Dissecting Trade and Business Cycle Co-movement*

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Abstract

International business cycles have become highly synchronized across countries in the past three decades, yet there is a lack of consensus on whether this is due to an increase in correlated, country-specific shocks or trade shocks through an increased economic integration. To understand this empirical phenomenon, I develop a multi-country real business cycle model with international trade that captures several potential explanations: shocks to productivity, demand, leisure, investment, sectoral expenditures, and trade. By matching the data exactly with the endogenous outcomes of the model, shocks fully account for the data such as GDP and trade shares. Calibrating the model to a panel of developed (G7) countries during 1992-2014, I find that country-pair trade shocks, which capture the increased economic integration and volatility of trade flows, are essential in synchronizing international business cycles. In contrast, other correlated country-specific shocks play relatively minor roles. This suggests that trade shocks through economic integration have been the primary driver of the co-movement of international business cycles. Furthermore, I use my model to address the *trade co-movement puzzle*, which states that international real business cycle models should be predicting a much stronger link between trade and cross-country GDP correlations. Once I account for the country-pair trade shocks, the model predicts a strong link between trade and business cycle co-movement. This finding suggests that incorporating the dynamics of trade shocks is crucial when studying international business cycles.

Keywords: International business cycles; Business cycle synchronization; International trade; Business cycle accounting

JEL Classifications: E32, E37, F15, F41, F44, F62

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1 Introduction

As world economies become more globalized, international business cycles have become more synchronized. On the one hand, with the growth of international trade, countries have been more economically integrated as they face trade shocks, which, in turn, synchronize their business cycles across trading partners. On the other hand, increased correlated country-specific shocks, such as productivity or demand, may have affected the increase in international business cycle synchronization. Understanding the process and the specific drivers of international business cycle synchronization is necessary to fully anticipate the effects of a country or regional specific policies on various countries and the global economy. However, there is still a lack of consensus on what is generating cross-country business cycle synchronization. Since both country-specific, correlated shocks and shocks through international trade linkages affect business cycles in each country, we need a *comprehensive* account of global business cycle co-movement that addresses all of these forces in a holistic way.

In this paper, I ask two fundamental questions: (1) What are the main drivers of the co-movement of international business cycles? (2) Does international trade play a crucial role in creating such an empirical phenomenon? To answer these questions, I construct a multi-country international real business cycle model with various shocks that completely account for the movements of gross domestic product (GDP) and trade in each country. The embedded shocks allow the model to be fully saturated so that it replicates the data exactly. By identifying shocks that are accountable for the movements in various macroeconomic aggregates, I seek to give a full account of international business cycles.

Disentangling the various forces that influence cross-country business cycle co-movement is not an easy task, since these forces affect business cycles simultaneously. Productivity shocks are one of the common potential explanations; as productivity shocks are more positively correlated across countries, output across countries will be more synchronized. However, a standard international real business cycle model that only accounts for productivity shocks does not generate as much of cross-country co-movement that we observe from the data.[1] Another explanation is the correlated demand shocks, where each country's increased consumptions generate higher outputs, thus leading to the positive business cycle co-movement. Correlated labor shocks are another potential explanation, as an increased correlation in employment can lead to the output correlation. Furthermore, investment-driven shocks may be the answer, as it creates higher correlations to the returns to capital in each country, which, in turn, drives the output co-movement. At the same time, we cannot ignore shocks to the composition of economic sectors as many developed countries experienced a gradual shift from manufacturing to services in the composition of their economies.

In addition to the country-specific correlated shocks, the world has experienced a considerable increase in international trade, as countries have become more economically integrated. There has been much evidence in the literature that country-pairs that trade more with each other indeed have a higher degree of business cycle synchronization, which suggests that countries may face common trade shocks with their trading partners. As the global trade to GDP ratio increased from 40% in the early 1990s to 60% in 2010s, countries are more susceptible to the common shocks through trade linkages. Therefore, disentangling these forces involves assimilating these economic shocks to understand their contributions to global business cycle synchronization.

To dissect the effects of various shocks on global business cycle synchronization, I build a two-sector

[1] This is a well documented result from a seminal paper on international business cycles by Backus et al. (1992).

multi-country international real business cycle (IRBC) model with input-output linkages by saturating the model with shocks to (i) productivities to merchandised goods and (ii) services, (iii) aggregate demand, (iv) leisure, (v) efficiency to investment, (vi) sectoral expenditure of the merchandised goods sector, and (vii) trade in the merchandised goods sector for each country-pair. Each country entirely specializes in two sectors: a tradable merchandised goods sector and a non-tradable services sector. Merchandised goods are then traded between countries to create a composite good by combining home and foreign merchandised goods, while services are only consumed inside the country. A fraction of traded composite goods and services are used as intermediate inputs, and the rest is used in the production of the final good. The final good production, which aggregates a portion of traded composite merchandised goods and non-tradable services, is either consumed or invested. All tradable goods are subject to iceberg costs, as is commonly assumed in the international trade literature. I take a bird's eye view of the global economy as I assume that there exists a global social planner who allocates all the endogenous outcomes, taking as given the initial parameters and structural shocks.[2] Since there is no market failure in this model, the allocations of the social planner are equivalent to the competitive equilibrium allocations with complete markets.

I then calibrate the model to a set of developed countries (G7), and the rest of the world (RoW) in which the endogenous outcomes of the model match the data exactly.[3] I assume that the global economy behaves as if it follows the international real business cycle framework of this paper. In other words, there is no unexplained variation in the data – the shocks that I extract capture all of the variations in the observables. With this assumption, I match the model moments of GDP, gross production in merchandised goods, consumption expenditure of households, labor hours, producer and consumer prices, and bilateral trade shares with the data counterpart. Subsequently, using the equilibrium conditions, I back out a set of structural shocks that completely describe the data between 1992 and 2014.

The business cycle accounting methodology I employ in my framework was first developed by Parkin (1988), Chari et al. (2007), and further extended by Eaton et al. (2016). Productivity shocks to the merchandised goods and non-tradable services sectors can be considered Solow residuals; they are obtained using GDP, gross production in merchandised goods, and the producer prices of merchandised goods. Aggregate demand shocks encapsulate the underlying fluctuations in consumption expenditures, which are embedded in the model as time-varying preference wedges. Moreover, leisure shocks, which are another type of preference shocks in the model, are backed out as a wedge from the marginal rate of substitution between consumption expenditure and labor. I obtain shocks to efficiency to investment by using investment and consumption expenditures. Sectoral shocks to the expenditure of merchandised goods capture the shocks to demand in the goods sector relative to the services sector, and these shocks are backed out using input (PPI) and output (CPI) prices. The important shock in this paper is the shock to bilateral trade costs. Bilateral trade cost shocks measure the underlying common shocks for each country-pair, which are attained by matching bilateral trade shares of merchandised goods. By definition, if I feed in the set of shocks that I back out into the model, it replicates the data exactly such as GDP, gross production of merchandised goods, consumption expenditure of households and government, labor hours, and traded import shares of merchandised goods.

Using the structural model, I back out various shocks which can be considered potential drivers of international business cycle co-movement. Equipped with the extracted shocks, I conduct counterfactual

[2]This methodology is first developed by Parkin (1988) in a closed economy real business cycle framework. The methodology that I closely follow is from Eaton et al. (2016).

[3]G7 countries are Canada, France, Germany, Italy, Japan, UK, and US.

analyses to understand how each set of shocks contributes to the cross-country business cycle synchronization. To isolate the contributions of each set of shocks, I re-calibrate the model by keeping one shock constant at a time, while feeding in the other shocks as they are backed out from the model. By performing these counterfactuals, I can understand how much each shock contributes to the cross-country GDP correlations by comparing the results to the cross-country GDP correlations from the data. The bigger the gap between the data and the counterfactual GDP correlations, higher the contribution is to the GDP co-movement.

I uncover several results. First, I find that country-pair trade shocks, which are captured by matching imported trade shares from the data, are primarily responsible for the global business cycle synchronization, from 1992 to 2014. Other country-specific correlated shocks play a minimal role in generating cross-country business cycle co-movement. A counterfactual scenario when country-pair trade shocks are kept constant yields the most significant drop in cross-country GDP correlations compared to those from the data. When the other country-specific shocks are constant, the counterfactual cross-country GDP correlations are not as responsive. I further analyze the results by dividing the time into three periods: 1990s, early 2000s, and the Great Recession and post recovery period. I further find that country-pair trade shocks had the strongest impact in synchronizing business cycles in early 2000s compared to the other periods.

The intuition behind why country-pair trade shocks affect the global business cycle co-movement the most is the following. Country-pair trade shocks that I back out capture two components in the data: an increase in economic integration as countries become more open to international trade (a trend component), and volatility in bilateral trade flows (a cyclical component). As countries are more economically integrated, they become more susceptible to common shocks with their trading partners; the degree of how much common shocks affect cross-country GDP correlations grows as the imported shares of countries' expenditures increase. As a country-pair trades more, its GDP moves in the same direction as the bilateral trade cost shocks hit the country-pair simultaneously. Therefore, trade dynamics over the long horizon in time series are necessary to capture the international business cycle co-movement. This intuition is consistent with my finding that the early 2000s were most affected by the country-pair trade shocks.

To delve more deeply into the causes of the country-pair shocks, I run counterfactual analyses where I fix one country's trade shocks constant while maintaining all the other country-pair trade shocks that do not involve the particular country of interest.[4] I find that the trade shocks coming from the RoW are substantial. I compare the effect of the RoW trade shocks for three different periods: the 1990s (1993 – 1999), early 2000s (2000 – 2007), and the Great Recession and the post-recovery period (2008 – 2014). I find that the RoW trade shocks impact becomes larger in 2000 and onwards, compared to the 1990s. As the world faced significant growth in international trade in 2000 accompanied by China's accession to the World Trade Organization (WTO), the RoW trade shock has become more important to international business cycle co-movement.

Last, I apply the results of my paper to show that the country-pair trade shocks are crucial in alleviating *the trade co-movement puzzle*, which could be a door for future researchers to understand other international macroeconomic puzzles. The trade co-movement puzzle, which was first proposed by Kose and Yi (2006), states that the standard international macroeconomic models cannot capture the associated increase in GDP correlations as trade increases between the trading partners. My findings suggest that standard macroeconomic models fail to resolve the puzzle because the previous literature has not accounted for the common

[4] The country-specific correlated shocks are still varying in these exercises.

trade shocks that hit the country-pairs. Other country-specific shocks, including productivity shocks, do not increase the GDP co-movement when trade increases. Through my model, I show that without country-pair trade shocks, the bilateral trade shares fall significantly short of the data. The changing nature of economic integration (increase in import shares) and the volatility of bilateral trade flows are the missing piece of the trade co-movement puzzle. In their original work that started this strand of literature, Frankel and Rose (1998) write, "the large IRBC literature, which endogenize output correlations ... does not focus on *the effects of changing economic integration*." Indeed, my results show that the previous literature has been missing the dynamic aspect of trade. Previous research has largely focused on calibrating models to steady-state import trade shares, which does not capture the complex evolutionary process of trade shocks.

Finally, I briefly discuss different classes of models that can potentially induce an increase in trade and trade volatility between countries. Two papers in particular closely align with my results. First, as Drozd et al. (2019) suggest, allowing for dynamic elasticities between home and foreign goods induces more trade volatility between countries. Furthermore, de Soyres and Gaillard (2019) also addresses the trade co-movement puzzle by introducing a model with extensive margin adjustments to entry and exit of firms and love for variety. This additional channel potentially creates extra volatility in bilateral trade.

This paper contributes mainly to the following three lines of literature. First, it contributes to the literature on the determinants of international business cycles and cross-country business cycle synchronization. Ever since Backus et al. (1992)'s seminal paper on an extension of a real business cycle model to a two-country, large open economy, research on international business cycles has diverged in two different ways.[5] A large portion of the literature studies how the correlated, country-specific shocks affect international business cycles. On the one hand, Imbs (2004) argues that country-specific correlated shocks are responsible for the business cycle co-movement and that trade does not play a significant role. There have been numerous studies on other types of correlated shocks besides correlated TFP shocks.[6] On the other hand, Frankel and Rose (1998) and Baxter and Kouparitsas (2005) find that countries that trade more with each other have a higher degree of business cycle synchronization, which suggests that trade may be essential in understanding international co-movements.[7] I contribute to this literature by incorporating all of the shocks in competing hypotheses to determine which story has the largest contribution to the international business cycle synchronization.

Secondly, this paper contributes to the literature on *the trade co-movement puzzle*. First proposed by Kose and Yi (2006), the trade co-movement puzzle describes standard IRBC models' inability to create higher cross-country GDP synchronization when country-pairs increase their trade. In standard IRBC models, Kose and Yi (2006) find that the cross-country GDP correlations do not respond very much to changes in trade costs. A large portion of the literature studies how to amplify the shock propagation from one country to the other by introducing different mechanisms.[8] I contribute to the literature by showing that we

[5] Backus et al. (1992) find that only with correlated TFP shocks, the two-country IRBC model does not match international co-movements of output and consumption very well, thus becoming a starting point of various international macroeconomic puzzles.

[6] Stockman and Tesar (1995) incorporate shocks to preferences to explain international co-movements of output and consumption across countries. Huo et al. (2019) embody non-TFP shocks in their model, which is similar to the labor/leisure shocks in my paper. Raffo (2010) argues that investment technology shocks drive international business cycles.

[7] Alessandria et al. (2013) and Engel and Wang (2011) allude that bilateral trade cost shocks are the key source of cross-country business cycles.

[8] Burstein et al. (2008) introduce Leontiff production with global value chains, and Johnson (2014) extends IRBC model with input-output linkages, but both of these papers fail to generate the coefficient (slope) that is seen from the data. Liao and Santacreu (2015) and de Soyres and Gaillard (2019) argue that Melitz-type of model, with extensive margin of trade and

need a mechanism in models that captures the dynamics of bilateral trade shocks between trading partners. Furthermore, I give a brief intuition where I link the results to the existing trade co-movement puzzle literature.

Finally, this paper adds to the vast literature studying the impacts of various shocks on macroeconomic aggregates by a business cycle accounting methodology first developed by Parkin (1988) and Chari et al. (2007). Since then, the methodology has been applied to open economy studies by Eaton et al. (2016), Reyes-Heroles (2017), and Ohanian et al. (2018).[9] Building on these previous studies, my paper further highlights that the trade shocks (trade-linkages) are important when considering international co-movements.

The rest of the paper proceeds as follows. Section 2 details the model of international real business cycles with international trade that incorporates various shocks, and section 3 describes the model calculation. Section 4 describes the accounting procedure and provides which variables are matched with the data. Section 5 describes the data and the main results. Section 6 discusses application to the trade co-movement puzzle and links the results to the existing literature. Section 7 details sensitivity analyses, and section 8 concludes.

2 A Dynamic IRBC Model with Trade in Merchandised Goods

To disentangle the effects of international trade from other country-specific shocks, I build an international real business cycle model that captures international trade in merchandised goods.[10]

There are N number of countries, and for each country, there is a representative utility for all agents. There are two sectors in each country: merchandised goods (G) and services (S) sectors. Merchandised goods are internationally traded while services goods are not tradable. Each country completely specializes in producing both tradable and non-tradable (service) goods.[11] Final good production is an aggregate of merchandised goods and services using Armington technology, and the final good produced is either consumed as current consumption good or invested as future capital stocks. There are multiple shocks in this economy: shocks to productivity in tradable and non-tradable sectors, aggregate demand, leisure, investment, sectoral to merchandised goods, and bilateral trade costs. I assume that the global economy has perfect foresight. With perfect foresight assumption, I interpret the model as a global social planner's problem and solve for planner's allocations. In this section, I discuss the details of the model and the solutions to the social planner's problem.

2.1 Production

Intermediate Production. There are two sectors in each country: merchandised goods (G) and services (S). Intermediate merchandised goods are traded across countries while goods produced in service sector are not traded.

love for variety, alleviate the puzzle more. Drozd et al. (2019) argue that when trade elasticity varies across time, the puzzle is alleviated.

[9] Eaton et al. (2016) studies the role of various macroeconomic shocks to the trade collapse during the Great Recession, and Reyes-Heroles (2017) studies the role of decline in trade costs to the global trade imbalances. Ohanian et al. (2018) studies the changes in capital flows in Latin America and Asia.

[10] Merchandised goods consist of primary (agriculture and mining) and manufacturing goods.

[11] This assumption arises from the IRBC framework.

$$y^j_{n,t} \;=\; z^j_{n,t}\left[\left(L^j_{n,t}\right)^{\alpha_n}\left(K^j_{n,t}\right)^{1-\alpha_n}\right]^{\beta^j_n}\left(\left(M^{jj}_{n,t}\right)^{\gamma^j_n}\left(M^{jj'}_{n,t}\right)^{1-\gamma^j_n}\right)^{1-\beta^j_n} \tag{1}$$

To produce good $y^j_{n,t}$ in sector $j \in \{G,S\}$, country n needs labor $L^j_{n,t}$, capital $K^j_{n,t}$, and a bundle of intermediates $\left(M^{jj}_{n,t}\right)^{\gamma^j_n}\left(M^{jj'}_{n,t}\right)^{1-\gamma^j_n}$. α_n dictates the country-specific share of labor of output in country n. β^j_n is the share of *value-added* in producing sectoral good j in country n. For a bundle of intermediate inputs, γ^j_n denotes the share of how much sector j's goods is used as its own intermediates. Hence, $1-\gamma^j_n$ portion of intermediate bundle in sector j comes from sector j'.

