textit{such that} \quad \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})=\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\alpha}}^C\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\alpha}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}), \tag{78}$$

where

$$\eta^R_{gi}(\hat{\boldsymbol{\tau}})=\sum_s \ell^0_{gi,s}(\mu_{i,s}(\hat{\boldsymbol{\tau}})+\sum_{j,k}b^U_{is,jk}\mu_{j,k}(\hat{\boldsymbol{\tau}})),\quad \eta^C_i(\hat{\boldsymbol{\tau}})=\sum_{s,o}x^C_{oi,s}\hat{\tau}_{oi,s},\quad \eta^M_{i,s}(\hat{\boldsymbol{\tau}})=\mu^M_{i,s}(\hat{\boldsymbol{\tau}})+\sum_{j,k}b^D_{is,jk}\mu^M_{j,k}(\hat{\boldsymbol{\tau}}) \tag{79}$$

such that

$$\mu_{i,s}(\hat{\boldsymbol{\tau}})\equiv\sum_{o,h}\sum_j(r^C_{ij,s}\chi^C_{ij,s,oh}+\sum_k r^U_{ij,sk}\chi^M_{ij,sk,oh})\hat{\tau}_{oj,h} \textit{ and } \mu^M_{i,s}(\hat{\boldsymbol{\tau}})\equiv\sum_{j,k}x^M_{ji,ks}\hat{\tau}_{ji,k}, \tag{80}$$

$$\bar{\boldsymbol{b}}^U\equiv\sum_{d=1}^{\infty}\left(\bar{\boldsymbol{r}}^U\right)^d \textit{ and } \bar{\boldsymbol{b}}^D\equiv\sum_{d=1}^{\infty}\left(\bar{\boldsymbol{x}}^D\right)^d, \tag{81}$$

with $\bar{\boldsymbol{r}}^U_{NS\times NS}\equiv[x^M_{ij,sk}a^M_{j,k}R_{j,k}/R_{i,s}]_{is,jk}$, $\bar{\boldsymbol{x}}^D_{NS\times NS}\equiv[x^M_{ji,ks}a^M_{j,k}]_{is,jk}$, $\boldsymbol{\mathcal{W}}^0\equiv\{\{x^C_{ij,s}\}_j,\{x^M_{ij,sk}\}_{j,k},\{a^L_{gi,s}\}_g\}_j$ and $\boldsymbol{\theta}\equiv\{\chi^M_{ji,ks,oh},\chi^C_{ji,s,oh},\psi^n_{is,gj},\psi^N_{is,gj},\phi^{Y,w}_{gi,fj},\phi^{Y,P}_{gi,j}\}$.

Appendix A.3.4 contains the proof of this proposition. It generalizes the reduced-form representation in Section 4 for the model with the non-parametric links in production and labor supply described in Appendix A.3.1. It maps wage changes to measures of shock exposure that depend on the initial bilateral trade shares for both final and intermediate consumption, the initial factor spending shares in production, and the elasticity matrices governing cross-market links in labor supply ($\phi^{Y,w}_{gi,fj}$ and $\phi^{Y,P}_{gi,j}$), productivity ($\psi^{n}_{is,gj}$ and $\psi^{N}_{is,gj}$) and trade demand ($\chi^{M}_{ji,ks,oh}$ and $\chi^{C}_{ji,s,oh}$). The definitions of consumption and input exposure are identical to those in Section 4 for arbitrary bilateral (final and intermediate) spending shares, $x^{M}_{ji,ks}$ and $x^{C}_{ji,s}$. However, the definition of revenue exposure needs to be extended to account for the more general demand for goods: the shock's impact on the sales of s in i, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$, is now a function of the cross-price demand elasticities, $\chi^{C}_{ij,s,oh}$ and $\chi^{M}_{ij,sk,oh}$. With the nested CES demand in Section 4, these elasticities take the simple form of $\chi^{C}_{ij,s,oh}=\chi^{M}_{ij,sk,oh}=(1-\sigma)(\mathbb{I}_{i=o}-x_{oj,s})\mathbb{I}_{s=k}$.

In Appendix A.3.4, we also characterize the reduced-form representation for labor market and price outcomes. For any $\hat{Y}_i\in\{\{\hat{w}_{gi},\hat{n}_{gi},\hat{N}_{gi},\hat{E}_{gi},\widehat{\overline{\ln w}}_{gi}\}_g,\hat{P}_i,\{\hat{P}^M_{i,s}\}_s\}$, we show that $\hat{Y}_i$ can be written as

$$\hat{Y}_i=\sum_{g,j}\beta^{Y,R}_{igj}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^R_{gj}(\hat{\boldsymbol{\tau}}|\boldsymbol{\theta})+\sum_{j}\beta^{Y,C}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^C_{j}(\hat{\boldsymbol{\tau}})+\sum_{j,s}\beta^{Y,M}_{ij,s}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^M_{j,s}(\hat{\boldsymbol{\tau}}). \tag{82}$$

The steps in Appendix A.2.4 imply that, under Assumption 1,

$$\Delta \ln Y_i=\alpha^Y+\sum_{g,j}\beta^{Y,R}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^R_{gj}(\hat{\boldsymbol{\tau}}^{\text{obs}})+\sum_{j}\beta^{Y,C}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^C_{j}(\hat{\boldsymbol{\tau}}^{\text{obs}})+\sum_{j,s}\beta^{Y,M}_{ij,s}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^M_{j,s}(\hat{\boldsymbol{\tau}}^{\text{obs}})+\nu^Y_i \tag{83}$$

such that, for $\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}}\equiv\hat{\boldsymbol{\tau}}^{\text{obs}}-\tau^{\text{obs}}$,

$$E\left[\nu^Y_i\,\hat{\eta}^R_{gj}(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}}|\boldsymbol{\theta})\right]=E\left[\nu^Y_i\,\hat{\eta}^C_{j}(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}})\right]=E\left[\nu^Y_i\,\hat{\eta}^M_{j,s}(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}})\right]=0\quad\text{for any } i,j,g,s. \tag{84}$$

A.3.3 Integration

The reduced-form representation in (82) is a first-order approximation for changes in the market-level outcomes: $\hat{Y}_i\in\{\{\hat{w}_{gi},\hat{n}_{gi},\hat{N}_{gi},\hat{E}_{gi},\widehat{\overline{\ln w}}_{gi}\}_g,\hat{P}_i,\{\hat{P}^M_{i,s}\}_s\}$. It can be integrated to compute exact changes in these outcomes using the following algorithm.

1. Consider $r=1,...,R$ repetitions. Let the initial conditions be $\boldsymbol{\mathcal{W}}^0\equiv\{\{x^C_{ij,s}\}_j,\{x^M_{ij,sk}\}_{s,j,k},\{a^L_{gi,s}\}_g\}_j$, and the initial elasticities be $\boldsymbol{\theta}^0\equiv\{\chi^M_{ji,ks,oh},\chi^C_{ji,s,oh},\psi^n_{is,gj},\psi^N_{is,gj},\phi^{Y,w}_{gi,fj},\phi^{Y,P}_{gi,j}\}$.

2. Given $\boldsymbol{\theta}^{r-1}$ and $\boldsymbol{\mathcal{W}}^{r-1}$, for $\hat{\boldsymbol{\tau}}^r=\hat{\boldsymbol{\tau}}/R$:

   (a) Compute $Y^r_i=Y^{r-1}_i\exp(\hat{Y}^r_i)$ where $\hat{Y}^r_i$ is given by (82);

   (b) Compute $p^r_{i,s}=p^{r-1}_{i,s}\exp(\hat{p}^r_{i,s})$ where $\hat{p}^r_{i,s}$ is given by (75);

   (c) Compute $\boldsymbol{\mathcal{W}}^r\equiv\{\{x^{C,r-1}_{ij,s}\exp(\hat{x}^{C,r}_{ij,s})\}_j,\{x^{M,r-1}_{ij,sk}\exp(\hat{x}^{M,r}_{ij,sk})\}_{j,k},\{a^L_{gi,s}\exp(\hat{a}^{L,r}_{gi,s})\}_g\}_j$ where $\hat{x}^{C,r}_{ij,s}$, $\hat{x}^{M,r}_{ij,sk}$, and $\hat{a}^{L,r}_{i,s}$ are respectively given by (72), (73) and (74);

   (d) Compute $\boldsymbol{\theta}^r\equiv\{\chi^{M,r}_{ji,ks,oh},\chi^{C,r}_{ji,s,oh},\psi^{n,r}_{is,gj},\psi^{N,r}_{is,gj},\phi^{Y,w,r}_{gi,fj},\phi^{Y,P,r}_{gi,j}\}$ using the definitions above and outcomes in iteration r.

3. Repeat step 2 for each r. The overall change in any outcome is $Y'_i/Y^0_i=Y^R_i/Y^0_i$.

A.3.4 Proof of Proposition 1

Step 1. We first derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (73) and (75),

$$\hat{P}^M_{i,s} = \sum_{j,k} x^M_{ji,ks}\left(\hat{\tau}_{ji,k} + \sum_g a^L_{gj,k}\hat{w}_{gj} + a^M_{j,k}\hat{P}^M_{j,k} - \sum_{g,o}\psi^n_{jk,go}\hat{n}_{go} - \sum_{g,o}\psi^N_{jk,go}\hat{N}_{go}\right)$$

which, by defining $\bar{\boldsymbol{x}}^D_{NS\times NS} \equiv [x^M_{ji,ks}a^M_{j,k}]_{is,jk}$, $\bar{\boldsymbol{x}}^M_{NS\times NS} \equiv [x^M_{ji,ks}]_{is,jk}$, $\bar{\boldsymbol{a}}^L_{NS\times NG} \equiv [a^L_{gj,k}\mathbb{I}_{i=j}]_{ik,gj}$, $\bar{\boldsymbol{\psi}}^n_{NS\times NG} = [\psi^n_{is,gj}]_{is,gj}$, $\bar{\boldsymbol{\psi}}^N_{NS\times NG} = [\psi^N_{is,gj}]_{is,gj}$ and $\boldsymbol{\mu}^M_{NS\times 1} \equiv [\hat{\mu}_{i,s}(\hat{\boldsymbol{\tau}})]_{i,s}$ with $\hat{\mu}_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,k} x^M_{ji,ks}\hat{\tau}_{ji,k}$, implies

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D\right)\hat{\boldsymbol{P}}^M = \boldsymbol{\mu}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right)$$

and, therefore,

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) \tag{85}$$

for $\bar{\boldsymbol{g}}^D \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D\right)^{-1} = \bar{\boldsymbol{I}} + \sum_{d=1}^{\infty}(\bar{\boldsymbol{x}}^D)^d$ and $\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}})$ in (79).

From (72),

$$\hat{P}_i = \sum_{j,s} x^C_{ji,s}\left(\hat{\tau}_{ji,s} + \sum_g a^L_{gj,s}\hat{w}_{gj} + a^M_{j,s}\hat{P}^M_{j,s} - \sum_{g,o}\psi^n_{jk,go}\hat{n}_{go} - \sum_{g,o}\psi^N_{jk,go}\hat{N}_{go}\right)$$

which, by defining $\bar{\boldsymbol{x}}^C_{N\times NS} \equiv [x^C_{ji,s}]_{i,js}$ and $\bar{\boldsymbol{a}}^M_{NS\times NS} \equiv [a^M_{j,k}\mathbb{I}_{is=jk}]_{is,jk}$,

$$\hat{\boldsymbol{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M$$

with $\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}})$ in (79).

Substituting (85) into this expression,

$$\hat{\boldsymbol{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right), \tag{86}$$

where $\bar{\boldsymbol{c}}^M \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{a}}^M\bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M$.

Using (71) for outcome Y,

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{\phi}}^{Y,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^{Y,P}\hat{\boldsymbol{P}}$$

where we define $\bar{\boldsymbol{\phi}}^{Y,w}_{GN\times GN} \equiv [\phi^{Y,w}_{gi,fj}]_{gi,fj}$ and $\bar{\boldsymbol{\phi}}^{Y,P}_{GN\times N} \equiv [\phi^{Y,P}_{gi,j}]_{gi,j}$.

To obtain an expression for the price index, we now combine this expression with (86) to derive

$$\hat{\boldsymbol{P}} = \bar{\boldsymbol{\rho}}^P\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{\alpha}}^{P,w}\hat{\boldsymbol{w}} \tag{87}$$

where $\bar{\boldsymbol{\rho}}^P \equiv \left[\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\phi}}^{n,P} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\phi}}^{N,P}\right)\right]^{-1}$ and $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \bar{\boldsymbol{\rho}}^P\bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L - \bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\phi}}^{n,w} - \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\phi}}^{N,w}\right)$.

Using (71) for outcome Y,

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{\alpha}}^{Y,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{Y,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{88}$$

where $\bar{\boldsymbol{\alpha}}^{Y,w} \equiv \bar{\boldsymbol{\phi}}^{Y,w} + \bar{\boldsymbol{\phi}}^{Y,P}\bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{Y,C} \equiv \bar{\boldsymbol{\phi}}^{Y,P}\bar{\boldsymbol{\rho}}^P$.

