General Equilibrium Effects in Space: Theory and Measurement*

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Abstract

We propose a methodology to recover the general equilibrium impact of an economic shock by aggregating its impact across regions. Theoretically, it is sufficient to measure i) shifts in regional excess labor demand curves, which are shift-share exposure variables, and (ii) the associated (direct and indirect) reduced-form elasticities determined by spatial links in excess labor demand. Empirically, our characterization yields a generalized shift-share strategy to either estimate parameters regulating the model's reduced-form elasticities, or test the model fit based on the shock's differential effect across regions. Studying the impact of the "China shock" on labor outcomes of U.S. Commuting Zones, we estimate large direct elasticities to regional shock exposure and reinforcing indirect elasticities to other regions' shock exposure, which lead to aggregate employment losses of 3.4 – 4.8 million jobs. Our estimates point out a disconnect: common assumptions in quantitative spatial frameworks yield differential effects that are too small compared to their empirical counterparts and, thus, are rejected by our model fit test.

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1 Introduction

Aggregate shocks do not affect all regions of a country in the same way. A recent wave of empirical work in international, macro, and urban economics has exploited variation in regional exposure to aggregate shocks to evaluate their differential impact on regional outcomes – for reviews, see Moretti (2011), Autor et al. (2016b), Nakamura and Steinsson (2018), Chodorow-Reich (2019; 2020). Such an empirical strategy became a popular tool to uncover causal evidence about the patterns of labor market adjustment to aggregate shocks. However, it suffers from the so-called "missing intercept" problem: it does not recover the aggregate general equilibrium impact of the shock when regions are spatially connected – for example, when there are spatial demand spillovers or upstream and downstream relationships.[1] How can we solve the aggregation problem of recovering the general equilibrium impact of economic shocks from their differential effects across regions? Moreover, how can we design an empirical strategy to measure the impact of such shocks that exploits credibly exogenous cross-regional variation in shock exposure?

We propose a new theoretical solution to the aggregation problem by expressing the changes in regional outcomes implied by spatial models in their reduced form: in terms of shifts in regional excess labor demand (i.e., regional shock exposure) and reduced-form elasticities, direct and indirect, to these shifts. We show that the reduced-form elasticities are sufficient statistics for aggregating the exposure of different regions in order to compute the shock's general equilibrium impact. We further characterize how these reduced-form elasticities depend on features of the initial network of trade and labor outcomes, as well as parameters governing adjustment channels in the economy.

Based on our theoretical results, we develop an empirical specification to measure the (differential and aggregate) impact of observed economic shocks on regional outcomes. The specification is a generalization of shift-share empirical strategies that identifies the parameters controlling the model's reduced-form elasticities from the response of regional outcomes to the exposure of different regions to exogenous observed shocks. Conversely, when parameters are known, our empirical specification yields over-identification moments for testing the fit of the model's predicted responses to observed exogenous shocks, providing a solution for the lack of a methodology to evaluate the fit of quantitative trade and spatial models (Kehoe, 2005; Kehoe et al., 2017; Antràs and Chor, 2021).

We then use our empirical specification to study the impact of the China shock on U.S. Commuting Zones (CZs). Our results indicate a disconnect between the large empirical estimates and the small quantitative predictions in the literature for the shock's differential impact across regions. Such a disconnect arises because common assumptions in quantitative spatial frameworks yield reduced-form elasticities that are too small compared to their empirical counterparts and, thus, are rejected by our model fit test. In contrast, our empirical specification yields large reduced-form

[1] This is related to the problem that difference-in-difference empirical strategies do not recover the general equilibrium effect of the "treated" on "non-treated" (Heckman et al., 1998). Muendler (2017) and Chodorow-Reich (2020) discuss this problem for specifications based on cross-regional variation in shock exposure.

elasticities that point to larger employment losses, both on average and differentially across CZs.

To guide our modeling choices, we start in Section 2 by extending the specification in Autor et al. (2013) (henceforth ADH) to document three facts about how different mechanisms shaped the regional responses to the China shock. First, spatial links propagated negative shocks in labor demand across regions: employment and wage growth were weaker in CZs geographically close to a CZ facing higher import competition. Second, stronger import growth in (final and intermediate) goods consumed in a CZ did not generate relative gains in employment and wages. Third, population did not respond to any measure of regional shock exposure. This suggests that spatial links in trade and migration flows did not offset, but instead amplified, the negative differential impact of higher regional exposure to import competition. We argue that this finding points to a *disconnect* between the large estimated differential effects of the China shock and their relatively small counterparts in quantitative models with rich spatial links – e.g., Caliendo et al. (2019).

We then build a tractable spatial model in Section 3, which we qualitatively connect to these facts and use to solve the aggregation problem. We consider a multi-region, multi-sector gravity trade model that features local agglomeration forces in production, as well as endogenous employment decisions based on the ratio of local wages to non-employment benefits. We show that, up to a first-order approximation, log-changes in labor outcomes of market i, $\hat{Y}_i$, following shocks in the fundamentals of the global economy $\hat{\boldsymbol{\tau}}$ (e.g., trade costs and productivity) are given by

$$\hat{Y}_i = \beta^Y_{ii}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_i(\hat{\boldsymbol{\tau}}|\boldsymbol{\mathcal{W}}^0) + \sum_{j\neq i}\beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j(\hat{\boldsymbol{\tau}}|\boldsymbol{\mathcal{W}}^0) \tag{1}$$

where $\hat{\eta}_i(\hat{\boldsymbol{\tau}}|\boldsymbol{\mathcal{W}}^0)$ is the shock-induced shift in each market's excess labor demand, and $\beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ is the reduced-form elasticity of market i's outcome to the shift in excess labor demand of another market j (including its own market i). In our model, $\hat{\eta}_i(\hat{\boldsymbol{\tau}}|\boldsymbol{\mathcal{W}}^0)$ captures the market's "revenue shock exposure", i.e. how much its revenue falls due to the shock (holding constant wages and employment). It takes a shift-share form: the summation of shocks in $\hat{\boldsymbol{\tau}}$ interacted with pre-shock regional exposure shares computed from the sectoral employment and trade outcomes in $\boldsymbol{\mathcal{W}}^0$. The reduced-form elasticities $\beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ capture how much the shock exposure of a market directly affects its own outcomes, and indirectly percolates to other markets. They depend on the same pre-shock variables in $\boldsymbol{\mathcal{W}}_0$ and the two parameters in $\boldsymbol{\theta}$ determining the curvatures of the regional supply and demand for labor.

This structural relationship forms the basis of how we recover the aggregate impact of shocks from their differential effects across regions. For any $\hat{\boldsymbol{\tau}}$, we measure the shock exposure of each market using outcomes observed prior to the shock. We then compute the shock's general equilibrium impact by aggregating the exposure of different markets using estimates of the direct and indirect reduced-form elasticities that determine the shock's differential effects across regions.[2]

Based on this theoretical result, we explain how the model can generate the facts above given that the shift-share measure of regional exposure in ADH resembles a negative revenue shock exposure in our model. First, indirect reduced-form elasticities are increasing in bilateral trade links and are positive when such links are strong enough. Thus, a negative revenue shock exposure in one market endogenously reduces labor demand in nearby regions for which trade links are stronger. Second, while higher pre-shock spending on imported goods that became cheaper directly affects the cost of living, it does not have any impact on employment and wages when import prices do not affect non-employment benefits and production costs. Finally, reduced-form elasticities are increasing in both the strength of agglomeration and labor supply responses. Thus, the fact that these forces are weak in recent Ricardian spatial models helps to explain the empirical disconnect highlighted above.

We move beyond qualitatively linking the model to evidence by explicitly using our reduced-form characterization to estimate the general equilibrium impact of observed shocks to the economy's fundamentals. Through the lens of our model, the observed changes in regional outcomes are the sum of the predicted response to the observed shock, given by (1), plus a constant and a residual solely determined by unobserved shocks. Therefore, if the observed shock is exogenous (i.e., orthogonal to all other unobserved shocks), this structural relationship yields an empirical specification for the estimation of the parameters in $\boldsymbol{\theta}$ and, therefore, the reduced-form elasticities $\beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$. Identification comes from estimated differential effects: how much regional outcomes directly and indirectly respond to higher exposure in different markets. Our empirical specification has two advantages. It transparently connects the aggregate impact of the observed shock to the magnitude and sign of the estimated reduced-form elasticities, because the constant and the residual in our specification do not depend on the observed shocks. In addition, it yields the most efficient estimator of $\boldsymbol{\theta}$ since it leverages all the channels through which the observed shock affects outcomes in general equilibrium, instead of the equilibrium relationships between endogenous variables (instrumented with exogenous shocks) traditionally used in structural estimation (e.g. Galle et al. (2017); Fajgelbaum et al. (2018); Faber and Gaubert (2019)).

Moreover, once the spatial model is fully specified (i.e., $\boldsymbol{\theta}$ is already known), our empirical specification yields additional over-identification moments that can be used to formally test whether one can reject that the model's predicted responses to observed shocks are consistent with actual responses in labor outcomes across markets. Importantly, the credibility of the model's predictions is undermined if it fails the test, since the targeted over-identification moments rely on the reduced-form elasticities that are sufficient for computing both the model's differential and aggregate predictions. Our test can be applied to traditional structural estimation procedures because it

[2] We further show how our formulas can be easily integrated to recover the exact impact of the shock, and document that the first-order approximation performs well in our empirical application.

uses the general equilibrium relationship in the model between each endogenous outcome and the exogenous shocks, which is different than using equilibrium relationships between endogenous variables (instrumented with exogenous shocks) typically targeted in structural estimation. Lastly, since our test leverages an exogenous observed shock, it is robust to unobserved shocks driving most of the variation in regional outcomes – a common critique against performance evaluations based on statistical decomposition, such as the one proposed by Kehoe et al. (2017) (see discussion in Antràs and Chor (2021)).

In Section 4, we generalize our results to a wider class of models that includes recent quantitative versions of trade and spatial models – for reviews, see Costinot and Rodríguez-Clare (2014) and Redding and Rossi-Hansberg (2017). We allow for trade in final and intermediate goods, as well as labor supply to depend on migration choices and import prices. These additional mechanisms yield an extension of (1) that entails three new insights. First, trade in intermediate goods introduces upstream production relationships into the measure of "revenue shock exposure" of each market. Second, higher usage of intermediates plays a similar role to stronger agglomeration forces in amplifying the reduced-form elasticities to revenue shock exposure. Third, the shift in excess labor demand now also incorporates two measures of "consumption shock exposure": one accounting for the downstream effect of import cost shocks on sales and another accounting for the effect of import price shocks on labor supply. Despite these additional considerations, our extended reduced-form characterization delivers a similar empirical specification that can be used for estimation and testing under the same exogeneity restriction on the observed shock.

The last part of the paper, Section 5, revisits the problem of estimating the impact of the "China shock" on U.S. CZs. In the baseline model, our empirical specification is a generalization of the shift-share specification in ADH for observed changes in wage and employment rates across regions. It accounts for general equilibrium spatial links through the heterogeneous (direct and indirect) reduced-form elasticities of these outcomes to the revenue exposure of different CZs to the China shock. We find that these reduced-form elasticities are large as a result of strong agglomeration forces and high sensitivity of employment to wages. The estimation of the extended model yields similar conclusions but also indicates that the two channels of consumption exposure channels in the model lead to relatively weak employment responses to import price shocks.

We then implement our model fit test for different specifications of spatial links. Our estimated specification yields predictions that are consistent with the observed differential responses in both outcomes used in estimation (i.e., wage and employment rates), as well as other outcomes not targeted in estimation (i.e., sectoral employment composition). We also implement the test for the predictions of the model under alternative calibrations based on quantitative spatial frameworks recently used to study the China shock – e.g., Galle et al. (2017) and Caliendo et al. (2019).[3] In

[3] In fact, under this alternative calibration, our model's predicted responses in the employment rate of U.S. states are similar to those reported in Caliendo et al. (2019).

line with the disconnect pointed out above, these alternative calibrations yield small differential predicted effects across CZs that are rejected by our model fit test. We identify the reason behind this disconnect to be the following features common to recent quantitative spatial frameworks: lack of agglomeration forces, weak employment responses to wage changes, and strong employment responses to import price shocks.

We conclude by aggregating the predicted responses implied by our estimates to compute the shock's general equilibrium impact. We find that, while most CZs experienced employment losses, the magnitude of those losses varied substantially. Aggregating responses for the entire U.S., our model predicts that the China shock eliminated 3.4 – 4.8 million jobs between 1990 and 2007, with half of this impact caused by the indirect effects of spatial links. When we account for the impact of the shock on the cost of living, our baseline model yields an average real wage loss of 2 p.p., but our extended model implies a smaller loss of 0.35 p.p. due to the downstream cost reduction caused by cheaper imports. Again, in both cases, we obtain large spatial dispersion in real wage responses. Compared to existing work, we find differential and aggregate losses in employment that are an order of magnitude larger than Caliendo et al. (2019), and negative and dispersed effects on real wages compared to the small average and differential gains reported by Galle et al. (2017) and Caliendo et al. (2019).

Our theoretical solution to the aggregation problem is a significant departure from the common approach of computing aggregate effects using models with rich calibrated spatial links (as in the literature summarized in Redding and Rossi-Hansberg (2017)). The key differentiation is that we express the model's predictions in terms of heterogeneous direct and indirect reduced-form elasticities and observable measures of shock exposure, and that we leverage this reduced-form representation for estimation and testing by linking the theoretical differential effects to their empirical counterparts. The heterogeneous indirect effects set our analysis apart from recent macroeconomic frameworks with regional responses featuring a "missing intercept" computed with calibrated spatial models – e.g. Nakamura and Steinsson (2014); Mian and Sufi (2014); Beraja et al. (2019); Chodorow-Reich (2019). As Chodorow-Reich (2020) points out, under such an approach, the measurement of the shock's aggregate impact heavily depends on particular modeling assumptions and the identification of its differential impact relies on the Stable Unit Treatment Value Assumption (SUTVA). This identification assumption rules out the type of heterogeneous indirect effects documented in our empirical analysis.[4] In fact, we show that a common "missing intercept" can arise only with restrictive symmetry assumptions on spatial links.

Our reduced-form characterization involves deriving the elasticity of regional wage and employment outcomes to shocks in the global economy. In our baseline model, this characterization exploits an intuitive supply and demand representation of spatial models similar to that in Allen

[4]It also rules out heterogeneity in the direct "treatment" effect of regional shocks, which also arises from spatial links as shown by Monte et al. (2018).

et al. (2020b) and Bartelme (2018), but in the context of a more general economy with multiple sectors, endogenous employment choice, and an arbitrary structure of trade costs. Furthermore, in the extended spatial model with intermediates in production, our characterization yields measures of upstream and downstream shock exposure that are generalizations of those for single market economies either in autarky (e.g., Acemoglu et al. (2016b) and Carvalho and Tahbaz-Salehi (2019)) or in partial equilibrium (e.g., Acemoglu et al. (2016a)). They provide a structural interpretation for measures of upstreamness and downstreamness (in levels) for open economies suggested by Fally (2012), and Antràs and Chor (2013, 2018). In contemporaneous work, Baqaee and Farhi (2019) provide a first-order approximation for the impact of productivity shocks on wages and welfare in open economies linked through final and intermediate trade. Our theoretical characterization not only leads itself to a specification for the estimation of the general equilibrium effects of economic shocks on regional markets in the presence of richer spatial links, but also uncovers how such effects depend on agglomeration forces and employment responses.[5]

Our empirical specification generalizes shift-share strategies such as those in the seminal contributions of Bartik (1991) and Blanchard and Katz (1992), and those used more recently in the international trade literature – see e.g. Topalova (2010), Autor et al. (2013), Kovak (2013), Dix-Carneiro and Kovak (2017), Autor et al. (2016a), Pierce and Schott (2020). By accounting for heterogeneous indirect effects across regions, it can be used for estimating regional responses to economic shocks through the model's general equilibrium mechanisms. Our empirical specification also complements structural estimation strategies based on equilibrium relationships between endogenous outcomes in spatial models.[6] It provides additional moments for both estimating the model-implied reduced-form elasticities and evaluating the fit of the model's predictions determined by them. The latter involves testing whether one can reject a unitary pass-though from the predicted effects of a general equilibrium spatial model to the actual changes in the corresponding outcome – similar testing strategies have been used in the international trade literature (e.g., Davis and Weinstein (2001), Costinot and Donaldson (2012), Kovak (2013), Dingel and Tiltenot (2020), Adao et al. (2020b)). Our paper is closest to Kovak (2013) in that we regress changes in regional outcome on their model-implied reduced-form response to an observed exogenous trade shock, but in addition we provide formal conditions for testing a wide class of models that allow for indirect effects arising from spatial links in general equilibrium.

[5]Our work is also related to the literature on sufficient statistics in international trade, such as Arkolakis et al. (2012), Bartelme et al. (2020), Kleinman et al. (2020), and Borusyak and Jaravel (2018).

[6]This includes the so-called "market access" approach (see e.g. Redding and Venables (2004); Donaldson and Hornbeck (2016); Alder et al. (2015); Bartelme (2018)), since it is based on the equilibrium relationship between endogenous regional outcomes and the endogenous market access. Notice also that our empirical specification remains valid under a flexible structure of spatial links and arbitrary unobserved shocks, while the measurement of market access requires restricting spatial links and observing all trade costs (before and after the shock).

2 Adjustment of U.S. Regional Markets to Trade Shocks: Three Stylized Facts

We begin with an extension of the empirical analysis in ADH aimed at evaluating the importance of a number of economic channels in regulating how regional economies adjusted to trade shocks, in general, and the China shock, in particular. We document three new stylized facts that guide our modeling choices in the next section.

2.1 Empirical Specification

Our empirical analysis evaluates the differential effect of the China shock across U.S. CZs on three labor market outcomes: log of average weekly wage, log of employment rate, and log of working-age population. We now present our empirical specification that introduces two new measures of shock exposure, in addition to the ADH employment exposure of CZ i to import competition at period t (IC_i^t). In particular, we also consider the impact of a geographic gravity-based measure of region i's indirect exposure to the rise in import competition faced by nearby CZs (GC_i^t), as well as the impact of a measure of CZ i's expenditure exposure to Chinese import growth (IE_i^t). Using these measures, we estimate the following specification:

$$\Delta Y_i^t = \alpha^t + \beta^{IC} IC_i^t + \beta^{GC} GC_i^t + \beta^{IE} IE_i^t + X_i^t \lambda + \epsilon_i^t \tag{2}$$

where Y_i^t is a labor market outcome, α^t is a time fixed-effect, and X_i^t is a set of regional controls.

We now define these exposure measures. The next sections show that they arise in various model specifications. As in ADH, CZ i's employment exposure to import competition is

$$IC_i^t \equiv \sum_s \ell_{i,s}^{t_0} \Delta M_s^t, \tag{3}$$

where ΔM_s^t is the change in imports from China in the 4-digit SIC sector s for a set of high-income countries divided by the U.S. initial employment in sector s, and $\ell_{i,s}^{t_0}$ is CZ i's ten-year-lagged employment share in sector s.[7] Our definition of IC_i^t is identical to the shift-share instrumental variable (IV) in ADH. Thus, β^{IC} is the *direct* differential impact on the CZ's labor market outcomes of higher employment exposure to the growth of Chinese imports in other developed economies.

Our gravity-based measure of indirect exposure to the import competition faced by other CZs is

$$GC_i^t \equiv \sum_{j \neq i} \frac{D_{ij}^{-\delta}}{\sum_{k \neq i} D_{ik}^{-\delta}} IC_j^t, \tag{4}$$

[7] We follow ADH by using 10-year equivalent changes in imports of eight high-income countries with trade data covering the sample period: Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland.

where D_{ij} is the bilateral distance between the population centroids of CZs i and j. Our specification has a "gravity" structure: GC_i^t is higher if i is near CZs with higher import competition exposure. The parameter δ controls how much indirect exposure declines with distance – in our baseline, we use typical estimates of the trade elasticity and set $\delta = 5$. Accordingly, conditional on i's import competition exposure, β^{GC} is the *indirect* differential effect of the shock exposure of nearby regions on i's labor market outcomes. It intuitively captures the net effect of different sources of spatial shock percolation in general equilibrium. For example, labor demand spillovers from lower domestic sales to nearby regions or labor supply spillovers due to in-migration from more negatively exposed CZs.[8]

Finally, our measure of the CZ's expenditure exposure to Chinese import growth is

$$IE_i^t \equiv \sum_s e_{i,s}^{t_0} \Delta M_s^t, \tag{5}$$

where $e_{i,s}^{t_0}$ is the share of sector s in the total gross spending of CZ i. IE_i^t captures the intuitive notion that the expenditure shock in CZ i is stronger if i has a higher spending share on a sector s, $e_{i,s}^{t_0}$, in which China expanded more the world output supply, as measured by ΔM_s^t. Thus, β^{IE} is the differential effect of higher expenditure exposure to the shift in world output supply caused by the China shock. Such an impact can be positive if either labor supply or labor demand rises when there is a positive shock in the supply of goods used for final or intermediate consumption. Alternatively, the impact can be negative if higher availability of Chinese imports in a sector induces firms in the region to strongly substitute local labor for imported inputs.[9]

2.2 Data

We follow ADH to measure labor market outcomes and import competition exposure (IC_i^t) for the same pooled sample of 722 CZs in mainland U.S. over 1990-2000 and 2000-2007. To compute IE_i^t, we follow Gervais and Jensen (2019) by measuring CZ i's share of gross spending in sector s as $e_{i,s}^{t_0} \equiv \frac{\xi_s^{t_0} + \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0}}{1 + \sum_k a_k^{t_0} \ell_{i,k}^{t_0}}$, where ξ_{sk}^{M,t_0} is the share of sector s in input spending of sector k, $a_k^{t_0}$ is the ratio of input-to-labor spending in sector k, and $\xi_s^{t_0}$ is the share of sector s in final consumption. We compute ξ_{sk}^{M,t_0} and $\xi_s^{t_0}$ from the BEA input-output table, and $a_k^{t_0}$ from the NBER manufacturing

[8]We use the gravity structure in (4) to approximate (and formalize in our model below) these two main sources of cross-regional links, highlighted in recent spatial gravity models – e.g., Allen and Arkolakis (2014) and Donaldson and Hornbeck (2016). Appendix B.1 shows that our results are robust to alternative specifications for GC_i^t.

[9]For example, import supply shocks can have a positive impact on labor supply because cheaper imports increase the opportunity cost of leisure by lowering the local price index. The ambiguous effect of input prices on labor demand arises from the productivity and substitution effects of higher foreign input supply – e.g., as in Feenstra and Hanson (1999), and Grossman and Rossi-Hansberg (2008). Our model below clarifies how these mechanisms affect regional exposure to trade shocks.

database for manufacturing sectors and from the WIOD database for non-manufacturing.[10]

Table B.1 and Figure B.1 in Appendix B.1 present moments of the main variables used in our empirical application. Our two new exposure measures vary considerably across CZs, but their standard deviations are around half of that of ADH's employment exposure to import competition. Despite being constructed with the same sector-level shifters, the different exposure shares used to compute each measure imply that regions are not equally exposed to them. The correlation across CZs is 0.53 between IC_i^t and GC_i^t, but it is only 0.16 between IC_i^t and IE_i^t.

2.3 Results

Table 1 reports our baseline results. Our baseline specification includes ADH's largest control set (described in Table 1's note), as well as two extra controls for the potential confounding effect of exposure through our two additional channels to the well-known secular manufacturing decline in the period: the share of gross spending on manufacturing, and the gravity-based measure of indirect exposure to the manufacturing employment share of nearby CZs.

In columns (1), (3) and (5), we first estimate the regression in (2) using only IC_i^t to replicate ADH's findings. The estimates indicate a relative decline in both the average wage and the employment rate of CZs with higher employment in industries experiencing stronger growth in Chinese import competition. Compared to the CZ in the 25^{th} percentile of the distribution of IC_i^t, the CZ in the 75^{th} percentile of the distribution experienced changes in the average wage and employment rate that were 1.8p.p. and 2.0p.p. lower, respectively. These are large differential effects when we consider that, over the two periods, the standard deviation across CZs of changes in the average wage and the employment rate were 6.5p.p. and 6.4p.p., respectively. As in ADH, we find that higher exposure to Chinese import competition did not reduce the CZ's population. This suggests weak migration responses to the China shock.

We then turn to the full specification in (2) that also includes our two additional measures of exposure to the China shock, GC_i^t and IE_i^t. In the second row of Table 1, we report the differential impact of being close to CZs with higher exposure to import competition. Columns (2) and (4) show that the negative impact of local shock exposure propagates to nearby regions: a CZ whose neighbors are more exposed to Chinese import competition experienced relative declines in its average wage and employment rate. The simultaneous reduction of wages and employment suggests that general equilibrium links spatially spread the decline in regional labor demand and reinforce the effects of the China shock. In column (6), we again report a non-significant impact on population.

[10] This procedure is valid if input and final spending shares are the same in all CZs, and trade is balanced. In Appendix C.1.2, we evaluate our procedure to construct $e_{i,s}^{t_0}$ by running a regression of gross spending shares implied by shipment inflows in the Commodity Flow Survey (CFS) on our measured spending shares when aggregated for states and CFS commodity groups. We obtain a coefficient close to 1 and a R^2 of 0.95.

Table 1: Impact of the China Shock on U.S. CZs

	Change in average weekly log-wage		Change in log of employment rate		Change in log of working-age population	
	(1)	(2)	(3)	(4)	(5)	(6)
IC_i^t	-0.471***	-0.383***	-0.519***	-0.369***	0.273	0.127
	(0.127)	(0.113)	(0.089)	(0.079)	(0.180)	(0.155)
GC_i^t		-0.606***		-0.691***		0.348
		(0.156)		(0.155)		(0.212)
IE_i^t		0.077		-0.154		0.418
		(0.164)		(0.143)		(0.294)
Differential treatment effect (percentage points):						
	-1.78	-3.52	-1.97	-4.16	1.03	2.44

Notes: Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All endogenous variables are multiplied by 100. All specifications include the following two sets of controls. Regional controls in ADH: period and census division dummies, manufacturing employment share in 1990, college-educated population share in 1990, foreign-born population share in 1990, employment share of women in 1990, employment share in routine occupations in 1990, and average offshorability in 1990. Additional controls: CZ's share of spending in manufacturing in 1990 ($\sum_s e_{i,s}^{t_0}$), and CZ's indirect exposure to manufacturing employment share in 1990 ($\sum_{j \neq i} z_{ij} \sum_s l_{js}^{t_0}$, with $z_{ij} \equiv D_{ij}^{-5} \sum_k D_{ik}^{-5}$). Differential effect: difference between the estimated treatment effects of CZs in the 75^{th} and 25^{th} percentiles of the empirical distribution of the estimated treatment effects. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

The third row of Table 1 reports the differential impact of higher spending exposure to the China shock, IE_i^t. For all outcomes, we find that the coefficients are not statistically different from zero. Importantly, this is driven by lower point estimates with standard errors whose magnitude are similar to those of the indirect effects. Since IE_i^t is based on gross expenditure shares, our findings are consistent with weak differential responses in labor market outcomes to higher exposure to the input supply expansion caused by the China shock. This is similar to the evidence in Pierce and Schott (2016a) and Acemoglu et al. (2016a) of no differential growth in the national employment of industries using more intensively inputs from sectors in which Chinese imports grew more.

Our full specification indicates that, compared to the CZ in the 25^{th} percentile of the treatment effect distribution, the CZ in the 75^{th} percentile of the distribution experienced 3.5p.p. and 4.2p.p. lower changes in its average wage and employment rate, respectively. These differential effects are twice those implied by the ADH specification (columns (1) and (3)). Hence, the two new adjustment margins that we study (namely, GC_i^t and IE_i^t) do not offset the differential impact of the China shock documented by ADH. Instead, these margins imply even larger differential effects on employment and wage rates across CZs, but no differential responses in regional population.

Our results contradict the finding of a small differential impact of the China shock across regions implied by quantitative frameworks in the literature featuring rich spatial links in trade and migration flows. For instance, Caliendo et al. (2019) (henceforth CDP) find that the China shock had a small impact on employment, both on aggregate and differentially across U.S. states. We compare the cross-state variation in the predicted employment rate changes in CDP to those implied by the estimates in Table 1 (see Figure B.2 in Appendix B.1), and identify a disconnect between the empirical estimates and the quantitative predictions for the shock's differential impact: those in CDP have a standard deviation of 0.05, while those implied by Table 1 have a ten-times larger standard deviation of 0.54.[11] Such a disconnect is problematic and to some extent surprising because the analysis of the spatial quantitative literature on the China shock is motivated exactly by the need to complement the evidence of the differential effect in ADH with the general equilibrium channels of adjustment that may affect the aggregate impact of the shock. However, the predicted differential effects in CDP are an order of magnitude smaller than their empirical counterparts. Our analysis below formalizes this discussion with a series of model fit tests.

To summarize, our empirical analysis documents three novel stylized facts. First, we show that spatial links amplify the negative impact of local exposure to import competition by generating relative reductions in the labor demand of other nearby regions. Second, we find no evidence of attenuating responses on employment and wages in regions more exposed to the positive shock in the supply of imported goods for (final and intermediate) consumption. Third, we find no evidence of population responses to the CZ's indirect exposure to the shock in nearby CZs, in addition to the lack of population responses to the CZ's own employment exposure documented in ADH. These empirical facts guide the specification of the general equilibrium spatial model that we use to study the impact of trade shocks on regional labor markets in the next section.

2.4 Robustness and Additional Results

We now discuss the robustness of our baseline results. Appendix B.1 displays all the tables.

Alternative Empirical Specifications. Table B.2 shows that estimates are similar when we consider only subsets of our exposure measures. We also document the absence of attenuating effects from indirect exposure to spending shocks in nearby CZs (i.e., the analog of (4) for IE_j^t). Column (2) of Table B.3 indicates that our estimates of the employment and wages responses to the CZ's direct and indirect shock exposure remain statistically significant at usual levels when we use the shift-share inference of Adão et al. (2019). Columns (3)–(4) of Table B.3 report similar results when we control for state fixed-effects and lagged population growth (as in Greenland et al. (2019)) to account for state-wide and persistent amenity shocks. Column (5) of Table B.3 controls for the CZ's initial manufacturing shares interacted with period dummies, which absorbs period-specific manufacturing shocks. As in Borusyak et al. (2018), this reduces the estimated impact of import competition on wages and employment, but only the direct effect on wages is not significant at 10%. Column (6) of Table B.3 reports similar results when we weigh CZs by their

[11] We obtain the predicted responses in Caliendo et al. (2019) from their replication files, and the state-level responses implied by Table 1 from the average shock exposure of the CZs in each state. Results are similar if we estimate (2) across states. A small dispersion in the effects of the China shock across regions is a common finding in recent quantitative frameworks (e.g., Galle et al. (2017)).

population.

Alternative Shock Exposure Measures. In Table B.4, we document the same reinforcing pattern of indirect responses to the shock exposure of nearby regions when we compute the gravity-based measure in (4) while setting the distance decay to one or eight (columns (2)-(3)), adjusting for the size of nearby CZs (column (4)), and excluding out-of-state CZs (column (5)).

Table B.5 considers alternative definitions of expenditure shock exposure. In column (2), we consider two separate exposure measures of the form in (5) built with sectoral spending shares out of final and intermediate expenditure (respectively, IEF_i^t and IEI_i^t).[12] We find that employment and wages do not differentially respond in CZs with higher shock exposure in terms of either final or intermediate expenditure. Column (3) reports similar estimates when we exclude input spending on the own sector in the computation of the intermediate spending shares. Lastly, column (4) reports estimates when we approximate for cross-industry supply links using the "Leontief expenditure shares" in Acemoglu et al. (2016a). In this case, we find that higher exposure to cheaper inputs from China causes a relative decline in the CZ's employment rate.

Table B.6 considers alternative measures of the China shock in each sector. This addresses concerns related to ADH's specification of the shifters in terms of import growth in other countries, which may be itself affected by productivity shocks in U.S. CZs or demand shocks in importing countries. In Panel A, we use China's exporter fixed-effect in each sector that we obtain from a gravity regression of log changes in bilateral trade shares on sector-origin and sector-destination fixed-effects. In Panel B, we construct exposure measures using the same sector-level NTR gaps used in Pierce and Schott (2016a). In both cases, we find similar qualitative patterns of responses to higher (direct or indirect) exposure to Chinese import competition.

Additional Outcomes. Table B.7 investigates the impact of the different exposure measures on several margins of employment adjustment. Higher direct and indirect exposure to Chinese import competition caused relative declines in both the share of individuals employed in manufacturing (column (2)) and non-manufacturing (column (3)). The counterpart of the employment rate reduction is an increase in the share of individuals either out of the labor force (column (4)) or unemployed (column (5)). For both direct and indirect responses, the main adjustment margin is labor force participation, accounting for roughly two-thirds of the employment rate fall.

We conclude by investigating the impact of the China shock on gross migration flows across U.S. CZs in Table B.8. We find that all measures of exposure to the China shock did not have statistically significant impacts on either the inflow or the outflow of migrants across CZs.

[12] As in the baseline, we construct intermediate spending shares using the national input-output table and the CZ's sectoral employment shares: the share of intermediate spending on sector s is $ei_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_k a_k^{t_0} \ell_{i,k}^{t_0}$. The share of final spending on sector s in CZ i, $ef_{i,s}^{t_0}$, is the share of average household expenditure in i's state across 3-digit SIC manufacturing sectors (constructed from the Consumer Expenditure Survey – see Appendix C.2.1).

3 Theory of General Equilibrium Effects in Space

We now build a simple general equilibrium model with sufficient features to generate the stylized facts documented above. We characterize the model-implied reduced-form elasticity of regional wage and employment outcomes to shocks in the global economy. This characterization not only delivers model-consistent measures of regional exposure to shocks in economic fundamentals, but also uncovers the attenuating and reinforcing effects created by different adjustment mechanisms. Finally, based on these theoretical results, we propose an empirical specification that allows us to recover the aggregate impact of macroeconomic shocks from estimates of their differential effect across markets. This specification also yields a test of whether one can reject that the model's predicted responses to observed shocks are consistent with the actual responses in outcomes across markets.

Environment. We consider a multi-sector gravity trade model with I segmented markets grouped into countries. Each market comprises a product and labor market with a set of consumers and workers that face the same product and labor prices.[13] Let $i \in \mathcal{I}_c$ denote a market in country c. In sector s of market i, a representative competitive firm uses labor to produce a differentiated good with an endogenous production cost of $p_{i,s}$, and faces exogenous iceberg trade costs for selling to different destinations j of $\tau_{ij,s}$. Each market is endowed with a mass of heterogeneous individuals, $\bar{N}_i$, that endogenously decide whether or not to work by comparing the market's wage rate w_i to a government non-employment transfer b_i. Residents of market i face an income tax rate of v_i.[14]

Gravity Trade Demand. All individuals in market j maximize the same nested Constant Elasticity of Substitution (CES) preferences. We consider a Cobb-Douglas aggregator of sector-specific composite goods where $\xi_{j,s}$ is the constant spending share on sector s. The sectoral composite good is a CES aggregator over the differentiated sector-specific products from different origins, with $\sigma > 1$ denoting the elasticity of substitution across origins.[15] Since markets are competitive, the price of market i's sector s differentiated good in market j is $\tau_{ij,s}p_{i,s}$. Thus, utility

[13] We define a product market as a set of consumers with access to the same products and prices, a common approach in industrial organization (e.g., Berry and Haile (2014)). Similarly, we define labor markets as sets of producers that face the same labor cost, as in neoclassical and gravity trade models (e.g., Dixit and Norman (1980), Costinot and Rodríguez-Clare (2014)). We incorporate wage differences across sectors when markets are groups of sectors within a region – for instance, when each region has two distinct markets, one for the set of manufacturing industries and another for the set of non-manufacturing industries. We return to this point in Section 4.

[14] Our baseline model does not entail endogenous changes in population and intermediate input costs, since we did not find evidence for the importance of these forces in the context of the China shock in Section 2. To formally investigate their role, we incorporate these channels in Section 4 and estimate their relevance in Section 5.

[15] This demand specification greatly simplifies exposition, but we show below that our insights do not rely on assumptions of either nested CES preferences or a single elasticity of substitution for all sectors.

maximization implies that the bilateral sales in sector s from i to j are

$$X_{ij,s} = x_{ij,s}\xi_{j,s}E_j = \frac{\left(\tau_{ij,s}p_{i,s}\right)^{1-\sigma}}{\sum_o \left(\tau_{oj,s}p_{o,s}\right)^{1-\sigma}}\xi_{j,s}E_j, \tag{6}$$

where E_j is j's total expenditure. The associated consumption price index in i is

$$P_i = \prod_s (P_{i,s})^{\xi_{i,s}}, \quad \text{with} \quad P_{i,s} = \left[\sum_o \left(\tau_{oi,s}p_{o,s}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{7}$$

This demand structure implies that market i's revenue is the sum of sectoral sales to different destinations, $R_i = \sum_{j,s} X_{ij,s}$. These sales, in turn, are a function of bilateral trade costs, $\tau_{ij,s}$, and the trade elasticity, $1-\sigma$. To the extent that $\tau_{ij,s}$ depends on distance, we show below that our model features the type of spatial percolation in regional labor demand shocks that we documented in Section 2. This multi-sector gravity-based demand has become a standard way of modeling spatial links in the trade literature – see e.g. Anderson (1979); Eaton and Kortum (2002); Costinot et al. (2010); Arkolakis et al. (2012) and, for a review, Costinot and Rodríguez-Clare (2014).

Labor Supply. Individuals are heterogeneous and choose whether to be employed or not. If employed, individual ι supplies $l(\iota)$ efficiency units, obtaining a net labor income of $(1-v_i)w_i l(\iota)$. If non-employed, individual ι's income is $(1-v_i)b_i u(\iota)$, with $u(\iota)$ denoting ι's non-employment income potential. The pair $(l(\iota), u(\iota))$ is drawn independently from a Frechet distribution with shape parameter $\phi > 1$ and scale 1, so that the employment share in market i is

$$n_i = \Pr\left[(1-v_i)\frac{w_i}{P_i}l(\iota) \geq (1-v_i)\frac{b_i}{P_i}u(\iota)\right] = \frac{w_i^\phi}{w_i^\phi + b_i^\phi}. \tag{8}$$

Up to a first order approximation, the log-change in the share of employed residents in market i is $\Delta \ln n_i = \phi(1-n_i)\Delta \ln(w_i/b_i)$ and, therefore, is proportional to the change in the ratio of the market's wage rate to the return of the non-employment outside option, with a sensitivity controlled by ϕ. Under this specification, a reduction in market's labor demand leads to a decline in both wages and employment rates, in line with the evidence in Section 2.[16] This structure of selection of heterogeneous individuals into employment is a standard way of modeling changes in the extensive margin of labor supply – e.g., see Heckman and Sedlacek (1985), Rogerson (1988),

[16]Kim and Vogel (2021) impose similar assumptions to model the choice of labor force participation of heterogeneous workers. The same employment rate expression arises if we relax the Frechet assumption as in Adão (2016), but that would introduce an additional parameter to control selection forces in wages. All our results are identical if $u(\iota)$ is a private benefit of not working rather than an income potential. Appendix A.5 shows that an expression for the change in the employment rate in terms of changes in w_i/b_i also arises in a competitive search environment in which firms post vacancies with a given wage and workers decide whether to search for a job. In this case, the employment rate elasticity also depends on the efficiency parameter of the matching function (as in Kim and Vogel (2021)).

Mulligan and Rubinstein (2008), and Chetty et al. (2013a). It is also consistent with the evidence in Autor et al. (2013) and Pierce and Schott (2020) that the number of recipients of different types of government transfers increases in regions more exposed to the China shock.

