Fiscal Multipliers in Integrated Local Labor Markets*

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Abstract

I reveal and analyze a spatial mechanism for generating the aggregate fiscal multiplier. Government budgets balance nationally. This permits the geographic distribution of a government spending stimulus to vary independently from the geographic distribution of the tax burden. Given asymmetric economic geography, the resulting wedge between local spending and local tax burdens can generate an increase in aggregate GDP. This effect is independent of the behavioral response to taxation, thereby providing a distinct mechanism to the canonical New Keynesian and Neoclassical models. To analyze this, I bring in techniques from the International Trade literature: I construct a tractable general equilibrium representation of the fiscal multiplier in a spatially rich framework, and prove isomorphisms to canonical macroeconomic amplification mechanisms. I exploit local fiscal multiplier moments for identification, and estimate my model using US Federal defense procurement in the late 20^{th} century and new interstate trade data for this period. I find that the internal geography of trade is a key determinant of the aggregate fiscal multiplier, characterizing a meaningful analogy to state-dependent fiscal multipliers with *geography-dependent* fiscal multipliers: greater changes in aggregate GDP are to be expected from spending that is concentrated in states that run a trade deficit.

Keywords: Fiscal Multiplier, Interregional Trade, Quantitative Trade Models
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1 Introduction

In response to the 2009 recession, the US Federal Government increased expenditure by 5% ($787 billion) of US GDP and increased its fiscal deficit to levels not seen since the Second World War.[1] Governments throughout the world face a difficult decision when considering the use of fiscal policy to stimulate GDP. The induced expansion in GDP must be weighed against the already extreme levels of public debt, and the wide aversion to a tax increase. The value of the (aggregate) *fiscal multiplier* — the dollar increase in national GDP caused by a one dollar increase in government spending — is a critical metric informing and guiding policy-makers in taking these decisions.

In this paper, I ask the question: how does the geography of trade between states in the US affect the value of the aggregate fiscal multiplier? Naturally, the resulting change in national GDP depends on where the dollar is spent: the change will be different if the government spends the dollar in California or in Illinois or any other state. This observation itself is important for guiding the policymaker. But a more profound consequence is that a transfer from a low-return state to a high-return state can generate a net increase in national GDP. That is, spending the tax revenue in a location different to where it was raised creates a net increase in the beneficiary location's local market demand. Through standard channels, this increase in local demand generates a fiscal multiplier effect on local GDP. With asymmetric geography between the locations receiving the spending and financing the spending, the effect on aggregate GDP can also be positive, despite the GDP of the financing location likely decreasing.

The dependence of this *spatial mechanism* on the internal geography of trade within a country forms the key result of this paper, and can be expressed by a meaningful analogy to the literature. An widely established result is of *state-dependent* fiscal multipliers: multipliers tend to be greater in times of recession, than in normal times.[2] My results suggest a parallel with *geography-dependent* fiscal multipliers, characterized by greater magnitudes being expected when spending is concentrated in states that run trade deficits with other states.[3]

This geographic dependence can be understood as follows. Trade imbalance is a central requirement, and the empirically relevant case for all states in the US. This permits import and export shares to differ; otherwise, the aggregate effect of a transfer between regions

[1] The number refers to the American Recovery and Reinvestment Act.

[2] See Auerbach and Gorodnichenko (2012).

[3] I abstract from international trade. See Auerbach and Gorodnichenko (2013) for the dependence of aggregate fiscal multipliers on international trade. See Serrato and Wingender (2016) for geographic heterogeneity in the local fiscal multiplier due to local GDP. See Batini et al. (2014) for a review of fiscal multiplier determinants.

is zero as the positive change in GDP in the beneficiary regions cancel with the negative changes in GDP in the financing regions. With trade imbalance this is no longer true. On the one hand, as the state's GDP increases, local expenditure and therefore imports from other states increases. This creates positive spillovers to other states, implying a positive dependence of the aggregate fiscal multiplier on state import share. On the other hand, the spending in a state increases local real wages which further increases the local GDP, which further increases local real wages, and so on, amplifying the local response. The strength of this amplification depends on the share of production sold locally — as it is the local demand that experiences the real wage increase — therefore implying a negative dependence of the aggregate fiscal multiplier on state import share. Taking these two together implies a negative dependence on the trade balance.