For intermediate input bundle, $M^{jj'}$ is the intermediates from sector j' that are used to make sector j goods. $z^j_{n,t}$ denotes productivity shock to sector j in country n. Labor is defined as $L^j_{n,t} = l^j_{n,t}N_{n,t}$, where $N_{n,t}$ is total population of country n at time t.

Composite Tradable Goods. A portion of merchandised goods are traded across countries to form a composite bundle of intermediate good $TR^G_{n,t}$. This composite traded bundle takes a form using Armington technology:

$$TR^G_{n,t} \;=\; \left[\sum_{i=1}^{N}\left(m^G_{ni,t}\right)^{\sigma}\right]^{\frac{1}{\sigma}} \tag{2}$$

where

$$m^G_{ni,t} \;\equiv\; \frac{1}{d^G_{ni,t}}y^G_{ni,t} \tag{3}$$

$y^G_{ni,t}$ is a quantity of merchandised goods exports that country i ships to country n, and $m^G_{ni,t}$ is a quantity of imports that arrive at n from i. Imported intermediates $m^G_{ni,t}$ are subject to trade frictions, which are specific to country pair (i,n). $d^G_{ni,t}$ is bilateral trade cost shocks for tradable merchandised goods, similar to the standard iceberg costs in Eaton and Kortum (2002), where $d^G_{ni,t} > 1$ if $n \neq i$ and $d_{ii,t} = 1$. Elasticity of substitution between home and foreign intermediate goods, which is characterized by $\frac{1}{1-\sigma}$, is a key parameter that determines how substitutable imported goods are from home goods.

Feasibility Conditions for Goods and Services Sectors. After merchandised goods are traded, merchandised goods bundle $TR^G_{n,t}$ is used to create final goods or used as intermediate inputs. Denoting $F^G_{n,t}$ as a portion of merchandised goods that are used to make final goods, the following equation has to hold for feasibility condition.

$$TR^G_{n,t} \;=\; F^G_{n,t} + M^{GG}_{n,t} + M^{SG}_{n,t} \tag{4}$$

The same logic applies to the service sector. Since service sector is non-tradable, the total service production

$y^S_{n,t}$ is used to to create final good ($F^S_{n,t}$), or are used as intermediate inputs to merchandised goods ($M^{GS}_{n,t}$) or itself (services) ($M^{SS}_{n,t}$).

$$y^S_{n,t} \; = \; F^S_{n,t} + M^{SS}_{n,t} + M^{GS}_{n,t} \tag{5}$$

The final feasibility condition is about the world usage of merchandised goods output. The use of merchandised goods sector in country n at time t has to be equal to what n produces.

$$y^G_{n,t} \; = \; \sum_{i=1}^{N} d^G_{in,t} m^G_{in,t} \tag{6}$$

The above equations are will be used as feasibility constraints in the global social planner's problem that will be discussed in the next section.

Final Goods Production. A portion of traded merchandised goods $F^G_{n,t}$ and a portion of non-traded services goods $F^S_{n,t}$ are aggregated to produce final goods which is used for consumption and investment every period. The final goods production function uses Armington technology to produce $F_{n,t}$:

$$F_{n,t} = \left[\left(\theta_{n,t}\right)^{1-\mu} \left(F^G_{n,t}\right)^{\mu} + \left(y^S_{n,t}\right)^{\mu} \right]^{\frac{1}{\mu}} \tag{7}$$

where $\theta_{n,t}$ governs shocks to relative demand for tradable merchandised goods sector to non-tradable services sector, which I call sectoral shocks. Elasticity of substitution between traded merchandised goods and non-tradable services is characterized by $\frac{1}{\mu-1}$.[12]

2.2 Households

In each period t the representative household in country n consumes some portion of the final goods, denoted as $C_{n,t}$, and chooses the amount of leisure time $(1 - l_{n,t})$. The lifetime utility of the representative agent in country n is

$$U_n = \sum_{t=0}^{\infty} \rho^t \phi_{n,t} \Big(\log \left(C_{n,t}\right) + \xi_{n,t} \log \left[N_{n,t}(1 - l_{n,t})\right] \Big) \tag{8}$$

where ρ is a constant discount factor, and $\phi_{n,t}$ is an aggregate demand shock. I denote $\xi_{n,t}$ as a leisure shock, which acts as a wedge between the marginal rate of substitution between consumption and leisure. $N_{n,t}$ is a total population of country n at time t. This is multiplied with leisure for accounting purposes.[13]

2.3 Capital Dynamics

The aggregate capital $K_{n,t}$ evolves according to the following capital dynamics equation.

[12] Sectoral shocks, $\theta_{n,t}$, have powers of $1 - \mu$ to simplify the algebra.

[13] Population is a shock that is treated as exogenous.

$$K_{n,t+1} = (1-\delta)K_{n,t} + \chi_{n,t} I_{n,t} \tag{9}$$

where δ is the depreciation rate of capital. Aggregate capital is defined as $K_{n,t} = K^M_{n,t} + K^S_{n,t}$. $\chi_{n,t}$ governs shock to efficiency of investment. This type of shock is first introduced by Greenwood et al. (1988), and has been discussed in the previous literature as the key driver of the closed-economy business cycles.

2.4 Solving Social Planner's Problem

I assume that markets are perfectly competitive, and allow to have perfect foresight into the future. This allows me to solve for the market allocations as a global social planner's problem in which the market allocations are efficient. Thus, the resource allocations by the social planner are supported by the competitive equilibrium allocations.

The planner's objective at time 0 is to maximize

$$W = \sum_{n=1}^{N} \omega_n U_n \tag{10}$$

given the initial levels of aggregate capital $K_{n,0}$ for each country n. where ω_n is a country weight that the social planner assigns to each country.[14] By solving this global social planner's problem, I implicitly assume that the financial markets are complete as the allocations are efficiently distributed among countries. All of the production functions, capital dynamics, and household maximizations are taken into account as feasibility constraints for the social planner's problem. The details of how I solve for the planner's allocations are in the appendix of this paper.

2.5 Linking Planner's Allocations to Equilibrium Conditions

In the appendix, I solve the global social planner's problem, which delivers the market allocations for each country. I interpret the shadow prices in the Lagrangian optimization problem as competitive intermediate prices $p^j_{n,t}$ for each sector $\{G, S\}$, composite tradable and non-tradable goods price indices $q^G_{n,t}$ and $q^S_{n,t}$, final good price index $q_{n,t}$, wages $w_{n,t}$, and rental rates $r_{n,t}$ for each country n. All allocations of the social planner's problem are equivalent to the competitive equilibrium allocations as shown in Lucas and Prescott (1971).[15]

(i) Intermediate price of non-tradable services sector of country n at time t is

$$p^S_{n,t} \;=\; \frac{B^S\left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta^S_n}\left[\left(p^S_{n,t}\right)^{\gamma^S_n}\left(q^G_{n,t}\right)^{1-\gamma^S_n}\right]^{1-\beta^S_n}}{z^S_{n,t}} \tag{11}$$

[14] Note that ω_n is time-invariant. This is an important feature in the model, as this assumption will be useful in invoking the business cycle accounting methodology later.

[15] This holds as I do not have any market failure in this model.

where $B^S = \left[\left(\alpha_n\beta_n^S\right)^{-\alpha_n}\left((1-\alpha_n)\beta_n^S\right)^{-(1-\alpha_n)}\right]^{\beta_n^S}\left[\left(\gamma_n^S(1-\beta_n^S)\right)^{-\gamma_n^S}\left((1-\gamma_n^S)(1-\beta_n^S)\right)^{-(1-\gamma_n^S)}\right]^{1-\beta_n^S}$.

Cost of a bundle of inputs for services sector is $B^S\left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta_n^S}\left[\left(p_{n,t}^S\right)^{\gamma_n^S}\left(q_{n,t}^G\right)^{1-\gamma_n^S}\right]^{1-\beta_n^S}$, which combines labor, capital, and the price bundles for the intermediate input usage. Treating $z_{n,t}^S$ as a TFP shock in services sector in country n at time t, the intermediate price index $p_{n,t}^S$ is equal to its marginal cost.

(ii) Intermediate price of tradable, merchandised goods sector of country n at time t is

$$p_{n,t}^G = \frac{B^G\left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta_n^G}\left[\left(q_{n,t}^G\right)^{\gamma_n^G}\left(p_{n,t}^S\right)^{1-\gamma_n^G}\right]^{1-\beta_n^G}}{z_{n,t}^G} \tag{12}$$

where $B^G = \left[\left(\alpha_n\beta_n^G\right)^{-\alpha_n}\left((1-\alpha_n)\beta_n^G\right)^{-(1-\alpha_n)}\right]^{\beta_n^G}\left[\left(\gamma_n^G(1-\beta_n^G)\right)^{-\gamma_n^G}\left((1-\gamma_n^G)(1-\beta_n^G)\right)^{-(1-\gamma_n^G)}\right]^{1-\beta_n^G}$.

Cost of a bundle of inputs for goods sector is $B^G\left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta_n^G}\left[\left(q_{n,t}^G\right)^{\gamma_n^G}\left(p_{n,t}^S\right)^{1-\gamma_n^G}\right]^{1-\beta_n^G}$, which combines wages, rental rates, and prices for the intermediate input usages. $z_{n,t}^G$ is a TFP shock in merchandised goods sector, so $p_{n,t}^G$ is equal to its marginal cost. Since labor and capital are freely mobile inside each country, wages and rental rates are equalized between the two sectors.

(iii) Composite tradable goods price index $q_{n,t}^G$ is given by

$$q_{n,t}^G = \left[\sum_{i=1}^{N}\left(p_{i,t}^G d_{ni,t}^G\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} \tag{13}$$

where $p_{i,t}^G$ is the intermediate price for merchandised goods defined in (ii). Merchandised goods price index of country n accounts for the production costs in each country subject to bilateral trade shocks that are country-pairwise. When there are shocks in foreign country, it propagates through the foreign price index $p_{i,t}^G$. Further, as trade frictions $d_{ni,t}^G$ increase, ceteris paribus, the intermediate input price index increases. I interpret the fluctuations in $d_{ni,t}^G$ as bilateral trade cost shocks between country-pair n and i.

(iv) Final goods price index $q_{n,t}$ for country n is given by

$$q_{n,t} = \left(\theta_{n,t}\left(q_{n,t}^G\right)^{\frac{\mu}{\mu-1}} + \left(p_{n,t}^S\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{\mu-1}{\mu}} \tag{14}$$

where $\theta_{n,t}$ represents sectoral shocks to traded merchandised goods. If θ increases, the share of merchandised goods sector increase relative to the non-tradable services sector.

(v) The share of country n's expenditure on merchandised goods sector G, imported from country i is

$$\pi^G_{ni,t} = \left(\frac{p^G_{i,t} d^G_{ni,t}}{q^G_{n,t}}\right)^{\frac{\sigma}{\sigma-1}} \tag{15}$$

$\pi^G_{ni,t}$ is defined as country n's imported trade shares coming from country i, where $\sum_{i=1}^N \pi^G_{ni,t} = 1$. The trade shares $\pi^G_{ni,t}$ depends on the intermediate price index $p^G_{n,t}$. The shock spillover of foreign shock, regardless of what it is, happens through $p^G_{i,t}$.[16] On the other hand, if there are shocks that affect the particular country-pair, changes would appear in bilateral trade shocks $d^G_{ni,t}$.

(vi) Marginal rate of substitution between consumption and leisure is given by

$$w_{n,t}(1 - l_{n,t})N_{n,t} = \xi_{n,t} q_{n,t} C_{n,t} \tag{16}$$

The leisure shock $\xi_{n,t}$ appears as part of the marginal rate of substitution between consumption and leisure. $\xi_{n,t}$ not only affect the allocations of labor hours and wages today but also across time.[17]

(vii) Household spending on consumption goods is

$$q_{n,t}C_{n,t} = \omega_n \phi_{n,t} \tag{17}$$

where it depends on the country weight ω_n, which is time-invariant, and aggregate demand shock $\phi_{n,t}$. $\phi_{n,t}$ captures changes to preferences of consumers in country n, or it could be interpreted as the changes in saving behaviors of consumers in country n. If the aggregate demand shock $\phi_{n,t}$ increases, the consumption expenditure of the households increases.

(viii) The Euler equation for each country n is

$$\frac{q_{n,t}}{\chi_{n,t}} = \rho\left[\frac{q_{n,t+1}}{\chi_{n,t+1}}(1 - \delta) + r_{n,t+1}\right] \tag{18}$$

where $q_{n,t}$ is final good price index, and $r_{n,t+1}$ is rental rate of capital at time $t+1$. The left-hand side of the Euler equation represents the value of the final good that should be given up to make future investments, and the right-hand side is the benefit of using that investment in the next period. $\chi_{n,t}$ is a shock to investment efficiency. If the investment is completely efficient (a standard IRBC model),

[16] This works as a terms-of-trade shock between countries n and i.
[17] This is possible due to the perfect foresight assumption.

then $\chi_{n,t}$ would disappear from the equation, and there would not be any distortion to the final good price index. There are no expectations on the right-hand side as the economy is assumed to be perfect foresight.

(ix) I define $Y_{n,t}^G = p_{n,t}^G y_{n,t}^G$ to be the total gross production of country n in merchandised goods, and $X_{n,t}^G = p_{n,t}^G x_{n,t}^G$ to be total value of expenditure (absorption) on merchandised goods of country n. In this way, I can write the world merchandised goods market clearing as the flow of traded intermediate merchandised goods in terms of their values.

$$Y_{n,t}^G \;=\; \sum_{i \in N} X_{in,t}^G \;=\; \sum_{i \in N} \pi_{in,t}^G X_{i,t}^G \tag{19}$$

where $X_{in,t}^G$ is how much country i spends on its traded merchandised goods that come from country n. Summing the expenditures on traded merchandised goods that come from country n, it must be equal to the total value of gross production merchandised goods of country n.

(x) Define $X_{n,t}^{F,G} = q_{n,t}^G F_{n,t}^G$ to be the expenditure on merchandised goods in country n that is used to produce final goods. The total expenditure on merchandised goods sector is the sum of country n's the spending on merchandised goods in producing final goods plus the spending on purchasing the intermediate input bundle in merchandised goods production.

$$X_{n,t}^G \;=\; X_{n,t}^{F,G} + \left(1-\beta_n^G\right)\left(\gamma_n^G\right)Y_{n,t}^G + \left(1-\beta_n^S\right)\left(1-\gamma_n^S\right)Y_{n,t}^S \tag{20}$$

(xi) Define $X_{n,t}^{F,S} = p_{n,t}^S F_{n,t}^S$ to be the expenditure on services in country n that is used to produce final goods. The total expenditure on services sector is the sum of country n's the spending on services in producing final goods plus the spending on purchasing the intermediate input bundle in services production.

$$X_{n,t}^S \;=\; X_{n,t}^{F,S} + \left(1-\beta_n^G\right)\left(1-\gamma_n^G\right)Y_{n,t}^G + \left(1-\beta_n^S\right)\left(\gamma_n^S\right)Y_{n,t}^S \tag{21}$$

(xii) Define $X_{n,t}^F = q_{n,t} F_{n,t}$ to be the total spending (absorption) on final goods which is equal to $X_{n,t}^F = X_{n,t}^{F,M} + X_{n,t}^{F,S}$. The total spending on producing final goods is either used as purchasing final consumption goods or as investing in future capital stocks:

$$X_{n,t}^F \;=\; X_{n,t}^C + X_{n,t}^I \tag{22}$$

where $X_{n,t}^C = q_{n,t} C_{n,t}$ and $X_{n,t}^I = q_{n,t} I_{n,t}$.

(xiii) Define $Y_{n,t}$ to be the *GDP* of country n at time t. I use *production approach* to calculate GDP in this model. The gross value of domestic output is equal to the sum of merchandised goods and services output $Y^G_{n,t} + Y^S_{n,t}$. To calculate GDP, we only need to take into account the *value-added* portion of domestic output in merchandised goods. Hence, I define GDP as

$$Y_{n,t} \;=\; \beta^G_n Y^G_{n,t} + \beta^S_n Y^S_{n,t} \tag{23}$$

where β^j_n is the value-added share of the gross production in sector j.

(xiv) *Labor market clearing.* The labor income is equal to labor demand every period.

$$w_{n,t} l_{n,t} N_{n,t} \;=\; \alpha_n \beta^G_n Y^G_{n,t} + \alpha_n \beta^S_n Y^S_{n,t} \;=\; \alpha_n Y_{n,t} \tag{24}$$

where $Y_{n,t} = \beta^G_n Y^G_{n,t} + \beta^S_n Y^S_{n,t}$.