Together with (85) and (86), equation (88) implies

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^{M,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{\alpha}}^{M,w}\hat{\boldsymbol{w}}, \tag{89}$$

where $\bar{\boldsymbol{\alpha}}^{M,C} \equiv \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,C} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,C}\right)$ and $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L - \bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,w} - \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,w}\right)$.

Step 2. We now solve for the change in revenue of sector-market pairs. From (76), by defining $\bar{\boldsymbol{r}}^C_{NS\times N} \equiv [x^C_{ij,s}E_j/R_{i,s}]_{is,j}$ and $\bar{\boldsymbol{r}}^U_{NS\times NS} \equiv [x^M_{ij,sk}a^M_{j,k}R_{j,k}/R_{i,s}]_{is,jk}$,

$$(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^U)\hat{\boldsymbol{R}} = \left[\sum_j r^C_{ij,s}\hat{x}^C_{ij,s} + \sum_{j,k} r^U_{ij,sk}\hat{x}^M_{ij,sk}\right]_{is} + \bar{\boldsymbol{r}}^U\hat{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

Using (72) and (73), this expression becomes

$$(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^U)\hat{\boldsymbol{R}} = \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\chi}}\hat{\boldsymbol{p}} + \bar{\boldsymbol{r}}^U\hat{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

where $\bar{\boldsymbol{\chi}}_{NS\times NS} \equiv [\sum_j(r^C_{ij,s}\chi^C_{ij,s,oh} + \sum_k r^U_{ij,sk}\chi^M_{ji,ks,oh})]_{is,oh}$ and $\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$ defined in (80).

From (74) and (75), we have that

$$\hat{\boldsymbol{p}} = \bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} + \bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}$$

$$\hat{\boldsymbol{a}}^M = \bar{\boldsymbol{\epsilon}}^{LM}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\epsilon}}^M\hat{\boldsymbol{P}}^M$$

where $\bar{\boldsymbol{\epsilon}}^{LM}_{NS\times NG} \equiv [\epsilon^{LM}_{gi,s}\mathbb{I}_{i=j}]_{is,gj}$ and $\bar{\boldsymbol{\epsilon}}^M \equiv [\epsilon^M_{i,s}\mathbb{I}_{is=jk}]_{is,jk}$.

By defining $\bar{\boldsymbol{g}}^U \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^U$, the expressions above imply that

$$\hat{\boldsymbol{R}} = \bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} + \bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^U\left(\bar{\boldsymbol{\epsilon}}^{LM}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\epsilon}}^M\hat{\boldsymbol{P}}^M\right) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

Using the expressions for $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{N}}$ in (88) and $\hat{\boldsymbol{E}}$ in (88),

$$\hat{\boldsymbol{R}} = \bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^{R,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{R,M}\hat{\boldsymbol{P}}^M + \bar{\boldsymbol{\alpha}}^{R,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{90}$$

where

$$\bar{\boldsymbol{\alpha}}^{R,w} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^L + \bar{\boldsymbol{r}}^U\bar{\boldsymbol{\epsilon}}^{LM} - \bar{\boldsymbol{\chi}}(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,w} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,w}) + \bar{\boldsymbol{r}}^C\bar{\boldsymbol{\alpha}}^{E,w}\right)$$

$$\bar{\boldsymbol{\alpha}}^{R,M} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^U\bar{\boldsymbol{\epsilon}}^M\right)$$

$$\bar{\boldsymbol{\alpha}}^{R,C} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{r}}^C\bar{\boldsymbol{\alpha}}^{E,C} - \bar{\boldsymbol{\chi}}(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,C} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,C})\right)$$

Step 3. The final step is characterizing the system in (49). From the labor market clearing condition in (77), $\hat{W}_{gi} = \sum_s \ell_{gi,s}(\hat{a}^L_{gi,s} + \hat{R}_{i,s})$. When combined with (74), we get that

$$\hat{W}_{gi} = \sum_s \ell_{gi,s}\left(\sum_{g'}\epsilon^L_{gi,s,g'}\hat{w}_{g'i} + \epsilon^{LM}_{gi,s}\hat{P}^M_{i,s} + \hat{R}_{i,s}\right)$$

which, in matrix notation, yields

$$\hat{\boldsymbol{W}} = \bar{\boldsymbol{\alpha}}^{\epsilon,L}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{\epsilon,M}\hat{\boldsymbol{P}}^M + \bar{\boldsymbol{\ell}}\hat{\boldsymbol{R}},$$

where $\bar{\boldsymbol{\alpha}}^{\epsilon,L}_{NG\times NG} \equiv \{\mathbb{I}_{i=j}\sum_s \ell_{gi,s}\epsilon^L_{gi,s,g'}]_{gi,g'j}$ is the matrix of cross-group elasticities of labor demand with respect to wages, $\bar{\boldsymbol{\alpha}}^{\epsilon,M}_{NG\times NS}\equiv\{\mathbb{I}_{i=j}\ell_{gi,s}\epsilon^{LM}_{gi,s}]_{gi,js}$ is the matrix of group elasticities of labor demand with respect to input cost, and $\bar{\boldsymbol{\ell}}_{GN\times NS}\equiv[\ell_{gi,s}\mathbb{I}_{[i=j]}]_{gi,js}$ is the matrix of the share of sector s in employment of group g in market i.

By applying (88) and (90) into the expression above, we get

$$\left(\bar{\boldsymbol{\alpha}}^{W,w}-\bar{\boldsymbol{\alpha}}^{\epsilon,L}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,w}\right)\hat{\boldsymbol{w}}=\hat{\boldsymbol{\eta}}^{R}+\left(\bar{\boldsymbol{\alpha}}^{\epsilon,M}+\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\hat{\boldsymbol{P}}^{M}+\left(\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,C}-\bar{\boldsymbol{\alpha}}^{W,C}\right)\left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}})\right),$$

where, by definition, $\hat{\boldsymbol{\eta}}^{R}\equiv\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^{U}\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$.

By applying (89) into this expression, we get that

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}}=\hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\alpha}}^{C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\alpha}}^{M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}), \tag{91}$$

where

$$\begin{aligned}\bar{\boldsymbol{\gamma}}&\equiv\bar{\boldsymbol{\alpha}}^{W,w}-\bar{\boldsymbol{\alpha}}^{\epsilon,L}-\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,w}-\left(\bar{\boldsymbol{\alpha}}^{\epsilon,M}+\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\bar{\boldsymbol{\alpha}}^{M,w},\\ \bar{\boldsymbol{\alpha}}^{C}&\equiv\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,C}-\bar{\boldsymbol{\alpha}}^{W,C}-\left(\bar{\boldsymbol{\alpha}}^{\epsilon,M}+\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\bar{\boldsymbol{\alpha}}^{M,C},\\ \bar{\boldsymbol{\alpha}}^{M}&\equiv\bar{\boldsymbol{\alpha}}^{\epsilon,M}+\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}+\bar{\boldsymbol{\alpha}}^{C}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}.\end{aligned}$$

The representation in (78) follows from the same steps in Part B of Appendix A.2.4 under the diagonal dominance condition in (62).

Step 4. We derive reduced-form expressions for all labor market outcomes $\hat{Y}_{gi}\in\{\hat{n}_{gi},\hat{N}_{gi},\hat{E}_{gi},\widehat{\overline{\ln w_{gi}}}\}$ using (88):

$$\hat{\boldsymbol{Y}}=\bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \tag{92}$$

where

$$\bar{\boldsymbol{\beta}}^{Y,R}\equiv\bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}},\quad \bar{\boldsymbol{\beta}}^{Y,C}\equiv\bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{C}+\bar{\boldsymbol{\alpha}}^{Y,C},\quad \bar{\boldsymbol{\beta}}^{Y,M}\equiv\bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{M}+\bar{\boldsymbol{\alpha}}^{Y,C}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}.$$

Using (87),

$$\hat{\boldsymbol{P}}=\bar{\boldsymbol{\beta}}^{C,R}\hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\beta}}^{C,C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}})+\bar{\boldsymbol{\beta}}^{C,M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}), \tag{93}$$

where

$$\bar{\boldsymbol{\beta}}^{C,R}\equiv\bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}},\quad \bar{\boldsymbol{\beta}}^{C,C}\equiv\bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{C}+\bar{\boldsymbol{\rho}}^{P},\quad \bar{\boldsymbol{\beta}}^{C,M}\equiv\bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{M}+\bar{\boldsymbol{\rho}}^{P}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}.$$

A.4 Proofs and Additional Results in Section 5

A.4.1 Proof of Expression (36)

The gravity trade demand $X_{ij,s}$ in (6) and the expression for $p_{i,s}$ in (10) imply that

$$\Delta\log X^{t}_{ij,s}=(1-\sigma)\hat{\tau}^{t}_{ij,s}-\kappa\hat{w}^{t}_{i,s}+\underbrace{\hat{E}^{t}_{j}-\left(\sum_{o}\widehat{\tau^{1-\sigma}_{oj,s}w^{-\kappa}_{o}\bar{b}^{\kappa-\sigma+1}_{o}}\right)}_{\equiv\Lambda^{t}_{j,s}}.$$

Up to a first order approximation, the definition of $\Delta M^{t}_{\text{China},s}\equiv\sum_{j}\frac{\Delta X^{t}_{\text{China}j,s}}{L^{t0}_{\text{US},s}}$ is equal to

$$\begin{aligned}\Delta M^{t}_{\text{China},s}&=\sum_{j}\frac{X^{t0}_{\text{China}j,s}}{L^{t0}_{\text{US},s}}\Delta\log X^{t}_{\text{China}j,s}\\ &=\sum_{j}\frac{X^{t0}_{\text{China}j,s}}{L^{t0}_{\text{US},s}}\left((1-\sigma)\hat{\tau}^{t}_{\text{China}j,s}-\kappa\hat{w}^{t}_{\text{China},s}+\Lambda^{t}_{j,s}\right).\end{aligned}$$

By setting the Chinese wage as the numeraire ($\hat{w}^t_{\text{China},s}=0$) without loss of generality, the expression above implies that

$$\Delta M^t_{\text{China},s}=\sum_j \frac{E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}}\left((1-\sigma)x^{t_0}_{\text{China}j,s}\hat{\tau}^t_{\text{China}j,s}\right)+\frac{\sum_j X^{t_0}_{\text{China}j,s}\Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}}.$$

The decomposition $(1-\sigma)x^{t_0}_{\text{China}j,s}\hat{\tau}^t_{\text{China}j,s}=\hat{\zeta}^t_{\text{China},s}+\hat{\varepsilon}^t_{\text{China}j,s}$ implies that

$$\Delta M^t_{\text{China},s}=\left(\frac{\sum_j E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}}\right)\left(\hat{\zeta}^t_{\text{China},s}+\sum_j \frac{E^{t_0}_{j,s}}{\sum_{j'}E^{t_0}_{j',s}}\hat{\varepsilon}^t_{\text{China}j,s}\right)+\frac{\sum_j X^{t_0}_{\text{China}j,s}\Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}},$$

which immediately yields expression (36) since $\sum_j \frac{E^{t_0}_{j,s}}{\sum_{j'}E^{t_0}_{j',s}}\hat{\varepsilon}^t_{\text{China}j,s}=0$.

A.4.2 Specification of Transfer Numeraire

The definition $\Omega(\boldsymbol{w})=(W_{\text{US}}(\boldsymbol{w}))^{\bar{\omega}}(W_{\text{W}}(\boldsymbol{w}))^{1-\bar{\omega}}$ implies that

$$\omega^{t_0}_j\equiv\frac{\partial \ln\Omega(\boldsymbol{w}^{t_0})}{\partial \ln w_j}=\bar{\omega}\frac{W^{t_0}_j}{\sum_{i\in\mathcal{I}_{\text{US}}}W^{t_0}_i}\mathbb{I}_{j\in\mathcal{I}_{\text{US}}}+(1-\bar{\omega})\frac{W^{t_0}_j}{\sum_c\sum_{i\in\mathcal{I}_c}W^{t_0}_i} \tag{94}$$

where $W^{t_0}_i$ is the GDP of market i in the initial period t_0.