This spatial mechanism is fundamentally distinct to the canonical fiscal multiplier models, for two main reasons. First, aggregate fiscal multiplier models in the macroeconomic literature almost always abstract from space. A country is modeled to be a single entity therefore assuming the spatial mechanism away.[4] Second, is the source of the initial change in market demand. In canonical New Keynesian and Neoclassical models, this is achieved through intertemporal substitution and income effects on households, derived from the associated tax burden. The spatial mechanism is independent of any behavioral response to taxation, requiring instead an increase in local market demand through an increase in the geographic concentration of spending.

Now, in order to reach these findings, one has to contend with the unfortunate reality: aggregate fiscal multipliers are challenging to identify, and this is exacerbated in my research question. The standard challenge is that government spending is endogenous with respect to GDP. It's the very definition of endogeneity; if it wasn't endogenous, we'd be concerned the government isn't doing its job. Finding a single source of exogenous variation at the national level has constituted entire papers in the literature.[5] For my research question, I would need multiple sources of exogeneity that exhibit sufficient variation across different geographies, and of sufficient magnitudes to affect national GDP. Together, with data limitations at granular geographic aggregations and at high frequencies, this becomes unrealistic.

To solve this challenge, I take an indirect approach. I exploit the fact that identification of *local fiscal multipliers* — the dollar increase in state GDP caused by a one dollar increase in government spending in that state — does not exhibit the same challenges. The sample size is multiplied by the size of the cross section, and Bartik instruments can be used. Interacting

[4]A similarity can be found in the literature with Ramey and Shapiro (1998), who look how industrial composition affects multiplier magnitudes. Nakamura and Steinsson (2014) have a two country fiscal multiplier model, but assume symmetric geography.

[5]See Ramey (2011) for a survey.

these instruments with state geographic characteristics, such as the import share, recovers the dependence of the local fiscal multiplier on geography.

I then map this geographic dependence of local fiscal multipliers to aggregate fiscal multipliers by appealing to economic theory. Because states in the US are economically integrated, this becomes a setting of high spatial dimensionality, described by the network of all state-state bilateral interactions. Motivated by the observation that states are deeply integrated through trade, I bring in machinery from the International Trade literature to the study of fiscal multipliers, and model these interactions using a general equilibrium model of trade.[6] In this framework, the effect of spending in a given state on another state's GDP, for all state-state pairs, is completely identified from observable interstate trade flows and two unobservable structural parameters of the model (a demand and a supply elasticity).

Conditional on observed trade flows, the model gives predictions for the dependence of local fiscal multipliers on geography as a function of the two structural parameters. I identify these parameters by matching the local fiscal multiplier predictions to those empirically identified using Bartik instruments.[7] I then use the identified model to derive the implied aggregate fiscal multiplier, and analyze its dependence on the geography of states.

Using a quantitative spatial trade model to characterize the fiscal multiplier is unique in the literature, and allows me to provide some interesting insights.[8] First, in measuring the spatial mechanism, I prove that I am able to be agnostic about how the spending is financed (e.g. the instrument of taxation or its timing) and what the behavioral responses to taxation are.[9] This is achieved because, by modeling the responses to first order, I can additively decompose the fiscal multiplier into the spatial channel and the taxation channel. Second, I prove that the trade model jointly captures various canonical fiscal multiplier amplification mechanisms in a single general equilibrium parameter: the supply elasticity (as alluded to above). This allows me to be agnostic on the operative microfoundation; I prove that standard amplification channels — countercyclical mark-ups, labor-consumption complementarities and scale economies — are all isomorphic in my framework. These amplification channels are present when the supply elasticity is negative, indicating a downward-sloping

[6] Looking at figure 1 the fraction of state production consumed outside the state is greater than 60% for 40 states; a similar finding is observed with imports too.

[7] This methodology makes analogy to many papers in macroeconomics using moments to calibrate models. See DeJong and Dave (2011).

[8] This is not to say trade is absent from the fiscal multiplier literature (see e.g. Nakamura and Steinsson (2014)) but the spatial aspects there are very restrictive (typically only two locations, and symmetric geography), and far from modern quantitative trade models (see Redding and Rossi-Hansberg (2017) for a review).