(xv) *Capital market clearing.* The capital income is equal to capital demand every period.

$$r_{n,t} K_{n,t} \;=\; \big(1-\alpha_n\big)\beta^G_n Y^G_{n,t} + \big(1-\alpha_n\big)\beta^S_n Y^S_{n,t} \;=\; \big(1-\alpha_n\big) Y_{n,t} \tag{25}$$

where $Y_{n,t} = \beta^G_n Y^G_{n,t} + \beta^S_n Y^S_{n,t}$.

The equations from (i) – (xv) reflect all of the equilibrium relationships and market clearing conditions from the global social planner's problem. To perform the accounting procedure, I allow for exogenous debt process for services sector, $D^S_{n,t}$, where $D^S_{n,t} = Y^S_{n,t} - X^S_{n,t}$ and $\sum_{i=1}^N \left(D^M_{n,t} + D^S_{n,t}\right) = 0$.

2.6 Steady State of the Equilibrium with Perfect Foresight

I define the steady state of the perfect foresight economy of the social planner's problem. First, I collect the shocks that I have defined in the model, which I define as Ψ_t:

$$\Psi_t = \Big\{ z^G_{n,t}, z^S_{n,t}, \phi_{n,t}, \xi_{n,t}, \chi_{n,t}, \theta_{n,t}, d^G_{ni,t}, D^S_{n,t}, N_{n,t} \Big\} \tag{26}$$

These shocks are defined to be time-varying in the model. To find the steady state, I assume that the shocks are constant and the capital stocks, $K_{n,t}$, have settled down to constant levels of K_n. Hence the steady state of the model has to satisfy the capital and investment spending ratio

$$\frac{r_n K_n}{X^I_n} = \frac{1-\rho+\rho\delta}{\rho\delta} \tag{27}$$

and the capital accumulation equation becomes

$$\frac{I_n}{K_n} = \frac{\delta}{\chi_n} \tag{28}$$

and all the outcomes from the equilibrium relationships are satisfied.

3 Model Calculation

In this section, I show how I calculate the model, given the initial parameters and exogenous shocks. I follow the hat-algebra method by Dekle et al. (2007) to solve the model exactly.[18] By solving the model exactly, it is possible to express all the shocks and the endogenous outcomes of the model in changes.

3.1 Initial Parameters and Exogenous Shocks

There are initial parameters that are time-invariant, which I denote them as Ω.

$$\Omega \; = \; \left\{ \alpha_n, \; \beta_n^j, \; \gamma_n^j, \; \sigma, \; \rho, \; \mu, \; \beta \right\} \tag{29}$$

I denote time-varying shocks to be $\widehat{\Psi}_{t+1}$.

$$\widehat{\Psi}_{t+1} \; = \; \left\{ \widehat{z}^G_{n,t+1}, \; \widehat{z}^S_{n,t+1}, \; \widehat{\phi}_{n,t+1}, \; \widehat{\xi}_{n,t+1}, \; \widehat{\chi}_{n,t+1}, \; \widehat{\theta}_{n,t+1}, \; \widehat{d}^G_{ni,t+1}, \; \widehat{N}_{n,t+1}, \; D^S_{n,t} \right\} \tag{30}$$

Using the set of time-invariant initial parameters Ω and the set of time-varying structural shocks $\widehat{\Psi}_{t+1}$, I show the algorithm to calculate the endogenous outcomes of the equilibrium.

3.2 Calculating Endogenous Outcomes of the Model

Equipped with the initial parameters and structural shocks, the paths of endogenous outcomes of the model can be calculated. Let's say that the initial period starts at time t, and we want to calculate the endogenous outcomes of the model in $t+1$. Given the initial values of GDP ($Y_{n,t}$), gross production in merchandised goods ($Y^G_{n,t}$), labor hours ($l_{n,t}$), trade shares of merchandised goods ($\pi^G_{ni,t}$), expenditure on consumption goods ($X^C_{n,t}$), and changes to capital ($\widehat{K}_{n,t+1}$), and the paths of shocks $\widehat{\Psi}_{t+1}$, I recover the next period's endogenous outcomes. In other words, using $\widehat{\Psi}_{t+1}$, I can calculate the next period's GDP ($Y_{n,t+1}$), gross production of merchandised goods ($Y^G_{n,t+1}$), labor hours ($l_{n,t+1}$), bilateral trade shares ($\pi^G_{n,t+1}$), and consumption spending ($X^C_{n,t+1}$), and changes to capital ($\widehat{K}_{n,t+2}$).

Suppose that the path of capital $\widehat{K}_{n,t+1}$ and the entire paths of shocks Ψ_{t+1} are given under perfect foresight equilibrium. Given the initial values of $Y_{n,t}$, $Y^G_{n,t}$, $l_{n,t}$, $X^C_{n,t}$, and $\pi^G_{ni,t}$, I obtain the next period's outcomes using the following conditions:

(i) Using TFP shocks to services sector $\widehat{z}^S_{n,t+1}$, changes to intermediate price index for services sector can be obtained using

[18] Solving dynamic macro or trade models recently have adopted this hat-algebra approach. Eaton et al. (2016) follow the same approach, but with a Ricardian framework.

$$\widehat{p}^S_{n,t+1} = \frac{\left[\left(\widehat{w}_{n,t+1}\right)^{\alpha_n}\left(\widehat{r}_{n,t+1}\right)^{1-\alpha_n}\right]^{\beta^S_n}\left[\left(\widehat{p}^S_{n,t+1}\right)^{\gamma^S_n}\left(\widehat{q}^G_{n,t+1}\right)^{1-\gamma^S_n}\right]^{1-\beta^S_n}}{\widehat{z}^S_{n,t+1}} \tag{31}$$

(ii) Using productivity shocks to tradable sector $\widehat{z}^G_{n,t+1}$, changes to price for merchandised goods is

$$\widehat{p}^G_{n,t+1} = \frac{\left[\left(\widehat{w}_{n,t+1}\right)^{\alpha_n}\left(\widehat{r}_{n,t+1}\right)^{1-\alpha_n}\right]^{\beta^G_n}\left[\left(\widehat{q}^G_{n,t+1}\right)^{\gamma^G_n}\left(\widehat{p}^S_{n,t+1}\right)^{1-\gamma^G_n}\right]^{1-\beta^G_n}}{\widehat{z}^G_{n,t+1}} \tag{32}$$

(iii) Changes to the composite tradable goods price index is

$$\widehat{q}^G_{n,t+1} = \left[\sum_{i=1}^{N}\pi^G_{ni,t+1}\left(\widehat{p}^G_{i,t}\widehat{d}^G_{ni,t+1}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} \tag{33}$$

(iv) Using shocks to tradable goods sector $\widehat{\theta}_{n,t+1}$, changes to final goods price index is

$$\widehat{q}_{n,t+1} = \left[\frac{X^{F,G}_{n,t}}{X^F_{n,t}}\widehat{\theta}_{n,t+1}\left(\widehat{q}^G_{n,t+1}\right)^{\frac{\mu}{\mu-1}} + \frac{X^{F,S}_{n,t}}{X^F_{n,t}}\left(\widehat{p}^S_{n,t+1}\right)^{\frac{\mu}{\mu-1}}\right]^{\frac{\mu-1}{\mu}} \tag{34}$$

where $X^{F,G}_{n,t}$ and $X^{F,S}_{n,t}$ are the total expenditure of merchandised goods and services in producing final goods. The two expenditures sum up to $X^F_{n,t}$.

(v) Using bilateral trade frictions $\widehat{d}^G_{ni,t+1}$, changes to shares of tradable merchandised goods in country n from country i is

$$\widehat{\pi}^G_{ni,t+1} = \left(\frac{\widehat{p}^G_{i,t+1}\widehat{d}^G_{ni,t+1}}{\widehat{q}^G_{n,t+1}}\right)^{\frac{\sigma}{\sigma-1}} \tag{35}$$

(vi) Using aggregate demand shocks $\widehat{\phi}_{n,t+1}$, changes to expenditure in consumption spending is

$$\widehat{X}^C_{n,t+1} = \widehat{q}_{n,t+1}\widehat{C}_{n,t+1} = \widehat{\phi}_{n,t+1} \tag{36}$$

(vii) Using leisure shocks $\widehat{\xi}_{n,t+1}$, changes to leisure is

$$1 - \widehat{l_{n,t+1}} = \frac{\widehat{\phi}_{n,t+1}\widehat{\xi}_{n,t+1}}{\widehat{w}_{n,t+1}\widehat{N}_{n,t+1}} \tag{37}$$

(viii) Using investment shocks $\widehat{\chi}_{n,t+1}$, changes to investment spending, $\widehat{X}^I_{n,t+1}$, satisfies the Euler equation

$$1 = \rho\frac{\widehat{q}_{n,t+1}}{\widehat{\chi}_{n,t+1}}(1-\delta) + \rho\frac{r_{n,t+1}K_{n,t+1}}{X^I_{n,t}}(1-(1-\delta)\widehat{K}^{-1}_{n,t+1}) \tag{38}$$

where the expression $r_{n,t+1}K_{n,t+1}$ comes from the market clearing condition for capital.

(ix) Using the capital market clearing condition, changes to rental rates and capital satisfy

$$\widehat{r}_{n,t+1}\widehat{K}_{n,t+1}r_{n,t}K_{n,t} = (1-\alpha_n)Y_{n,t+1} \tag{39}$$

(x) Using the labor market clearing condition, changes to wages and labor satisfy

$$\widehat{w}_{n,t+1}\widehat{l}_{n,t+1}\widehat{N}_{n,t+1}w_{n,t}l_{n,t}N_{n,t} = (\alpha_n)Y_{n,t+1} \tag{40}$$

(xi) And the market clearing conditions hold from the previous section.

Given the initial values of GDP ($Y_{n,t}$), gross production in merchandised goods ($Y^G_{n,t}$), labor hours ($l_{n,t}$), trade shares of merchandised goods ($\pi^G_{ni,t}$), expenditure on consumption goods ($X^C_{n,t}$), and changes to capital ($\widehat{K}_{n,t+1}$), and equipped with equations (i) – (xi), I can calculate the next period's GDP ($Y_{n,t+1}$), gross production in merchandised goods and services ($Y^j_{n,t+1}$ for $j\{G,S\}$), labor hours ($l_{n,t+1}$), consumption spending ($X^C_{n,t+1}$), imported trade shares $\pi_{ni,t+1}$), and subsequent capital stocks in changes ($\widehat{K}_{n,t+2}$). I repeat this process for the next period.

4 Accounting Procedure

The allocations from the social planner's problem, with the market clearing conditions, completely describe the market allocations of the global economy. Using the methodology developed by Parkin (1988) and Eaton et al. (2016), I use the endogenous outcomes of the model to match the data perfectly to back out the set of structural shocks that are embedded in the economy. I discuss how I acquire the set of structural shocks using the data.

4.1 Initial Parameters of the Model

To perform the business cycle accounting procedure, I first calibrate the initial parameters of the model. The discount factor for the utility function ρ is 0.96, which is taken directly from Backus et al. (1992)

Parameters	Description	Value	Source
α_n	Labor share in production	-	Data (PWT 9.1)
β_n^j, $j \in \{G, S\}$	value-added to gross output ratio	-	Data (OECD STAN)
γ_n^j, $j \in \{G, S\}$	Input-output shares	-	Data (WIOT)
δ	Depreciation rate	0.05	Standard annual rate
ρ	Discount factor	0.96	Kose & Yi (2006)
σ	Elasticity of substitution between foreign intermediates	0.333	Kose & Yi (2006)
η	Goods vs. services elasticity	0.5	Johnson (2014)

Table 1: Time-Invariant Parameters of the Model

and Kose and Yi (2006). The labor shares for production functions (Cobb-Douglas labor share) are taken from Penn World Table 9.1. I choose $\delta = 0.05$ to correspond to the annual depreciation rate in the data. The value-added shares for each sector in each country, β_n^j, is calibrated using OECD STAN database by calculating the ratio between value-added to gross output in each sector. The input-output shares of each sector, γ_n^j, is also calibrated using the world input-output tables. Although the labor shares (α_n), value-added shares (β_n^j), and input-output shares (γ_n^j) change every year in the data, I fix them in the model and hold them to be time-invariant, and take the average of the values between 1992 and 2014.

There are two elasticities of substitution in the model. The first elasticity σ, which determines the elasticity of substitution between domestic and foreign merchandised goods, is set to match the short-run elasticity of substitution between goods in standard international macroeconomic literature (elasticity of 1.5). For elasticity of substitution between final tradable merchandised goods and final non-tradable services, I use $\eta = 0.5$, where I take the number from Johnson (2014).

4.2 Paths of Capital

Equipped with the initial parameters, I now discuss how I determine the paths of capital before backing out the shocks in the model. This procedure is critical in the accounting procedure, as I require shocks to satisfy the perfect foresight equilibrium path of the global economy. Here, I make a critical assumption; GDP, production of merchandised goods, consumption expenditure, labor hours, and bilateral trade shares from each country lie on the dynamic perfect foresight equilibrium path of the specified model. Essentially, the assumption is that the global economy behaves as if it follows the model described in this paper.

With this assumption, I want the endogenous variables such as GDP, production in merchandised goods, consumption expenditure, labor hours, and bilateral trade shares to follow the dynamics of the model. First, I assume that the shocks Ψ_{t+1} remain unchanged after 2014, as we do not have any information on the global economy beyond the period of the data. In other words, the constant shocks can be described as

$$\{\widehat{\Psi}_{t+1}\}_{2015}^{\infty} = 1 \qquad \text{and} \qquad \{D_{n,t}^S\}_{2015}^{\infty} = D_{n,2014}^S \tag{41}$$

Then the world with unchanging shocks converges to the steady state in which all the endogenous variables are time-invariant. Given the data in 2014, I find the changes to capital, $\widehat{K}_{n,2015}$ that guarantees all the endogenous outcomes converge to a steady state. Note that the Euler equation from the social planner's allocation is

$$\frac{q_{n,t}}{\chi_{n,t}} = \rho\left[\frac{q_{n,t+1}}{\chi_{n,t+1}}(1-\delta) + r_{n,t+1}\right] \tag{42}$$

By multiplying $\frac{\chi_{n,t}}{q_{n,t}}$ on both sides and using the definition that $X^I_{n,t} \equiv q_{n,t}I_{n,t}$, I can rewrite the Euler equation as

$$1 = \rho\left(\frac{\widehat{q}_{n,t+1}}{\widehat{\chi}_{n,t+1}}\right)(1-\delta) + \rho\left(\frac{r_{n,t+1}K_{n,t+1}}{X^I_{n,t}}\right)\left(1-(1-\delta)\widehat{K}^{-1}_{n,t+1}\right) \tag{43}$$

The Euler equation (43) still requires knowledge of $\widehat{\chi}_{n,t+1}$, which is unknown at this point. To circumvent this problem, I incorporate the capital accumulation equation and combine it with the previous equation to obtain:

$$\frac{\widehat{K}_{n,t}}{\widehat{K}_{n,t} - (1-\delta)} = \rho\left(\frac{1-\delta}{\widehat{K}_{n,t+1} - (1-\delta)}\right)\widehat{X}^I_{n,t} + \rho\left(\frac{r_{n,t}K_{n,t}}{X^I_{n,t-1}}\right) \tag{44}$$

Now, the Euler equation (44) does not require any knowledge of shocks as I eliminated $\widehat{\chi}_{n,t+1}$. What is required to back out the path of capital, is the data on investment spending for each country, $X^I_{n,t}$, and $r_{n,t}K_{n,t}$. From the capital market clearing condition, we know that

$$r_{n,t}K_{n,t} = (1-\alpha)Y_{n,t} \tag{45}$$

Therefore, knowing GDP and investment spending is enough to deliver the path of capital that satisfies the perfect foresight equilibrium of this model.

4.3 Paths of Shocks

By expressing the shocks using the hat algebra, I can express endogenous variables and shocks in their changes. In this subsection, I discuss how I recover each set of shocks.

4.3.1 TFP Shocks to Goods and Services

TFP shocks to merchandised goods and services sectors can be directly taken from combining the marginal costs of intermediate producers and market clearing conditions.

$$\widehat{z}^{G}_{n,t+1} \;=\; \frac{\left[\left(\widehat{w}_{n,t+1}\right)^{\alpha_n}\left(\widehat{r}_{n,t+1}\right)^{1-\alpha_n}\right]^{\beta^G_n}\left[\left(\widehat{q}^{G}_{n,t+1}\right)^{\gamma^G_n}\left(\widehat{p}^{S}_{n,t+1}\right)^{1-\gamma^G_n}\right]^{1-\beta^G_n}}{\widehat{p}^{G}_{n,t+1}} \tag{46}$$

$$\widehat{z}^{S}_{n,t+1} \;=\; \frac{\left[\left(\widehat{w}_{n,t+1}\right)^{\alpha_n}\left(\widehat{r}_{n,t+1}\right)^{1-\alpha_n}\right]^{\beta^S_n}\left[\left(\widehat{p}^{S}_{n,t+1}\right)^{\gamma^S_n}\left(\widehat{q}^{G}_{n,t+1}\right)^{1-\gamma^S_n}\right]^{1-\beta^S_n}}{\widehat{p}^{S}_{n,t+1}} \tag{47}$$

where $\widehat{p}^{G}_{n,t+1}$, is obtained by directly matching changes to producer price indices for merchandised goods (PPI) for each country n, and $\widehat{w}_{n,t+1}$ and $\widehat{r}_{n,t+1}$ are determined from the market clearing conditions, using GDP, labor hours, and population data.[19]

4.3.2 Aggregate Demand Shocks

Given that the planner's weights are time-invariant, the aggregate demand shock captures the fluctuations in consumption spending of a country.

$$\widehat{\phi}_{n,t+1} \;=\; \widehat{q}_{n,t+1}\widehat{C}_{n,t+1} \equiv \widehat{X}^{C}_{n,t+1} \tag{48}$$

where $\widehat{X}^{C}_{n,t+1}$ is defined as nominal consumption spending in changes. The world goods market-clearing conditions determine the total expenditure. I match $\widehat{C}_{n,t+1}$ with the real consumption expenditure of the households and government from Penn World Table 9.1, so $\widehat{\phi}_{n,t+1}$ captures the volatility in which nominal consumption changes across time.

