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Measuring What Matters in Global Value Chains and Value-Added Trade

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Abstract

The spread of global value chains (GVCs) has given rise to new statistical tools, the Inter-Country Input-Output tables and new analytical frameworks aimed at properly identifying production linkages between and within economies. However, several important questions remain unaddressed. This paper proposes a new toolkit for value-added accounting of trade flows at the aggregate, bilateral, and sectoral levels that can be used to investigate a broad set of empirical questions—including an assessment of the share of trade related to GVCs. The paper shows how different empirical issues require distinct accounting perspectives, and maps these methodologies onto the economic questions they are best suited to address. In this way, in addition to providing novel tools, the paper brings a large part of the related literature under one comprehensive framework.

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Measuring What Matters in Global Value Chains and Value-Added Trade

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1 Introduction

The international fragmentation of production processes has challenged the capability of standard trade statistics to truly represent supply and demand linkages between economies. In general, bilateral exports differ from the portion of a country's GDP related to the production of goods and services shipped to a certain foreign market. On the one hand, exports also embed imported intermediate inputs and on the other, the directly importing country often differs from the product's ultimate destination. Whenever production is organized in sequential processing stages in different countries, trade statistics repeatedly double count the same value-added. The diffusion of global value chains (GVC) has therefore deepened the divergence between gross flows, as recorded by traditional trade statistics, and the data on production and final demand as accounted for in statistics based on value-added (above all GDP).

In this new context, we must reconsider how to track production-demand linkages, assess the spillovers of macroeconomic shocks, and evaluate the impact of trade policies. We also need to assess countries' participation in the international sharing of production. New, specific empirical questions have also emerged such as: What part of a country's exports can be ascribed to the value-added produced at home or abroad? How can we allocate this value-added across the different bilateral and sectoral trade flows? Which markets absorb this production as final demand? How important is GVC-oriented trade for a country's exports and imports? To what extent is a country's production affected by macroeconomic shocks emanating from other countries? How can it be affected by trade policies implemented in a given country on a given sector, and/or vis-à-vis certain partners? This is just a subset of all the key questions raised by the prevalence of GVCs.

This paper develops a new toolkit of methodologies for value-added accounting of trade flows at the country-, bilateral- and sectoral-levels by using Inter-Country Input-Output (ICIO) tables. The measures we propose can be used to properly address important empirical questions, such as those mentioned above, and to evaluate countries' and sectors' participation in GVCs. We argue that different empirical issues require distinct methodological approaches, and provide a mapping of the questions under investigation and the accounting methods needed to address them.

ICIO tables, like the WIOD, the OECD-WTO TiVA or the EORA, combine national input-output data with detailed trade statistics to trace cross-country and cross-sector interconnections (Timmer et al., 2015; OECD-WTO, 2014; Lenzen et al., 2013), and represent a key statistical tool for macroeconomic analysis that takes the international fragmentation of production into account. New analytical methodologies have been proposed to exploit these data, like those developed by Hummels et al. (2001), Johnson and Noguera (2012) and Koopman et al. (2014). These seminal contributions have significantly improved the investigation of production-demand linkages and knowledge of the value-added composition of gross trade flows when productions are unbundled across different countries and industries.[1] Nevertheless, relevant questions remain unanswered or, in some cases, are only partially addressed with the above methodologies. In general, bilateral exporter-importer relations and the sectoral dimensions of trade flows are overlooked in these works. This impairs a deeper analysis of the direct and indirect trade linkages between countries and sectors.

We fill this gap with a more detailed examination of the various dimensions of trade flows, which is also useful for arriving at a more precise representation of certain aggregate-level phenomena (e.g. exporting/importing country, region, world). There are several reasons why the bilateral and sectoral dimensions matter. First, firms trade with bilateral partners, even when participating in more complex multi-country production networks, and trade policies are usually implemented at the bilateral and sectoral level. Second, when studying the implications of GVCs, it is relevant to consider the position of a country (or sector) within the production chain and identify its direct upstream and downstream trade partners. This may be relevant in order to geographically map the production networks and analyze the international propagation of macroeconomic shocks, such as exchange rates' variations and inflation spillovers (Patel et al. 2016, Auer et al. 2017, De Soyres et al. 2018). Deepening the knowledge of these mechanisms is also useful for interpreting the short-term dynamics of trade flows. Finally, macroeconomic shocks and trade policies may produce heterogeneous effects on the different trade components, depending on the extent to which they are involved in GVCs (Ruta, 2017; Hofmann et al., 2018; Cook and Patel, 2018).

Through the decomposition of bilateral trade flows, we also provide a novel measure of the share of trade components related to GVCs, which consistently refines the vertical specialization index proposed by Hummels et al. (2001). This can be important, among other things, to assess how the evolution of GVCs influences

[1] Another strand of the literature has focused on the length of production chains and on how countries and sectors are specialized in the different stages of the production process (Fally, 2012; Antras et al., 2012; Antras and Chor 2013; Wang et al. 2016).

the long-term dynamics of trade (Borin et al. 2017, De Vries et al. 2017), or how countries' participation in GVCs affects bilateral trade balances—an aspect often at the center of international trade negotiations (Nagengast and Stehrer, 2016).

In general, we present detailed breakdowns of aggregate/bilateral/sectoral trade flows that offer broad scope for empirical investigations into global production networks both at the macro and micro levels. Other contributions, some developed in parallel with this work, tackle similar aspects (Wang et al., 2013; Los et al., 2016; Nagengast and Stehrer, 2016; Johnson, 2018; Miroudot and Ye, 2018; Los and Timmer, 2018). The presence of different concurring methodologies has raised doubts about the appropriate way to measure certain trends. We argue there is no unique correct methodology to address all possible empirical questions related to the value-added accounting of trade. Indeed, we show how different questions call for distinct approaches.

A key issue in the value-added accounting of trade regards the definition of 'double counted' components, i.e. items that are recorded several times in a given gross trade flow due to the back-and-forth shipments that occur in a cross-national production process (Koopman et al. 2014). We suggest the definition of double counting can change depending on the specific phenomenon under investigation. This gives rise to what we call alternative 'perspectives'. We argue that each one is conceptually suited to addressing different types of empirical issues. We also show how some of the methodologies proposed in other contributions fit into these different accounting perspectives. Indeed, our work manages to reconcile a large part of the existing literature under one comprehensive framework, noting the relations between a specific approach and the questions it is best suited to address. We also discuss a series of shortcomings and limitations that affect some of the techniques proposed in the literature. We show how this leads to imprecise or incorrect evaluations compared with those presented in this paper and in other contributions that entirely square with the proposed framework (e.g. Los et al. 2016; Johnson, 2018; Miroudot and Ye, 2018). Although there are several possible accounting decompositions, this does not mean all of them are conceptually correct or economically meaningful. Each method finds its theoretical justification in measuring a certain phenomenon properly and/or addressing an economically relevant question. Therefore, every decomposition of trade flows should meet two basic requirements: i) accuracy (i.e. each component should correctly identify what it is supposed to measure); and ii) internal consistency (i.e. each component should be consistent with the specific approach adopted). We show that, in some cases, the value-added decompositions proposed in the literature do not meet these criteria.

Yet this paper's contribution extends well beyond a critical review and systematic reorganization of existing methodologies, since most of the measures we propose do not find any correspondence in previous literature. For instance, we take into account novel 'perspectives' that allow us to properly address important empirical issues that have not yet been touched. Even the decompositions that share the same perspective as other contributions present relevant novelties that make the measures more accurate and/or exhaustive. For example, for each approach we also provide a consistent measure of the value-added generated in countries other than the exporter (i.e. the so-called 'foreign value added'). This aspect, overlooked in the literature, is key to assessing indirect linkages and spillovers through the supply chains.

The rest of the paper is organized as follows. Section 2 reviews the main existing methods to measure the origin and final destination of the value-added embedded in trade flows; we clarify how we can improve these measures by focusing on bilateral and sectoral relationships; we then discuss a number of key methodological issues and point out how different approaches can be adopted to address different questions. Section 3 presents two novel breakdowns of the domestic and foreign GDP in exports by bilateral partners, and shows how to include different sectoral dimensions in these decompositions; we also derive new indicators of GVC-related trade. Section 4 explores alternative perspectives for the value-added accounting of trade at the bilateral, sectoral and sectoral-bilateral levels. Section 5 examines in greater detail the relationship between our methodologies and other accounting frameworks proposed in the literature and discusses some critical aspects of the latter. Section 6 concludes.

2 Trade in value-added: Seminal contributions and methodological issues

A useful start is to review what we know about trade-production-demand nexuses from the existing input-output literature.[2] All the measures are based on a standard Inter-Country Input-Output (ICIO) model with G countries and N sectors. Appendix A gives an exhaustive definition of the notation and, for this reason, here

[2]When possible, we consider a formulation of these measures suitable for bilateral trade flows, so we can easily compare them with the methodologies proposed in the following sections. However, there are measures that can be applied only at the multilateral level (e.g. for the total exports of a country).

we only mention that $\mathbf{E}_{sr}$ is the $N\times 1$ vector of exports of country s to country r, $\mathbf{X}_s$ is the $N\times 1$ vector of gross output produced by country s, $\mathbf{A}$ is the $GN\times GN$ global matrix of input coefficients, $\mathbf{B}$ is the global Leontief inverse matrix for the entire inter-country model and $\mathbf{V}_s$ is the $1\times N$ vector that incorporates the value-added shares embedded in each unit of gross output produced by country s.

In their seminal contribution, Hummels et al. (2001) propose to split gross exports between a share of domestically produced items and a share of imported inputs embedded in exports, where the latter takes into account both the direct and indirect production linkages within the domestic market. Hummels et al. (2001) define the share of import content in exports as the 'Vertical Specialization Index' (**VS**) which can also be computed for bilateral exports between s and r. In an ICIO framework the $\mathbf{VS}_{sr}$ indicator can be expressed as:[3]

$$\mathbf{VS}_{sr} = \mathbf{u}_N \sum_{t\neq s}^{G} \mathbf{A}_{ts}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr}/\mathbf{u}_N\mathbf{E}_{sr}, \tag{1}$$

where $\mathbf{u}_N$ is the $1\times N$ unit row vector and $(\mathbf{I} - \mathbf{A}_{ss})^{-1}$ represents the domestic inverse Leontief matrix.

Although the **VS** indicator provides valuable information for several empirical applications,[4] there are relevant aspects that are not taken into account. In particular, imported inputs in exports are considered as a single category, without distinguishing between the part that originated abroad and the part that was originally produced by s itself and then re-imported.

Indeed, by exploiting Inter-Country Input-Output tables, gross exports can be broken down according to the country that initially produced each component. Koopman et al. (2010) use the global Leontief inverse matrix to trace back the total gross output produced by each country j to deliver one unit of country s exports ($\mathbf{B}_{js}$), and the related value-added shares ($\mathbf{V}_j$). The part that originated in country s is referred to as the 'domestic content of exports' ($\mathbf{DC}_{sr}$), whereas the remaining

[3]Hummels et al. (2001) measures can be computed without resorting to an Inter-Country Input-Output database since they were developed for national Input-Output tables that distinguish only between imported and domestic inputs.

[4]In particular, later in the paper we show that the **VS** indicator is a good measure of the participation of a country in the downstream phases of international production chains. Moreover, the complementary part of the **VS** share of exports is one of the possible measures of domestic value-added embedded in exports (see also Johnson, 2018, on the same point).

part is called the 'foreign content of exports' ($\mathbf{FC}_{sr}$, Koopman et al. 2014):

$$\mathbf{u}_N\mathbf{E}_{sr} = \underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{sr}}_{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}}_{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{2}$$

In the same way gross exports are decomposed by country of origin in (2), they can be classified according to the country of ultimate absorption in final demand. $\mathbf{E}_{sr}$ consists of final goods ($\mathbf{Y}_{sr}$) and intermediate inputs for the production of gross output in country r ($\mathbf{X}_r$):

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}\mathbf{X}_r, \tag{3}$$

where $\mathbf{Y}_{sr}$ indicates the $N\times1$ vector of final goods completed in s and consumed in r. The intermediate inputs imported by country r ($\mathbf{X}_r$) can be linked with the country of final completion and the market of ultimate demand. According to basic I-O accounting relations (see equation A.2 in the Appendix), all the remaining (and potentially infinite) stages of production are accounted for by the Leontief inverse matrix $\mathbf{B}$:

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{rk}\mathbf{Y}_{kl}, \tag{4}$$

where k and l might be the exporting country s itself, and the generic $\mathbf{Y}_{ij}$ indicates the final goods completed in country i and finally sold in country j.

Relationships in (2) and in (4) add relevant information to traditional gross trade statistics (i.e. the very origin and the ultimate destination of exports). Nevertheless, it is still 'gross accounting', in the sense that these measures include the items that cross country s's borders several times along the production process. Only by isolating these double counted items, we can measure the 'net' production (value-added) embedded in exports, akin to countries' GDP.

Johnson and Noguera (2012) contribute to filling this gap by proposing a way to measure the share of a country's GDP that is absorbed abroad. In particular, in a global input-output framework the 'net' value-added produced by s and absorbed abroad (the so-called 'value-added export', $\mathbf{VAX}_s$) can be computed as follows:

$$\mathbf{VAX}_s = \mathbf{V}_s\sum_{k}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{sk}\mathbf{Y}_{kl}. \tag{5}$$

On the one hand, this measure is comparable to value-added statistics, like GDP, and it is useful to link the 'net' production with the specific market of final absorption; on the other hand, the $\mathbf{VAX}_s$ is only a fraction of the country s's GDP embedded in its exports since it does not consider the portion that is later re-imported and absorbed at home; moreover, this approach does not allow us to identify the trade linkages through which the value produced in s reaches the market of final destination l. The latter aspect could be relevant in many cases, including the analyses of international supply networks and evaluation of trade policies.

Koopman et al. (2014) combine the measures of the content of exports (i.e. (2)) with the index of 'Value Added Exports' proposed by Johnson and Noguera (2012). More generally, their accounting framework allows to single out the entire domestic and foreign value-added embedded in the aggregate exports of country s, as well as the double counted items originally produced at home and abroad. In particular, they highlight that some trade flows are purely double-counted, such as when intermediate inputs cross a country's borders several times during the different stages of production. The complete decomposition proposed by Koopman et al. (2014) is reported in Appendix B whereas Figure 1 shows a simplified scheme of their breakdown of aggregate exports. Notice that $\mathbf{VAX}_s$ is a subcomponent of the domestic value-added embedded in gross exports, the remaining part being the value-added that is finally absorbed by the exporting country itself (labeled 'Reflection' by Koopman et al., 2014).