The presence of heterogeneous individuals allows us to incorporate in our analysis a salient feature of the data: individuals with lower initial income are more likely to become non-employed when exposed to higher Chinese import competition (see Autor et al. (2014)). This is true in our model because individuals differ in their efficiency, implying that the wage rate w_i is not identical to the observable average log of labor earning, $\overline{\ln w_i}$, used to document the wage responses in Section 2.[17] Instead, our model yields the following equation:

$$\Delta\overline{\ln w_i} = \Delta \ln w_i - \frac{1}{\phi}\Delta \ln n_i. \tag{9}$$

The decision of non-employment in our model depends on the reservation wage, b_i, that needs to be specified in terms of a numeraire. This is similar to the specification of the numeraire of international transfers in Dekle et al. (2007) (see Ossa (2014) for a discussion) and akin to the specification of the outside numeraire good in industrial organization. We assume that, in every market i, non-employment benefits are set in terms of a common numeraire function of wages: $b_i = \bar{b}_i \Omega(\{w_j\}_j)$, where $\Omega(.)$ is homogeneous of degree one and $\omega_i \equiv \frac{\partial \ln \Omega(\{w_j\}_j)}{\partial \ln w_i}$. This specification accounts for the evidence in Chodorow-Reich and Karabarbounis (2016) that changes in the aggregate opportunity cost of employment – in our model the average change in b_i/P_i across $i \in \mathcal{I}_c$ – are positively, but only partially, correlated with changes in the aggregate real wage – in our model the average change of w_i/P_i across $i \in \mathcal{I}_c$. In Section 5, we use their evidence to specify the numeraire function $\Omega(\{w_j\}_j)$ in the non-employment payoff, $b_i/P_i = \bar{b}_i\Omega(\{w_j\}_j)/P_i$, in terms of output of different markets.[18]

Production Technology. We start with a simple structure of production whereby, in each sector s of market i, output is proportional to the representative firm's endogenous employment choice, $L_{i,s}$, as well as to a term capturing economies of scale that are external to the firm and increasing in the market's employment rate. Specifically, the production function is $Q_{i,s} = (\frac{n_i}{1-n_i})^{\psi} L_{i,s}$ and, thus, the unit production cost is

$$p_{i,s} = w_i^{1-\psi\phi} b_i^{\psi\phi}. \tag{10}$$

[17] Several models used to study the impact of trade shocks on regional economies cannot account for this fact since they miss either non-employment or heterogeneity in worker efficiency – e.g., Burstein et al. (2019); Galle et al. (2017); Caliendo et al. (2019). Adão (2016) and Kim and Vogel (2021) constitute recent exceptions.

[18] In Section 4, we specify $b_i = \bar{b}_i P_i^{\lambda}(\Omega(\{w_j\}_j))^{1-\lambda}$ and show that the impact of import expenditure exposure on labor market outcomes is increasing in λ. Given the evidence in Section 2, our estimated λ is close to zero in Section 5, which roughly corresponds to our baseline specification of b_i. Thus, our estimates and the evidence in Chodorow-Reich and Karabarbounis (2016) reject that the non-employment payoff is invariant to shocks (i.e., that b_i/P_i is constant as imposed in Caliendo et al. (2019), in which case λ would be one).

Agglomeration forces may arise from a variety of economic mechanisms such as entry externalities (e.g., Krugman (1991)), Marshallian production externalities (e.g., Ethier (1982) and Kucheryavyy et al. (2016)), and search frictions (see Appendix A.5). The importance of this mechanism to analyze regional responses to local shocks in labor demand has been emphasized by several recent papers – e.g., Greenstone et al. (2010), Kline and Moretti (2014), Dix-Carneiro and Kovak (2017), and Peters (2019).[19] Our specification captures the combination of these economic forces in a reduced-form way through the combined strength of agglomeration and labor supply forces in $\psi\phi$ and thus our functional form choice is guided by its convenient implication that the pass-through from wages to prices is the constant $1 - \psi\phi$. As shown below, this links directly the combined strength of agglomeration and labor supply forces in $\psi\phi$ to the curvature of the regional labor demand function. In Section 4, we substantially generalize the structure of production by introducing intermediate inputs. Such an extension implies a non-unitary pass-through from wages to prices that is decreasing in the share of inputs in production.

Equilibrium: Market-level Labor Demand and Supply. To analyze the equilibrium, we start by characterizing the labor demand in market i. Since labor is the only factor of production, this is simply given by the sum of sectoral revenues in equation (6) (after substituting for the production cost in (10)):

$$R_i = \sum_s \sum_j \frac{\tau_{ij,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\kappa-\sigma+1}}{\sum_o \tau_{oj,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\kappa-\sigma+1}} \xi_{j,s} E_j, \tag{11}$$

where $\kappa \equiv (\sigma - 1)(1 - \psi\phi)$ is a parameter determining the sensitivity of labor demand to changes in the wage rate of different markets (conditional on total spending). As such, we show below that κ is a central determinant of the magnitude of the differential responses in wages and employment to shocks in economic fundamentals. In our model, the labor demand elasticity is lower if the trade elasticity, $(\sigma - 1)$, is lower, or the combined strength of the agglomeration and labor supply elasticities, $\psi\phi$, is higher.

To solve for the equilibrium and simplify our analysis, we impose that benefits are financed by a local income tax v_i that is set such that trade is balanced in equilibrium: $v_i(W_i + B_i) = B_i$, with W_i and B_i denoting total wage and benefit payments in market i, respectively. The market level spending is thus $E_i = (1 - v_i)(W_i + B_i) = W_i$.[20] Given our labor supply structure, total income in market i is given by $W_i = w_i^{\phi}(w_i^{\phi} + b_i^{\phi})^{\frac{1-\phi}{\phi}} \bar{N}_i \varrho$ where $\varrho \equiv \Gamma(1 - 1/\phi)$ and $\Gamma(.)$ is the

[19] This channel is absent in recent quantitative spatial frameworks based on the Ricardian model of Eaton and Kortum (2002) used to quantify the impact of trade shocks on regional economies – e.g. Caliendo et al. (2019), Lyon and Waugh (2019), Galle et al. (2017), and Kim and Vogel (2021).

[20] This assumption is not important for our results. In Section 4, we show that an arbitrary structure of (endogenous and exogenous) transfers across markets only determines how E_i depends on wages in different markets, generating similar qualitative insights to those below. In addition, we show in Section 5 that our empirical findings are similar when we allow for fiscal transfers across U.S. CZs to finance changes in non-employment benefit payments caused by the China shock (as specified in Appendix A.2.6).

gamma function. This indicates that, in our model, ϕ determines the elasticity of both employment and spending in each market to changes in the local wage rate. For this reason, ϕ is also key to determine how labor market outcomes respond to shocks in economic fundamentals.

We then define the equilibrium as a wage vector that yields an excess labor demand of zero in every market. Formally, consider a wage vector $\boldsymbol{w} \equiv \{w_o\}_o$ with $w_m \equiv 1$ for an arbitrary numeraire market m. It is an equilibrium if $D_i(\boldsymbol{w}|\boldsymbol{\tau}) = 0$ for all i, such that

$$D_i\left(\boldsymbol{w}|\boldsymbol{\tau}\right) \equiv \sum_j \left(\sum_s \frac{\tau_{ij,s}^{1-\sigma} w_i^{-\kappa} \bar{b}_i^{\kappa-\sigma+1}}{\sum_o \tau_{oj,s}^{1-\sigma} w_o^{-\kappa} \bar{b}_o^{\kappa-\sigma+1}} \xi_{j,s} - \mathbb{I}_{i=j} \right) w_j^{\phi} \left(w_j^{\phi} + \bar{b}_j^{\phi} \left(\Omega(\boldsymbol{w})\right)^{\phi} \right)^{\frac{1-\phi}{\phi}} \bar{N}_j \varrho, \tag{12}$$

where $\boldsymbol{\tau} \equiv \{\tau_{id,s}\}_{ids}$ is a vector of bilateral trade costs, and $\mathbb{I}_{i=j}$ is an indicator function that equals one if, and only if, $i = j$.

Lastly, we should point out that when $\psi = 0$ and $\phi \to 1$, equation (12) becomes isomorphic to the excess demand function implied by a multi-sector gravity trade model with a fixed labor supply (see e.g. Costinot et al. (2010) and Costinot and Rodríguez-Clare (2014)), and thus all our theoretical results below apply also to gravity trade models. In this case, the labor demand elasticity equals the trade elasticity ($\kappa = \sigma - 1$) and the elasticity of market-level spending to wages equals one ($E_j = w_j \bar{N}_j$).

3.1 General Equilibrium Effects of Trade Shocks in Space

We now study how arbitrary changes in trade costs $\tau_{ij,s}$ affect outcomes in different markets. Given our definition of $\tau_{ij,s}$, our analysis may include productivity shocks, i.e. when trade costs change only for an origin market. We use 0 superscripts to denote variables in the initial equilibrium, z_j^0; hats to denote log changes in variables between the initial and new equilibria, $\hat{z}_j \equiv \ln(z_j/z_j^0)$; bold variables to denote stacked vectors of market outcomes, $\boldsymbol{z} \equiv \{z_i\}_i$; and bar bold variables to denote matrices with bilateral variables associated with origin market i and destination market j, $\bar{\boldsymbol{z}} \equiv \{z_{ij}\}_{i,j}$.

The response of the wage rate in each market to changes in trade costs follows directly from the total differentiation of the equilibrium definition in terms of excess labor demand. This yields the two key objects in our analysis. The first is the partial equilibrium shift in the excess labor demand caused by the shock (holding wages constant),

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \equiv (\bar{\boldsymbol{R}}^0)^{-1} \left(\nabla_{\ln \boldsymbol{\tau}} \boldsymbol{D}\left(\boldsymbol{w}^0|\boldsymbol{\tau}^0\right)\right) \hat{\boldsymbol{\tau}}, \tag{13}$$

where $\bar{\boldsymbol{R}}^0$ is the diagonal matrix of initial revenues. The second is the "spatial links" matrix,

$$\bar{\boldsymbol{\gamma}}^0 \equiv -(\bar{\boldsymbol{R}}^0)^{-1} \left(\nabla_{\ln w} \boldsymbol{D}\left(\boldsymbol{w}^0|\boldsymbol{\tau}^0\right)\right), \tag{14}$$

which captures the elasticity of a market's excess labor demand to wages in different markets. Written as such, our analysis is a traditional comparative statics exercise in general equilibrium.[21]

We therefore can express the wage response to trade shocks as

$$\bar{\boldsymbol{\gamma}}^0 \hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}). \tag{15}$$

In the rest of this section, we first establish that the excess demand shift in each market, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$, takes the form of a shift-share variable based on the sum of trade shocks interacted with market-specific exposure shares. We then characterize the sources of spatial links embedded in $\bar{\boldsymbol{\gamma}}^0$. We finally invert expression (15) to characterize the reduced-form elasticities that are sufficient statistics for the computation of the general equilibrium impact of the shock exposure vector, $\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$, on market-level outcomes.

3.1.1 A Shift-Share Measure for Shocks in Excess Labor Demand

The expression in (12) implies that $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ takes the form of a shift-share variable:

$$\hat{\eta}_i(\hat{\boldsymbol{\tau}}) = (1-\sigma) \sum_s \ell_{i,s}^0 \mu_{i,s}(\hat{\boldsymbol{\tau}}), \tag{16}$$

where $\ell_{i,s}^0$ is the initial share of labor in market i employed in sector s, and $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ is the shift in the demand for i's goods in sector s,

$$\mu_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_j r_{ij,s}^0 \left(\hat{\tau}_{ij,s} - \sum_o x_{oj,s}^0 \hat{\tau}_{oj,s} \right), \tag{17}$$

with $r_{ij,s}^0 \equiv X_{ij,s}^0 / \sum_d X_{id,s}^0$ denoting the initial share of market j in market i's sales in sector s. In our baseline model, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ corresponds to the market's "revenue shock exposure" since it is the sum across sectors of the shock to the demand for i's goods in each sector, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$, weighted by the sector's initial share in i's employment $\ell_{i,s}^0$. The sector-level demand shock $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ itself is the sum across destinations j of the impact of market i's own trade shock on the demand for its goods minus the demand shift caused by competitors' trade shocks in that sector, weighted by the revenue importance of each destination $r_{ij,s}^0$. Note that all components of $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ can be computed with measures of the bilateral trade shocks and information on initial bilateral trade flows.[22]

The excess labor demand shift in (16) is closely related to shift-share measures of exposure to sectoral shocks used in the literature (such as that used in Section 2). To see this, consider a

[21] For example, see sections 10.2 in Arrow and Hahn (1971) and 17.G in Mas-Colell et al. (1995).

[22] It is also worth noting that, with a single sector, $\hat{\eta}_i(\hat{\boldsymbol{\tau}})$ is the partial equilibrium (i.e. holding wages constant in all markets) change in firm market access. The concept of firm market access introduced in Anderson and Van Wincoop (2003) and Redding and Venables (2004) is widely used to measure the revenue potential of a location in the literature (e.g., Redding and Sturm (2008), Donaldson and Hornbeck (2016), Bartelme (2018)).

foreign shock with an identical impact on the sectoral demand of all destinations: formally, $\hat{\tau}_{oj,s} = 0$ for all $o \neq \mathrm{F}$ and $\hat{\zeta}_{F,s} \equiv (1-\sigma)x^0_{Fj,s}\hat{\tau}_{Fj,s}$ for all j. Then, $\hat{\zeta}_{F,s}$ is the common impact, the "shift", that the foreign country's trade shock has on the sectoral demand of every other market, and thus

$$\hat{\eta}_i(\hat{\boldsymbol{\tau}}) = -\sum_s \ell^0_{i,s}\hat{\zeta}_{\mathrm{F},s}. \tag{18}$$

If the foreign country becomes more productive in sector s ($\hat{\zeta}_{\mathrm{F},s} > 0$), then every other market suffers a negative shift in its excess labor demand, $\hat{\eta}_i < 0$ for $i \neq \mathrm{F}$. The size of this shift is proportional to the initial share of sector s in i's labor demand, as measured by the "share" $\ell^0_{i,s}$. In Section 5, we use the common component of the growth in sectoral Chinese imports across destinations to link the movement in the regional excess labor demand to the shift-share exposure to import competition used in ADH and in Section 2.

3.1.2 Spatial Links in General Equilibrium

We proceed with the characterization of the spatial links in the economy, i.e. $\bar{\boldsymbol{\gamma}}^0$ in (14). This matrix summarizes the spatial percolation of shocks in our model as it regulates how much wage changes in one market affect excess labor demand in other markets. By defining $\phi^0_i \equiv \phi - (\phi-1)n^0_i$, we establish in Appendix A.1 that

$$\gamma^0_{ij} = \left(\phi^0_i + \kappa\right)\mathbb{I}_{[i=j]} - \rho^0_{ij} \quad \text{where} \quad \rho^0_{ij} \equiv r^0_{ij}\phi^0_j + \kappa\sum_s\sum_d \ell^0_{i,s}r^0_{id,s}x^0_{jd,s} + \omega^0_j\sum_d r^0_{id}(\phi^0_i - \phi^0_d). \tag{19}$$

The first component of this expression is the own-elasticity of i's excess labor demand to its wage, which corresponds to the sum of the labor demand and labor supply elasticities, regulated by κ and ϕ^0_i, respectively. Following the usual logic in supply-demand frameworks, a lower value of $\phi^0_i + \kappa$ implies stronger responses in all outcomes conditional on the same shock.

The second component is the cross-wage elasticity of excess labor demand, ρ^0_{ij}, which has three terms. The term $r^0_{ij}\phi^0_j$ captures the positive impact that an increase on j's wage has on its total expenditure (proportional to ϕ^0_j) and, consequently, on the sales of i (proportional to the share of j in i's revenue, r^0_{ij}). The next term captures endogenous changes in excess labor demand arising from demand substitution across suppliers due to changes in competitor j's labor cost. It is proportional to the sensitivity of demand to wages κ and, importantly, to the covariance between i's sales $\ell^0_{i,s}r^0_{id,s}$ and j's market share $x^0_{jd,s}$ across sectors and destinations. The last term is the impact on excess labor demand of changes in labor supply due to the non-employment benefit's numeraire and arises because of the heterogeneity in the labor supply elasticity across markets – in fact, it is zero if $n^0_i = n^0$ and, thus, $\phi^0_i = \phi^0$ for all i.

3.1.3 General Equilibrium Effects in Space and their Determinants

We now characterize the "reduced-form" elasticity of wages to trade shocks in general equilibrium. This is a "sufficient statistics" characterization: it yields responses in terms of market-level measures of shock exposure (determined by $\hat{\eta}_i$ in (16)) and market-to-market reduced-form elasticities to these measures (determined by γ_{ij} in (19)). Both components are functions of variables observed in the initial equilibrium, as well as parameters controlling the elasticities in the model. Appendix A.1 contains the proofs of the results in this section.

Throughout our analysis, we impose sufficient conditions for equilibrium uniqueness given any set of exogenous trade shifters $\boldsymbol{\tau}$. This guarantees that our counterfactual analysis yields unambiguous predictions for the impact of shocks in economic fundamentals. Following Arrow and Hahn (1971) T.9.12 (p. 234), we assume that the excess demand system satisfies diagonal dominance: there exists $\{h_i\}_{i\neq m} \gg 0$ such that, for all $i \neq m$,[23]

$$h_i \gamma_{ii}^0 > \sum_{j\neq m,i} h_j |\gamma_{ij}^0|. \tag{20}$$

Theorem 1. *(Sufficient Statistics for Reduced-Form Responses) Consider any shock to bilateral shifters $\hat{\boldsymbol{\tau}}$. If condition (20) holds, then (up to a first-order approximation)*

$$\hat{w}_i = \underbrace{\beta_{ii}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_i(\hat{\boldsymbol{\tau}})}_{\text{Direct effect}} + \underbrace{\sum_{j\neq i}\beta_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j(\hat{\boldsymbol{\tau}})}_{\text{Indirect effect}}, \quad \text{with} \quad \beta_{ij} = \frac{1}{\phi_j^0+\kappa}\left(\mathbb{I}_{[i=j]} + \tilde{\gamma}_{ij} + \sum_{d=2}^{\infty}\tilde{\gamma}_{ij}^{(d)}\right), \tag{21}$$

$\tilde{\gamma}_{ij}^{(d)}$ is the i-j entry of $(\bar{\tilde{\boldsymbol{\gamma}}})^d$ such that $\tilde{\gamma}_{ij} \equiv (\phi_i^0+\kappa)^{-1}\rho_{ij}^0\mathbb{I}_{[i,j\neq m]}$, $\boldsymbol{\theta} \equiv (\phi,\kappa)$ is a parameter vector, and $\boldsymbol{\mathcal{W}}^0 \equiv \{n_i^0, \omega_i^0, \{X_{ij,s}^0\}_{j,s}\}_i$ is a matrix of initial conditions.

Theorem 1 yields a set of sufficient statistics for counterfactual analysis in general equilibrium: the vector of excess labor demand shifts (i.e, $\hat{\eta}_i$ in (16)), as well as the reduced-form elasticities to such measures (i.e., β_{ij} in (21)). The formula for wage changes in (21) *aggregates* the direct effect of the market's own shock exposure and the indirect effect of the shock exposure of all other markets, weighted by the reduced-form elasticities β_{ii} and β_{ij}, respectively. The aggregation formula thus maps measures of shock exposure in partial equilibrium for all markets (i.e., the shifts in excess labor demand) into general equilibrium responses of wages in each market. As a special case, it provides a closed-form characterization (up to a first-order approximation) for the solution of the non-linear system of equations for counterfactuals in gravity trade models (see e.g. Proposition 2 in Arkolakis et al. (2012)).

[23] This assumption is weaker than the gross substitution property (i.e., $\gamma_{ii} > 0$ and $\gamma_{ij} < 0$ for all $i \neq j$) that yields uniqueness of one-sector gravity trade models with exogenous labor supply (Alvarez and Lucas, 2007).

The reduced-form elasticity β_{ij} is a series expansion of the spatial links matrix $\bar{\boldsymbol{\gamma}}^0$. Thus, spatial spillovers are stronger between markets with tighter ties in terms of bilateral sales or competition, as captured by ρ^0_{ij}, and in terms of third-market connections in the network, as captured by the power series term. Intuitively, any wage change necessary to restore market clearing in market j following an exogenous shock in its labor demand endogenously shifts the labor demand in all other markets i through changes in both j's demand for i products and j's market share in other markets served by i. These endogenous shifts in the labor demand of other markets must also be corrected in general equilibrium, triggering the multiple rounds of adjustment summarized in the higher-order terms of the power series. This generates a pattern of spatial percolation of regional shocks that is similar to that of the percolation of shocks across production networks (Acemoglu et al. (2016b) and Carvalho and Tahbaz-Salehi (2019)), since spatial models inherit the mathematical architecture of network models (see e.g Allen et al. (2020a)).

Importantly, the representation in (21) links our model to the evidence in Section 2: for foreign shocks in which $\hat{\eta}_i$ takes the shift-share form in (18), the direct effect, $\beta_{ii}\hat{\eta}_i$, is related to the direct impact of the market's employment exposure to import competition IC^t_i, while the indirect effect, $\sum_{j\neq i}\beta_{ij}\hat{\eta}_j$, is related to the impact of the gravity-based measure of exposure to shocks in other markets, GC^t_i. We now exploit this connection to provide a rationale through the lens of the model for our empirical findings regarding the sign and size of the direct and indirect effects of regional exposure to import competition, and the importance of expenditure shock exposure.

We first show that cross-market trade links generate the type of reinforcing indirect effects documented in Section 2 – that is, direct and indirect reduced-form elasticities have the same sign.

Corollary 1. *If $\kappa > 0$ and $\max_{i,j}|n^0_i - n^0_j|$ is low enough, then $\tilde{\gamma}_{ij} \geq 0$ and $\beta_{ij} \geq 0$ $\forall i, j$.*

In this result, a foreign productivity gain ($\hat{\zeta}_{\mathrm{F},s} > 0$) leads to a negative shift in i's excess demand ($\hat{\eta}_i < 0$), creating a negative direct effect on the labor demand in that market, but also in all other markets due to $\beta_{ij} \geq 0$. Intuitively, the negative shift pushes down i's wage (relative to the foreign country) and, consequently, also the trade demand in all other markets j through losses in both their sales to i (captured by $r^0_{ji}\phi^0_i$) and their market share in all destinations (captured by $\sum_s\sum_d \ell^0_{j,s} r^0_{jd,s} x^0_{id,s}$). The upper bound on the dispersion of n^0_i guarantees that these demand channels are not overturned by labor supply responses created by the impact of wages on the payoff of not working.

Second, we investigate the determinants of the size of the reduced-form elasticities to understand the drivers of the large differential effects estimated in Section 2. To do so, it is useful to focus on the special case in which the indirect effects are identical, which arises when labor supply elasticities and trade links are the same in all markets.

Corollary 2. *Assume that markets have the same labor supply elasticity ($\phi^0_j = \phi^0$) and trade links*

($\xi_{j,s} = \xi_s$, $x^0_{ij,s} = x^0_{i,s}$, and $\frac{\sum_s \xi_s x^0_{i,s} x^0_{j,s}}{\sum_s \xi_s x^0_{i,s}} = \chi_j$). Thus,

$$\hat{w}_i \quad = \quad \tfrac{1}{\kappa+\phi^0}\hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \bar{\eta} \quad \textit{such that} \quad \bar{\eta} \equiv \textstyle\sum_j \tfrac{\beta_j}{\kappa+\phi^0}\hat{\eta}_j(\hat{\boldsymbol{\tau}}) \ . \tag{22}$$

The direct reduced-form elasticity $(\kappa + \phi^0)^{-1}$ is positive, increasing in $\psi\phi$, and decreasing in σ, and the indirect reduced-form elasticity β_j is positive and increasing in j's size.

The differential direct impact of a market's own shock exposure on its wage, $(\kappa + \phi^0)^{-1}$, is decreasing on the labor demand elasticity, κ. This underscores the importance of the labor demand elasticity for the magnitude of the predicted responses to higher shock exposure. The corollary also indicates that market j's (symmetric) impact on other markets is proportional to its size.

In addition, the symmetry in spatial links gives rise to an "endogenous" fixed-effect, $\bar{\eta}$, comprising all the indirect effects of the shock in general equilibrium. Hence, Corollary 2 establishes sufficient conditions for wage changes in a market to be a linear combination of its shift-share shock exposure plus a common fixed-effect. This special case thus yields a tight connection between our characterization and empirical shift-share specifications that followed Bartik (1991). The frameworks proposed in Nakamura and Steinsson (2014), Chodorow-Reich (2019) and Beraja et al. (2019), given the absence of trade costs, are akin the case of identical spatial linkages across markets considered in Corollary 2.

Lastly, we characterize the importance of expenditure shock exposure. While it does not matter for responses in wages and employment, it does affect changes in the consumption price index.

Corollary 3. *Consider any shock to bilateral shifters $\hat{\boldsymbol{\tau}}$. If condition (20) holds, then (up to a first-order approximation)*

$$\hat{P}_i = \sum_j \beta^C_{ij}\hat{\eta}_j(\hat{\boldsymbol{\tau}}) + \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) \quad \textit{where} \tag{23}$$

$$\hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) = \sum_{s,o} \xi_{i,s} x^0_{oi,s} \hat{\tau}_{oi,s}, \quad \textit{and} \quad \beta^C_{ij} \equiv \sum_o \left(x^0_{oi}\frac{\kappa}{\sigma-1} + \left(1 - \frac{\kappa}{\sigma-1}\right)\omega^0_o \right) \beta_{oj}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0). \tag{24}$$

The price index change combines two effects. The first term, $\sum_j \beta^C_{ij}\hat{\eta}_j(\hat{\boldsymbol{\tau}})$, measures the impact of the shock on the market's consumption cost through the endogenous changes in production costs arising from the wage responses in Theorem 1. The second term, $\hat{\eta}^C_i(\hat{\boldsymbol{\tau}})$, instead measures the shock's impact on the exogenous component of consumption costs. It is the average change in bilateral trade shifters of a destination market, weighted by its final spending share across sectors and origins. To gain intuition for this term, consider again the foreign sectoral shock introduced in Section 3.1.1 for which $\hat{\eta}^C_i(\hat{\boldsymbol{\tau}})$ is a shift-share variable based on sectoral spending shares, $\hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) \propto -\sum_s \xi_{i,s}\hat{\zeta}_{\mathrm{F},s}$. In this case, the price index falls more in markets with a higher initial spending share on sectors in which the foreign country experienced stronger productivity growth. In the absence of intermediate goods, final and gross spending shares are equal, implying that $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ is proportional to the import expenditure exposure IE_i used in Section 2.

We conclude with two comments. First, in our baseline model, consumption cost exposure does not affect wages and employment across markets. While this is consistent with the evidence in Section 2, Section 4 shows that the sensitivity of labor supply to the consumption price index controls how much $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ affects labor market outcomes. Second, changes in the real wage, w_i/P_i, combine the direct impact of the shock on consumption costs, measured by $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$, with the terms-of-trade effects implied by the shock, measured by $\sum_j(\beta_{ij}-\beta_{ij}^C)\hat{\eta}_j(\hat{\boldsymbol{\tau}})$.[24]

3.2 From Theory to an Empirical Specification

We use Theorem 1 to derive a model-consistent empirical specification to measure the general equilibrium impact of observed trade shocks on the labor market outcomes of each market. It allows us to move beyond the qualitative investigation of the adjustment channels driving the differential regional responses to trade shocks that motivated the specification of our spatial model.

Consider two observed equilibria that differ because of the realization of random shocks, $\hat{\tau}_{ij,s}$, and assume that we observe a component of these shocks, $\hat{\tau}_{ij,s}^{\text{obs}}$. Without loss of generality, we can define the unobserved component of shocks as $\hat{\boldsymbol{\tau}}^{\text{unbs}} = \hat{\boldsymbol{\tau}} - \hat{\boldsymbol{\tau}}^{\text{obs}}$, implying that

$$\hat{\eta}_i(\hat{\boldsymbol{\tau}}) = \sum_s \ell_{i,s}^0 \hat{z}_{i,s}^{\text{obs}} + \hat{\eta}_i(\hat{\boldsymbol{\tau}}^{\text{unbs}}), \tag{25}$$

where $\hat{z}_{i,s}^{\text{obs}} \equiv (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}^{\text{obs}})$ is the impact of $\hat{\boldsymbol{\tau}}^{\text{obs}}$ on market i's sector s demand (defined in (17)).

We show in Appendix A.1.6 that by combining the decomposition in (25), the wage response in (21), and the supply relationships in (8)–(9), we obtain a structural relationship between changes in observed labor market outcomes, i.e. average log-wages and log employment rates, and measures of market-level exposure to observed and unobserved shocks:

$$\begin{bmatrix} \Delta \overline{\ln w_i} \\ \Delta \ln n_i \end{bmatrix} = \begin{bmatrix} \alpha^w \\ \alpha^n \end{bmatrix} + \sum_j \begin{bmatrix} \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \\ \beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \end{bmatrix} \left(\sum_s \ell_{j,s}^0 \hat{z}_{j,s}^{\text{obs}} \right) + \begin{bmatrix} \nu_i^w \\ \nu_i^n \end{bmatrix}, \tag{26}$$

where $\beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \equiv (n_i^0\beta_{ij} + (1-n_i^0)\sum_d \omega_d^0 \beta_{dj})$ and $\beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) \equiv \phi(1-n_i^0)(\beta_{ij} - \sum_d \omega_d^0\beta_{dj})$, with $\beta_{ij} = \beta_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ given by (21). In this expression, α^w and ν_i^w are, respectively, the average and idiosyncratic changes in wages generated by the unobserved component of trade shocks $\hat{\boldsymbol{\tau}}^{\text{unbs}}$.[25] α^n and ν_i^n are similarly defined for changes in the employment rate.

[24] Note that in our framework the welfare of an individual corresponds to her real wage (if working), or to her real benefit from non-employment (if not working). Even in a setting with a representative agent with endogenous labor supply, it is easy to show that the equivalent variation associated with a trade shock is proportional to the change in the real wage (see Appendix B.3.1 of the old version of our paper, Adao et al. (2020a)).

[25] Formally, $\alpha^w \equiv I^{-1}\sum_i\sum_j \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)E[\hat{\eta}_j(\hat{\boldsymbol{\tau}}^{\text{unbs}})]$ and $\nu_i^w \equiv \sum_j \beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j(\hat{\boldsymbol{\tau}}^{\text{unbs}}) - \alpha^w$.

Through the lens of our model, both the residuals (ν_i^w, ν_i^n) and the constants (α^w, α^n) are not functions of the observed shocks in $\hat{z}_{i,s}^{\text{obs}}$. Because of this property, knowledge of the reduced-form elasticities $\beta_{ij}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ and $\beta_{ij}^n(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ is sufficient to compute *both* the differential *and* the aggregate impact in general equilibrium of observed shock exposure across markets, $\sum_s \ell_{j,s}^0 \hat{z}_{j,s}^{\text{obs}}$, on employment and wages. This has two important implications. First, in contrast to common empirical specifications in the literature (for example, ADH's specification in Section 2), our model provides a set of conditions that allows estimates of the reduced-form elasticities based on equation (26) to be aggregated for computation of the general equilibrium impact of the observed shock. Second, one can also use equation (26) to test whether the model's predicted responses to observed shocks are consistent with observed responses in the data. The credibility of the model's predictions is severely curtailed if its reduced-form elasticities do not generate differential responses consistent with those observed in the data, since these elasticities are sufficient determinants of both differential and aggregate predicted effects with respect to shocks in economic fundamentals.

To take equation (26) to the data, we need to impose further restrictions on the data generating process of shocks to economic fundamentals. Assume that, given initial conditions, observed and unobserved components of these shocks are uncorrelated: for all markets and sectors,

$$Cov(\hat{\tau}_{ij,s}^{\text{obs}}, \hat{\tau}_{od,k}^{\text{unbs}}|\boldsymbol{\mathcal{W}}^0) = 0. \tag{27}$$

This is a standard type of orthogonality assumption for shocks to bilateral trade costs. It guarantees the causal interpretation of estimates in the literature of the impact of different observed measures of trade costs on trade flows, or the impact of changes in import tariffs or foreign productivity on firms, industries and regions (see e.g. Autor et al. (2013), Kovak (2013) and Pierce and Schott (2016a)). Notice that the orthogonality condition is stated in terms of shocks to fundamentals (instead of measures of shock exposure, e.g. $\hat{\eta}_j$ in our model). It is therefore a version of the quasi-random assignment of shocks that was recently used in the context of shift-share instrumental variables by Borusyak et al. (2018) and Adão et al. (2019). Since this assumption is not testable, how reasonable it is must be evaluated in each particular application. We return to this point below in the context of the China shock.[26]

As shown in Appendix A.1.7, the orthogonality assumption in (27) implies that the unobserved residuals in (26) are orthogonal to measures of market-level exposure to the observed shocks:

$$E\left[\nu_i^w \sum_j h_{ij}^w Z_j\right] = E\left[\nu_i^n \sum_j h_{ij}^n Z_j\right] = 0 \quad \text{for any real matrices} \quad \{h_{ij}^w, h_{ij}^n\}_j, \tag{28}$$

where $Z_j \equiv \sum_s \ell_{j,s}^0 (\hat{z}_{j,s}^{\text{obs}} - \bar{z}_{j,s}^{\text{obs}})$ is market j's exposure to the de-meaned shock with $\bar{z}_{j,s}^{\text{obs}} \equiv$

[26]It is easy to allow for shocks in $\bar{b}_i$ (akin to labor supply or amenities shocks) to affect outcomes through the definitions of ν_i^w and ν_i^n. In this case, on top of condition (27), we must assume that $Cov(\hat{\tau}_{ij,s}^{\text{obs}}, \hat{\bar{b}}_o|\boldsymbol{\mathcal{W}}^0) = 0$.

$(1-\sigma)\mu_{j,s}(E[\hat{\tau}^{\text{obs}}_{od,k}|\boldsymbol{\mathcal{W}}^0])$ computed from the mean observed shock. The use of de-meaned shifters avoids identification threats arising, even under (27), from markets being more exposed to all types of random shocks (observed or unobserved) – see Borusyak and Hull (2020) for a general treatment of non-random exposure to random shocks.

We now discuss a number of advantages of using (26) and (28) for empirical analyses of the aggregate and differential effects of observed trade shocks. First, our specification links in a transparent way the shock's impact in general equilibrium to exposure measures and the magnitude and sign of indirect reduced-form effects (as determined by the economy spatial links). Equations (26) and (28) then connect such an impact to moments in the data associated with the elasticity of market-level outcomes to the observed shock exposure of different markets. The empirical content of (26) and (28) is a significant departure from the common approach of computing the shock's general equilibrium impact using calibrated spatial models – either in quantitative frameworks with rich calibrated spatial links (as in Redding and Rossi-Hansberg (2017)), or in frameworks combining an empirical strategy of the form in (22) and a calibrated spatial model to quantify the common "missing intercept" (as in Kovak (2013); Nakamura and Steinsson (2014); Mian and Sufi (2014); Beraja et al. (2019); Chodorow-Reich (2019)). As Chodorow-Reich (2020) points out, this common approach has the cost of generating an aggregate impact that "depends heavily, and sometimes non-transparently, on the ingredients in the model as well as the particular parameterization."

Second, (26) and (28) can be used to estimate the parameter vector $\boldsymbol{\theta}$ and, therefore, $\beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$ and $\beta^n_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)$. Intuitively, identification comes from how market-level outcomes directly and indirectly respond to the shock exposure of markets with stronger (bilateral and higher-order) cross-market links in γ_{ij} (as defined in (19)). Formally, it follows from applying the usual rank condition for non-linear moment conditions in Newey and McFadden (1994) and Chen et al. (2014) to the specification in (26) that is non-linear in $\boldsymbol{\theta}$.[27] In addition, under the assumption that the general equilibrium model is well specified, the use of (26)–(28) for estimating $\boldsymbol{\theta}$ also has the advantage of generating a more efficient estimator than those implied by structural estimation approaches based on moments associated with equilibrium relationships between the endogenous outcomes in the left hand side of (26) (as in e.g. Faber and Gaubert (2019); Galle et al. (2017); Allen and Donaldson (2017); Fajgelbaum et al. (2018)). This is because the proper selection of $\{h^w_{ij}, h^n_{ij}\}_j$ yields an estimator that relies on all sources of variation associated with $\boldsymbol{\theta}$ in general

[27] Leveraging the facts that (26) is additive in the residual and that $\boldsymbol{\theta}$ only enters (26) through functions that are multiplicative on the random variables $\hat{z}^{\text{obs}}_{j,s}$, identification of $\boldsymbol{\theta}$ follows from the rank of $\sum_{i,j,d}\left(h^w_{ij}\nabla_{\boldsymbol{\theta}}\beta^w_{id}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}), h^n_{ij}\nabla_{\boldsymbol{\theta}}\beta^n_{id}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\right)E[Z_jZ_d|\boldsymbol{\mathcal{W}}^0]$ being equal to $\dim(\boldsymbol{\theta})$. Notice that, since $E[Z_jZ_d|\boldsymbol{\mathcal{W}}^0]\neq 0$ for some j and d is a weak condition, identification essentially relies on all entries of $\boldsymbol{\theta}$ being associated with heterogeneous (direct and indirect) reduced-form elasticities across markets. In other words, we cannot identify parameters that are *only* associated with a common component of the reduced-form elasticities on all markets i. This condition is weaker than the Stable Unit Treatment Value Assumption (SUTVA) that yields identification of the direct reduced-form elasticity to local shock exposure in structural models with a common "missing intercept" – see result 2 of Chodorow-Reich (2020). SUTVA rules out that shock exposure of a region differentially affects outcomes in other regions.

equilibrium. Formally, in Appendix A.1.8, we apply the results in Chamberlain (1987) to derive the optimal moment conditions in the context of our general equilibrium model: that is, we characterize the weights, $\{h^w_{ij}, h^n_{ij}\}_j$, that minimize the variance of the GMM estimator of $\boldsymbol{\theta}$ based on (28) for the model's reduced-form specification in (26).[28] We implement this estimation strategy to study the general equilibrium impact of the China shock on U.S. CZs in Section 5.3.

Third, the combination of the reduced-form predictions in (26) and the moment condition in (28) yields testable predictions for fully specified spatial models (i.e., $\boldsymbol{\theta}$ is already known). In this case, (26) and (28) imply additional moments that make the model over-identified and thus can be used for testing. This is important because traditional structural approaches that estimate $\boldsymbol{\theta}$ from equilibrium relationships between endogenous variables in the model do not necessarily generate predicted responses to observed shocks that are consistent with estimates of such responses across markets – for example, see the discussion in Section 2.3. If $\boldsymbol{\theta}$ is known, the predicted response in any labor market outcome Y_i to the observed shock can be written as $\hat{Y}_i(\boldsymbol{Z}|\boldsymbol{\theta}, \boldsymbol{W}^0) \equiv \sum_j \beta^Y_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0) Z_j$. Thus,

$$\hat{Y}_i = \alpha^Y + \rho^Y \hat{Y}_i(\boldsymbol{Z}|\boldsymbol{\theta}, \boldsymbol{W}^0) + \nu^Y_i \quad \text{with} \quad E[\nu^Y_i \hat{Y}_i(\boldsymbol{Z}|\boldsymbol{\theta}, \boldsymbol{W}^0)] = 0. \tag{29}$$

Under the null hypothesis that the model is well specified, the pass-though coefficient from predicted to actual changes in any outcome Y_i is one (i.e., $\rho^Y = 1$). The test retains its validity even if other shocks may drive much of the cross-market variation in the outcome of interest, because the orthogonality condition in (27) guarantees the identification of the impact of the observed shock while holding other unobserved shocks constant. In this sense, our procedure is a clear improvement to statistical decomposition methods (such as the one proposed by Kehoe et al. (2017)) whose conclusions are dependent on the importance of other unobserved shocks (see, for example, the discussion in Antràs and Chor (2021)).

Importantly, this additional moment has the advantage of relying exactly on the reduced-form elasticities that are sufficient for the computation of the model's counterfactual predictions in general equilibrium. Therefore, if one rejects the model's predicted responses using equation (29), the credibility of the model's counterfactual predictions is undermined. Intuitively, as long as the orthogonality condition in (27) holds, an estimated coefficient much larger than one suggests that the predicted responses in the model need to be re-scaled by a large coefficient to match the differential impact of the observed shock across markets and, therefore, are too small. The opposite is true if the estimated fit coefficient is small and non-significant. Since we show below that a version of (26)–(28) holds in a general class of spatial models, this discussion applies to a growing literature on quantitative spatial economics whose ultimate goal is measuring the general

[28] We also apply the results in Borusyak and Hull (2020) to derive the optimal moment conditions under the assumption of independence of observed shocks $\hat{\tau}^{\text{obs}}_{od,s}$ across markets and sectors (for arbitrary cross-market correlation in (ν^w_i, ν^n_i)). This extension yields similar weights for the exposure of different markets j for the observed responses of any given market i, but introduces an extra term to account for the correlation in the residuals across markets.

equilibrium impact of shocks in economic fundamentals across different markets. Note that our test can be applied to traditional structural estimation procedures because it uses the general equilibrium relationship in the model between each endogenous outcome and the exogenous shocks in (29), which is different than the equilibrium relationships between endogenous variables (instrumented with exogenous shocks) typically targeted in structural estimation. In Section 5.4, we use (29) to both (i) evaluate which specifications of spatial links in the literature are rejected by the differential responses of employment and wages to the China shock, and (ii) test the fit of our model for changes in outcomes not used in the estimation of $\boldsymbol{\theta}$ (e.g., sectoral employment composition).