To compute $\omega^{t_0}_j$, we need to specify $\bar{\omega}$. We do so using the series of the opportunity cost of not working in Chodorow-Reich and Karabarbounis (2016). In our model, the average change in the payoff of not working in the U.S. is $\hat{z}^t_{\text{US}}\equiv\sum_{i\in\mathcal{I}_{\text{US}}}\frac{N_i}{N_{\text{US}}}(\hat{b}_i-\hat{P}_i)$. By defining the U.S. price index change as $\hat{P}^t_{\text{US}}\equiv\sum_{i\in\mathcal{I}_{\text{US}}}\frac{N_i}{N_{\text{US}}}\hat{P}_i$, the fact that $\hat{b}^t_i=\hat{\Omega}^t$ yields

$$\begin{aligned}\hat{z}^t_{\text{US}} &= \bar{\omega}\hat{W}^t_{\text{US}}+(1-\bar{\omega})\hat{W}^t_{\text{W}}-\hat{P}^t_{\text{US}}\\ &= \left(\hat{W}^t_{\text{US}}-\hat{P}^t_{\text{US}}\right)-(1-\bar{\omega})\left(\hat{W}^t_{\text{US}}-\hat{W}^t_{\text{W}}\right)\\ &= \left(\hat{W}^t_{\text{US}}-\hat{P}^t_{\text{US}}\right)-(1-\bar{\omega})\left(\left(\hat{W}^t_{\text{US}}-\hat{P}^t_{\text{US}}\right)-\left(\hat{W}^t_{\text{W}}-\hat{P}^t_{\text{W}}\right)+\left(\hat{P}^t_{\text{US}}-\hat{P}^t_{\text{W}}\right)\right).\end{aligned}$$

By defining the real income of a country as $\widehat{RW}^t_c=\hat{W}^t_c-\hat{P}^t_c$ and the relative price as $\hat{\mathcal{P}}^t_c=\hat{P}^t_c-\hat{P}^t_{\text{W}}$, this expression is equivalent to

$$\hat{z}^t_{\text{US}}=\widehat{RW}^t_{\text{US}}-(1-\bar{\omega})\left(\widehat{RW}^t_{\text{US}}-\widehat{RW}^t_{\text{W}}+\hat{\mathcal{P}}^t_{\text{US}}\right).$$

Thus,

$$Cov\left(\widehat{RW}^t_{\text{US}},\hat{z}^t_{\text{US}}\right)=Var\left(\widehat{RW}^t_{\text{US}}\right)-(1-\bar{\omega})Cov\left(\widehat{RW}^t_{\text{US}},\widehat{RW}^t_{\text{US}}-\widehat{RW}^t_{\text{W}}+\hat{\mathcal{P}}^t_{\text{US}}\right)$$

and, therefore,

$$\bar{\omega}=1-\frac{Var\left(\widehat{RW}^t_{\text{US}}\right)-Cov\left(\widehat{RW}^t_{\text{US}},\hat{z}^t_{\text{US}}\right)}{Cov\left(\widehat{RW}^t_{\text{US}},\widehat{RW}^t_{\text{US}}-\widehat{RW}^t_{\text{W}}+\hat{\mathcal{P}}^t_{\text{US}}\right)}. \tag{95}$$

We obtain $\bar{\omega}=0.62$ using expression (95) computed with year-to-year log-changes in every variable between 1961 and 2012. To measure $\hat{z}^t_{\text{US}}$, we use the series in Chodorow-Reich and Karabarbounis (2016) of the opportunity cost of employment implied by their separable utility specification at the first quarter of each year (available for 1961-2012). We measure all other variables using data from the Penn World Tables produced with the methodology in Feenstra et al. (2015). Specifically, we measure $\widehat{RW}^t_c$ using the annual series of the real domestic absorption at current PPPs (CDA) divided by population (pop), and $\hat{P}_c^t$ using the annual series of the Price level of CDA (PPP/XR). We compute the world average of the log-change in each variable as the average log-change in that variable across all countries, weighted by the country's share in world GDP in the previous year.

A.5 Adding Frictional Unemployment to Model of Section 3

We now outline an extension of the model in Section 3 featuring frictional unemployment. It yields an expression for the change in the employment rate in terms of changes in w_i/b_i with an elasticity that combines the parameters controlling responses in both the labor force participation and the unemployment rate.

Environment. We consider the same preferences as in our baseline model, with $l(\iota)$ and $u(\iota)$ denoting ι's efficiency units and non-employment income. As in the baseline, individuals draw $(l(\iota),u(\iota))$ independently from a Frechet distribution with shape parameter $\phi > 1$ and scale 1. Given the uncertainty in the job search process, we assume that individuals are risk neutral.

As in our baseline, each sector s of market i has a representative firm that produces a differentiated good subject to iceberg trade costs. We now assume that production depends on a CES aggregator of the continuum of non-traded inputs available in the market, $\nu \in \mathcal{V}_i$:

$$Q_{i,s} = \left[\int_{\nu \in \mathcal{V}_i} (q_{i,s}(\nu))^{\frac{\mu-1}{\mu}} d\nu \right]^{\frac{\mu}{\mu-1}}, \tag{96}$$

where $\mu > 1$ is the elasticity of substitution between non-traded varieties.

We assume that the economy has a fixed pool of potential producers of the non-traded inputs that operate in monopolistic competition. In order to produce, firms need to get matched with a worker. If the owner of the firm does not post a vacancy, she gets an outside option payoff of $\bar{\nu}_i$. We consider a competitive search environment in which firm ν posts a wage offer $w_i(\nu)$. We analyze a symmetric equilibrium in which all firms post the same wage (i.e., $w_i(\nu) = w_i$), and then are randomly matched with a worker in the economy. Conditional on being matched to individual ι, intermediate producers have a linear production function such that $y_i(\nu) = l(\iota)$. The matching technology is such that, if V_i vacancies are posted and N_i^p workers search for a job, the number of matches is

$$M_i = (V_i)^{\alpha} (N_i^p)^{1-\alpha}. \tag{97}$$

Labor Force Participation. We first solve for the share of individuals in market i that look for a job given an offered wage rate of w_i. Consider the case in which individual ι searches for a job. With probability M_i/N_i^p, she finds a job and has a payoff of $(1-v_i)w_i l(\iota)/P_i$; with probability $1-M/N_i^p$, she does not find a job and has a payoff of $(1-v_i)b_i u(\iota)/P_i$. If the same individual ι does not search for a job, she gets a payoff of $(1-v_i)b_i u(\iota)/P_i$. Thus, the maximization of expected utility implies that the market's labor force participation is

$$n_i^p = \Pr\left[\frac{M_i}{N_i^p} w_i l(\iota) + \left(1 - \frac{M_i}{N_i^p}\right) b_i u(\iota) > b_i u(\iota) \right] = \Pr[w_i l(\iota) > b_i u(\iota)] \quad \Rightarrow \quad n_i^p = \frac{w_i^{\phi}}{w_i^{\phi} + b_i^{\phi}}. \tag{98}$$

As in our baseline, the mean efficiency of those searching for jobs is $\bar{l}_i = \varrho (n_i^p)^{-\frac{1}{\phi}}$.

Unemployment Rate. Given the cost minimization problem of the representative firm in market i, the demand for the output of the intermediate producer ν is

$$q_i(\nu)=\frac{(p_i(\nu))^{-\mu}}{\int_{\tilde{\nu}\in\mathcal{V}_i}(p_i(\tilde{\nu}))^{1-\mu}d\tilde{\nu}}R_i.$$

Thus, the profit maximization problem of firm ν yields the typical constant markup expression for the price of the intermediate good:

$$\tilde{p}_i(\nu)=\frac{\mu}{\mu-1}w_i \quad \forall\nu\in\mathcal{V}_i.$$

This implies that the production cost of firms in market i is $p_{i,s}\equiv\left[\int_{\nu\in\mathcal{V}_i}(\tilde{p}_i(\nu))^{1-\mu}d\nu\right]^{\frac{1}{1-\mu}}=\frac{\mu}{\mu-1}w_i(M_i)^{\frac{1}{1-\mu}}$. In equilibrium, the number of successful matches must be equal to the number of employed individuals ($M_i=L_i$), so

$$p_{i,s}=\frac{\mu}{\mu-1}w_i(L_i)^{-\psi} \quad \text{such that} \quad \psi\equiv\frac{1}{\mu-1}. \tag{99}$$

Finally, the free entry condition implies that the expected profit of posting a vacancy must be equal to the outside option of not posting it. Given that the probability of filling a vacancy is M_i/V_i and that the expected efficiency of a match is $\bar{l}_i$, we have that

$$\bar{\nu}_i=(\tilde{p}_i(v)-w_i)\bar{l}_i\frac{M_i}{V_i}=\frac{1}{\mu-1}w_i\bar{l}_i\left(\frac{N_i^p}{V_i}\right)^{1-\alpha} \quad\Rightarrow\quad \frac{N_i^p}{V_i}=\left(\frac{(\mu-1)\bar{\nu}_i}{w_i\bar{l}_i}\right)^{\frac{1}{1-\alpha}}$$

This expression determines the share of individuals searching for a job that get matched to a producer:

$$n_i^m=\frac{M_i}{N_i^p}=\left(\frac{V_i}{N_i^p}\right)^{\alpha}=\left(\frac{w_i\bar{l}_i}{(\mu-1)\bar{\nu}_i}\right)^{\frac{\alpha}{1-\alpha}}=\left(\frac{w_i\varrho(n_i^p)^{-\frac{1}{\phi}}}{(\mu-1)\bar{\nu}_i}\right)^{\frac{\alpha}{1-\alpha}}.$$

Assuming that the outside option of producers is proportional to the non-employment transfer ($\bar{\nu}_i=\nu_i b_i$), we derive our main expression for the share of individuals in market i that are employed:

$$n_i=n_i^m n_i^p=\left(\frac{\varrho}{(\mu-1)\nu_i}\frac{w_i}{b_i}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{(w_i/b_i)^{\phi}}{1+(w_i/b_i)^{\phi}}\right)^{\left(1-\frac{\alpha}{1-\alpha}\frac{1}{\phi}\right)}. \tag{100}$$

Up to a first order approximation, this expression implies that

$$\hat{n}_i=\hat{n}_i^m+\hat{n}_i^p=\left(\frac{\alpha}{1-\alpha}n_i^p+\phi(1-n_i^p)\right)(\hat{w}_i-\hat{b}_i).$$

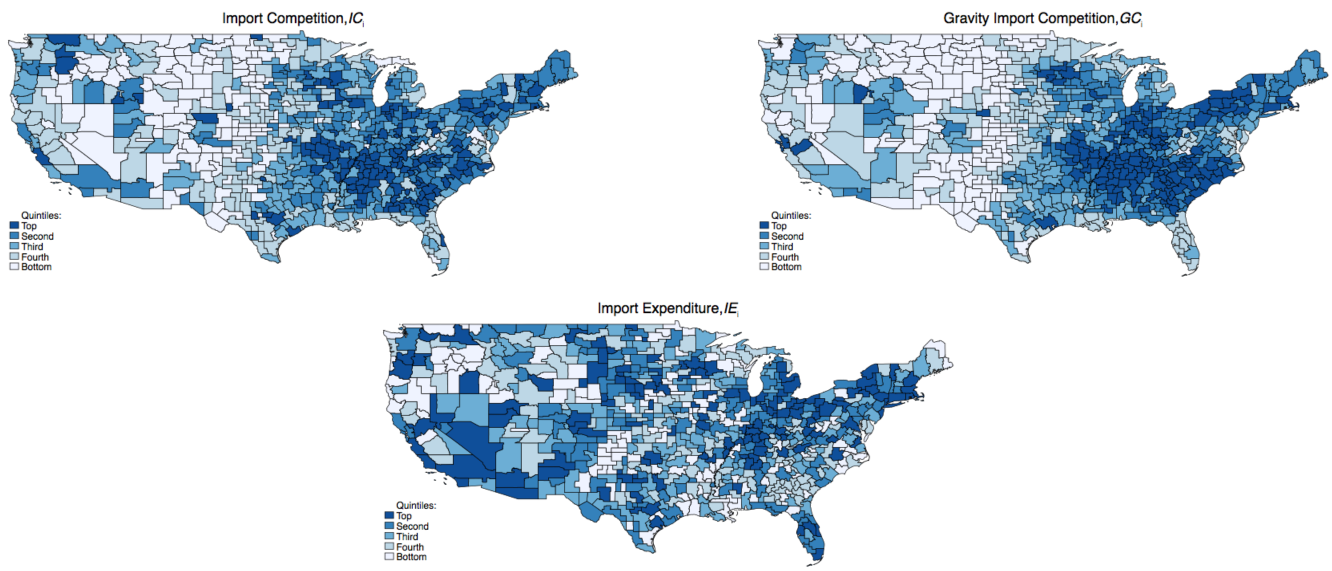
The elasticity of the employment rate to the wage rate has two components. As before, it entails the elasticity of the labor force participation margin, $\phi(1-n_i^p)$. But now it also encompasses the elasticity of the matching rate, $\frac{\alpha}{1-\alpha}n_i^p$, which depends on the matching technology parameter α. Whenever $\alpha=0$, all individuals searching for a job get a match and this term disappears.

B Appendix: Additional Empirical Results (Not for publication)

B.1 Adjustment of U.S. Regional Markets to Trade Shocks: Three Stylized Facts

This appendix presents additional empirical results that complement those in Section 2.

Figure B.1: Regional Exposure to the China Shock, 1990-2007



Notes: For each CZ, the left panel reports IC_i^t, the right panel reports GC_i^t, and the bottom panel reports IE_i^t.