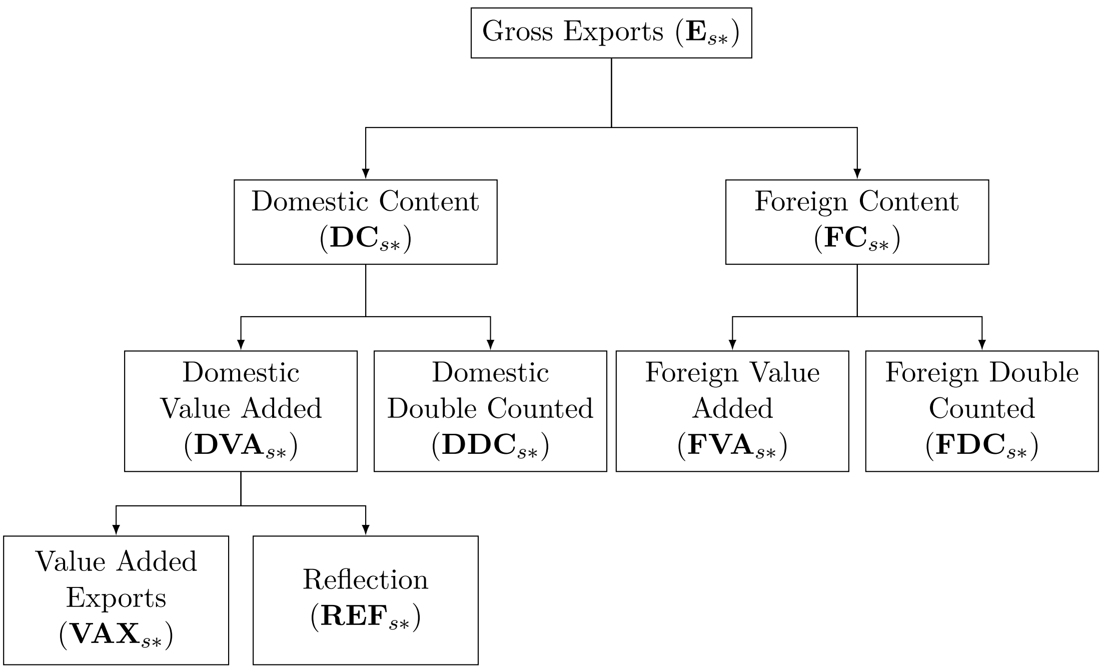

Figure 1: A basic scheme of the Koopman et al. (2014) decomposition of total exports

The Koopman et al. (2014) breakdown applies only to aggregate exports, whereas investigating the value-added content of trade at the bilateral level (and/or sectoral level) is crucial in many cases. For instance, this method has limited scope of application in the assessment of trade policies and in many analyses of international production linkages. Moreover, some of the components of the Koopman et al. (2014) breakdown are imprecisely defined, as extensively discussed in Section 5.2.

Our main contribution is to provide exhaustive and rigorous value-added decompositions of exports (and imports) at the aggregate, bilateral and sectoral levels. Some specific methodological issues arise when considering disaggregated shipments, instead of aggregate exports. Our claim is that there is no such thing as a unique method to account for value-added in disaggregated trade flows, and we must consider different measures to address different empirical questions.

A key matter is the choice of perspective the analysis should take to address a particular problem. This perspective defines the perimeter according to which something is classified as 'value-added' or 'double counted', the latter being the items that cross this perimeter more than once. For instance, the boundaries may be defined at the level of the exporting country (or the importing one), or of a specific bilateral relation, or of a single exporting sector within a bilateral flow. For instance, one of the most common questions is 'what part of a country's GDP is exported?' This can be answered with a 'country-level perspective': the exporting country's frontiers as a whole constitute the perimeter that matters in in deciding whether or not a certain item has to be classified as domestic value-added (GDP) or double-counted.[5] This country-level perspective should also be considered in the decomposition of disaggregated trade flows (bilateral and sectoral) when we seek a measure of 'net' trade that can be added up to the exporter's GDP in its total exports.

Alternative perspectives are better suited to address other issues. For instance, suppose a tariff is imposed on the imports of a given sector from a certain partner, and we are interested in evaluating how this policy affects the GDP of the exporting country. In this case we want to consider as 'value-added' the entire GDP that is involved in this sectoral-bilateral relationship, even if part of that was previously exported to other countries/sectors (i.e. double counted in a 'country-level

[5]Indeed, the definition of domestic value-added in Koopman et al. (2014) follows precisely what we call a 'country-level perspective'. It is worth noting that the accounting of the foreign content in exports in Koopman et al. (2014) follows a different perspective. This issue is discussed in more detail in Section 5.1.

perspective'). The specific sectoral-bilateral relationship become the new relevant perimeter, and only the items that enter multiple times in this trade flow are considered as 'double counted'. Indeed, this is what we call a pure 'sectoral-bilateral' perspective.[6]

We consider the perspectives that seem more economically important. Thus, we show how domestic and foreign 'value-added' and domestic and foreign 'double counted' terms should be accounted for when the perimeter of interest is defined at the levels of: i) exporting country as a whole, ii) a bilateral relationship, iii) sectoral-bilateral one, iv) sector of export, v) importing country as a whole, vi) sector of import.[7] Table 2 in Appendix E summarizes all the measures proposed in the paper.

It is worth noting that the accounting perspective employed does not necessarily coincide with the level of disaggregation of trade flows considered in the analysis. For instance, in Section 3, we show that in some cases it may be interesting to consider a value-added decomposition of bilateral and sectoral trade flows while employing the perspective of the exporting country as a whole (that we call 'country-level perspective').

3 The decomposition of gross exports in a country-level perspective

The country-level perspective presented in this section is suited to measuring the GDP of a country that is embedded in its own total exports ('domestic value added') or in the total exports of any other country ('foreign value added'). In this framework, value-added indicators measured at the bilateral and/or sectoral level can be added up to the total GDP exported by a country. For this reason, we examine the value-added decompositions of disaggregated trade flows, since the correspondent components for the total exports of a country can be simply obtained by summing across the different bilateral (or sectoral-bilateral) trade flows.

A specific methodological issue arises when measuring the value-added in

[6]See also Johnson (2018) and Los and Timmer (2018) on a similar point.

[7]In Section 5.1, we discuss the accounting of foreign value-added based on a 'world level perspective' (i.e. when a given item is accounted for as 'foreign value added' only once in all the possible trade flows), since it has been employed in other contributions (Koopman et al. 2014; Wang et al. 2013; Miroudot and Ye, 2018). However, we argue that the exports' decompositions based on this perspective do not provide any economically meaningful information in addition to what we obtain through the other perspectives.

bilateral exports by using a country-level perspective. In this case, when a certain portion of value-added crosses the border of the same country more than once, it has to be assigned to a particular gross bilateral trade flow whereas it is recorded as double counted in the others. The question is: when should we consider it as 'domestic value-added' versus 'double counted'? Nagengast and Stehrer (2016) propose two alternative approaches: the first takes the perspective of the country where the value-added originates (the source-based approach), the second that of the country that ultimately absorbs it in final demand (the sink-based approach).

A concrete example can help clarify this point. In the production-trade-demand scheme depicted in Figure 2 the 1 USD of value-added originally produced in A is first exported to B as intermediate inputs, processed there, then shipped back to A and used to produce final goods for re-export to C. In this case, the value-added generated in the very first stage of production in A is counted twice in its aggregate gross exports (the first time in the bilateral exports toward B, the second time in the shipments to C).

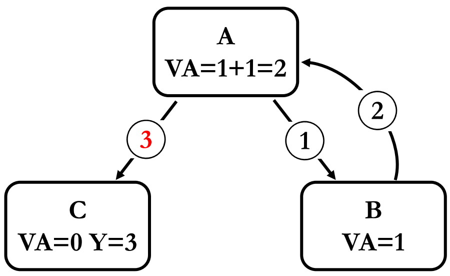

Figure 2: Value-added and double counting in bilateral trade flows

According to the source-based approach, the original 1 USD of production of country A would be considered as 'domestic value-added' in A's exports to B (and 'double counted' in its shipments to C); vice versa, using the sink-based approach it would be considered as 'domestic value-added' in A's exports to C (and 'double counted' in its shipments to B). Table 1 shows the complete value-added accounting of country A's exports in the source and sink logic.

In short, we can say that the source-based method accounts for the value-added the first time it leaves the country of origin, whereas the sink-based approach considers it the last time it crosses the national borders. Again, the choice between the two frameworks depends on the particular empirical issue we want to address.

Table 1: Source and sink accounting of country A's exports in Figure 2

Trade flow →	A→ B				A→ C				
	DVA	**DDC**	**FVA**	**FDC**	**DVA**	**DDC**	**FVA**	**FDC**	**Gross exports**
Source	1	0	0	0	1	1	1	0	4
Sink	0	1	0	0	2	0	1	0	4

In the source-based approach the value-added is recorded as closely as possible to the moment when it is produced. Then, it is designed to examine the production linkages and the country/sector participation to different types of production processes and to study the features of the production processes in which export flows are involved. For instance, by drawing on this breakdown of gross trade, we derive a set of indicators of GVC-related trade that consistently extend the 'Vertical Specialization' index (**VS**) proposed by Hummels et al. (2001).

Conversely, the value-added in the sink-based approach is recorded as closely as possible to the moment when it is ultimately absorbed. This makes it more suited to studying the relationship between production and final demand. The sink-based approach should be adopted to study the value-added composition of final goods' exports or examine the role of a country's final demand in activating productions and trade flows, such as, for instance, in an analysis of bilateral trade balances (Borin and Mancini, 2016a, 2016b; Nagengast and Stehrer, 2016).

We provide both source-based and sink-based decompositions of bilateral exports that are accurate and internally consistent. Since they are based on a country-level perspective, in both the source and the sink decompositions the sum of the domestic value-added in exports across all the bilateral flows yields the total GDP embedded in exports.

Operationally, we develop the two breakdowns of bilateral trade flows starting from the common basic scheme of Koopman et al. (2014), depicted in Figure 1. This means the gross exports from country s to country r ($\mathbf{E}_{sr}$) are decomposed according to the country of origin (e.g. the domestic content $\mathbf{DC}_{sr}$ and the foreign content $\mathbf{FC}_{sr}$), further distinguishing between the components of 'net' production (e.g. the domestic value-added $\mathbf{DVA}_{sr}$ and the foreign value-added $\mathbf{FVA}_{sr}$) and the double counted items (domestic, $\mathbf{DDC}_{sr}$, and foreign, $\mathbf{FDC}_{sr}$). Then, further details are added to take the final market dimension into account. By focusing on bilateral trade flows, we can follow the pattern of value-added in exports along the different phases of the value chain. However, the input-output framework potentially allows for infinite rounds of production. Hence, we face a trade-off between adding details on the international production linkages and providing an analytically tractable and conceptually intelligible framework. Our compromise is to track the direct importing country, then—if the value-added is not absorbed there—we consider the additional destinations of re-export from the direct importers.

In summary, our strategy is to decompose gross bilateral trade flows by identifying the following actors: *i*) the country of origin of value-added; *ii*) the direct importers; *iii*) the (eventual) second destination of re-export; *iv*) the country of completion of final products; *v*) the final destination market.

We apply the same notion of double counting both for the domestic value-added and for the value-added originated abroad, i.e. they are both defined according to the (exporting) country perspective. In this way by summing the foreign value-added terms across the different bilateral destinations (r) we obtain the total value-added (i.e. GDP) produced by a given country t and exported by country s. This notion differs from the definition of foreign value-added and foreign double counted most commonly used in the literature (Koopman et al., 2014; Wang et al., 2013; Nagengast and Stehrer, 2016; Miroudot and Ye, 2018). We discuss this issue in more detail in Section 5.1.

In Section 3.3 we show how the decompositions of exports based on a country-level perspective (both source and sink) can be extended to take into account the sectoral dimension. In particular, we consider three different sectoral breakdowns: i) by sector of origin; ii) by sector of export; iii) by sector of final absorption.

3.1 Bilateral source-based breakdown

In the source-based breakdown of bilateral exports, the value produced in a country is accounted for as 'value-added' the first time it crosses the exporter's national borders. For instance, in the basic production process depicted in Figure 3, the value produced in 'phase 1' is accounted for as 'value-added' in the first shipment from s, whereas in the last shipment it is considered as 'double counted'. Indeed, in the decomposition of 'shipment 2' from s, only the value produced by s and r in 'phase 2' is accounted for as 'value-added'. In other words, starting from 'shipment 2', we should go back up the production chain to 'shipment 1' (i.e. up to the point in which country r imports intermediate inputs from country s). In general the distinction between 'value-added' and 'double counted' can be made by splitting the production chain in phases, each one delimited by an export flow of country s: what is generated within that particular production phase is accounted for as 'vale added' in exports, what comes from further upstream production stages is 'double counted'.

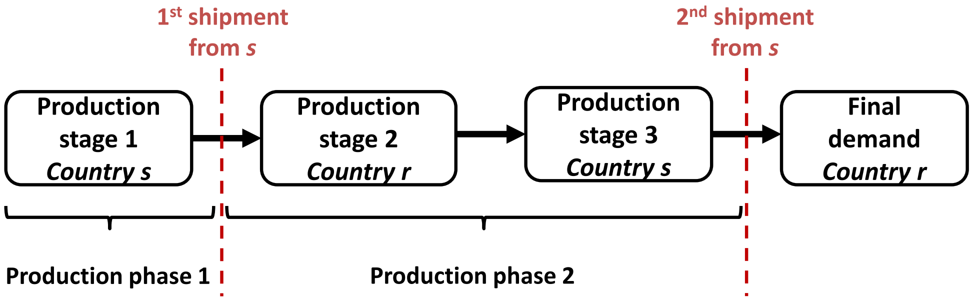

Figure 3: Breakdown of production phases

How can we implement this partition of the production process in a general ICIO framework?

In the decomposition of gross exports in equation (2) the Leontief inverse matrix $\mathbf{B}$ takes into account all the backward production linkages that precede a certain export flow, and this leads to the double counting issue. We propose singling out the value-added components by modifying the matrix $\mathbf{B}$ in such a way that we can slice down the production process along the outward boundaries of the exporting country s. To implement this operationally, it's useful to consider the representation of the global Leontief inverse as a sum of infinite series of the gross output generated in all the upstream stages of the production process:

$$\mathbf{B} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^n \quad n \to \infty. \tag{6}$$

We can split the production process along country s's borders by cutting out its intermediate export linkages in any stage of the above series. Algebraically, we set to zero the coefficients of matrix $\mathbf{A}$ that identify the direct requirement of intermediate inputs from country s (i.e. $\mathbf{A}_{sj} = 0 \; \forall \; j \neq s$):

$$\mathbf{A}^{\not s} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1s} & \cdots & \mathbf{A}_{1G} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{ss} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{G1} & \mathbf{A}_{G2} & \cdots & \mathbf{A}_{Gs} & \cdots & \mathbf{A}_{GG} \end{bmatrix}. \tag{7}$$

Then, the corresponding inverse Leontief matrix is:

$$\mathbf{B}^{\nexists} = (\mathbf{I} - \mathbf{A}^{\nexists})^{-1}. \tag{8}$$

In Appendix B we show how $\mathbf{B}^{\nexists}$ is related to the original global Leontief inverse matrix. In particular, we can see that the following relation holds:

$$\mathbf{B}_{is} = \mathbf{B}^{\nexists}_{is} + \mathbf{B}^{\nexists}_{is} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js}, \tag{9}$$

where i could be either s or a different country.