4.3.3 Leisure Shocks

Leisure shock comes from marginal rate of substitution between consumption and leisure.[20]

$$\widehat{\xi}_{n,t+1} \;=\; \frac{\widehat{w}_{n,t+1}(\widehat{1-l_{n,t+1}})\widehat{N}_{n,t+1}}{\widehat{\phi}_{n,t+1}} \tag{49}$$

To back out leisure shocks, it is crucial to match the average labor hours $l_{n,t}$ to calculate the changes to leisure, $\widehat{1-l_{n,t+1}}$. $\widehat{N}_{n,t+1}$, population in changes, is exogenous and directly taken from the data. The changes to wages $\widehat{w}_{n,t+1}$, is an unmatched moment and it is solved in the model.

[19]I use changes to PPI as a proxy to match the changes to intermediate prices.

[20]In standard IRBC models, the marginal rate of substitution between consumption and leisure is equal to 1. $\widehat{\xi}_{n,t+1}$ is the distortion that arises from the possible misspecification arising from standard macroeconomic models.

4.3.4 Investment Shocks

Investment shocks is backed out from the capital accumulation equation (9). By reformulating equation (9),

$$\chi_{n,t+1} = \frac{K_{n,t+2} - (1-\delta)K_{n,t+1}}{I_{n,t+1}} \tag{50}$$

Taking the ratio of $\chi_{n,t+1}$ and $\chi_{n,t}$, the following expression for changes to investment shocks can be obtained:

$$\widehat{\chi}_{n,t+1} = \left(\frac{K_{n,t+2} - (1-\delta)K_{n,t+1}}{K_{n,t+1} - (1-\delta)K_{n,t}} \right) \frac{1}{\widehat{I}_{n,t+1}} \tag{51}$$

dividing by $\frac{K_{n,t+1}}{K_{n,t+1}}$ and using the definition $X^I_{n,t} \equiv q_{n,t} I_{n,t}$, the right-hand side can be re-written as

$$\widehat{\chi}_{n,t+1} = \left(\frac{\widehat{K}_{n,t+2} - (1-\delta)}{1 - (1-\delta)\widehat{K}^{-1}_{n,t+1}} \right) \frac{\widehat{q}_{n,t+1}}{\widehat{X}^I_{n,t+1}} = \left(\frac{\widehat{K}_{n,t+2} - (1-\delta)}{\widehat{K}_{n,t+1} - (1-\delta)} \right) \left(\frac{\widehat{q}_{n,t+1}\widehat{K}_{n,t+1}}{\widehat{X}^I_{n,t+1}} \right) \tag{52}$$

From the previous section, I already know the path of capital $\widehat{K}_{n,t+1}$. Using the investment spending from the market clearing condition, I back out $\widehat{\chi}_{n,t+1}$. Note that investment shocks involve $\widehat{K}_{n,t+2}$ term; investment shocks directly affect intertemporal choices of the social planner.

4.3.5 Sectoral Shocks to Merchandised Goods Expenditure

Changes to sectoral shocks $\widehat{\theta}_{n,t+1}$ is backed out using the following relationship:

$$\widehat{\theta}_{n,t+1} = \frac{\widehat{X}^G_{n,t+1}}{\widehat{X}_{n,t+1}} \left(\frac{\widehat{q}^G_{n,t+1}}{\widehat{q}_{n,t+1}} \right)^{\frac{\mu}{1-\mu}} \tag{53}$$

where $\widehat{X}^G_{n,t+1}$ and $\widehat{X}_{n,t+1}$ are changes to final absorption (expenditure) in merchandised goods and in total. $\widehat{q}^G_{n,t+1}$ and $\widehat{q}_{n,t+1}$ are changes to the price indices for tradable goods and final goods in total. I match Consumer Price Index (CPI) with $q_{n,t}$, so the changes to the final good price index is equivalent to the changes in CPI for each country n. This shock affects the share of tradable merchandised goods.

4.3.6 Country-pair Trade Shocks

To back out country-pair trade shocks, I start from changes to imported trade shares, $\widehat{\pi}^G_{ni,t+1}$, when $n \neq i$ and $n = i$:

$$\widehat{\pi}^G_{ni,t+1} = \left(\frac{\widehat{cost}^G_{i,t+1}\widehat{d}^G_{ni,t+1}}{\widehat{z}^G_{i,t+1}\widehat{q}^G_{n,t+1}}\right)^{\frac{\sigma}{\sigma-1}} \qquad \widehat{\pi}^G_{nn,t+1} = \left(\frac{\widehat{cost}^G_{n,t+1}\widehat{d}^G_{nn,t+1}}{\widehat{z}^G_{n,t+1}\widehat{q}^G_{n,t+1}}\right)^{\frac{\sigma}{\sigma-1}} \tag{54}$$

Taking the ratio of these the above equations, and using the fact that $\widehat{p}^G_{n,t+1} = \frac{\widehat{cost}^G_{n,t+1}}{\widehat{z}^G_{n,t+1}}$, I find that

$$\left(\frac{\widehat{\pi}^G_{ni,t+1}}{\widehat{\pi}^G_{nn,t+1}}\right)^{\frac{\sigma-1}{\sigma}} = \left(\frac{\widehat{p}^G_{i,t+1}}{\widehat{p}^G_{n,t+1}}\right)\left(\frac{\widehat{d}^G_{ni,t+1}}{\widehat{d}^G_{nn,t+1}}\right) \tag{55}$$

Furthermore, using the assumption that there is no cost in consuming your own goods ($d^G_{ni,t} = 1$ when $n = i$), I back out the bilateral trade frictions from the following equation:

$$\widehat{d}^G_{ni,t+1} = \left(\frac{\widehat{\pi}^G_{ni,t+1}}{\widehat{\pi}^G_{nn,t+1}}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\widehat{p}^G_{n,t+1}}{\widehat{p}^G_{i,t+1}}\right) \tag{56}$$

Using the data on bilateral trade volumes and producer price index (PPI), $\widehat{d}^G_{ni,t+1}$ can be directly backed out from matching the data.

4.3.7 Population and Trade Deficit Shocks

Population growth $\widehat{N}_{n,t+1}$ and trade deficits $D^S_{n,t}$ are directly taken from the data.

4.4 Mechanism of How Each Shocks Affect GDP Co-movement

In this section, I discuss how each set of shocks can induce positive cross-country GDP co-movement. For the sake of simplicity, I consider a simple scenario in which there are only two countries (home and foreign), and the other shocks remain as constant as I consider each set of shock one by one.

4.4.1 TFP Shocks to Merchandised Goods

To understand how productivity shocks work, consider that there are only two countries, home and foreign. Let's consider a simple scenario in which productivity for the merchandised goods sector increases for home country, and the other shocks are unchanged.[21] Then, the gross output for merchandised goods for home increases as productivity increases. This induces the prices to merchandised goods, $p^G_{n,t}$, to fall and the terms of trade depreciate, which makes the home country's merchandised good more attractive. If the trade costs are unchanged with given Armington elasticity, foreign countries would demand more of home merchandised goods and increase the imports from home country. Due to complementarity between imported home and foreign goods with given elasticities σ and μ, a foreign country increases its factor supplies and

[21] An increase in $z^G_{n,t}$.

intermediate inputs to produce more of its own merchandised goods. This, in turn, increases the GDP of the foreign country, hence inducing the positive co-movement of both countries' GDP.

4.4.2 Aggregate Demand Shocks

Suppose there is a positive aggregate demand shock at home country. Home country then increases its demand for both merchandised goods and services. This would induce higher output in gross production in merchandised goods and services, which requires more intermediate input bundle. This causes an increase in the production of merchandised goods for foreign countries, which creates positive growth in outputs.

4.4.3 Leisure Shocks

Understanding leisure shock is more involving than the other shocks. Let's assume that there is a positive shock to leisure in home country. This can be interpreted as people are "enjoying" more leisure or taxes to income, given that the wages and consumption levels are constant. First, if people enjoy leisure more, then the real wages $w_{n,t}$ would increase, but labor would fall.[22] Since people work less, the factor supply decreases overall, and output decreases at home. Less output at home is linked to how much home can export to foreign countries, and the amount of exports decrease. Due to complementarity between home and foreign tradable sector, the overall expenditure on tradable sector decreases as the foreign output decreases, which leads to the co-movement of home and foreign GDP.

4.4.4 Investment Shocks

Let's assume that there is an increase in investment shock at home country. As this shock determines the efficiency to investment, the return on the investment tomorrow, $I_{n,t+1}$, increases. On the other hand, the benefit of another unit of capital in $t+1$ decreases ($K_{n,t+1}$ falls), which leads to the benefit of the capital $K_{n,t+2}$ increasing. Since the efficiency to investment tomorrow increases, the factor supply of capital tomorrow, $K_{n,t+1}$ falls and the cost of unit of capital or the final good price index, $q_{n,t+1}$, increases. This leads to a decrease in the production of intermediate and final goods sectors. Decrease in intermediate sector decreases the output for home country, which decreases exports to foreign countries. Furthermore, decrease in final goods sector at home decrease the amount of imported goods from foreign countries. The total effect leads to a decrease in foreign output, thus leading to a positive output co-movement across countries.

4.4.5 Sectoral Shocks

To understand how the sectoral shocks work, suppose there is a positive shock to tradable sector in country n, as $\theta_{n,t+1}$ increases. This gives higher share of tradable merchandised goods compared to non-tradable service sector. This induces more tradable goods to be made in country n, and it demands more of home and foreign intermediate merchandised goods. This creates positive output co-movement between home and foreign countries.

[22] This can be interpreted as people working less. Since there is no unemployment in the model, it can appear as working less and "enjoying" more leisure.

4.4.6 Country-pair Trade Shocks

To understand how country-pair trade shocks affect GDP co-movement, I discuss a simple scenario where there are only two symmetric countries, home (H) and foreign (F). I assume that all other shocks are constant, and only allow $d_{HF,t}$ and $d_{FH,t}$ to decrease (It is easier to trade between H and F). This increases the amount of imported merchandised goods from foreign country. Depending on the value of the elasticity of substitution between home and foreign merchandised goods, this increases the total expenditure on merchandised goods ($X^G_{H,t}$ and $X^G_{F,t}$). Since the demand for the merchandised goods have increased globally, each country increases its merchandised goods productions ($Y^G_{H,t}$ and $Y^G_{F,t}$). Therefore, both home and foreign GDP ($Y_{H,t}$ and $Y_{F,t}$) increase.[23] How much impact country-pair trade shocks have ultimately depend on the elasticity of substitution, σ. If $\sigma < 1$, both countries increase their output, which leads to the positive GDP co-movement.[24]

4.5 Replicating the Data

The set of shocks that I back out using the data, by definition, captures GDP and other macroeconomic aggregate variables for each country. I list the set of shocks and the data that I use to match with in Table (2), where I use 7 sets of data to back out 7 sets of shocks from the model. Therefore, the model is exactly identified and the seven sets of shocks completely describe the data. To check that this holds quantitatively, I perform a quantitative exercise in which I feed in all the shocks that I obtained using the data and simulate the model. This simulation replicates the paths of GDP and other macroeconomic variables in levels and in changes exactly.

4.6 Counterfactuals

In the counterfactual analysis, I ask whether international trade (bilateral trade shocks), or correlated, country-specific shocks affect cross-country GDP co-movement. To answer this question, we need to understand how the global economy would have evolved in a counterfactual world in which various shocks are not existent. Therefore, for each counterfactual scenario, I turn off a set of shocks at a time while keeping other shocks on, or I keep one shock at a time. Counterfactual analyses tell us how each country's macroeconomic variables would have evolved given the set of counterfactual shocks. It is important to discuss how counterfactual shocks arrive and how the social planner views them.

Any counterfactual shocks to the social planner are unexpected. When the social planner wakes up in the year of 1992, she has already allocated all of the endogenous variables such as GDP, consumption expenditure, changes to capital, and bilateral trade shares between countries based on the shocks that she had expected to come in 1993. If there are no changes to shocks, then the social planner allocates the

[23] You can technically interpret country-pair trade shocks as bilateral technology shocks that home and foreign countries experience simultaneously.

[24] Note, that if you multiply $\widehat{d}^G_{ni,t+1}$ and $\widehat{d}^G_{in,t+1}$, then I can obtain

$$\widehat{d}^G_{ni,t+1}\widehat{d}^G_{in,t+1} = \left(\frac{\widehat{\pi}^G_{ni,t+1}\widehat{\pi}^G_{in,t+1}}{\widehat{\pi}^G_{nn,t+1}\widehat{\pi}^G_{ii,t+1}}\right)^{\frac{\sigma-1}{\sigma}} \tag{57}$$

which cancels the price terms. Hence I can obtain the two-way trade costs that was proposed by Head and Ries (2001). Instead of Head and Ries Index, I use the asymmetric bilateral trade costs in equation (55).

Shocks	Data	Source
TFP shock to merchandised goods ($z^G_{n,t}$)	Gross production in merchandised goods ($Y^G_{n,t}$)	OECD STAN
TFP shock to services ($z^S_{n,t}$)	GDP ($Y_{n,t}$)	PWT 9.1
Aggregate demand shock ($\phi_{n,t}$)	Consumption expenditure ($C_{n,t}$)	PWT 9.1
Leisure shock ($\xi_{n,t}$)	Labor hours	PWT 9.1
Investment shock ($\chi_{n,t}$)	PPI in merchandised goods ($p^G_{n,t}$)	OECD STAN
Sectoral shock ($\theta_{n,t}$)	CPI ($q_{n,t}$)	OECD STAN
Bilateral trade shock ($d_{ni,t}$)	Bilateral trade volumes ($X_{ni,t}$)	OECD STAN
Population ($N_{n,t}$)	-	PWT 9.1
Services deficit ($D^S_{n,t}$)	-	OECD STAN

Table 2: Data and Shocks

macroeconomic variables as they are observed in the data. However, in a counterfactual world, a set of shocks changes in the middle of 1992, and the planner is caught by surprise. As she already allocated each country's outcomes for 1992 and the future capital path between 1992 and 1993. Therefore, for the year of 1993, the planner *inefficiently* allocates each country's GDP, consumption, labor, and the amount of bilateral trade between countries. However, after 1993 and onwards, she learns that the counterfactual shocks have arrived, and re-optimizes all the allocations based on the new set of shocks starting from 1993 and onwards.[25] In subsection D.1, I describe how the social planner allocates endogenous outcomes between 1992 and 1993.

5 Main Results

In this section, I analyze how each set of shocks affect international business cycle synchronization between 1993 and 2014 (cross-country real GDP correlations) by running different counterfactual analyses. I focus on two counterfactuals. First, I fix one shock to be constant at at a time, while feeding in the other shocks in the estimation. This is to understand how each shock affects the cross-country GDP co-movement when it is kept constant at initial levels. Second set of counterfactuals is to understand multilateral transmission, by having country-pair trade shocks to be fixed at initial (1992) levels for each country. Since the model outcomes produce counterfactual paths of real GDP in each country, any difference between the counterfactual cross-country GDP correlations and those from the data is the contribution of the set of shocks that are fixed as constant.

5.1 Contributions of Each Shocks on Business Cycle Synchronization

The main question of this paper is to understand the primary driver(s) of international business cycle synchronization. With the shocks backed out in hand, I now answer the main question. In the main counterfactuals, I keep one set of shock as constant in initial (1992) levels of the data period, while feeding in

[25] I take this approach because we need to know the initial levels of the endogenous outcomes of the model to start the algorithm. Hence, if a set of counterfactual shocks are not a surprise to the social planner, the initial outcomes of 1992 also cannot hold as they could not have been on the perfect foresight equilibrium path chosen by the social planner.

other shocks as they are backed out using the model. Does international trade have a big role in synchronizing international business cycles, or is it country-specific correlated shocks that are primarily responsible?

Table 3 summarizes the result. It shows the average of the cross-country correlations of real GDP growth rates between G7 countries plus the rest of the world (RoW). The first row is the average of the cross-country correlations of real GDP growth rates, from 1993 to 2014. The average correlation from the data is 0.5108, which shows that the business cycles of G7 countries and the rest of the world (RoW) are quite synchronized in this period. Each of the rows in the table shows the counterfactual correlations where each specified shocks are kept constant at initial years and the remainder of the shocks are fed in the model: (i) productivity shocks to merchandised goods (tradable) sector; (ii) productivity shocks to services (non-tradable) sector; (iii) aggregate demand shocks; (iv) leisure shocks; (v) investment shocks; (vi) sectoral shocks to merchandised goods expenditure; and (vii) country-pair trade shocks.