Table B.1: Summary Statistics of Outcomes for U.S. CZs

	1990-2000		2000-2007		1990-2007	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
100 x Change in average weekly log-wage	12.39	4.65	3.84	5.51	16.23	6.47
100 x Change in log of employment rate	1.27	4.23	1.71	5.31	2.98	6.42
IC_i^t	1.01	1.06	2.52	2.54	3.52	3.35
IE_i^t	2.51	0.58	7.39	1.31	9.90	1.62
GC_i^t	1.03	0.88	2.60	1.99	3.64	2.73

Notes: Sample of 722 Commuting Zones.

Table B.2: Differential Impact of the China Shock on U.S. CZs, Employment Outcomes

	Change in the share of working-age population by category				
	Employed	Emp. in Manuf	Emp. in Non-Manuf	Unemp.	Out of labor force
	(1)	(2)	(3)	(4)	(5)
IC_i^t	-0.253***	-0.166***	-0.087**	0.095***	0.159***
	(0.055)	(0.047)	(0.037)	(0.026)	(0.039)
GC_i^t	-0.482***	-0.210***	-0.272***	0.186***	0.297***
	(0.109)	(0.053)	(0.087)	(0.046)	(0.077)
IE_i^t	-0.120	-0.102**	-0.018	0.034	0.086
	(0.101)	(0.043)	(0.090)	(0.039)	(0.075)
R^2	0.322	0.550	0.225	0.282	0.293

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.3: Differential Impact of the China Shock on U.S. CZs, Employment Outcomes II

	Change in the share of working-age population by category				
	Employed	Emp. in Manuf	Emp. in Non-Manuf	Unemp.	Out of labor force
	(1)	(2)	(3)	(4)	(5)
IC_i^t	-0.451***	-0.366***	-0.084	0.126***	0.325***
	(0.069)	(0.044)	(0.077)	(0.032)	(0.066)
GC_i^t	-0.353***	-0.086	-0.267***	0.123***	0.230***
	(0.091)	(0.056)	(0.078)	(0.038)	(0.078)
R^2	0.364	0.488	0.389	0.460	0.454

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls used in ADH, and weight the observations by the initial population share. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.4: Differential Impact of the China Shock on U.S. CZs, Alternative Specifications I

	(1)	(2)	(3)	(4)	(5)
Panel A: **Change in average log weekly wage**					
IC_i^t	-0.471***	-0.368***	-0.475***	-0.383***	-0.383***
	(0.127)	(0.104)	(0.138)	(0.113)	(0.114)
GC_i^t		-0.601***		-0.606***	-0.600***
		(0.155)		(0.156)	(0.174)
IE_i^t			0.023	0.077	0.079
			(0.168)	(0.164)	(0.145)
$\sum_{j\neq i} z_{ij} IE_j^t$					-0.025
					(0.310)
R^2	0.517	0.526	0.517	0.527	0.527
Panel B: **Change in log of employment rate**					
IC_i^t	-0.519***	-0.400***	-0.474***	-0.369***	-0.363***
	(0.089)	(0.075)	(0.095)	(0.079)	(0.079)
GC_i^t		-0.700***		-0.691***	-0.582***
		(0.156)		(0.155)	(0.158)
IE_i^t			-0.216	-0.154	-0.106
			(0.146)	(0.143)	(0.140)
$\sum_{j\neq i} z_{ij} IE_j^t$					-0.516*
					(0.261)
R^2	0.300	0.326	0.302	0.327	0.330
Panel C: **Change in log of working-age population**					
IC_i^t	0.273	0.209	0.180	0.127	0.118
	(0.180)	(0.159)	(0.172)	(0.155)	(0.152)
GC_i^t		0.372*		0.348	0.191
		(0.217)		(0.212)	(0.204)
IE_i^t			0.449	0.418	0.349
			(0.292)	(0.294)	(0.277)
$\sum_{j\neq i} z_{ij} IE_j^t$					0.739
					(0.469)
R^2	0.309	0.310	0.310	0.312	0.313

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Spatial indirect effects computed as in Table 1: $z_{ij} \equiv D_{ij}^{-5} / \sum_k D_{ik}^{-5}$ where D_{ij} is the distance between CZs i and j. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.5: Differential Impact of the China Shock on U.S. CZs, Alternative Specifications II

	(1)	(2)	(3)	(4)	(5)	(6)
***Panel A:* Change in average log weekly wage**						
IC_i^t	-0.383***	-0.383***	-0.357***	-0.426***	-0.104	-0.283*
	(0.113)	(0.135)	(0.107)	(0.112)	(0.106)	(0.158)
GC_i^t	-0.606***	-0.606**	-0.528***	-0.720***	-0.284***	-0.670***
	(0.156)	(0.262)	(0.125)	(0.174)	(0.103)	(0.187)
IE_i^t	0.077	0.077	0.062	0.070	-0.043	-0.346
	(0.164)	(0.092)	(0.160)	(0.164)	(0.143)	(0.252)
R^2	0.527	0.527	0.538	0.563	0.578	0.592
***Panel B:* Change in log of employment share**						
IC_i^t	-0.369***	-0.369***	-0.352***	-0.365***	-0.159**	-0.567***
	(0.079)	(0.107)	(0.076)	(0.077)	(0.060)	(0.141)
GC_i^t	-0.691***	-0.691***	-0.641***	-0.717***	-0.449***	-0.792***
	(0.155)	(0.258)	(0.159)	(0.153)	(0.139)	(0.222)
IE_i^t	-0.154	-0.154	-0.153	0.070	-0.244**	-0.180
	(0.143)	(0.078)	(0.142)	(0.164)	(0.118)	(0.225)
R^2	0.327	0.327	0.332	0.395	0.383	0.383
***Panel C:* Change in log of working-age population**						
IC_i^t	0.127	0.127	0.014	0.065	0.176	-0.301
	(0.155)	(0.124)	(0.123)	(0.145)	(0.165)	(0.434)
GC_i^t	0.348	0.348	0.040	0.111	0.404*	-0.078
	(0.212)	(0.133)	(0.129)	(0.223)	(0.213)	(0.453)
IE_i^t	0.418	0.418	0.194	0.523*	0.397	0.707
	(0.294)	(0.330)	(0.220)	(0.284)	(0.288)	(0.574)
R^2	0.312	0.312	0.442	0.442	0.312	0.444
Control set:						
Baseline controls	Y	Y	Y	Y	Y	Y
Lagged population growth	N	N	Y	N	N	N
State dummies	N	N	N	Y	N	N
Manuf share x period dummy	N	N	N	N	Y	N
Observations weights:						
Population	N	N	N	N	N	Y
No weights	Y	Y	Y	Y	Y	N
Inference:						
State clustered	Y	N	Y	Y	Y	Y
Adão et al. (2019)	N	Y	N	N	N	N

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Baseline controls defined in Table 1. Lagged population growth from Greenland et al. (2019): growth of population with 15-34 years old and 35-64 years old in the previous 10-year period. Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.6: Differential Impact of the China Shock on U.S. CZs, Alternative Spatial Indirect Effects

	(1)	(2)	(3)	(4)	(5)
Panel A: **Change in average log weekly wage**					
IC_i^t	-0.383***	-0.321***	-0.425***	-0.403***	-0.441***
	(0.113)	(0.110)	(0.125)	(0.122)	(0.129)
GC_i^t	-0.606***	-7.647***	-0.457***	-0.956***	-1.623**
	(0.156)	(2.365)	(0.136)	(0.297)	(0.790)
IE_i^t	0.077	0.111	0.054	0.052	0.014
	(0.164)	(0.166)	(0.165)	(0.163)	(0.161)
R^2	0.527	0.536	0.521	0.523	0.525
Panel B: **Change in log of employment share**					
IC_i^t	-0.369***	-0.265***	-0.410***	-0.389***	-0.439***
	(0.079)	(0.069)	(0.086)	(0.089)	(0.087)
GC_i^t	-0.691***	-10.39***	-0.586***	-1.138***	-1.663***
	(0.155)	(1.464)	(0.145)	(0.224)	(0.508)
IE_i^t	-0.154	-0.096	-0.176	-0.181	-0.225
	(0.143)	(0.129)	(0.145)	(0.144)	(0.137)
R^2	0.327	0.370	0.315	0.320	0.319
Panel C: **Change in log of working-age population**					
IC_i^t	0.127	0.071	0.138	0.129	0.163
	(0.155)	(0.141)	(0.160)	(0.159)	(0.173)
GC_i^t	0.348	5.430	0.388*	0.685	0.789
	(0.212)	(3.421)	(0.230)	(0.484)	(0.959)
IE_i^t	0.418	0.387	0.423	0.428	0.454
	(0.294)	(0.303)	(0.291)	(0.292)	(0.293)
R^2	0.312	0.314	0.312	0.312	0.311
Spatial Indirect Effect Specification:					
Definition of z_{ij}	$\frac{D_{ij}^{-5}}{\sum_k D_{ik}^{-5}}$	$\frac{D_{ij}^{-1}}{\sum_k D_{ik}^{-1}}$	$\frac{D_{ij}^{-8}}{\sum_k D_{ik}^{-8}}$	$\frac{L_j^0 D_{ij}^{-5}}{\sum_k L_k^0 D_{ik}^{-5}}$	$\frac{L_j^0 St_{ij}}{\sum_k L_k^0 St_{ik}}$

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Spatial indirect effects given by $GC_i^t \equiv \sum_{j \neq i} z_{ij} IC_j^t$ where z_{ij} is specified in each column, D_{ij} is the distance between CZs i and j, L_j^0 is the population of CZ j in 1990, and St_{ij} is a dummy that equals one if CZs i and j belong to the same state. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.7: Differential Impact of the China Shock on U.S. CZs, Alternative Spending Shares

	(1)	(2)	(3)	(4)
***Panel A:* Change in average log weekly wage**				
IC_i^t	-0.383***	-0.397***	-0.396***	-0.348***
	(0.113)	(0.114)	(0.112)	(0.112)
GC_i^t	-0.606***	-0.589***	-0.590***	-0.563***
	(0.156)	(0.154)	(0.155)	(0.156)
IE_i^t	0.077			
	(0.164)			
IEI_i^t		0.110	0.101	-0.548
		(0.107)	(0.140)	(0.452)
IEF_i^t		0.267	0.246	0.239
		(0.757)	(0.755)	(0.754)
R^2	0.527	0.530	0.532	0.532
***Panel B:* Change in log of employment share**				
IC_i^t	-0.369***	-0.395***	-0.393***	-0.344***
	(0.079)	(0.081)	(0.078)	(0.073)
GC_i^t	-0.691***	-0.681***	-0.680***	-0.652***
	(0.155)	(0.151)	(0.151)	(0.147)
IE_i^t	-0.154			
	(0.143)			
IEI_i^t		-0.003	-0.014	-0.804**
		(0.091)	(0.111)	(0.389)
IEF_i^t		0.175	0.174	0.182
		(0.489)	(0.487)	(0.484)
R^2	0.327	0.329	0.329	0.332
***Panel C:* Change in log of working-age population**				
IC_i^t	0.127	0.122	0.157	0.188
	(0.155)	(0.149)	(0.149)	(0.157)
GC_i^t	0.348	0.326	0.334	0.348
	(0.212)	(0.211)	(0.212)	(0.218)
IE_i^t	0.418			
	(0.294)			
IEI_i^t		0.307*	0.195	0.041
		(0.181)	(0.205)	(0.844)
IEF_i^t		0.291	0.272	0.267
		(0.722)	(0.718)	(0.713)
R^2	0.312	0.315	0.314	0.313
Construction of IEI_i^t:				
Drop final spending	N	Y	Y	Y
Drop own industry spending	N	N	Y	N
Use Leontief IO shares	N	N	N	Y

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Col. (1) is the baseline specification in which $IE_i^t \equiv \sum_s e_{i,s}^{t_0} \Delta M_{\text{China},s}^t$, where $e_{i,s}^{t_0}$ is the share of gross spending in sector s (as defined in Section 2.2). In cols. (2)-(4), $IEF_i^t \equiv \sum_s ef_{i,s}^{t_0} \Delta M_{\text{China},s}^t$ is the exposure to the shock in final import expenditure, where $ef_{i,s}^{t_0}$ is the share of household spending on sector s in CZ i constructed from the Consumer Expenditure Survey (as described in Appendix C.2.1). In col. (2), $IEI_i^t \equiv \sum_s ei_{i,s}^{t_0} \Delta M_{\text{China},s}^t$ is the exposure to the shock in intermediate import expenditure, where $ei_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_k a_k^{t_0} \ell_{i,k}^{t_0}$ is the share of intermediate spending on sector s in CZ i. In col. (3), we compute IEI_i^t using $ei_{i,s}^{t_0} \equiv \sum_{k \neq s} \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_{k \neq s} a_k^{t_0} \ell_{i,k}^{t_0}$ that ignores the own industry input spending. In col. (4), $IEI_i^t \equiv \sum_s eiL_{i,s}^{t_0} \Delta M_{\text{China},s}^t$ where $eiL_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{L,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_{s,k} \xi_{sk}^{L,t_0} a_k^{t_0} \ell_{i,k}^{t_0}$ is the share of total intermediate spending on sector s in CZ i, with ξ_{sk}^{L,t_0} defined as the Leontief input spending shares from Acemoglu et al. (2016a). Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.8: Differential Impact of the China Shock on U.S. CZs, Alternative Sectoral Shifters