The relation in equation (9) can be used to refine the bilateral version of the decomposition in equation (2) so that we can single out the 'value-added' and 'double counted' terms within each component:

$$\mathbf{u}_N \mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s \mathbf{B}^{\nexists}_{ss} \mathbf{E}_{sr}}_{\substack{\text{domestic value}\\ \text{added}\\ (\mathbf{DVA}source_{sr})}} + \underbrace{\mathbf{V}_s \mathbf{B}^{\nexists}_{ss} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js} \mathbf{E}_{sr}}_{\substack{\text{domestic double}\\ \text{counted}\\ (\mathbf{DDC}source_{sr})}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}^{\nexists}_{ts} \mathbf{E}_{sr}}_{\substack{\text{foreign value}\\ \text{added}\\ (\mathbf{FVA}source_{sr})}} + \underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}^{\nexists}_{ts} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js} \mathbf{E}_{sr}}_{\substack{\text{foreign double}\\ \text{counted}\\ (\mathbf{FDC}source_{sr})}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{10}$$

Equation (10) reports a source-based breakdown of bilateral exports according to the main items identified in Koopman et al. (2014) for aggregate flows (see Figure 1). The double counted items are measured by isolating the portion of country s exports to r that have been already exported by s in a previous stage of the production process.[8]

As regards the domestic components, it is worth noting that $\mathbf{B}^{\nexists}_{ss}$ corresponds to the so-called local Leontief matrix $(\mathbf{I} - \mathbf{A}_{ss})^{-1}$ (see equation (D.9) in Appendix B). This means the domestic value-added in exports in the source-based approach is obtained by isolating all the domestic stages of production needed to produce the

[8]Part of the intermediate goods exported by country s ($\sum_{j \neq s}^{G} \mathbf{A}_{sj} \mathbf{X}_j$) are later re-imported by s itself and enter again its exports, generating in this sense a double-counted item in a source based framework. In particular, we are interested in the intermediate goods shipped abroad that re-enter in the exports from s to r, following any possible production pattern ($\sum_{j \neq s}^{G} \mathbf{A}_{sj} \mathbf{X}_j^{(\rightarrow \mathbf{E}_{sr})}$). This can be computed as: $\sum_{j \neq s}^{G} \mathbf{A}_{sj} \mathbf{B}_{js} \mathbf{E}_{sr}$.

exported goods, while ignoring the domestic content of imported inputs:

$$\mathbf{DVA}source_{sr} = \mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr}. \tag{11}$$

Notably, this measure of domestic value-added in exports represents the complement to the 'import content of exports' proposed by Hummels et al. (2001) (i.e. the numerator of the 'Vertical Specialization' index $\mathbf{VS}_{sr}$ of equation (1), see proof in Appendix C). It also corresponds to the measure of domestic value-added proposed in Johnson (2018), but only in the two-country world the author considers. In a more general framework with several countries and sectors, the approach suggested by Johnson (2018) seems different from the country-level source-based one presented here. We discuss this point more extensively in Section 4.1, where we propose an alternative decomposition of gross exports that generalizes to an n-country context the approach suggested by Johnson (2018).

In addition to the breakdown of the value-added by country of origin, the literature has also considered the relationship with the market of final absorption (Johnson and Noguera, 2012, Koopman et al., 2014). In a bilateral context we can dig deeper into the forward production linkages and into the connections with final demand. Since infinite rounds of production could occur before an intermediate product reaches the final demand, we stress that our choice is to identify the direct importer, the (potential) second destination of re-export, the country of completion of final products and the ultimate destination market. As a first step, we can split bilateral exports $\mathbf{E}_{sr}$ into final goods ($\mathbf{Y}_{sr}$) and intermediate inputs required by the production of gross output of country r ($\mathbf{X}_r$):

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}\mathbf{X}_r. \tag{12}$$

In country r, in turn, the intermediate inputs imported from s undergo one or more processing phases to produce final products for domestic consumption or goods for re-export (both intermediate and final):

$$\mathbf{A}_{sr}\mathbf{X}_r = \mathbf{A}_{sr}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I} - \mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}. \tag{13}$$

Then, (re)exports from country r can be split into intermediate goods and final products:

$$\mathbf{E}_{r*} = \sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{X}_j. \tag{14}$$

At this point, we can link the intermediate inputs imported by country j with the country of final completion and the market of ultimate demand. According to basic I-O accounting relations (see equation A.2 in the appendix), all the remaining (and potentially infinite) stages of production are accounted for by the Leontief inverse matrix $\mathbf{B}$:

$$\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{X}_{j} \quad = \quad \sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}. \tag{15}$$

By combining equations (11) to (15), we can obtain a comprehensive source-based decomposition of domestic and foreign value-added in bilateral exports:

$$\begin{aligned}\mathbf{DVA}source_{sr} = \mathbf{V}_{s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[&\mathbf{Y}_{sr}+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}\\ &+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg],\end{aligned} \tag{16}$$

$$\begin{aligned}\mathbf{FVA}source_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_{t}\mathbf{B}_{ts}^{\not s}\Bigg[&\mathbf{Y}_{sr}+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}\\ &+\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg].\end{aligned} \tag{17}$$

It is worth recalling that the two subscripts on final demand matrix $\mathbf{Y}$ refer to the country of final completion and the market of final absorption.[9] We can then consider specific subportions of the final demand matrix, for instance, to distinguish between the domestic value-added ultimately absorbed in the country of origin s (i.e. the 'reflection' terms in Koompan et al., 2014 terminology) or in a foreign market (i.e. the 'value-added exports', or $\mathbf{VAX}_{sr}$, in Johnson and Noguera, 2012 nomenclature):[10]

[9]For instance $\mathbf{Y}_{kl}$ identifies the vector of goods finalized in k and sold in l.

[10]We present some specific classification of the value-added embedded in bilateral trade flows (e.g. according to whether it originates at home or abroad, or according to whether it is finally absorbed domestically or in another country). In fact, through the selection of some particular subcomponent from equations (16) and (17), it is possible to isolate any specific country of origin (s and t) and market of final absorption (r, j and l); in addition, we could also identify the countries of re-export (j) and those where final products are completed (s, r and k).

$$\mathbf{REF}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\left[\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rs} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\mathbf{B}_{jk}\mathbf{Y}_{ks}\right] \quad (18)$$

$$\mathbf{VAX}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r,s}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\right]. \quad (19)$$

The first two terms of equation (19) represent the value-added generated in s and absorbed directly by the importer country r without any further re-export (i.e. the 'directly absorbed value-added in exports', or $\mathbf{DAVAX}_{sr}$):[11]

$$\mathbf{DAVAX}_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{sr} + \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} \quad (20)$$

Measuring Global Value Chain-related trade

The **DAVAX** is a measure of what is produced entirely at home and consumed abroad ($\mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{sr}$), and of the intermediate inputs that are (entirely) produced at home and used by the importing country to produce final goods for its internal market ($\mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}$). In this sense it identifies the 'traditional' type of trade/production as opposed to the international shipments that take place under the global sharing of production ('GVC-related trade'). In other words, the 'GVC-related trade' includes all the traded items that cross at least two international borders, i.e. that are re-exported at least once before being absorbed in final demand. This can be considered as a sufficient condition for an exported good to be part of an international production network.

In a bilateral trade flow, the 'GVC-related trade' can be measured simply by excluding from country s's gross exports the domestic value-added absorbed

[11] This indicator can be computed only in a source-based approach and differs from the sum of the first two terms in the Koopman et al. (2014) decomposition, even when summing across all the bilateral destinations. Indeed, although Koopman et al. (2014) refer to these terms as 'exports absorbed by direct importers', in Section 5.2 we show that these subcomponents are not correctly identified in their decomposition.

directly by its importer ($\mathbf{DAVAX}_{sr}$):

$$\mathbf{GVCX}_{sr} = \mathbf{u}_N \mathbf{E}_{sr} - \mathbf{DAVAX}_{sr}. \tag{21}$$

Therefore, the GVC share in bilateral exports is

$$\mathbf{GVC}_{sr} = \frac{\mathbf{GVCX}_{sr}}{\mathbf{u}_N \mathbf{E}_{sr}}, \tag{22}$$

which can be computed for the exporting country s as a whole:

$$\mathbf{GVC}_{s} = \frac{\sum_{r \neq s}^{G} \mathbf{GVCX}_{sr}}{\mathbf{u}_N \mathbf{E}_{s*}}, \tag{23}$$

or even at world level:

$$\mathbf{GVC}_{world} = \frac{\sum_{s}^{G} \sum_{r \neq s}^{G} \mathbf{GVCX}_{sr}}{\sum_{s}^{G} (\mathbf{u}_N \mathbf{E}_{s*})}. \tag{24}$$

The GVC-related trade indicator proposed above is not the first measure based on ICIO tables that has been developed to gauge the relevance of GVCs in international shipments. Following the seminal article of Hummels et al. (2001), various measures of a country or a region's integrations in international production networks have been proposed (Johnson and Noguera, 2012; Rahman and Zhao, 2013; Los et al., 2018). The 'vertical specialization' index (**VS**) of Hummels et al. (2001) is probably one of the first and most popular of these measures. However, as pointed out by the authors themselves, this is a partial measure of participation in global value chains, since it only considers the backward linkages (i.e. it measures the import content of a country's exports, see equation (1)). In order to take forward linkages into account, Hummels et al. (2001) also suggest considering the exports of intermediate products that later are further processed and re-exported (they label it **VS1**). However, they do not propose a precise formulation of this measure, since it can be implemented only in a fully-fledged ICIO framework that was not available at the time of writing.

By exploiting the bilateral source-based decomposition presented here above, we can provide a precise measure of the share of exports related to 'forward' supply linkages ($\mathbf{GVC}forward$ or **VS1** in Hummels et al., 2001 nomenclature), which can be computed as the difference between $\mathbf{DVA}source$ and **DAVAX**. Then, the overall **GVC** indicator of equation (22) can be decomposed into a 'backward' component, corresponding to the **VS** Index (see proof in Appendix C) and 'forward' component, which can be considered the first correct implementation of the **VS1** indicator suggested by Hummels et al. (2001):

$$\mathbf{GVC}_{sr} = \underbrace{\mathbf{GVC}backward_{sr}}_{\mathbf{VS}_{sr}} + \underbrace{\mathbf{GVC}forward_{sr}}_{\mathbf{VS1}_{sr}} \tag{25}$$

where

$$\mathbf{GVC}backward_{sr} = \frac{\mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\sum_{j\neq s}^{G}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr} + \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}}{\mathbf{u}_N\mathbf{E}_{sr}} \tag{26}$$

and

$$\mathbf{GVC}forward_{sr} = \frac{\mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}(\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl})}{\mathbf{u}_N\mathbf{E}_{sr}}. \tag{27}$$

As for the overall indicator of 'GVC-related trade', its subcomponents can also be computed for the total exports of a country s (or even at the world level).

Notably, $\mathbf{GVC}forward_s$ differs from the version of the $\mathbf{VS1}_s$ Index proposed by Koopman et al. (2014), since they compute it by aggregating the content of a country's production embedded in other countries' exports (i.e. $\mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{sr}\mathbf{E}_{r*}$). While the $\mathbf{GVC}forward$ index is a portion of country s's exports (like **VS**), this does not necessarily hold true for the measure proposed by Koopman et al. (2014). Suppose, for instance, that a certain intermediate component exported by country s later undergoes other processing phases in different countries; the original component will be double-counted several times in the summation of country s's content in other countries' exports. The discrepancy between the original value of goods exported by s and the related amount that enters in Koopman et al.'s (2014) indicator increases with the relative 'upstreamness' of country s's production. This is a feature that refers to the relative positioning of a country in GVCs and that has been specifically addressed in the literature through proper tools.[12] Moreover, this positioning does not directly influence the **VS** indicator which is commonly used as the 'backward'-participation counterpart of the **VS1** indicator proposed by Koopman et al. (2014).[13] Conversely, the $\mathbf{GVC}forward$ of equation (27) measures the share of a country's exports related to forward GVC linkages in a way that

[12]Indicators of relative upstreamness/downstreamness in GVCs have been proposed by Fally, 2012; Antras et al., 2012; Antras and Chor 2013; Wang et al. 2016.

[13]The $\mathbf{VS}_s$ Index does not vary with the number of borders crossed by a certain item before being imported by country s. In other words, the relative 'downstreamness' of country s does not influence the $\mathbf{VS}_s$ indicator in the same way as its relative 'upstreamness' influences the $\mathbf{VS1}_s$ indicator in the formulation of Koopman et al. (2014).

is consistent with how the **GVC***backward* (i.e. **VS**) measures the portion that is related to backward GVC connections.

Finally, other studies have measured a country's forward GVC participation by identifying the export components that are later re-exported by the direct importer, as we propose here (see, among others, Rahman and Zhao, 2013; Cappariello and Felettigh, 2014; Ahmed et al., 2017; Altomonte et al. 2018). However, these contributions rely on the decomposition of gross exports of Koopman et al. (2014) or, alternatively, on that of Wang et al. (2013). The problem, discussed in more detail in Section 5.2, is that these methodologies do not properly allocate countries' exports between the share that is directly absorbed by importers and the one that is re-exported abroad. The resulting measures of GVC participation are also imprecise.

3.2 Bilateral sink-based breakdown

While the source-based approach discussed above is most suited to examining the production linkages, the sink-based approach is most appropriate when the focus is on final demand and how it is related to the total value-added produced in a country. That's because in the sink-based decomposition a given item is accounted for as 'value-added' the last time it leaves national borders, and, in the case of multiple crossing, it is considered 'double counted' in prior shipments. Reconsidering, for instance, the illustrative example of Figure 3, the whole value-added generated in phases 1 and 2 is accounted for as such in the last shipment from country s (i.e. in shipment 2), whereas the value of shipment 1 is entirely attributed to the double-counted term. Then, in order to single out the 'value-added' components in a sink-based framework, it is necessary to isolate the portion of ultimate shipments within a certain bilateral trade flow. These 'ultimate exports' ($\mathbf{E}_{sr}^{(\overset{\not{s}}{\longrightarrow}\mathbf{Y}_*)}$) are made up of final goods ($\mathbf{Y}_{sr}$) and of intermediate goods that do not re-enter country s's exports, before reaching the ultimate destination ($\mathbf{A}_{sr}\mathbf{X}_j^{(\overset{\not{s}}{\longrightarrow}\mathbf{Y}_*)}$). Since the latter are commensurate with final goods as concerns the exporting country s, the overall value-added can be computed by pre-multiplying the vector of 'ultimate exports' by the **VB** matrix. In other words, once the part of 'ultimate exports' is singled out, the value-added in exports can be computed in the same way as how the **VBY** matrix is used to measure the total value-added in final demand (see Appendix A). In particular, the global Leontief inverse matrix **B** takes into account all the upstream production stages, as required by the sink-based approach. Conceptually, assuming that we can split the bilateral exports between ultimate shipments ($\mathbf{E}_{sr}^{(\overset{\nexists}{\rightarrow}\mathbf{Y}_*)}$) and exports of intermediates that later on will be re-exported by s itself ($\mathbf{E}_{sr}^{(\rightarrow\mathbf{E}_{s*})}$), the essential value-added breakdown of bilateral exports in a sink-based framework can be expressed as follows:

$$\mathbf{u}_N\mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{sr}^{(\overset{\nexists}{\rightarrow}\mathbf{Y}_*)}}_{\substack{\text{domestic value}\\ \text{added }(\mathbf{DVA}sink_{sr})}} + \underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{sr}^{(\rightarrow\mathbf{E}_{s*})}}_{\substack{\text{domestic double}\\ \text{counted }(\mathbf{DDC}sink_{sr})}}}^{\substack{\text{domestic}\\ \text{content }(\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}^{(\overset{\nexists}{\rightarrow}\mathbf{Y}_*)}}_{\substack{\text{foreign value}\\ \text{added }(\mathbf{FVA}sink_{sr})}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}^{(\rightarrow\mathbf{E}_{s*})}}_{\substack{\text{foreign double}\\ \text{counted }(\mathbf{FDC}sink_{sr})}}}^{\substack{\text{foreign}\\ \text{content }(\mathbf{FC}_{sr})}}. \tag{28}$$

In order to make the breakdown in equation (28) operational, we have to identify the ‘ultimate shipping’. We then proceed by disentangling the bilateral flow $\mathbf{E}_{sr}$, as we did for the source-based approach to identify the downstream linkages with final demand. By making use of the relations in equations (12)–(14), we can express the bilateral exports as follows:

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{X}_j. \tag{29}$$

The first three terms in the right-hand side of equation (29) are clearly part of the ‘ultimate shipment’ of country s, since the value-added reaches final demand without any re-shipment from s. At this point, we need to define the part of country j’s output that passes through country s’s export borders ($\mathbf{X}_j^{(\rightarrow\mathbf{E}_{s*})}$) and the part that reaches the final demand without any re-shipment from s ($\mathbf{X}_j^{(\overset{\nexists}{\rightarrow}\mathbf{Y}_*)}$). The problem is very similar to the one we faced in the source-based decomposition, when we singled out the portion of exports that crossed country s’s export border for the first time. Also in this case, the problem can be solved algebraically by recurring to the modified version of the Leontief inverse matrix that excludes the intermediate export linkages from country s ($\mathbf{B}^{\nexists}$). In this way, we can take into account all the possible patterns through which country j’s output reaches the final demand, with the exception of those that involve a re-export from s. It is worth noting that in this case country s’s exports of final goods also have to be excluded.