[9] This is particularly helpful for my empirical application in defense procurement as contracts are most likely procured through deficit financing during my time period. Do taxpayers internalize the resulting future tax liabilities? I am able to remain completely silent on this.

supply curve. As this may sound unorthodox, it's important to emphasize that this is the general equilibrium supply curve facing the demand curve. Therefore, an outward-shift of the demand curve will only ever cause a more-than-proportionate response in equilibrium output if the supply is downward-sloped. An insight concealed in traditional macroeconomic models as a result of their complexity.[10]

I structurally identify my framework using data on US Federal Defense Procurement during the period 1966-2006. Military spending is the canonical source of government spending used in the estimation of fiscal multipliers as it tends to suffer from fewer identification issues than other forms of government spending, and, in the US, it is a large subset of total government spending.[11] Defense procurement is a large subset of military expenditure, and I particularly focus on this as there is a large variation in its geographic distribution of spending. The data on US defense procurement is well-documented and publicly available for accountability and transparency reasons.[12] The data is at the contract level. A contract is between the federal government and a firm, with the firm producing a good or providing a service for the government. The contracts can range from the construction of aircraft carriers to wheels for tanks, to factory maintenance and catering. I aggregate the contract values to the state-year level for my analysis.

This source of spending is not free from endogeneity concerns: the state allocation of contracts is notoriously political (see, e.g., Russett et al. (1992)), and thus likely to be endogenous to state economic conditions. For example, if Illinois has a senator on the Senate Committee on Armed Services, then the senator may be able to influence the allocation of contracts and allow firms in IL to receive more in years when the economy in IL is struggling. To mitigate this concern, I follow Nakamura and Steinsson (2014) and use a Bartik (shift-share) instrumental variable for state spending. This strategy is based on two characteristics of military spending: first, national military spending is dominated by geopolitical events; second, given a rise in national expenditure, there is a differential increase in some states — such as California — relative to others — such as Illinois — consistently across the sample timeframe. The identification assumption is that the US did not embark on military buildups or drawdowns — such as those associated with the Vietnam War — to differentially benefit those states that consistently receive more of the spending (e.g. California) relative to those that consistently receive less (e.g. Illinois). A typical endogeneity concern with military

[10] I see this as entirely complementary to the macroeconomic models. Their complexity allows for many important insights and results which mine cannot speak to.

[11] To put this into context: US government spending is about 20% of US GDP; Military spending is about 5% of US GDP; defense procurement is about 2% of US GDP.

[12] For the electronic database, see Record Group 330: Records of the Office of the Secretary of Defense at https://www.archives.gov/research/electronic-records/reference-report/federal-contracts.

spending is that the US went to war to benefit the domestic economy. With this Baritk-style identification strategy, this would not invalidate the identification assumption. What would be problematic, is if the US went to war to benefit California relative to Illinois. This violation seems less plausible.

I also require data on US interstate trade flows during this period. However, the standard data source for interstate trade within the US — the Commodity Flow Survey — only began publishing data in 1993, which is towards the end of my sample timeframe.[13] Fortunately, I discovered a precursor survey — the Commodity Transportation Survey — that published interstate manufacturing trade flows for the year 1977. Perhaps a reason why this data has not been used in the literature is due to it only existing in scanned-image form on the US National Archives.[14] I transcribed this data to electronic form and use this for my measure of interstate trade.

The structural estimation results are consistent with the fiscal multiplier mechanism, revealing a negative supply elasticity. The demand elasticity is estimated to be close to one, reflective of the fact that the military procurement spending shocks are transitory, and the industrial composition of the procurement contracts span across manufacturing and services. I confirm that the estimated model does match the local fiscal multiplier moments, and provide validation by looking at a non-matched moment.

Regarding the aggregate fiscal multiplier, I find large heterogeneity in the effect of spending by state on national GDP. At the median, concentrating spending in a state as opposed to distributing the spending across all states, generates an aggregate fiscal multiplier that is 0.28 larger. To put this into context, the consensus in the literature on the magnitude of the aggregate fiscal multiplier is between 0.5 to 1.[15] Geographic heterogeneity arising from the internal geography of interstate trade is thus an important determinant of the aggregate fiscal multiplier magnitude.

Precisely how the fiscal multiplier depends on the geography is a complex relation, depending on the entire network of interstate trade flows, with each state being a node in a 51×51 network.[16] To distill some helpful regularities, and to arrive at the qualitative findings and mechanism discussed at the start of the introduction, I regress the aggregate fiscal multiplier by state on state import and export shares, finding a positive dependence on the former and negative on the latter. This inverse dependence suggests using the trade balance as a single, convenient, state-specific statistic of geography. I find this geographic

[13] See: https://www.census.gov/programs-surveys/cfs.html.

[14] Weiss (1972) is the only paper I'm aware of. They use the much more limited 1963 iteration of the survey.

[15] See Ramey (2011) for a survey.