	Change in average weekly log-wage		Change in log of employment rate		Change in log of working-age population	
	(1)	(2)	(3)	(4)	(5)	(6)
***Panel A:* China exporter-sector gravity fixed-effect**						
IC_i^t	-0.177**	-0.094	-0.267***	-0.157***	0.115	0.064
	(0.083)	(0.081)	(0.064)	(0.052)	(0.113)	(0.110)
GC_i^t		-0.455***		-0.490***		0.022
		(0.134)		(0.126)		(0.159)
IE_i^t		-0.270		-0.557***		0.586
		(0.248)		(0.202)		(0.566)
R^2	0.506	0.513	0.281	0.298	0.308	0.308
***Panel B:* NTR gap**						
IC_i^t	-0.466***	-0.256***	-0.393***	-0.195***	0.0252	-0.0281
	(0.073)	(0.040)	(0.045)	(0.034)	(0.087)	(0.079)
GC_i^t		-0.351***		-0.355***		-0.0392
		(0.092)		(0.084)		(0.098)
IE_i^t		-0.0749		-0.013		0.324**
		(0.120)		(0.103)		(0.134)
R^2	0.570	0.584	0.360	0.388	0.307	0.309
***Panel C:* Sectoral demand shift, $\hat{\zeta}^t_{\mathbf{China},s}$**						
IC_i^t	-0.857***	-0.463***	-0.894***	-0.536***	0.161	0.049
	(0.186)	(0.143)	(0.128)	(0.118)	(0.284)	(0.199)
GC_i^t		-1.022***		-0.920***		0.292
		(0.232)		(0.203)		(0.376)
IE_i^t		-0.353		-0.065		0.167
		(0.216)		(0.175)		(0.553)
R^2	0.524	0.536	0.311	0.330	0.307	0.308

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Each panel presents estimates of regression (2) with different exposure measures. All panels use the same exposure measures as in (3)-(5), but built with an alternative definition of the sectoral shifter $\Delta M^t_{\text{China},s}$. In panel A, the shifter is $\hat{\Gamma}^t_{China,s} M^{t_0}_{\text{China},s}/L^{t_0}_{\text{US},s}$, where $\hat{\Gamma}^t_{China,s}$ is the sector-origin fixed-effect for China obtained from the estimation for the periods 1991-2000 and 2000-2007 of $\Delta \log X^t_{ij,s} = \Lambda^t_{j,s} + \Gamma^t_{i,s} + \varepsilon^t_{ij,s}$ in the same sample of high-income countries used to compute the ADH IV, plus U.S. and China; $M^{t_0}_{\text{China},s}$ is the initial level of imports in sector s of the eight high-income countries used to compute the ADH IV; and $L^{t_0}_{\text{US},s}$ is the initial level of U.S. employment in sector s. In Panel B, the shfiter is 100 times the average NTR Gap in sector s obtained from the replication package of Pierce and Schott (2016b). In panel C, the shifter is $\hat{\zeta}^t_{\text{China},s}$ defined in Section 5.1. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table B.9: Differential Impact of the China Shock on U.S. CZs, Migration Outcomes

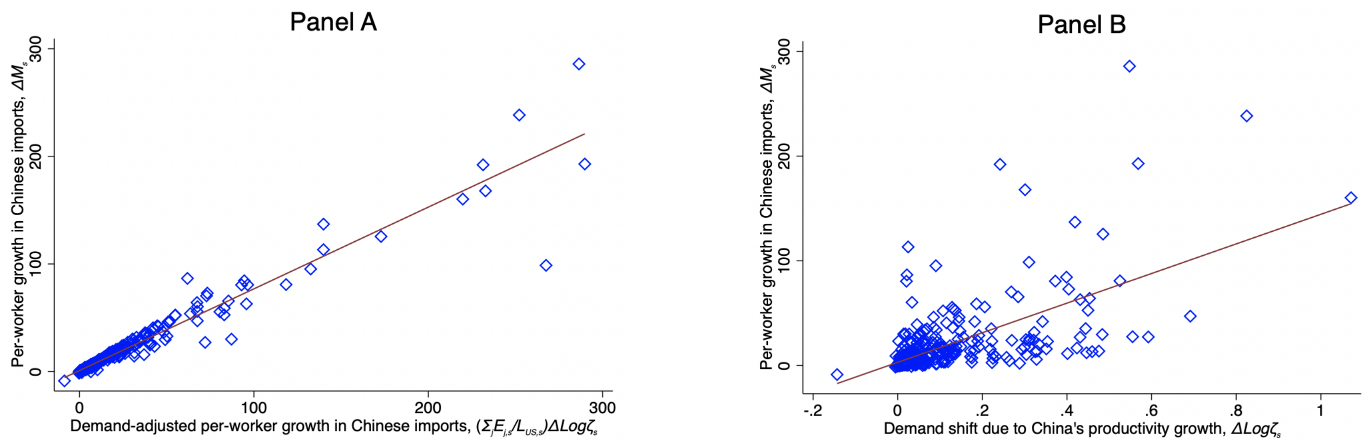
	Change in the log of			
	Population	In-migration	Out-migration	Net migration
	(1)	(2)	(3)	(4)
IC_i^t	0.127	0.152	0.063	-0.090
	(0.155)	(0.184)	(0.184)	(0.114)
GC_i^t	0.348	0.052	0.077	0.025
	(0.212)	(0.325)	(0.340)	(0.113)
IE_i^t	0.418	0.670	0.515	-0.155
	(0.294)	(0.410)	(0.351)	(0.155)
R^2	0.312	0.885	0.817	0.116

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

B.2 The General Equilibrium Effect of The China Shock

B.2.1 Measuring the China Shock

Figure B.2: Measures of the China Shock across Sectors, 1991-2007



Notes: The graph on the left plots the per-worker growth in Chinese imports by the eight developed countries used in ADH ($\Delta M^t_{\text{China},s}$) against its component associated with China's productivity growth ($\sum_j E^{t_0}_{j,s}\hat{\zeta}^t_{\text{China},s}/L^{t_0}_{\text{US},s}$) across sectors. The graph on the right plots $\Delta M^t_{\text{China},s}$ agaist the sectoral demand shift implied by the China shock ($\hat{\zeta}^t_{\text{China},s}$) across sectors. All variables are computed over the entire period between 1991 and 2007. The red line is the best linear fit.

Table B.10: Sectors with the Highest and Lowest Exposure to the China Shock

Sectors most affected by the China shock	Sectors least affected by the China shock
Rubber and plastics footwear	Tobacco stemming and redrying
Games, toys, and children's vehicles	Malt
Housefurnishings, n.e.c.	Primary copper
Cement, hydraulic	Industrial gases
Waterproof outerwear	Vegetable oil mills
Bags: plastics, laminated, and coated	Ordinance and accessories
Printing trades machinery	Soybean oil mills
Cut stone and stone products	Logging
Girls' and children's outerwear	Cane sugar refining
Men's and boys' shirts	Cottonseed oil mills

Notes: The table reports the sectors with the highest (left column) and lowest (right column) values of the shift in demand caused by China's productivity growth as measured by $\hat{\zeta}^t_{\text{China},s}$ with equation (36) between 1991 and 2007.

B.2.2 Estimation: Robustness and Additional Results

Evaluating the First-Order Approximation. In this appendix, we implement the two procedures described in the end of Section 3.3 to evaluate the quality of the first-order approximation for the model's predicted impact of the China shock. They apply the integration algorithm in Appendix A.3.3 to the context of the China shock, where $\hat{\zeta}^t_{\text{China},s}$ is the observed trade shock. We focus on the simple model in Section 3 because of the heavy computation burden involved with manipulating, in each integration step, the high-dimensional matrices in the formulas for the model with intermediate inputs that characterize responses in labor market outcomes and bilateral trade flows in all region-sector pairs.

We start by comparing the first-order approximation and the non-linear solution for the model's predictions (given the estimates in Panel C of Table 2). Specifically, we define the integral of our reduced-form formulas with R partitions of the shock as

$$\hat{Y}^M_i(\hat{\boldsymbol{\zeta}}^t|\boldsymbol{\theta},\boldsymbol{\mathcal{W}}^0,R) \equiv \sum_{r=1}^{R}\sum_j \beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^{r-1})\sum_s \ell^{r-1}_{j,s}\left(\hat{\zeta}^t_{\text{China},s}/R\right),$$

where $\boldsymbol{\mathcal{W}}^r = \boldsymbol{H}(\hat{\boldsymbol{\zeta}}^t/R|\boldsymbol{\theta},\boldsymbol{\mathcal{W}}^{r-1})$ defines the law of motion for the endogenous variables in $\boldsymbol{\mathcal{W}}^0$ following the shifts in sectoral demand caused by Chinese cost shocks, $\hat{\zeta}^t_{\text{China},s}/R$. When R is large enough, this algorithm converges to the exact solution for the model's predicted impact of the China shock.

Figure B.3 reports the predicted changes in the employment rate that we obtain with different partitions R. The top left panel shows that the first-order approximation yields predictions that are similar to the ones implied by the integration algorithm with $R=5$. The correlation between them is close to one. Notice however that the first-order approximation slightly over-predicts the impact of the shock. This is because it fails to capture that employment in more affected sectors falls along the path to the new equilibrium, attenuating the shift in labor demand caused by the same sectoral demand shock $\hat{\zeta}^t_{\text{China},s}/R$. The remaining panels show that there is very little gain in accuracy from using more than five partitions of the shock.

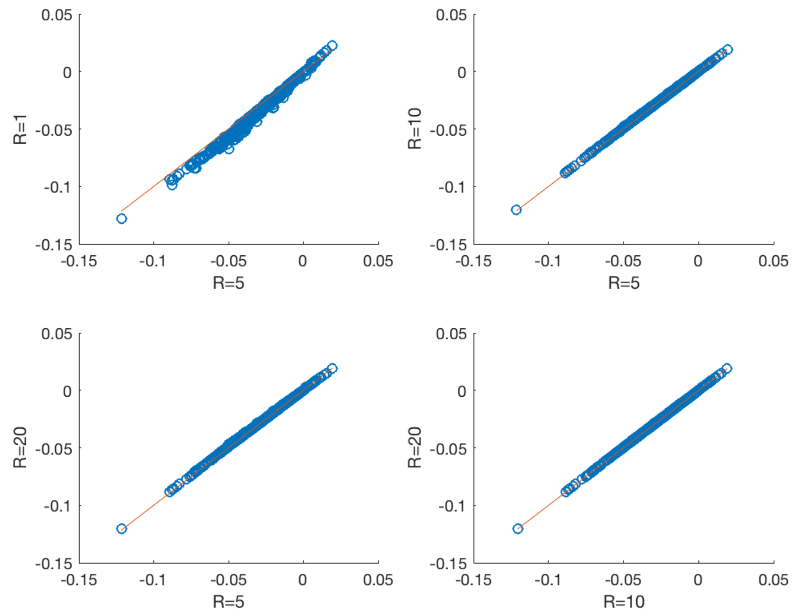
We then implement the second procedure, which consists of estimating $\boldsymbol{\theta}$ with the exact solution for the predicted impact of the observed shock, instead of the first-order approximation in (26). Specifically, we estimate $\boldsymbol{\theta}$ with the same moment conditions in (28), but we use the following specification for changes in outcome Y (that is, log average wage or log employment rate):

$$\hat{Y}_i = \alpha^Y + \sum_{r=1}^{R}\sum_j \beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^{r-1})\sum_s \ell^{r-1}_{j,s}\left(\hat{\zeta}^t_{\text{China},s}/R\right) + \nu^Y_i, \tag{B.1}$$

where we now rely on the integral of our formulas to obtain the predicted impact of the shock for any given value of $\boldsymbol{\theta}$. Relative to the structural residual in (26), ν^Y_i in the specification above does not include the higher-order terms of the Taylor expansion of the model's predicted response to the China shock. Thus, whenever these higher-order terms are not important, we should obtain similar estimates for $\boldsymbol{\theta}$ with either (26) or (B.1).

We implement this alternative estimation procedure using five partitions of the shock. We obtain estimates of $\hat{\phi}=2.16$ and $\hat{\psi}=0.43$, with standard errors of 0.33 and 0.06 respectively. These estimates are not statistically different at usual significance levels from those reported in Panel C of Table 2 that we obtained using the model's first-order approximation in (26). This reflects the fact that the first-order approximation performs well in this context, as illustrated by Figure B.3.

Figure B.3: Impact of the China Shock on the Employment Rate in General Equilibrium, Integral of First-Order Approximation



Notes: Each dot is the predicted change in the employment rate of each of the 722 CZs in 2000-2007 that we compute using the integration algorithm in Appendix A.3.3 for R partitions of the shift in sectoral demand caused by the China shock, $\hat{\zeta}^{t}_{\text{China},s}$. Predictions computed for the simple model of Section 3 with parameters in Panel C of Table 2.