In order to derive the required splitting of country j’s output, we can re-express the general relationship of production and trade in our global I-O setting (see A.1 in Appendix A) by separating the export flows from country s as follows:

$$\mathbf{X} = \mathbf{A}^{\not s}\mathbf{X} + \mathbf{A}^{s}\mathbf{X} + \mathbf{Y}^{\not s} + \mathbf{Y}^{s}, \tag{30}$$

where $\mathbf{A}^{s} = (\mathbf{A} - \mathbf{A}^{\not s})$, $\mathbf{Y}^{\not s}$ is the final demand matrix $\mathbf{Y}$ with the block matrix corresponding to exports of final goods from s equal to 0 (but including domestic final demand $\mathbf{Y}_{ss}$), and $\mathbf{Y}^{s}$ is simply equal to $(\mathbf{Y} - \mathbf{Y}^{\not s})$. Given that the sum of $\mathbf{A}^{s}\mathbf{X}$ and $\mathbf{Y}^{s}$ is a $GN{\times}N$ matrix with the total exports from country s in the corresponding block submatrix and zeros elsewhere ($\mathbf{E}^{s}$), we can re-arrange (30) as follows:

$$\mathbf{X} = \mathbf{B}^{\not s}\mathbf{Y}^{\not s} + \mathbf{B}^{\not s}\mathbf{E}^{s}, \tag{31}$$

where $\mathbf{B}^{\not s} \equiv (\mathbf{I} - \mathbf{A}^{\not s})^{-1}$ is the Leontief inverse matrix derived from the new input coefficient matrix $\mathbf{A}^{\not s}$, which excludes the input requirement of other economies from country s.[14]

By applying the new accounting relationship in (31), country j's gross production can be decomposed as:

$$\mathbf{X}_j = \underbrace{\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\not s}_{jk}\mathbf{Y}_{kl} + \mathbf{B}^{\not s}_{js}\mathbf{Y}_{ss}}_{\mathbf{X}_j^{(\overset{\not s}{\rightarrow}\mathbf{Y}_*)}} + \underbrace{\mathbf{B}^{\not s}_{js}\mathbf{E}_{s*}}_{\mathbf{X}_j^{(\rightarrow\mathbf{E}_{s*})}}. \tag{32}$$

The decomposition in equation (32) allows us to identify the part of country j's production that is not part of country s's exports before reaching the ultimate destination ($\mathbf{X}_j^{(\overset{\not s}{\rightarrow}\mathbf{Y}_*)}$), whereas the remaining component identifies the double counted terms ($\mathbf{X}_j^{(\rightarrow\mathbf{E}_{s*})}$). By combining equations (28), (29) and (32) we can express the main terms of the sink-based breakdown of bilateral exports (i.e. the domestic value-added, $\mathbf{DVA}_{sr}$, the foreign value-added, $\mathbf{FVA}_{sr}$, the domestic double counted, $\mathbf{DDC}_{sr}$, and the foreign double counted, $\mathbf{FDC}_{sr}$)

[14] Note that the domestic input coefficient matrix $\mathbf{A}_{ss}$ is part of the $\mathbf{A}^{\not s}$ matrix in which only the other $\mathbf{A}_{st}$ submatrices, with $t \neq s$, have all the elements equal to zero. This allows us to include in the domestic value-added in exports the goods that undergo a final processing stage in country s and are ultimately used there.

$$\mathbf{DVA}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\nexists}_{jk}\mathbf{Y}_{kl} + \mathbf{B}^{\nexists}_{js}\mathbf{Y}_{ss}\right)\Bigg] \tag{33}$$

$$\mathbf{FVA}sink_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\nexists}_{jk}\mathbf{Y}_{kl} + \mathbf{B}^{\nexists}_{js}\mathbf{Y}_{ss}\right)\Bigg] \tag{34}$$

$$\mathbf{DDC}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\nexists}_{js}\mathbf{E}_{s*} \tag{35}$$

$$\mathbf{FDC}sink_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\nexists}_{js}\mathbf{E}_{s*}. \tag{36}$$

As in the case of the source-based decomposition, equations (33) and (34) can be used to isolate the value-added in exports for one or more of the highlighted actors (i.e. the exporter, the importer, the origin of the value-added, the market of re-export, the country of final completion and that of ultimate absorption). For instance, also in this case we can distinguish between domestic value-added that is finally absorbed at home ($\mathbf{REF}_{sr}$) or abroad ($\mathbf{VAX}_{sr}$):

$$\mathbf{REF}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\Bigg[\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rs} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\nexists}_{jk}\mathbf{Y}_{ks} + \mathbf{B}^{\nexists}_{js}\mathbf{Y}_{ss}\right)\Bigg] \tag{37}$$

$$\mathbf{VAX}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r,s}^{G}\mathbf{Y}_{rj}\right.$$
$$\left. + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{kl}\right]. \tag{38}$$

A subcomponent of the $\mathbf{VAX}sink_{sr}$ that may be economically interesting is the domestic value-added that is finally absorbed by the importer country r itself:

$$\mathbf{VAXIM}_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{kr}\right]. \tag{39}$$

The $\mathbf{VAXIM}_{sr}$ includes the value-added produced in s and absorbed in r and that factors in the bilateral exports between the two countries. The measure can be used for instance in the analysis of bilateral trade balances to assess how reciprocal final demand helps activate bilateral exports and production in the two countries.[15] The $\mathbf{DAVAX}_{sr}$ measure developed within the source-based approach is a subcomponent of the $\mathbf{VAXIM}_{sr}$ as it includes only the value-added produced in s and directly absorbed in r, without any possible re-export (and re-import) by r.[16]

The source-based measures and the sink-based ones can be used in different contexts to address different issues. Nevertheless, it is important to reaffirm that i) at the bilateral level, the domestic and foreign contents are the same in the two breakdowns;[17] ii) the 'value-added' and 'double counted' terms of the two decompositions differ only at the bilateral level and when summing across the destinations of a given exporter we obtain exactly the same results (i.e. see Appendix C for a for-

[15]The $\mathbf{VAXIM}_{sr}$ indicator, as any other measure based on a country-level perspective can be summed across bilateral importing partners to get an aggregate indicator for the exporting country s. Then, the $\mathbf{VAXIM}_{sr}$ can be particularly useful, for instance, when we are interested in decomposing the overall trade balance of a country by its bilateral positions. When the analysis focuses exclusively on a given bilateral relation, it might be more appropriate to resort to the approach presented in Section 4.1.

[16]Neither the $\mathbf{VAXIM}_{sr}$, nor the $\mathbf{DAVAX}_{sr}$ correspond to what Koopman et al. (2014) define as the 'exports absorbed by direct importers.' Indeed, as we show in Section 5, these subcomponents are not correctly identified in Koopman et al.'s (2014) decomposition.

[17]In both the decompositions the domestic content of exports (i.e. the sum of the domestic value-added and the domestic double counted) corresponds to the ones defined by Koopman et al. (2010).

mal proof).[18] This property does not apply to other value-added decompositions of trade that have been presented in the literature and, as far as the value-added originated abroad is concerned, this feature stems from the specific notions of 'foreign value-added' and 'foreign double counted' we adopted.[19]

3.3 Country-level perspective sectoral breakdown

The bilateral decompositions presented above can be easily extended to consider the sectoral dimension.[20] It is worth recalling that here we do not change the perimeter for defining double counting, which is represented by the exporter's borders as a whole. Thus, an item first exported by a certain sector and then re-exported by a different sector is accounted for as 'value-added' on one occasion and as 'double counted' on another.[21] By keeping this country-level perspective, we preserve the additivity of value-added components (i.e. the sum across all sectors and all bilateral partners yields the total GDP embedded in a country's exports).[22]

Here we focus on three different sectoral breakdowns: *i*) by origin of the value-added, either domestic or foreign, *ii*) by exports (the only one considered in the work of Wang et al., 2013) and *iii*) by final absorption.

To get a decomposition by sectors of origin, it is necessary to substitute in all the indicators of Sections 3.1 and 3.2 the $1 \times N$ vector $\mathbf{V}_j$ $(j = s, t)$ with its diagonalized form $\widehat{\mathbf{V}}_j$ (i.e. the $N \times N$ diagonal matrix with the direct value-added coefficients along the principal diagonal and zeros elsewhere):

$$\widehat{\mathbf{V}}_j \equiv \begin{bmatrix} v_{j,1} & 0 & \cdots & 0 \\ 0 & v_{j,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_{j,N} \end{bmatrix}.$$

[18]More specifically all the following equivalences hold true: $\sum_{r \neq s}^{G} \mathbf{DVA}source_{sr} = \sum_{r \neq s}^{G} \mathbf{DVA}sink_{sr}$, $\sum_{r \neq s}^{G} \mathbf{FVA}source_{sr} = \sum_{r \neq s}^{G} \mathbf{FVA}sink_{sr}$, $\sum_{r \neq s}^{G} \mathbf{VAX}source_{sr} = \sum_{r \neq s}^{G} \mathbf{VAX}sink_{sr}$ $\sum_{r \neq s}^{G} \mathbf{REF}source_{sr} = \sum_{r \neq s}^{G} \mathbf{REF}sink_{sr}$.

[19]See Section 5.1 for more detailed discussion on this issue.

[20]See also Borin and Mancini (2016) on this point.

[21]Depending on the choice between the source and the sink based decomposition, the 'value-added' is attributed to a certain export flow or to another.

[22]In Section 4 we present a decomposition based on a pure sectoral-bilateral perspective which means that a certain item is considered as double counted only when it is exported multiple times to the same partner within the same sector. In this framework, the additivity property does not hold, but these alternative measures may be very useful when addressing some specific issues such as trade policy analysis.

Similarly, the decomposition by export sectors is obtained by substituting vectors $\mathbf{V}_j\mathbf{B}_{js}$ and $\mathbf{V}_j\mathbf{B}^{\not s}_{js}$ in the DVA and FVA terms of equations (10) and (28) (and the following ones) with their $N \times N$ diagonalized forms, $\widehat{\mathbf{V}_j\mathbf{B}_{js}}$ and $\widehat{\mathbf{V}_j\mathbf{B}^{\not s}_{js}}$:

$$\widehat{\mathbf{V}_j\mathbf{B}_{js}} \equiv \begin{bmatrix} \sum_n^N v_{j,n}b_{js,n1} & 0 & \cdots & 0 \\ 0 & \sum_n^N v_{j,n}b_{js,n2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_n^N v_{j,n}b_{js,nN} \end{bmatrix}.$$

Finally the decomposition by sectors of final absorption is obtained by replacing the vector of final demand with its diagonalized form. For instance, for goods completed in country k and absorbed in country l, the $N \times N$ diagonal matrix of final demand is as follows:

$$\widehat{\mathbf{Y}}_{kl} \equiv \begin{bmatrix} y_{kl,1} & 0 & \cdots & 0 \\ 0 & y_{kl,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & y_{kl,N\cdot} \end{bmatrix}.$$

Depending on the specific empirical application, it will be possible to choose the best suited bilateral sectoral decomposition. For instance, if the focus is on the origin of production, the natural choice is to extend a source-based decomposition with a breakdown by sector of origin.

The different sectoral breakdowns can also be combined. For instance, in order to measure simultaneously the value-added embedded in a bilateral trade flow in a specific sector of origin destined for a particular sector of absorption, we can use $\widehat{\mathbf{V}}_j$ and $\widehat{\mathbf{Y}}_{kl}$ at the same time.

4 Decompositions of bilateral trade based on different perspectives

4.1 Bilateral-level perspective

The country-level perspective considered so far ensures that each component of domestic—or foreign—value-added corresponds to mutually exclusive portions of the GDP produced by a country and embedded in its own exports, or in another country's. This approach can be useful when addressing several different empirical questions and we also showed that it is at the bottom of some key indicators of GVC participation (see Section 3.1). However, there are other potential issues that require an accounting framework based on a different perspective. For instance, this occurs when we want to measure the GDP of a country which, at any point in time, passes through a certain bilateral trade flow. Suppose, for example, that there is a deterioration in the trade relationship between country s and country r; in this case we might be interested in measuring the total value-added that crosses this specific bilateral border, regardless of whether the same components are also part of the exports of s (or r) to other countries or not (i.e. they are double counted items from a country-level perspective).

To address this problem, we need an accounting method for value-added in bilateral exports that excludes from gross trade figures only the items that are double counted in the very same bilateral flow. In other words, the specific bilateral relation represents the new perimeter for defining double-counted flows in gross exports.

By proceeding as for the derivation of the source-based decomposition (see Section 3.1), we can modify the input coefficient matrix $\mathbf{A}$ to split the production process along the new perimeter and single out the 'value-added' and 'double counted' items. While in the country-level perspective we set to zero the coefficients that identify the direct requirement of intermediate inputs from country s to all the other countries, here we only set to zero the bilateral coefficient matrix $\mathbf{A}_{sr}$:

$$\mathbf{A}^{\not{sr}} = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1s} & \cdots & \mathbf{A}_{1r} & \cdots & \mathbf{A}_{1G} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{A}_{s1} & \cdots & \mathbf{A}_{ss} & \cdots & \mathbf{0} & \cdots & \mathbf{A}_{sG} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{A}_{G1} & \cdots & \mathbf{A}_{Gs} & \cdots & \mathbf{A}_{Gr} & \cdots & \mathbf{A}_{GG} \end{bmatrix}. \tag{40}$$

Then, the corresponding inverse Leontief matrix can be defined as:

$$\mathbf{B}^{\not{sr}} = (\mathbf{I} - \mathbf{A}^{\not{sr}})^{-1}. \tag{41}$$

The submatrix $\mathbf{B}_{is}^{\not{sr}}$ measures the gross output produced by i to deliver one unit of production to s, with the exclusion of the production linkages that pass through the bilateral flow s-r; in this way, all the production stages that antecede the export from s to r are taken into account. It is also easy to show that the following relationship holds true:

$$\mathbf{B}_{is}^{\cancel{sr}} = \mathbf{B}_{is}^{\cancel{sr}} + \mathbf{B}_{is}^{\cancel{sr}} \mathbf{A}_{sr} \mathbf{B}_{rs}. \tag{42}$$

By analogy, with the derivation of the source-based decomposition in (10), we can express the complete decomposition of bilateral exports based on a pure bilateral perspective:

$$\mathbf{u}_N \mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s \mathbf{B}_{ss}^{\cancel{sr}} \mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{DVA}_{sr}^{*}}} + \underbrace{\mathbf{V}_s \mathbf{B}_{ss}^{\cancel{sr}} \mathbf{A}_{sr} \mathbf{B}_{rs} \mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{DDC}_{sr}^{*}}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts}^{\cancel{sr}} \mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FVA}_{sr}^{*}}} + \underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts}^{\cancel{sr}} \mathbf{A}_{sr} \mathbf{B}_{rs} \mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FDC}_{sr}^{*}}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{43}$$

The measures of 'domestic value-added' and 'foreign value-added' in (43) correspond to those proposed by Johnson (2018) in a two-country context; the same measure of 'domestic value-added' in bilateral exports is also obtained by Los et al. (2016) by using a hypothetical extraction procedure (see Table 2 in Appendix E for a classification of the main measures proposed in the literature according to the framework proposed in this paper).