From Table 3, I find that the country-pair trade shocks, $d_{ni,t}$, on average, affects the international business cycle synchronization, as the counterfactual GDP correlation gives the largest gap with the data correlation. Most of the country-specific correlated shocks play minimal roles, as the counterfactual GDP correlations are not very different from the data correlations. Sectoral shocks to tradable merchandised goods seems to be the second most important shock. To understand exactly what is going on, I divide the data period into three waves: 1993 – 1999 (the 1990s), 2000 – 2007 (the early 2000s), and 2008 – 2014 (The Great Recession and post recovery period).

Shocks	Avg. GDP Correlations
Data	**0.5108**
no $z^G_{n,t}$ (tradable TFP)	0.4799
no $z^S_{n,t}$ (services TFP)	0.5436
no $\phi_{n,t}$ (agg. demand shock)	0.6349
no $\xi_{n,t}$ (leisure shock)	0.4473
no $\chi_{n,t}$ (investment shock)	0.5213
no $\theta_{n,t}$ (sectoral shock)	0.2339
no $d_{ni,t}$ (country-pair trade shocks)	**0.1358**

Table 3: Average Cross-country Correlations of Real GDP Growth Rates, 1993 – 2014

Table 4 shows the results for the three different waves. We observe that, on average, the international business cycles were not synchronized. After 2000, on the other hand, the cross-country GDP correlations jump to 0.58 and it rises more after the Great Recession. We calculate the counterfactual real GDP correlations for each period, and compare which set of shocks were important.

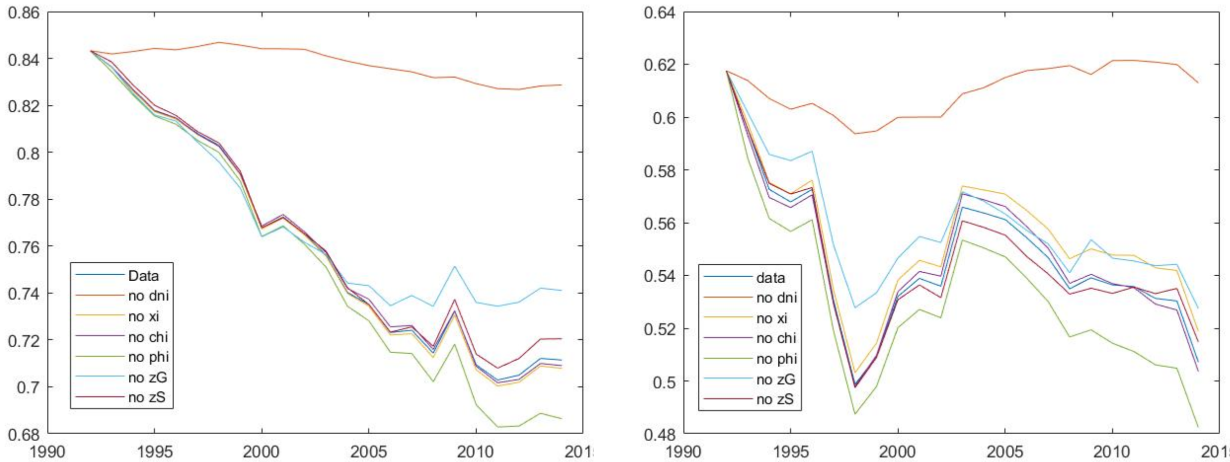
I find that none of the shocks were affecting the cross-country GDP correlations significantly. If any, productivity shocks to services and sectoral shocks would have synchronized cross-country business cycles more when those shocks were kept constant at initial levels. The early 2000s, on the other hand, was very different from the 1990s; the global business cycles were much more positively synchronized. Comparing the counterfactual results, I find that the country-pair trade shocks are substantially more important than the other country-specific correlated shocks. Having kept the country-pair shocks to be constant, the cross-country real GDP correlations drop to 0.18, which is about 70% drop from the correlations in the data. The Great Recession and the post recovery period synchronized the cross-country business cycles even more. Country-pair trade shocks, and sectoral shocks to tradable sector seemed to be the primary drivers of the GDP co-movement in this period.

These results tell us that shocks through international trade linkages have affected the global business cycle co-movement, but there are more intricate stories involved. There has been a stark difference in global business cycle co-movement between the 1990s and 2000s; country-pair trade shocks do not seem to matter all that much before 2000. In fact, services sector productivity shocks hindered the global economy from synchronizing. However, in the early 2000s, the world experienced a boom in international trade, accompanied by China's accession to the WTO. This large increase in international trade made countries to be more susceptible to shocks through trade linkages, and these expanded trade channels created much higher degree of business cycle synchronization after 2000s. Trade shocks were important after the Great Recession and the post recovery period, but the sectoral shocks to tradable sector become similarly important. The emergence of the importance of sectoral shocks to tradable sector comes from an abnormal decline in manufacturing, particularly durable goods sector following the recession. Furthermore, the global economy faced the international trade collapse between 2008 and 2010, which makes trade to be one of the primary source of the business cycle synchronization.

The main takeaways of the counterfactual analyses are first, trade shocks have been important in synchronizing cross-country business cycles. Second, the effects of the trade shocks have been dynamic as the global economy has become more economically integrated, and the country-specific correlated shocks have become less important. These two results suggest that any policies that affect trade directly, such as trade agreements and trade wars, will have larger impact to international business cycles.

Shocks	1993 – 1999	2000 – 2007	2008 – 2014
Data	**-0.0442**	**0.5809**	**0.6456**
no $z_{n,t}^G$ (tradable TFP)	-0.0480	0.7152	0.5917
no $z_{n,t}^S$ (services TFP)	0.1967	0.5045	0.7042
no $\phi_{n,t}$ (agg. demand shock)	-0.0323	0.6096	0.7926
no $\xi_{n,t}$ (leisure shock)	-0.0008	0.5557	0.5251
no $\chi_{n,t}$ (investment shock)	0.0306	0.5506	0.5097
no $\theta_{n,t}$ (sector shock to tradables)	0.1709	0.4334	0.1726
no $d_{ni,t}$ (country-pair trade shocks)	0.0548	0.1859	0.1283

Table 4: Avg. Cross-country Correlations of Real GDP Growth Rates, Three Time Waves



The United States Canada

Figure 1: Domestic Shares of Merchandised Goods ($\pi^G_{nn,t}$): Data vs. Counterfactuals

5.2 Country-Pair Level Results

In this section, I delve deeper into the results and look at how each set of shocks affect the cross-country GDP co-movement for each country-pairs. Does the average result that country-pair trade shocks are important carry over to individual country-pairs as well?

Table 5 shows the individual country-pair results. The left-most column that is denoted as "all shocks" is the actual real GDP correlation between specified country-pairs. For columns on the right, it denotes the country-pair real GDP correlations for each counterfactual scenario, in which the specified shock is fixed at constant levels of 1992. The correlations that are highlighted with bold are the counterfactual results that create the largest gap compared to the actual real GDP correlations in the data.

Among the G7 countries, I observe that having constant country-pair trade shocks overwhelmingly give the largest gap between the data and counterfactual real GDP correlations. Even the country-pairs that trade shocks did not produce the largest gap compared to the data, the trade shocks still had non-negligible amount of impact. The shocks that had the second largest impact is the sectoral shocks to merchandised goods. Other country-specific shocks, such as TFP, demand, leisure, and investment shocks had relatively minor roles in creating the co-movement between the country-pairs.

Overall, the effects of various shocks on the cross-country GDP co-movement are heterogeneous across country-pairs. Nevertheless, fixing country-pair trade shocks overwhelmingly affected the GDP co-movement for the majority of country-pairs.

5.2.1 Importance of Country-pair Trade Shocks

What are country-pair trade shocks capturing in this dynamic model? In this section, I show why country-pair trade shocks are important in more detail.

Figure 1 shows the importance of what happens when the country-pair trade shocks are fixed. The left

		Counterfactual Cross-country GDP Correlations (1993 - 2014)						
Country-pairs	All shocks (data)	w/o merchandised goods TFP ($z^G_{n,t}$)	w/o services TFP ($z^S_{n,t}$)	w/o aggregate demand shocks ($\phi_{n,t}$)	w/o leisure shocks ($\xi_{n,t}$)	w/o investment shocks ($\chi_{n,t}$)	w/o sectoral shocks ($\theta_{n,t}$)	w/o trade shocks ($d_{ni,t}$)
US-Canada	0.7644	0.8176	0.6799	0.8531	0.5198	0.6671	0.3517	**-0.0906**
US-Germany	0.7000	0.7104	0.5159	0.7427	0.6220	0.7908	0.4735	**0.4214**
US-France	0.5991	0.6487	0.4825	0.6376	0.5066	0.7135	**0.3718**	0.6018
US-UK	0.6749	0.6954	0.4004	0.7731	0.3659	0.6764	**-0.1875**	0.5297
US-Italy	0.6363	0.6749	0.5796	0.6330	**0.3873**	0.5496	0.4905	0.6155
US-Japan	0.6135	**0.2361**	0.5984	0.7182	0.5440	0.5601	0.3540	0.6940
US-ROW	0.0907	0.4254	0.5623	0.4965	-0.0114	0.0156	0.0971	**-0.5616**
Canada-Germany	0.7645	0.6481	0.4270	0.7862	0.7069	0.8133	0.2645	**0.1801**
Canada-France	0.4895	0.5187	0.4664	0.5847	0.4514	0.6247	0.2135	**-0.1525**
Canada-UK	0.6624	0.6363	0.5531	0.7451	0.5640	0.7441	**-0.1494**	0.0028
Canada-Italy	0.5152	0.5579	0.5494	0.6038	0.5204	0.5796	0.1818	**-0.2720**
Canada-Japan	0.5666	0.0809	0.5132	0.7372	0.4440	0.5611	0.1832	**-0.0744**
Canada-ROW	0.3745	0.4124	0.3300	0.5460	0.3348	0.1367	**-0.2833**	0.1822
Germany-France	0.7019	0.8908	0.8866	0.7203	0.7879	0.8198	0.8771	**0.2649**
Germany-UK	0.5340	0.4656	0.3357	0.6846	0.4433	0.7793	0.0607	**-0.0013**
Germany-Italy	0.7579	0.6974	0.7887	0.8384	0.7439	0.7788	0.7348	**0.0869**
Germany-Japan	0.4109	**0.2315**	0.6277	0.5185	0.5330	0.6038	0.3353	0.4132
Germany-ROW	0.3423	0.3439	0.5656	0.4540	0.2926	0.1586	0.4356	**-0.1209**
France-UK	0.4820	0.5495	0.5190	0.6624	0.3994	0.6758	**0.1325**	0.5361
France-Italy	0.7347	0.7763	0.8617	0.7968	0.6798	0.8063	0.6158	**0.2861**
France-Japan	0.2571	0.2363	0.6448	0.3290	0.3849	0.4467	0.3765	**0.6512**
France-ROW	0.2625	0.4224	0.6523	0.3744	0.1337	0.0616	0.5009	**-0.2499**
UK-Italy	0.5272	0.5942	0.5013	0.6531	0.3455	0.4671	**-0.1267**	0.2594
UK-Japan	0.7616	0.3087	0.2801	0.7587	0.7519	0.8094	**0.1540**	0.5803
UK-ROW	0.1655	0.4005	0.3521	0.5295	0.1595	0.1005	-0.1549	**-0.3285**
Italy-Japan	0.2714	**-0.0990**	0.4372	0.4042	0.2925	0.1537	-0.0411	0.1349
Italy-ROW	0.2184	0.3979	0.6053	0.4155	0.2353	0.3474	0.4028	**-0.4229**
Japan-ROW	0.4238	0.1540	0.5046	0.7818	0.3852	0.1739	0.0054	**-0.3626**

Table 5: Largest Contributing Shocks for Each Country-pairs

panel of Figure 1 is the evolution of domestic shares of merchandised goods for the United States, and the right one is the evolution the domestic shares for Canada. For both countries, blue line depicts the evolution of the domestic shares from the data, and the rest are counterfactual scenarios in which the specified shock is constant while the other shocks are fed in to the model. In the data, we observe that the domestic shares of the United States declined from 0.84 to 0.7, and the decline for Canada was from 0.6 to 0.5. This tells us that there has been a significant decline in international trade costs between early 1990s and 2014. The global decline of trade barriers have been widely noted in previous studies such as Santacreu and Zhu (2018), and the this appears in the data as the decline in domestic shares of merchandised goods expenditure.

When country-pair trade shocks are fixed at constant levels ($\widehat{d}_{ni,t+1} = 1$), the counterfactual domestic shares do not show any significant decline compared to the data. However, the counterfactuals that involve correlated, country-specific shocks still exhibit the decline in domestic shares of merchandised goods for each country. This shows that the increase in openness to trade and globalization is captured by country-pair trade shocks.

Digging deeper into the movements of country-pair trade shocks, I find that there are two components at work: a trend component and a cyclical component. The trend component of the country-pair trade shocks is the global decline of the trade costs, which is not captured by any other country-specific shocks. Furthermore, there are also the cyclical component of the country-pair trade shocks, which affects volatility of bilateral trade flows between countries. Figure 2 shows this result. The blue lines in Figure 2 show the changes to bilateral trade shares in merchandised goods between selected country-pairs. (US-Canada and US-UK). The country-pair trade shares fluctuate quite significantly from 1992 to 2014. However, when the trade shocks are fixed, the trade shares are not very responsive compared to the data, which is evident from the red lines in Figure 2. Without the fluctuations in country-pair trade shocks, the only potential source of fluctuations of trade flows are created by terms-of-trade shocks (through intermediate and final good price indices). However, much of the literature, first noted by **?**, have shown that terms-of-trade shocks are not very large when we consider cross-country GDP co-movement.[26] This is what we observe from Figure 2, where accounting for these country-pair trade shocks that captures the trade flows from the data, has larger effects to countries' GDP.

The yellow lines on Figure 2 shows another set of counterfactuals, in which only productivity shocks to merchandised goods are fed in, while the other shocks are kept fixed at 1992 levels. This exercise is analogous to a canonical international macroeconomic models such as Backus et al. (1992). This shows that, only with the TFP shocks to tradable sector, we cannot capture the increase in international trade and trade flows endogenously.

Main implications in this section are that, accounting for the trade shocks that are common to the country-pairs are important to the GDP co-movements, and one such way to capture them was to match the trade data exactly. The second implication is that, an increased international trade also creates higher GDP co-movement as countries become more receptive in facing the common shocks through trade. This implication is in line with Frankel and Rose (1998), where higher trade does indeed synchronize international business cycles. Understanding the relationship between trade and cross-country co-movement of business cycles has been studied a lot in international economics literature. Next, I use my results to apply it to one such strand of literature in studying the relationship between trade and GDP co-movement.

[26] **?** show that foreign shocks through terms-of-trade effects do not have a lot of effect on home GDP.