Alternative Specification of Sectoral Shifters. In this appendix, we consider an alternative specification of the sectoral demand shifters. In our baseline estimates, consistent with the implicit assumption embedded in the ADH specification, we demean the shifters using the average shock across all sectors over the two periods (1990-2000 and 2000-2007). As discussed in Section 5.1, this approach may capture period-specific unobserved shocks whose exposure is correlated with the exposure to the China shock. Instead we now implement an alternative specification that is robust to such a concern by relying on the period-specific mean of the sectoral shifters; that is, we set $(1-\sigma)x^{t_0}_{\text{China}j,s}\ddot{\tau}^{\text{obs},t}_{\text{China}j,s}=\hat{\zeta}^{t}_{\text{China},s}-(1/S)\sum_{s'}\hat{\zeta}^{t}_{\text{China},s'}$ for each of the two periods t. We estimate this alternative specification while setting $\lambda=\vartheta=0$, because of the weak evidence in favor of these channels in Section 2 and their estimates close to zero in Section 5.3.

Table B.11 reports the point estimates of ϕ and ψ that we obtain with this alternative specification. For both versions of the model, compared to our baseline estimates in Table 2, we obtain similar point estimates for the two parameters, but much higher standard errors. This imprecision is a consequence of the fact that this alternative specification yields less precise estimates of the impact of the China shock across CZs. This is particularly severe for wage responses, which has a larger impact on our estimates of ψ. See Borusyak et al. (2018) for a detailed analysis of how alternative assumptions about the shock's distribution affect the estimates in ADH.

Table B.12 reports our estimates of the fit coefficient obtained from the specification in (29) with the alternative definition of the demeaned sectoral demand shifters. Reassuringly, we obtain point estimates close to one for all outcomes and specifications. Note however that the fit coefficient for the wage responses in column (1) is imprecise, which implies that we cannot reject a wide range of fit coefficients. This follows again from the fact that this alternative shifter definition leads to imprecise estimates of the impact of the China shock on wages across CZs.

Table B.11: Estimates of the Structural Parameters, Alternative Shifter Specification

ϕ	ψ
Panel A: Model without intermediates in Section 3	
3.74	0.19
(0.75)	(0.04)
Panel B: Model with intermediates in Section 4	
5.73	0.05
(2.28)	(0.05)

Notes: Panels A reports GMM estimates of $\boldsymbol{\theta}$ implied by the specification in (26) and (28), where we define the (de-meaned) observed shock as $(1-\sigma)x^{t_0}_{\text{China}j,s}\ddot{\tau}^{\text{obs},t}_{\text{China}j,s}=\hat{\zeta}^t_{\text{China},s}-(1/S)\sum_{s'}\hat{\zeta}^t_{\text{China},s'}$ for each t. Panel B reports GMM estimates implied by the specification in (34) and (35), using the same (de-meaned) observed shock as in Panel A. Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications also include the baseline control vector used in Table 1, and impose that $\lambda=\vartheta=0$. Standard errors in parentheses are clustered by state.

Table B.12: Fit of the Model for Labor Market Outcomes across U.S. CZs, Alternative Shifter Specification

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in working-age population	Share of Manufacturing in employed population
	(1)	(2)	(3)	(4)
Panel A: Model without intermediates in Section 3 (estimates of Panel A of Table B.11)				
Fit Coef. (ρ^Y)	1.22	1.02	1.26	1.12
	(0.46)	(0.16)	(0.15)	(0.17)
p-value of $H_0:\rho^Y=1$	64.1%	91.5%	08.6%	49.1%
Panel B: Model with intermediates in Section 3 (estimates of Panel B of Table B.11)				
Fit Coef. (ρ^Y)	0.98	0.98	1.16	1.11
	(0.79)	(0.15)	(0.19)	(0.19)
p-value of $H_0:\rho^Y=1$	97.5%	87.4%	39.0%	55.1%

Notes: Estimation of (29) in the pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. The regressor is the predicted impact of the exposure to the alternative shifter specification considered in Table B.11, obtained from the model without intermediate production in Section 3 in Panel A, and from the model with intermediate production in Section 4 in Panel B. Each panel uses parameter estimates indicated in the panel's label, while setting $\lambda=\vartheta=0$. Standard errors in parentheses are clustered by state.

Additional Model Fit Results. In Panels A–C of Table B.13, we report the fit coefficient estimated with (29) when we consider the alternative model specifications in Panels A–C of Table 2. We cannot reject a fit coefficient of one for all specifications and outcomes. Panel D implements the same specification in Panel C, but uses instead the alternative inference procedure for shift-share specifications in Adão et al. (2019) that is robust to any pattern of spatial correlation in residuals. We obtain similar standard errors for all outcomes.

In Table B.14, we estimate the fit coefficient of our model for log-changes in exports and imports aggregated for the entire U.S. across sectors ($EXP_{\text{US},s} \equiv \sum_{i\in\mathcal{I}_{\text{US}}}\sum_{j\notin\mathcal{I}_{\text{US}}} X_{ij,s}$ and $IMP_{\text{US},s} \equiv \sum_{i\in\mathcal{I}_{\text{US}}}\sum_{j\notin\mathcal{I}_{\text{US}}} X_{ji,s}$, respectively). That is, we regress the observed changes in U.S. exports and U.S. imports on their analogs predicted by our estimated model in response to the China shock across sectors. Following the same steps in Section 3.3, one can show that the fit coefficient must be equal to one under the null that the model is well-specified and the observed shock is exogenous. Again, because of the computational burden, we consider the model without intermediate production in Section 3, using the parameter estimates reported in Panel C of Table 2. Reassuringly, we cannot reject a fit coefficient of one for sectoral responses in both exports and imports of the U.S. (aggregated for all CZs). This indicates that our model predicts responses in sectoral trade outcomes for the U.S. that are consistent with those observed in the data.

In Figure B.4, we investigate how the values of the parameters ϕ and κ affect the fit coefficient of the model without intermediate production in Section 3. We report the fit coefficients for wages (left panel) and employment (center panel) given different values of (ϕ,κ), as well as the values (ϕ,κ) not rejected by the joint test that these fit coefficients are equal to one (right panel). For high values of ϕ, the predicted responses in employment are too large (fit coefficient is below one), and those for wages are too small (fit coefficient is above one). The right panel shows that we reject any value of ϕ that is either below one or above five. For high values of κ, the predicted responses in both wages and employment become too small (fit coefficients are much higher than one). We reject values of κ above 1.8 and, therefore, values of $\psi\phi$ below 0.64 (for a trade elasticity of five). Thus, given any value of ϕ, we reject the predicted responses implied by a multi-sector Ricardian production framework without agglomeration and intermediate production (in which $\psi = 0$ and $\kappa = \sigma - 1$).

Finally, Figure B.5 investigates how the predictions in CDP (as reported in their replication package) is related to actual changes in employment rates across U.S. states. This is a graphical representation of the fit coefficient estimation based on (29) where the regressor is the predicted effect in CDP. In this case, the figure shows that the fit coefficient between actual and predicted responses across U.S. states is much larger than one.

Table B.13: Fit of the Model for Labor Market Outcomes across U.S. CZs, Alternative Specifications

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in	
			working-age population	employed population
	(1)	(2)	(3)	(4)
Panel A: Model with intermediates in Section 3 (estimates of Panel A of Table 2)				
Fit Coef. (ρ^Y)	1.16	1.07	0.86	0.79
	(0.48)	(0.20)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	73.9%	70.5%	42.6%	21.4%
Panel B: Model with intermediates in Section 3 (estimates of Panel B of Table 2)				
Fit Coef. (ρ^Y)	1.11	1.10	0.80	0.71
	(0.48)	(0.22)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	82.6%	65.1%	22.7%	08.0%
Panel C: Model without intermediates in Section 3 (estimates of Panel C of Table 2)				
Fit Coef. (ρ^Y)	0.97	0.90	0.95	0.82
	(0.25)	(0.15)	(0.11)	(0.13)
p-value of $H_0 : \rho^Y = 1$	91.5%	51.1%	63.9%	16.0%
Panel D: Model without intermediates in Section 3 (estimates of Panel C of Table 2)				
Fit Coef. (ρ^Y)	0.97	0.90	0.95	0.82
	(0.19)	(0.20)	(0.10)	(0.14)
p-value of $H_0 : \rho^Y = 1$	88.7%	61.2%	60.7%	19.0%

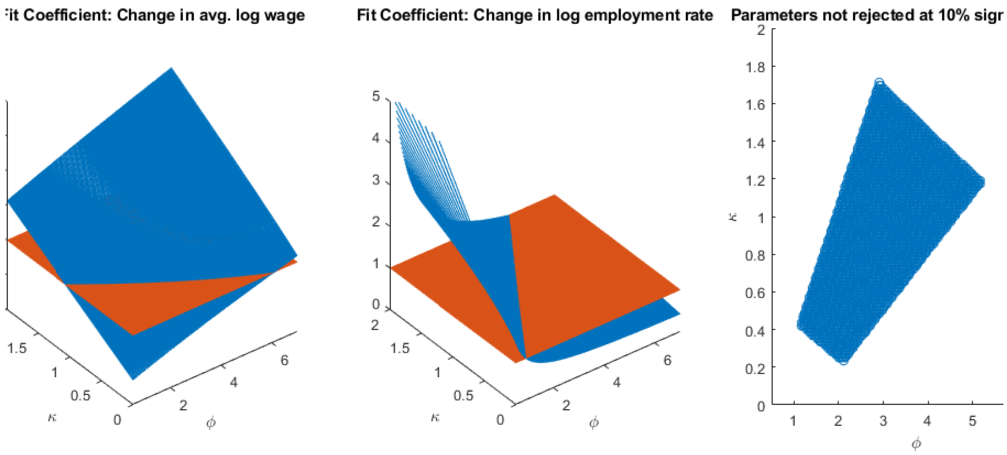
Notes: Estimation of (29) in the pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. The regressor is the predicted impact of the (de-meaned) exposure to the China shock obtained from the model with intermediate production in Section 4 in Panels A and B, and from the model without intermediate production in Section 3 in Panels C and D. Each panel uses parameter estimates indicated in the panel's label. In Panels A, B and C standard errors in parentheses are clustered by state. In Panel D, standard errors in parentheses are computed with the inference procedure for shift-share specifications in Adão et al. (2019).

Table B.14: Fit of the Model for U.S. Trade Outcomes across Sectors

	Dependent variable: Change in	
	Log of Exports	Log of Imports
	(1)	(2)
Fit Coef. (ρ^Y)	1.39	0.95
	(0.42)	(0.26)
p-value of $H_0 : \rho^Y = 1$	35.0%	84.4%

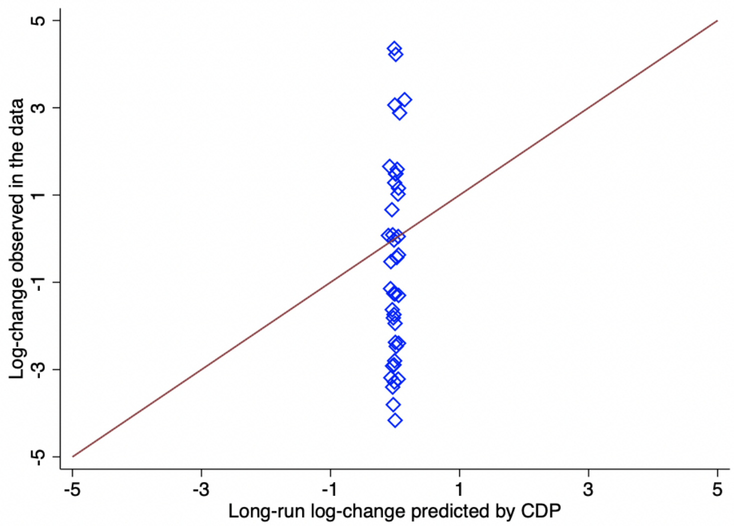
Notes: Column (1) reports the fit coefficient of a regression of observed log-changes in U.S. exports (aggregated for all CZs) on its model-predicted analog in response to the China shock across manufacturing sectors in the periods of 1991-2000 and 2000-2007, weighted by initial exports. Columns (2) reports the corresponding fit coefficient for U.S. imports (aggregated for all CZs). Regressors are computed using the (de-meaned) observed shock, $\ddot{\zeta}_{\text{China},s}$, and the model in Section 3 for the estimates in Panel C of Table 2. Robust standard errors in parentheses are clustered by sectors.

Figure B.4: Fit Coefficient for Alternative Parameter Values



Notes: In left and center panels, the blue area shows the fit coefficient implied by the estimation of (29) for different values of the parameters (ϕ,κ), and the orange area illustrates a fit coefficient of one. The right panel reports the set of parameters for which we fail to reject at a 10% significance level the hypothesis that the fit coefficient is one in the estimation of (29) for either average log wage or log employment rate (using standard errors clustered by state). The regressor is the predicted impact of the (de-meaned) exposure to the China shock obtained from the model in Section 3. All specifications include the set of baseline controls in Table 1.