The components in equation (43) are uniquely defined for a certain bilateral flow s-r, because the perimeter for the definition of double counted items is the bilateral relationship itself.[23] It might also be interesting to consider how they are related to the terms in the bilateral decompositions based on a country-level perspective presented in Section 3. First, since the breakdown in equation (43) hinges on a less restrictive notion of 'double counting' as compared to the country-level perspective, both the sink and source measures of domestic and foreign value-added are part of the corresponding terms derived in the bilateral perspective decomposition (i.e. $\mathbf{DVA}source_{sr} \subset \mathbf{DVA}_{sr}^{*}$, $\mathbf{FVA}source_{sr} \subset \mathbf{FVA}_{sr}^{*}$; $\mathbf{DVA}sink_{sr} \subset \mathbf{DVA}_{sr}^{*}$, $\mathbf{FVA}sink_{sr} \subset \mathbf{FVA}_{sr}^{*}$). We can then break down the double counted terms of the decompositions based on a country level perspective in order to single out the subcomponents that are classified differently according to the perspective adopted. For

[23] As for any other case in which the perspective coincides with the considered trade flow, there is no distinction between a source- and sink-based approach. Although equation (43) has been obtained proceeding as for the derivation of the source-based decomposition of Section 3.1, we could have obtained the same result by exploiting a sink-based algebra, as in Section 3.2.

instance, if we consider the domestic double-counted component ($\mathbf{DDC}_{sr}$), we can re-express the original indicators as follows:

$$\mathbf{DDC}source_{sr} = \underbrace{\mathbf{V}_s \mathbf{B}^{\not s}_{ss} \sum_{j \neq s,r} \mathbf{A}_{sj} \mathbf{B}^{\not{sr}}_{js} \mathbf{E}_{sr}}_{\mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr}} + \mathbf{DDC}^*_{sr} \tag{44}$$

$$\mathbf{DDC}sink_{sr} = \underbrace{\mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \sum_{i \neq s,r}^{G} \left[\mathbf{Y}_{si} + \mathbf{A}_{si} \mathbf{B}^{\not{sr}}_{is} \sum_{l \neq r}^{G} \mathbf{Y}_{sl} + \mathbf{A}_{si} \sum_{k \neq s}^{G} \sum_{l}^{G} \mathbf{B}^{\not{sr}}_{ik} \mathbf{Y}_{kl} \right]}_{\mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}} + \mathbf{DDC}^*_{sr}, \tag{45}$$

where $\mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr}$ and $\mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}$ are the components that are classified as 'double counted' respectively in the source and sink decompositions based on a country-level perspective, but are classified as 'domestic value-added' in a purely bilateral perspective. How these components should be considered depends on the specific economic issue under investigation. It is also worth noting that the following relationship holds true:

$$\mathbf{DVA}^*_{sr} = \mathbf{DVA}source_{sr} + \mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr} = \mathbf{DVA}sink_{sr} + \mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}. \tag{46}$$

The derivation of equations (44)–(46) can also be applied to single out the differences between the foreign value added in a bilateral-level perspective ($\mathbf{FVA}^*_{sr}$) and those defined in a country-level perspective ($\mathbf{FVA}source_{sr}$ and $\mathbf{FVA}sink_{sr}$).

By following the same scheme in Section 3.1, equation (43) can be further developed to consider all the forward production linkages, as well as the countries of completion and the markets of final absorption. We can also extend the breakdown to take into account the sectors of original production of the value-added and those of ultimate absorption (see Section 3.3). Conversely, the decomposition by sector of export is not univocal even in the bilateral-level perspective and will change depending on whether we employ a source-based or a sink-based approach. However, in this context we do not consider this type of breakdown particularly meaningful from an economic standpoint. It is more useful to analyze the case in which a specific exporting sector, within a bilateral relationship, is the focus of the analysis.

4.2 Sectoral-bilateral perspective

It is particularly relevant for some empirical analysis to measure the value-added of a country that enters in the exports between two countries in a specific sector. For instance, trade policies are often applied by the importing country on the basis of a particular exporting partner and sector (or product) of export. In order to assess how this type of policy affects the GDP produced in a given country, we need an accounting framework for the value-added in which the single sectoral-bilateral flow is the new perimeter used to define the value-added accounting of exports.

By proceeding in the same way as the other decompositions, we can define a modified version of the input requirement matrix in which all the coefficients corresponding to the intermediate exports from s to r in (exporting) sector n are set to zero:

$$\mathbf{A}^{\cancel{sr,n}} = \begin{bmatrix} a_{11,11} & \cdots & a_{1r,11} & \cdots & a_{1r,1n} & \cdots & a_{1r,1N} & \cdots & a_{1G,1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{s1,11} & \cdots & a_{sr,11} & \cdots & a_{sr,1n} & \cdots & a_{sr,1N} & \cdots & a_{sG,1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{s1,n1} & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & a_{sG,nN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{G1,N1} & \cdots & a_{Gr,N1} & \cdots & a_{Gr,Nn} & \cdots & a_{Gr,NN} & \cdots & a_{GG,NN} \end{bmatrix}. \tag{47}$$

The corresponding inverse Leontief matrix is defined as:

$$\mathbf{B}^{\cancel{sr,n}} = (\mathbf{I} - \mathbf{A}^{\cancel{sr,n}})^{-1}. \tag{48}$$

As in previous cases, we can obtain a breakdown of exports from a sectoral-bilateral perspective:

$$e_{sr,n} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}_{ss}^{\cancel{sr,n}}E_{sr,n}}_{\substack{\text{sectoral-bilateral}\\ \text{perspective}\\ \mathbf{DVA}_{sr}^{\S}}} + \underbrace{\mathbf{V}_s\mathbf{B}_{ss}^{\cancel{sr,n}}\mathbf{A}_{sr}\mathbf{B}_{rs}E_{sr,n}}_{\substack{\text{sectoral-bilateral}\\ \text{perspective}\\ \mathbf{DDC}_{sr}^{\S}}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\cancel{sr,n}}E_{sr,n}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FVA}_{sr}^{\S}}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\cancel{sr,n}}\mathbf{A}_{sr}\mathbf{B}_{rs}E_{sr,n}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FDC}_{sr}^{\S}}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}, \tag{49}$$

where $E_{sr,n}$ is an $N\times1$ vector with the scalar corresponding to the gross exports from s to r in position n (i.e. $e_{sr,n}$) and zeros elsewhere. As in previous cases, we can consider the forward production-demand linkages up to the market of final destination as well as a breakdown by sector of origin/final-destination as described in Section 3.3.

Here, the definition of double counted items is even less restrictive than in the bilateral perspective case, which means the $\mathbf{DVA}^*_{sr}$ and the $\mathbf{FVA}^*_{sr}$ are smaller than the sum of the value-added terms in equation (49) across the different exporting sectors. Notably, this sum does not have an economically meaningful interpretation, due to double counted items.

Since tariffs are usually imposed at the sectoral-bilateral level, this perspective is suited to evaluating the incidence of tariffs on the value-added produced by different countries. The ICIO framework allows to single out the countries/sectors that originated the value embedded in the goods on which a tariff is levied. By boosting flows in intermediate goods, across countries and stages of production, GVCs tend to amplify the costs generated by border barriers (Rouzet and Miroudot, 2013; Vandenbussche et al. 2017). Since tariffs are applied on gross trade flows, the value-added that crosses the targeted importer border several times is subject to an 'inflated' tariff as compared to the original nominal one.

Given the nominal tariff $\tau_{sr,n}$ imposed by country r on the goods imported from s in sector n, the 'effective tariff' on the value-added produced by a certain country j con be computed as follows:

$$\mathbf{ET}^j_{sr,n} = \frac{\mathbf{V}_j \mathbf{B}_{js} E_{sr,n} \tau_{sr,n}}{\mathbf{V}_j \mathbf{B}^{\cancel{sr,n}}_{js} E_{sr,n}} \tag{50}$$

while the incidence of the tariff on the total value-added produced by j can be computed multiplying $\mathbf{ET}^j_{sr,n}$ by the ratio between $\mathbf{V}_j \mathbf{B}^{\cancel{sr,n}}_{js} E_{sr,n}$ and $\mathbf{V}_j \sum_k \sum_l \mathbf{B}_{jk} Y_{kl}$.

4.3 Other accounting perspectives

Additional breakdowns of exports can be obtained by using further accounting perspectives. In this section we consider those that may be useful in addressing the most common empirical issues.

Sectoral-exporter perspective

In some cases, economic shocks or policy measures affect all the exports of a country in a given sector, irrespective of the specific bilateral importer. An example could be the so-called 'dieselgate' involving the main German car producers in 2015. All the German exports in the automotive sector became vulnerable to the shock. In order to assess spillovers from this shock into different countries/sectors, we can compute the value-added content embedded in German exports in that specific sector. This can be singled out by deriving a decomposition of a country's aggregate sectoral exports on a 'sectoral-level perspective'.[24]

The domestic and foreign value-added embedded in total exports of country s and sector n ($\sum_{r\neq s} e_{sr,n}$) can be computed in a manner similar to the 'sectoral-bilateral' decomposition (see equation (49)). The only difference is that the matrix of technical coefficients $\mathbf{A}$ has to be modified such that $a_{sj,n}$ is set to zero $\forall\ j \neq s$; thus the inverse Leontief matrix is computed accordingly.

Importer perspective

Another issue is worth considering is when we define the relevant perimeter at the level of the importing country's borders as a whole. This 'importer perspective' should be adopted to compute the GDP of a given country j that enters, directly or indirectly, in the total imports of a given country r. This measure can be interesting, for instance, when a certain country is going to adopt a general protectionist stance (i.e. vis-à-vis all the exporting partners) and we want to compute the portion of other countries' GDP at stake.

By following a procedure similar to that used to derive the (exporting) 'country perspective' of Section 3, the total gross imports of country r can be decomposed as follows:

$$\mathbf{u}_N \mathbf{E}_{*r} = \underbrace{\sum_{j}^{G} \mathbf{V}_j \sum_{s\neq r} \widetilde{\mathbf{B}}^{\not r}_{js} \mathbf{E}_{sr}}_{\substack{\text{importer}\\ \text{perspective}\\ \text{Value-Added}}} + \underbrace{\sum_{j}^{G} \mathbf{V}_j \sum_{t\neq r} \widetilde{\mathbf{B}}^{\not r}_{jt} \sum_{s\neq r} \mathbf{A}_{tr} \mathbf{B}_{rs} \mathbf{E}_{sr}}_{\substack{\text{importer}\\ \text{perspective}\\ \text{Double-Counted}}} \tag{51}$$

where $\widetilde{\mathbf{B}}^{\not r}_{js}$ is the Leontief inverse block matrix derived from a technical coefficient matrix $\mathbf{A}$ in which the sub-blocks $\mathbf{A}_{tr}$ are set to zero $\forall\ t \neq r$.

[24] In this perspective, a certain item is considered as 'double counted' only when it is re-exported by the same country and sector.

In equation (51) we do not use the distinction between 'domestic' and 'foreign' value-added as this notion is related to the exporting country (i.e. the domestic one). However, we can still identify 'value-added' and 'double counted' according to a specific country of origin, labeled j in equation (51).

Sectoral-importer perspective

The focus can also be on a particular sector of a given importing country. This occurs when a certain shock affects only the imports of a country of a specific sector, as for the tariff hikes imposed in 2018 by the U.S. administration on the steel products imported from all the trading partners.

We can easily modify the decomposition in (51) in order to isolate the value-added generated in a given country j that is embedded in the total import of a country r in a given sector n. This can be obtained by substituting the matrix $\widetilde{\mathbf{B}}_{js}^{\not{r}}$ with a Leontief inverse $\widetilde{\mathbf{B}}_{js}^{\not{r,n}}$ based on a technical coefficient matrix in which only the elements $a_{tr,n}$ are set to zero $\forall\ t \neq r$.

Clearly when the focus is on a country's imports from a specific exporting partner the appropriate decomposition is the one based on the bilateral-level perspective (see Section 4.1) or the sectoral-bilateral one (see Section 4.2) when the bilateral trade flows in the spotlight are only those of a particular sector.

5 Comparison with other methodologies

In previous sections we point out how some of the measures we propose relate to other contributions in the literature. For instance, we show that the **VS** indicator proposed by Hummels et al. (2001) and the **VAX** index by Johnson and Noguera (2012) correspond to specific subcomponents of the country-perspective breakdowns in Section 3. Similarly, we recall that the decomposition based on a bilateral-perspective of Section 4.1 shares the same rationale as the measure of domestic value-added proposed by Los et al. (2016) and the breakdown of bilateral exports developed by Johnson (2018) in a two-country framework.

In this section we discuss in more detail the relationship with other measures of value-added in gross exports proposed in the literature. Table 2 in Appendix E summarizes the different measures proposed in this paper, whereas Table 3 classifies the other contributions according to the different perspectives and approach they adopt. Although these measures can be attributed to one specific case of the general scheme we propose, in most of the cases there is not a perfect correspondence with the methodologies presented in this paper. The reason for this is that some contributions present specific conceptual drawbacks which we will discuss in detail further on in the section.

5.1 Breakdown of the foreign content of exports

We begin with a more detailed look at the relationship with other methodologies by considering the breakdown of the foreign content of exports since our approach differs from most of the other contributions in that respect. We then deal with other aspects that are more specific to each alternative measure proposed in the literature.

As already mentioned, the breakdowns proposed in Section 3 use a notion of 'foreign double counted' (FDC) that shares the same rationale as the 'domestic double counted': we only include the items that cross the same (i.e. the exporter's) border more than once in the FDC. In other words, we use a 'country-level perspective' for both the accounting of domestic and foreign content of exports.

Other methodologies, which also use a country-level perspective for the domestic component, take a different approach for the foreign content of exports:[25] a certain item is considered as value-added only the first (or the last) time it crosses a foreign border, whereas all the other times it crosses any foreign border it is classified as double counted. We can label this approach a 'world-level perspective', since all trade flows—not only the exports of a single country—are considered in order to single out the items that are exported multiple times. In other words, with the 'country-level perspective' a certain item is accounted for as FVA only once in the total exports of a country, whereas the 'world-level perspective' requires it to be accounted for as FVA only once in total world exports.

In order to better appreciate the difference between the two approaches, it is useful to re-express the decompositions of the foreign content of export according to a 'world-level perspective'. The distinction between the source- and sink-based approaches also applies in this case. The source-based approach requires a certain item to be recorded as 'foreign value-added' the first time it is re-exported by a country other than the country of origin and it is the one followed by Miroudot and

[25] The following contributions fit into this category: Koopman et al. (2014), Wang et al. (2013), Nagengast and Sterher (2016), Miroudot and Ye (2018).