Fourth, it is worth mentioning that the estimation methodology based on (26)–(28) remains valid under a flexible structure of spatial links and arbitrary unobserved shocks. Such a flexibility is in contrast with the "market access" approach in Donaldson and Hornbeck (2016). In such setting, market access is an endogenous variable obtained from solving the general equilibrium model under restrictive assumptions on the economy's spatial links – specifically, a single sector with symmetric trade costs that are fully observed before and after the shock.[29] Even under these assumptions, one cannot simply aggregate the empirical specification to compute the general equilibrium impact of changes in market access as it also involves an endogenous common component that is not separately identified from the constant.

So far, we have discussed the advantages of using equation (26) for empirical analysis. The use of this expression is, however, subject to three important caveats. The first is that the separability of the unobserved residuals (ν_i^w, ν_i^n) is a consequence of the log-linearization around the initial equilibrium. So, equation (26) may be a poor approximation for the model's predictions depending on the application. Given this concern, one can use the integral of our formulas described in Appendix A.3.3 to evaluate the quality of the approximation in each particular application – for example, see the robustness exercises in Section 5.5. The second is that we specify the reduced-form elasticities as parametric functions of the data in $\boldsymbol{\mathcal{W}}^0$ and the parameters in $\boldsymbol{\theta}$. We follow this approach because a type of dimensionality curse prevents the non-parametric estimation of the reduced-form elasticities in (26), as we only observe outcomes for I markets, but (37) has I^2 reduced-form elasticities.[30] The last one is that, as in any structural framework, the derivation of (26) requires the spatial model to be well specified. In case it is not, additional channels will be included in the residuals and the constant, which would lead to the violation of the exclusion restriction and the mis-measurement of the aggregate effects. For that purpose, we selected the

[29] Donaldson and Hornbeck (2016) point out that "the calculation of market access (via equation (9)) requires the measurement of all trade costs." This is true even if one extends their environment to obtain expressions in terms of changes in market access. In this case, knowledge of initial trade flows subsumes knowledge of initial trade costs, but it is still necessary to observe all components of bilateral trade shocks (in our notation, $\hat{\boldsymbol{\tau}}^{\text{unbs}} = 0$). In gravity trade models, identifying $\hat{\boldsymbol{\tau}}$ typically requires assuming symmetric shocks, as in Head and Ries (2001).

[30] This procedure effectively projects the reduced-form elasticities onto observable variables regulating the strength of spatial links. It is similar to the common practice in demand estimation of specifying cross-price demand elasticities in terms of observable variables (Berry, 1994; Berry et al., 1995).

channels in our baseline model motivated by the evidence in Section 2. To explore additional channels previously highlighted in the literature, we extend our methodology to a broader set of models in the next section.

4 Additional Margins of General Equilibrium Effects in Space

We now extend the empirical specification in Section 3.2 for an economy with trade in intermediate goods as well as a labor supply that depends on migration choices and consumption prices, three features widely present in quantitative trade and spatial models. Our main result is that the measures of market-level shock exposure must incorporate i) the upstream and downstream exposure of labor demand to shocks in the revenue and expenditure associated with intermediate goods, and ii) the exposure of labor supply to shocks in import prices. We also show how these mechanisms modify the initial conditions and parameters that are sufficient for computing the reduced-form elasticities to the different measures of shock exposure.

Labor Supply with Endogenous Population. Each country c has a continuum $\bar{N}_c$ of workers. Individuals have heterogeneous preferences for the amenities of different markets and draw market-specific amenities $\{a_i(\iota)\}_{i\in\mathcal{I}_c}$ independently from a Frechet distribution with shape parameter ϑ and scale $\bar{\nu}_j$. As before, we assume that, conditional on residing in a market i, individuals independently draw a realization of their income potentials $(l(\iota), u(\iota))$ from the same Frechet distribution used in Section 3. Thus, the employment rate is given by n_i in (8), and the log average income by $\overline{\ln w_i}$ in (9). Worker ι chooses in which market $i \in \mathcal{I}_c$ to reside based on her expected payoffs, $U_i(\iota) = a_i(\iota)\varrho w_i^{\phi}(w_i^{\phi} + b_i^{\phi})^{\frac{1-\phi}{\phi}}/P_i$. This implies a location choice similar to that of recent spatial frameworks (Allen and Arkolakis, 2014; Redding, 2016; Allen et al., 2020b):

$$N_i = \frac{\bar{\nu}_i P_i^{-\vartheta} w_i^{\phi\vartheta}(w_i^{\phi} + b_i^{\phi})^{\vartheta\frac{1-\phi}{\phi}}}{\sum_{j\in\mathcal{I}_{c(i)}} \bar{\nu}_j P_j^{-\vartheta} w_j^{\phi\vartheta}(w_j^{\phi} + b_j^{\phi})^{\vartheta\frac{1-\phi}{\phi}}}\bar{N}_c. \tag{30}$$

Equation (30) indicates that population in market i (and consequently labor supply) is higher whenever the real income in that market is higher relative to the average real income in other markets of the country. The parameter ϑ controls the sensitivity of a market's population to changes in its relative average real income and, as we formally show below, the type of responses in population to regional shock exposure studied in Section 2.

We further generalize the model by introducing a parameter that controls the sensitivity of the payoff of not working to local prices: $b_i = \bar{b}_i P_i^{\lambda}(\Omega(\{w_j\}_j))^{1-\lambda}$. When λ is higher, the same decline in the price of imported goods will have a stronger positive impact on the relative payoff of working and, consequently, on labor supply. Thus, λ determines the magnitude of the responses of wages and employment to shocks in the supply of imported goods (such as those that we investigated in Section 2). Note that, in the limit case of $\lambda = 1$, labor supply becomes a function of the market's real wage.

Gravity Trade in Final and Intermediate Goods. We follow the gravity trade framework with intermediate inputs of Caliendo and Parro (2015) and Costinot and Rodríguez-Clare (2014). We maintain sectoral gravity trade links across markets: sector s of origin i has a representative competitive firm that produces a differentiated tradable good at a cost of $p_{i,s}$ and faces iceberg trade costs of $\tau_{ij,s}$ to sell in j. In each sector and destination, the differentiated products of all origins are combined to produce a composite non-tradable good, using a constant elasticity aggregator with elasticity σ. These sectoral composite goods are inputs for the production in each market of the final consumption good and the tradable differentiated goods of each sector.

The production function of the final consumption good is a Cobb-Douglas aggregator of the sectoral non-tradable composite goods with shares $\xi_{i,s}$, so that the final good price is still given by (7). In addition, we assume that the production function of the differentiated good of sector s is Cobb-Douglas between labor and an intermediate input aggregator, with spending shares of $a^L_{i,s}$ and $a^M_{i,s}$, respectively. The intermediate input aggregator in sector s, $M_{i,s}$, is also a Cobb-Douglas function of the sectoral non-tradable composite goods, with $\xi^M_{i,ks}$ denoting the share of intermediate spending on sector k ($\xi^M_{i,ks} > 0$ and $\sum_k \xi^M_{i,ks} = 1$). We maintain the assumption of external economies of scale associated with the market's employment rate (as regulated by an elasticity ψ).[31] From cost minimization, the production cost in sector s of market i is

$$p_{i,s} = (w_i)^{1-\psi\phi-a^M_{i,s}} (P^M_{i,s})^{a^M_{i,s}} (b_i)^{\psi\phi}, \quad \text{with} \quad P^M_{i,s} = \Pi_k (P_{i,k})^{\xi^M_{i,ks}}. \tag{31}$$

Notice that, relative to the model of Section 3, the pass-through of wages to production costs is now a function of the share of intermediate goods in production. Given the same value of $\psi\phi$, a higher $a^M_{i,s}$ will lower the sensitivity of labor demand to the local wage, since input prices also depend on the labor cost in other markets through input purchases. As we formally show below, this mechanism generates wage responses to a given shift in excess labor demand that are larger when the share of intermediate goods in production is higher.

[31] The general specification of the model in Appendix A.3 also entails an elasticity of productivity to population. We set this elasticity to zero in this section because we cannot estimate it given the lack of population responses to the China shock documented in Section 2.

Equilibrium. In equilibrium, good market clearing requires the gross revenue $\{R_{i,s}\}_{i,s}$ to solve

$$R_{i,s} = \sum_j \frac{\left(\tau_{ij,s} p_{i,s}\right)^{1-\sigma}}{\sum_o \left(\tau_{oj,s} p_{o,s}\right)^{1-\sigma}} \left(\xi_{j,s} E_j + \sum_k \xi^M_{j,sk} a^M_{j,k} R_{j,k}\right) \quad \text{for all} \quad (i,s), \tag{32}$$

where $E_i = W_i = \varrho w_i^{\phi}(w_i^{\phi} + b_i^{\phi})^{\frac{1-\phi}{\phi}} N_i$. In Appendix A.2.1, we define the equilibrium wage vector in terms of an excess labor demand system: $D_i(\boldsymbol{w}|\boldsymbol{\tau}) = 0$ for all i, such that

$$D_i(\boldsymbol{w}|\boldsymbol{\tau}) \equiv \sum_s a^L_{i,s} R_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}) - W_i(\boldsymbol{w}|\boldsymbol{\tau}). \tag{33}$$

4.1 An Extended Reduced Form Representation

We now extend the empirical specification in Section 3.2, with all proofs presented in Appendix A.2. We start by characterizing the implications of trade in intermediate goods for the measures of shock exposure. Consider first the shift in market-level sales caused by the shock (holding constant all endogenous variables), the "revenue shock exposure" defined as $\eta^R_i(\hat{\boldsymbol{\tau}}) \equiv \sum_{s,o,d} \frac{\partial \ln R_i}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s}$:

$$\eta^R_i(\hat{\boldsymbol{\tau}}) = (1-\sigma) \sum_s \ell^0_{i,s} \left(\mu_{i,s}(\hat{\boldsymbol{\tau}}) + \mu^U_{i,s}(\hat{\boldsymbol{\tau}})\right). \tag{34}$$

Here, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$ is the same shock to the demand for goods of sector s of market i defined in (17). The new component, $\mu^U_{i,s}(\hat{\boldsymbol{\tau}})$, measures the upstream shock exposure of sector s of market i,

$$\mu^U_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,k} b^U_{is,jk} \mu_{j,k}(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \bar{\boldsymbol{b}}^U \equiv \sum_{d=1}^{\infty} \left(\bar{\boldsymbol{r}}^U\right)^d, \tag{35}$$

and $\bar{\boldsymbol{r}}^U \equiv [r^U_{is,jk}]_{is,jk}$ with $r^U_{is,jk} \equiv X^M_{ij,sk}/R_{i,s}$ denoting the share of revenue in sector s of market i, $R_{i,s}$, coming from its intermediate sales to sector k of market j, $X^M_{ij,sk}$. Since the demand shift for the products of a sector-market affects its input purchases, it also generates revenue shifts for upstream sectors and markets that we capture in the series expansion of the upstream matrix of revenue shares, $\bar{\boldsymbol{r}}^U$.

We also consider the shock's impact on input costs, $\eta^M_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{k,o,d} \frac{\partial \ln P^M_{i,s}}{\partial \ln \tau_{od,k}} \hat{\tau}_{od,k}$:

$$\eta^M_{i,s}(\hat{\boldsymbol{\tau}}) = \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k} b^D_{is,jk} \mu^M_{j,k}(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,k} x^0_{ji,k} \xi^M_{i,ks} \hat{\tau}_{ji,k}, \quad \bar{\boldsymbol{b}}^D \equiv \sum_{d=1}^{\infty} \left(\bar{\boldsymbol{x}}^D\right)^d, \tag{36}$$

and $\bar{\boldsymbol{x}}^D \equiv [x^D_{is,jk}]_{is,jk}$ with $x^D_{is,jk} \equiv a^M_{j,k} X^M_{ji,ks}/a^M_{i,s} R_{i,s}$ denoting the share of input expenditure in sector s from i that corresponds to input purchases from sector k of market j. This "input shock exposure" has again two terms. The first is the direct impact of the shock on the unit input cost of sector s from market i, which by Shepard's lemma is simply an average of the shocks across sectors and markets, weighted by the spending shares on them. In addition, cost shocks in other sectors and markets have a downstream impact on the cost of production in sector s from i through its intermediate input purchases, with weights given by the series expansion of the matrix of intermediate cost shares, $\bar{\boldsymbol{x}}^D$.[32]

Appendix A.2 shows that Theorem 1 still holds in this general setting with the shift in excess labor demand now depending on the measures of revenue and cost exposure defined above. Thus, as in Section 3.2, when we consider observed and unobserved components of trade shocks, we can also show that the changes in any labor market outcome $\hat{Y}_i \in \{\Delta\overline{\ln w_i}, \Delta \ln n_i, \Delta \ln N_i\}$ have the following reduced-form representation:

$$\hat{Y}_i = \alpha^Y + \sum_j \beta_{ij}^{Y,R}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j^R(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_j \beta_{ij}^{Y,C}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_j^C(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_{j,s} \beta_{ij,s}^{Y,M}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}_{j,s}^M(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \nu_i^Y, \quad (37)$$

where we now define the parameter vector and the matrix of initial conditions as $\boldsymbol{\theta} \equiv (\phi, \psi, \lambda, \vartheta, \sigma)$ and $\boldsymbol{\mathcal{W}}^0 \equiv \{n_i^0, \omega_i^0, \{X_{ij,s}^0\}_{j,s}, \{\xi_{i,s}, a_{i,s}^L\}_s, \{\xi_{i,ks}^M\}_{k,s}\}_i$. Under the exogeneity assumption in (27),

$$E\left[\nu_i^Y \sum_j h_{ij}^{Y,R}\hat{\eta}_j^R(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}})\right] = E\left[\nu_i^Y \sum_j h_{ij}^{Y,C}\hat{\eta}_j^C(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}})\right] = E\left[\nu_i^Y \sum_{j,s} h_{ij,s}^{Y,M}\hat{\eta}_{j,s}^M(\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}})\right] = 0. \quad (38)$$

for the de-meaned shock, $\ddot{\hat{\boldsymbol{\tau}}}^{\text{obs}} \equiv \hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\tau}^{\text{obs}}$, and any real matrices $\{h_{ij}^{Y,R}, h_{ij}^{Y,C}, \{h_{ij,s}^{Y,M}\}_s\}_j$.

Equations (37) and (38) generalize the empirical specification in Section 3.2. As such, (37)–(38) inherit all the properties outlined in Section 3.2 that allow their use for both estimation and testing. However, there are three additional implications of the extended model embedded in (37).

The first term in (37) is the analog for this general model of the reduced-form responses in (26) for the simpler model of Section 3. Not only trade in intermediate goods requires the measurement of upstream revenue exposure (i.e., $\sum_s \ell_{i,s}^0 \mu_{i,s}^U(\hat{\boldsymbol{\tau}})$ in (34)), but it also alters the reduced-form elasticities to revenue shock exposure (i.e., $\boldsymbol{\mathcal{W}}^0$ includes final and intermediate spending shares). In fact, higher intermediate input usage plays a similar role to stronger agglomeration forces in amplifying reduced-form elasticities due to a lower pass-through from wages to prices and, thus, a flatter labor demand curve. We formalize this intuition in Appendix A.2.5 for a symmetric economy with intermediate inputs in production in which wage changes are still given by (22) but the labor demand elasticity is instead $\kappa = (\sigma - 1)(1 - \psi\phi - a^M)$.

The second term indicates that shocks in the price of imported final goods, $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}^{\text{obs}})$, also affect labor market outcomes in this more general framework. This follows from the impact that

[32] Our theoretical exposure measures are closely related to empirical measures of upstreamness and downstreamness (in levels) for open economies suggested by Fally (2012), Antràs and Chor (2013), and Antràs and Chor (2018). Our measures are the open economy analogs of the Leontief matrices controlling shock percolation across sectors in a closed economy network model (see Acemoglu et al. (2016b) and Carvalho and Tahbaz-Salehi (2019)), and related to the forces highlighted in the open economy model of Baqaee and Farhi (2019).

such shocks have on both the non-employment payoff (as regulated by λ) and the allocation of individuals across markets (as regulated by ϑ). Formally, we can write $\beta_{ij}^{Y,C} = \lambda\tilde{\beta}_{ij}^{Y,C\lambda} + \vartheta\tilde{\beta}_{ij}^{Y,C\vartheta}$. Thus, when λ and ϑ are higher, the impact of consumption exposure on labor market outcomes is also stronger. In fact, $\beta_{ij}^{Y,C} = 0$ for the labor supply structure of Section 3 that entails $\lambda = \vartheta = 0$.

The last term captures how outcomes respond to shocks in the cost of imported inputs, $\hat{\eta}_{i,s}^{M}(\hat{\boldsymbol{\tau}}^{\text{obs}})$. Such responses arise from two channels. When input costs fall in a market, its labor demand increases due to market share gains in all destinations. Moreover, input cost shocks affect labor supply through its impact on the consumption price index across markets.

Notice that the representation in (37) links our model to the evidence in Section 2 documenting small responses of wages and employment to higher exposure to the China shock in terms of gross expenditure. For the same foreign shock $\hat{\zeta}_{\text{F},s}$, the shift-share exposure in (5) is a weighted average of the exposure to shocks in the cost of final and intermediate goods.[33] Thus, the evidence in Section 2 suggests that responses of wages and employment to exposure to the China shock in terms of (final and intermediate) consumption costs, $\hat{\eta}_i^C$ and $\hat{\eta}_{i,s}^M$, are not strong enough to offset the negative impact caused by revenue losses due to Chinese import competition, $\hat{\eta}_i^R$. We return to this point in the next section.

Generality of the Empirical Specification. In Appendix A.3.1, we show that our results hold for a general class of models encompassing most of the recent quantitative trade and spatial models reviewed by Costinot and Rodríguez-Clare (2014) and Redding and Rossi-Hansberg (2017). We outline general conditions that yield (37) with the same shift-share measures of exposure $\{\hat{\eta}_j^R, \hat{\eta}_j^C, \hat{\eta}_{j,s}^M\}$ that satisfy (38).[34] This characterization follows three steps: (i) specifying the observed and unobserved trade shocks, (ii) solving for the first-order approximation of log-changes in observable outcomes, and (iii) defining the reduced-form elasticities as a function of initial conditions and elasticities in the model.

[33] Formally, $\sum_s e_{i,s}\hat{\zeta}_{F,s} \propto \sum_k \ell_{i,k} a_{i,k} \mu_{i,k}^M(\hat{\boldsymbol{\tau}}) + (1 - \sum_k \ell_{i,k} a_{i,k})\eta_i^C(\hat{\boldsymbol{\tau}})$ with $a_{i,k} = a_{i,k}^M / a_{i,k}^L$.

[34] We consider a market definition based on factors and sets of sectors in a region. Equation (37) holds when, at least locally, (i) different adjustment margins of factor supply are a function of wage and price index vectors, (ii) the consumption price index is a function of production costs across sectors and markets, and (iii) production is a function of the firm's usage of factors and intermediate goods, as well as employment in different markets. Similar results hold in non-competitive environments whenever it is possible to define bilateral price indices in terms of factor prices, input prices, and labor supply outcomes (as in Costinot and Rodríguez-Clare (2014)). Alternative modeling choices may bring additional measures of exposure, but our methodology continues to hold.

5 Application: Measuring the General Equilibrium Effect of The China Shock

Our theory has established that the general equilibrium impact of trade shocks on local labor markets is intrinsically related to the direct and indirect reduced-form elasticities of labor market outcomes to measures of shock exposure of different markets. We now use our characterization of these elasticities in the context of the spatial models described above to empirically investigate how U.S. CZs were affected by the China shock.

5.1 Measuring the China Shock

We back out model-consistent sectoral demand shifts from ADH's measure of the China shock in each sector – that is, the per-worker growth in Chinese imports in developed countries, ΔM_s^t. We consider without loss of generality a decomposition of the shift in sectoral demand triggered by the China shock into a common component and destination-specific components: $(1-\sigma)x^{t_0}_{\text{China}j,s}\hat{\tau}^t_{\text{China}j,s} = \hat{\zeta}^t_{\text{China},s} + \hat{\varepsilon}^t_{\text{China}j,s}$ for all j. We set our observed measure of the China shock to be the common sectoral component $\hat{\zeta}^t_{\text{China},s}$, which we can recover from ΔM_s^t under the assumption that the size-weighted mean of $\hat{\varepsilon}^t_{\text{China}j,s}$ is zero, $\sum_j \frac{E^{t_0}_{j,s}}{\sum_{j'} E^{t_0}_{j',s}} \hat{\varepsilon}^t_{\text{China}j,s} \approx 0$ for each s. Without loss of generality, we set China's wage as the economy's numeraire (e.g., $\hat{w}^t_{\text{China}} = 1$) and show in Appendix A.4.1 that

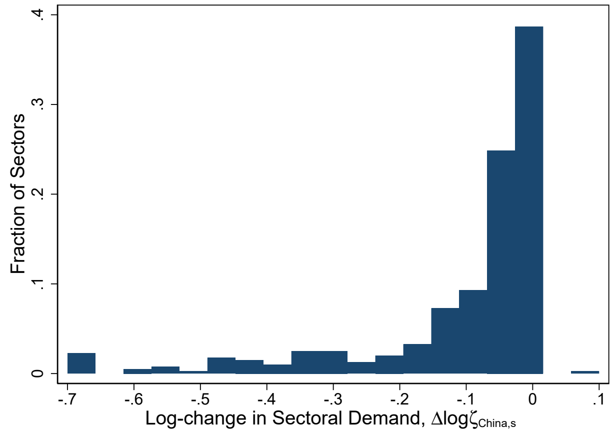
$$\Delta M_s^t = \left(\frac{\sum_j E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}} \right) \hat{\zeta}^t_{\text{China},s} + \frac{\sum_j X^{t_0}_{\text{China}j,s} \Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}}, \tag{39}$$

where $\Lambda^t_{j,s}$ is the destination-sector fixed-effect in a sector-level gravity regression for log-changes in bilateral trade flows, and $L^{t_0}_{\text{US},s}$ is the initial U.S. employment in sector s (as in ADH). This expression indicates that the sectoral shifter used in ADH combines two components: the sectoral China shock $\hat{\zeta}^t_{\text{China},s}$, and the average across destinations of the endogenous changes in their sectoral demand $\Lambda^t_{j,s}$.

In our empirical analysis, we use (39) to compute $\hat{\zeta}^t_{\text{China},s}$ from the sectoral shifter in ADH, ΔM_s^t. Thus, the exogeneity assumption in (27) requires the sectoral component of China's productivity shocks, $\hat{\zeta}^t_{\text{China},s}$, to be uncorrelated with all other unobserved shocks in economic fundamentals (given initial conditions). This is reasonable because the recent productivity growth in China was largely driven by internal reforms implemented as China transitioned to a market-oriented economy and by China's accession to the WTO in 2001. For discussions, see Naughton (2006), Hsieh and Klenow (2009), Brandt et al. (2012) and Autor et al. (2013).

Figure 1 displays the histogram of the impact of the China shock on the demand for manufacturing products of U.S. CZs between 1991 and 2007 – that is, $\hat{z}^{\text{obs},t}_{i,s} = -\hat{\zeta}^t_{\text{China},s}$ for all i. The China shock caused demand declines in essentially all sectors, but the intensity of the shock varied

Figure 1: Histogram of Sectoral Demand Shifts Induced by the China Shock



Notes: Histogram of log-changes in sectoral demand implied by the China shock for the set of 4-digit SIC sectors in ADH between 1991 and 2007, computed as $\hat{z}_s^{\text{obs},t} = -\hat{\zeta}^t_{\text{China},s}$ with equation (39).

substantially across sectors. This is the variation that we exploit in our estimation strategy.

The fact that ΔM_s^t has an endogenous demand component implies that the shifter used in ADH (and thus that used in Section 2) may be affected by other confounding shocks in U.S. CZs. However, this concern is likely of second-order for two reasons. First, the demand-adjusted growth in per-worker Chinese imports, $\Delta M_s^t - \sum_j X^{t_0}_{\text{China}j,s} \Lambda^t_{j,s} / L^{t_0}_{\text{US},s}$, has a cross-sector correlation of 0.95 with the growth of per-worker Chinese imports, ΔM_s^t. This means that cross-sector variation in ΔM_s^t is essentially driven by the sectoral shocks $\hat{\zeta}^t_{\text{China},s}$. Second, due to this high correlation, Panel C of Table B.6 shows that the results in Table 1 are qualitatively similar when we compute the shift-share exposure variables in (3)–(5) using the (de-meaned) sectoral demand shock $\hat{\zeta}^t_{\text{China},s}$ instead of the per-worker import growth ΔM_s^t.

5.2 Measuring the Spatial Links

We now discuss the specification of the variables in $\boldsymbol{\mathcal{W}}^0$ necessary to compute the reduced-form elasticities for any given $\boldsymbol{\theta}$. Appendix C presents details about the data construction procedure.

We first construct sectoral trade flows between the 722 U.S. CZs and 52 foreign countries. We use trade data from UN Comtrade assembled by CEPII to measure country-to-country trade flows for 4-digit SIC sectors. We use the gravity structure of our model to impute domestic sales in each 4-digit SIC sector by combining bilateral trade flows and information on domestic sales in aggregate sectors obtained from Eora MRIO. Second, we distribute U.S. domestic and international trade flows across CZs using again the gravity structure of our model. Specifically, we first split U.S. Census data on imports and exports for each industry-country across CZs using measures of each CZ's share in that industry's national spending and production. We then impute bilateral trade shares across CZs using a gravity specification estimated with bilateral shipment data from the Commodity Flow Survey (CFS). Since our baseline model imposes trade balance, we adjust market sizes to balance trade flows given the bilateral trade shares.[35]

The extended empirical specification in Section 4 requires, in addition, the shares of final and intermediate spending for each sector. For U.S. CZs, as discussed in Section 2.4, we measure final expenditure shares using the Public-use Micro-data from the Consumer Expenditure Surveys. For foreign countries, we use the final spending shares from the BEA input-output matrix. We also use the BEA input-output matrix to specify the sectoral intermediate cost shares for all markets. Finally, we set the share of intermediate inputs in total cost in each sector and market by assuming that $a^M_{j,k} = a_j a^M_k$ with a^M_k obtained from the NBER Manufacturing database, and selecting a_j to match observed value-added in each market.[36]

Finally, we specify the numeraire function of non-employment benefits, $\Omega(\boldsymbol{w})$. We use the evidence in Chodorow-Reich and Karabarbounis (2016) that, for annual fluctuations in the U.S., the non-employment payoff (the average change in b_i/P_i across U.S. CZs) has a correlation of 0.64 with per-capita real income (the average change in w_i/P_i across U.S. CZs). To match this correlation, we set $b_i/P_i = \bar{b}_i \Omega(\boldsymbol{w})/P_i$ such that $\Omega(\boldsymbol{w})$ is the geometric average of the per-capita income in the U.S. and the World, $\Omega(\boldsymbol{w}) = (W_{\text{US}}(\boldsymbol{w}))^{\bar{\omega}}(W_{\text{W}}(\boldsymbol{w}))^{1-\bar{\omega}}$ with $\bar{\omega} = 0.62$. See Appendix A.4.2 for details.

5.3 Estimation of Model Parameters and Reduced-Form Elasticities

Table 2 presents the estimates of $\boldsymbol{\theta}$ that we obtain using a GMM estimator based on (26)–(28) with $\hat{z}^{\text{obs},t}_{i,s} = -\hat{\zeta}^t_{\text{China},s}$ and the pooled sample of 722 U.S. CZs in 1990-2000 and 2000-2007. Because $\hat{\zeta}^t_{\text{China},s}$ already accounts for the trade elasticity, we do not estimate this parameter and set it to five (i.e., $\sigma - 1 = 5$), a typical value in the literature (see Costinot and Rodríguez-Clare (2014)).[37] In all specifications, we use the same control set in Table 1, and use the moment weights suggested by the optimal approach described in Appendix A.1.8,

$$h^{w,t}_{ij} = \nabla_{\boldsymbol{\theta}} \beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{W}^{t_0}) \quad \text{and} \quad h^{n,t}_{ij} = \nabla_{\boldsymbol{\theta}} \beta^n_{ij}(\boldsymbol{\theta}|\boldsymbol{W}^{t_0}). \tag{40}$$

We start in Panel A with the baseline model of Section 3 that only depends on the elasticity parameters of labor supply (ϕ) and labor demand (κ). The first column reports an estimate of ϕ equal

[35] Table C.3 in Appendix C.1.2 reports validation tests using the CFS data. Regressions of actual on predicted trade flows across states and SCTGs yield coefficients close to 1 and R^2 of 0.48–0.83.

[36] We impose that final and intermediate spending shares are the same across countries because we are not aware of any comprehensive dataset that includes this information for all countries and 4-digit SIC sectors considered in our empirical application. Figure C.1 in Appendix C.2.2 shows that our calibration procedure almost exactly matches the observed shares of value added across U.S. CZs and foreign countries.

[37] This is without loss of generality for the baseline model as reduced-form elasticities only depend on the labor demand elasticity $\kappa \equiv (\sigma - 1)(1 - \psi\phi)$. The choice of σ affects the estimate of ψ, but does not alter the model's predictions.

Table 2: Estimates of the Structural Parameters

ϕ	ψ	λ	ϑ
Panel A: Baseline Model of Section 3			
2.53	0.35	-	-
(0.37)	(0.05)	-	-
Panel B: Extended Model of Section 4, no intermediates			
2.51	0.36	0.19	-
(0.41)	(0.05)	(0.32)	-
Panel C: Extended Model of Section 4, no intermediates			
2.42	0.35	-	-0.12
(0.41)	(0.06)	-	(0.14)
Panel D: Extended Model of Section 4			
4.16	0.05	-	-
(1.23)	(0.01)	-	-
Panel E: Extended Model of Section 4			
4.39	0.05	0.21	-0.19
(1.28)	(0.02)	(0.32)	(0.27)

Notes: GMM Estimates of $\boldsymbol{\theta}$ implied by the specification in (26) and (28) for Panel A and (37) and (38) for Panels B-D, with the shock $\hat{z}_{i,s}^{\text{obs},t} = -\hat{\zeta}_{\text{China},s}^{t}$ in (39) and the weight matrix in (40). Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications also include the baseline control vector used in Table 1. Standard errors in parentheses are clustered by state.

to 2.5.[38] It implies a median labor supply elasticity across U.S. CZs of $\phi(1-n_i) = 2.53 \times (1-0.7) = 0.75$. Thus, our estimate yields a Marshallian elasticity of labor supply that is consistent with values required to match employment fluctuations over the business cycle and across countries in the macro literature, but that is higher than micro estimates of the Hicksian elasticity implied by individual-level responses (Chetty et al., 2013b). The second column reports a value of ψ equal to 0.35. This follows from the low value of the labor demand elasticity that we estimate from the reduced-form responses to revenue shock exposure across CZs, $\kappa = (\sigma-1)(1-\psi\phi) = 5 \times 0.12 = 0.6$. Our estimate of the labor demand elasticity is much lower than that implied by typical calibrations of a Ricardian model without agglomeration forces ($\psi = 0$) and input-output links – in this case, a trade elasticity of 5 yields $\kappa = 5$. As we formally show in Section 5.4, the high values of both ϕ and ψ are central for a model without intermediates in production to replicate the large differential effects created by the China shock across U.S. CZs.

In Panels B and C, we allow for the additional margins of labor supply adjustment introduced in Section 4, but maintain the same baseline production structure of Section 3. In both cases, the estimates of ϕ and ψ remain similar to those reported in Panel A. Panel B reports an estimate

[38] Our estimate of ϕ is closely related to the evidence in Table 1. Note that (8)–(9) imply that $\phi = \frac{d \ln n_i}{d\overline{\ln w_i}} \frac{n_i}{1-n_i}$, with $d \ln n_i$ and $d\overline{\ln w_i}$ denoting respectively the responses in the employment rate and average log-wage caused by a regional demand shock orthogonal to $\hat{b}_i$. Applying this formula to the estimates in columns (1) and (3) of Table 1, $\phi = \frac{d \ln n_i}{d\overline{\ln w_i}} \frac{n_i}{1-n_i} = 1.1 \times 0.7/0.3 \approx 2.6$ for the median U.S. CZ, a value similar to the estimated ϕ.

of λ of 0.19, implying that a decrease of 1% in the local price index is associated with a median increase in labor supply across U.S. CZs of $\lambda\phi(1-n_i) \approx 0.19 \times 2.51 \times (1-0.7) = 0.14\%$. In line with the discussion in Section 4, our low estimate of λ follows from the evidence in Table 1 that higher expenditure exposure to the China shock had small, non-significant impacts on wages and employment across CZs. Although we are not aware of estimates of this parameter in the literature, the fact that we reject $\lambda = 1$ indicates that our estimate is consistent with the evidence in Chodorow-Reich and Karabarbounis (2016) that the non-employment payoff in the U.S., b_i/P_i, responds to shocks in labor demand and labor supply.

In addition, Panel C reports a negative point estimate for the elasticity of location choice to real wages, ϑ. Since ϑ is proportional to the reduced-form response of population to regional shock exposure (see Part C of Appendix A.2.4), our estimate of ϑ follows from the evidence in Table 1 that the differential impact of higher exposure to Chinese import competition on regional population was not statistically different from zero, with a positive point estimate. Our result is consistent with a growing body of literature documenting that recent shocks in regional labor demand in the U.S. triggered weak population responses over ten year horizons – see Molloy et al. (2011), Autor et al. (2013), Cadena and Kovak (2016), Yagan (2019), and Benguria (2020).[39]

In Panels D and E, we consider the richer production structure with input-output linkages introduced in Section 4. In this case, we obtain different estimates of ϕ and ψ, but similar estimates of λ and ϑ. The estimate of ϕ points to a higher median labor supply elasticity of 1.2, but it is now estimated with higher standard error. In fact, we cannot reject that the estimates of ϕ in Panels A and D are equal at usual significance levels.[40] In addition, the second column reports a lower value of ψ equal to 0.05. This is the result of both the higher estimate of ϕ and the fact that, as discussed in Section 4, a higher share of intermediates in production yields a flatter labor demand function for any given value of $\psi\phi$. However, our lower estimate of ψ still implies strong agglomeration forces: the median elasticity of production costs to regional employment across CZs is $\psi/(1-n_i) = 0.17$, a value similar to that implied by the models in Krugman (1980) and Krugman (1991) for a trade elasticity of five. The strong agglomeration forces that we find are consistent with the evidence of responses to regional demand shocks in the U.S. and Brazil (Kline and Moretti, 2014; Dix-Carneiro and Kovak, 2017) and regional labor supply shocks in Germany (Peters, 2019), and are in the upper range of the sectoral scale elasticities at the country-level reported by Bartelme et al. (2019).

[39] Greenland et al. (2019) find that the population response to the China shock is weak for all working-age individuals, but it is statistically different from zero for those aged below 30 years old. Cadena and Kovak (2016) find weak responses in the U.S. native population to local labor demand shocks, but positive responses for the U.S. immigrant population.

[40] We obtain a higher estimate of ϕ because this version of the model incorporates upstream revenue exposure to the China shock. In fact, when we estimate, in Panel D of Table B.6, the specification in (2) with IC_i^t constructed as the measure in (35) that accounts for upstream exposure, we obtain a higher estimated response of employment relative to that of wages. This helps explain the higher estimated ϕ given the argument in footnote 38.

Table 3 reports the reduced-form elasticities, β_{ij}, and the shifts in excess labor demand, $\hat{\eta}_i$, that we obtain with the estimated parameters in Table 2. In the top panel, we first report percentiles of the empirical distribution in 2000 of β_{ij} and $\hat{\eta}_i$ for the baseline model of Section 3 with the parameters in Panel A of Table 2. The first column indicates that, for the median U.S. CZ, a 1% increase in its excess labor demand triggers an increase in the local wage of 0.67%. There is substantial heterogeneity in this direct reduced-form elasticity across CZs, as it can be seen from the value of the 99$^{\text{th}}$ percentile, due to their distinct conditions before the shock (e.g., employment rate, openness and size). The second column shows that the indirect reduced-form elasticities are positive and, thus, imply a reinforcing spatial propagation of regional demand shocks. The median indirect elasticity of 0.003 is small, but the combined indirect effect may be relatively large as there are 721 CZs indirectly affecting each region. A small subset of large or centrally-connected CZs create much stronger indirect effects: the 99$^{\text{th}}$ percentile of the indirect reduced-form elasticity is 0.301. Lastly, the third column of Table 3 reports the percentiles of the shift in excess labor demand across CZs. Although the China shock reduced the excess labor demand in most CZs, the extent of this reduction varied substantially across markets because of the existing cross-regional variation in the initial sectoral employment composition.

In the bottom panel of Table 3, we investigate how β_{ij} and $\hat{\eta}_i$ implied by the different versions of the model in Panels B-E of Table 2 compare to those implied by the baseline model in Panel A of Table 2. We document a pattern of strong correlation of β_{ij} and $\hat{\eta}_i$ across the different versions. The correlation of one in the first and second rows indicate that, given the estimates in Table 2, spatial links and demand shifts in the baseline model are not affected when we allow for labor supply to endogenously respond to import prices and location choices. Adding intermediate inputs reduces the correlation slightly, but it remains above 0.84. Intuitively, the similarity in β_{ij} and $\hat{\eta}_i$ follows from the fact that our empirical strategy precisely targets the reduced-form elasticities to observed measures of exposure, and thus it adjusts the parameters (as reported in Panels D and E of Table 2) to generate similar reduced-form responses to the China shock across CZs.

5.4 Evaluating the Fit of Different Specifications of Spatial Links

Our next goal is to evaluate which specifications of spatial links imply predicted responses to the China shock that are aligned with those observed across U.S. CZs. To do so, we estimate the fit coefficient in (29) for different outcomes and specifications.

We start in Table 4 with the predicted responses implied by our estimates: the baseline model of Section 3 and the extended model of Section 4 with the estimates in Panels A and E of Table 2, respectively. We again use the controls in Table 1, and the pooled sample of 722 U.S. CZs in 1990-2000 and 2000-2007.[41] Columns (1) and (2) present the estimates for the two labor market

[41] Table B.9 in Appendix B.2 shows that results are similar for the alternative versions in Table 2. We use

Table 3: Reduced-form Elasticities and Shifts in Excess Labor Demand for U.S. CZs, 2000-2007

	Reduced-form Elasticity		Shift in Excess Labor Demand
	Direct	Indirect	
	β_{ii}	β_{ij}	$\hat{\eta}_i$
Percentiles of empirical distribution, baseline model of Section 3			
10^{th} percentile	0.503	0.000	-0.060
50^{th} percentile	0.666	0.003	-0.020
90^{th} percentile	1.576	0.031	-0.002
99^{th} percentile	4.212	0.301	0.000
Correlation between outcomes implied by each specification and the baseline model of Section 3			
Panel B of Table 2	1.000	1.000	0.999
Panel C of Table 2	0.999	1.000	0.997
Panel D of Table 2	0.854	1.000	0.883
Panel E of Table 2	0.842	1.000	0.883

Notes: The top panel reports the percentiles of the empirical distribution for the 722 U.S. CZs of the reduced-form elasticities in 2000 and the shift in excess labor demand in 2000-2007 implied by the baseline model in Section 3 and the estimates in Panel A of Table 2. Each row of the bottom panel reports the correlation between the outcome implied by the baseline model of Section 3 (for estimates in Panel A of Table 2) and the same outcome implied by the alternative specification of the model described in that row (for estimates of the corresponding panel of Table 2) with β_{ij} and $\hat{\eta}_i$ given by (56) and (57).

outcomes used in the estimation of the reduced-form elasticities in Section 5.3: the changes in the average log wage and the log of the employment rate. It is thus not surprising that, both for our baseline model and for the extended specification, we cannot reject that the fit coefficients are one for these two outcomes.