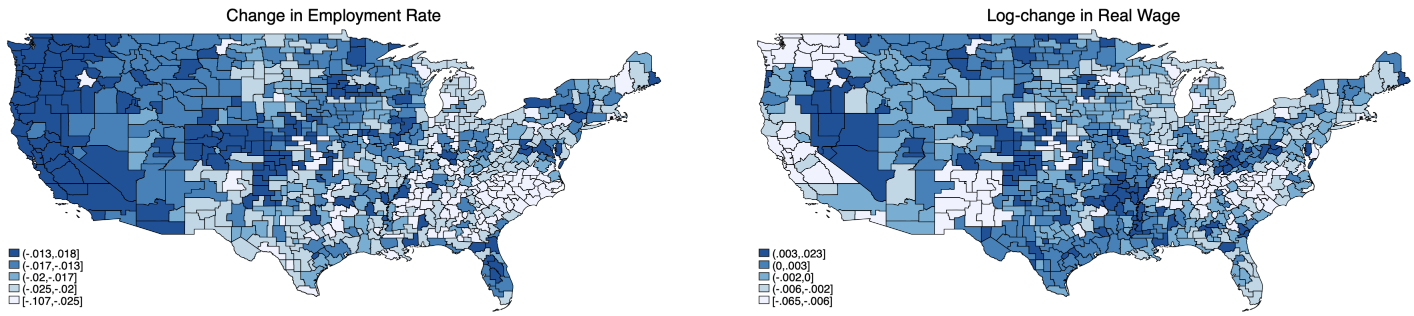
Figure B.5: Log-change in Employment Rate across U.S. States, 2000-2007



Notes: The figure plots the log-change in the employment rate (multiplied by 100) of U.S. states observed in the data between 2000 and 2007 (vertical axis), against the corresponding log-change predicted by the quantitative spatial model in CDP after the China shock (horizontal axis). The red line is the 45 degree line. We obtain the predicted responses of CDP from their replication files. All variables are normalized to have mean zero.

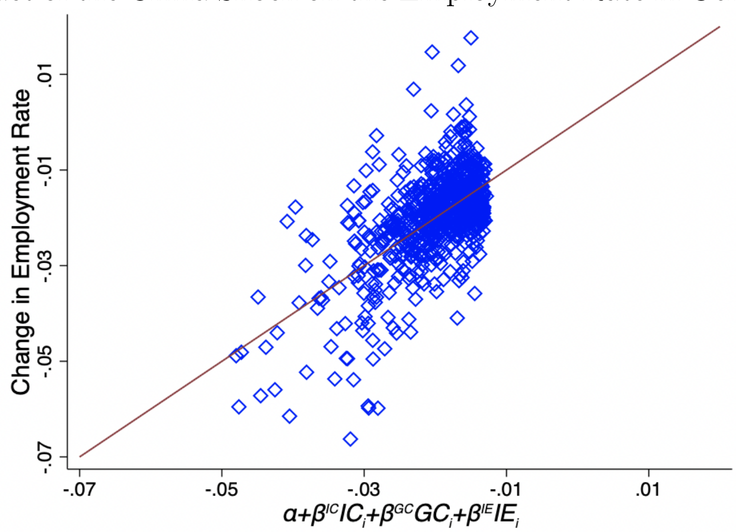
B.2.3 Aggregate Effects: Robustness and Additional Results

Figure B.6: Impact of the China Shock in General Equilibrium



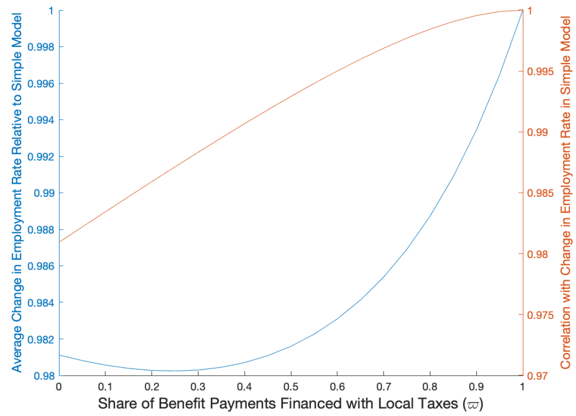
Notes: The map on the left displays the response of the employment rate to the China shock for each of the 722 CZs that we compute with the sum of the predicted effects for that CZ in 1990-2000 and 2000-2007 implied by the estimated specification in (34) for the parameters in Panel A of Table 2. The map on the right displays the analog for the predicted response of the log of the real wage.

Figure B.7: Impact of the China Shock on the Employment Rate in General Equilibrium



Notes: We plot on the vertical axis the change in the employment rate for each of the 722 CZs in 1990-2007 implied the estimated specification in (34) using the parameters in Panel A of Table 2. The x-axis is the fitted value obtained by regressing the predicted effects of the model on a constant, IC_i^t, GC_i^t, and IE_i^t. We compute the exposure measures using (3)-(5) with the sectoral shifter defined as $\hat{\zeta}^t_{\text{China},s}$ (instead of $\Delta M^t_{\text{China},s}$). The red line is the 45-degree line.

Figure B.8: Impact of the China Shock on the Employment Rate in General Equilibrium, Alternative Regional Transfers Schemes



Notes: For each value of the share of benefit payments financed with local taxes (ϖ) in the alternative specification of the model in Appendix A.2.6, the figure reports the average employment rate change across U.S. CZs relative to the average change predicted by the baseline model (blue line), and the correlation with the employment rate change implied by the baseline model (in which $\varpi = 1$). Predictions computed with the simple model of Section 3 with estimated in Panel C of Table 2.

C Appendix: Data Construction (Not for publication)

This appendix describes the procedure to construct the data used in Section 5.

C.1 Bilateral Trade Matrix

C.1.1 Data Construction

Country-to-country bilateral trade matrix. We start by creating a country-to-country matrix of trade flows at the 4-digit SIC classification. We consider the countries listed in Table C.1. We obtain international trade flows at the product-country level from the BACI dataset, assembled by CEPII, which we aggregate at the 4-digit SIC level. Since the starting year of the BACI dataset is 1995, we use the trade flows for 1995 and 2000.[48] To obtain domestic spending shares for each country, we note first that our gravity model implies that $X_{ij,s}^t = (\tau_{ij,s}^t p_{i,s}^t)^{1-\sigma} (P_{j,s}^t)^{\sigma-1} E_{j,s}^t$. For any sector s within an aggregate sector S, assume that, for $i \neq j$, $\tau_{ij,s}^t = \tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{j,S}^{D,t} e^{\tilde{\tau}_{ij,s}^t}$. Thus,

$$\ln X_{ij,s}^t = \tilde{\tau}_{ij,s}^t + \alpha_{i,s}^t + \varphi_{j,s}^t, \tag{B.2}$$

where $\alpha_{i,s}^t \equiv \ln\left(\left(\tilde{\tau}_{i,S}^{O,t} p_{i,s}^t\right)^{1-\sigma}\right)$ and $\varphi_{j,s}^t \equiv \ln\left(\left(\tilde{\tau}_{j,S}^{D,t} P_{j,s}^t\right)^{\sigma-1} E_{j,s}^t\right)$.

To get the domestic trade flows, notice that $X_{ii,s}^t = (p_{i,s}^t)^{1-\sigma} (P_{i,s}^t)^{\sigma-1} E_{i,s}^t = \left(e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t}\right) / \left(\tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{i,S}^{D,t}\right)$. Since $X_{ii,S}^t = \sum_{k \in S} X_{ii,k}^t$,

$$X_{ii,s}^t = X_{ii,S}^t \frac{e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t}}{\sum_{k \in S} e^{\alpha_{i,k}^t} e^{\varphi_{i,k}^t}} \tag{B.3}$$

We use (B.3) to compute $X_{ii,s}^t$. In each year t, we obtain $\alpha_{i,s}^t$ and $\varphi_{j,s}^t$ from the estimation of (B.2) with bilateral trade flows by sector, and $X_{ii,S}^t$ from the domestic sales in two aggregate sectors in the Eora MRIO dataset: manufacturing and non-manufacturing.

Table C.1: Sample of Countries

Argentina	Czech Republic	Malaysia	Singapore
Australia	Denmark	Mexico	Slovakia
Austria	Finland	Netherlands	South Africa
Baltic Republics	France	New Zealand	South Korea
Belarus	Germany	Norway	Spain
Benelux	Greece	Pakistan	Sweden
Brazil	Hungary	Philippines	Switzerland
Bulgaria	India	Poland	Taiwan
Canada	Indonesia	Portugal	Thailand
Chile	Ireland	Rest of World	Ukraine
China	Italy	Romania	United Kingdom
Colombia	Japan	Russia	Uruguay
Croatia	Kazakhstan	Saudi Arabia	Venezuela

Notes: Baltic Republics includes Estonia, Lithuania and Latvia.

[48] Although trade data is available for 1990 from UN Comtrade, it is quite sparse across countries and industries.

CZ employment share. We use the same imputation procedure of ADH to compute employment in each 4-digit SIC manufacturing industry for 1980, 1990 and 2000 using the County Business Pattern (CBP). In year t, we use $L^t_{i,s}$ to denote employment in CZ i and 4-digit SIC industry s and $\ell^t_{i,s} = L^t_{i,s}/L^t_i$ to denote the associated employment share.

CZ gross spending shares. We construct gross spending by sector and CZ, $e^t_{i,s}$, using

$$e^t_{i,s} \equiv \frac{E^t_{i,s}}{E^t_i} = \frac{\xi^t_{i,s} + \sum_k \xi^{M,t}_{i,sk} a^t_k \ell^t_{i,k}}{1 + \sum_k a^t_k \ell^t_{i,k}}. \tag{B.4}$$

where, in year t, $\xi^{M,t}_{i,sk}$ is the share of spending on intermediates of sector s by sector k (common to all CZs, $\xi^{M,t}_{i,sk} = \xi^{M,t}_{sk}$), a^t_k is the ratio of intermediate cost to labor cost of sector k (common to all CZs), and $\xi^t_{i,s}$ is consumers' spending share on final goods of sector s (common to all CZs, $\xi^t_{i,s} = \xi^t_s$). We compute $\xi^t_{sk} \equiv \frac{M^t_{sk}}{\sum_{s'} M^t_{s'k}}$ where M^t_{sk} is the spending of industry k on industry s in the BEA 1992 U.S. Input-Output table used in Acemoglu et al. (2016a). For manufacturing SIC-4 industries, we compute a^t_k using total material costs divided by payroll in the NBER manufacturing database for year t. For non-manufacturing industries, we compute a^t_k as average the material to payroll ratio across all U.S. non-manufacturing industries in the WIOD database. Finally, we obtain ξ^t_s from the BEA 1992 U.S. Input-Output table.

CZ exports and imports. We follow three steps to create exports and imports for each CZ and industry. First, we compute the CZ spending on sector s as $E^t_{i,s} = e^t_{i,s} L^t_i$ where $e^t_{i,s}$ is the sectoral spending share described above and L^t_i is the total employment in the CZ. Second, for each sector s, we compute the share of CZ i in national spending, $\tilde{e}^t_{i,s} = E^t_{i,s}/\sum_j E^t_{j,s}$, and in national employment, $\tilde{\ell}^t_{i,s} = L^t_{i,s}/\sum_j L^t_{j,s}$. Third, we use the US Census data at the state-sector level for 1997 to compute the share of each state in the exports/imports to/from each foreign country in a SCTG category, which is the 40-sector classification used by the US Census.[49] This yields $\beta_{state,i,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign importer, and $\beta_{i,state,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign exporters. We use the same share $\beta_{state,i,s}$ and $\beta_{state,i,s}$ for all SIC-4 industries within the same SCTG category. Finally, in each year t, we take US imports $X^t_{i,US,s}$ and US exports $X^t_{US,i,s}$ in each sector s and foreign country i, and split them across CZs using the following expressions:

$$X^t_{ij,s} = \frac{\tilde{e}^t_{j,s}}{\sum_{j' \in state} \tilde{e}^t_{j',s}} \beta_{i,state,s} X^t_{i,US,s} \quad \text{and} \quad X^t_{ji,s} = \frac{\tilde{\ell}^t_{j,s}}{\sum_{j' \in state} \tilde{\ell}^t_{j',s}} \beta_{state,i,s} X^t_{US,i,s}.$$

CZ-to-CZ bilateral trade matrix. We follow three steps to impute trade flows across CZs using the gravity trade structure of our model. First, for each SCTG category, we use state-to-state shipment data from the Commodity Flow Survey in 1997 to estimate

$$\ln X_{ij,s} = \delta_s + \beta_1 \ln D_{ij} + \beta_2 \ln E_{j,s} + \beta_3 \ln R_{i,s} + \beta_4 d_{i=j} + \varepsilon_{ij,s} \tag{B.5}$$

where i is the origin state, j is the destination state, s is the SCTG category, D_{ij} is the bilateral distance between the population centroids of states i and j, $E_{j,s}$ are expenditures, $R_{i,s}$ are revenues, $d_{i=j}$ is a dummy that equals 1 when $i = j$.