Ye (2018). Here we also consider the country of final completion and the market of final destination:

$$\mathbf{FVA}source_{sr}^{WP} = \sum_{t\neq s}^{G} \mathbf{V}_t(\mathbf{I}-\mathbf{A}_{tt})^{-1}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg] \quad (52)$$

$$\mathbf{FDC}source_{sr}^{WP} = \sum_{t\neq s}^{G} \mathbf{V}_t(\mathbf{I}-\mathbf{A}_{tt})^{-1}\Bigg[\sum_{j\neq t,s}^{G}\mathbf{A}_{tj}\mathbf{B}_{js}\mathbf{E}_{sr} + \mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})\sum_{t\neq s}^{G}\mathbf{A}_{st}\mathbf{B}_{ts}\mathbf{E}_{sr}\Bigg]. \quad (53)$$

The foreign content of exports can also be decomposed using a sink-based approach, while maintaining a 'world-level perspective'. In this case, a given item is accounted for as 'foreign value-added' the last time it is exported by a country that is not the country of origin. This logic is also followed by Koopman et al. (2014), however, this part of their decomposition is affected by some specific drawbacks that we will discuss in more detail in the following paragraph. The value-added and double-counted components of the foreign content of exports in a world-perspective/sink-based approach can be expressed as follows:

$$\mathbf{FVA}sink_{sr}^{WP} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\Bigg[\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}\Bigg] \quad (54)$$

$$\mathbf{FDC}sink_{sr}^{WP} = \sum_{t\neq s,r}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*} + \mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{E}_{j*}. \quad (55)$$

If we consider an exporter s and we sum across the bilateral destinations r

the FVA and FDC indicators in equations (52)–(55) we obtain different values for the FVA and the FDC. Indeed, whenever the first re-exporting country does not coincide with the last, the source-based indicators of foreign content and the sink-based ones differ when computed at the level of the exporting country. However, both approaches record a given item as FVA only once in world trade flows. This means that when aggregating at the world level the components in equations (52)–(55) (i.e. summing across all the exporters and the importers) we obtain exactly the same measures of FVA and FDC.

Conversely the source and sink decompositions of the foreign content of exports based on a 'country-level perspective' are completely consistent with each other when we sum across all destination markets (i.e. considering equations (17) and (34), $\sum_{r \neq s}^{G} \mathbf{FVA}sink_{sr} = \sum_{r \neq s}^{G} \mathbf{FVA}source_{sr}$). Thus, the country-level approach leads to a unique measure of FVA (and FDC) at the country level.

The decompositions based on a 'world-level perspective' can be used to address important questions regarding the breakdown of total world trade. For instance, we can measure the share of world GDP that enters the exports of some other country. However, these measures seem particularly unsatisfactory when analyzing the exports of a given country. Indeed, in the breakdown of a country's exports, this distinction between FVA and FDC turns out to be totally arbitrary. Consider the following example: China imports intermediates directly from Germany and indirectly from France; according to a world-perspective source-based approach, the German VA is considered as FVA in Chinese exports, whereas the French VA is classified as FDC, even if the two components contribute in a very similar way to the value embedded in Chinese exports.[26]

From the perspective of the exporting country, we are usually interested in measuring the share of exports that can be traced back to the domestic and foreign GDP, irrespective of the number of upstream or downstream production stages that separate the exporter from the country of origin and/or the market of final destination. Indeed the 'foreign value-added' indicators based on a 'country-level perspective' address the following questions: which part of a country's exports can be traced back to another country's GDP? Alternatively, what is the portion of a country's GDP that is embedded into the exports of a certain other country or of a certain bilateral flow? These questions might be particularly relevant from a policy point of view, for instance when discussing the impact of a trade policy on third-country production.

[26] Likewise the sink-based classification produces arbitrary allocations too.

In principle, the FVA calculated according to the world-level perspective can be used to address the following question: 'what part of country s's GDP enters other countries' exports?' In particular, it can be computed through equation (52) (or in (54)) by tracking a specific country of origin of the value-added t in the exports of all the other exporting countries $s \neq t$ toward all the bilateral partners r. However, the same question can be addressed directly in the country-level source-based framework by subtracting the component of direct absorption from the total DVA of the exporting country itself (i.e. $\sum_{r \neq s}^{G} \mathbf{DVA}source_{sr} - \sum_{r \neq s}^{G} \mathbf{DAVAX}_{sr}$, see equations (16) and (20)). Notably, this is the numerator of the $\mathbf{GVC}forward$ indicator of equation (27) computed for the total exports of a country (i.e. summed across bilateral importers).

Finally, it is worth noting that at world level all the FVA components are also recorded as DVA in the flows of other countries, meaning that they are already double counted GDP in exports. It is thus questionable to argue a certain item should be recorded as FVA only once in world trade flows in order to avoid double counting of the same production in trade.

5.2 Critical aspects in other methodologies

Different approaches and perspectives can be used in the value-added accounting of gross exports, the choice depending on the specific phenomenon under investigation. However, it does not mean any alternative accounting decomposition should be considered completely correct or sound from an economic point of view. First of all, each component should *measure* precisely what it is supposed to *measure*. Then, since each approach is suited to gauging a certain phenomenon properly, every decomposition of trade flows should maintain an internal consistency (i.e. each component should be consistent with the specific approach adopted). Falling short of any of these requirements leads to inaccurate measures and/or impairs the possibility to answer a particular question accurately.

Here we discuss some critical aspects of other accounting frameworks based on a 'country-level perspective,' by comparing them with the decompositions presented in Section 3. We first consider Koopman et al. (2014) and then extend the analyses to other contributions. To make the discussion clearer in equation (D.2) in Appendix D, we re-write our bilateral breakdown so that the components match directly those proposed in Koopman et al. (2014) (see equation (D.1) in Appendix D).[27] Notably, for the domestic content we consider our bilateral sink-based decomposition presented in Section 3.2, since the original expressions proposed in Koopman et al. (2014) resemble this approach more closely.[28] By the same token, in equation (D.2) the decomposition of the foreign content of exports follows the sink-based-world-level perspective of equations (54) and (55) so that the whole decomposition is conceptually consistent with Koopman et al. (2014).

Despite the algebraic consistency between the two classifications, there are few discrepancies due to two main criticisms that affect the breakdown by Koopman et al. (2014):[29] i) their classification does not properly allocate the domestic value-added in exports between the part eventually absorbed by direct importers and the part absorbed in third markets;[30] ii) a portion of the foreign content of exports is erroneously classified as 'double counted' whereas it should be allocated to the 'foreign value-added'.

The first issue arises from the fact that the bilateral dimension of trade flows is generally overlooked in Koopman et al. (2014). More specifically they calculate the 'domestic value-added in intermediate exports absorbed by direct importers' as: $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rr}$; they also claim that the 'domestic value-added in intermediate goods re-exported to third countries' is given by: $\mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt}$. In fact, both the expressions refer to the domestic value-added absorbed in any foreign market; the difference between the two components is that, in the first case, final goods are completed and consumed in the same country whereas in the latter goods are completed in a foreign country and absorbed in a different one. The problem is that the global inverse Leontief matrix $\mathbf{B}_{sr}$ employed by Koopman et al. (2014) does not trace a bilateral exporter-importer linkage.

The mismeasurement in Koopman et al. (2014) can be gauged more precisely by considering the decomposition of bilateral exports in equation (D.2). Koopman et al. (2014) allocate the second term of their decomposition to the bilateral importers' final demand; in reality, only sub-items **2a** and **2b** of equation (D.2) can

[27]Equation (D.2) is derived by extending in a very intuitive way our bilateral sink-based decomposition presented in Section 3.2. The original Koopman et al. (2014) components can be obtained as a simple summation over the importing countries r of the corresponding items of equation (D.2) (e.g. the second term in Koopman et al. (2014) is equal to the sum across the r destinations, $\mathbf{E}_{s*} = \sum_{r \neq s}^{G} \mathbf{E}_{sr}$, of **2a**+**2b**+**2c**).

[28]The terms in Koopman et al. (2014) can also be retrieved from the source-based decomposition of Section 3.1, since the two bilateral breakdowns yield to the same results when considering the aggregate exports of a country.

[29]The differences between our results and those in Koopman et al. (2014) emerge by comparing the definitions of the items reported below equation (D.2) with those originally assigned by Koopman et al. (2014) and quoted below equation (D.1).

[30]See also Nagengast and Sterher (2016) on the same point.

be defined as such (while sub-item **2c** is not). Conversely, part of the third term (**3c**) should also be classified as 'direct importers' final absorption', instead of third countries'.

Measuring the value-added absorbed, directly and indirectly, by the importing country could be relevant to various empirical issues, including an analysis of bilateral trade balances (Nagengast and Steherer, 2016), evaluation of trade policies (Cappariello et al. 2018) and assessment of countries' involvement in global value chains (see Section 3.1). To this end, the breakdowns of gross bilateral exports presented in Section 3 single out—among other components—the total domestic value-added absorbed by bilateral importers (**VAXIM**), and the part that is directly absorbed by the importing country, without any other processing stage abroad or at home (**DAVAX**). The latter is at the basis of the GVC indicators proposed in Section 3.1. These measures cannot be obtained either from the decomposition by Koopman et al. (2014), or from similar breakdowns of bilateral exports proposed in the literature (e.g. Wang et al., 2013), as discussed in the final part of this section.

The second critical aspect of Koopman et al.'s methodology is the incorrect classification of part of the foreign value-added in exports. The problem is that they classify the whole foreign content of country s's exports that the importing country r re-exports abroad as 'foreign double counted' (i.e. $\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*}$). In doing so, Koopman et al. (2014) fail to record as 'foreign value-added' the items, originally generated in the importing country r, that r itself re-exports to the market of final destination. The following example can better clarify this point: the US produces components that are processed in Mexico, before being sent back to the US and exported to their final market destination. Koopman et al. (2014) classify the US content in Mexican exports as 'foreign double counted', even if in the very last shipment from the US these components are not part of the 'foreign value-added', since they are included in the domestic content. The sink-based-world-level perspective adopted by Koopman et al. (2014) for the foreign content of exports envisages that a given item is classified as 'foreign value-added' only once in global trade flows (i.e. the last time it is exported by a country other than the country of origin). However, in Koopman at al. (2014) some items can be classified as 'foreign double counted' even if they are never recorded as 'foreign value-added'. The terms **9a** and **9b** in equation (D.2) quantify the amount that is misallocated which leads to a systematic underestimation of the 'foreign value-added'.

This criticism applies also to other decompositions that adopt the same breakdown of the foreign content of exports. In particular this mismeasurement is also evident in the decomposition of bilateral exports proposed by Wang et al. (2013). Another critical issue of this methodology is that it uses different approaches (sink and source) at the same time to single out various components, then it suffers from internal inconsistency. This makes the different items not fully comparable with each other and, in turn, could lead to inaccurate evaluations when comparing the value-added structure of two (or more) distinct trade flows, as in the analysis of bilateral trade balances.

Moreover, since a particular approach is appropriate for addressing a specific issue, the decomposition Wang et al. (2013) propose is unsuited to measuring some phenomena. In other cases, it does not provide the most precise measure available. For instance, it is not possible to compute the **DAVAX** indicator or the measures of GVC related trade in Section 3.1.[31]

Despite having first introduced the concepts of source-based and sink-based approaches, not even Nagengast and Sterher (2016) propose fully fledged breakdowns according to these different logics. On a more general level, Nagengast and Sterher (2016) focus on the value-added components that are produced and absorbed by the two bilateral partners in order to evaluate how these items contribute to bilateral trade balances since this is the aim of their analysis. For the remaining part of their decompositions, they do not employ a specific strategy to single out value-added and double-counted terms. Instead, they just distinguish between items that belong to the 'domestic content of exports' and items that are part of the 'foreign content of exports,' by using expressions that resemble those in equation (2).[32] Nagengast and Sterher (2016) label these terms 'value-added' as well, instead of 'content', which can lead to a misinterpretation of results. For instance, their definitions of the domestic value-added finally absorbed at home and in third countries produce an overestimation of the domestic value-added in exports, since double counted components are also included.[33]

[31] Wang et al. (2013) do not identify the domestic value-added absorbed by the bilateral importer as *i*- the DVA in exports of final goods is measured using a sink-based accounting (i.e. $\mathbf{V}_s\mathbf{B}_{ss}\mathbf{Y}_{sr}$); *ii*- it does not single out the intermediate goods absorbed by the bilateral importer without additional processing stages abroad. Indeed, in a revised version of their work, Wang et al. (2018), the authors themselves acknowledge this limitation and refer to our source-based decomposition, presented here in Section 3.1, to measure GVC-related trade.

[32] These components are computed in exactly the same way in the source-based breakdown and sink-based one.

[33] For example in Nagengast and Sterher (2016) the domestic value-added absorbed in third countries is calculated as $\mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}\mathbf{B}_{rr}\sum_{j\neq s,r}\mathbf{Y}_{rj}$. The simultaneous application of the $\mathbf{B}_{ss}$ Leontief inverse matrix and the $\mathbf{B}_{rr}$ one, leads to a double counting of the same value-added.

Neither methodology proposed by Nagengast and Sterher (2016) manages to break up the whole domestic and foreign value-added exported by a country across the different bilateral flows. However, this probably goes beyond the scope of their analysis, which focuses on the value-added components produced and absorbed by two bilateral partners. Since an important aspect in their study is the role of final demand in generating bilateral trade balances, a sink-based approach seems ideally suited. As mentioned, the proper measure of this component is given by the **VAXIM** of equation (39), which differs from the measure proposed by Nagengast and Sterher (2016). In fact, they only classify as 'domestic value-added absorbed by direct importers' what is embedded in goods that do not leave this country again, assigning the remainder to the double counted component. In this way, they do not take into account what we have classified in the **2b** and **3c** components in equation (D.2), underestimating the domestic value-added absorbed by direct importers.

6 Concluding remarks

The diffusion of international production networks over the last three decades calls for new tools in order to evaluate supply and demand relationships between countries which can no longer be adequately gauged by gross trade flows. For this reason, inter-country input-output tables and new methodologies that exploit these data have been developed to measure trade in value-added terms and countries' participation in GVCs. These tools have been extensively used in a large number of applications. However, on the one hand, several empirical issues have been not properly addressed so far; on the other hand, the emergence of different methodologies for the value-added accounting of trade flows has raised doubts as to the correct way of measuring the phenomena.

In this paper we propose a general scheme for breaking down bilateral and sectoral exports flows according to the source and the destination of their value-added content. This framework can be differentiated according to alternative approaches and perspectives that are instrumental to addressing different empirical issues. These different approaches are reflected in distinct ways to distinguish between 'value-added' and 'double-counted' terms in gross trade flows. Operationally, this differentiation is obtained by changing the definition of the sectoral-geographical perimeters according to which the items that cross these boundaries more than once are classified as 'double counted'. We consider more in detail three alternative cases: in the first one, the boundaries are defined at the level of the exporting county, in the second one, at the level of a specific bilateral trade relation and in the third one, at the level of a single exporting sector within a bilateral flow. We also derive the accounting methods for other relevant perspectives. We argue that each perspective is conceptually suited to addressing a different class of empirical issues. For instance, the country-level approach is the proper one when allocating a country's GDP across the different trade flows, whereas the bilateral perspective is best suited to evaluating the extent to which the GDP of a country is involved in the commercial interchange between two economies.