In columns (3) and (4), we present estimates of the fit coefficient for the predicted responses in the CZ's sectoral employment composition (as derived in Appendix A.1.9). Since these outcomes were not used in the estimation of $\boldsymbol{\theta}$, the fit coefficient of one is an over-identification restriction that we now use for testing.[42] Results indicate that our estimated models generate differential responses in sectoral employment composition that are consistent with those observed following the China shock. Column (3) shows that the estimated fit coefficients are close to one for the change in the share of the CZ's working-age population employed in manufacturing – the main dependent variable in ADH. Finally, in column (4), we estimate the fit coefficient for the change in the share of manufacturing in the CZ's total employment. We again obtain a fit coefficient close to one indicating that the results in column (3) are not only driven by the changes in the

standard errors clustered by state that impose independence of residuals across states. Because predictions in our model take a shift-share form, Table B.10 shows that standard errors are similar when we allow for arbitrary cross-market correlation in the residuals using the inference procedure in Adão et al. (2019). Unfortunately, we cannot implement this inference procedure for the extended model with intermediate goods because we cannot separately compute the exposure measures and the matrices in the reduced-form elasticities given the computational burden of inverting and manipulating the high-dimension matrices evolved.

[42] Note that there could be many reasons why our model may fail to match these non-targeted moments, as it does not feature search frictions (e.g. as in Helpman and Itskhoki (2010)), mobility costs and amenity preferences (e.g. as in Caliendo et al. (2019)), or sector-specific human capital (e.g. as in Burstein et al. (2019); Galle et al. (2017)).

Table 4: Fit of the Model across U.S. CZs

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in working-age population	Share of Manufacturing in employed population
	(1)	(2)	(3)	(4)
Panel A: Baseline Model of Section 3				
Fit Coef. (ρ^Y)	0.97	0.90	0.95	0.82
	(0.25)	(0.15)	(0.11)	(0.13)
p-value of $H_0 : \rho^Y = 1$	91.5%	51.1%	63.9%	16.0%
Panel B: Extended Model of Section 4				
Fit Coef. (ρ^Y)	1.16	1.07	0.86	0.79
	(0.48)	(0.20)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	73.9%	70.5%	42.6%	21.4%

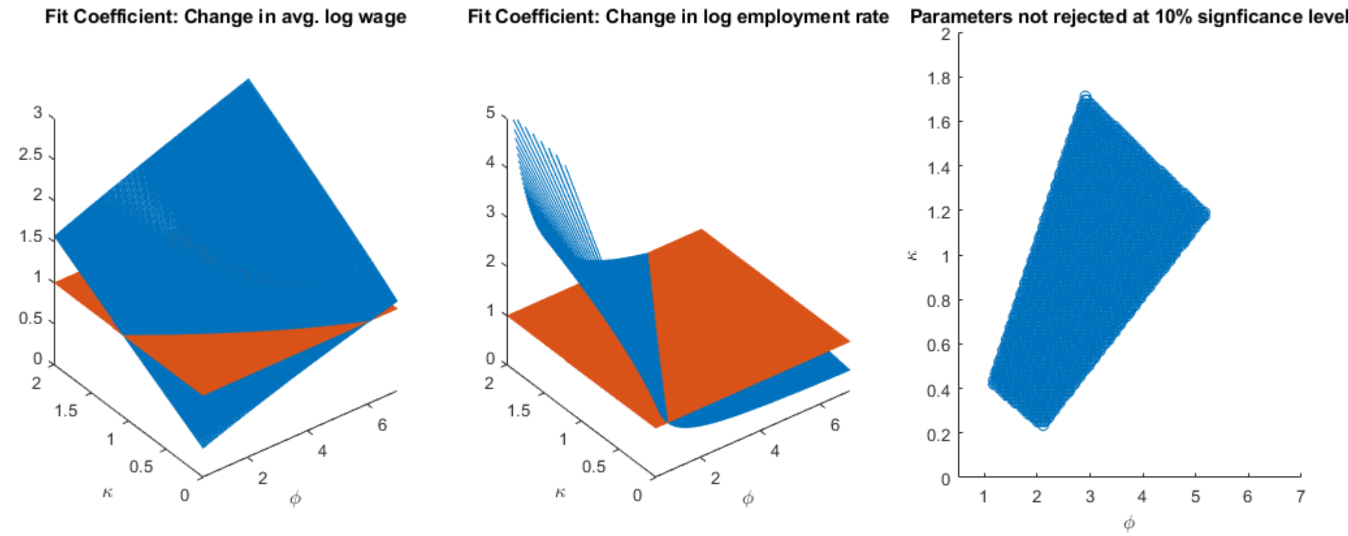
Notes: Estimation of (29) with the shock $\hat{z}_{i,s}^{\text{obs},t} = -\hat{\zeta}_{\text{China},s}^{t}$. Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Panel A presents the fit coefficient of the baseline model of Section 3 for the estimates in Panel A of Table 2. Panel B presents the fit coefficient of the extended model of Section 4 for the estimates in Panel E of Table 2. Robust standard errors in parentheses are clustered by state.

employment rate that was targeted in estimation.

Having established the good fit of our baseline, we investigate which alternative calibrations of our model are rejected by the model fit test. We first consider alternative specifications of the parameters of labor supply and labor demand in the baseline model of Section 3. Figure 2 reports the fit coefficients for wages (left panel) and employment (center panel) given different values of (ϕ, κ), as well as the values (ϕ, κ) not rejected by the joint test that these fit coefficients are equal to one (right panel). For low values of ϕ, the predicted responses in employment are too small (fit coefficient is above one), and those for wages are too large (fit coefficient is below one). The right panel shows that we reject any value of ϕ below one and thus a median labor supply elasticity below 0.3. For high values of κ, the predicted responses in both wages and employment become too small (fit coefficients are much higher than one). We reject values of κ above 1.8 and, therefore, values of $\psi\phi$ below 0.64 (for a trade elasticity of five). Thus, given any value of ϕ in our model, the test rejects the predicted responses implied by a multi-sector Ricardian production framework without agglomeration and input-output linkages – such as the frameworks in Galle et al. (2017) and Kim and Vogel (2021).

Lastly, in Table 5, we investigate the fit of alternative specifications of the extended model of Section 4 with input-output links, endogenous population mobility, and import price effects in labor supply. Our benchmark is a calibration of the model that is consistent with the long-run predicted responses in CDP: a multi-sector Ricardian framework with input-output links and no agglomeration forces ($\psi = 0$), isomorphic employment and location choices ($\phi = \vartheta$), and

Figure 2: Fit Coefficient for Alternative Parameter Values



Notes: In left and center panels, the blue area shows the fit coefficient implied by the estimation of (29) for different values of the parameters (ϕ, κ), and the orange area illustrates a fit coefficient of one. The right panel reports the set of parameters for which we fail to reject at a 10% significance level the hypothesis that the fit coefficient is one in the estimation of (29) for either average log wage or log employment rate (using standard errors clustered by state).

non-employment payoff proportional to the price index ($\lambda = 1$). We set $\phi = 1.5$ so that the median labor supply elasticity is 0.5 across CZs.[43] Under this calibration, the model of Section 4 yields predicted changes in the employment rate for U.S. states in 2000-2007 that are similar to the long-run predicted changes in CDP: they have a correlation of 0.5, and almost identical standard deviations of 0.05%. Column (1) of Table 5 shows that this alternative specification implies responses that are substantially smaller than those in the data: the fit coefficient is 4.5 for the employment rate and 1.9 for the average log wage. While the fit is also less precise than that for our estimated specifications, one can reject this alternative specification based on the fit for the employment rate at a 5% significance level.[44]

The remaining columns of Table 5 investigate why this alternative specification is rejected by the model fit test, while our estimated model is not. We estimate the model fit by sequentially modifying our baseline estimates, such that the migration choice elasticity is 1.5 in column (2), the non-employment payoff is proportional to the price index in column (3), the agglomeration elasticity is zero in column (4), and the labor supply elasticity parameter is 1.5 in column (5).[45]

[43]In CDP, ϕ and ϑ correspond to β/ν, which is estimated to be 0.2 or 0.5 at the quarterly or annual frequencies, respectively. They caution readers that this parameter should be higher at longer horizons. So, given that we implement our model for changes over ten years, we prefer to calibrate this parameter using the estimates for the long-run (Hicksian) elasticity of labor supply in Chetty et al. (2013b).

[44]One may still be concerned that our results are driven by differences that remain between the specification we consider and that in CDP, despite the similarity between the predicted responses implied by the two models. To ease such concerns, the right panel of Figure B.2 in Appendix B.1 depicts a version of the model fit test based directly on the predictions in CDP (as reported in their replication package). It shows that the slope coefficient between actual and predicted responses is much larger than one.

[45]In columns (3)–(5), we set the location choice elasticity to zero, as our negative point estimate is not statistically

Table 5: Fit of the Model across U.S. CZs – Alternative Specifications

	(1)	(2)	(3)	(4)	(5)
Dependent variable: Change in log of employment rate					
Fit Coef. (ρ^Y)	4.48	1.93	1.43	1.82	3.70
	(1.42)	(0.35)	(0.28)	(0.33)	(0.75)
p-value of $H_0 : \rho^Y = 1$	1.5%	0.7%	12.1%	1.3%	0.0%
Dependent variable: Change in log of average weekly wage					
Fit Coef. (ρ^Y)	1.89	1.88	2.08	2.07	1.46
	(0.96)	(0.84)	(0.65)	(0.86)	(0.64)
p-value of $H_0 : \rho^Y = 1$	35.2%	29.1%	9.7%	21.2%	47.8%
Parameters:					
ϕ	1.50	4.40	4.40	4.40	1.50
ψ	0.00	0.05	0.05	0.00	0.05
λ	1.00	0.21	1.00	0.21	0.21
ϑ	1.50	1.50	0.00	0.00	0.00

Notes: Estimation of (29) using the predictions of the extended model of Section 4 for $\hat{z}^{\text{obs},t}_{i,s} = -\hat{\zeta}^t_{\text{China},s}$. Each column considers predictions obtained with the indicated set of parameters. Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state.

For all alternative specifications, the fit coefficients are higher than those in Table 4 and are statistically different from one for either wages or employment at a 10% significance level.

To summarize, our results help identifying the roots of the disconnect documented in Section 2 between the small differential effects implied by quantitative spatial models in the literature and the large differential effects implied by the empirical specification in ADH and our extension of it. Such a disconnect disappears when we consider (i) the combination of strong agglomeration forces and high sensitivity of employment to wages, and (ii) weak responses of employment to the price of imported consumption goods.

5.5 The Impact of the China Shock in General Equilibrium

We conclude by presenting the predictions of our estimated model for the impact of the China shock on U.S. CZs. This section has two additional objectives. The first is to compare the differential and aggregate predictions of our estimated model to special cases of it motivated by prior quantitative work. The second is to show how the predictions of our estimated model are connected to those implied by simpler empirical specifications, such as the extension of ADH in Section 2 .

Table 6 summarizes the impact of the China shock on U.S. CZs for different specifications of the model. The top two panels report predicted responses in percentage points (p.p.) that we obtain from our estimates of the specifications in (26) and (37). For the baseline model, Panel A

distinguishable from zero and a negative value would not be used in the literature.

Table 6: Impact of China Shock on U.S. CZs in General Equilibrium

	Employment Rate			Log of Real Wage		
	Average	Standard Deviation	Correlation w/ Baseline	Average	Standard Deviation	Correlation w/ Baseline
Panel A: Baseline Model of Section 3						
	-2.71	1.77	1.00	-2.00	2.08	1.00
Panel B: Extended Model of Section 4						
	-1.88	0.90	0.63	-0.35	0.56	0.27
Panel C: Alternative Specification in column (1) of Table 5						
	-0.06	0.10	0.30	-0.14	0.23	0.30

Notes: Change in outcome for each CZ is the sum of the predicted effects for that CZ in 1990-2000 and 2000-2007. Panel A presents moments for baseline model of Section 3 with the estimates in Panel A of Table 2. Panel B presents moments for the extended model of Section 4 with the estimates in Panel E of Table 2. Panel C presents moments for the extended model of Section 4 with the calibration in column (1) of Table 5. Average and standard deviations are weighted by the CZ employment in 1990.

indicates that the China shock caused declines of 2.7 p.p. and 2 p.p. in the employment rate and real wage, respectively. The impact of the shock varied substantially across CZs: the standard deviation of predicted changes was 1.8 p.p. for the employment rate and 2.1 p.p. for the real wage. In Panel B, when we account for intermediate goods in production, we estimate that the China shock had a smaller impact on CZs, despite exhibiting a high cross-regional correlation with the baseline responses. The average real wage decline is only 0.3 p.p., but employment losses are still close to 2 p.p.. This specification yields smaller losses because it incorporates the compensating effect on consumption of the input cost reduction caused by the China shock. Overall, our model predicts employment losses between 3.4 and 4.8 million jobs in 1990–2007.[46]

Panel C of Table 6 tackles the first goal of comparing our predictions to those implied by a calibration of our model motivated by existing quantitative frameworks – specifically, that in column (1) of Table 5. This alternative calibration generates small predicted responses, both on average and differentially across CZs. The standard deviation of employment responses is only 0.1 p.p., a value much lower than those predicted by both our estimated model and ADH's empirical specification in Section 2. The results in Section 5.4 indicate that such small predicted effects are a consequence of the small reduced-form elasticities implied by the combination of weak agglomeration forces, low sensitivity of employment to wages, and high sensitivity of labor supply to import prices.

We next turn to our second objective. Figure 3 compares the predicted log-changes in the employment rate implied by two types of reduced-form specifications estimated from cross-regional variation in shock exposure. On the vertical axis, it plots the fitted values from the model-implied

[46] Table B.11 in Appendix B.2 presents moments for all the versions of the model in Table 2, along with the 95% confidence interval of each moment implied by a bootstrap procedure based on draws of the parameters from the estimator's asymptotic distribution. Figure B.3 in Appendix B.2 displays maps with the impact of the shock on the employment rate and the real wage of U.S. CZs.

specification in (26) which delivers the general equilibrium impact of the shock for the economy in Section 3. On the horizontal axis, it plots the fitted values from the simpler specification in (2) which parametrizes the direct elasticities to be constant and the indirect elasticities to be proportional to the gravity-based measure in (4).[47]

In the left panel, we consider first the specification in ADH that only includes the direct effect of the CZ's employment exposure to Chinese import competition, $\beta^{IC} IC_i^t$.It generates a pattern of cross-regional variation in fitted values that is similar to that implied by our baseline model – the correlation between them is 0.77. However, most of the points are below the red 45-degree line and, thus, the intercept of our model's predictions is lower than that of the fitted values from ADH's specification. The right panel shows that this is a consequence of the reinforcing indirect effects created by the shock exposure of nearby CZs. In fact, the difference in intercepts becomes smaller when we consider our extension of ADH's specification also featuring a gravity-based indirect exposure measure (i.e., $\beta^{IC} IC_i^t + \beta^{GC} GC_i^t$). This extension of ADH's specification approximates well the predicted responses of our baseline model, both for the differential and the average impact of the China shock. Figure B.4 in Appendix B.2 shows that results are similar when we consider instead the predictions of the richer model of Section 4.[48]

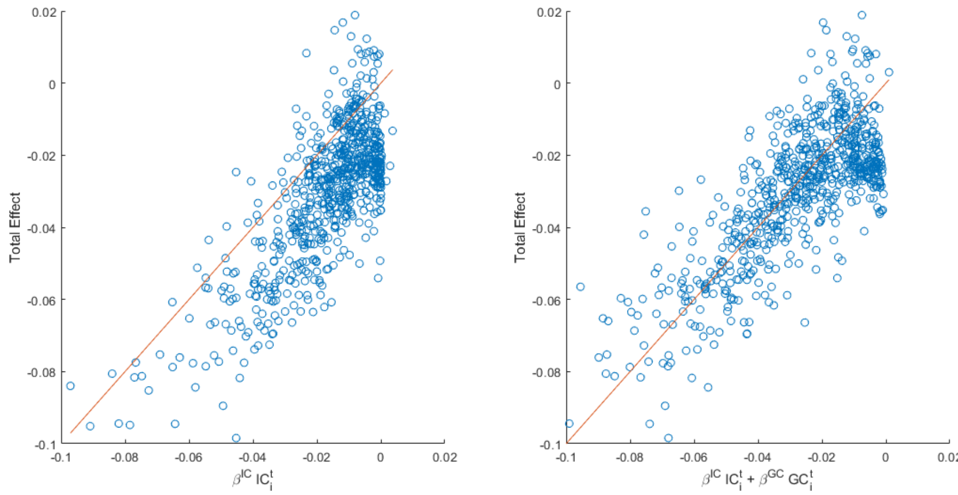
It is worth pointing out that the difference in the intercepts in the right panel does not arise because the indirect effects in our model are constant across CZs – in fact, their standard deviation is 1.32 p.p. Instead, it arises because the cross-regional variation in indirect effects is positively correlated with shock exposure (the correlation is 0.35). Thus, while some CZs are not affected by the China shock, some others not only are highly exposed to Chinese import competition, but also suffer a proportionally large indirect shock from the decline in the demand of the CZs they are directly trading with (due to the spatial correlation in sectoral employment composition).

Robustness. Appendix B.2 reports additional results attesting the robustness of our estimates of the impact of the China shock on U.S. CZs. Figure B.5 shows that we obtain similar employment responses to $\hat{\zeta}^t_{\text{China},s}$ when we use either our empirical specification based on the first-order approximation for the model's predictions or the integration algorithm in Appendix A.3.3 to recover the non-linear predictions of the model. Figure B.6 shows that predicted responses are again similar when we consider the alternative specification in Appendix A.2.6 allowing for different fiscal transfer schemes to fund the changes in non-employment benefit payments caused by the China shock. Lastly, Figure B.7 shows that employment responses in U.S. CZs are similar if we also take into account the exposure of foreign countries to the China shock.

[47]To make results comparable, when we compute the predicted effects based on the empirical specification in (2), we use the exposure measures built with $\hat{\zeta}^t_{\text{China},s}$ (instead of ΔM_s^t) and the estimates in Panel C of Table B.6.

[48]Note that these conclusions may be specific to the context of the China shock and the gravity-based models we consider in this paper. This relationship may be weaker when other mechanisms are more relevant quantitatively.

Figure 3: Impact of the China Shock on the Log Employment Rate in General Equilibrium



Notes: The y-axis on both graphs is the log-change in the employment rate for each of the 722 CZs in 2000-2007 implied by the model of Section 3 with parameters from Panel A of Table 2. The x-axis in the right panel is the change predicted by the specification in column (3) of Panel C in Table B.6, $\beta^{IC} IC_i^t$. The x-axis in the left panel is the change predicted by the specification in column (4) of Panel C in Table B.6, $\beta^{IC} IC_i^t + \beta^{GC} GC_i^t$. In both cases, we compute the exposure measures with $\hat{\zeta}^t_{\text{China},s}$ (instead of ΔM_s^t).

6 Conclusion

The use of cross-regional variation in shock exposure to study how labor markets adjust to economic shocks has become an important part of the toolkit of researchers in international, macro and urban economics. This approach has an important shortcoming, though: estimates of the differential responses of local outcomes to the market's shock exposure may not fully capture all the adjustment channels operating in general equilibrium. In this paper, we propose a new theoretical and empirical methodology for recovering the aggregate impact of economic shocks from their differential impact across local labor markets. Our methodology relies on the reduced-form characterization of a general class of spatial models whose predictions can be expressed in terms of shifts in regional excess labor demand and reduced-form elasticities, direct and indirect, to these shifts. These reduced-form elasticities are sufficient for aggregating the exposure of different markets in order to compute the shock's general equilibrium impact. We then exploit our reduced-form characterization to develop an empirical specification – a generalization of shift-share empirical strategies – that can be used for either estimating the parameters of the model's reduced-form elasticities or testing the model's differential predictions.

Our methodology fills an important gap in the literature. A class of quantitative spatial models has emerged precisely motivated by the critique that empirical strategies exploiting cross-regional variation in shock exposure can recover the shock's differential effect but not its aggregate effect. The ultimate goal of these papers is to use instead their spatial frameworks to quantify the general equilibrium impact of economic shocks by aggregating the predicted responses of regional outcomes.

Despite matching any cross-section of observed regional outcomes with free parameters, a priori these frameworks are not guaranteed to generate predicted responses of regional outcomes to observed shocks that are consistent with the shock's actual differential effect across markets. What we argue in this paper is that quantitative spatial models should be held to the same standard as the empirical strategies that they are supposed to complement, by generating differential responses to economic shocks that are credibly supported by evidence. This is important because, as our theoretical results show, the model's differential predicted responses depend on the same reduced-form elasticities that determine the model's predicted aggregate impact.

For that purpose, our methodology does allow the evaluation of the empirical content of spatial general equilibrium models in terms of their implications for the differential impact of exogenous shocks across markets. It thus makes progress in achieving the standards set by Kehoe (2005): *"Such evaluations also help make applied GE analysis a scientific discipline in which there are well-defined puzzles with clear successes and failures for competing theories"*. The advantage of our unified theoretical and empirical approach is also evinced by our findings that spatial models anchored to the reduced-form moments in the data imply a larger and more dispersed impact of the China shock on U.S. CZs when compared to alternative specifications whose differential predictions are not consistent with their empirical counterparts.

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A Appendix: Proofs and Additional Results (Not for publication)

A.1 Proofs for Section 3

A.1.1 Proof of Equation (14)

The definition of γ_{ij} in (14) and $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (12) immediately imply that

$$\gamma_{ij} = \mathbb{I}_{i=j}\left(\phi - (\phi-1)n_i\right) - (\phi-1)(1-n_i)\omega_j^0 - \frac{1}{R_i^0}\frac{\partial R_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j}$$

where

$$\frac{1}{R_i^0}\frac{\partial R_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_j} = \sum_s \sum_d \ell_{i,s}^0 r_{id,s}^0 \left[\mathbb{I}_{d=j}\left(\phi - (\phi-1)n_d\right) - (\phi-1)(1-n_d)\omega_j^0 - (\sigma-1)(1-\psi\phi)\left(\mathbb{I}_{i=j} - x_{jd,s}^0\right)\right].$$

Thus, using the definitions of $\kappa \equiv (\sigma-1)(1-\psi\phi)$, $\phi_i^0 \equiv \phi - (\phi-1)n_i$ and $r_{ij}^0 \equiv \sum_s \ell_{i,s}^0 r_{ij,s}^0$, we can re-write this expression as

$$\begin{aligned}
\gamma_{ij} &= \mathbb{I}_{i=j}\phi_i^0 + \left(1-\phi_i^0\right)\omega_j^0 - \textstyle\sum_s\sum_d \ell_{i,s}^0 r_{id,s}^0\left[\mathbb{I}_{d=j}\phi_d^0 + \left(1-\phi_d^0\right)\omega_j^0 - \kappa\left(\mathbb{I}_{i=j} - x_{jd,s}^0\right)\right] \\
&= \mathbb{I}_{i=j}\left(\phi_i^0+\kappa\right) - \textstyle\sum_s \ell_{i,s}^0 r_{ij,s}^0\phi_j^0 - \kappa\sum_s\sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0\left(\phi_i^0 - \sum_s\sum_d \ell_{i,s}^0 r_{id,s}^0\phi_d^0\right) \\
&= \mathbb{I}_{i=j}\left(\phi_i^0+\kappa\right) - r_{ij}^0\phi_j^0 - \kappa\textstyle\sum_s\sum_d \ell_{i,s}^0 r_{id,s}^0 x_{jd,s}^0 - \omega_j^0\left(\phi_i^0 - \sum_d r_{id}^0\phi_d^0\right)
\end{aligned}$$

which is equivalent to (19) since $\sum_d r_{id}^0 = 1$.

A.1.2 Proof of Theorem 1

We re-define the system in (15) to set the change in the wage of market m to zero. Consider the matrix $\bar{\boldsymbol{M}}$ obtained by deleting the m-th row from the identity matrix with dimension equal to the number of markets. If $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is nonsingular, then we can write

$$\bar{\boldsymbol{M}}\hat{\boldsymbol{w}} = \left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}},$$

which yields the representation in (21) when we define $\bar{\boldsymbol{\beta}} \equiv \bar{\boldsymbol{M}}'(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}')^{-1}\bar{\boldsymbol{M}}$.

In the rest of the proof, we first show that $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is nonsingular and then establish that $\bar{\boldsymbol{\beta}}$ admits the series representation in (21). To simplify exposition, we abuse notation by defining

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}', \quad \hat{\boldsymbol{w}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{w}} \qquad \text{and} \qquad \hat{\boldsymbol{\eta}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}}.$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\lambda}}$ be the diagonal matrix defined by the vector of $\lambda_i^0 \equiv \kappa + \phi_i$, and $\tilde{\boldsymbol{\gamma}}$ be the matrix with entries $\tilde{\gamma}_{ij} \equiv \rho_{ij}/(\kappa+\phi_i)$, so that

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\boldsymbol{\gamma}}}\right).$$

Consider the vector $\{h_i\}_{i\neq m} \gg 0$ that guarantees the diagonal dominance of $\bar{\boldsymbol{\gamma}}$ in the initial equilibrium. Let $\bar{\boldsymbol{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i. Thus, the system in (15) is equivalent to

$$\begin{aligned}
\bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{I}}-\bar{\tilde{\gamma}}\right)\left(\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^{-1}\right)\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\
\bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{h}}-\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\
\left(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\right)\left(\bar{\boldsymbol{I}}-\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}
\end{aligned}$$

which implies that

$$\hat{\boldsymbol{w}}=\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}}\right)^{-1}\left(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\right)^{-1}\hat{\boldsymbol{\eta}},\quad \bar{\bar{\tilde{\gamma}}}\equiv\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}. \tag{41}$$

Notice that, for all i, $\tilde{\tilde{\gamma}}_{ij}=\tilde{\gamma}_{ij}h_j/h_i=\rho_{ij}h_j/(\kappa+\phi_i)h_i$.

First, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is non-singular, so that we can write the expression in (41). We proceed by contradiction. Suppose that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is singular, so $\mu=0$ is an eigenvalue of $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$. Take the eigenvector $\boldsymbol{x}$ associated with the zero eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})\boldsymbol{x}=0$, so that the i-row of this system is

$$1-\sum_{j\neq i,m}\tilde{\tilde{\gamma}}_{ij}x_j=0 \quad\Longrightarrow\quad 1-\frac{\rho_{ii}}{\kappa+\phi_i}-\sum_{j\neq i,m}\frac{\rho_{ij}}{\kappa+\phi_i}\frac{h_j}{h_i}x_j=0$$

Thus, because $|x_j|\leq 1$ and $h_j>0$ for all j,

$$\left(\kappa+\phi_i-\rho_{ii}\right)h_i=\sum_{j\neq i,m}\rho_{ij}h_jx_j\leq\sum_{j\neq i,m}|\rho_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\rho_{ij}||h_j|,$$

which contradicts (20).

Second, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}$ admits the series representation in (21). This is true whenever the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\boldsymbol{x}$ associated with the largest eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $\mu\boldsymbol{x}=\bar{\bar{\tilde{\gamma}}}\boldsymbol{x}$ so that the i-row of this system is

$$1\leq\mu=\sum_{j\neq m}\frac{\rho_{ij}}{\kappa+\phi_i}\frac{h_j}{h_i}x_j.$$

Since $\kappa+\phi_i$ and h_i are positive, the same steps used above imply that

$$\left(\kappa+\phi_i-\rho_{ii}\right)h_i\leq\sum_{j\neq i,m}|\rho_{ij}|h_j,$$

which contradicts the assumption of diagonal dominance. Thus, the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one, allowing us to write $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}=\sum_{d=0}^{\infty}(\bar{\bar{\tilde{\gamma}}})^d$. Substituting this series expansion into (41) yields

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we now show that $\bar{\boldsymbol{h}}(\bar{\bar{\tilde{\gamma}}})^d\bar{\boldsymbol{h}}^{-1}=(\bar{\tilde{\gamma}})^d$. We proceed by induction. For $d=1$, it is trivial to see that $\bar{\boldsymbol{h}}(\bar{\bar{\tilde{\gamma}}})\bar{\boldsymbol{h}}^{-1}=\bar{\tilde{\gamma}}$. Then,

$$\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^{d+1}\bar{\boldsymbol{h}}^{-1}=\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\left(\bar{\boldsymbol{h}}\bar{\bar{\tilde{\gamma}}}\bar{\boldsymbol{h}}^{-1}\right)=\left(\bar{\tilde{\gamma}}\right)^d\left(\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\right)=\left(\bar{\tilde{\gamma}}\right)^{d+1}.$$

Thus,

$$\hat{\boldsymbol{w}} = \sum_{d=0}^{\infty} (\bar{\tilde{\boldsymbol{\gamma}}})^{d} \bar{\boldsymbol{\lambda}}^{-1} \hat{\boldsymbol{\eta}},$$

which immediately implies the result.

A.1.3 Proof of Corollary 1

The series expansion representation of β_{ij} indicates that $\beta_{ij} > 0$ if $\tilde{\gamma}_{ij} > 0$ for all i and j. We now show that $\tilde{\gamma}_{ij} > 0$ whenever $\max_{o,d} |n_o - n_d|$ is low enough. Since $\sum_d r^0_{id}(\phi^0_i - \phi^0_d) = (\phi - 1)\sum_d r^0_{id}(n^0_d - n^0_i) > -(\phi - 1)\max_{o,d} |n_o - n_d|$,

$$\tilde{\gamma}_{ij} > \frac{r^0_{ij}\phi^0_j + \kappa \sum_s \sum_d \ell^0_{i,s} r^0_{id,s} x^0_{jd,s} - \omega_j(\phi - 1)\max_{o,d} |n_o - n_d|}{\phi^0_i + \kappa}$$

and, thus,

$$\max_{o,d} |n_o - n_d| < \frac{r^0_{ij}\phi^0_j + \kappa \sum_s \sum_d \ell^0_{i,s} r^0_{id,s} x^0_{jd,s}}{\omega_j(\phi - 1)} \Rightarrow \tilde{\gamma}_{ij} > 0.$$

Since the numerator is positive in our model, there exists $\max_{o,d} |n_o - n_d| \geq 0$ such that the condition above holds for all i and j.

A.1.4 Proof of Corollary 2

We establish this result in two steps.

Step 1. First we show that, if $\bar{\boldsymbol{\gamma}} \equiv \lambda^0 \left(\bar{\boldsymbol{I}} - \mathbf{1}\boldsymbol{\rho}'\right)$ where $\mathbf{1}$ is a column vector of ones and $\boldsymbol{\rho} \equiv \{\rho_j\}_{j \neq m}$ is column vector, then $\bar{\boldsymbol{\gamma}}^{-1} = (\lambda^0)^{-1} \left(\bar{\boldsymbol{I}} + \rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'\right)$.

$$\begin{aligned}
\bar{\boldsymbol{\gamma}}^{-1}\bar{\boldsymbol{\gamma}} &= \bar{\boldsymbol{I}} + \rho_m^{-1}\mathbf{1}\boldsymbol{\rho}' - \mathbf{1}\boldsymbol{\rho}' - \rho_m^{-1}\mathbf{1}\boldsymbol{\rho}'\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}} + \rho_m^{-1}\mathbf{1}\boldsymbol{\rho}' - \mathbf{1}\boldsymbol{\rho}' - \left(\rho_m^{-1}\textstyle\sum_{j\neq m}\rho_j\right)\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}} + \left(\rho_m^{-1}(1 - \textstyle\sum_{j\neq m}\rho_j) - 1\right)\mathbf{1}\boldsymbol{\rho}' \\
&= \bar{\boldsymbol{I}}
\end{aligned}$$

where the second equality follows from $\boldsymbol{\rho}'\mathbf{1} = \sum_{j\neq m}\rho_j$, and the fourth from $\sum_j \rho_j = 1$ (since $\sum_j \gamma_{ij} = 0$ for all i).

Step 2. We now establish conditions that allow us to write $\gamma_{ij} = \lambda^0 \left(\mathbb{I}_{i=j} - \rho_j\right)$. Assume that $n^0_i = n^0$, so $\phi^0_i = \phi + (1 - \phi)n^0$ and we can define $\lambda^0 \equiv \kappa + \phi^0$. We now use the fact that $x_{ij,s} = x_{i,s}$ and $\xi_{j,s} = \xi_s$ so that

$$r^0_{ij} = \frac{\sum_s x^0_{ij,s}\xi_{j,s}E^0_j}{\sum_j \sum_s x^0_{ij,s}\xi_{j,s}E^0_j} = \frac{\sum_s x^0_{i,s}\xi_s E^0_j}{\sum_j \sum_s x^0_{i,s}\xi_s E^0_j} = \frac{E^0_j}{\sum_j E^0_j} = e^0_{Wj}.$$

In addition,

$$\kappa \sum_s \sum_d \ell^0_{i,s} r^0_{id,s} x^0_{jd,s} = \kappa \sum_s \sum_d \frac{x^0_{i,s}\xi_s E^0_d}{\sum_j \sum_s x^0_{i,s}\xi_s E^0_j} x^0_{j,s} = \kappa \frac{\sum_s \xi_s x^0_{i,s} x^0_{j,s}}{\sum_s \xi_s x^0_{i,s}} = \kappa\chi_j$$

Since $n^0_i = n^0$, then $\sum_d r^0_{id}(\phi^0_i - \phi^0_d) = 0$. This implies that $\rho^0_{ij} = e^0_{Wj}\phi^0 + \kappa\chi_j$, which we can use to define $\rho_j \equiv (e^0_{Wj}\phi^0_j + \kappa\chi_j)/(\kappa + \phi^0)$ and $\beta_j \equiv \mathbb{I}_{j\neq m}\rho_j/\rho_m$.

A.1.5 Proof of Corollary 3

By Shepard's lemma, the price index expression in (7) implies that

$$\hat{P}_i = \sum_{s,o} \xi_{i,s} x^0_{oi,s} \left(\hat{\tau}_{oi,s} + \hat{p}_o\right) = \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) + \sum_o x^0_{oi} \hat{p}_o$$

where $x^0_{oi} \equiv \sum_s \xi_{i,s} x^0_{oi,s}$ is the share of o in the total spending of i. Thus,

$$\begin{array}{rcl} \hat{P}_i & = & \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) + \sum_o x^0_{oi} \left((1-\psi\phi)\hat{w}_o + \psi\phi\hat{\Omega}\right) \\ & = & \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) + \sum_o \left(x^0_{oi}(1-\psi\phi) + \psi\phi\omega^0_o\right)\hat{w}_o, \\ & = & \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) + \sum_o \left(x^0_{oi}\frac{\kappa}{\sigma-1} + \left(1 - \frac{\kappa}{\sigma-1}\right)\omega^0_o\right)\hat{w}_o \end{array}$$

where the first expression follows from $\hat{p}_o$ in (10), the second from $\hat{b}_i = \hat{\Omega} = \sum_o \omega^0_o \hat{w}_o$, and the last from the definition of $\kappa = (\sigma-1)(1-\psi\phi)$. The combination of this expression and expression for $\hat{w}_o$ in (21) immediately implies (23).

A.1.6 Proof of Equation (26)

The labor supply equation in (8) with $\hat{b}_i = \sum_d \omega^0_d \hat{w}_d$ implies that $\hat{n}_i = \phi(1-n^0_i)(\hat{w}_i - \sum_d \omega^0_d \hat{w}_d)$. Using (21), we get that

$$\hat{n}_i = \phi(1-n^0_i)\sum_j \left(\beta_{ij} - \sum_d \omega^0_d \beta_{dj}\right)\hat{\eta}_j, \tag{42}$$

which immediately implies the second expression in (26) when combined with (25).

The combination of (9), (21), and (42) implies that

$$\Delta\overline{\ln w_i} = \sum_j \beta_{ij}\hat{\eta}_j - (1-n^0_i)\sum_j \left(\beta_{ij} - \sum_d \omega^0_d \beta_{dj}\right)\hat{\eta}_j,$$

and, thus,

$$\Delta\overline{\ln w_i} = \sum_j \left[n^0_i \beta_{ij} + (1-n^0_i)\sum_d \omega^0_d \beta_{dj}\right]\hat{\eta}_j.$$

This immediately implies the first expression in (26) when combined with (25).

A.1.7 Proof of Equation (28)

To establish condition (28) first notice that, by definition,

$$\begin{array}{rcl} \nu^w_i & = (1-\sigma) & \sum_j \beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\sum_s \ell^0_{j,s}\left(\sum_d r^0_{jd,s}\hat{\tau}^{\text{unbs}}_{jd,s} - \sum_o \sum_d r^0_{jd,s} x^0_{od,s}\hat{\tau}^{\text{unbs}}_{od,s}\right) - \alpha^w \\ & = & \sum_{s,d,o}\left[\sum_j (1-\sigma)\beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\ell^0_{j,s} r^0_{jd,s}\left(\mathbb{I}_{o=j} - x^0_{od,s}\right)\right]\hat{\tau}^{\text{unbs}}_{od,s} - \alpha^w. \end{array}$$

Using the definition $Z_j \equiv \sum_s \ell^0_{j,s}(\hat{z}^{\text{obs}}_{j,s} - \bar{z}^{\text{obs}}_{j,s})$, we have that

$$\begin{array}{rcl} Z_j & = (1-\sigma) & \sum_s \ell^0_{j,s}\left(\sum_d r^0_{jd,s}\left(\hat{\tau}^{\text{obs}}_{jd,s} - \bar{\tau}^{\text{obs}}\right) - \sum_o \sum_d r^0_{jd,s} x^0_{od,s}\left(\hat{\tau}^{\text{obs}}_{od,s} - \bar{\tau}^{\text{obs}}\right)\right) \\ & = & \sum_{s,d,o}\left[(1-\sigma)\ell^0_{j,s} r^0_{jd,s}\left(\mathbb{I}_{o=j} - x^0_{od,s}\right)\right]\left(\hat{\tau}^{\text{obs}}_{od,s} - \bar{\tau}^{\text{obs}}\right). \end{array}$$

For arbitrary i and j,

$$E\left[\nu_i^w Z_j|\boldsymbol{\mathcal{W}}^0\right] = E\left[\left(\sum_{s,d,o}\left[\sum_h \beta_{ih}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\ell_{h,s}^0 r_{hd,s}^0\left(\mathbb{I}_{o=h} - x_{od,s}^0\right)\right]\hat{\tau}_{od,s}^{\text{unbs}} - \alpha^w\right)\left(\sum_{s',d',o'}\ell_{j,s'}^0 r_{jd',s'}^0\left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0\right)\left(\hat{\tau}_{o'd',s'}^{\text{obs}} - \bar{\tau}^{\text{obs}}\right)\right)|\boldsymbol{\mathcal{W}}^0\right]$$

and, thus,

$$\begin{aligned} E\left[\nu_i^w Z_j|\boldsymbol{\mathcal{W}}^0\right] &= \textstyle\sum_{s,d,o,s',d',o'}\left(\sum_h \beta_{ih}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\ell_{h,s}^0 r_{hd,s}^0\left(\mathbb{I}_{o=h} - x_{od,s}^0\right)\right)\ell_{j,s'}^0 r_{jd',s'}^0\left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0\right)E\left[\hat{\tau}_{od,s}^{\text{unbs}}\left(\hat{\tau}_{o'd',s'}^{\text{obs}} - \bar{\tau}^{\text{obs}}\right)|\boldsymbol{\mathcal{W}}^0\right] \\ &- \quad \alpha^w\left(\textstyle\sum_{s',d',o'}\ell_{j,s'}^0 r_{jd',s'}^0\left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0\right)\right)E\left[\left(\hat{\tau}_{o'd',s'}^{\text{obs}} - \bar{\tau}^{\text{obs}}\right)|\boldsymbol{\mathcal{W}}^0\right]. \end{aligned}$$

Since $E\left[\left(\hat{\tau}_{o'd',s'}^{\text{obs}} - \bar{\tau}^{\text{obs}}\right)|\boldsymbol{\mathcal{W}}^0\right] = 0$,

$$E\left[\nu_i^w Z_j|\boldsymbol{\mathcal{W}}^0\right] = \textstyle\sum_{s,d,o}\sum_{s',d',o'}\left(\sum_h \beta_{ih}^w(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\ell_{h,s}^0 r_{hd,s}^0\left(\mathbb{I}_{o=h} - x_{od,s}^0\right)\right)\ell_{j,s'}^0 r_{jd',s'}^0\left(\mathbb{I}_{o'=j} - x_{o'd',s'}^0\right)Cov\left(\hat{\tau}_{od,s}^{\text{unbs}}, \hat{\tau}_{o'd',s'}^{\text{obs}}|\boldsymbol{\mathcal{W}}^0\right).$$

Thus, by the assumption in (27), $E\left[\nu_i^w Z_j|\boldsymbol{\mathcal{W}}^0\right] = 0$ and, therefore, $E\left[\nu_i^w Z_j\right] = E\left[E\left[\nu_i^w Z_j|\boldsymbol{\mathcal{W}}^0\right]\right] = 0$. This immediately establishes that, for any real matrix h_{ij}^w, $E\left[\nu_i^w \sum_j h_{ij}^w Z_j\right] = \sum_j h_{ij}^w E\left[\nu_i^w Z_j\right] = 0$. We can follow the same steps to show that $E\left[\nu_i^n \sum_j h_{ij}^n Z_j\right] = 0$.