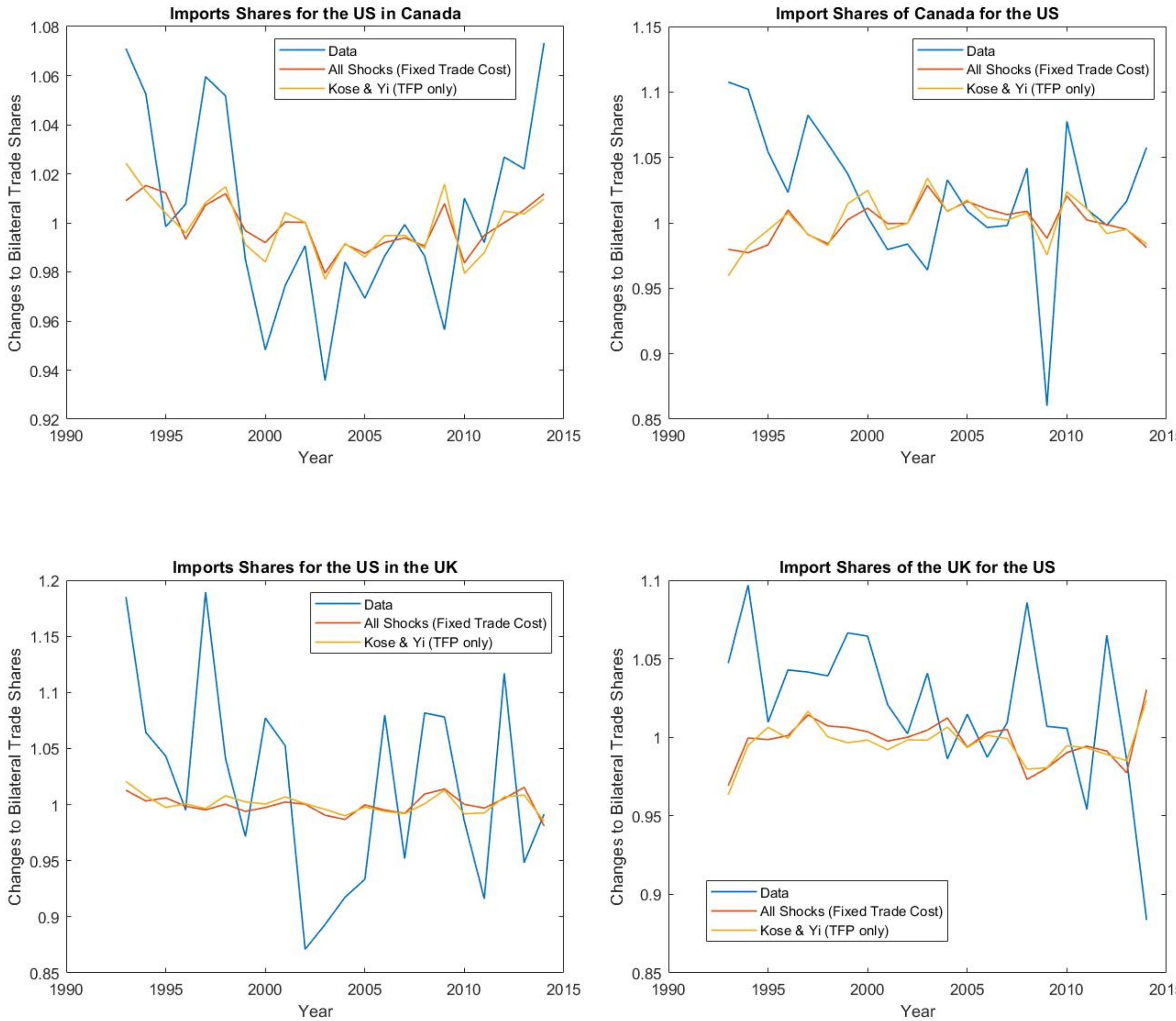


Figure 2: Changes to Trade Shares ($\widehat{\pi}^{G}_{ni,t}$): Data vs. Counterfactuals

Note: I compare bilateral trade shares between the US and other countries (Canada and the UK). Blue line is the data. Red lines represent counterfactual trade shares in which all shocks are fed in and having trade frictions to be fixed in 1992 levels. Yellow lines represent another counterfactual in which only TFP shocks with fixed trade frictions are included.

6 Applications: The Trade Co-movement Puzzle

In this section, I analyze *trade co-movement puzzle* proposed by Kose and Yi (2006), which considers the relationship between trade and cross-country business cycle co-movement. The notion of the trade co-movement puzzle is the following: standard international real business cycle (IRBC) models are not associated with increase in the cross-country GDP co-movement when involved country-pairs increase their trade. In this paper, I find that trade linkages do indeed synchronize cross-country business cycles. So why do we observe a puzzle in standard international macroeconomic models? Since the seminal paper by Kose and Yi (2006), there has been many attempts to solve this puzzle, and many have proposed different solutions; yet we still do not have a clear, satisfying answer why this happens. To answer this question, I re-run the regression first proposed by Frankel and Rose (1998) and Kose and Yi (2006) with my counterfactual economies to see which set of shocks are responsible for alleviating the trade co-movement puzzle.

I run the instrumental variables (IV) estimation on the log of bilateral trade intensity,

$$corr(\widetilde{GDP}_i, \widetilde{GDP}_j) \;=\; \beta_0 + \beta_1 \log(Trade_{ij}) + \varepsilon_{ij} \tag{58}$$

where $\widetilde{GDP}_i$ is the de-trended, first-difference growth rates of real GDP in country i. The definition of $\log(Trade_{ij})$ is taken from Frankel and Rose (1998), which is defined as the sum of each country's imports from the other, divided by the sum of the the countries' GDP, averaged over the entire period of the data.[27]

$$\log(Trade_{ij}) \;\equiv\; \log\left(\frac{X^G_{ij} + X^G_{ji}}{GDP_i + GDP_j}\right) \tag{59}$$

where X^G_{ji} is defined as the value of imports that country j receives from country i.[28] Instrumental variables for the regressions are the log of weighted distances between countries, dummy variables for whether the country-pair share a border (contiguity), and for the country-pair sharing a common language.[29]

Table 6 compares the regression from Kose and Yi (2006) and the dataset that I use. The estimates that I obtain running the instrumental variables (IV) regression is consistent with the literature. From Kose and Yi (2006), the slope coefficient for $\log(Trade_{ij})$ is 0.078. This estimate is from 21 OECD countries between 1970 and 2000. The dataset that I use is G7 countries from 1992 to 2014. The estimated slope coefficient that I obtain from my paper is 0.072 with the standard error of 0.015, which is consistent with their paper. I estimate the same regression with the allocations in the counterfactual economies. Table 7 shows the results. Each column in the table represents each counterfactuals in which the specified shocks are kept constant, while the other shocks are fed in the model as they are backed out from the data. Several results stand out.

First, without TFP shocks to either merchandised goods or services, there are higher cross-country business cycle co-movement. In fact, productivity shocks work on an opposite direction in which they dampen the GDP co-movement. Furthermore, I find that shocks to aggregate demand, leisure, investment, and merchandised goods sector do not impact the Frankel & Rose (FR) regression. However, when the

[27]Note that the data period of comparison is from 1993 to 2014.

[28]This is a cross-sectional regression, as they are averaged over the time between 1992 and 2014.

[29]The set of instrumental variables that I use here are the same as Frankel and Rose (1998) and Kose and Yi (2006).

country-pair trade shocks are kept constant, the positive relationship between trade and cross-country GDP correlations disappears. It is worthwhile to mention again that country-pair trade shocks capture both the gradual openness to trade and factors that create volatility in trade flows. As countries become more open to trade, countries' GDP movements are more affected by the trade shocks, which are not captured by any of the country-specific correlated shocks defined in my model. This is why GDP in standard IRBC models are not responsive to trade changes, as they do not allow for the trade shock that hits both countries simultaneously. Therefore, shocks to country-pair trade is crucial in alleviating *the trade co-movement puzzle*, and we need to include mechanism in the model that can capture these country-pair common trade shocks.

	(1) Kose & Yi (2006)	(2) Data (1992-2014)
$log(trade_{ij})$	0.078***	0.072***
	(0.014)	(0.015)
Observations	21 OECD	G7
IV	Yes	Yes

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 6: Kose & Yi (2006) Regression

	w/o goods TFP	w/o services TFP	w/o agg. demand shocks	w/o leisure shocks	w/o investment shocks	w/o sectoral shocks	w/o bilat. trade shocks
$log(Trade_{ij})$	0.207***	0.122***	0.062***	0.064***	0.075***	0.097*	-0.066
	(0.026)	(0.03)	(0.017)	(0.028)	(0.019)	(0.05)	(0.048)
Observations	G7	G7	G7	G7	G7	G7	G7
IV	Yes	Yes	Yes	Yes	Yes	Yes	Yes

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 7: Trade Co-movement Puzzle Counterfactuals

6.1 Connecting to the Trade Co-movement Puzzle Literature

In this section, I discuss how my findings are linked to the current literature on the trade co-movement puzzle. How do you induce more trade shocks that can make countries' GDP to be more responsive? I go back to the imported trade shares equation in the model.

$$\pi^G_{ni,t} = \left(\frac{p^G_{i,t} d^G_{ni,t}}{q^G_{n,t}}\right)^{\frac{\sigma}{\sigma-1}} \tag{60}$$

6.1.1 Time-Varying Armington Weights (Non-Tariff Trade Barriers)

We tend to think that bilateral trade costs are linked to bilateral tariffs, which do not fluctuate significantly every year in general.[30] However, there are trade frictions involving non-tariff barriers, or country-specific demand shocks that potentially appear as country-pair trade shocks in a model. What are the potential bilateral non-tariff barriers in reality? They can be in forms of import licensing, rules of origin, import-country specific quotas, product quality standards, or anti-dumping laws. These are, by no means, an exhaustive list of non-tariff barriers to trade.

To understand how non-tariff barriers can appear in a standard IRBC model, I consider a model in which the composite production function for the tradable merchandised goods is

$$TR^G_{n,t} = \sum_{i=1}^{N} \left(\omega^{\sigma}_{ni,t} (m^G_{ni,t})^{\sigma}\right)^{\frac{1}{\sigma}} \tag{61}$$

This production function is different from the original model as I impose country-pair specific weights $\omega_{ni,t}$ that vary across time. $\omega_{nn,t}$ can be interpreted as time-varying Armington weights that dictate home-bias in intermediate goods. Also, $\omega_{ni,t}$ can be country-pair specific demand shocks that can be interpreted as non-tariff barriers that make bilateral trade volumes to be more volatile as we observe in the data. Using this production function, I can re-write the bilateral trade shares as

$$\pi^G_{ni,t} = \left(\frac{\omega_{ni,t} p^G_{i,t} d^G_{ni,t}}{q^G_{n,t}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\omega_{ni,t}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{p^G_{i,t} d^G_{ni,t}}{q^G_{n,t}}\right)^{\frac{\sigma}{\sigma-1}} \tag{62}$$

These time-varying non-tariff barriers (time-varying Armington weights) can induce volatile bilateral trade shares when the model is calibrated. The non-tariff barriers are also country-pair specific so that they can appear as bilateral trade shocks in a standard real business cycle model. What can be the specific examples in reality that can appear as bilateral non-tariff barriers, that can be interpreted as country-pair specific demand shocks?

A recent example of non-tariff barriers is South Korea - Japan trade war in 2019. In July 2019, the Japanese government announced that it would tighten the controls on exporting chemicals. Even though Japan did not explicitly state whether the tighter control was targeted toward South Korea, the chemicals

[30] This may not be true given the recent trade wars between various countries.

that went on the list were essential in Korean semiconductor industries.[31] Moreover, Japan took of South Korea on the "white list," which is Japan's list of trusted trade partners. South Korea is now retaliating; the government of South Korea is considering removing Japan from the preferred trade partners, and the citizens of South Korea are boycotting Japanese brands. These are the examples of non-tariff-based barriers in which there can be represented as bilateral trade frictions in the model.

6.1.2 Dynamic Trade Elasticity

Another way in which a model can capture trade shocks is to allow for trade elasticities to vary across time. Even though we do not usually calibrate trade elasticities to vary every period, we still perceive that they do indeed change across time. In the calibration, I set the trade elasticity $\frac{1}{1-\sigma}$ to be equal to 1.5, which is a standard number between home and foreign goods in international macroeconomics literature. However, standard international trade literature calibrate the trade elasticity to be much larger.[32] Ruhl (2008) points out that the measured elasticities can be different when the countries face temporary or permanent shocks. Using this argument, I explain how time-varying Armington elasticity can appear in a standard IRBC model, and how it can generate volatility in bilateral trade.

Suppose that trade elasticity, σ_t, now has a subscript t where it is allowed to be dynamic. Then from the original model, the composite merchandised goods production function now becomes

$$TR^G_{n,t} = \sum_{i=1}^{N} \left(\left(m^G_{ni,t} \right)^{\sigma_t} \right)^{\frac{1}{\sigma_t}} \tag{63}$$

where $\frac{1}{1-\sigma_t}$ is the dynamic trade elasticity. Using the new composite merchandised goods production function, bilateral trade shares can be written as

$$\pi^G_{ni,t} = \left(\frac{p^G_{i,t} d^G_{ni,t}}{q^G_{n,t}} \right)^{\frac{\sigma_t}{\sigma_t - 1}} \tag{64}$$

The expression for the bilateral trade shares is similar to the original model in this paper, except for the dynamic trade elasticity. If the trade elasticity is dynamic, it makes the imported trade shares, $\pi^G_{ni,t}$, to be more volatile, which induces more shocks through GDP. Suppose we consider a typical shock process in which we impose correlated TFP shocks across countries. Suppose there is an increase in productivity in country i. This increase in productivity makes the price of intermediate tradables from country i to be cheaper, which drives country n to import more of country i's tradable merchandised goods.[33] This mechanism does not give enough volatility in trade as it has been shown that terms-of-trade shocks do not affect each country's GDP as much as we expect. However, when trade elasticity is dynamic, this creates an additional channel that makes each country's GDP to be more susceptible to trade shocks.

Drozd et al. (2019) argue that having dynamic trade elasticity is a potential source in which trade co-

[31] https://www.vox.com/world/2019/8/9/20758025/trade-war-south-korea-japan

[32] For example, Eaton and Kortum (2002) estimates the trade elasticities to be between 6 and 12.

[33] $\pi^G_{ni,t}$ increases as country i's share increases in the total merchandised goods imports of country n.

movement puzzle can be solved. In their paper, they introduce dynamic trade elasticity by incorporating a convex adjustment cost on imported trade shares. The convex adjustment cost depends on the previous period's imported trade shares, which is paid in some portion of final goods. To check whether trade elasticity is dynamic, one could also perform a similar accounting procedure laid out in this paper and back out σ_t by matching the bilateral trade costs taken from the data.[34]

6.1.3 Extensive Margin Adjustments

Another way in which we can capture trade shocks is by incorporating the *extensive margin of trade*, where we explicitly model the entry and exit of firms. There has been growing evidence that entry and exit of firms is an essential source of macroeconomic aggregate fluctuations. Hummels and Klenow (2005) finds that as countries are larger and richer, they have higher extensive margins. Using their findings, Liao and Santacreu (2015) and de Soyres and Gaillard (2019) show that incorporating extensive margins of trade in a monopolistic competitive model is a significant step toward solving the trade co-movement puzzle. I discuss how dynamic entry and exit decisions of exporting firms can create an additional channel for trade shocks in a model.

Suppose we have a monopolistic competitive economy à la Melitz. I re-define the composite tradable good production function as

$$TR^G_{n,t} = \sum_{i=1}^{N} \left(\int_{\Omega_{i,t}} \left(m^G_{ni,t}(\omega) \right)^{\sigma} d\omega \right)^{\frac{1}{\sigma}} \tag{65}$$

where $\Omega_{i,t}$ is defined to be a set of firms that are located in sourcing country i. Suppose in equilibrium, the number of firms that export from country i at time t is $b_{i,t}$. Then, the bilateral trade shares can be expressed as

$$\pi^G_{ni,t} = \left(\frac{b^G_{i,t} cost^G_{i,t} d^G_{ni,t}}{z^G_{i,t} q^G_{n,t}} \right)^{\frac{\sigma}{\sigma-1}} = \underbrace{\left(b^G_{i,t} \right)^{\frac{\sigma}{\sigma-1}}}_{\text{extensive margin}} \underbrace{\left(\frac{cost^G_{i,t} d^G_{ni,t}}{z^G_{i,t} q^G_{n,t}} \right)^{\frac{\sigma}{\sigma-1}}}_{\text{intensive margin}} \tag{66}$$

From equation (66), the intensive margin component of imported trade shares is equivalent to the trade shares from this paper. However, there is an additional component where an extensive margin adjustment that generates a channel where trade shocks can go through. This additional channel induces country's GDP to be more responsive to changes to trade.

7 Conclusion

In this paper, I highlight the effects of various shocks on cross-country business cycle synchronization. I find that cross-country GDP co-movement has been increasing for the past two decades due to trade shocks

[34] Bilateral trade costs are not measured well. However, OECD publishes its bilateral trade costs that they measure that potentially captures the actual trade costs between countries.

through an increased economic integration, instead of country-specific correlated shocks. Furthermore, I show that without country-pair trade shocks, the model cannot capture the observed decline in domestic shares in merchandised goods, even when other shocks are present. These findings show that increased economic integration creates a larger conduit for trade shocks that are common to country-pairs, which creates a higher GDP co-movement.

Motivated by the findings that an increased trade does indeed create higher GDP co-movement, I analyze the trade co-movement puzzle, which describes an inability of standard international macroeconomic models to capture the increase in GDP correlations when trade increases. In this exercise, I find that incorporating country-pair trade shocks is essential in solving the trade co-movement puzzle. The country-pair trade shocks, which are manifested as bilateral trade costs in the model, create an additional channel where each country's GDP becomes more responsive when trade shares change. Then, connecting my results to the existing literature, I discuss three ways in which this mechanism can be implemented. First, allowing for time-varying Armington weights may capture the country-pair trade shocks, as it can be interpreted as variations in non-tariff barriers or import-specific demand shocks. Second, dynamic trade elasticities can create more responsiveness in imported trade shares. Lastly, extensive margin adjustments in entry and exit of firms can also create a larger conduit through trade linkages.

Three assumptions in this paper can be relaxed for further analysis. First, by assuming perfect foresight, I limit the analysis to the structural shocks that are already known by the social planner. Allowing for stochasticity in the model can pave different behaviors in the paths of shocks that could result in different outcomes. Second, using other international trade models may bring in a different set of shock decomposition to address movements in macro aggregates.[35] By comparing different classes of models, we can gain a more profound understanding of how shocks affect outcomes based on various modeling assumptions. Third, relaxing the asset market completeness assumption can address the role of financial frictions in international business cycle co-movement. I plan to address these issues for future research.