We show how the main methodologies proposed in other contributions fit into this framework. Then, on the one hand, we reconcile a large part of the existing literature under a unique comprehensive scheme; on the other hand, we show the main shortcomings and limitations that affect some of the techniques that have been proposed in the literature.

In general, the detailed breakdowns of bilateral/sectoral exports we present find a broad scope for empirical investigations on global production networks both at macro and micro level. Indeed, there is a fast expanding empirical literature that uses tools similar to those proposed in this paper. Our contribution to this literature is twofold: i) we extend the set of possible measures, including a new indicator of GVC-related trade, in order to address a wider range of empirical issues; ii) we improve the accuracy of some of the existing measures. These improvements are likely to become increasingly relevant from a quantitative point of view since the inter-country input-output data will become more and more detailed, eventually relaxing some of the simplifying assumptions that characterize current databases (de Gortari, 2018).

Finally, an aspect that is currently debated in the literature is how to measure the impact of trade policies on countries' productions by taking all international supply linkages into account. The 'sectoral-bilateral' perspective of value-added accounting proposed in this paper offers a promising conceptual framework to address this issue, since it provides a precise measurement of the GDP of a country that is exposed to a trade policy implemented by any economy for a given sector and a given trade partner. We exploit this measure to derive an indicator of the effective incidence of tariffs on value-added. This may contribute to development of more refined measures of the spillovers of trade policies in global value chains using ICIO data.

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A Appendix: Notation and basic I-O relations

This appendix simply recalls our notation, which is broadly the same as Koopman et al. (2014), together with some basic accounting relationships.

We consider the general case of G countries producing N goods that are internationally traded both as intermediate inputs and as final goods. Thus, $\mathbf{X}_s = (x_1^s \ x_2^s \ \cdots \ x_N^s)'$ is the $N \times 1$ vector of the gross output of country s and Y_s is the $N \times 1$ vector of final goods, which is equal to the final demand for goods produced in s in each country of destination r: $\sum_r^G \mathbf{Y}_{sr}$. To produce one unit of gross output of good i a country uses a certain amount a of intermediate good j produced at home or imported from other countries. Thus, each unit of gross output can be either consumed as a final good or used as an intermediate good at home or abroad:

$$\mathbf{X}_s = \sum_r^G (\mathbf{A}_{sr}\mathbf{X}_r + \mathbf{Y}_{sr})$$

where $\mathbf{A}_{sr}$ is the $N \times N$ matrix of coefficients for intermediate inputs produced in s and processed further in r:

$$\mathbf{A}_{sr} = \begin{bmatrix} a_{sr,11} & a_{sr,12} & \cdots & a_{sr,1N} \\ a_{sr,21} & a_{sr,22} & \cdots & a_{sr,2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{sr,N1} & a_{sr,N2} & \cdots & a_{sr,NN}. \end{bmatrix}$$

Using the block matrix notation, the general setting of production and trade with G countries and N goods can be expressed as follows:

$$\underset{(NG\times 1)}{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix}} = \underset{(NG\times NG)}{\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1G} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{G1} & \mathbf{A}_{G2} & \cdots & \mathbf{A}_{GG} \end{bmatrix}} \underset{(NG\times 1)}{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix}} + \underset{(NG\times G)}{\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1G} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{G1} & \mathbf{Y}_{G2} & \cdots & \mathbf{Y}_{GG} \end{bmatrix}} \underset{(G\times 1)}{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}} \quad \text{(A.1)}$$

from which it is straightforward to derive the following relationship between gross output and final demand:

$$\begin{bmatrix}\mathbf{X}_1\\\mathbf{X}_2\\\vdots\\\mathbf{X}_G\end{bmatrix} = \begin{bmatrix}\mathbf{I}-\mathbf{A}_{11} & -\mathbf{A}_{12} & \cdots & -\mathbf{A}_{1G}\\ -\mathbf{A}_{21} & \mathbf{I}-\mathbf{A}_{22} & \cdots & -\mathbf{A}_{2G}\\ \vdots & \vdots & \ddots & \vdots\\ -\mathbf{A}_{G1} & -\mathbf{A}_{G2} & \cdots & \mathbf{I}-\mathbf{A}_{GG}\end{bmatrix}^{-1}\begin{bmatrix}\sum_r^G \mathbf{Y}_{1r}\\ \sum_r^G \mathbf{Y}_{2r}\\ \vdots\\ \sum_r^G \mathbf{Y}_{1G}\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1N}\\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{B}_{G1} & \mathbf{B}_{G2} & \cdots & \mathbf{B}_{GG}\end{bmatrix}\begin{bmatrix}\sum_r^G \mathbf{Y}_{1r}\\ \sum_r^G \mathbf{Y}_{2r}\\ \vdots\\ \sum_r^G \mathbf{Y}_{1G}\end{bmatrix} \tag{A.2}$$

where $\mathbf{B}_{sr}$ denotes the $N \times N$ block of the Leontief inverse matrix in a global IO setting. It indicates how much of country s's gross output of a certain good is required to produce one unit of country r's final production.

The direct value-added share in each unit of gross output produced by country s is equal to one minus the sum of the direct intermediate input share of all the domestic and foreign suppliers:

$$\mathbf{V}_s = \mathbf{u}_N(\mathbf{I} - \sum_r^G \mathbf{A}_{rs}) \tag{A.3}$$

where $\mathbf{u}_N$ is the $1 \times N$ unit row vector. Thus, the $G \times GN$ direct domestic value-added matrix for all countries can be defined as:

$$\mathbf{V} = \begin{bmatrix}\mathbf{V}_1 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_2 & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_G\end{bmatrix}$$

while the overall $G \times GN$ value-added share matrix is obtained by multiplying the $\mathbf{V}$ matrix by the Leontief inverse $\mathbf{B}$:

$$\mathbf{VB} = \begin{bmatrix}\mathbf{V}_1\mathbf{B}_{11} & \mathbf{V}_1\mathbf{B}_{12} & \cdots & \mathbf{V}_1\mathbf{B}_{1G}\\ \mathbf{V}_2\mathbf{B}_{21} & \mathbf{V}_2\mathbf{B}_{22} & \cdots & \mathbf{V}_2\mathbf{B}_{2G}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{V}_G\mathbf{B}_{G1} & \mathbf{V}_G\mathbf{B}_{G2} & \cdots & \mathbf{V}_G\mathbf{B}_{GG}\end{bmatrix}$$

Since the value-added shares of different countries in final goods have to sum to one, the following property holds:

$$\sum_{t}^{G} \mathbf{V}_t \mathbf{B}_{tr} = \mathbf{u}_N \tag{A.4}$$

Defining the $GN \times G$ final demand matrix as:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1G} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{G1} & \mathbf{Y}_{G2} & \cdots & \mathbf{Y}_{GG} \end{bmatrix}$$

we can derive the $G \times G$ value-added matrix by pairs of source-absorption countries:

$$\overline{\mathbf{VA}} \equiv \mathbf{VBY} =$$

$$= \begin{bmatrix} \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r} \mathbf{Y}_{r1} & \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r} \mathbf{Y}_{r2} & \cdots & \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r} \mathbf{Y}_{rG} \\ \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r} \mathbf{Y}_{r2} & \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r} \mathbf{Y}_{r2} & \cdots & \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r} \mathbf{Y}_{rG} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr} \mathbf{Y}_{rG} & \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr} \mathbf{Y}_{rG} & \cdots & \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr} \mathbf{Y}_{rG} \end{bmatrix} \tag{A.5}$$

B Appendix: Useful equivalences in ICIO models

The following two equivalences are often used in ICIO modeling to put different objects in relation to each other.

Considering the following property of inverse matrix B:

$$\mathbf{B}(\mathbf{I}-\mathbf{A}) = (\mathbf{I}-\mathbf{A})\mathbf{B} = \mathbf{I}$$

it is easily shown that the generic block diagonal element $\mathbf{B}_{ss}$ may be expressed as follows:

$$\begin{aligned} \mathbf{B}_{ss} &= \sum_{t\neq s}^{G} \mathbf{B}_{st}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1} + (\mathbf{I}-\mathbf{A}_{ss})^{-1} = \\ &= (\mathbf{I}-\mathbf{A}_{ss})^{-1} + (\mathbf{I}-\mathbf{A}_{ss})^{-1}\sum_{t\neq s}^{G} \mathbf{A}_{st}\mathbf{B}_{ts} \end{aligned} \tag{B.1}$$

while the generic off-diagonal block element $\mathbf{B}_{rs}$ corresponds to:

$$\begin{aligned} \mathbf{B}_{rs} &= \sum_{t\neq s}^{G} \mathbf{B}_{rt}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1} = \\ &= (\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{t\neq r}^{G} \mathbf{A}_{rt}\mathbf{B}_{ts} \end{aligned} \tag{B.2}$$

In deriving our bilateral decompositions, we introduce a modified version of the Leontief inverse matrix, $\mathbf{B}^{\not s}$. Here we show some key relationships between the elements of this matrix and those of the 'traditional' Leontief inverse matrix ($\mathbf{B}$) and the input requirement matrix ($\mathbf{A}$).

First, we recall that $\mathbf{B}^{\not s}$ is obtained by setting equal to 0 the coefficients that identify the requirement of inputs from country s in the $\mathbf{A}$ matrix (excepting only the domestic input requirement matrix $\mathbf{A}_{ss}$). Thus, the modified matrix of input requirements can be expressed as follows:

$$\mathbf{A}^{\not s} = \mathbf{A} - \mathbf{A}^{s} \tag{B.3}$$

where $\mathbf{A}^{s}$ is the $GN\times GN$ matrix with the coefficients of intermediate inputs imported from s in the corresponding sub-matrices and zero elsewhere. Since $\mathbf{B}^{\not s}$ is the inverse of $(\mathbf{I}-\mathbf{A}^{\not s})$, the following relationships hold:

$$(\mathbf{I}-\mathbf{A}^{\not s})\mathbf{B}^{\not s} = \mathbf{B}^{\not s}(\mathbf{I}-\mathbf{A}^{\not s}) = \mathbf{I} \tag{B.4}$$

Substituting (B.3) into (B.4) we get:

$$(\mathbf{I}-\mathbf{A})\mathbf{B}^{\not s}+\mathbf{A}^{s}\mathbf{B}^{\not s}=\mathbf{B}^{\not s}(\mathbf{I}-\mathbf{A})+\mathbf{B}^{\not s}\mathbf{A}^{s}=\mathbf{I} \tag{B.5}$$

and multiplying both sides of (B.5) by $\mathbf{B}\equiv(\mathbf{I}-\mathbf{A})^{-1}$ we obtain the following equivalence:

$$\mathbf{B}=\mathbf{B}^{\not s}+\mathbf{B}\mathbf{A}^{s}\mathbf{B}^{\not s}=\mathbf{B}^{\not s}+\mathbf{B}\mathbf{A}^{s}\mathbf{B}^{\not s} \tag{B.6}$$

Then we focus on the off-diagonal block element $\mathbf{B}_{sr}$ that identifies the gross output generated in s necessary to produce one unit of r final good. According to equation (B.6) this sub-matrix can be expressed as follows:

$$\mathbf{B}_{sr}=\mathbf{B}^{\not s}_{sr}+\mathbf{B}_{ss}\sum_{t\neq s}\mathbf{A}_{st}\mathbf{B}^{\not s}_{tr} \tag{B.7}$$

where $\mathbf{B}^{\not s}_{sr}$ is equal to $\mathbf{0}$ for each $r\neq s$, since it corresponds to a summation of infinite terms all equal to the null matrix. Therefore, if we single out the $\mathbf{B}^{\not s}_{rr}$ element from the final summation of the right-hand side of equation (B.7) we get:

$$\mathbf{B}_{sr}=\mathbf{B}_{ss}\mathbf{A}_{sr}\mathbf{B}^{\not s}_{rr}+\mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}\mathbf{B}^{\not s}_{tr} \tag{B.8}$$

Then applying to the elements of matrix $\mathbf{B}^{\not s}$ the properties of $\mathbf{B}$ sub-matrices illustrated in (B.1) and (B.2):

$$\mathbf{B}^{\not s}_{rr} = (\mathbf{I}-\mathbf{A}_{rr})^{-1}+(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}^{\not s}_{jr} \tag{B.9}$$

$$\mathbf{B}^{\not s}_{tr} = (\mathbf{I}-\mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}^{\not s}_{jr} \tag{B.10}$$

C Appendix: Proofs

Equivalence between the domestic value-added source-based and the complement of the 'import content of exports' in Hummels et al. (2001)

This equivalence can be proved by showing that the numerator of the $\mathbf{VS}_{sr}$ indicator in equation (1) is equal to the complement of the domestic value-added ($\mathbf{DVA}source_{sr}$) in the source-based decomposition of bilateral exports of equation (10). It means that

$$\mathbf{u}_N \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr} \tag{C.1}$$

should be equal to

$$\mathbf{V}_s\mathbf{B}_{ss}^{\not s} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}^{\not s}\mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}^{\not s} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr} \tag{C.2}$$

Since $\mathbf{u}_N = \sum_t \mathbf{V}_t\mathbf{B}_{tj}$ (see (A.4)), we can rewrite the expression in (C.1) as:

$$\mathbf{V}_s\mathbf{B}_{sj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{tj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr} \tag{C.3}$$

While, from equation (9) it follows that the expression in (C.2) is equal to:

$$\mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}, \tag{C.4}$$

where in (C.4) we make use of the equivalence between $\mathbf{B}_{ss}^{\not s}$ and $(\mathbf{I} - \mathbf{A}_{ss})^{-1}$. From equivalence in (B.1) we know that:

$$\mathbf{V}_s\mathbf{B}_{sj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr} = \mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr} \tag{C.5}$$

while, from (B.2)

$$\sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{tj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{sr} = \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr} \tag{C.6}$$

Then, it is proved that the expression in (C.1) is equal to that in (C.2). Q.E.D.

Equivalence between the source-based value-added in exports and the sink-based one for aggregate exports of a country

The domestic (foreign) value-added in the aggregate exports of country s can be obtained by summing across the bilateral importers r the expressions in equation (16) (equation (17)), for the source-based approach, and in equation (33) (equation (34)), for the sink-based one. Simplifying the expressions by abstaining from keeping trace of the country of re-export, the total value-added originated in a given country j and exported by country s can be expressed as follows:

$$\mathbf{VA}source_s^j = \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl} \tag{C.7}$$

$$\mathbf{VA}sink_s^j = \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}^{\nsharp}\mathbf{Y}_{ss} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}_{rk}^{\nsharp}\mathbf{Y}_{kl} \tag{C.8}$$

.

From equation (B.6) it follows that $\mathbf{B}_{js} = \mathbf{B}_{js}^{\nsharp} + \mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}$; then equation (C.8) can be re-expressed as:

$$\begin{aligned}\mathbf{VA}sink_s^j = {} & \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}\sum_{l\neq s}^{G}\mathbf{Y}_{sl} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}^{\nsharp}\mathbf{Y}_{ss} \\ & + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}_{rk}^{\nsharp}\mathbf{Y}_{kl},\end{aligned} \tag{C.9}$$

where we used the equivalence $\sum_{l\neq s}^{G}\mathbf{Y}_{sl} \equiv \sum_{r\neq s}^{G}\mathbf{Y}_{sr}$.