A.1.8 Model-Implied Optimal Moment Condition

To simplify exposition without loss of generality, we assume that all variables are demeaned, so that (26) and (28) can be written in the following vector form:

$$v_i(\boldsymbol{\theta}) = Y_i - (\beta_i(\boldsymbol{\theta}|\boldsymbol{W}))'\boldsymbol{Z} \quad \text{such that} \quad E\left[v_i(\boldsymbol{\theta})\sum_j h_{ij}Z_j\right] = 0. \tag{43}$$

Define $\boldsymbol{H}_i = \left[\left(\boldsymbol{h}_i^k\boldsymbol{Z}\right)'\right]_{k=1}^{dim(\boldsymbol{\theta})}$ where $\boldsymbol{h}_i^k\boldsymbol{Z}$ has dimension $dim(v_i)\times 1$. Thus, for any $\boldsymbol{h}_i^k$, the condition above is equivalent to

$$E[\boldsymbol{H}_i v_i(\boldsymbol{\theta})] = 0,$$

which yields the following class of GMM estimators of $\boldsymbol{\theta}$,

$$\hat{\boldsymbol{\theta}}_H \equiv \arg min_{\boldsymbol{\theta}}\left[\sum_i \boldsymbol{H}_i v_i(\boldsymbol{\theta})\right]'\left[\sum_i \boldsymbol{H}_i v_i(\boldsymbol{\theta})\right].$$

Optimal Moment Conditions with Independent Residuals. We follow Chamberlain (1987) to derive the optimal moment conditions under the assumption that v_i is independent across (clusters of) markets. In this case, the usual optimal IV formula in Chamberlain (1987) holds, so that the optimal moment condition is

$$H_i^* \equiv \left(E\left[v_i(\boldsymbol{\theta})v_i(\boldsymbol{\theta})'|\boldsymbol{Z}\right]\right)^{-1}\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$$

where, given (43),

$$\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta}) = -\nabla_{\boldsymbol{\theta}}\beta_i(\boldsymbol{\theta}|\boldsymbol{W})\boldsymbol{Z}.$$

The term $(E\left[v_i(\boldsymbol{\theta})v_i(\boldsymbol{\theta})'|\boldsymbol{Z}\right])^{-1}$ adjusts the weight of each observation to the inverse of the variance of its residuals. It is the usual adjustment that arises in generalized least squares under heteroskedasticity. This term is irrelevant under homoskedasticity. For each observation, $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ attributes a higher weight to the exposure of the markets whose bilateral reduced-form elasticities are more sensitive to changes in each parameter.

Optimal Moment Conditions with Correlated Residuals. We also derive the optimal moment conditions using the results in Borusyak and Hull (2020) that allow for arbitrary cross-market correlation in v_i but assume that the observed shocks are independent from each other. In this case, Borusyak and Hull (2020) show that

$$\boldsymbol{H}^* \equiv \left(E\left[\boldsymbol{v}(\boldsymbol{\theta})\boldsymbol{v}(\boldsymbol{\theta})'|\boldsymbol{\mathcal{W}}\right]\right)^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{v}(\boldsymbol{\theta}).$$

We then apply this formula to our reduced-form representation of the predictions of general equilibrium spatial models: using (43),

$$\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta}) = -\nabla_{\boldsymbol{\theta}}\beta_i(\boldsymbol{\theta}|\boldsymbol{W})\boldsymbol{Z}.$$

This formula is similar to the one above. The term $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ is identical in the two formulas. The only difference is the first term, which now accounts for the potential covariance between the residuals of different markets. In our model, such a correlation may arise if markets are exposed to similar unobserved shocks in economic fundamentals – a point recently raised by Adão et al. (2019) in the context of shift-share specifications. To see this, assume that $\hat{\tau}^{\text{unbs}}_{od,s}$ are independently drawn from an arbitrary distribution with mean zero and variance of σ^2_τ. As shown in Appendix A.1.7, the residual can be written in a general form, $v_i(\boldsymbol{\theta}) = \sum_{s,d,o}\boldsymbol{\beta}^v_{i,ods}(\boldsymbol{\theta}|\boldsymbol{W})\hat{\tau}^{\text{unbs}}_{od,s}$ where $\beta^v_{i,ods} = \sum_j[\beta^w_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}), \beta^n_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}})]'\ell_{j,s}y_{jd,s}(\mathbb{I}_{o=j} - x_{od,s})$. Thus,

$$E\left[v_i(\boldsymbol{\theta})v_j(\boldsymbol{\theta})'|\boldsymbol{\mathcal{W}}\right] = \sigma^2_\tau\sum_{s,d,o}\left(\boldsymbol{\beta}^v_{i,ods}(\boldsymbol{\theta}|\boldsymbol{W})\right)\left(\boldsymbol{\beta}^v_{j,ods}(\boldsymbol{\theta}|\boldsymbol{W})\right)'.$$

This expression shows that the correlation between the residuals of markets i and j is higher if they have higher exposure to the same productivity shocks. This is the case if the two markets are similar in terms of employment shares across sectors and/or within-sector revenue shares across destinations.

Implementation Comments. We conclude with two comments on implementation. First, while it is trivial to compute $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ because of our reduced-form characterization in (43), it is much harder to compute the variance adjustment term as it requires knowledge of the unobserved residuals. For this reason, it is common to ignore this adjustment term in practice by constructing moment conditions with $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$. This yields a consistent estimator of $\boldsymbol{\theta}$, but it is possible that implementing the variance correction term could further improve the estimator's efficiency. Second, $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta})$ must be evaluated at the true value of $\boldsymbol{\theta}$. To simplify the estimator's implementation, one can adopt an asymptotically equivalent two-step GMM estimator of $\boldsymbol{\theta}$ where, in the first-stage, we obtain a consistent estimator $\hat{\boldsymbol{\theta}}_1$ using $\nabla_{\boldsymbol{\theta}}v_i(\boldsymbol{\theta}_0)$ computed with an arbitrary guess $\boldsymbol{\theta}_0$ and, in the second-stage, we estimate $\hat{\boldsymbol{\theta}}_2$ using $\nabla_{\boldsymbol{\theta}}v_i(\hat{\boldsymbol{\theta}}_1)$ computed with the first-stage estimate $\hat{\boldsymbol{\theta}}_1$.

A.1.9 Reduced-form Responses in Sectoral Employment Outcomes

To derive the change in sectoral employment, recall that, in our model, the share of employment in sector s is equal to the share of revenue in that sector, so that $\hat{\ell}_{i,s} = \hat{R}_{i,s} - \sum_k \ell^0_{i,k}\hat{R}_{i,k}$. The definitions $R_{i,s} \equiv \sum_j x_{ij,s}\xi_{j,s}E_j$, $r_{ij,s} \equiv x_{ij,s}\xi_{j,s}E_j/R_{i,s}$ and $\mu_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_j r^0_{ij,s}(\hat{\tau}_{ij,s} - \sum_o x^0_{oj,s}\hat{\tau}_{oj,s})$ imply

$$\hat{R}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) + (1-\sigma)\hat{p}_i + (\sigma-1)\sum_o\left(\sum_j r^0_{ij,s}x^0_{oj,s}\right)\hat{p}_o + \sum_j r^0_{ij,s}\hat{E}_j.$$

Thus, since $\hat{\eta}_i(\hat{\boldsymbol{\tau}}) \equiv (1-\sigma)\sum_k \ell_{i,k}\mu_{i,k}(\hat{\boldsymbol{\tau}})$,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + (\sigma-1)\sum_o \left(\sum_j r^0_{ij,s}x^0_{oj,s} - \sum_k \ell^0_{i,k}\sum_j r^0_{ij,k}x^0_{oj,k}\right)\hat{p}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right)\hat{E}_j.$$

Using the fact that $\hat{p}_o \equiv (1-\psi\phi)\hat{w}_o + \psi\phi\hat{\Omega}$, this expression is equivalent to

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \kappa\sum_o \left(\sum_j r^0_{ij,s}x^0_{oj,s} - \sum_k \ell^0_{i,k}\sum_j r^0_{ij,k}x^0_{oj,k}\right)\hat{w}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right)\hat{E}_j$$

and, therefore,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \kappa\sum_o \left(\chi^0_{io,s} - \chi^0_{io}\right)\hat{w}_o + \sum_j \left(r^0_{ij,s} - r^0_{ij}\right)\hat{E}_j$$

where $\chi^0_{io,s} \equiv \sum_d r^0_{id,s}x^0_{od,s}$ and $\chi^0_{io} \equiv \sum_s \ell^0_{i,s}\chi^0_{io,s}$.

Recall that $E_j = W_j = w_i(n_i)^{\frac{\phi-1}{\phi}}N_i\varrho$ and $\hat{E}_j = \phi^0_j\hat{w}_j + (1-\phi^0_j)\hat{\Omega}$. Thus,

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \sum_j \left[\left(\chi^0_{ij,s} - \chi^0_{ij}\right)\kappa + \left(r^0_{ij,s} - r^0_{ij}\right)\phi^0_j\right]\hat{w}_j + \rho^0_{i,s}\hat{\Omega}$$

where $\rho^0_{i,s} \equiv -\sum_d (r^0_{id,s} - r^0_{id})\phi^0_d$. Since $\hat{\Omega} = \sum_j \omega^0_j\hat{w}_j$, this expression becomes

$$\hat{\ell}_{i,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) - \hat{\eta}_i(\hat{\boldsymbol{\tau}}) + \sum_j \pi^0_{ij,s}\hat{w}_j \tag{44}$$

where $\pi^0_{ij,s} \equiv \left(\chi^0_{ij,s} - \chi^0_{ij}\right)\kappa + \left(r^0_{ij,s} - r^0_{ij}\right)\phi^0_j + \rho^0_{i,s}\omega^0_j$. Notice that, up to a first-order approximation, the change in the share of employment in sector s is $\Delta\ell_{i,s} = \ell^0_{i,s}\hat{\ell}_{i,s}$ and share of population employed in sector s is $\Delta n_{i,s} = \ell^0_{i,s}n^0_i(\hat{\ell}_{i,s} + \hat{n}_i)$.

A.2 Proofs for Section 4

A.2.1 Proof of Excess Labor Demand in (33)

Step 1. We now implicitly characterize $P_{j,s}(\boldsymbol{w})$ from the combination of $P_{j,s}$ in (7), $p_{i,s}$ in (31), n_i in (8), N_i in (30) with $b_i = \bar{b}_i(P_i)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}$. Thus, any $\{P_{j,s}\}_{j,s} \in \{P_{j,s}(\boldsymbol{w})\}_{j,s}$ solves

$$(P_{j,s})^{1-\sigma} = \sum_i (\tau_{ij,s})^{1-\sigma}\left(\frac{w_i}{\bar{b}_i\left(\Pi_k(P_{i,k})^{\xi_{i,k}}\right)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}}\right)^{\phi\psi(\sigma-1)}\left[(w_i)^{a^L_{i,s}}\left(\Pi_k(P_{i,k})^{\xi^M_{i,ks}}\right)^{a^M_{i,s}}\right]^{1-\sigma}.$$

Step 2. For any $\boldsymbol{w}$, take $\{P_{j,s}\}_{j,s} \in \{P_{j,s}(\boldsymbol{w})\}_{j,s}$ to compute $P_i(\boldsymbol{w}|\boldsymbol{\tau}) = \Pi_k(P_{i,k})^{\xi_{i,k}}$, and $P^M_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}) = \Pi_k(P_{i,k})^{\xi^M_{i,ks}}$. We then obtain $p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau})$ from (31), $n_i(\boldsymbol{w}|\boldsymbol{\tau})$ from (8), $N_i(\boldsymbol{w}|\boldsymbol{\tau})$ from (30), and

$$E_j(\boldsymbol{w}|\boldsymbol{\tau}) = W_j(\boldsymbol{w}|\boldsymbol{\tau}) = w_i^\phi\left(w_i^\phi + \left(\bar{b}_i\left(\Pi_k(P_{i,k})^{\xi_{i,k}}\right)^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}\right)^\phi\right)^{\frac{1-\phi}{\phi}}N_i(\boldsymbol{w}|\boldsymbol{\tau})\varrho.$$

Step 3. To define the revenue function, we solve for

$$R_{i,s} - \sum_k \sum_j \frac{(\tau_{ij,s} p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi^M_{j,sk} a^M_{j,k} R_{j,k} = \sum_j \frac{(\tau_{ij,s} p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,s} E_j(\boldsymbol{w}|\boldsymbol{\tau})$$

This system has a unique solution because, for every (j, k),

$$\sum_s \sum_i \frac{(\tau_{ij,s} p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi^M_{j,sk} a^M_{j,k} = \sum_s \xi^M_{j,sk} a^M_{j,k} = a^M_{j,k} < 1,$$

so that

$$[R_{i,s}(\boldsymbol{w}|\boldsymbol{\tau})]_{i,s} = \sum_{d=0}^{\infty} \left(\bar{\boldsymbol{A}}(\boldsymbol{w}|\boldsymbol{\tau})\right)^d \left[\sum_j \frac{(\tau_{ij,s} p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi_{j,s} E_j(\boldsymbol{w}|\boldsymbol{\tau})\right]_{i,s}$$

where

$$\bar{\boldsymbol{A}}(\boldsymbol{w}|\boldsymbol{\tau}) \equiv \left[\frac{(\tau_{ij,s} p_{i,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}}{\sum_o (\tau_{oj,s} p_{o,s}(\boldsymbol{w}|\boldsymbol{\tau}))^{1-\sigma}} \xi^M_{j,sk} a^M_{j,k}\right]_{is,jk}.$$

A.2.2 Proof of Equation (34)

In all the remaining proofs of this section, we simplify notation by omitting the superscript 0. The system in (32) implicitly defines $\{R_{i,s}\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{p_{i,s}\}_{i,s}$. We can then use the implicit function theorem to write

$$\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} = \mathbb{I}_{s=k} \frac{X_{id,s}}{R_{i,s}} (1-\sigma) \left(\mathbb{I}_{i=o} - x_{od,s}\right) + \sum_{k'} \sum_j \frac{x_{ij,s} \xi^M_{j,sk'} a^M_{j,k'} R_{j,k'}}{R_{i,k}} \frac{\partial \ln R_{j,k'}}{\partial \ln \tau_{od,s}}$$

which implies that

$$\left[\frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}}\right]_{ik} = (1-\sigma) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^U\right) \left[\mathbb{I}_{s=k} \frac{X_{id,s}}{R_{i,s}} \left(\mathbb{I}_{i=o} - x_{od,s}\right)\right]_{ik}$$

where we have used the fact that $r^U_{is,jk} \equiv \frac{X_{ij,sk}}{R_{i,s}}$ and the definition of $\bar{\boldsymbol{b}}^U$ from (35).

Thus,

$$\begin{aligned}
\left[\sum_{s,o,d} \frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s}\right]_{ik} &= (1-\sigma) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^U\right) \left[\sum_{o,d} \frac{X_{id,k}}{R_{i,k}} \left(\mathbb{I}_{i=o} - x_{od,k}\right) \hat{\tau}_{od,k}\right]_{ik} \\
&= (1-\sigma) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^U\right) [\mu_{i,k}(\hat{\boldsymbol{\tau}})]_{ik} \\
&= (1-\sigma) \left[\mu_{i,k}(\hat{\boldsymbol{\tau}}) + \sum_{j,k'} b^U_{ik,jk'} \mu_{j,k'}(\hat{\boldsymbol{\tau}})\right]_{ik} \\
&= (1-\sigma) \left[\mu_{i,k}(\hat{\boldsymbol{\tau}}) + \mu^U_{i,k}(\hat{\boldsymbol{\tau}})\right]_{ik}
\end{aligned}$$

This immediately implies (34) because $\hat{\eta}^R_i(\hat{\boldsymbol{\tau}}) \equiv \sum_{s,o,d} \frac{\partial \ln R_i}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} = \sum_k \ell_{i,k} \sum_{s,o,d} \frac{\partial \ln R_{i,k}}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s}$.

A.2.3 Proof of Equation (36)

Equations (7) and (31) define $\{P^M_{i,s}\}_{i,s}$ as a function of $\boldsymbol{\tau}$ for any given $\{w_i, L_i\}_i$ from the solution of

$$P^M_{i,s} = \Pi_k \left(\sum_o (\tau_{oi,k})^{1-\sigma} \left(\frac{w_o}{b_o}\right)^{\phi\psi(\sigma-1)} w_o^{(1-\sigma)a^L_{o,s}} \left(P^M_{o,k}\right)^{(1-\sigma)a^M_{o,s}} \right)^{\frac{\xi^M_{i,ks}}{1-\sigma}}.$$

Using the implicit function theorem, we have that

$$\frac{\partial \ln P^M_{i,s}}{\partial \ln \tau_{od,k}} = \mathbb{I}_{i=d}\xi^M_{i,ks}x_{oi,k} + \sum_{k'}\sum_j \xi^M_{i,k's}x_{ji,k'}a^M_{j,k'}\frac{\partial \ln P^M_{j,k'}}{\partial \ln \tau_{od,k}}$$

which, by defining $\bar{\boldsymbol{g}}^D \equiv (\bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^D)$ implies that

$$\left[\frac{\partial \ln P^M_{i,s}}{\partial \ln \tau_{od,k}}\right]_{is} = \bar{\boldsymbol{g}}^D \left[\mathbb{I}_{i=d}\xi^M_{i,ks}x_{oi,k}\right]_{is}.$$

Thus, $[\hat{\eta}^M_{i,s}(\hat{\boldsymbol{\tau}})]_{i,s} = \bar{\boldsymbol{g}}^D[\mu^M_{i,s}(\hat{\boldsymbol{\tau}})]_{is} = \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k} b^D_{is,jk}\mu^M_{j,k}(\hat{\boldsymbol{\tau}})$.

A.2.4 Proof of Equations (37)–(38)

There are four parts to the derivation of equations (37)–(38). The first part is to characterize the matrix of spatial links, $\bar{\boldsymbol{\gamma}}$, and the vector of shifts in excess labor demand, $\hat{\boldsymbol{\eta}}$ (as in Sections 3.1.1 and 3.1.2). The second part is to characterize the reduced-form response of wages to shifts in excess labor demand (as in Section 3.1.3). The third is the derivation of the reduced-form responses in labor market outcomes to shocks in bilateral productivity. The last part is the derivation of (37) for observed and unobserved shocks, and the associated moment conditions in (38) (as in Section 3.2).

Part A: Derivation of the matrix of spatial links, $\bar{\boldsymbol{\gamma}}$, and the vector of shifts in excess labor demand, $\hat{\boldsymbol{\eta}}$. We first establish that, by totally differentiating the equilibrium conditions, we can write them as

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}. \tag{45}$$

Step 1. We first derive changes in labor market outcomes as a function of $\hat{\boldsymbol{w}}$ and $\hat{\boldsymbol{P}}$. Expression (8) with $b_i = \bar{b}_i P_i^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}$ implies that $\hat{n}_i = \phi(1-n_i)(\hat{w}_i - \lambda\hat{P}_i - (1-\lambda)\sum_j \omega_j\hat{w}_j)$. So,

$$\hat{\boldsymbol{n}} = \phi\bar{\boldsymbol{\phi}}^{n,w}\hat{\boldsymbol{w}} + \phi\lambda\bar{\boldsymbol{\phi}}^{n,P}\hat{\boldsymbol{P}} \tag{46}$$

such that $\phi^{n,w}_{ij} \equiv (1-n_i)\left(\mathbb{I}_{i=j} - (1-\lambda)\omega_j\right)$ and $\phi^{n,p}_{ij} \equiv -(1-n_i)\mathbb{I}_{i=j}$.

Expression (30) with $b_i = \bar{b}_i P_i^\lambda(\Omega(\boldsymbol{w}))^{1-\lambda}$ implies

$$\begin{aligned}\hat{N}_i &= \vartheta \textstyle\sum_j \left(\mathbb{I}_{i=j} - \frac{N_j}{N_{c(i)}}\mathbb{I}_{c(i)=c(j)}\right)\left(\hat{w}_j - \hat{P}_j + (\phi-1)(1-n_j)\left(\hat{w}_j - \lambda\hat{P}_j\right)\right) \\ &+ (\phi-1)(1-\lambda)\left(n_i - \textstyle\sum_{j\in c(i)}\frac{N_j}{N_{c(j)}}n_j\right)\textstyle\sum_d \omega_d\hat{w}_d.\end{aligned}$$

Thus,

$$\hat{\boldsymbol{N}} = \vartheta\bar{\boldsymbol{\phi}}^{N,w}\hat{\boldsymbol{w}} + \vartheta\bar{\boldsymbol{\phi}}^{N,P}\hat{\boldsymbol{P}}, \tag{47}$$

where $\tilde{\boldsymbol{N}} \equiv \left[\mathbb{I}_{i=j} - \mathbb{I}_{c(i)=c(j)}N_j/N_{c(j)}\right]_{i,j}$, $\bar{\boldsymbol{n}} \equiv [n_i\mathbb{I}_{i=j}]_{i,j}$,

$$\bar{\boldsymbol{\phi}}^{N,w} \equiv \tilde{\boldsymbol{N}}\left(\bar{\boldsymbol{I}} + (\phi-1)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{n}}\right)\right) + (\phi-1)(1-\lambda)\tilde{\boldsymbol{N}}\boldsymbol{n}\boldsymbol{\omega}', \quad \text{and} \quad \bar{\boldsymbol{\phi}}^{N,P} \equiv -\tilde{\boldsymbol{N}}\left(\bar{\boldsymbol{I}} + \lambda(\phi-1)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{n}}\right)\right).$$

Expressions (46) and (47) imply that $\hat{W}_i = \hat{w}_i + \hat{n}_i(\phi-1)/\phi + \hat{N}_i$ is given by

$$\hat{\boldsymbol{E}} = \hat{\boldsymbol{W}} = \bar{\boldsymbol{\phi}}^{W,w}\hat{\boldsymbol{w}} + \left((\phi-1)\lambda\bar{\boldsymbol{\phi}}^{n,P} + \vartheta\bar{\boldsymbol{\phi}}^{N,P}\right)\hat{\boldsymbol{P}}, \tag{48}$$

where $\bar{\boldsymbol{\phi}}^{W,w} \equiv \bar{\boldsymbol{I}} + (\phi-1)\bar{\boldsymbol{\phi}}^{n,w} + \vartheta\bar{\boldsymbol{\phi}}^{N,w}$.

Step 2. We now derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (31),

$$\hat{P}^M_{i,s} - \sum_{j,k}\xi^M_{i,ks}x_{ji,k}a^M_{j,k}\hat{P}^M_{j,k} = \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k}\xi^M_{i,ks}x_{ji,k}\left(a^L_{j,k}\hat{w}_j - \frac{\psi}{1-n_j}\hat{n}_j\right)$$

which, by defining $\bar{\boldsymbol{x}}^D_{NS\times NS} \equiv [a^M_{j,k}\xi^M_{i,ks}x_{ji,k}]_{is,jk}$, $\bar{\boldsymbol{x}}^M_{NS\times NS} \equiv [\xi^M_{i,ks}x_{ji,k}]_{is,jk}$, $\bar{\boldsymbol{a}}^L_{NS\times N} \equiv [a^L_{i,k}\mathbb{I}_{i=j}]_{ik,j}$, $\bar{\boldsymbol{v}}^n_{NS\times N} = [\mathbb{I}_{i=j}(1-n_i)^{-1}]_{ik,j}$, implies

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D\right)\hat{\boldsymbol{P}}^M = \boldsymbol{\mu}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \psi\bar{v}^n\hat{\boldsymbol{n}}\right)$$

and, therefore,

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \psi\bar{v}^n\hat{\boldsymbol{n}}\right). \tag{49}$$

From Shepard's lemma, $\hat{P}_i = \sum_s \xi_{i,s}\sum_j x_{ji,s}(\hat{\tau}_{ji,s} + \hat{p}_{j,s})$, which implies that

$$\hat{P}_i = \sum_s \xi_{i,s}\sum_j x_{ji,s}\left(\hat{\tau}_{ji,s} + a^L_{j,s}\hat{w}_j + a^M_{j,s}\hat{P}^M_{j,s} - \frac{\psi}{1-n_j}\hat{n}_j\right)$$

which, by defining $\bar{\boldsymbol{x}}^C_{N\times NS} \equiv [\xi_{i,s}x_{ji,s}]_{i,js}$, $\bar{\boldsymbol{a}}^M_{NS\times NS} \equiv [a^M_{j,k}\mathbb{I}_{is=jk}]_{is,jk}$,

$$\hat{\boldsymbol{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \psi\bar{v}^n\hat{\boldsymbol{n}}\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M.$$

Substituting (49) into this expression,

$$\hat{\boldsymbol{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \psi\bar{v}^n\hat{\boldsymbol{n}}\right),$$

where $\bar{\boldsymbol{c}}^M \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{a}}^M\bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M$.

By plugging (46) into this expression,

$$\begin{aligned}\hat{\boldsymbol{P}} &= \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} \\ &- \psi\bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\bar{v}^n\left(\phi\bar{\boldsymbol{\phi}}^{n,w}\hat{\boldsymbol{w}} + \phi\lambda\bar{\boldsymbol{\phi}}^{n,P}\hat{\boldsymbol{P}}\right)\end{aligned}$$

and, therefore,

$$\hat{\boldsymbol{P}} = \bar{\boldsymbol{\alpha}}^{P,w}\hat{\boldsymbol{w}} + \boldsymbol{\rho}^P\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right), \tag{50}$$

with $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \boldsymbol{\rho}^P\bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L - \phi\psi\bar{v}^n\bar{\boldsymbol{\phi}}^{n,w}\right)$ and $\boldsymbol{\rho}^P \equiv \left(\bar{\boldsymbol{I}} + \phi\psi\lambda\bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\bar{v}^n\bar{\boldsymbol{\phi}}^{n,P}\right)^{-1}$.

Substituting $\hat{\boldsymbol{P}}$ in (50) into (46), we have that

$$\hat{\boldsymbol{n}} = \phi \bar{\boldsymbol{\alpha}}^{n,w} \hat{\boldsymbol{w}} + \phi\lambda \bar{\boldsymbol{\alpha}}^{n,P} \left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^{C} \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \right) \tag{51}$$

with $\bar{\boldsymbol{\alpha}}^{n,w} \equiv \bar{\boldsymbol{\phi}}^{n,w} + \lambda \bar{\boldsymbol{\phi}}^{n,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{n,P} \equiv \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^{P}$.

Substituting $\hat{\boldsymbol{P}}$ in (50) into (47), we have that

$$\hat{\boldsymbol{N}} = \vartheta \bar{\boldsymbol{\alpha}}^{N,w} \hat{\boldsymbol{w}} + \vartheta \bar{\boldsymbol{\alpha}}^{N,P} \left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^{C} \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \right) \tag{52}$$

with $\bar{\boldsymbol{\alpha}}^{N,w} \equiv \bar{\boldsymbol{\phi}}^{N,w} + \bar{\boldsymbol{\phi}}^{N,P} \bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{N,P} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^{P}$.

Substituting $\hat{\boldsymbol{P}}$ in (50) into (48), we have that

$$\hat{\boldsymbol{W}} = \bar{\boldsymbol{\alpha}}^{W,w} \hat{\boldsymbol{w}} + \left(\lambda \bar{\boldsymbol{\alpha}}^{W,P}_{\lambda} + \vartheta \bar{\boldsymbol{\alpha}}^{W,P}_{\vartheta} \right) \left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^{C} \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \right) \tag{53}$$

with $\bar{\boldsymbol{\alpha}}^{W,w} \equiv \bar{\boldsymbol{\phi}}^{W,w} + \left((\phi-1)\lambda \bar{\boldsymbol{\phi}}^{n,P} + \vartheta \bar{\boldsymbol{\phi}}^{N,P} \right) \bar{\boldsymbol{\alpha}}^{P,w}$, $\bar{\boldsymbol{\alpha}}^{W,P}_{\lambda} \equiv (\phi-1) \bar{\boldsymbol{\phi}}^{n,P} \boldsymbol{\rho}^{P}$, and $\bar{\boldsymbol{\alpha}}^{W,P}_{\vartheta} \equiv \bar{\boldsymbol{\phi}}^{N,P} \boldsymbol{\rho}^{P}$.

Finally, we can solve for the change in the input price index using (49):

$$\hat{\boldsymbol{P}}^{M} = \bar{\boldsymbol{\alpha}}^{M,w} \hat{\boldsymbol{w}} + \hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) - \psi\phi\lambda \bar{\boldsymbol{\alpha}}^{M,P}_{\lambda} \left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^{C} \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \right) \tag{54}$$

where $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\boldsymbol{g}}^{D} \bar{\boldsymbol{x}}^{M} \left(\bar{\boldsymbol{a}}^{L} - \phi\psi \bar{\upsilon}^{n} \bar{\boldsymbol{\alpha}}^{n,w} \right)$, and $\bar{\boldsymbol{\alpha}}^{M,P}_{\lambda} \equiv \bar{\boldsymbol{g}}^{D} \bar{\boldsymbol{x}}^{M} \bar{\upsilon}^{n} \bar{\boldsymbol{\alpha}}^{n,P}$.

Step 3. We now solve for the change in revenue of sector-market pairs. From (32), by defining $\bar{\boldsymbol{r}}^{C}_{NS\times N} \equiv [x_{ij,s}\xi_{j,s}E_j/R_{i,s}]_{is,j}$,

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^{U} \right) \hat{\boldsymbol{R}} = \left[\sum_{d} r_{id,s} \hat{x}_{id,s} \right]_{is} + \bar{\boldsymbol{r}}^{C} \hat{\boldsymbol{W}},$$

where, from (6),

$$\sum_{d} r_{id,s} \hat{x}_{id,s} = (1-\sigma)\mu_{i,s}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \sum_{d} r_{id,s} \left(\mathbb{I}_{i=j} - x_{jd,s} \right) \left(a^{L}_{j,s} \hat{w}_j + a^{M}_{j,s} \hat{P}^{M}_{j,s} - \frac{\psi}{1-n_j} \hat{n}_j \right).$$

By defining $\bar{\boldsymbol{\chi}}_{NS\times NS} \equiv [\mathbb{I}_{s=k} \sum_d r_{id,s}(\mathbb{I}_{i=j} - x_{oj,s})]_{is,jk}$ and $\bar{\boldsymbol{g}}^{U} \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^{U}$, we can then write

$$\hat{\boldsymbol{R}} \;\; = \;\; (1-\sigma) \bar{\boldsymbol{g}}^{U} \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\boldsymbol{g}}^{U} \bar{\boldsymbol{\chi}} \left(\bar{\boldsymbol{a}}^{L} \hat{\boldsymbol{w}} - \psi \bar{\upsilon}^{n} \hat{\boldsymbol{n}} + \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{P}}^{M} \right) + \bar{\boldsymbol{g}}^{U} \bar{\boldsymbol{r}}^{C} \hat{\boldsymbol{W}}. \tag{55}$$

Step 4. We now characterize the system in (45). The equilibrium definition in (33) implies that $\hat{W}_i = \sum_s \ell_{i,s} \hat{R}_{i,s}$, which, by defining $\bar{\boldsymbol{\ell}}_{N\times NS} \equiv [\ell_{j,s} \mathbb{I}_{[i=j]}]_{i,js}$, and $\bar{\boldsymbol{\upsilon}}_{NS\times N} = [\mathbb{I}_{i=j}]_{ik,j}$, can be written as

$$\hat{\boldsymbol{W}} = \bar{\boldsymbol{\ell}} \hat{\boldsymbol{R}}.$$

Plugging (55) into this expression, we have that

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\ell}} \bar{\boldsymbol{g}}^{U} \bar{\boldsymbol{r}}^{C} \right) \hat{\boldsymbol{W}} \;\; = \;\; (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\boldsymbol{g}}^{U} \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma) \bar{\boldsymbol{\ell}} \bar{\boldsymbol{g}}^{U} \bar{\boldsymbol{\chi}} \left(\bar{\boldsymbol{a}}^{L} \hat{\boldsymbol{w}} - \psi \bar{\upsilon}^{n} \hat{\boldsymbol{n}} + \bar{\boldsymbol{a}}^{M} \hat{\boldsymbol{P}}^{M} \right) \; .$$

Using (51)–(53),

$$\bar{\boldsymbol{\gamma}}^w \hat{\boldsymbol{w}} = (1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + (1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma \hat{\boldsymbol{P}}^M + \left(\lambda\bar{\boldsymbol{\gamma}}^P_\lambda + \vartheta\bar{\boldsymbol{\gamma}}^P_\vartheta\right)\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C \bar{\boldsymbol{a}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right),$$

where

$$\bar{\boldsymbol{\gamma}}^w \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\right)\bar{\boldsymbol{\alpha}}^{W,w} - (1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\left(\bar{\boldsymbol{a}}^L - \psi\phi\bar{v}^n\bar{\boldsymbol{\alpha}}^{n,w}\right),$$

$$\bar{\boldsymbol{\gamma}}^P_\lambda \equiv -\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\right)\bar{\boldsymbol{\alpha}}^{W,P}_\lambda - (1-\sigma)\psi\phi\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\bar{v}^n\bar{\boldsymbol{\alpha}}^{n,P},$$
$$\bar{\boldsymbol{\gamma}}^P_\vartheta \equiv -\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{y}}^C\right)\bar{\boldsymbol{\alpha}}^{W,P}_\vartheta,$$
$$\bar{\boldsymbol{\alpha}}^M_\sigma \equiv \bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^M.$$

Substituting $\hat{\boldsymbol{P}}^M$ with (54),

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + (1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \left(\lambda\bar{\boldsymbol{\alpha}}^P_\lambda + \vartheta\bar{\boldsymbol{\alpha}}^P_\vartheta\right)\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right)$$

where

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{\gamma}}^w - (1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma\bar{\boldsymbol{\alpha}}^{M,w} \tag{56}$$
$$\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) \equiv (1-\sigma)\bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}),, \quad \bar{\boldsymbol{\alpha}}^P_\lambda \equiv \bar{\boldsymbol{\gamma}}^P_\lambda - (1-\sigma)\psi\phi\bar{\boldsymbol{\alpha}}^M_\sigma\bar{\boldsymbol{\alpha}}^{M,P}_\lambda, \quad \bar{\boldsymbol{\alpha}}^P_\vartheta \equiv \bar{\boldsymbol{\gamma}}^P_\vartheta.$$

Thus,

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$$

with

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \equiv \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \left(\lambda\bar{\boldsymbol{\alpha}}^P_\lambda + \vartheta\bar{\boldsymbol{\alpha}}^P_\vartheta\right)\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \left((1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma + \lambda\bar{\boldsymbol{\alpha}}^M_\lambda + \vartheta\bar{\boldsymbol{\alpha}}^M_\vartheta\right)\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{57}$$
$$\bar{\boldsymbol{\alpha}}^M_\lambda \equiv \bar{\boldsymbol{\alpha}}^P_\lambda\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M, \quad \text{and} \quad \bar{\boldsymbol{\alpha}}^M_\vartheta \equiv \bar{\boldsymbol{\alpha}}^P_\vartheta\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M.$$

Part B: Derivation of the reduced-form response of wages to shifts in excess labor demand. We now establish that the system above, $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$, yields a reduced-form representation for wage changes, $\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$, where $\bar{\boldsymbol{\beta}}$ has a series expansion representation. We impose the same diagonal dominance condition: In any equilibrium, there is a vector $\{h_i\}_{i\neq m} \gg 0$ such that, for all $i \neq m$,

$$h_i\gamma_{ii} > \sum_{j\neq i,m} |\gamma_{ij}|h_j. \tag{58}$$

The proof now proceeds in the same way as the proof in Section A.1.2. We start by redefining the system to set the change in the wage of market m to zero. Using the same matrix $\bar{\boldsymbol{M}}$ defined in Section A.1.2, we show that, if $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is nonsingular, then $\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}\boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$ for $\bar{\boldsymbol{\beta}} \equiv \bar{\boldsymbol{M}}'(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}')^{-1}\bar{\boldsymbol{M}}$. In the rest of the proof, we first show that $\bar{\boldsymbol{\beta}}$ exists and then that it admits a series representation. To simplify exposition, we again abuse notation by defining

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}', \quad \hat{\boldsymbol{w}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{w}} \qquad \text{and} \qquad \hat{\boldsymbol{\eta}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}).$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\lambda}}$ be the diagonal matrix with the diagonal elements of $\bar{\boldsymbol{\gamma}}$: $\bar{\boldsymbol{\lambda}}$ s.t. $\lambda_{ii} = \gamma_{ii}$ and $\lambda_{ij} = 0$ for $i \neq j$. Thus, we can write the system as

$$\bar{\gamma} = \bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\gamma}}\right) \quad st \quad \bar{\tilde{\gamma}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\lambda}}^{-1}\bar{\gamma},$$

which implies that $\tilde{\gamma}_{ii} = 0$ and $\tilde{\gamma}_{ij} = -\gamma_{ij}/\gamma_{ii}$. Let $\bar{\boldsymbol{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i. Thus, $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$ is equivalent to

$$\begin{aligned}\bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{I}}-\bar{\tilde{\gamma}}\right)\left(\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^{-1}\right)\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\ \bar{\boldsymbol{\lambda}}\left(\bar{\boldsymbol{h}}-\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}}\\ \left(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\right)\left(\bar{\boldsymbol{I}}-\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} &= \hat{\boldsymbol{\eta}},\end{aligned}$$

which implies that

$$\hat{\boldsymbol{w}}=\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}}\right)^{-1}\left(\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{h}}\right)^{-1}\hat{\boldsymbol{\eta}},\quad \bar{\bar{\tilde{\gamma}}}\equiv\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}. \tag{59}$$

Notice that, for all i, $\bar{\tilde{\gamma}}_{ii}=0$ and $\bar{\tilde{\gamma}}_{ij}=-\gamma_{ij}h_j/\gamma_{ii}h_i$.

First, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is non-singular, so that we can write the expression in (59). We proceed by contradiction. Suppose that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$ is singular, so $\mu=0$ is an eigenvalue of $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})$. Take the eigenvector $\boldsymbol{x}$ associated with the zero eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})\boldsymbol{x}=0$, so that the i-row of this system is

$$1+\sum_{j\neq i,m}-\bar{\tilde{\gamma}}_{ij}x_j=0 \quad\Longrightarrow\quad 1+\sum_{j\neq i,m}\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j=0.$$

Thus,

$$\gamma_{ii}h_i=-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq|\sum_{j\neq i,m}\gamma_{ij}h_jx_j|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$$

where the last inequality holds because $|x_j|\leq 1$ and $h_j>0$. Thus, $\gamma_{ii}h_i\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts (58).