In conclusion, the main takeaway is that understanding international business cycles relies on capturing the increased economic integration along with the volatility of trade shocks, which has not been studied in-depth in an open economy framework. Looking through the lens of this paper, in light of the ongoing backlash on globalization and the rise of isolationistic policies, we can expect that higher trade barriers and higher uncertainty in trade will significantly affect international business cycles. Whether the findings that I find in this paper only holds in the past two decades or also in the future is a question that will be answered as the global economy continues its cycles.

[35] Dynamic Eaton and Kortum (2002) or Ghironi and Melitz (2007) models can be used to back out shocks as well.

References

Alessandria, G., J. Kaboski, and V. Midrigan (2013). Trade wedges, inventories, and international business cycles. *Journal of Monetary Economics 60*(1), 1–20.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International real business cycles. *Journal of political Economy 100*(4), 745–775.

Baxter, M. and M. A. Kouparitsas (2005). Determinants of business cycle comovement: a robust analysis. *Journal of Monetary Economics 52*(1), 113–157.

Burstein, A., C. Kurz, and L. Tesar (2008). Trade, production sharing, and the international transmission of business cycles. *Journal of Monetary Economics 55*(4), 775–795.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007). Business cycle accounting. *Econometrica 75*(3), 781–836.

de Soyres, F. and A. Gaillard (2019). Value added and productivity linkages across countries. *Working paper*.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced trade. *American Economic Review 97*(2), 351–355.

Drozd, L. A., F. Philadelphia, S. Kolbin, and J. B. Nosal (2019). Long-run trade elasticity and the trade-comovement puzzle.

Eaton, J. and S. Kortum (2002). Technology, geography, and trade. *Econometrica 70*(5), 1741–1779.

Eaton, J., S. Kortum, B. Neiman, and J. Romalis (2016). Trade and the global recession. *American Economic Review 106*(11), 3401–3438.

Engel, C. and J. Wang (2011). International trade in durable goods: Understanding volatility, cyclicality, and elasticities. *Journal of International Economics 83*(1), 37–52.

Frankel, J. A. and A. K. Rose (1998). The endogenity of the optimum currency area criteria. *The Economic Journal 108*(449), 1009–1025.

Ghironi, F. and M. J. Melitz (2007). Trade flow dynamics with heterogeneous firms. *American Economic Review 97*(2), 356–361.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, 402–417.

Head, K. and J. Ries (2001). Increasing returns versus national product differentiation as an explanation for the pattern of us-canada trade. *American Economic Review 91*(4), 858–876.

Hummels, D. and P. J. Klenow (2005). The variety and quality of a nation's exports. *American Economic Review 95*(3), 704–723.

Huo, Z., A. A. Levchenko, and N. Pandalai-Nayar (2019). The global business cycle: measurement and transmission. *Working paper*.

Imbs, J. (2004). Trade, finance, specialization, and synchronization. *Review of Economics and Statistics 86*(3), 723–734.

Johnson, R. C. (2014). Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics 6*(4), 39–83.

Kose, M. A. and K.-M. Yi (2006). Can the standard international business cycle model explain the relation between trade and comovement? *Journal of international Economics 68*(2), 267–295.

Levchenko, A. A., L. T. Lewis, and L. L. Tesar (2010). The collapse of international trade during the 2008–09 crisis: in search of the smoking gun. *IMF Economic review 58*(2), 214–253.

Liao, W. and A. M. Santacreu (2015). The trade comovement puzzle and the margins of international trade. *Journal of International Economics 96*(2), 266–288.

Lucas, R. E. and E. C. Prescott (1971). Investment under uncertainty. *Econometrica: Journal of the Econometric Society*, 659–681.

Ohanian, L. E., P. Restrepo-Echavarria, and M. L. Wright (2018). Bad investments and missed opportunities? postwar capital flows to asia and latin america. *American Economic Review 108*(12), 3541–82.

Parkin, M. (1988). A method for determining whether parameters in aggregate models are structural. *Carnegie-Rochester Conference Series on Public Policy 29*, 215–252.

Raffo, A. (2010). Technology shocks: Novel implications for international business cycles.

Reyes-Heroles, R. (2017). The role of trade costs in the surge of trade imbalances. *Working paper*.

Ruhl, K. J. (2008). The international elasticity puzzle.

Santacreu, A. M. and H. Zhu (2018). Domestic innovation and international technology diffusion as sources of comparative advantage.

Stockman, A. C. and L. L. Tesar (1995). Tastes and technology in a two-country model of the business cycle: Explaining international comovements. *The American Economic Review*, 168–185.

A Tables and Figures

Countries
Canada (CAN)
France (FRA)
Germany (DEU)
Italy (ITA)
United Kingdom (GBR)
United States (USA))
Rest of the World (ROW)

Table 8: List of Countries

B Data Description

My analysis includes G7 countries plus the Rest of World (RoW), across two sectors (traded merchandised goods and non-traded services). All macroeconomic variables are in real terms, deflated by the United States GDP deflator, constant in 2011 prices. (US GDP deflator works as a price normalization.) All of the units are in terms of U.S. dollars.

B.1 GDP ($Y_{n,t}$)

The most crucial data to match in this paper is the GDP of each country. Using Penn World Table 9.1, I match GDP at current PPP (purchasing power parity). Rest of the World (RoW) GDP is taken as the sum of the countries' GDP that are not part of G7.

B.2 Gross Production in Merchandised Goods ($Y^G_{n,t}$)

The annual gross production of merchandised goods are taken from OECD Structural Analysis Database (STAN), with ISIC Revision 4 categories. I define merchandised goods as the sum of three different sectors as noted in Table 9. To create the gross production for merchandised goods for each country, I obtain gross production for Agriculture, forestry and fishing, gross production for mining and quarrying, and gross production for manufacturing and add them to create total gross production of merchandised goods for each country.

Industries
Agriculture, Forestry & Fishing (D01T03)
Mining & Quarrying (D05T09)
Manufacturing (D10T33)

Table 9: Merchandised Goods

B.2.1 Exchange Rates

Gross production for merchandised goods are reported in countries' own currencies. To match the annual gross production values with GDP and bilateral trade data, I convert the production values using nominal exchange rates so that all gross production values for each country are in terms of U.S. dollars

B.3 Labor hours ($l_{n,t}$)

From Penn World Table 9.1, I take the average amount of hours worked for each country to match with the endogenous labor hours in the model. We know that one year consists of 8760 hours and I assume that people on average sleep 2920 hours per year, which amounts to 8 hours of sleep per day. Each representative consumer in country n has to allocate 5840 hours either to work or to spend them on leisure. To have the data fit the model, I set 5840 hours to be the total amount of hours that each representative consumers in country n are endowed with. Then, I match the endogenous labor choice, $l_{n,t}$, with the model such that

$$l_{n,t} \equiv \frac{\text{avg. labor hours}_{n,t}}{5840} \tag{B.1}$$

where average labor hours is taken directly from the data. Therefore, leisure, by definition, is equal to $1 - l_{n,t}$ and the sum of the time available for workers in each country is normalized to 1. For the rest of the world (ROW), I take the average of the remaining countries from Penn World Table 9.1 besides G7 countries.

B.4 Consumption expenditure ($X^C_{n,t}$)

I match nominal consumption of households and government, at current PPP (purchasing power parity) from Penn World Table 9.1. For the rest of the world (RoW) consumption expenditure, I take the sum of the rest of the countries in Penn World Table 9.1 excluding G7.

B.5 Producer Price Index (PPI) ($p^G_{n,t}$)

For intermediate good prices, I match them with domestic producer price index (PPI) in manufacturing. Producer Price Index for agriculture and Mining & Quarrying sectors are not available, so I use manufacturing producer price index to match the intermediate price index. For the rest of the world (RoW), I use producer price index for total OECD countries from OECD Structural Analysis Database (STAN). The base year of PPI for each country from OECD STAN is 2015. To match with other macroeconomic aggregates, I convert each country's PPI to be in terms of 2011 base year.

B.6 Consumer Price Index (CPI) ($q_{n,t}$)

Consumer Price Indices (CPI) for each country are taken directly from OECD Structural Analysis Database (STAN). All of the countries have readily available CPI. For the rest of the world (RoW) CPI, I use average CPI for all of OECD countries. CPI is also deflated using 2011 as a base year.

B.7 Bilateral Trade Shares ($\pi_{ni,t}$)

B.7.1 Total Bilateral Trade

I take bilateral trade data from OECD Bilateral Trade by Industry and End-use ISIC Rev. 4, in annual levels. The total bilateral trade data are taken from the categories D01T99, and they are needed to calculate net exports and imports of each countries, and exogenous service deficits, which will be discussed in the next section.

B.7.2 Merchandised Goods Bilateral Trade

I divide the model into tradable (merchandised goods) and non-tradable (services) sectors. I take primary and manufactured goods bilateral trade data from 1992 to 2014.[36] I take both bilateral exports and imports of primary and manufactured goods to calculate import shares for each bilateral country-pairs. I denote the

[36] This is categorized by (AM) in the data, and it includes Agriculture, Mining & Quarrying, and Manufacturing industries (D01T32).

amount of imports that country n receives from country i as $X_{ni,t}^{IMP,M}$, where the superscript IMP represents "imports" and the superscript "G" represents merchandised tradable goods. Net exports of primary and manufactured goods are calculated using the total exports and imports of country n:

$$NX_{n,t}^{G} = X_{n,t}^{EXP,G} - X_{n,t}^{IMP,G} \tag{B.2}$$

Equipped with data for gross production in merchandised goods ($Y_{n,t}^{G}$), I can calculate total domestic absorption of tradable merchandised goods by

$$X_{n,t}^{G} \;=\; Y_{n,t}^{G} - NX_{n,t}^{G} \;=\; Y_{n,t}^{G} - X_{n,t}^{EXP,G} + X_{n,t}^{IMP,G} \tag{B.3}$$

which satisfies the global goods market clearing condition. To calculate bilateral import shares, I take the ratio between country n's imports from country i over the total absorption (expenditure) of country n:

$$\pi_{ni,t}^{G} = \frac{X_{ni,t}^{IMP,G}}{X_{n,t}^{G}} \tag{B.4}$$

To complete the bilateral trade share matrix, I need to calculate country n's *own* absorption, which is the gross production of merchandised goods that is consumed domestically. I denote this as $X_{nn,t}^{G}$.

$$X_{nn,t}^{G} = Y_{n,t}^{G} - X_{n,t}^{EXP,G} \tag{B.5}$$

Therefore, country n's own share is:

$$\pi_{nn,t}^{G} = \frac{X_{nn,t}^{G}}{X_{n,t}^{G}} \tag{B.6}$$

This allows us to have $\sum_{i=1}^{N} \pi_{ni,t}^{G} = 1$.

B.7.3 Rest of the World (RoW)

Bilateral exports and imports for the rest of the world (RoW) are calculated in the following way. First, I assume that the rest of the world itself is a country, and no countries that consist of the rest of the world do not trade with each other. Suppose I want to calculate $X_{US\leftarrow ROW,t}^{IMP,G}$, which is the amount of US imports from the rest of the world. I treat this as a residual from the total imports that the United States receive from the other G7 countries. Using the equation,

$$X_{US\leftarrow ROW,t}^{IMP,G} \;=\; X_{US,t}^{IMP,G} - \sum_{i\neq ROW} X_{US\leftarrow i,t}^{IMP,G} \tag{B.7}$$

Using this formulation, the total exports that the rest of the world (RoW) make is:

$$X_{ROW,t}^{EXP,G} \;=\; \sum_{n=1}^{N} X_{n\leftarrow ROW,t}^{IMP,G} \tag{B.8}$$

On the other hand, I also need to know how much the rest of the world (RoW) import from each of the G7, which I define it as $X_{ROW\leftarrow i,t}^{IMP,G}$. Similarly, I treat this as a residual by subtracting the sum of country i's exports to G7, from the total exports of country i:

$$X_{ROW\leftarrow i,t}^{IMP,G} = X_{i,t}^{EXP,G} - \sum_{n\neq ROW} X_{n\leftarrow i,t}^{IMP,G} \tag{B.9}$$

So the total imports by the rest of the world (ROW) is:

$$X_{ROW,t}^{IMP,G} = \sum_{i=1}^{N} X_{ROW\leftarrow i,t}^{IMP,G} \tag{B.10}$$

Using this method, I calculate the rest of the world's exports and imports for primary and manufacturing goods, $X_{ROW,t}^{EXP,G}$ and $X_{ROW,t}^{IMP,G}$ as well.

B.8 Service Deficits

I assume that the service sector is non-tradable in this paper. However, in reality, services are traded between countries. To avoid discrepancies in matching the data, I need to take into account for the services trade which does not appear in the model. For each country n, I use its GDP ($Y_{n,t}$) and gross production in merchandised goods ($Y_{n,t}^{G}$) to calculate gross production in services, using the GDP equation:

$$Y_{n,t}^{S} = \frac{Y_{n,t} - \beta_n^G Y_{n,t}^G}{\beta_n^S} \tag{B.11}$$

From total exports and imports between countries, I subtract bilateral exports and imports for merchandised goods:

$$X_{n,t}^{EXP,S} = X_{n,t}^{EXP} - X_{n,t}^{EXP,G} \tag{B.12}$$

$$X_{n,t}^{IMP,S} = X_{n,t}^{IMP} - X_{n,t}^{IMP,G} \tag{B.13}$$

Since I do not allow for service sector to be traded, I take exports and imports of services as a residual to match the data. Therefore, I define service deficits as:

$$D_{n,t}^{S} \equiv X_{n,t}^{EXP,S} - X_{n,t}^{IMP,S} \tag{B.14}$$

which is treated as exogenous. I obtain domestic absorption (expenditure) of services sector by

$$X_{n,t}^{S} = Y_{n,t}^{S} - D_{n,t}^{S} \tag{B.15}$$

C Solving for Social Planner's Allocations

I solve for the global social planner's problem in this section. The methodology listed out in this section is similar to Eaton et al. (2016). The global social planner's problem is:

$$
\begin{aligned}
\mathcal{L} = \sum_{n=1}^{N}\sum_{t=0}^{\infty}\rho^t \Bigg\{ & \omega_n \phi_{n,t}\Big[\log\left(c_{n,t}\right) + \xi_{n,t}\log\Big(N_{n,t}n_{n,t}\Big)\Big] + \\
& + \lambda_{n,t}^{INT,G}\left[z_{n,t}^{G}\left(\left(L_{n,t}^{G}\right)^{\alpha_n}\left(K_{n,t}^{G}\right)^{1-\alpha_n}\right)^{\beta_n^G}\left(\left(M_{n,t}^{GG}\right)^{\gamma_n^G}\left(M_{n,t}^{GS}\right)^{1-\gamma_n^G}\right)^{1-\beta_n^G} - y_{n,t}^{G}\right] \\
& + \lambda_{n,t}^{INT,S}\left[z_{n,t}^{S}\left(\left(L_{n,t}^{S}\right)^{\alpha_n}\left(K_{n,t}^{S}\right)^{1-\alpha_n}\right)^{\beta_n^S}\left(\left(M_{n,t}^{SS}\right)^{\gamma_n^S}\left(M_{n,t}^{SG}\right)^{1-\gamma_n^S}\right)^{1-\beta_n^S} - y_{n,t}^{S}\right] \\
& + \lambda_{n,t}^{FG}\left[\left(\sum_{i=1}^{N}\left(m_{ni,t}^{G}\right)^{\sigma}\right)^{\frac{1}{\sigma}} - TR_{n,t}^{G}\right] \\
& + \lambda_{n,t}^{F}\left[\left(\left(\theta_{n,t}\right)^{1-\mu}\Big(F_{n,t}^{G}\Big)^{\mu} + \Big(F_{n,t}^{S}\Big)^{\mu}\right)^{\frac{1}{\mu}} - F_{n,t}\right] \\
& + \widetilde{\lambda}_{n,t}^{INT}\left[y_{n,t}^{G} - \sum_{i=1}^{N} d_{in,t}^{G} m_{in,t}^{G}\right] \\
& + \lambda_{n,t}^{V}\Big[(1-\delta)K_{n,t} + \chi_{n,t}I_{n,t} - K_{n,t+1}\Big] \\
& + \lambda_{n,t}^{MC,F}\Big[F_{n,t} - C_{n,t} - I_{n,t}\Big] \\
& + \lambda_{n,t}^{MC,G}\Big[TR_{n,t}^{G} - F_{n,t}^{G} - M_{n,t}^{GG} - M_{n,t}^{SG}\Big] \\
& + \lambda_{n,t}^{MC,S}\Big[y_{n,t}^{S} - F_{n,t}^{S} - M_{n,t}^{SS} - M_{n,t}^{GS}\Big] \\
& + \widetilde{\lambda}_{n,t}^{L}\Big[N_{n,t}(1 - \sum_{j\in\{G,S\}}\frac{L_{n,t}^{j}}{N_{n,t}} - n_{n,t})\Big]\Bigg\}
\end{aligned}
\tag{C.1}
$$

where each λ is the Lagrangian multiplier associated with the corresponding constraint. The transversality condition is:

$$
\lim_{t\to\infty}\rho^t\lambda_{n,t}^{V}K_{n,t+1} = 0 \tag{C.2}
$$

for each country n.

Equipped with the Lagrangian equation (C.1), I can solve for all the allocations in the global economy that are equivalent to the perfect foresight competitive equilibrium.