Since $\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rk}^{\nsharp} = \mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rk}$, we can re-write equation (C.9) as:

$$\mathbf{VA}sink_s^j = \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}_{js}^{\nsharp}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl} \tag{C.10}$$

which is equal to the value-added originated in j and exported by s according to the source-based approach of equation (C.7), Q.E.D.

D Appendix: Comparison with Koopman et al. (2014) and other related contributions

The Koompan et al. (2014) decomposition of total exports of country s ($\mathbf{u}_N\mathbf{E}_{s*}$) is summarized by the following accounting relationship:

$$\begin{aligned}
\mathbf{u}_N\mathbf{E}_{s*} &= \left\{\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{ss}\mathbf{Y}_{sr} + \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rr} + \mathbf{V}_s\sum_{r\neq s}^{G}\sum_{t\neq s,r}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rt}\right\} \\
&+ \left\{\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rs} + \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss}\right\} \\
&+ \mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} \\
&+ \left\{\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}\right\} \\
&+ \sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*} \qquad \text{(D.1)}
\end{aligned}$$

KWW defines the nine items in equation (D.1) as follows:

1. $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{ss}\mathbf{Y}_{sr}$: domestic value-added in direct final goods exports;
2. $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rr}$: domestic value-added in intermediate exports absorbed by direct importers;
3. $\mathbf{V}_s\sum_{r\neq s}^{G}\sum_{t\neq s,r}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rt}$: domestic value-added in intermediate goods re-exported to third countries;
4. $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{Y}_{rs}$: domestic value-added in intermediate exports reimported as final goods;
5. $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss}$: domestic value-added in intermediate inputs reimported as intermediate goods and finally absorbed at home;
6. $\mathbf{V}_s\sum_{r\neq s}^{G}\mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{E}_{s*}$: double-counted intermediate exports originally produced at home;
7. $\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr}$: foreign value-added in exports of final goods;

8. $\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}$: foreign value-added in exports of intermediate goods;

9. $\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}$: double-counted intermediate exports originally produced abroad.

Sink-based breakdown of bilateral exports

A full sink-based decomposition of bilateral exports from country s to country r can be expressed by the following accounting relationship:

$$
\begin{aligned}
\mathbf{u}_N\mathbf{E}_{sr} = {}& \overset{\textbf{1}}{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{Y}_{sr}} \\
&+ \mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\Bigg[\overset{\textbf{2a}}{\mathbf{Y}_{rr}} + \overset{\textbf{2b}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\sharp}_{jr}\mathbf{Y}_{rr}} + \overset{\textbf{2c}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s,r}^{G}\mathbf{B}^{\sharp}_{jk}\mathbf{Y}_{kk}}\Bigg] \\
&+ \mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\Bigg[\overset{\textbf{3a}}{\sum_{j\neq r,s}^{G}\mathbf{Y}_{rj}} + \overset{\textbf{3b}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{l\neq s,r}^{G}\mathbf{B}^{\sharp}_{jr}\mathbf{Y}_{rl}} \\
&\qquad + \overset{\textbf{3c}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s,r}^{G}\mathbf{B}^{\sharp}_{jk}\mathbf{Y}_{kr}} + \overset{\textbf{3d}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s,r,l}^{G}\sum_{l\neq s,r}^{G}\mathbf{B}^{\sharp}_{jk}\mathbf{Y}_{kl}}\Bigg] \\
&+ \mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\Bigg[\overset{\textbf{4a}}{\mathbf{Y}_{rs}} + \overset{\textbf{4b}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\sharp}_{jr}\mathbf{Y}_{rs}} + \overset{\textbf{4c}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s,r}^{G}\mathbf{B}^{\sharp}_{jk}\mathbf{Y}_{ks}}\Bigg] \\
&+ \overset{\textbf{5}}{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\sharp}_{js}\mathbf{Y}_{ss}} \\
&+ \overset{\textbf{6}}{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\sharp}_{js}\mathbf{E}_{s*}} \\
&+ \overset{\textbf{7}}{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr}} + \overset{\textbf{8}}{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}} \\
&+ \mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\Bigg[\overset{\textbf{9a}}{\sum_{j\neq r}^{G}\mathbf{Y}_{rj}} + \overset{\textbf{9b}}{\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}}\Bigg] \\
&+ \overset{\textbf{9c}}{\sum_{t\neq s,r}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}}
\end{aligned}
$$

$$+\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\overset{\textbf{9d}}{\sum_{j\neq r}^{G}}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{E}_{j*} \tag{D.2}$$

We can define the items that form the bilateral decomposition of gross exports as follows:

1 domestic value-added (VA) in direct final good exports;

2a domestic VA in intermediate exports absorbed by direct importers as local final goods;

2b domestic VA in intermediate exports absorbed by direct importers as local final goods only after additional processing stages abroad;

2c domestic VA in intermediate exports absorbed by third countries as local final goods;

3a domestic VA in intermediate exports absorbed by third countries as final goods from direct bilateral importers;

3b domestic VA in intermediate exports absorbed by third countries as final goods from direct bilateral importers only after further processing stages abroad;

3c domestic VA in intermediate exports absorbed by direct importers as final goods from third countries;

3d domestic VA in intermediate exports absorbed by third countries as final goods from other third countries;

4a domestic VA in intermediate exports absorbed at home as final goods of the bilateral importers;

4b domestic VA in intermediate exports absorbed at home as final goods of the bilateral importers after additional processing stages abroad;

4c domestic VA in intermediate exports absorbed at home as final goods of a third country;

5 domestic VA in intermediate exports absorbed at home as domestic final goods;

6 double-counted intermediate exports originally produced at home;

7 foreign VA in exports of final goods;

8 foreign VA in exports of intermediate goods directly absorbed by the importing country r;

9a and 9b foreign VA in exports of intermediate goods re-exported by r directly to the country of final absorption.

9c and 9d double-counted intermediate exports originally produced abroad.

The enumeration of the items recalls the original Koopman et al. (2014) components, which can be obtained as a simple summation over the importing countries r of the corresponding items in our bilateral decomposition (e.g. the second term in KWW is equal to the sum across the r destinations, $\mathbf{E}_{s*} = \sum_{r \neq s}^{G} \mathbf{E}_{sr}$, of **2a**+**2b**+**2c**).

We can then provide formal proof of this equivalence for each item in equation (D.2).

For items **1**, **7** and **8** the original KWW components can be obtained as a simple sum over the importing countries r of the corresponding items in our bilateral decomposition.

As already mentioned, despite being labeled by KWW as foreign double counted, item **9** in the original Koopman et al. (2014) decomposition (D.1) includes both foreign value-added—which corresponds to terms **9a** and **9b** in equation (D.2)—and foreign double counted components—i.e. terms **9c** and **9d**. Then, item **9** of the KWW decomposition can be obtained by aggregating terms **9a**–**9d** in (D.2), and summing across all the importing partners r. To show this, we can first split term **9** of equation (D.1) into the part originally produced by the importing country r and that produced elsewhere:

$$\begin{aligned}\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} &= \sum_{t \neq s,r}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} \\ &+ \sum_{r \neq s}^{G} \mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*}. \qquad \text{(D.3)}\end{aligned}$$

The first item on the right-hand side of equation (D.3) corresponds to the sum across bilateral partners of the item **9c**. Items **9a**, **9b** and **9d** can be easily identified starting from the second item on the right-hand side of equation (D.3) and expressing the exports from countries r ($\mathbf{E}_{r*}$) according to equations (12)–(14):

$$\begin{aligned}
\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*} \;=\;& \sum_{t\neq s,r}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}\\
&+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}\\
&+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{E}_{j*}\\
&+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}. \qquad \text{(D.4)}
\end{aligned}$$

For the remaining components, a few more steps are needed to prove the equivalence between the two expressions.

Plugging (B.9) and (B.10) into (B.8), we obtain the following expression for $\mathbf{B}_{sr}$:

$$\begin{aligned}
\mathbf{B}_{sr} \;=\;& \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} + \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}_{jr}^{\not s}\\
&+ \mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}(\mathbf{I}-\mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}_{jr}^{\not s}. \qquad \text{(D.5)}
\end{aligned}$$

Finally, we can sum across the $G-1$ foreign countries (i.e. $\sum_{r\neq s}^{G}$) to show that the remaining items in the accounting of bilateral trade flows in equation (D.2) can be mapped into the corresponding components of the original KWW decomposition of aggregate exports. For instance, pre-multiplying by matrix $\mathbf{V}_s$, post-multiplying by $\mathbf{Y}_{rr}$ and summing across r both sides of equation (D.5) we exactly retrieve the second component of the KWW decomposition:

$$\begin{aligned}
\mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{sr}\mathbf{Y}_{rr} =\;& \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}\\
&+\mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}_{jr}^{\not s}\mathbf{Y}_{rr} \qquad \text{(D.6)}\\
&+\mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}(\mathbf{I}-\mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}_{jr}^{\not s}\mathbf{Y}_{rr}
\end{aligned}$$

where the left-hand side of equation (D.6) corresponds to the sum across all direct importers (r) of the components **2a**, **2b** and **2c** in equation (D.2):

$$\sum_{r \neq s}(\mathbf{2a}+\mathbf{2b}+\mathbf{2c}) = \mathbf{V}_s \sum_{r \neq s} \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{V}_s \sum_{r \neq s} \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj}\mathbf{B}^{\not s}_{jr}\mathbf{Y}_{rr} + \mathbf{V}_s \sum_{r \neq s} \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj} \sum_{k \neq s,r}^{G} \mathbf{B}^{\not s}_{jk}\mathbf{Y}_{kk} \tag{D.7}$$

The first two terms on the left-hand side are clearly identical, and the equivalence between the last items is readily verified by replacing the subscript k with r and the subscript r with t in the last term of equation (D.7). However, it should be noticed that for this last term the single addends in the summation across the r foreign countries differ between the two equations. This is because this portion of domestic value-added produced in s for final use in r gets to the final destination markets by passing through one or more third countries; that is, it is not part of the bilateral exports from s to r.

Starting from the definition of the $\mathbf{B}_{rs}$ matrix in equation (D.5) and following the same procedure employed for the second item of the KWW decomposition, it is easy to prove that the third and fourth components too can be obtained as the sum of the corresponding items in our bilateral decomposition across all the destinations.

Finally, we use a slightly different procedure to show that also the fifth and sixth terms in the KWW main accounting relationship are exactly mapped within the bilateral exports. We start by singling out the block matrix $\mathbf{B}_{ss}$ from the principal diagonal of the $\mathbf{B}$ matrix. According to equation (B.6) this matrix is equal to:

$$\mathbf{B}_{ss} = \mathbf{B}^{\not s}_{ss} + \mathbf{B}_{ss} \sum_{r \neq s} \mathbf{A}_{sr}\mathbf{B}^{\not s}_{rs} \tag{D.8}$$

We can then apply to the $\mathbf{B}^{\not s}$ the property of the block diagonal elements of the $\mathbf{B}$ matrix illustrated in (B.1):

$$\mathbf{B}^{\not s}_{ss} = (\mathbf{I}-\mathbf{A}^{\not s}_{ss})^{-1} + \sum_{t \neq s} \mathbf{B}^{\not s}_{st}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}^{\not s}_{ss})^{-1} = (\mathbf{I}-\mathbf{A}_{ss})^{-1} \tag{D.9}$$

where the last equality follows from the fact that, by construction, $\mathbf{B}^{\not s}_{st}$ is equal to

$\mathbf{0}$ for each $t \neq s$. Therefore (D.8) can be rewritten as follows:

$$\mathbf{B}_{ss} = (\mathbf{I} - \mathbf{A}_{ss})^{-1} + \mathbf{B}_{ss} \sum_{r \neq s} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \tag{D.10}$$

Then, applying the same property of the block diagonal elements of the $\mathbf{B}$ matrix to the left-hand side of (D.10) and rearranging we obtain:

$$\sum_{r \neq s} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} = \sum_{r \neq s} \mathbf{B}_{ss} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \tag{D.11}$$

Finally, using the property presented in (B.2) to the $\mathbf{B}^{\not s}_{rs}$ matrix we get:

$$\sum_{r \neq s} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} = \sum_{r \neq s} \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{js} \tag{D.12}$$

Now it is straightforward to see that the fifth and sixth terms in the KWW decomposition are simply the sum of the same terms in equation (D.2) across all the bilateral destinations.

E Appendix: Summary tables

Table 2: Summary of the different measures of value-added in exports proposed in the work

	Content	Trade flow decomposed	Perspective	Approach
Section 3.1	Domestic	total/bilateral exp. (1)	exp. country	source
	Foreign	total/bilateral exp. (1)	exp. country	source
Section 3.2	Domestic	total/bilateral exp. (1)	exp. country	sink
	Foreign	total/bilateral exp. (1)	exp. country	sink
Section 4.1	Domestic	bilateral	bilateral	
	Foreign	bilateral	bilateral	
Section 4.2	Domestic	sectoral-bilateral	sectoral-bilateral	
	Foreign	sectoral-bilateral	sectoral-bilateral	
Section 4.3	Domestic	sectoral exp.	exp. sector	
	Foreign	sectoral exp.	exp. sector	
Section 4.3	Country of origin	total imports	imp. country	
Section 4.3	Country of origin	sectoral imports	imp. sector	
Section 5.1	Domestic	n.a.	n.a.	n.a.
	Foreign	bilateral (1)	world	source
	Foreign	bilateral (1)	world	sink

(1) Correspondent decompositions for total exports can be obtained summing across importing countries. In this case, sink and source breakdowns provide the same results for DVA and FVA.

Table 3: A classification of the main measures proposed in the literature

	Content	Export flow decomposed	Perspective	Approach
Hummels et al. (2001) (1)	Domestic	total/bilateral	country	source
	Foreign	n.a.	n.a.	n.a.
Johnson and Noguera (2012) (2)	Domestic	total	exp. country	
	Foreign	n.a.	n.a.	n.a.
Koopman et al. (2014)	Domestic	total	exp. country	
	Foreign	total	world	sink (3)
Wang et al. (2013)	Domestic	total/bilateral	exp. country	mixed
	Foreign	total/bilateral	world	sink (3)
Los et al. (2016) - section I	Domestic	total	exp. country	
	Foreign	n.a.	n.a.	n.a.
Los et al. (2016) - section III	Domestic	bilateral	bilateral	
	Foreign	n.a.	n.a.	n.a.
Nagengast and Stehrer (2016) - sec. A3a	Domestic	bilateral	exp. country	source/mixed (4)
	Foreign	bilateral	exp. country	mixed (4)
Nagengast and Stehrer (2016) - sec. A3b	Domestic	bilateral	exp. country	sink/mixed (4)
	Foreign	bilateral	exp. country	mixed (4)
Johnson (2018)	Domestic	bilateral	bilateral	
	Foreign	bilateral	bilateral	
Miroudot and Ye (2018)	Domestic	total	exp. country	source
	Foreign	total	word	source

(1) For Hummels et al. (2001) we are considering the complement to the import content of exports (i.e. VS). (2) Johnson and Noguera (2012) single out only the part of DVA in total exports that is absorbed abroad (VAX). Then, they do not consider the "reflection" part of DVA. (3) Koopman et al. (2014) and Wang et al. (2013) underestimate the correct measure of foreign value-added in exports, see discussion in paragraph 5.2 (4) In Nagengast and Stehrer (2016) the distinction between sink and source is implemented only for a sub-portion of their decompositions (i.e. for the direct absorption of DVA by the bilateral partner). Moreover, even for this sub-portion, the sink decomposition is inaccurately specified.