[49] We construct state-sector exports and imports as follows. First, we use the US Merchandise Trade Data for 1997 released by the US Census to create a mapping from each of the 44 US districts to the 50 US states, in terms of share of imports and exports to each foreign country. Note that this is done at the aggregate level as this information is not available at the industry-level. We then use US Census data to create district-level exports and imports at the HS-6 level for 1997. Finally, we use the mapping previously constructed to obtain state-HS6, and then state-SIC 4 digit, trade flows with our sample of foreign countries.

Second, we use the estimated coefficients to impute trade flows across CZs with the following gravity specification:

$$\ln X_{ij,s}^t \equiv \hat{\beta}_1 \ln D_{ij} + \hat{\beta}_2 \ln \tilde{e}_{j,s}^t + \hat{\beta}_3 \ln \tilde{\ell}_{i,s}^t + \hat{\beta}_4 d_{state(i)=state(j)} \tag{B.6}$$

where D_{ij} is the distance between the population centroids of CZs i and j, and $d_{state(i)=state(j)}$ is a dummy equal 1 if i and j belong to the same state.

Lastly, we re-scale the imputed CZ-to-CZ trade flows so that the sum of the bilateral flows in each SIC sector across all CZs is equal to the total U.S. domestic sales in each SIC sector in the country-to-country trade matrix.

Trade balance. Finally, we impose that trade is balanced at the regional level, as in the baseline model. We use the trade flows obtained above to compute matrix $\bar{\boldsymbol{x}}^t$ whose entries correspond to the share of spending of each region j on another region i. Under trade balance, the vector of total revenue in the world economy, $\boldsymbol{R}^t$, must satisfy $\bar{\boldsymbol{x}}^t \boldsymbol{R}^t = \boldsymbol{R}^t$ and, therefore, $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)\boldsymbol{R}^t = 0$. Notice that it is always possible to find a vector $\boldsymbol{R}^t$ that satisfies this system since $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ is singular ($\sum_i x_{ij}^t = 1$ for every j). Thus, we find the vector $\boldsymbol{R}^t$ as the eigenvector of $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ associated with the eigenvalue of zero. Without loss of generality, we then normalize it such that world GDP is one, $\sum_i R_i^t = 1$.

C.1.2 Validation Tests

We first evaluate the correlation between the expenditure shares $e_{i,s}^t$ constructed in equation (B.4) and the spending shares implied by the shipment data for U.S. states. To this end, for each of the 40 SCTG categories, we compute state-level total shipment inflow in the Commodity Flow Survey (CFS) for 1997. We then construct state-level spending shares at each SCTG category using the expenditure shares $e_{i,s}^t$ in equation (B.4) for the CZs in the state. Specifically, we first aggregate our expenditure shares at the SCTG level using a crosswalk between SIC-4 and SCTG categories, and then compute total spending by SCTG in each state using the total expenditure of the CZs in that state. Table C.2 reports the result of a regression of the expenditure shares computed from the CFS on our constructed gross spending shares in 1990 and 2000. We can see that they are positively and significantly correlated, with an OLS coefficient close to 1 and a R2 of 0.95.

We then proceed to assess whether our constructed CZ-level trade matrix reproduces the patterns of observed trade flows for U.S. states. We use the CFS to measure bilateral shipments between U.S. states in each SCTG category for 1997, 2002 and 2007. To obtain comparable data, we aggregate the bilateral trade flows for the CZs in the same state and the SIC sectors in the same SCTG category. Table C.3 reports the results of regressing actual shipment data on the corresponding trade flow obtained from our trade matrix. Column (1) considers domestic flows between U.S. states, column (2) considers export flows from U.S. states to foreign countries, and column (3) considers import flows from foreign countries to U.S. states. All specifications include sector fixed-effects. We can see that the predicted trade flows are significantly and positively related to the actual flows, with coefficients close to 1. Notice also that our imputed data captures a large share of the variation in bilateral trade flows. The R2 is above 0.8 for exports and imports of U.S. states, and around 0.5 for domestic flows between U.S. states.

Table C.2: Validation Test – Gross Expenditure Shares

Dependent variable:	Observed expenditure shares, 1997	
	(1)	(2)
Constructed expenditure shares, 1990	1.275***	
	(0.01)	
Constructed expenditure shares, 2000		1.265***
		(0.01)
Constant	-0.009***	-0.009***
	(0.00)	(0.00)
Observations	1,392	1,392
R^2	0.95	0.95

Notes: Sample of 1,392 state-SCTG pairs, where SCTG is the industry classification used in the CFS. Dependent variable is the observed expenditure share in 1997 computed from the CFS. The regressors are the expenditure shares computed in equation (B.4), aggregated at the state-SCTG level. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

Table C.3: Validation Test – Bilateral Trade Flows

	(1)	(2)	(3)
***Panel A:* Log of Actual Flows in 1997**			
Log of Predicted Flows in 1997	1.068***	0.973***	0.993***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.512	0.950	0.950
***Panel B:* Log of Actual Flows in 2002**			
Log of Predicted Flows in 2002	1.024***	0.847***	0.884***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.509	0.816	0.837
***Panel C:* Log of Actual Flows in 2007**			
Log of Predicted Flows in 2007	1.047***	0.797***	0.861***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.477	0.806	0.827
Flow type:			
U.S. state to U.S. state	Yes	No	No
U.S. state to Country	No	Yes	No
Country to U.S. state	No	No	Yes

Notes: The dependent variable in column (1) is the actual shipment flow reported in the CFS for state-state-SCTG triples. The dependent variables in columns (2) and (3) are trade flows constructed from the US Census trade data for state-country-SCTG triples. The regressors are the trade flows constructed using our methodology for the years 1997, 2002 and 2007, aggregated at the state-state-SCTG or state-country-SCTG level. All regressions include sector fixed effects. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.10$

C.2 Trade in Intermediate and Final Goods

The methodology described in the previous section yields a bilateral matrix of *gross* trade flows between 722

U.S. CZs and 52 countries. While this is enough to implement the model of Section 3, the estimation of the more general model with input-output links of Section 4 requires bilateral trade flows in intermediate and final goods. We now describe how we proceed to construct such data. Our procedure relies on the fact that, in our model, trade flows in final goods and intermediate inputs between two markets i and j can be written respectively as $X^C_{ij,s} = x_{ij,s}\xi_{j,s}E_j$ and $X^M_{ij,sk} = x_{ij,s}\xi^M_{j,sk}a^M_{j,k}R_{j,k}$, where $x_{ij,s}$ is the matrix of *gross* trade shares within sector s. This property is the by-product of the assumption that the elasticity of substitution between products of different origins is the same for final consumption and intermediate consumption in all sectors, as in Caliendo and Parro (2015) and in the literature reviewed by Costinot and Rodríguez-Clare (2014). Therefore, we only need to complement the bilateral matrix of trade shares described above with data on the sectoral spending shares of final and intermediate expenditures. To reduce the computational burden, we construct the bilateral trade flows in intermediate and final goods at the 3-digit sector level.

C.2.1 Final Spending Shares

Our main data source is the Consumer Expenditure Survey (CEX) Public-use Micro-data from the U.S. Bureau of Labor Statistics for the years of 1996 and 2000. We first combine the individual-level information in the interview and diary databases to generate annual average household expenditure in each U.S. state on the different product categories in the CEX (i.e., the UCC codes). We then construct a crosswalk from the UCC product classification used in the CEX to 3-digit SIC sectors, using the UCC description provided by the BLS. For the states without data in the CEX, we assign the final expenditure shares of the US Census division to which that state belongs. For all foreign countries, we set the share of final spending on each SIC sector to be the same as that reported in the 1992 U.S. IO table from the BEA.

C.2.2 Intermediate Spending Shares

We measure the sectoral intermediate spending shares in each CZ and country by assuming that $a^M_{j,k} = a_j\left(1-a^L_k\right)$ where a^L_k is the share of labor in sector k's total cost (common to all countries). We first describe how we calibrate a_i and then how we construct each variable.

First, from the good market clearing condition,

$$R_{i,s} = \sum_j x_{ij,s}\left(\xi_{j,s}E_j + \sum_k \xi^M_{j,sk}a_j a^M_k R_{j,k}\right),$$

where E_j is market j's expenditure on final goods, and $\xi_{j,s}$ and $\xi^M_{j,sk}$ are market j's final and intermediate spending shares. We can write this expression in matrix form and invert it:

$$\boldsymbol{R}(\boldsymbol{a}) = \sum_{d=0}^{\infty}\left(\bar{\boldsymbol{A}}(\boldsymbol{a})\right)^d \boldsymbol{F}, \tag{B.7}$$

where $\boldsymbol{F} \equiv \left[\sum_j x_{ij,s}\xi_{j,s}E_j\right]_{is}$, $\bar{\boldsymbol{A}}(\boldsymbol{a}) = \bar{\boldsymbol{x}}^A diag(a_j)$ and $\bar{\boldsymbol{x}}^A \equiv [x_{ij,s}\xi^M_{j,sk}a^M_k]_{is,jk}$.

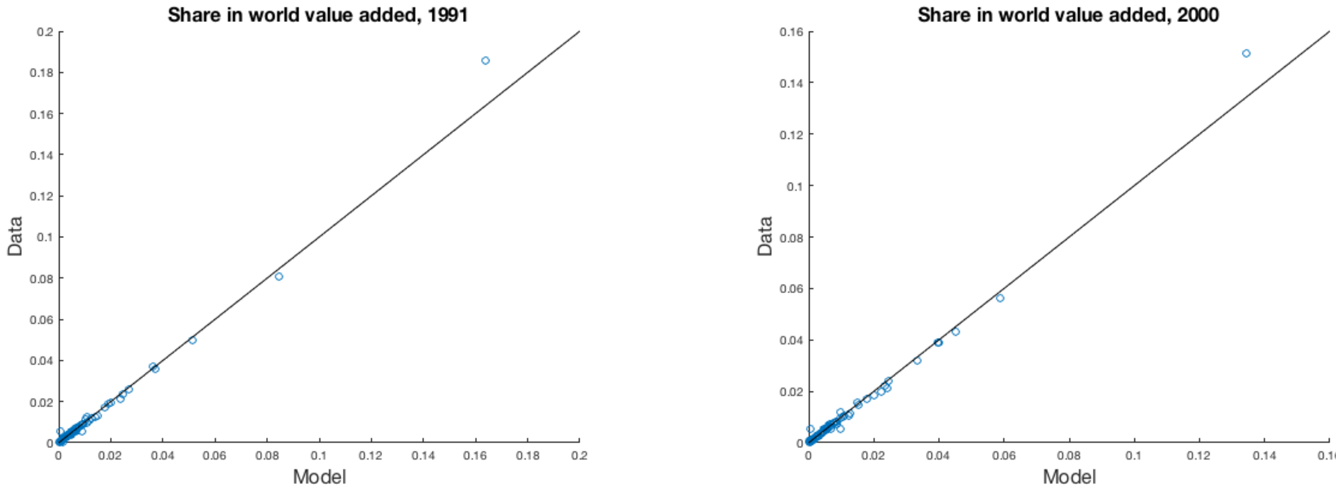
Second, from the labor market clearing condition,

$$W_i(\boldsymbol{a}) = \sum_s (1 - a^M_s a_j) R_{i,s}(\boldsymbol{a}), \tag{B.8}$$

where $R_{i,s}(\boldsymbol{a})$ is given by (B.7).

Finally, we calibrate $\boldsymbol{a}$ to minimize the difference between the observed value-added in market i, W_i, and the one predicted by equation (B.8), $W_i(\boldsymbol{a})$:

Figure C.1: Share of World Value Added



Notes: For each CZ and foreign country, each graph plots the share of world value added observed in the data against the corresponding share predicted by our calibration procedure.

$$\boldsymbol{a}^* = \arg \min_{a_j \in \left(0, 1/\max_k \left\{a_k^M\right\}\right)} \sqrt{\sum_i (W_i - W_i(\boldsymbol{a}))^2}.$$

To implement this calibration, we use the within-sector bilateral trade shares, $x_{ij,s}$, that we constructed with the methodology described in Appendix C.1. The labor shares a_k^L are obtained from the NBER Manufacturing database. For all markets, we use the 1992 BEA IO table to measure final and intermediate spending share. We aggregate all the shares at the 3-digit sector level. For all countries, we measure value-added from the WIOT. For the U.S., we split value-added across CZs by setting value-added in CZ i to $W_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P}\right) W_{US}$, where W_i^P is the CZ's wage bill in the CBP. Similarly, we use the WIOT to measure aggregate final expenditure in each country, and split total final expenditure in the U.S. across CZs using the same payroll shares, $E_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P}\right) E_{US}$. Figure C.1 shows that our calibration procedure almost exactly matches the observed shares of value added across U.S. CZs and foreign countries.