C.1 Costs of (Tradable) Merchandised Goods and (Non-tradable) Services

Primary inputs of the economy in each country are labor, capital, and intermediate inputs. In the subsections below, I take the first-order conditions for the each of the inputs in sector j.

C.1.1 Costs of Labor

I take the first-order condition with respect to labor $L^j_{n,t}$ for each sector $j \in \{G, S\}$:

$$\widetilde{\lambda}^L_{n,t} \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\Big(\alpha_n \beta^j_n\Big)\Big(L^j_{n,t}\Big)^{\alpha_n\beta^j_n - 1}\Big(K^j_{n,t}\Big)^{(1-\alpha^j_n)\beta^j_n}\left[\Big(M^{jj}_{n,t}\Big)^{\gamma^j_n}\Big(M^{jj'}_{n,t}\Big)^{1-\gamma^j_n}\right]^{1-\beta^j_n} \tag{C.3}$$

Multiplying $L^j_{n,t}$ on both sides of (C.3) and collecting the terms on the right-hand side,

$$\widetilde{\lambda}^L_{n,t} \;=\; \lambda^{INT,j}_{n,t}\big(\alpha_n\beta^j_n\big)\frac{y^j_{n,t}}{L^j_{n,t}} \tag{C.4}$$

The Lagrangian multiplier $\widetilde{\lambda}^L_{n,t}$ can be interpreted as wages for an individual worker in country n at time t.

C.1.2 Costs of Capital

I take the first-order condition with respect to capital $K^j_{n,t}$ for each sector $j \; in\{G, S\}$:

$$\widetilde{\lambda}^K_{n,t} \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\Big((1-\alpha^j_n)\beta^j_n\Big)\Big(L^j_{n,t}\Big)^{\alpha_n\beta^j_n}\Big(K^j_{n,t}\Big)^{(1-\alpha^j_n)\beta^j_n - 1}\left[\Big(M^{jj}_{n,t}\Big)^{\gamma^j_n}\Big(M^{jj'}_{n,t}\Big)^{1-\gamma^j_n}\right]^{1-\beta^j_n} \tag{C.5}$$

Multiplying $K^j_{n,t}$ on both sides of (C.5) and collecting the terms on the right-hand side,

$$\widetilde{\lambda}^K_{n,t} \;=\; \lambda^{INT,j}_{n,t}\big((1-\alpha_n)\beta^j_n\big)\frac{y^j_{n,t}}{K^j_{n,t}} \tag{C.6}$$

The Lagrangian multiplier $\widetilde{\lambda}^K_{n,t}$ can be interpreted as rental rates of a unit of capital in country n at time t.

C.1.3 Costs of Intermediate Inputs

I take the first-order condition with respect to the intermediate inputs $M^{jj}_{n,t}$ and $M^{jj'}_{n,t}$ for each sector $j \; in\{G, S\}$:

$$\widetilde{\lambda}^{F,j}_{n,t} \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\Big(\gamma^j_n(1-\beta^j_n)\Big)\left[\Big(L^j_{n,t}\Big)^{\alpha_n}\Big(K^j_{n,t}\Big)^{1-\alpha^j_n}\right]^{\beta^j_n}\Big(M^{jj}_{n,t}\Big)^{\gamma^j_n(1-\beta^j_n)-1}\Big(M^{jj'}_{n,t}\Big)^{(1-\gamma^j_n)(1-\beta^j_n)} \tag{C.7}$$

$$\widetilde{\lambda}^{F,j'}_{n,t} \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\Big((1-\gamma^j_n)(1-\beta^j_n)\Big)\Big[\Big(L^j_{n,t}\Big)^{\alpha_n}\Big(K^j_{n,t}\Big)^{1-\alpha^j_n}\Big]^{\beta^j_n}\Big(M^{jj}_{n,t}\Big)^{\gamma^j_n(1-\beta^j_n)}\Big(M^{jj'}_{n,t}\Big)^{(1-\gamma^j_n)(1-\beta^j_n)-1} \tag{C.8}$$

Multiplying $M^{jj}_{n,t}$ and $M^{jj'}_{n,t}$ on both sides of (C.7) and (C.8), and collecting the terms on the right-hand side,

$$\widetilde{\lambda}^{F,j}_{n,t} \;=\; \lambda^{INT,j}_{n,t}\big((\gamma^j_n)(1-\beta^j_n)\big)\frac{y^j_{n,t}}{M^{jj}_{n,t}} \tag{C.9}$$

$$\widetilde{\lambda}^{F,j'}_{n,t} \;=\; \lambda^{INT,j}_{n,t}\big((1-\gamma^j_n)(1-\beta^j_n)\big)\frac{y^j_{n,t}}{M^{jj'}_{n,t}} \tag{C.10}$$

C.1.4 Calculating a Unit Cost Bundle

We need to relate the shadow cost of producing a good to the shadow costs of the primary inputs (labor, capital, and other intermediate inputs). To do this, I define $Y^j_{n,t}$ as the value of the production of a good in country n at time t.[37]

$$Y^j_{n,t} \;=\; \lambda^{INT,j}_{n,t} y^j_{n,t} \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\Bigg(\big(L^j_{n,t}\big)^{\alpha_n}\big(K^j_{n,t}\big)^{1-\alpha_n}\Bigg)^{\beta^j_n}\Bigg(\big(M^{jj}_{n,t}\big)^{\gamma^j_n}\big(M^{jj'}_{n,t}\big)^{1-\gamma^j_n}\Bigg)^{1-\beta^j_n} \tag{C.11}$$

Replacing $L^j_{n,t}$, $K^j_{n,t}$, $M^{jj}_{n,t}$, and $M^{jj'}_{n,t}$ from using (C.4), (C.6), (C.9), and (C.10),

$$Y^j_{n,t} \;=\; \lambda^{INT,j}_{n,t} y^j_{n,t}\lambda^{INT,j}_{n,t} z^j_{n,t}\big(Cons\big)^j_n\Big(\widetilde{\lambda}^L_{n,t}\Big)^{-\alpha_n\beta^j_n}\Big(\widetilde{\lambda}^K_{n,t}\Big)^{-(1-\alpha_n)\beta^j_n}\Big(\widetilde{\lambda}^{F,j}_{n,t}\Big)^{-\gamma^j_n(1-\beta^j_n)}\Big(\widetilde{\lambda}^{F,j'}_{n,t}\Big)^{-(1-\gamma^j_n)(1-\beta^j_n)} \tag{C.12}$$

where $\big(Cons\big)^j_n = \big(\alpha_n\beta^j_n\big)^{\alpha_n\beta^j_n}\big((1-\alpha_n)\beta^j_n\big)^{(1-\alpha_n)\beta^j_n}\big(\gamma^j_n(1-\beta^j_n)\big)^{\gamma^j_n(1-\beta^j_n)}\big((1-\gamma^j_n)(1-\beta^j_n)\big)^{(1-\gamma^j_n)(1-\beta^j_n)}$. Since $Y^j_{n,t} \;=\; \lambda^{INT,j}_{n,t} y^j_{n,t}$, it is canceled out from both sides:

$$1 \;=\; \lambda^{INT,j}_{n,t} z^j_{n,t}\big(Cons\big)^j_n\Big(\widetilde{\lambda}^L_{n,t}\Big)^{-\alpha_n\beta^j_n}\Big(\widetilde{\lambda}^K_{n,t}\Big)^{-(1-\alpha_n)\beta^j_n}\Big(\widetilde{\lambda}^{F,j}_{n,t}\Big)^{-\gamma^j_n(1-\beta^j_n)}\Big(\widetilde{\lambda}^{F,j'}_{n,t}\Big)^{-(1-\gamma^j_n)(1-\beta^j_n)} \tag{C.13}$$

[37] This is matched with the gross production of merchandised goods from the data.

Then, we can re-write the shadow price for a good $y^j_{n,t}$ as

$$\lambda^{INT,j}_{n,t} \;=\; \frac{(Cons)^j_n \left[\left(\widetilde{\lambda}^L_{n,t}\right)^{\alpha_n}\left(\widetilde{\lambda}^K_{n,t}\right)^{1-\alpha_n}\right]^{\beta^j_n}\left[\left(\widetilde{\lambda}^{F,j}_{n,t}\right)^{\gamma^j_n}\left(\widetilde{\lambda}^{F,j'}_{n,t}\right)^{1-\gamma^j_n}\right]^{1-\beta^j_n}}{z^j_{n,t}} \tag{C.14}$$

Changing the notations as

$$p^j_{n,t} \;\equiv\; \lambda^{INT,j}_{n,t} \qquad w_{n,t} \;\equiv\; \widetilde{\lambda}^L_{n,t} \qquad r_{n,t} \;\equiv\; \widetilde{\lambda}^K_{n,t} \qquad \widetilde{\lambda}^{F,j}_{n,t} \;\equiv\; q^{F,j}_{n,t}$$

We can re-write the above equation for goods and services:

$$p^G_{n,t} \;=\; \frac{(Cons)^G_n \left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta^G_n}\left[\left(q^G_{n,t}\right)^{\gamma^G_n}\left(p^S_{n,t}\right)^{1-\gamma^G_n}\right]^{1-\beta^G_n}}{z^G_{n,t}} \tag{C.15}$$

$$p^S_{n,t} \;=\; \frac{(Cons)^S_n \left[\left(w_{n,t}\right)^{\alpha_n}\left(r_{n,t}\right)^{1-\alpha_n}\right]^{\beta^S_n}\left[\left(p^S_{n,t}\right)^{\gamma^S_n}\left(q^G_{n,t}\right)^{1-\gamma^S_n}\right]^{1-\beta^S_n}}{z^S_{n,t}} \tag{C.16}$$

C.2 Demand for Unit Merchandised (Tradable) Goods

I take the first-order condition with respect to $y^M_{n,t}$. Then,

$$\lambda^{INT,M}_{n,t} \;=\; \widetilde{\lambda}^{INT}_{n,t} \tag{C.17}$$

C.3 Demand for Composite Merchandised (Tradable) Goods and Services

I take the first-order conditions with respect to $F^M_{n,t}$ and $y^S_{n,t}$.

$$\lambda^{FM}_{n,t} \;=\; \lambda^F_{n,t}\left(F_{n,t}\right)^{1-\mu}\left(\theta_{n,t}\right)^{\mu-1}\left(F^M_{n,t}\right)^{\mu-1} \tag{C.18}$$

$$\lambda^{INT,S}_{n,t} \;=\; \lambda^F_{n,t}\left(F_{n,t}\right)^{1-\mu}\left(1-\theta_{n,t}\right)^{\mu-1}\left(y^S_{n,t}\right)^{\mu-1} \tag{C.19}$$

Defining the Lagrangian prices as the following

$$p^S_{n,t} \;\equiv\; \lambda^{INT,S}_{n,t} \qquad q^M_{n,t} \;\equiv\; \lambda^{FM}_{n,t} \qquad q_{n,t} \;\equiv\; \lambda^F_{n,t}$$

we can calculate final good price index by combining (C.15) and (C.16).

C.4 Merchandised Goods Trade

In this subsection, I derive the bilateral imports demand for merchandised goods. I take the first-order condition with respect to $m_{ni,t}$.

$$\widetilde{\lambda}_{i,t}^{INT} d_{ni,t} \;=\; \lambda_{n,t}^{FM}\left(F_{n,t}^{M}\right)^{\frac{1-\sigma}{\sigma}}\left(m_{ni,t}\right)^{\sigma-1} \tag{C.20}$$

Since $\widetilde{\lambda}_{n,t}^{INT} = \lambda_{n,t}^{INT,M}$, we can re-write the previous equation as

$$p_{i,t} d_{ni,t} \;=\; q_{n,t}^{M}\left(F_{n,t}^{M}\right)^{\frac{1-\sigma}{\sigma}}\left(m_{ni,t}\right)^{\sigma-1} \tag{C.21}$$

Using this equation, we can obtain composite merchandised goods price index $q_{n,t}^{M}$.

C.5 Demand for Final Goods

Taking the first-order condition with respect to $F_{n,t}$, we get

$$\lambda_{n,t}^{F} \;=\; \lambda_{n,t}^{MC} \tag{C.22}$$

and this tells us that both of these prices are equal to the final good price index, and I define it as $q_{n,t}$.

C.6 Consumption

I take the first-order condition with respect to $c_{n,t}$:

$$\widetilde{\lambda}_{n,t}^{C} c_{n,t} N_{n,t} \;=\; \omega_n \phi_{n,t} \tag{C.23}$$

Letting $q_{n,t} \equiv \widetilde{\lambda}_{n,t}^{C}$, we can re-write this as:

$$q_{n,t} c_{n,t} \;=\; \frac{\omega_n \phi_{n,t}}{N_{n,t}} \tag{C.24}$$

C.7 Leisure

I derive the marginal rate of substitution between consumption and leisure. I take the first-order condition with respect to $n_{n,t}$, which is equal to $1 - l_{n,t}$.

$$\widetilde{\lambda}_{n,t}^{L} N_{n,t} \;=\; \omega_n \phi_{n,t} \xi_{n,t} \frac{1}{1 - l_{n,t}} \tag{C.25}$$

Replacing $w_{n,t} \equiv \widetilde{\lambda}_{n,t}^{L}$, we have

$$w_{n,t}\left(1 - l_{n,t}\right) N_{n,t} \;=\; \omega_n \phi_{n,t} \xi_{n,t} \tag{C.26}$$

C.8 Investment

In this subsection, I derive the Euler equation of the model. First, I take the first-order condition with respect to $I_{n,t}$:

$$\lambda_{n,t}^{MC} = \lambda_{n,t}^{V} \chi_{n,t} \tag{C.27}$$

Now we take the first-order condition with respect to $K_{n,t+1}$:

$$\lambda_{n,t}^{V} = \rho \lambda_{n,t+1}^{V} \big(1-\delta\big) + \rho \widetilde{\lambda}_{n,t+1}^{K} \tag{C.28}$$

Using equation (C.16), I can replace $\lambda_{n,t}^{V}$ and $\lambda_{n,t+1}^{V}$.

$$\frac{\lambda_{n,t}^{MC}}{\chi_{n,t}} = \rho \left[\frac{\lambda_{n,t+1}^{MC}}{\chi_{n,t+1}} \big(1-\delta\big) + \widetilde{\lambda}_{n,t+1}^{K} \right] \tag{C.29}$$

Letting

$$q_{n,t} \equiv \lambda_{n,t}^{MC} \qquad \widetilde{\lambda}_{n,t}^{K} \equiv r_{n,t}$$

I obtain the Euler equation from the model.

$$\frac{q_{n,t}}{\chi_{n,t}} = \rho \left[\frac{q_{n,t+1}}{\chi_{n,t+1}} \big(1-\delta\big) + r_{n,t+1} \right] \tag{C.30}$$

D Numerical Algorithm

In this section of the appendix, I write down the algorithm that solves for the social planner's problem. Expressing all the variables in changes is the key to this algorithm as I do not have to find everything in levels.

D.1 Calculating the Counterfactual Equilibrium between 1992-1993

We start with the economic outcomes in 1992: $Y_{n,92}, Y_{n,92}^{G}, l_{n,92}, \pi_{ni,92}, X_{n,92}^{C}$, and $\widehat{K}_{n,93}$. Even though a set of counterfactual shocks arrive between 1992 and 1993, changes to the capital, $\widehat{K}_{n,93}$, had been decided by the social planner in the beginning of 1992. After the planner has allocated the endogenous outcomes in the beginning of 1992, the counterfactual bilateral trade shocks arrive

$$\widehat{d}_{ni,1993} = 1 \quad \text{if} \quad n \neq i$$

This means that the bilateral trade shocks remain constant (fixed) at 1992 levels. Meanwhile, all other country-specific shocks are left unchanged.

Between 1992 and 1993, the shocks were not anticipated by the social planner. Therefore, the Euler equation cannot hold as the capital that the planner allocated is *is not efficiently allocated.* Instead, the planner solves for the changes in investment spending:

$$\widehat{X}^I_{n,93} = \left(\frac{\widehat{K}_{n,94} - (1-\delta)}{\widehat{K}_{n,93} - (1-\delta)} \right) \frac{\widehat{q}_{n,93}\widehat{K}_{n,93}}{\widehat{\chi}_{n,93}}$$

and solve for the other equilibrium outcomes, while minimizing the market clearing conditions:

$$\widehat{w}_{n,t+1}\widehat{l}_{n,t+1}\widehat{N}_{n,t+1} = \widehat{Y}_{n,t+1}$$

$$\widehat{r}_{n,t+1}\widehat{K}_{n,t+1} = \widehat{Y}_{n,t+1}$$

Then, I obtain $Y_{n,93}, Y^G_{n,93}, l_{n,93}, \pi_{ni,93}, X^C_{n,93}$, and $\widehat{K}_{n,94}$. Starting from 1994, however, the counterfactual shocks are all known to the planner as she now fully realizes the altered paths of shocks. Therefore, the planner re-optimizes by taking the set of counterfactual shocks as given and optimally allocate all the outcomes for each country.[38] All of the counterfactual exercises are done based on this philosophy.

[38] Starting from 1994, the Euler equation holds every period.