Second, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}$ admits a series representation. This is true whenever the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue μ is weakly greater than one. Take the eigenvector $\boldsymbol{x}$ associated with the largest eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $\mu\boldsymbol{x}=\bar{\bar{\tilde{\gamma}}}\boldsymbol{x}$ so that the i-row of this system is

$$1\leq\mu=\sum_{j\neq i,m}-\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j$$

Since γ_{ii} and h_i are positive,

$$\gamma_{ii}h_i\leq-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq|\sum_{j\neq i,m}\gamma_{ij}h_jx_j|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|$$

Since $|x_j|\leq 1$ and $h_j>0$, $\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$. Thus, $\gamma_{ii}h_i\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts (58). Thus, the largest eigenvalue of $\bar{\bar{\tilde{\gamma}}}$ is below one, allowing us to write $(\bar{\boldsymbol{I}}-\bar{\bar{\tilde{\gamma}}})^{-1}=\sum_{d=0}^{\infty}(\bar{\bar{\tilde{\gamma}}})^d$. Substituting this series expansion into (59) yields

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we now show that $\bar{\boldsymbol{h}}(\bar{\bar{\tilde{\gamma}}})^d\bar{\boldsymbol{h}}^{-1}=(\bar{\tilde{\gamma}})^d$. We proceed by induction. For $d=1$, it is trivial to see that $\bar{\boldsymbol{h}}(\bar{\bar{\tilde{\gamma}}})\bar{\boldsymbol{h}}^{-1}=\bar{\tilde{\gamma}}$. Then,

$$\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^{d+1}\bar{\boldsymbol{h}}^{-1}=\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\tilde{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\left(\bar{\boldsymbol{h}}\bar{\bar{\tilde{\gamma}}}\bar{\boldsymbol{h}}^{-1}\right)=(\bar{\tilde{\gamma}})^d\left(\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\right)=(\bar{\tilde{\gamma}})^{d+1}.$$

Thus,

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}\hat{\boldsymbol{\eta}} = \sum_{d=0}^{\infty} (\bar{\tilde{\gamma}})^d \, \bar{\boldsymbol{\lambda}}^{-1}\hat{\boldsymbol{\eta}},$$

which immediately implies that

$$\hat{w}_i = \sum_j \beta_{ij}\hat{\eta}_j(\hat{\boldsymbol{\tau}}) \quad \text{where} \quad \beta_{ij} = \frac{1}{\gamma_{ii}}\left(\mathbb{I}_{[i=j]} - \frac{\gamma_{ij}}{\gamma_{jj}}\mathbb{I}_{[i\neq j]}\right) + \sum_{d=2}^{\infty}\frac{\tilde{\gamma}_{ij}^{(d)}}{\gamma_{jj}} \tag{60}$$

with $\tilde{\gamma}_{ij}^{(d)}$ denoting the i-j entry of $(\bar{\tilde{\gamma}})^d$ defined as $\tilde{\gamma}_{ij} \equiv -\frac{\gamma_{ij}}{\gamma_{ii}}\mathbb{I}_{[i\neq j;\ i,j\neq m]}$.

Part C: Reduced-form representation for changes in labor market outcomes. We start by writing the reduced-form response in wages:

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}^R\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^C\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{61}$$

with

$$\bar{\boldsymbol{\beta}}^R = \bar{\boldsymbol{\beta}}, \quad \bar{\boldsymbol{\beta}}^C \equiv \bar{\boldsymbol{\beta}}\left(\lambda\bar{\boldsymbol{\alpha}}^P_\lambda + \vartheta\bar{\boldsymbol{\alpha}}^P_\vartheta\right), \quad \boldsymbol{\beta}^M \equiv \boldsymbol{\beta}\left((1-\sigma)\bar{\boldsymbol{\alpha}}^M_\sigma + \lambda\bar{\boldsymbol{\alpha}}^M_\lambda + \vartheta\bar{\boldsymbol{\alpha}}^M_\vartheta\right).$$

The combination of (61) and (51)–(52) implies

$$\hat{\boldsymbol{n}} = \bar{\boldsymbol{\beta}}^{n,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{62}$$

$$\hat{\boldsymbol{N}} = \bar{\boldsymbol{\beta}}^{n,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{n,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{63}$$

where

$$\bar{\boldsymbol{\beta}}^{n,R} \equiv \phi\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^R, \quad \bar{\boldsymbol{\beta}}^{n,C} \equiv \phi\left(\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^C + \lambda\bar{\boldsymbol{\alpha}}^{n,P}\right), \quad \bar{\boldsymbol{\beta}}^{n,M} \equiv \phi\left(\bar{\boldsymbol{\alpha}}^{n,w}\bar{\boldsymbol{\beta}}^M + \lambda\bar{\boldsymbol{\alpha}}^{n,P}\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\right)$$

$$\bar{\boldsymbol{\beta}}^{N,R} \equiv \vartheta\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^R, \quad \bar{\boldsymbol{\beta}}^{n,C} \equiv \vartheta\left(\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^C + \bar{\boldsymbol{\alpha}}^{N,P}\right), \quad \bar{\boldsymbol{\beta}}^{N,M} \equiv \vartheta\left(\bar{\boldsymbol{\alpha}}^{N,w}\bar{\boldsymbol{\beta}}^M + \bar{\boldsymbol{\alpha}}^{N,P}\bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\right)$$

Finally, from the expression for $\Delta\overline{\ln w_i}$ in (9), $\Delta\overline{\ln \boldsymbol{w}} = \hat{\boldsymbol{w}} - (1/\phi)\hat{\boldsymbol{n}}$. By substituting (60) and (62),

$$\Delta\overline{\ln \boldsymbol{w}} = \bar{\boldsymbol{\beta}}^{w,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{w,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{w,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) \tag{64}$$

where

$$\bar{\boldsymbol{\beta}}^{w,R} \equiv \bar{\boldsymbol{\beta}}^R - (1/\phi)\bar{\boldsymbol{\beta}}^{n,R}, \quad \bar{\boldsymbol{\beta}}^{w,C} \equiv \bar{\boldsymbol{\beta}}^C - (1/\phi)\bar{\boldsymbol{\beta}}^{n,C}, \quad \bar{\boldsymbol{\beta}}^{w,M} \equiv \bar{\boldsymbol{\beta}}^M - (1/\phi)\bar{\boldsymbol{\beta}}^{n,M}.$$

Part D: Empirical specification in (37)–(38). For outcomes $\Delta\ln Y_i \in \{\Delta\ln n_i, \Delta\ln N_i, \Delta\overline{\ln \boldsymbol{w}}\}$, equations (62)–(64) imply that, up to a first-order approximation,

$$\Delta\ln\boldsymbol{Y} = \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}).$$

By definition, $\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}})$, $\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}})$ and $\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})$ are linear combinations of $\hat{\tau}_{ij,s}$. Thus, $\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{\tau}}^{\text{obs}} + \hat{\boldsymbol{\tau}}^{\text{unbs}}$ implies that

$$\Delta\ln\boldsymbol{Y} = \alpha^Y + \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \boldsymbol{\nu}^Y$$

where $\boldsymbol{\nu} \equiv \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^{\text{unbs}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^{\text{unbs}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}^{\text{unbs}})$, $\alpha^Y \equiv I^{-1}\sum_i E[\nu_i|\boldsymbol{\mathcal{W}}^0]$, and $\nu_i^Y \equiv \nu_i - \alpha^Y$.

We now establish that, if $Cov(\hat{\tau}^{\text{obs}}_{ij,s}, \hat{\tau}^{\text{unbs}}_{od,k}|\boldsymbol{\mathcal{W}}^0) = 0$, then $E[\nu_i^Y\hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\boldsymbol{\tau}}^{\text{obs}})] = 0$ for $E \in \{R, C, M\}$.

The proof is analogous to that in Appendix A.1.7. First notice that the definitions of $\bar{\boldsymbol{\beta}}^{Y,E}$ and $\hat{\boldsymbol{\eta}}^{E}(\hat{\boldsymbol{\tau}})$ imply that we can write

$$\nu_i^Y = \sum_{s,d,o} \beta_{i,sdo}^{Y,\tau}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\tau}_{od,s}^{\text{unbs}} - \alpha^Y, \quad \text{and} \quad \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\boldsymbol{\tau}}^{\text{obs}}) = \sum_{s,d,o} H_{j,sdo}^{E,\tau}(\boldsymbol{\mathcal{W}}^0)\left(\hat{\tau}_{od,s}^{\text{obs}} - \bar{\tau}^{\text{obs}}\right).$$

Thus,

$$\begin{aligned} E\left[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\boldsymbol{\tau}}^{\text{obs}})|\boldsymbol{\mathcal{W}}^0\right] &= \textstyle\sum_{s,d,o}\sum_{s',d',o'} \beta_{i,sdo}^{Y,\tau}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)H_{j,s'd'o'}^{\tau}(\boldsymbol{\mathcal{W}}^0)E\left[\hat{\tau}_{od,s}^{\text{unbs}}(\hat{\tau}_{od,s}^{\text{obs}} - \bar{\tau}^{\text{obs}})|\boldsymbol{\mathcal{W}}^0\right] \\ &- \alpha^Y \textstyle\sum_{s,d,o} H_{j,sdo}^{\tau}(\boldsymbol{\mathcal{W}}^0)E\left[\hat{\tau}_{od,s}^{\text{obs}} - \bar{\tau}^{\text{obs}}|\boldsymbol{\mathcal{W}}^0\right] \end{aligned}$$

Note that $E[\hat{\tau}_{od,s}^{\text{obs}} - \bar{\tau}^{\text{obs}}|\boldsymbol{\mathcal{W}}^0] = 0$ and $E[\hat{\tau}_{od,s}^{\text{unbs}}(\hat{\tau}_{od,s}^{\text{obs}} - \bar{\tau}^{\text{obs}})|\boldsymbol{\mathcal{W}}^0] = 0$ from $Cov(\hat{\tau}_{ij,s}^{\text{obs}}, \hat{\tau}_{od,k}^{\text{unbs}}|\boldsymbol{\mathcal{W}}^0) = 0$. Hence, $E[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\boldsymbol{\tau}}^{\text{obs}})] = E[E[\nu_i^Y \hat{\eta}_j^E(\hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\boldsymbol{\tau}}^{\text{obs}})|\boldsymbol{\mathcal{W}}^0]] = 0$ for any i and j. This immediately implies that (38) holds for any real matrix h_{ij}^E.

A.2.5 Extension of Reduced-Form Response in (22) for a Symmetric Economy with Intermediate Inputs in Production

There are no trade costs, $\tau_{ij,s} = \tau_{i,s}$ for all j, the input spending shares are the same in all sectors and markets, $a_{i,s}^M = a^M$ and $\xi_{i,ks}^M = \xi_k^M$. We consider the same labor supply structure in Section 3 where $\lambda = \vartheta = 0$. Thus,

$$P_{i,s}^M = P^M = \Pi_k \left[\sum_o (\tau_{o,k} p_o)^{1-\sigma}\right]^{\frac{\xi_k}{1-\sigma}},$$

$$p_i = (w_i)^{1-\psi\phi-a^M}\left(P^M\right)^{a^M} \bar{b}_i^{\psi\phi}\left(\Omega(\boldsymbol{w})\right)^{\psi\phi}.$$

By defining $\kappa \equiv (\sigma-1)(1-\psi\phi-a^M)$, these expressions imply that

$$x_{ij,s} = x_{i,s} = \frac{(\tau_{i,s}p_i)^{1-\sigma}}{\sum_o(\tau_{o,s}p_o)^{1-\sigma}} = \frac{\tau_{i,s}^{1-\sigma}w_i^{-\kappa}\bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o \tau_{o,s}^{1-\sigma}w_o^{-\kappa}\bar{b}_o^{\psi\phi(1-\sigma)}}.$$

In this case, labor market clearing requires that

$$\begin{aligned} W_i &= (1-a^M)\textstyle\sum_s\sum_j x_{i,s}\left(\xi_s W_j + \sum_k \xi_s^M a^M R_k\right) \\ W_i &= \textstyle\sum_s\sum_j x_{i,s}\left((1-a^M)\xi_s W_j + a^M\xi_s^M \sum_k W_k\right) \\ W_i &= \textstyle\sum_s\sum_j x_{i,s}\left((1-a^M)\xi_s + a^M\xi_s^M\right)W_j \\ W_i &= \textstyle\sum_s\sum_j x_{i,s}e_s W_j \end{aligned}$$

where $e_s \equiv (1-a^M)\xi_s + a^M\xi_s^M$ is the share of gross spending on sector s (common to all markets i).
Finally, the supply of labor efficiency units is

$$W_j = w_j^\phi\left(w_j^\phi + \bar{b}_j^\phi\left(\Omega(\boldsymbol{w})\right)^\phi\right)^{\frac{1-\phi}{\phi}}\bar{N}_j\varrho.$$

The combination of the equilibrium conditions above implies that the equilibrium wage vector solves $D_i(\boldsymbol{w}|\boldsymbol{\tau}) = 0$ for all i such that

$$D_i(\boldsymbol{w}|\boldsymbol{\tau}) \equiv \sum_s\sum_j\left[\frac{\tau_{i,s}^{1-\sigma}w_i^{-\kappa}\bar{b}_i^{\psi\phi(1-\sigma)}}{\sum_o \tau_{o,s}^{1-\sigma}w_o^{-\kappa}\bar{b}_o^{(1-\sigma)\psi\phi}}e_s - \mathbb{I}_{i=j}\right]w_j^\phi\left(w_j^\phi + \bar{b}_j^\phi\left(\Omega(\boldsymbol{w})\right)^\phi\right)^{\frac{1-\phi}{\phi}}\bar{N}_j\varrho.$$

Given the alternative definition of the labor demand parameter κ, this excess labor demand system is isomorphic to that in (12) for the model of Section 3 in the special case of $\tau_{ij,s} = \tau_{i,j}$ and $\xi_{j,s} = e_s$ for all j.This implies that Corollary 2 holds for this economy.

A.2.6 Adding Endogenous Transfers to the Model

We now extend our model to allow for endogenous transfers across markets to finance the non-employment benefits. We maintain the same assumptions of Section 4 for production and labor supply. In this case, it is useful to write the location choice in terms of per-capita spending in each market, so that

$$\frac{N_i}{N_j} = \frac{P_i^{-\vartheta}\left(E_i/N_i\right)^{\vartheta}}{P_j^{-\vartheta}\left(E_j/N_j\right)^{\vartheta}}$$

and, therefore,

$$N_i = \frac{\left(E_i/P_i\right)^{\frac{\vartheta}{1+\vartheta}}}{\sum_{j\in\mathcal{I}_{c(i)}}\left(E_j/P_j\right)^{\frac{\vartheta}{1+\vartheta}}}. \tag{65}$$

We further assume that a fraction ϖ of non-employment benefits is financed with local taxes and that the remaining balance is financed with a common national income tax. Hence,

$$E_i = W_i + B_i - v_i(W_i + B_i) - v_c(W_i + B_i)$$

such that, in equilibrium,

$$v_i(W_i + B_i) = \varpi B_i, \quad \text{and} \quad v_c \sum_{i\in\mathcal{I}_c}(W_i + B_i) = (1-\varpi)\sum_{i\in\mathcal{I}_c} B_i.$$

Using the fact that $\frac{W_i}{B_i} = \frac{n_i}{1-n_i}$, this expression is equivalent to

$$E_i = W_i + (1-\varpi)\left[1 - n_i - \frac{\sum_{j\in\mathcal{I}_{c(i)}}(1-n_j)(W_j/n_j)}{\sum_{j\in\mathcal{I}_{c(i)}}(W_j/n_j)}\right]\frac{W_i}{n_i}. \tag{66}$$

Notice that, as in our baseline specification, $E_i = W_i$ if $\varpi = 1$. For any $\varpi \in [0,1]$, equations (8), (65), and (66) constitute a system of equations that can be locally solved as a function of $\boldsymbol{w}$ and $\boldsymbol{P}$. Conditional on these expressions, we can follow exactly the same steps in Appendix A.2.4 to characterize the reduced-form responses in the model.

A.3 Proofs of General Version of the Model in Section 4

A.3.1 Environment

Suppose that each country c is populated by a continuum of individuals divided into multiple groups indexed by $g = 1, ..., G$. Workers of group g in market i receive a wage of w_{gi} for each efficiency unit supplied. As in the baseline, each market has a competitive representative firm that produces a differentiated tradable intermediate good in each sector s, whose endogenous production cost is $p_{i,s}$ and iceberg trade cost of selling in j is $\tau_{ij,s}$.[49] There is a representative firm that produces a single non-tradable final consumption good in each market, and sells it at price of P_i.

[49] It is straightforward to define markets as groups of sectors by assuming that, for a subset of sectors, $\tau_{ij,s} = \infty$ for all j (including i).

We now present the three central parts of the model that, up to a first-order approximation, yield log-linear equilibrium relationships that are sufficient to derive the reduced-form representation for the model's predictions.

Labor Supply. Assume that worker preferences and efficiency implicitly define all labor outcomes as a (local) function of the wage rate and the price of the consumption good across markets. That is, for any group g in market i, we have the following local representation for one outcome Y_{gi} out of the employment rate n_{gi}, population N_{gi}, employment L_{gi}, spending E_{gi}, wage bill W_{gi}, or log average wage $\Delta\overline{\ln w}_{gi}$:

$$Y_{gi} = \Phi^Y_{gi}(\{w_{g'j}\}_{g'j}, \{P_j\}_j).$$

Notice that the models in Sections 3 and 4 satisfy this general restriction. It allows for (endogenous or exogenous) transfers across markets (for example, as in Appendix A.2.6). The general representation also covers a generalized Roy model with arbitrary individual heterogeneity in market-specific efficiency and preferences (for example, as in Adão (2016)). In addition, it allows for a rich structure of preferences for leisure and home production (as specified in Appendix B of the old version of our paper, Adao et al. (2020a)), as well as competitive search environments (such as the one described in Appendix A.5).

The first-order approximation for changes in any outcome Y_{gi} is

$$\hat{Y}_{gi} = \sum_{f,j} \phi^{Y,w}_{gi,fj} \hat{w}_{g'j} + \sum_j \phi^{Y,P}_{gi,j} \hat{P}_j, \tag{67}$$

where $\phi^{Y,w}_{gi,fj} \equiv \frac{\partial \ln \Phi^Y_{gi}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln w_{fj}}$ and $\phi^{Y,P}_{gi,j} \equiv \frac{\partial \ln \Phi^Y_{gi}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln P_j}$ are the labor supply elasticities with respect to wages and prices, respectively.

Final Consumption Good. Assume that the production function for the final good combines the differentiated good from all origins: $C_j = F^C_j(\{c_{ij,s}\}_{i,s})$ where $c_{ij,s}$ is the quantity of the differentiated good of sector s from i used to produce the final good in market j. Assume that F^C_j is continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. Thus, cost minimization and zero profit imply that

$$P_j\left(\{\tau_{oj,k} p_{o,k}\}_{o,k}\right) \equiv \min_{\{c_{ij,s}\}_{i,s}} \sum_{o,k} \tau_{oj,k} p_{o,k} \quad \text{such that} \quad F^C_j(\{c_{ij,s}\}_{i,s}) = 1.$$

The first-order approximation for changes in the final good price and final spending shares are given by

$$\hat{P}_j = \sum_{o,s} x^C_{ij,s}\left(\hat{\tau}_{ij,s} + \hat{p}_{i,s}\right), \quad \text{and} \quad \hat{x}^C_{ij,s} = \sum_{o,k} \chi^C_{ij,s,ok}\left(\hat{\tau}_{oj,k} + \hat{p}_{o,k}\right), \tag{68}$$

where $x^C_{ij,s} \equiv \frac{\partial \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s})}$ is the share of good s from i in final spending of market j, and $\chi^C_{ij,s,ok} \equiv \frac{\partial^2 \ln P_j(\{\tau_{oj,k} p_{o,k}\}_{o,k})}{\partial \ln(\tau_{ij,s} p_{i,s}) \partial \ln(\tau_{oj,k} p_{i,k})}$ is the elasticity of $x^C_{ij,s}$ to changes in the cost of good k from o. This final good production structure allows for arbitrary final spending shares and cross-price elasticities in the initial equilibrium. It is equivalent to allowing individuals to have arbitrary homothetic preferences for the differentiated products of different sectors and origins.

Differentiated Good. Assume that the production function for the differentiated good of sector s from market i is subject to external economies of scale and combines labor of different groups and inputs from different sectors and origins. That is, $Q_{i,s} = \Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j}) F_{i,s}\left(\{L_{gi,s}\}_g, F^M_{i,s}\left(\{M_{oi,ks}\}_{o,k}\right)\right)$, where $L_{gi,s}$ is the number of efficiency units employed in sector s of market i, $M_{oi,ks}$ is the quantity of the differentiated good of sector k from o used to produce good s in market i, and $\Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})$ is the endogenous productivity term (but external to the firm) in sector s of market i that depends on employment outcomes across groups and markets. Assume that $F_{i,s}$ and $F^M_{i,s}$ are continuous, twice differentiable, increasing in all arguments, strictly quasi-concave, and homogeneous of degree one. This production function allows us to solve the firm's cost minimization problem in two stages.

Consider first the cost minimization problem of selecting intermediate inputs:

$$P^M_{i,s}\left(\{\tau_{oi,k}p_{o,k}\}_{o,k}\right) \equiv \min_{\{M_{oi,ks}\}_{o,k}} \sum_{o,k} \tau_{oi,k}p_{o,k}M_{oi,ks} \quad \text{s.t.} \quad F^M_{i,s}\left(\{M_{oi,ks}\}_{o,k}\right) = 1,$$

which implies that

$$\hat{P}^M_{i,s} = \sum_{j,k} x^M_{ji,ks}\left(\hat{\tau}_{ji,k} + \hat{p}_{j,k}\right), \quad \text{and} \quad \hat{x}^M_{ji,ks} = \sum_{o,h} \chi^M_{ji,ks,oh}\left(\hat{\tau}_{oi,h} + \hat{p}_{o,h}\right), \tag{69}$$

where $x^M_{ji,ks} = \frac{\partial \ln P^M_{i,s}\left(\{\tau_{oi,k}p_{o,k}\}_{o,k}\right)}{\partial \ln(\tau_{ji,k}p_{j,k})}$ is the share of spending on sector k from j in the total input spending of sector s in market j, and $\chi^M_{ji,ks,oh} \equiv \frac{\partial^2 \ln P^M_{i,s}\left(\{\tau_{oi,k}p_{o,k}\}_{o,k}\right)}{\partial \ln(\tau_{ji,k}p_{j,k})\partial \ln(\tau_{oi,h}p_{o,h})}$ is the elasticity of $x^M_{ji,ks}$ to changes in the cost of the good from sector h of market o.

We then solve the firm's optimal spending on labor and inputs:

$$c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right) = \min_{\{L_{gi,s}\}_g, M_{i,s}} \sum_g w_{gi}L_{gi,s} + P^M_{i,s}M_{i,s} \quad \text{s.t.} \quad F_{i,s}\left(\{L_{gi,s}\}_g, M_{i,s}\right) = 1,$$

which implies that

$$\hat{c}_{i,s} = \sum_g a^L_{gi,s}\hat{w}_{gi} + a^M_{i,s}\hat{P}^M_{i,s}, \quad \hat{a}^L_{gi,s} = \sum_{g'} \epsilon^L_{gi,s,g'}\hat{w}_{g'i} + \epsilon^{LM}_{gi,s}\hat{P}^M_{i,s}, \quad \text{and} \quad \hat{a}^M_{i,s} = \sum_g \epsilon^{LM}_{gi,s}\hat{w}_{gi} + \epsilon^M_{i,s}\hat{P}^M_{i,s} \tag{70}$$

where $a^L_{gi,s} \equiv \frac{\partial \ln c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right)}{\partial \ln w_{gi}}$ and $a^M_{i,s} = \frac{\partial \ln c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right)}{\partial \ln P^M_{i,s}}$ are the shares of labor and inputs on the total cost of sector s from i, and $\epsilon^L_{gi,s,g'} \equiv \frac{\partial^2 \ln c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right)}{\partial \ln w_{gi}\partial \ln w_{g'i}}$, $\epsilon^{LM}_{gi,s} \equiv \frac{\partial^2 \ln c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right)}{\partial \ln w_{gi}\partial \ln P^M_{i,s}}$ and $\epsilon^M_{i,s} \equiv \frac{\partial \ln c_{i,s}\left(\{w_{gi}\}_g, P^M_{i,s}\right)}{\partial \ln P^M_{i,s}\partial \ln P^M_{i,s}}$ are the elasticities of labor and input cost shares with respect to changes in wages and input prices.

Expressions in (69)–(70) allow a flexible structure of production. In the initial equilibrium, each sector-market pair can have arbitrary spending shares on labor of different groups and intermediate goods from different sectors and origins. In addition, we flexibly allow for a nested elasticity structure in the labor and input demand functions. We do not impose parametric restrictions on the cross-price elasticity matrix for intermediate spending, allowing for different substitution patterns across goods from different sectors and markets. This structure also yields an arbitrary demand substitution pattern for labor of different groups. Importantly, while changes in the unit input cost can differentially affect demand for different labor groups, the nested production function imposes that the cost of inputs of distinct sectors and origins only affect factor demand through a single unit input cost index.

Finally, production cost of s in i is $p_{i,s} = c_{i,s}(\{w_{gi}\}_g, P^M_{i,s})/\Psi_{i,s}(\{n_{gj}, N_{gj}\}_{g,j})$, which implies that

$$\hat{p}_{i,s} = \sum_g a^L_{gi,s}\hat{w}_{gi} + a^M_{i,s}\hat{P}^M_{i,s} - \sum_{g,j} \psi^n_{is,gj}\hat{n}_{gj} - \sum_{g,j} \psi^N_{is,gj}\hat{N}_{gj}, \tag{71}$$

where $\psi^n_{is,gj} \equiv \frac{\partial \ln \Psi_{i,s}(\{n_{gj},N_{gj}\}_{g,j})}{\partial \ln n_{gj}}$ and $\psi^N_{is,gj} \equiv \frac{\partial \ln \Psi_{i,s}(\{n_{gj},N_{gj}\}_{g,j})}{\partial \ln N_{gj}}$. This general agglomeration elasticity matrix allows for technology diffusion between regions, as in Fujita et al. (1999) and Lucas and Rossi-Hansberg (2003), and differences across sectors in economies of scale – e.g., Krugman and Venables (1995), Balistreri et al. (2010), Kucheryavyy et al. (2016). In addition, the cross-market elasticity of labor productivity may also incorporate congestion forces implied by the re-allocation of other factors of production, like land and capital (see Appendix B of the old version of our paper, Adao et al. (2020a)).

Equilibrium. The equilibrium requires both goods and labor markets to clear. For every sector s and market i, the vector of gross revenues $\{R_{i,s}\}_{i,s}$ must solve

$$R_{i,s} = \sum_j x^C_{ij,s} E_j + \sum_j x^M_{ij,sk} a^M_{j,k} R_{j,k}. \tag{72}$$

For every group g and market i, the vector of wages must guarantee that

$$W_{gi} = \sum_s a^L_{gi,s} R_{i,s}. \tag{73}$$

A.3.2 Reduced-form Representation

The following proposition characterizes the reduced-form representation of the wage change for each group and market as a function of measures of market-level shock exposure.

Proposition 1. *For an arbitrary* $\hat{\boldsymbol{\tau}} \equiv \{\hat{\tau}_{ij,s}\}_{ijs}$*, the vector of wage change,* $\hat{\boldsymbol{w}} \equiv \{\hat{w}_{gi}\}_{gi}$*, solves* $\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \boldsymbol{\eta}(\hat{\boldsymbol{\tau}})$*. Under the diagonal dominance condition in (58),* $\hat{\boldsymbol{w}}$ *has a representation of the form:*

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) \quad \textit{such that} \quad \hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) = \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^C \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^M \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}), \tag{74}$$

where

$$\eta^R_{gi}(\hat{\boldsymbol{\tau}}) = \sum_s \ell^0_{gi,s}(\mu_{gi,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k} b^U_{is,jk}\mu_{gj,k}(\hat{\boldsymbol{\tau}})), \quad \eta^C_i(\hat{\boldsymbol{\tau}}) = \sum_{s,o} x^C_{oi,s}\hat{\tau}_{oi,s}, \quad \eta^M_{i,s}(\hat{\boldsymbol{\tau}}) = \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) + \sum_{j,k} b^D_{is,jk}\mu^M_{j,k}(\hat{\boldsymbol{\tau}}) \tag{75}$$

such that

$$\mu_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{o,h}\sum_j (r^C_{ij,s}\chi^C_{ij,s,oh} + \sum_k r^U_{ij,sk}\chi^M_{ij,sk,oh})\hat{\tau}_{oj,h} \;\; \textit{and} \;\; \mu^M_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,k} x^M_{ji,ks}\hat{\tau}_{ji,k}, \tag{76}$$

$$\bar{\boldsymbol{b}}^U \equiv \sum_{d=1}^{\infty} \left(\bar{\boldsymbol{r}}^U\right)^d \;\; \textit{and} \;\; \bar{\boldsymbol{b}}^D \equiv \sum_{d=1}^{\infty} \left(\bar{\boldsymbol{x}}^D\right)^d, \tag{77}$$

with $\bar{\boldsymbol{r}}^U_{NS\times NS} \equiv [x^M_{ij,sk}a^M_{j,k}R_{j,k}/R_{i,s}]_{is,jk}$*,* $\bar{\boldsymbol{x}}^D_{NS\times NS} \equiv [x^M_{ji,ks}a^M_{j,k}]_{is,jk}$*,* $\boldsymbol{\mathcal{W}}^0 \equiv \{\{x^C_{ij,s}\}_j, \{x^M_{ij,sk}\}_{j,k}, \{a^L_{gi,s}\}_g\}_j$ *and* $\boldsymbol{\theta} \equiv \{\{\chi^M_{ji,ks,oh}\}_{o,h}, \{\chi^C_{ji,s,oh}\}_{j,k}, \psi_{is,gj}, \phi^{Y,w}_{gi,fj}, \phi^{Y,P}_{gi,j}\}_j$.

Appendix A.3.4 contains the proof of this proposition. It generalizes the reduced-form representation in Section 4 for the model with the non-parametric links in production and labor supply described in Appendix A.3.1. It maps wage changes to measures of shock exposure that depend on the initial bilateral trade shares for both final and intermediate consumption, the initial factor spending shares in production, and the elasticity matrices governing cross-market links in labor supply ($\phi^{Y,w}_{gi,fj}$ and $\phi^{Y,P}_{gi,j}$), productivity ($\psi^n_{is,gj}$ and $\psi^N_{is,gj}$) and trade demand ($\chi^M_{ji,ks,oh}$ and $\chi^C_{ji,s,oh}$). The definitions of consumption and input exposure are identical to those in Section 4 for arbitrary bilateral (final and intermediate) spending shares,

$x^M_{ji,ks}$ and $x^C_{ji,s}$. However, the definition of revenue exposure needs to be extended to account for the more general demand for goods: the shock's impact on the sales of s in i, $\mu_{i,s}(\hat{\boldsymbol{\tau}})$, is now a function of the cross-price demand elasticities, $\chi^C_{ij,s,oh}$ and $\chi^M_{ij,sk,oh}$. With the nested CES demand in Section 4, these elasticities take the simple form of $\chi^C_{ij,s,oh} = \chi^M_{ij,sk,oh} = (1-\sigma)(\mathbb{I}_{i=o} - x_{oj,s})\mathbb{I}_{s=k}$.

In Appendix A.3.4, we also characterize the reduced-form representation for labor market and price outcomes. For any $\hat{Y}_j \in \{\{\hat{w}_{gi}, \hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \widehat{\overline{\ln w_{gi}}}\}_g, \hat{P}_i, \{\hat{P}^M_{i,s}\}_s\}$, we show that $\hat{Y}_i$ can be written as

$$\hat{Y}_i = \sum_{g,j} \beta^{Y,R}_{igj}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^R_{gj}(\hat{\boldsymbol{\tau}}|\boldsymbol{\theta}) + \sum_j \beta^{Y,C}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^C_j(\hat{\boldsymbol{\tau}}) + \sum_{j,s} \beta^{Y,M}_{ij,s}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^M_{j,s}(\hat{\boldsymbol{\tau}}). \tag{78}$$

The steps in Appendix A.2.4 imply that, if $\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{\tau}}^{\text{obs}} + \hat{\boldsymbol{\tau}}^{\text{unbs}}$ with $Cov(\hat{\tau}^{\text{obs}}_{ij,s}, \hat{\tau}^{\text{unbs}}_{od,k}|\boldsymbol{\mathcal{W}}^0) = 0$, then

$$\Delta \ln Y_i = \alpha^Y + \sum_{g,j} \beta^{Y,R}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^R_{gj}(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_j \beta^{Y,C}_{ij}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^C_j(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \sum_{j,s} \beta^{Y,M}_{ij,s}(\boldsymbol{\theta}|\boldsymbol{\mathcal{W}}^0)\hat{\eta}^M_{j,s}(\hat{\boldsymbol{\tau}}^{\text{obs}}) + \nu^Y_i \tag{79}$$

such that, for $\ddot{\boldsymbol{\tau}}^{\text{obs}} \equiv \hat{\boldsymbol{\tau}}^{\text{obs}} - \bar{\tau}^{\text{obs}}$,

$$E\left[\nu^Y_i \hat{\eta}^R_{gj}(\ddot{\boldsymbol{\tau}}^{\text{obs}}|\boldsymbol{\theta})\right] = E\left[\nu^Y_i \hat{\eta}^C_j(\ddot{\boldsymbol{\tau}}^{\text{obs}})\right] = E\left[\nu^Y_i \hat{\eta}^M_{j,s}(\ddot{\boldsymbol{\tau}}^{\text{obs}})\right] = 0 \quad \text{for any } i,j,g,s. \tag{80}$$

A.3.3 Integration

The reduced-form representation in (78) is a first-order approximation for changes in the market-level outcomes: $\hat{Y}_j \in \{\{\hat{w}_{gi}, \hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \widehat{\overline{\ln w_{gi}}}\}_g, \hat{P}_i, \{\hat{P}^M_{i,s}\}_s\}$. It can be integrated to compute exact changes in these outcomes using the following algorithm.

1. Consider $r = 1, ..., R$ repetitions. Let the initial conditions be $\boldsymbol{\mathcal{W}}^0 \equiv \{\{x^C_{ij,s}\}_j, \{x^M_{ij,sk}\}_{j,k}, \{a^L_{gi,s}\}_g\}_j$, and the initial elasticities be $\boldsymbol{\theta}^0 \equiv \{\{\chi^M_{ji,ks,oh}\}_{o,h}, \{\chi^C_{ji,s,oh}\}_{j,k}, \psi^n_{is,gj}, \psi^N_{is,gj}, \phi^{Y,w}_{gi,fj}, \phi^{Y,P}_{gi,j}\}$.
2. Given $\boldsymbol{\theta}^{r-1}$ and $\boldsymbol{\mathcal{W}}^{r-1}$, for $\hat{\boldsymbol{\tau}}^r = \hat{\boldsymbol{\tau}}/R$:
   (a) Compute $Y^r_i = Y^{r-1}_i \exp(\hat{Y}^r_i)$ where $\hat{Y}^r_i$ is given by (78);
   (b) Compute $p^r_{i,s} = p^{r-1}_{i,s}\exp(\hat{p}^r_{i,s})$ where $\hat{p}^r_{i,s}$ is given by (71);
   (c) Compute $\boldsymbol{\mathcal{W}}^r \equiv \{\{x^{C,r-1}_{ij,s}\exp(\hat{x}^{C,r}_{ij,s})\}_j, \{x^{M,r-1}_{ij,sk}\exp(\hat{x}^{M,r}_{ij,sk})\}_{j,k}, \{a^L_{gi,s}\exp(\hat{a}^{L,r}_{gi,s})\}_g\}_j$ where $\hat{x}^{C,r}_{ij,s}$, $\hat{x}^{M,r}_{ij,sk}$, and $\hat{a}^{L,r}_{i,s}$ are respectively given by (68), (69) and (70);
   (d) Compute $\boldsymbol{\theta}^r \equiv \{\{\chi^M_{ji,ks,oh}\}_{o,h}, \{\chi^C_{ji,s,oh}\}_{j,k}, \psi_{is,gj}, \phi^{Y,w}_{gi,fj}, \phi^{Y,P}_{gi,j}\}_j$ using the definitions above and outcomes in iteration r.
3. Repeat step 2 for each r. The overall change in any outcome is $Y'_i/Y^0_i = Y^R_i/Y^0_i$.

A.3.4 Proof of Proposition 1

Step 1. We first derive expressions for changes in price indices as a function of wages, and exogenous shocks. From (69) and (71),

$$\hat{P}^M_{i,s} = \sum_{j,k} x^M_{ji,ks}\left(\hat{\tau}_{ji,k} + \sum_g a^L_{gj,k}\hat{w}_{gj} + a^M_{j,k}\hat{P}^M_{j,k} - \sum_{g,o}\psi^n_{jk,go}\hat{n}_{go} - \sum_{g,o}\psi^N_{jk,go}\hat{N}_{go}\right)$$

which, by defining $\bar{\boldsymbol{x}}^D_{NS\times NS} \equiv [x^M_{ji,ks} a^M_{j,k}]_{is,jk}$, $\bar{\boldsymbol{x}}^M_{NS\times NS} \equiv [x^M_{ji,ks}]_{is,jk}$, $\bar{\boldsymbol{a}}^L_{NS\times NG} \equiv [a^L_{gj,k}\mathbb{I}_{i=j}]_{ik,gj}$, $\bar{\boldsymbol{\psi}}^n_{NS\times NG} = [\psi^n_{is,gj}]_{is,gj}$, $\bar{\boldsymbol{\psi}}^N_{NS\times NG} = [\psi^N_{is,gj}]_{is,gj}$ and $\boldsymbol{\mu}^M_{NS\times 1} \equiv [\hat{\mu}_{i,s}(\hat{\boldsymbol{\tau}})]_{i,s}$ with $\hat{\mu}_{i,s}(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,k} x^M_{ji,ks}\hat{\tau}_{ji,k}$, implies

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D\right)\hat{\boldsymbol{P}}^M = \boldsymbol{\mu}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right)$$

and, therefore,

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) \tag{81}$$

for $\bar{\boldsymbol{g}}^D \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^D\right)^{-1} = \bar{\boldsymbol{I}} + \sum_{d=1}^{\infty}(\bar{\boldsymbol{x}}^D)^d$ and $\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}})$ in (75).

From (68),

$$\hat{P}_i = \sum_{j,s} x^C_{ji,s}\left(\hat{\tau}_{ji,s} + \sum_g a^L_{gj,s}\hat{w}_{gj} + a^M_{j,s}\hat{P}^M_{j,s} - \sum_{g,o}\psi^n_{jk,go}\hat{n}_{go} - \sum_{g,o}\psi^N_{jk,go}\hat{N}_{go}\right)$$

which, by defining $\bar{\boldsymbol{x}}^C_{N\times NS} \equiv [x^C_{ji,s}]_{i,js}$ and $\bar{\boldsymbol{a}}^M_{NS\times NS} \equiv [a^M_{j,k}\mathbb{I}_{is=jk}]_{is,jk}$,

$$\hat{\boldsymbol{P}} = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M$$

with $\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}})$ in (75).

Substituting (81) into this expression,

$$\hat{\boldsymbol{P}} = \left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right), \tag{82}$$

where $\bar{\boldsymbol{c}}^M \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{a}}^M\bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M$.

Using (67) for outcome Y,

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{\phi}}^{Y,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^{Y,P}\hat{\boldsymbol{P}}$$

where we define $\bar{\boldsymbol{\phi}}^{Y,w}_{GN\times GN} \equiv [\phi^{Y,w}_{gi,fj}]_{gi,fj}$ and $\bar{\boldsymbol{\phi}}^{Y,P}_{GN\times N} \equiv [\phi^{Y,P}_{gi,j}]_{gi,j}$.

To obtain an expression for the price index, we now combine this expression with (82) to derive

$$\hat{\boldsymbol{P}} = \bar{\boldsymbol{\rho}}^P\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{\alpha}}^{P,w}\hat{\boldsymbol{w}} \tag{83}$$

where $\bar{\boldsymbol{\rho}}^P \equiv \left[\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\phi}}^{n,P} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\phi}}^{N,P}\right)\right]^{-1}$ and $\bar{\boldsymbol{\alpha}}^{P,w} \equiv \bar{\boldsymbol{\rho}}^P\bar{\boldsymbol{x}}^C\bar{\boldsymbol{c}}^M\left(\bar{\boldsymbol{a}}^L - \bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\phi}}^{n,w} - \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\phi}}^{N,w}\right)$.

Using (67) for outcome Y,

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{\alpha}}^{Y,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{Y,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{84}$$

where $\bar{\boldsymbol{\alpha}}^{Y,w} \equiv \bar{\boldsymbol{\phi}}^{Y,w} + \bar{\boldsymbol{\phi}}^{Y,P}\bar{\boldsymbol{\alpha}}^{P,w}$ and $\bar{\boldsymbol{\alpha}}^{Y,C} \equiv \bar{\boldsymbol{\phi}}^{Y,P}\bar{\boldsymbol{\rho}}^P$.

Together with (81) and (82), equation (84) implies

$$\hat{\boldsymbol{P}}^M = \hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^{M,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) + \bar{\boldsymbol{\alpha}}^{M,w}\hat{\boldsymbol{w}}, \tag{85}$$

where $\bar{\boldsymbol{\alpha}}^{M,C} \equiv \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,C} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,C}\right)$ and $\bar{\boldsymbol{\alpha}}^{M,w} \equiv \bar{\boldsymbol{g}}^D\bar{\boldsymbol{x}}^M\left(\bar{\boldsymbol{a}}^L - \bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,w} - \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,w}\right)$.

Step 2. We now solve for the change in revenue of sector-market pairs. From (72), by defining $\bar{\boldsymbol{r}}^C_{NS\times N} \equiv [x^C_{ij,s}E_j/R_{i,s}]_{is,j}$ and $\bar{\boldsymbol{r}}^U_{NS\times NS} \equiv [x^M_{ij,sk}a^M_{j,k}R_{j,k}/R_{i,s}]_{is,jk}$,

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^U\right)\hat{\boldsymbol{R}} = \left[\sum_j r^C_{ij,s}\hat{x}^C_{ij,s} + \sum_{j,k} r^U_{ij,sk}\hat{x}^M_{ij,sk}\right]_{is} + \bar{\boldsymbol{r}}^U\hat{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

Using (68) and (69), this expression becomes

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{r}}^U\right)\hat{\boldsymbol{R}} = \boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\chi}}\hat{\boldsymbol{p}} + \bar{\boldsymbol{r}}^U\hat{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

where $\bar{\boldsymbol{\chi}}_{NS\times NS} \equiv [\sum_j (r^C_{ij,s}\chi^C_{ij,s,oh} + \sum_k r^U_{ij,sk}\chi^M_{ji,ks,oh})]_{is,oh}$ and $\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$ defined in (76).

From (70) and (71), we have that

$$\hat{\boldsymbol{p}} = \bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} + \bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}$$

$$\hat{\boldsymbol{a}}^M = \bar{\boldsymbol{\epsilon}}^{LM}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\epsilon}}^M\hat{\boldsymbol{P}}^M$$

where $\bar{\boldsymbol{\epsilon}}^{LM}_{NS\times NG} \equiv [\epsilon^{LM}_{gi,s}\mathbb{I}_{i=j}]_{is,gj}$ and $\bar{\boldsymbol{\epsilon}}^M \equiv [\epsilon^M_{i,s}\mathbb{I}_{is=jk}]_{is,jk}$.

By defining $\bar{\boldsymbol{g}}^U \equiv \bar{\boldsymbol{I}} + \bar{\boldsymbol{b}}^U$, the expressions above imply that

$$\hat{\boldsymbol{R}} = \bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{\chi}}\left(\bar{\boldsymbol{a}}^L\hat{\boldsymbol{w}} + \bar{\boldsymbol{a}}^M\hat{\boldsymbol{P}}^M - \bar{\boldsymbol{\psi}}^n\hat{\boldsymbol{n}} - \bar{\boldsymbol{\psi}}^N\hat{\boldsymbol{N}}\right) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^U\left(\bar{\boldsymbol{\epsilon}}^{LM}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\epsilon}}^M\hat{\boldsymbol{P}}^M\right) + \bar{\boldsymbol{g}}^U\bar{\boldsymbol{r}}^C\hat{\boldsymbol{E}}.$$

Using the expressions for $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{N}}$ in (84) and $\hat{\boldsymbol{E}}$ in (84),

$$\hat{\boldsymbol{R}} = \bar{\boldsymbol{g}}^U\boldsymbol{\mu}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^{R,w}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{R,M}\hat{\boldsymbol{P}}^M + \bar{\boldsymbol{\alpha}}^{R,C}\left(\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^C\bar{\boldsymbol{a}}^M\hat{\boldsymbol{\eta}}^M(\hat{\boldsymbol{\tau}})\right) \tag{86}$$

where

$$\bar{\boldsymbol{\alpha}}^{R,w} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^L + \bar{\boldsymbol{r}}^U\bar{\boldsymbol{\epsilon}}^{LM} - \bar{\boldsymbol{\chi}}(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,w} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,w}) + \bar{\boldsymbol{r}}^C\bar{\boldsymbol{\alpha}}^{E,w}\right)$$

$$\bar{\boldsymbol{\alpha}}^{R,M} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{\chi}}\bar{\boldsymbol{a}}^M + \bar{\boldsymbol{r}}^U\bar{\boldsymbol{\epsilon}}^M\right)$$

$$\bar{\boldsymbol{\alpha}}^{R,C} \equiv \bar{\boldsymbol{g}}^U\left(\bar{\boldsymbol{r}}^C\bar{\boldsymbol{\alpha}}^{E,C} - \bar{\boldsymbol{\chi}}(\bar{\boldsymbol{\psi}}^n\bar{\boldsymbol{\alpha}}^{n,C} + \bar{\boldsymbol{\psi}}^N\bar{\boldsymbol{\alpha}}^{N,C})\right)$$

Step 3. The final step is characterizing the system in (45). From the labor market clearing condition in (73), $\hat{W}_{gi} = \sum_s \ell_{gi,s}(\hat{a}^L_{gi,s} + \hat{R}_{i,s})$. When combined with (70), we get that

$$\hat{W}_{gi} = \sum_s \ell_{gi,s}\left(\sum_{g'}\epsilon^L_{gi,s,g'}\hat{w}_{g'i} + \epsilon^{LM}_{gi,s}\hat{P}^M_{i,s} + \hat{R}_{i,s}\right)$$

which, in matrix notation, yields

$$\hat{\boldsymbol{W}} = \bar{\boldsymbol{\alpha}}^{\epsilon,L}\hat{\boldsymbol{w}} + \bar{\boldsymbol{\alpha}}^{\epsilon,M}\hat{\boldsymbol{P}}^M + \bar{\boldsymbol{\ell}}\hat{\boldsymbol{R}},$$

where $\bar{\boldsymbol{\alpha}}^{\epsilon,L}_{NG\times NG} \equiv \{\mathbb{I}_{i=j}\sum_s \ell_{gi,s}\epsilon^L_{gi,s,g'}]_{gi,g'j}$ is the matrix of cross-group elasticities of labor demand with respect to wages, $\bar{\boldsymbol{\alpha}}^{\epsilon,M}_{NG\times NS} \equiv \{\mathbb{I}_{i=j}\ell_{gi,s}\epsilon^{LM}_{gi,s}]_{gi,js}$ is the matrix of group elasticities of labor demand with respect to input cost, and $\bar{\boldsymbol{\ell}}_{GN\times NS} \equiv [\ell_{gi,s}\mathbb{I}_{[i=j]}]_{gi,js}$ is the matrix of the share of sector s in employment of group g in market i.

By applying (84) and (86) into the expression above, we get

$$\left(\bar{\boldsymbol{\alpha}}^{W,w} - \bar{\boldsymbol{\alpha}}^{\epsilon,L} - \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,w}\right)\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}^{R} + \left(\bar{\boldsymbol{\alpha}}^{\epsilon,M} + \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\hat{\boldsymbol{P}}^{M} + \left(\bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,C} - \bar{\boldsymbol{\alpha}}^{W,C}\right)\left(\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}})\right),$$

where, by definition, $\hat{\boldsymbol{\eta}}^{R} \equiv \bar{\boldsymbol{\ell}}\bar{\boldsymbol{g}}^{U}\boldsymbol{\mu}(\hat{\boldsymbol{\tau}})$.

By applying (85) into this expression, we get that

$$\bar{\boldsymbol{\gamma}}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^{C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\alpha}}^{M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}), \tag{87}$$

where

$$\begin{aligned} \bar{\boldsymbol{\gamma}} &\equiv \bar{\boldsymbol{\alpha}}^{W,w} - \bar{\boldsymbol{\alpha}}^{\epsilon,L} - \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,w} - \left(\bar{\boldsymbol{\alpha}}^{\epsilon,M} + \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\bar{\boldsymbol{\alpha}}^{M,w}, \\ \bar{\boldsymbol{\alpha}}^{C} &\equiv \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,C} - \bar{\boldsymbol{\alpha}}^{W,C} - \left(\bar{\boldsymbol{\alpha}}^{\epsilon,M} + \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M}\right)\bar{\boldsymbol{\alpha}}^{M,C}, \\ \bar{\boldsymbol{\alpha}}^{M} &\equiv \bar{\boldsymbol{\alpha}}^{\epsilon,M} + \bar{\boldsymbol{\ell}}\bar{\boldsymbol{\alpha}}^{R,M} + \bar{\boldsymbol{\alpha}}^{C}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}. \end{aligned}$$

The representation in (74) follows from the same steps in Part B of Appendix A.2.4 under the diagonal dominance condition in (58).

Step 4. We derive reduced-form expressions for all labor market outcomes $\hat{Y}_{gi} \in \{\hat{n}_{gi}, \hat{N}_{gi}, \hat{E}_{gi}, \widehat{\overline{\ln w_{gi}}}\}$ using (84):

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{\beta}}^{Y,R}\hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{Y,M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}) \tag{88}$$

where

$$\bar{\boldsymbol{\beta}}^{Y,R} \equiv \bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}}, \quad \bar{\boldsymbol{\beta}}^{Y,C} \equiv \bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{C} + \bar{\boldsymbol{\alpha}}^{Y,C}, \quad \bar{\boldsymbol{\beta}}^{Y,M} \equiv \bar{\boldsymbol{\alpha}}^{Y,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{M} + \bar{\boldsymbol{\alpha}}^{Y,C}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}.$$

Using (83),

$$\hat{\boldsymbol{P}} = \bar{\boldsymbol{\beta}}^{C,R}\hat{\boldsymbol{\eta}}^{R}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{C,C}\hat{\boldsymbol{\eta}}^{C}(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\beta}}^{C,M}\hat{\boldsymbol{\eta}}^{M}(\hat{\boldsymbol{\tau}}), \tag{89}$$

where

$$\bar{\boldsymbol{\beta}}^{C,R} \equiv \bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}}, \quad \bar{\boldsymbol{\beta}}^{C,C} \equiv \bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{C} + \bar{\boldsymbol{\rho}}^{P}, \quad \bar{\boldsymbol{\beta}}^{C,M} \equiv \bar{\boldsymbol{\alpha}}^{P,w}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\alpha}}^{M} + \bar{\boldsymbol{\rho}}^{P}\bar{\boldsymbol{x}}^{C}\bar{\boldsymbol{a}}^{M}.$$

A.4 Proofs and Additional Results in Section 5

A.4.1 Proof of Expression (39)

The gravity trade demand $X_{ij,s}$ in (6) and the expression for $p_{i,s}$ in (10) imply that

$$\Delta \log X^{t}_{ij,s} = (1-\sigma)\hat{\tau}^{t}_{ij,s} - \kappa\hat{w}^{t}_{i,s} + \underbrace{\hat{E}^{t}_{j} - \left(\widehat{\sum_{o}\tau^{1-\sigma}_{oj,s}w_{o}^{-\kappa}\bar{b}_{o}^{\kappa-\sigma+1}}\right)}_{\equiv\Lambda^{t}_{j,s}}.$$

Up to a first order approximation, the definition of $\Delta M^{t}_{s} \equiv \sum_{j}\frac{\Delta X^{t}_{\text{China}j,s}}{L^{t_0}_{\text{US},s}}$ is equal to

$$\begin{aligned} \Delta M^{t}_{s} &= \sum_{j}\frac{X^{t_0}_{\text{China}j,s}}{L^{t_0}_{\text{US},s}}\Delta\log X^{t}_{\text{China}j,s} \\ &= \sum_{j}\frac{X^{t_0}_{\text{China}j,s}}{L^{t_0}_{\text{US},s}}\left((1-\sigma)\hat{\tau}^{t}_{\text{China}j,s} - \kappa\hat{w}^{t}_{\text{China},s} + \Lambda^{t}_{j,s}\right). \end{aligned}$$

By setting the Chinese wage as the numeraire ($\hat{w}^t_{\text{China},s} = 0$), the expression above implies that

$$\Delta M^t_s = \sum_j \frac{E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}} \left((1-\sigma) x^{t_0}_{\text{China}j,s} \hat{\tau}^t_{\text{China}j,s} \right) + \frac{\sum_j X^{t_0}_{\text{China}j,s} \Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}}.$$

The decomposition $(1-\sigma) x^{t_0}_{\text{China}j,s} \hat{\tau}^t_{\text{China}j,s} = \hat{\zeta}^t_{\text{China},s} + \hat{\varepsilon}^t_{\text{China}j,s}$ implies that

$$\Delta M_s = \left(\frac{\sum_j E^{t_0}_{j,s}}{L^{t_0}_{\text{US},s}} \right) \left(\hat{\zeta}^t_{\text{China},s} + \sum_j \frac{E^{t_0}_{j,s}}{\sum_{j'} E^{t_0}_{j',s}} \hat{\varepsilon}^t_{\text{China}j,s} \right) + \frac{\sum_j X^{t_0}_{\text{China}j,s} \Lambda^t_{j,s}}{L^{t_0}_{\text{US},s}},$$

which immediately yields expression (39) under the assumption that $\sum_j \frac{E^{t_0}_{j,s}}{\sum_{j'} E^{t_0}_{j',s}} \hat{\varepsilon}^t_{\text{China}j,s} \approx 0$.

A.4.2 Specification of $\Omega(\boldsymbol{w})$

The definition $\Omega(\boldsymbol{w}) = (W_{\text{US}}(\boldsymbol{w}))^{\bar{\omega}} (W_{\text{W}}(\boldsymbol{w}))^{1-\bar{\omega}}$ implies that

$$\omega^{t_0}_j \equiv \frac{\partial \ln \Omega(\boldsymbol{w}^{t_0})}{\partial \ln w_j} = \bar{\omega} \frac{W^{t_0}_j}{\sum_{i \in \mathcal{I}_{\text{US}}} W^{t_0}_i} \mathbb{I}_{i \in \mathcal{I}_{\text{US}}} + (1-\bar{\omega}) \frac{W^{t_0}_j}{\sum_c \sum_{i \in \mathcal{I}_c} W^{t_0}_i} \tag{90}$$

where $W^{t_0}_i$ is the GDP of market i in the initial period t_0.

To compute $\omega^{t_0}_j$, we need to specify $\bar{\omega}$. We do so using the series of the opportunity cost of not working in Chodorow-Reich and Karabarbounis (2016). In our model, the average change in the payoff of not working in the U.S. is $\hat{z}^t_{\text{US}} \equiv \sum_{i \in \mathcal{I}_{\text{US}}} \frac{N_i}{N_{\text{US}}} (\hat{b}_i - \hat{P}_i)$. By defining the U.S. price index change as $\hat{P}^t_{\text{US}} \equiv \sum_{i \in \mathcal{I}_{\text{US}}} \frac{N_i}{N_{\text{US}}} \hat{P}_i$, the fact that $\hat{b}^t_i = \hat{\Omega}$ yields

$$\begin{aligned} \hat{z}^t_{\text{US}} &= \bar{\omega} \hat{W}^t_{\text{US}} + (1-\bar{\omega}) \hat{W}^t_{\text{W}} - \hat{P}^t_{\text{US}} \\ &= \left(\hat{W}^t_{\text{US}} - \hat{P}^t_{\text{US}} \right) - (1-\bar{\omega}) \left(\hat{W}^t_{\text{US}} - \hat{W}^t_{\text{W}} \right) \\ &= \left(\hat{W}^t_{\text{US}} - \hat{P}^t_{\text{US}} \right) - (1-\bar{\omega}) \left(\left(\hat{W}^t_{\text{US}} - \hat{P}^t_{\text{US}} \right) - \left(\hat{W}^t_{\text{W}} - \hat{P}^t_{\text{W}} \right) + \left(\hat{P}^t_{\text{US}} - \hat{P}^t_{\text{W}} \right) \right). \end{aligned}$$

By defining the real income of a country as $\widehat{RW}^t_c = \hat{W}^t_c - \hat{P}^t_c$ and the relative price as $\hat{\mathcal{P}}^t_c = \hat{P}^t_c - \hat{P}^t_{\text{W}}$, this expression is equivalent to

$$\hat{z}^t_{\text{US}} = \widehat{RW}^t_{\text{US}} - (1-\bar{\omega}) \left(\widehat{RW}^t_{\text{US}} - \widehat{RW}^t_{\text{W}} + \hat{\mathcal{P}}^t_{\text{US}} \right).$$

Thus,

$$Cov\left(\widehat{RW}^t_{\text{US}}, \hat{z}^t_{\text{US}} \right) = Var\left(\widehat{RW}^t_{\text{US}} \right) - (1-\bar{\omega}) Cov\left(\widehat{RW}^t_{\text{US}}, \widehat{RW}^t_{\text{US}} - \widehat{RW}^t_{\text{W}} + \hat{\mathcal{P}}^t_{\text{US}} \right)$$

and, therefore,

$$\bar{\omega} = 1 - \frac{Var\left(\widehat{RW}^t_{\text{US}} \right) - Cov\left(\widehat{RW}^t_{\text{US}}, \hat{z}^t_{\text{US}} \right)}{Cov\left(\widehat{RW}^t_{\text{US}}, \widehat{RW}^t_{\text{US}} - \widehat{RW}^t_{\text{W}} + \hat{\mathcal{P}}^t_{\text{US}} \right)}. \tag{91}$$

We obtain $\bar{\omega} = 0.62$ using expression (91) computed with year-to-year log-changes in every variable between 1961 and 2012. To measure $\hat{z}^t_{\text{US}}$, we use the series in Chodorow-Reich and Karabarbounis (2016) of the opportunity cost of employment implied by their separable utility specification at the first quarter of each year (available for 1961-2012). We measure all other variables using data from the Penn World Tables produced with the methodology in Feenstra et al. (2015). Specifically, we measure $\widehat{RW}_c^t$ using the annual series of the real domestic absorption at current PPPs (CDA) divided by population (pop), and $\hat{P}_c^t$ using the annual series of the Price level of CDA (PPP/XR). We compute the world average of the log-change in each variable as the average log-change in that variable across all countries, weighted by the country's share in world GDP in the previous year.

A.5 Adding Frictional Unemployment to Model of Section 3

We now outline an extension of the model in Section 3 featuring frictional unemployment. It yields an expression for the change in the employment rate in terms of changes in w_i/b_i with an elasticity that combines the parameters controlling responses in both the labor force participation and the unemployment rate.

Environment. We consider the same preferences as in our baseline model, with $l(\iota)$ and $u(\iota)$ denoting ι's efficiency units and non-employment income. As in the baseline, individuals draw $(l(\iota), u(\iota))$ independently from a Frechet distribution with shape parameter $\phi > 1$ and scale 1. Given the uncertainty in the job search process, we assume that individuals are risk neutral.

As in our baseline, each sector s of market i has a representative firm that produces a differentiated good subject to iceberg trade costs. We now assume that production depends on a CES aggregator of the continuum of non-traded inputs available in the market, $\nu \in \mathcal{V}_i$:

$$Q_{i,s} = \left[\int_{\nu \in \mathcal{V}_i} (q_{i,s}(\nu))^{\frac{\mu-1}{\mu}}\, d\nu\right]^{\frac{\mu}{\mu-1}}, \tag{92}$$

where $\mu > 1$ is the elasticity of substitution between non-traded varieties.

We assume that the economy has a fixed pool of potential producers of the non-traded inputs that operate in monopolistic competition. In order to produce, firms need to get matched with a worker. If the owner of the firm does not post a vacancy, she gets an outside option payoff of $\bar{\nu}_i$. We consider a competitive search environment in which firm ν posts a wage offer $w_i(\nu)$. We analyze a symmetry equilibrium in which all firms post the same wage (i.e., $w_i(\nu) = w_i$), and then are randomly matched with a worker in the economy. Conditional on being matched to individual ι, intermediate producers have a linear production function such that $y_i(\nu) = l(\iota)$. The matching technology is such that, if V_i vacancies are posted and N_i^p workers search for a job, the number of matches is

$$M_i = (V_i)^{\alpha} (N_i^p)^{1-\alpha}. \tag{93}$$

Labor Force Participation. We first solve for the share of individuals in market i that look for a job given an offered wage rate of w_i. Consider the case in which individual ι searches for a job. With probability M_i/N_i^p, she finds a job and has a payoff of $(1-v_i)w_i l(\iota)/P_i$; with probability $1 - M/N_i^p$, she does not find a job and has a payoff of $(1-v_i)b_i u(\iota)/P_i$. If the same individual ι does not search for a job, she gets a payoff of $(1-v_i)b_i u(\iota)/P_i$. Thus, the maximization of expected utility implies that the market's labor force participation is

$$n_i^p = \Pr\left[\frac{M_i}{N_i^p} w_i l(\iota) + \left(1 - \frac{M_i}{N_i^p}\right) b_i u(\iota) > b_i u(\iota)\right] = \Pr\left[w_i l(\iota) > b_i u(\iota)\right] \quad \Rightarrow \quad n_i^p = \frac{w_i^{\phi}}{w_i^{\phi} + b_i^{\phi}}. \tag{94}$$

As in our baseline, the mean efficiency of those searching for jobs is $\bar{l}_i = \varrho (n_i^p)^{-\frac{1}{\phi}}$.

Unemployment rate. Given the cost minimization problem of the representative firm in market i, the demand for the output of the intermediate producer ν is

$$q_i(\nu) = \frac{(p_i(\nu))^{-\mu}}{\int_{\tilde{\nu}\in\mathcal{V}_i} (p_i(\tilde{\nu}))^{1-\mu}\, d\tilde{\nu}} R_i.$$

Thus, the profit maximization problem of firm ν yields the typical constant markup expression for the price of the intermediate good:

$$\tilde{p}_i(\nu) = \frac{\mu}{\mu-1} w_i \quad \forall \nu \in \mathcal{V}_i.$$

This implies that the production cost of firms in market i is $p_{i,s} \equiv \left[\int_{\nu\in\mathcal{V}_i} (\tilde{p}_i(\nu))^{1-\mu}\, d\nu\right]^{\frac{1}{1-\mu}} = \frac{\mu}{\mu-1} w_i\,(M_i)^{\frac{1}{1-\mu}}$. In equilibrium, the number of successful matches must be equal to the number of employed individuals ($M_i = L_i$), so

$$p_{i,s} = \frac{\mu}{\mu-1} w_i\,(L_i)^{-\psi} \quad \text{such that} \quad \psi \equiv \frac{1}{\mu-1}. \tag{95}$$

Finally, the free entry condition implies that the expected profit of posting a vacancy must be equal to the outside option of not posting it. Given that the probability of filling a vacancy is M_i/V_i and that the expected efficiency of a match is $\bar{l}_i$, we have that

$$\bar{\nu}_i = (\tilde{p}_i(v) - w_i)\bar{l}_i \frac{M_i}{V_i} = \frac{1}{\mu-1} w_i \bar{l}_i \left(\frac{N_i^p}{V_i}\right)^{1-\alpha} \quad \Rightarrow \quad \frac{N_i^p}{V_i} = \left(\frac{(\mu-1)\bar{\nu}_i}{w_i\bar{l}_i}\right)^{\frac{1}{1-\alpha}}$$

This expression determines the share of individuals searching for a job that get matched to a producer:

$$n_i^m = \frac{M_i}{N_i^p} = \left(\frac{V_i}{N_i^p}\right)^{\alpha} = \left(\frac{w_i\bar{l}_i}{(\mu-1)\bar{\nu}_i}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{w_i\varrho(n_i^p)^{-\frac{1}{\phi}}}{(\mu-1)\bar{\nu}_i}\right)^{\frac{\alpha}{1-\alpha}}.$$

Assuming that the outside option of producers is proportional to the non-employment transfer ($\bar{\nu}_i = \nu_i b_i$), we derive our main expression for the share of individuals in market i that are employed:

$$n_i = n_i^m n_i^p = \left(\frac{\varrho}{(\mu-1)\nu_i}\frac{w_i}{b_i}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(w_i/b_i)^{\phi}}{1+(w_i/b_i)^{\phi}}\right)^{\left(1-\frac{\alpha}{1-\alpha}\frac{1}{\phi}\right)}. \tag{96}$$

Up to a first order approximation, this expression implies that

$$\hat{n}_i = \hat{n}_i^m + \hat{n}_i^p = \left(\frac{\alpha}{1-\alpha} n_i^p + \phi(1-n_i^p)\right)(\hat{w}_i - \hat{b}_i).$$

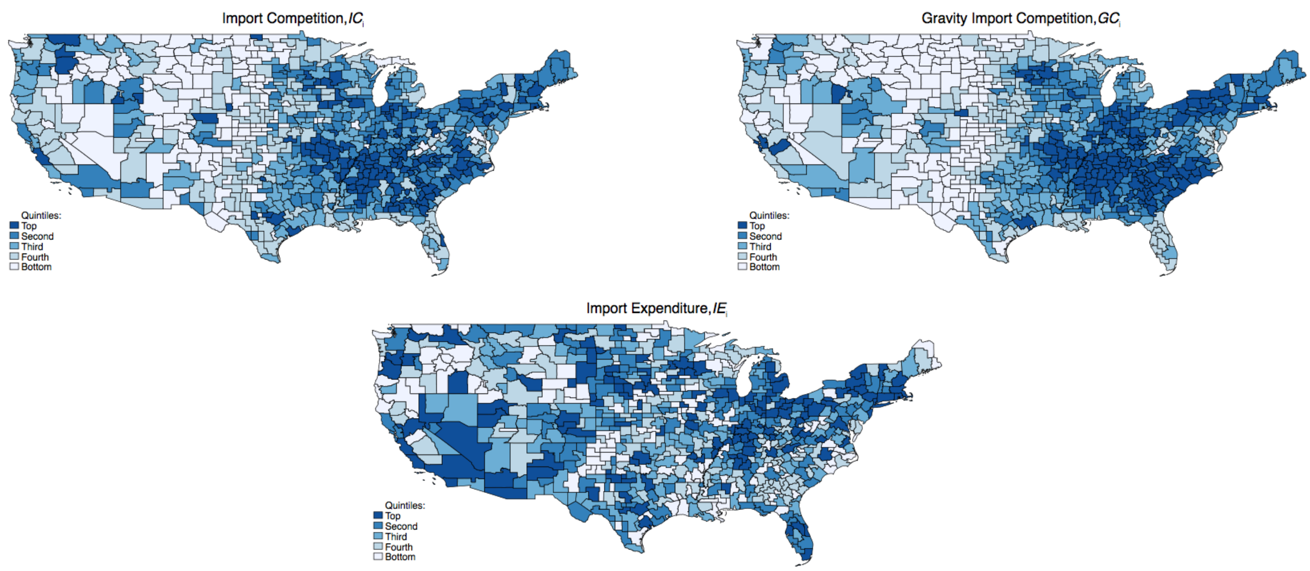
The elasticity of the employment rate to the wage rate has two components. As before, it entails the elasticity of the labor force participation margin, $\phi(1-n_i^p)$. But now it also encompasses the elasticity of the matching rate, $\frac{\alpha}{1-\alpha} n_i^p$, which depends on the matching technology parameter α. Whenever $\alpha = 0$, all individuals search for a job get a match and this term disappears.

B Appendix: Additional Empirical Results (Not for publication)

B.1 Adjustment of U.S. Regional Markets to Trade Shocks: Three Stylized Facts

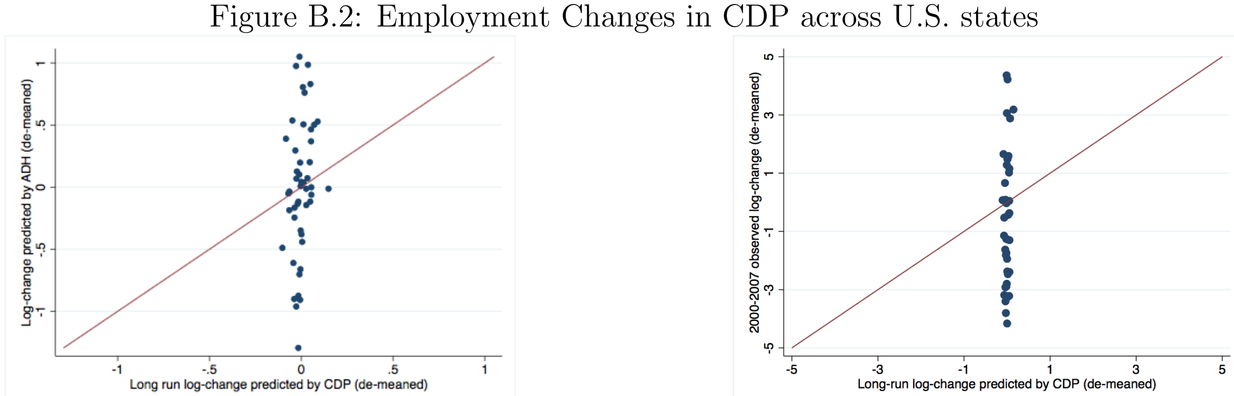
This appendix presents additional empirical results that complement those in Section 2.

Figure B.1: Regional Exposure to the China Shock, 1990-2007



Notes: For each CZ, the left panel reports IC_i^t, the right panel reports GC_i^t, and the bottom panel reports IE_i^t.

Figure B.2: Employment Changes in CDP across U.S. states



Notes: The figure on the left displays a scatter plot of log-changes in employment rate between 2000 and 2007 predicted by the ADH model (shown in column (1) of Table 1) and aggregated at the state level, against the corresponding log-change predicted by Caliendo et al (2019), together with the 45 degree line. The figure on the right displays a scatter plot of changes in employment rate observed between 2000 and 2007 against the corresponding log-change predicted by Caliendo et al (2019), together with the 45 degree line.

Table B.1: Summary Statistics of Outcomes for U.S. CZs

	1990-2000		2000-2007		1990-2007	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
100 x Change in average weekly log-wage	12.39	4.65	3.84	5.51	16.23	6.47
100 x Change in log of employment rate	1.27	4.23	1.71	5.31	2.98	6.42
IC_i^t	1.01	1.06	2.52	2.54	3.52	3.35
IE_i^t	2.51	0.58	7.39	1.31	9.90	1.62
GC_i^t	1.03	0.88	2.60	1.99	3.64	2.73

Notes: Sample of 722 Commuting Zones.

Table B.2: Impact of the China Shock on U.S. CZs, Alternative Specifications I

	(1)	(2)	(3)	(4)	(5)
***Panel A:* Change in average log weekly wage**					
IC_i^t	-0.471***	-0.368***	-0.475***	-0.383***	-0.383***
	(0.127)	(0.104)	(0.138)	(0.113)	(0.114)
GC_i^t		-0.601***		-0.606***	-0.600***
		(0.155)		(0.156)	(0.174)
IE_i^t			0.023	0.077	0.079
			(0.168)	(0.164)	(0.145)
$\sum_{j\neq i} z_{ij} IE_j^t$					-0.025
					(0.310)
R^2	0.517	0.526	0.517	0.527	0.527
***Panel B:* Change in log of employment rate**					
IC_i^t	-0.519***	-0.400***	-0.474***	-0.369***	-0.363***
	(0.089)	(0.075)	(0.095)	(0.079)	(0.079)
GC_i^t		-0.700***		-0.691***	-0.582***
		(0.156)		(0.155)	(0.158)
IE_i^t			-0.216	-0.154	-0.106
			(0.146)	(0.143)	(0.140)
$\sum_{j\neq i} z_{ij} IE_j^t$					-0.516*
					(0.261)
R^2	0.300	0.326	0.302	0.327	0.330
***Panel C:* Change in log of working-age population**					
IC_i^t	0.273	0.209	0.180	0.127	0.118
	(0.180)	(0.159)	(0.172)	(0.155)	(0.152)
GC_i^t		0.372*		0.348	0.191
		(0.217)		(0.212)	(0.204)
IE_i^t			0.449	0.418	0.349
			(0.292)	(0.294)	(0.277)
$\sum_{j\neq i} z_{ij} IE_j^t$					0.739
					(0.469)
R^2	0.309	0.310	0.310	0.312	0.313

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Indirect effects computed as in Table 1: $z_{ij} \equiv D_{ij}^{-5} / \sum_k D_{ik}^{-5}$ where D_{ij} is the distance between CZs i and j. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.3: Impact of the China Shock on U.S. CZs, Alternative Specifications II

	(1)	(2)	(3)	(4)	(5)	(6)
***Panel A:* Change in average log weekly wage**						
IC_i^t	-0.383***	-0.383***	-0.357***	-0.426***	-0.104	-0.283*
	(0.113)	(0.135)	(0.107)	(0.112)	(0.106)	(0.158)
GC_i^t	-0.606***	-0.606**	-0.528***	-0.720***	-0.284***	-0.670***
	(0.156)	(0.262)	(0.125)	(0.174)	(0.103)	(0.187)
IE_i^t	0.077	0.077	0.062	0.070	-0.043	-0.346
	(0.164)	(0.092)	(0.160)	(0.164)	(0.143)	(0.252)
R^2	0.527	0.527	0.538	0.563	0.578	0.592
***Panel B:* Change in log of employment share**						
IC_i^t	-0.369***	-0.369***	-0.352***	-0.365***	-0.159**	-0.567***
	(0.079)	(0.107)	(0.076)	(0.077)	(0.060)	(0.141)
GC_i^t	-0.691***	-0.691***	-0.641***	-0.717***	-0.449***	-0.792***
	(0.155)	(0.258)	(0.159)	(0.153)	(0.139)	(0.222)
IE_i^t	-0.154	-0.154	-0.153	0.070	-0.244**	-0.180
	(0.143)	(0.078)	(0.142)	(0.164)	(0.118)	(0.225)
R^2	0.327	0.327	0.332	0.395	0.383	0.383
***Panel C:* Change in log of working-age population**						
IC_i^t	0.127	0.127	0.014	0.065	0.176	-0.301
	(0.155)	(0.124)	(0.123)	(0.145)	(0.165)	(0.434)
GC_i^t	0.348	0.348	0.040	0.111	0.404*	-0.078
	(0.212)	(0.133)	(0.129)	(0.223)	(0.213)	(0.453)
IE_i^t	0.418	0.418	0.194	0.523*	0.397	0.707
	(0.294)	(0.330)	(0.220)	(0.284)	(0.288)	(0.574)
R^2	0.312	0.312	0.442	0.442	0.312	0.444
Control set:						
Baseline controls	Y	Y	Y	Y	Y	Y
Lagged population growth	N	N	Y	N	N	N
State dummies	N	N	N	Y	N	N
Manuf share x period dummy	N	N	N	N	Y	N
Observations weights:						
Population	N	N	N	N	N	Y
No weights	Y	Y	Y	Y	Y	N
Inference:						
State clustered	Y	N	Y	Y	Y	Y
Adão et al. (2019)	N	Y	N	N	N	N

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Baseline controls defined in Table 1. Lagged population growth from Greenland et al. (2019): growth of population with 15-34 years old and 35-64 years old in the previous 10-year period. Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.4: Impact of the China Shock on U.S. CZs, Alternative Indirect Effects

	(1)	(2)	(3)	(4)	(5)
Panel A: **Change in average log weekly wage**					
IC_i^t	-0.383***	-0.321***	-0.425***	-0.403***	-0.441***
	(0.113)	(0.110)	(0.125)	(0.122)	(0.129)
GC_i^t	-0.606***	-7.647***	-0.457***	-0.956***	-1.623**
	(0.156)	(2.365)	(0.136)	(0.297)	(0.790)
IE_i^t	0.077	0.111	0.054	0.052	0.014
	(0.164)	(0.166)	(0.165)	(0.163)	(0.161)
R^2	0.527	0.536	0.521	0.523	0.525
Panel B: **Change in log of employment share**					
IC_i^t	-0.369***	-0.265***	-0.410***	-0.389***	-0.439***
	(0.079)	(0.069)	(0.086)	(0.089)	(0.087)
GC_i^t	-0.691***	-10.39***	-0.586***	-1.138***	-1.663***
	(0.155)	(1.464)	(0.145)	(0.224)	(0.508)
IE_i^t	-0.154	-0.096	-0.176	-0.181	-0.225
	(0.143)	(0.129)	(0.145)	(0.144)	(0.137)
R^2	0.327	0.370	0.315	0.320	0.319
Panel C: **Change in log of working-age population**					
IC_i^t	0.127	0.071	0.138	0.129	0.163
	(0.155)	(0.141)	(0.160)	(0.159)	(0.173)
GC_i^t	0.348	5.430	0.388*	0.685	0.789
	(0.212)	(3.421)	(0.230)	(0.484)	(0.959)
IE_i^t	0.418	0.387	0.423	0.428	0.454
	(0.294)	(0.303)	(0.291)	(0.292)	(0.293)
R^2	0.312	0.314	0.312	0.312	0.311
Indirect Effect Specification:					
Definition of z_{ij}	$\frac{D_{ij}^{-5}}{\sum_k D_{ik}^{-5}}$	$\frac{D_{ij}^{-1}}{\sum_k D_{ik}^{-1}}$	$\frac{D_{ij}^{-8}}{\sum_k D_{ik}^{-8}}$	$\frac{L_j^0 D_{ij}^{-5}}{\sum_k L_k^0 D_{ik}^{-5}}$	$\frac{L_j^0 St_{ij}}{\sum_k L_k^0 St_{ik}}$

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Indirect effects given by $GC_i^t \equiv \sum_{j \neq i} z_{ij} IC_j^t$ where z_{ij} is specified in each column, D_{ij} is the distance between CZs i and j, L_j^0 is the population of CZ j in 1990, and St_{ij} is a dummy that equals one if CZs i and j belong to the same state. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.5: Impact of the China Shock on U.S. CZs, Alternative Spending Shares

	(1)	(2)	(3)	(4)
***Panel A:* Change in average log weekly wage**				
IC_i^t	-0.383***	-0.397***	-0.396***	-0.348***
	(0.113)	(0.114)	(0.112)	(0.112)
GC_i^t	-0.606***	-0.589***	-0.590***	-0.563***
	(0.156)	(0.154)	(0.155)	(0.156)
IE_i^t	0.077			
	(0.164)			
IEI_i^t		0.110	0.101	-0.548
		(0.107)	(0.140)	(0.452)
IEF_i^t		0.267	0.246	0.239
		(0.757)	(0.755)	(0.754)
R^2	0.527	0.530	0.532	0.532
***Panel B:* Change in log of employment share**				
IC_i^t	-0.369***	-0.395***	-0.393***	-0.344***
	(0.079)	(0.081)	(0.078)	(0.073)
GC_i^t	-0.691***	-0.681***	-0.680***	-0.652***
	(0.155)	(0.151)	(0.151)	(0.147)
IE_i^t	-0.154			
	(0.143)			
IEI_i^t		-0.003	-0.014	-0.804**
		(0.091)	(0.111)	(0.389)
IEF_i^t		0.175	0.174	0.182
		(0.489)	(0.487)	(0.484)
R^2	0.327	0.329	0.329	0.332
***Panel C:* Change in log of working-age population**				
IC_i^t	0.127	0.122	0.157	0.188
	(0.155)	(0.149)	(0.149)	(0.157)
GC_i^t	0.348	0.326	0.334	0.348
	(0.212)	(0.211)	(0.212)	(0.218)
IE_i^t	0.418			
	(0.294)			
IEI_i^t		0.307*	0.195	0.041
		(0.181)	(0.205)	(0.844)
IEF_i^t		0.291	0.272	0.267
		(0.722)	(0.718)	(0.713)
R^2	0.312	0.315	0.314	0.313
Construction of* IEI_i^t*:				
Drop final spending	N	Y	Y	Y
Drop own industry spending	N	N	Y	N
Use Leontief IO shares	N	N	N	Y

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Col. (1) is the baseline specification in which $IE_i^t \equiv \sum_s e_{i,s}^{t_0} \Delta M_s^t$, where $e_{i,s}^{t_0}$ is the share of gross spending in sector s (as defined in Section 2.2). In cols. (2)-(4), $IEF_i^t \equiv \sum_s ef_{i,s}^{t_0} \Delta M_s^t$ is the exposure to the shock in final import expenditure, where $ef_{i,s}^{t_0}$ is the share of household spending on sector s in CZ i constructed from the Consumer Expenditure Survey (as described in Appendix C.2.1). In col. (2), $IEI_i^t \equiv \sum_s ei_{i,s}^{t_0} \Delta M_s^t$ is the exposure to the shock in intermediate import expenditure, where $ei_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_k a_k^{t_0} \ell_{i,k}^{t_0}$ is the share of intermediate spending on sector s in CZ i. In col. (3), we compute IEI_i^t using $ei_{i,s}^{t_0} \equiv \sum_{k \neq s} \xi_{sk}^{M,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_{k \neq s} a_k^{t_0} \ell_{i,k}^{t_0}$ that ignores the own industry input spending. In col. (4), $IEI_i^t \equiv \sum_s eiL_{i,s}^{t_0} \Delta M_s^t$ where $eiL_{i,s}^{t_0} \equiv \sum_k \xi_{sk}^{L,t_0} a_k^{t_0} \ell_{i,k}^{t_0} / \sum_{s,k} \xi_{sk}^{L,t_0} a_k^{t_0} \ell_{i,k}^{t_0}$ is the share of total intermediate spending on sector s in CZ i, with ξ_{sk}^{L,t_0} defined as the Leontief input spending shares from Acemoglu et al. (2016a). Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.6: Impact of the China Shock on U.S. CZs, Alternative Sectoral Shifters

	Change in average weekly log-wage		Change in log of employment rate		Change in log of working-age population	
	(1)	(2)	(3)	(4)	(5)	(6)
***Panel A:* China exporter-sector gravity fixed-effect**						
IC_i^t	-0.177**	-0.094	-0.267***	-0.157***	0.115	0.064
	(0.083)	(0.081)	(0.064)	(0.052)	(0.113)	(0.110)
GC_i^t		-0.455***		-0.490***		0.022
		(0.134)		(0.126)		(0.159)
IE_i^t		-0.270		-0.557***		0.586
		(0.248)		(0.202)		(0.566)
R^2	0.506	0.513	0.281	0.298	0.308	0.308
***Panel B:* NTR gap**						
IC_i^t	-0.466***	-0.256***	-0.393***	-0.195***	0.0252	-0.0281
	(0.073)	(0.040)	(0.045)	(0.034)	(0.087)	(0.079)
GC_i^t		-0.351***		-0.355***		-0.0392
		(0.092)		(0.084)		(0.098)
IE_i^t		-0.0749		-0.013		0.324**
		(0.120)		(0.103)		(0.134)
R^2	0.570	0.584	0.360	0.388	0.307	0.309
***Panel C:* Sectoral demand shift, $\hat{\zeta}^t_{\mathbf{China},s}$**						
IC_i^t	-0.857***	-0.463***	-0.894***	-0.536***	0.161	0.049
	(0.186)	(0.143)	(0.128)	(0.118)	(0.284)	(0.199)
GC_i^t		-1.022***		-0.920***		0.292
		(0.232)		(0.203)		(0.376)
IE_i^t		-0.353		-0.065		0.167
		(0.216)		(0.175)		(0.553)
R^2	0.524	0.536	0.311	0.330	0.307	0.308
***Panel D:* Sectoral demand shift, $\hat{\zeta}^t_{\mathbf{China},s}$, with upstream exposure**						
IC_i^t	-0.740***	-0.402***	-0.819***	-0.507***	0.147	0.0104
	(0.151)	(0.116)	(0.102)	(0.090)	(0.252)	(0.175)
GC_i^t		-0.804***		-0.735***		0.326
		(0.188)		(0.176)		(0.307)
IE_i^t		-0.358*		-0.0727		0.169
		(0.212)		(0.173)		(0.550)
R^2	0.526	0.537	0.321	0.339	0.308	0.308

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Each panel presents estimates of regression (2) with different exposure measures. Panels A, B and C use the same exposure measures as in (3)-(5), but built with an alternative definition of the sectoral shifter ΔM_s^t. In panel A, the shifter is $\hat{\Gamma}^t_{i,s} M_s^{t_0}/L_s^{t_0}$, where $\hat{\Gamma}^t_{i,s}$ is the sector-origin fixed-effect for China obtained from the estimation for the periods 1991-2000 and 2000-2007 of $\Delta \log X^t_{ij,s} = \Lambda^t_{j,s} + \Gamma^t_{i,s} + \varepsilon^t_{ij,s}$ in the same sample of high-income countries used to compute the ADH IV, plus U.S. and China; $M_s^{t_0}$ is the initial level of imports in sector s of the eight high-income countries used to compute the ADH IV; and $L_s^{t_0}$ is the initial level of U.S. employment in sector s. In Panel B, the shfiter is 100 times the average NTR Gap in sector s obtained from the replication package of Pierce and Schott (2016b). In panel C, the shifter is $\hat{\zeta}^t_{\text{China},s}$ defined in Section 5.1. In panel D, the shifter is the same as in panel C, but IC_i^t is computed using equation (34) in Section 4, and GC_i^t is computed using this modified IC_i^t and equation (4) in Section 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.7: Impact of the China Shock on U.S. CZs, Employment Outcomes

	Change in the share of working-age population by category				
	Employed	Emp. in Manuf	Emp. in Non-Manuf	Unemp.	Out of labor force
	(1)	(2)	(3)	(4)	(5)
IC_i^t	-0.253***	-0.166***	-0.087**	0.095***	0.159***
	(0.055)	(0.047)	(0.037)	(0.026)	(0.039)
GC_i^t	-0.482***	-0.210***	-0.272***	0.186***	0.297***
	(0.109)	(0.053)	(0.087)	(0.046)	(0.077)
IE_i^t	-0.120	-0.102**	-0.018	0.034	0.086
	(0.101)	(0.043)	(0.090)	(0.039)	(0.075)
R^2	0.322	0.550	0.225	0.282	0.293

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table B.8: Impact of the China Shock on U.S. CZs, Migration Outcomes

	Change in the log of			
	Population	In-migration	Out-migration	Net migration
	(1)	(2)	(3)	(4)
IC_i^t	0.127	0.152	0.063	-0.090
	(0.155)	(0.184)	(0.184)	(0.114)
GC_i^t	0.348	0.052	0.077	0.025
	(0.212)	(0.325)	(0.340)	(0.113)
IE_i^t	0.418	0.670	0.515	-0.155
	(0.294)	(0.410)	(0.351)	(0.155)
R^2	0.312	0.885	0.817	0.116

Notes: Sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

B.2 Measuring the General Equilibrium Effect of The China Shock

Table B.9: Fit of the Model across U.S. CZs, all Panels

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in working-age population	Share of Manufacturing in employed population
	(1)	(2)	(3)	(4)
Panel A: Baseline Model of Section 3 (estimates of Panel A of Table 2)				
Fit Coef. (ρ^Y)	0.97	0.90	0.95	0.82
	(0.25)	(0.15)	(0.11)	(0.13)
p-value of $H_0 : \rho^Y = 1$	91.5%	51.1%	63.9%	16.0%
Panel B: Extended Model of Section 4, no intermediates (estimates of Panel B of Table 2)				
Fit Coef. (ρ^Y)	0.99	0.95	0.97	0.84
	(0.25)	(0.16)	(0.12)	(0.13)
p-value of $H_0 : \rho^Y = 1$	96.9%	73.6%	77.2%	19.4%
Panel C: Extended Model of Section 4, no intermediates (estimates of Panel C of Table 2)				
Fit Coef. (ρ^Y)	0.96	0.96	0.96	0.83
	(0.25)	(0.16)	(0.12)	(0.13)
p-value of $H_0 : \rho^Y = 1$	87.2%	81.4%	75.0%	18.3%
Panel D: Extended Model of Section 4 (estimates of Panel D of Table 2)				
Fit Coef. (ρ^Y)	1.11	1.10	0.80	0.71
	(0.48)	(0.22)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	82.6%	65.1%	22.7%	08.0%
Panel E: Extended Model of Section 4 (estimates of Panel E of Table 2)				
Fit Coef. (ρ^Y)	1.16	1.07	0.86	0.79
	(0.48)	(0.20)	(0.17)	(0.17)
p-value of $H_0 : \rho^Y = 1$	73.9%	70.5%	42.6%	21.4%

Notes: Estimation of (29) with the shock $\hat{z}_{i,s}^{\text{obs},t} = -\hat{\zeta}_{\text{China},s}^{t}$ and estimates in Table 2. Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state.

Table B.10: Fit of the Model across U.S. CZs – Inference Procedure in Adão et al. (2019)

	Dependent variable: Change in			
	Average weekly log-wage	Log of employment rate	Share of Manufacturing in working-age population	Share of Manufacturing in employed population
	(1)	(2)	(3)	(4)
Panel A: Baseline Model of Section 3 (estimates of Panel A of Table 2)				
Fit Coef. (ρ^Y)	0.97	0.90	0.95	0.82
	(0.19)	(0.20)	(0.10)	(0.14)
p-value of $H_0 : \rho^Y = 1$	88.7%	61.2%	60.7%	19.0%
Panel B: Extended Model of Section 4, no intermediates (estimates of Panel B of Table 2)				
Fit Coef. (ρ^Y)	0.99	0.95	0.97	0.84
	(0.19)	(0.27)	(0.11)	(0.14)
p-value of $H_0 : \rho^Y = 1$	95.9%	84.2%	75.5%	23.1%
Panel C: Extended Model of Section 4, no intermediates (estimates of Panel C of Table 2)				
Fit Coef. (ρ^Y)	0.96	0.96	0.96	0.83
	(0.18)	(0.22)	(0.11)	(0.14)
p-value of $H_0 : \rho^Y = 1$	83.0%	86.1%	73.4%	21.9%

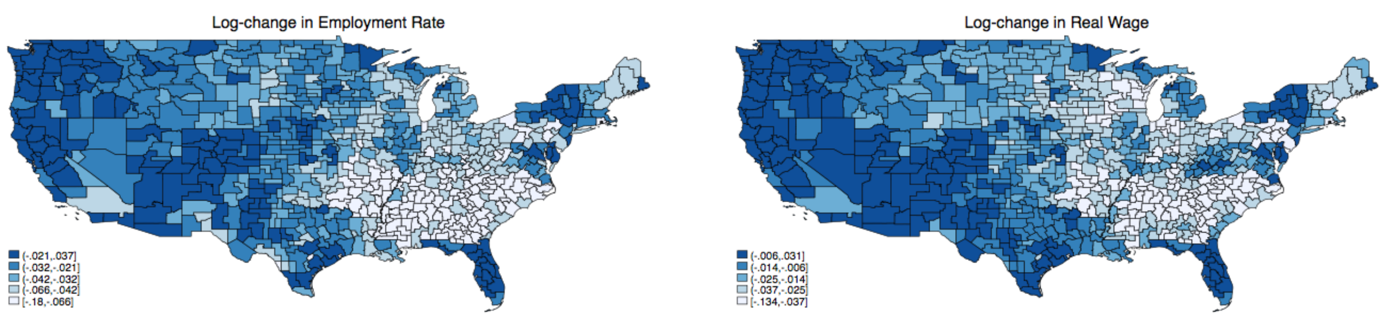
Notes: Estimation of (29) with the shock $\hat{z}_{i,s}^{\text{obs},t} = -\hat{\zeta}_{\text{China},s}^{t}$ and estimates in Table 2. Pooled sample of 1,444 CZs in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Standard errors in parentheses computed with the inference procedure for shift-share specifications in Adão et al. (2019).

Table B.11: Impact of China Shock on U.S. CZ in General Equilibrium

	Employment Rate			Log of Real Wage		
	Average	Standard Deviation	Correlation w/ Baseline	Average	Standard Deviation	Correlation w/ Baseline
Panel A: Baseline Model of Section 3 (estimates of Panel A of Table 2)						
	−2.71	1.77	1	−2.00	2.08	1
	[−2.98, −1.72]	[1.16, 1.90]		[−2.11, −1.20]	[1.55, 2.16]	
Panel B: Extended Model of Section 4, no intermediates (estimates of Panel B of Table 2)						
	−2.34	1.68	1	−1.69	2.03	1
	[−2.84, −0.01]	[1.05, 1.81]		[−1.97, 0.89]	[1.26, 2.11]	
Panel C: Extended Model of Section 4, no intermediates (estimates of Panel C of Table 2)						
	−2.34	1.65	1	−1.59	1.93	1
	[−2.65, −1.26]	[0.96, 1.90]		[−1.68, −0.57]	[1.26, 2.08]	
Panel D: Extended Model of Section 4 (estimates of Panel D of Table 2)						
	−2.21	1.02	0.63	−0.43	0.60	0.21
	[−2.88, −1.13]	[0.49, 1.18]		[−0.47, −0.10]	[0.47, 0.62]	
Panel E: Extended Model of Section 4 (estimates of Panel E of Table 2)						
	−1.88	0.90	0.63	−0.35	0.56	0.27
	[−2.64, −0.64]	[0.48, 1.10]		[−0.48, 0.13]	[0.46, 0.66]	

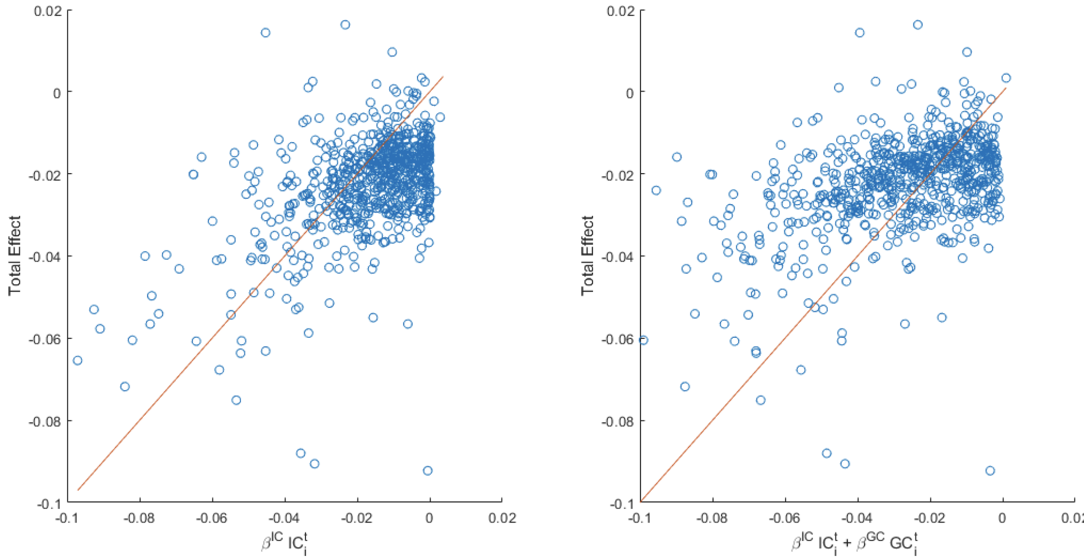
Notes: Change in outcome for each CZ is the sum of the predicted effects for that CZ in 1990-2000 and 2000-2007. Each panel presents moments based on the estimates of the corresponding specification in Table 2. Average and standard deviations are weighted by the CZ employment in 1990. Correlation is the correlation with the baseline model of Panel A. 95% Confidence Intervals in brackets computed with a bootstrap procedure on the asymptotic distribution of the estimator of the corresponding panel in Table 2.

Figure B.3: Impact of the China Shock in General Equilibrium



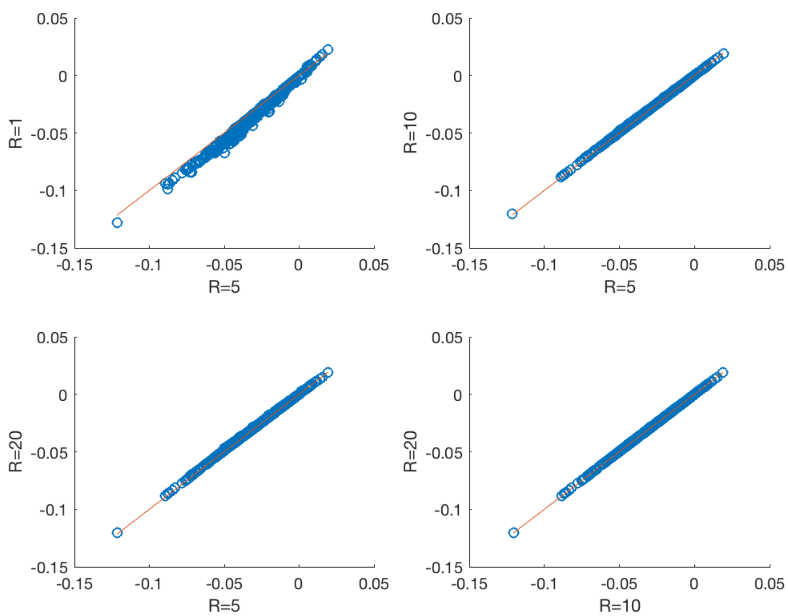
Notes: The map on the left displays the change in the log of the employment rate for each of the 722 CZs computed with the sum of the predicted effects for that CZ in 1990-2000 and 2000-2007 as implied by the model of Section3 with parameters in Panel A of Table 2. The map on the right displays the corresponding change in the log of the real wage.

Figure B.4: Impact of the China Shock on the Log of Employment Rate in General Equilibrium, Extended Model of Section 4



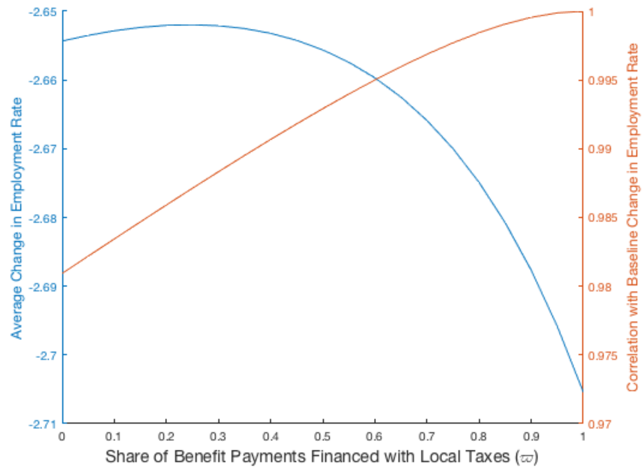
Notes: The y-axis on both graphs is the log-change in the employment rate for each of the 722 CZs in 2000-2007 implied by the model of Section 4 with parameters from Panel E of Table 2. The x-axis in the right panel is the change predicted by the specification in column (3) of Panel C in Table B.6, $\beta^{IC}IC_i^t$. The x-axis in the left panel is the change predicted by the specification in column (4) of Panel C in Table B.6, $\beta^{IC}IC_i^t + \beta^{GC}GC_i^t$. In both cases, we compute the exposure measures with $\hat{\zeta}^t_{\text{China},s}$ (instead of ΔM_s^t).

Figure B.5: Impact of the China Shock on the Employment Rate in General Equilibrium, Integral of First-Order Approximation



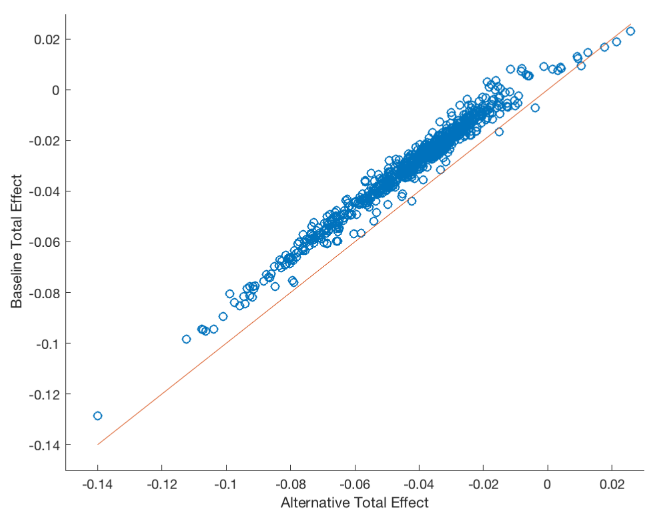
Notes: Each dot is the predicted change in the employment rate of each of the 722 CZs in 2000-2007 that we compute using the integration algorithm in Appendix A.3.3 for R partitions of the shift in sectoral demand caused by the China shock, $\hat{\zeta}^t_{\text{China},s}$. Baseline model of Section 3 with parameters in Panel A of Table 2.

Figure B.6: Impact of the China Shock on the Employment Rate in General Equilibrium, Alternative Regional Transfers Schemes



Notes: For each value of the share of benefit payments financed with local taxes (ϖ) in the alternative specification of the model in Appendix A.2.6, the figure reports the average employment rate change across U.S. CZs (blue line), and the correlation with the employment rate change implied by the baseline model (in which $\varpi = 1$).

Figure B.7: Impact of the China Shock on the Employment Rate in General Equilibrium, Alternative Assumptions about Shock Exposure of Foreign Countries



Notes: Each dot is the predicted change in the employment rate of each of the 722 CZs computed with different assumptions about the shock exposure of foreign countries (excluding China). Baseline model of Section 3 with parameters in Panel A of Table 2. The shock exposure of foreign countries (excluding China) is set to zero in the y-axis (as in our baseline), and it is set to $-\sum_s \ell^0_{i,s} \hat{\zeta}^t_{\text{China},s}$ in the x-axis (with $\ell^0_{i,s}$ measured as the share of setor s in country i's output).

C Appendix: Data Construction (Not for publication)

This appendix describes the procedure to construct the data used in Section 5.

C.1 Bilateral Trade Matrix

C.1.1 Data Construction

Country-to-country bilateral trade matrix. We start by creating a country-to-country matrix of trade flows at the 4-digit SIC classification. We consider the countries listed in Table C.1. We obtain international trade flows at the product-country level from the BACI dataset, assembled by CEPII, which we aggregate at the 4-digit SIC level. Since the starting year of the BACI dataset is 1995, we use the trade flows for 1995 and 2000.[50] To obtain domestic spending shares for each country, we note first that our gravity model implies that $X^t_{ij,s} = (\tau^t_{ij,s} p^t_{i,s})^{1-\sigma} (P^t_{j,s})^{\sigma-1} E^t_{j,s}$. For any sector s within an aggregate sector S, assume that, for $i \neq j$, $\tau^t_{ij,s} = \tilde{\tau}^{O,t}_{i,S} \tilde{\tau}^{D,t}_{j,S} e^{\tilde{\tau}^t_{ij,s}}$. Thus,

$$\ln X^t_{ij,s} = \tilde{\tau}^t_{ij,s} + \alpha^t_{i,s} + \varphi^t_{j,s}, \tag{C.1}$$

where $\alpha^t_{i,s} \equiv \ln \left(\left(\tilde{\tau}^{O,t}_{i,S} p^t_{i,s} \right)^{1-\sigma} \right)$ and $\varphi^t_{j,s} \equiv \ln \left(\left(\tilde{\tau}^{D,t}_{j,S} P^t_{j,s} \right)^{\sigma-1} E^t_{j,s} \right)$.

To get the domestic trade flows, notice that $X^t_{ii,s} = (p^t_{i,s})^{1-\sigma} (P^t_{i,s})^{\sigma-1} E^t_{i,s} = \left(e^{\alpha^t_{i,s}} e^{\varphi^t_{i,s}} \right) / \left(\tilde{\tau}^{O,t}_{i,S} \tilde{\tau}^{D,t}_{i,S} \right)$. Since $X^t_{ii,S} = \sum_{k \in S} X^t_{ii,k}$,

$$X^t_{ii,s} = X^t_{ii,S} \frac{e^{\alpha^t_{i,s}} e^{\varphi^t_{i,s}}}{\sum_{k \in S} e^{\alpha^t_{i,k}} e^{\varphi^t_{i,k}}} \tag{C.2}$$

We use (C.2) to compute $X^t_{ii,s}$. In each year t, we obtain $\alpha^t_{i,s}$ and $\varphi^t_{j,s}$ from the estimation of (C.1) with bilateral trade flows by sector, and $X^t_{ii,S}$ from the domestic sales in two aggregate sectors in the Eora MRIO dataset: manufacturing and non-manufacturing.

CZ employment share. We use the same imputation procedure of ADH to compute employment in each 4-digit SIC manufacturing industry for 1980, 1990 and 2000 using the County Business Pattern (CBP). In year t, we use $L^t_{i,s}$ to denote employment in CZ i and 4-digit SIC industry s and $\ell^t_{i,s} = L^t_{i,s} / L^t_i$ to denote the associated employment share.

CZ gross spending shares. We construct gross spending by sector and CZ, $e^t_{i,s}$, using

$$e^t_{i,s} \equiv \frac{E^t_{i,s}}{E^t_i} = \frac{\xi^t_{i,s} + \sum_k \xi^{M,t}_{i,sk} a^t_k \ell^t_{i,k}}{1 + \sum_k a^t_k \ell^t_{i,k}}. \tag{C.3}$$

where, in year t, $\xi^{M,t}_{i,sk}$ is the share of spending on intermediates of sector s by sector k (common to all CZs, $\xi^{M,t}_{i,sk} = \xi^{M,t}_{sk}$), a^t_k is the ratio of intermediate cost to labor cost of sector k (common to all CZs), and $\xi^t_{i,s}$ is consumers' spending share on final goods of sector s (common to all CZs, $\xi^t_{i,s} = \xi^t_s$). We compute $\xi^t_{sk} \equiv \frac{M^t_{sk}}{\sum_{s'} M^t_{s'k}}$ where M^t_{sk} is the spending of industry k on industry s in the BEA 1992 U.S. Input-Output table used in Acemoglu et al. (2016a). For manufacturing SIC-4 industries, we compute a^t_k using total material costs divided by payroll in the NBER manufacturing database for year t. For non-manufacturing

[50] Although trade data is available for 1990 from UN Comtrade, it is quite sparse across countries and industries.

Table C.1: Sample of Countries

Argentina	Czech Republic	Malaysia	Singapore
Australia	Denmark	Mexico	Slovakia
Austria	Finland	Netherlands	South Africa
Baltic Republics	France	New Zealand	South Korea
Belarus	Germany	Norway	Spain
Benelux	Greece	Pakistan	Sweden
Brazil	Hungary	Philippines	Switzerland
Bulgaria	India	Poland	Taiwan
Canada	Indonesia	Portugal	Thailand
Chile	Ireland	Rest of World	Ukraine
China	Italy	Romania	United Kingdom
Colombia	Japan	Russia	Uruguay
Croatia	Kazakhstan	Saudi Arabia	Venezuela

Notes: Baltic Republics includes Estonia, Lithuania and Latvia.

industries, we compute a_k^t as average the material to payroll ratio across all U.S. non-manufacturing industries in the WIOD database. Finally, we obtain ξ_s^t from the BEA 1992 U.S. Input-Output table.

CZ exports and imports. We follow three steps to create exports and imports for each CZ and industry. First, we compute the CZ spending on sector s as $E_{i,s}^t = e_{i,s}^t L_i^t$ where $e_{i,s}^t$ is the sectoral spending share described above and L_i^t is the total employment in the CZ. Second, for each sector s, we compute the share of CZ i in national spending, $\tilde{e}_{i,s}^t = E_{i,s}^t / \sum_j E_{j,s}^t$, and in national employment, $\tilde{\ell}_{i,s}^t = L_{i,s}^t / \sum_j L_{j,s}^t$. Third, we use the US Census data at the state-sector level for 1997 to compute the share of each state in the exports/imports to/from each foreign country in a SCTG category, which is the 40-sector classification used by the US Census.[51] This yields $\beta_{state,i,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign importer, and $\beta_{i,state,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign exporters. We use the same share $\beta_{state,i,s}$ and $\beta_{state,i,s}$ for all SIC-4 industries within the same SCTG category. Finally, in each year t, we take US imports $X_{i,US,s}^t$ and US exports $X_{US,i,s}^t$ in each sector s and foreign country i, and split them across CZs using the following expressions:

$$X_{ij,s}^t = \frac{\tilde{e}_{j,s}^t}{\sum_{j' \in state} \tilde{e}_{j',s}^t} \beta_{i,state,s} X_{i,US,s}^t \quad \text{and} \quad X_{ji,s}^t = \frac{\tilde{\ell}_{j,s}^t}{\sum_{j' \in state} \tilde{\ell}_{j',s}^t} \beta_{state,i,s} X_{US,i,s}^t.$$

CZ-to-CZ bilateral trade matrix. We follow three steps to impute trade flows across CZs using the gravity trade structure of our model. First, for each SCTG category, we use state-to-state shipment data from the Commodity Flow Survey in 1997 to estimate

$$\ln X_{ij,s} = \delta_s + \beta_1 \ln D_{ij} + \beta_2 \ln E_{j,s} + \beta_3 \ln R_{i,s} + \beta_4 d_{i=j} + \varepsilon_{ij,s} \tag{C.4}$$

[51]We construct state-sector exports and imports as follows. First, we use the US Merchandise Trade Data for 1997 released by the US Census to create a mapping from each of the 44 US districts to the 50 US states, in terms of share of imports and exports to each foreign country. Note that this is done at the aggregate level as this information is not available at the industry-level. We then use US Census data to create district-level exports and imports at the HS-6 level for 1997. Finally, we use the mapping previously constructed to obtain state-HS6, and then state-SIC 4 digit, trade flows with our sample of foreign countries.

where i is the origin state, j is the destination state, s is the SCTG category, D_{ij} is the bilateral distance between the population centroids of states i and j, $E_{j,s}$ are expenditures, $R_{i,s}$ are revenues, $d_{i=j}$ is a dummy that equals 1 when $i = j$.

Second, we use the estimated coefficients to impute trade flows across CZs with the following gravity specification:

$$\ln X^t_{ij,s} \equiv \hat{\beta}_1 \ln D_{ij} + \hat{\beta}_2 \ln \tilde{e}^t_{j,s} + \hat{\beta}_3 \ln \tilde{\ell}^t_{i,s} + \hat{\beta}_4 d_{state(i)=state(j)} \tag{C.5}$$

where D_{ij} is the distance between the population centroids of CZs i and j, and $d_{state(i)=state(j)}$ is a dummy equal 1 if i and j belong to the same state.

Lastly, we re-scale the imputed CZ-to-CZ trade flows so that the sum of the bilateral flows in each SIC sector across all CZs is equal to the total U.S. domestic sales in each SIC sector in the country-to-country trade matrix.

Trade balance. Finally, we impose that trade is balanced at the regional level, as in the baseline model. We use the trade flows obtained above to compute matrix $\bar{\boldsymbol{x}}^t$ whose entries correspond to the share of spending of each region j on another region i. Under trade balance, the vector of total revenue in the world economy, $\boldsymbol{R}^t$, must satisfy $\bar{\boldsymbol{x}}^t \boldsymbol{R}^t = \boldsymbol{R}^t$ and, therefore, $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)\boldsymbol{R}^t = 0$. Notice that it is always possible to find a vector $\boldsymbol{R}^t$ that satisfies this system since $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ is singular ($\sum_i x^t_{ij} = 1$ for every j). Thus, we find the vector $\boldsymbol{R}^t$ as the eigenvector of $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ associated with the eigenvalue of zero. Without loss of generality, we then normalize it such that world GDP is one, $\sum_i R^t_i = 1$.

C.1.2 Validation Tests

We first evaluate the correlation between the expenditure shares $e^t_{i,s}$ constructed in equation (C.3) and the spending shares implied by the shipment data for U.S. states. To this end, for each of the 40 SCTG categories, we compute state-level total shipment inflow in the Commodity Flow Survey (CFS) for 1997. We then construct state-level spending shares at each SCTG category using the expenditure shares $e^t_{i,s}$ in equation (C.3) for the CZs in the state. Specifically, we first aggregate our expenditure shares at the SCTG level using a crosswalk between SIC-4 and SCTG categories, and then compute total spending by SCTG in each state using the total expenditure of the CZs in that state. Table C.2 reports the result of a regression of the expenditure shares computed from the CFS on our constructed gross spending shares in 1990 and 2000. We can see that they are positively and significantly correlated, with an OLS coefficient close to 1 and a R2 of 0.95.

We then proceed to assess whether our constructed CZ-level trade matrix reproduces the patterns of observed trade flows for U.S. states. We use the CFS to measure bilateral shipments between U.S. states in each SCTG category for 1997, 2002 and 2007. To obtain comparable data, we aggregate the bilateral trade flows for the CZs in the same state and the SIC sectors in the same SCTG category. Table C.3 reports the results of regressing actual shipment data on the corresponding trade flow obtained from our trade matrix. Column (1) considers domestic flows between U.S. states, column (2) considers export flows from U.S. states to foreign countries, and column (3) considers import flows from foreign countries to U.S. states. All specifications include sector fixed-effects. We can see that the predicted trade flows are significantly and positively related to the actual flows, with coefficients close to 1. Notice also that our imputed data captures a large share of the variation in bilateral trade flows. The R2 is above 0.8 for exports and imports of U.S. states, and around 0.5 for domestic flows between U.S. states.

Table C.2: Validation Test – Gross Expenditure Shares

Dependent variable:	Observed expenditure shares, 1997	
	(1)	(2)
Constructed expenditure shares, 1990	1.275***	
	(0.01)	
Constructed expenditure shares, 2000		1.265***
		(0.01)
Constant	-0.009***	-0.009***
	(0.00)	(0.00)
Observations	1,392	1,392
R^2	0.95	0.95

Notes: Sample of 1,392 state-SCTG pairs, where SCTG is the industry classification used in the CFS. Dependent variable is the observed expenditure share in 1997 computed from the CFS. The regressors are the expenditure shares computed in equation (C.3), aggregated at the state-SCTG level. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table C.3: Validation Test – Bilateral Trade Flows

	(1)	(2)	(3)
***Panel A:* Log of Actual Flows in 1997**			
Log of Predicted Flows in 1997	1.068***	0.973***	0.993***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.512	0.950	0.950
***Panel B:* Log of Actual Flows in 2002**			
Log of Predicted Flows in 2002	1.024***	0.847***	0.884***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.509	0.816	0.837
***Panel C:* Log of Actual Flows in 2007**			
Log of Predicted Flows in 2007	1.047***	0.797***	0.861***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.477	0.806	0.827
Flow type:			
U.S. state to U.S. state	Yes	No	No
U.S. state to Country	No	Yes	No
Country to U.S. state	No	No	Yes

Notes: The dependent variable in column (1) is the actual shipment flow reported in the CFS for state-state-SCTG triples. The dependent variables in columns (2) and (3) are trade flows constructed from the US Census trade data for state-country-SCTG triples. The regressors are the trade flows constructed using our methodology for the years 1997, 2002 and 2007, aggregated at the state-state-SCTG or state-country-SCTG level. All regressions include sector fixed effects. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

C.2 Trade in Intermediate and Final Goods

The methodology described in the previous section yields a bilateral matrix of *gross* trade flows between 722 U.S. CZs and 52 countries. While this is enough to implement the baseline model of Section 3, the estimation of the more general model with input-output links of Section 4 requires bilateral trade flows in intermediate and final goods. We now describe how we proceed to construct such data. Our procedure relies on the fact that, in our model, trade flows in final goods and intermediate inputs between two markets i and j can be written respectively as $X^C_{ij,s} = x_{ij,s}\xi_{j,s}E_j$ and $X^M_{ij,sk} = x_{ij,s}\xi^M_{j,sk}a^M_{j,k}R_{j,k}$, where $x_{ij,s}$ is the matrix of *gross* trade shares within sector s. This property is the by-product of the assumption that the elasticity of substitution between products of different origins is the same for final consumption and intermediate consumption in all sectors, as in Caliendo and Parro (2015) and in the literature reviewed by Costinot and Rodríguez-Clare (2014). Therefore, we only need to complement the bilateral matrix of trade shares described above with data on the sectoral spending shares of final and intermediate expenditures.

C.2.1 Final Spending Shares

Our main data source is the Consumer Expenditure Survey (CEX) Public-use Micro-data from the U.S. Bureau of Labor Statistics for the years of 1996 and 2000. We first combine the individual-level information in the interview and diary databases to generate annual average household expenditure in each U.S. state on the different product categories in the CEX (i.e., the UCC codes). We then construct a crosswalk from the UCC product classification used in the CEX to 3-digit SIC sectors, using the UCC description provided by the BLS. For the states without data in the CEX, we assign the final expenditure shares of the US Census division to which that state belongs. For all foreign countries, we set the share of final spending on each SIC sector to be the same as that reported in the 1992 U.S. IO table from the BEA.

C.2.2 Intermediate Spending Shares

We measure the sectoral intermediate spending shares in each CZ and country by assuming that $a^M_{j,k} = a_j\left(1 - a^L_k\right)$ where a^L_k is the share of labor in sector k's total cost (common to all countries). We first describe how we calibrate a_i and then how we construct each variable.

First, from the good market clearing condition,

$$R_{i,s} = \sum_j x_{ij,s}\left(\xi_{j,s}E_j + \sum_k \xi^M_{j,sk}a_j a^M_k R_{j,k}\right),$$

where E_j is market j's expenditure on final goods, and $\xi_{j,s}$ and $\xi^M_{j,sk}$ are market j's final and intermediate spending shares. We can write this expression in matrix form and invert it:

$$\boldsymbol{R}(\boldsymbol{a}) = \sum_{d=0}^{\infty}\left(\bar{\boldsymbol{A}}(\boldsymbol{a})\right)^d \boldsymbol{F}, \tag{C.6}$$

where $\boldsymbol{F} \equiv \left[\sum_j x_{ij,s}\xi_{j,s}E_j\right]_{is}$, $\bar{\boldsymbol{A}}(\boldsymbol{a}) = \bar{\boldsymbol{x}}^A diag(a_j)$ and $\bar{\boldsymbol{x}}^A \equiv [x_{ij,s}\xi^M_{j,sk}a^M_k]_{is,jk}$.

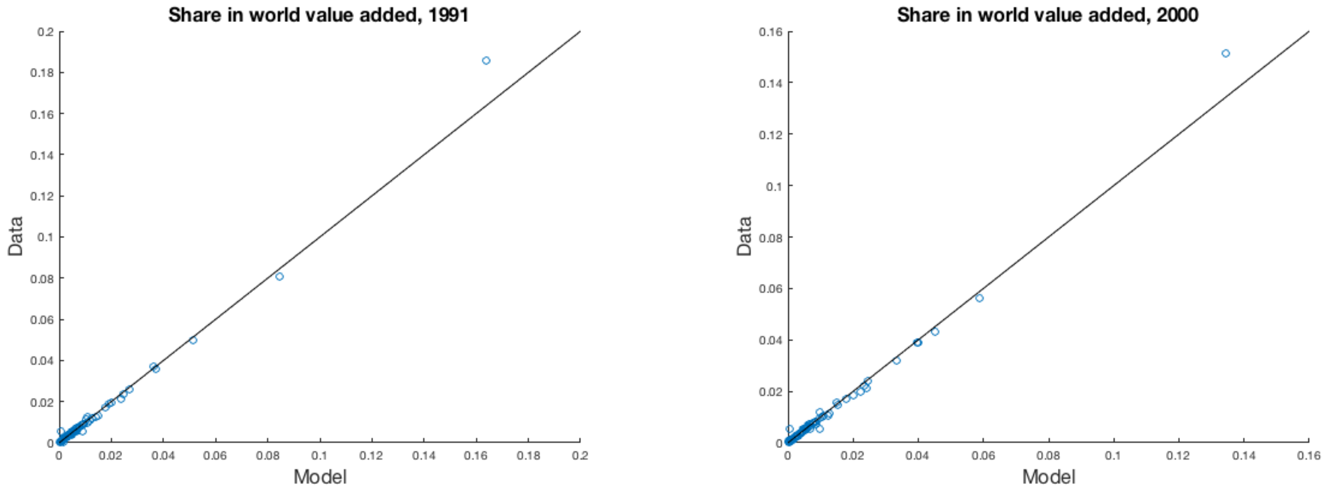
Second, from the labor market clearing condition,

$$W_i(\boldsymbol{a}) = \sum_s (1 - a^M_s a_j)R_{i,s}(\boldsymbol{a}), \tag{C.7}$$

where $R_{i,s}(\boldsymbol{a})$ is given by (C.6).

Finally, we calibrate $\boldsymbol{a}$ to minimize the difference between the observed value-added in market i, W_i, and the one predicted by equation (C.7), $W_i(\boldsymbol{a})$:

Figure C.1: Share of World Value Added



Notes: For each CZ and foreign country, each graph plots the share of world value added observed in the data against the corresponding share predicted by our calibration procedure.

$$\boldsymbol{a}^* = \arg \min_{a_j \in \left(0, 1/\max_k \left\{a_k^M\right\}\right)} \sqrt{\sum_i \left(W_i - W_i(\boldsymbol{a})\right)^2}.$$

To implement this calibration, we use the within-sector bilateral trade shares, $x_{ij,s}$, that we constructed with the methodology described in Appendix C.1. The labor shares a_k^L are obtained from the NBER Manufacturing database. For all markets, we use the 1992 BEA IO table to measure final and intermediate spending share. For all countries, we measure aggregate value-added using the WIOT. For the U.S., we split value-added across CZs by setting value-added in CZ i to $W_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P}\right) W_{US}$, where W_i^P is the CZ's wage bill in the CBP. Similarly, we use the WIOT to measure aggregate final expenditure in each country, and split total final expenditure in the U.S. across CZs using the same payroll shares, $E_i = \left(\frac{W_i^P}{\sum_{j \in US} W_j^P}\right) E_{US}$. Figure C.1 shows that our calibration procedure almost exactly matches the observed shares of value added across U.S. CZs and foreign countries.