[16] Including Washington DC.

dependency to be large: an increase in the trade balance of a state by 5 percentage points decreases the aggregate fiscal multiplier from spending in that state by 0.63.

From this follows the key result of the paper. When it comes to fiscal multipliers, the geography of spending matters a lot. The spatial mechanism – the result of distinct financing and spending locations of asymmetric geography – can generate large variation in the fiscal multiplier. A *geography-dependent* fiscal multiplier can be characterized with greater magnitudes expected in states that are more open to imports from other states, and less open to exports from other states.

1.1 Outline

The rest of this paper is organized as follows. I first discuss the literature in section 1.2. In section 2, I explain why a structural approach is necessary. Then, in section 3 I develop the theoretical framework, derive the fiscal multipliers as implied by the theory and present the sufficient statistics for identification. In section 4, I present the empirical framework, detailing the structural parameter identification strategy, the data, the estimation, results and discussion. In section 5, I present the main results of the paper - the fiscal multiplier estimates - provide intuition on the mechanisms in the model underlying their values, and discuss the resulting fiscal policy implications. In section 6, I conclude.

1.2 Literature

This paper relates to a large literature on fiscal multipliers, both theoretical and empirical — see Ramey (2011) for an excellent review. More specifically, to those that use regional variation in military expenditure to make inference, such as Dupor and Guerrero (2017), Chodorow-Reich (2017), Auerbach et al. (2019), and, most closely, Nakamura and Steinsson (2014).[17] The pivotal difference with Nakamura and Steinsson (2014) is the research question: they provide an answer to what can be inferred about aggregate fiscal multipliers from only local fiscal multipliers are identified. They do not try to answer how the geography of spending affects the multiplier magnitude. In fact, their model assumes geographic asymmetry and therefore is uninformative on how the aggregate fiscal multiplier depends on geography, which is the research question I provide an answer to.

Within the fiscal multiplier literature, my research question shares analogy to papers looking at determinants of fiscal multiplier magnitudes. I look at geography-dependence,

[17] The research on using regional variation to identify fiscal multipliers is not restricted to military expenditure; see, for example, Demyanyk et al. (2016), Serrato and Wingender (2016), and Crucini and Vu (2017).

whereas many papers look at state dependence, such as Auerbach and Gorodnichenko (2012). Ramey and Shapiro (1998) analyzes spending distributions across industries — and find that costly capital reallocation can magnify the multiplier — whereas I look at spending across regions. Outside of the fiscal multiplier literature, this dependency of federal policy on geography shares similarity with the literature on optimal spatial policy, see Fajgelbaum and Gaubert (2018).

Bridging the fiscal multiplier and trade literatures, I analyze the fiscal multiplier mechanism in a *gravity* model of trade.[18] Building off the generalized spatial framework of Allen et al. (2014), I integrate a rich - yet tractable - spatial analysis, going beyond the standard two-region small-open economy model typical in macroeconomic theory. My approach connects the fiscal multiplier mechanism to the Home Market Effect, a mechanism studied extensively in trade, that generates multipliers from the interaction of scale economies and segmented spending.[19] One conceptual difference is that the fiscal multiplier mechanisms tend to focus on a temporal shift of expenditure, whereas the home market effect focuses on a spatial shift of expenditure.

My analysis more broadly relates to the study of local labor market shocks, a growing trend in the trade literature with early work by Autor et al. (2013); more recent studies take greater focus on heterogenous response - see Monte et al. (2015) - and the role of spillovers - see Adao et al. (2018) and Stumpner (2019). For a study on the aggregation of local shocks, see Carvalho et al. (2016). Similar to papers in this area, my paper utilizes the exact hat algebra as pioneered by Dekle et al. (2008), which allows one to avoid explicit estimation of many parameters of the model if one is only interested in counterfactuals in changes, rather than levels.[20]

2 Why use a Structural Solution?

The contribution of this paper is to identify the dependence of the aggregate fiscal multiplier on the geography of spending. This cannot be done directly, and in this paper I develop a structural methodology to get identification indirectly. Before turning to my methodology, in this section I make clear why a direct, non-structural approach is not feasible.

Consider what the non-structural research design would require. The following relation

[18] For seminal contributions to the gravity models of trade see Anderson (1979), Eaton and Kortum (2002), and Melitz (2003).

[19] See Krugman (1979) and Helpman and Krugman (1985) for seminal work on the home-market effect in two-region models; see Behrens et al. (2009) and Matsuyama (2017) for many-region theoretical analysis. See Costinot et al. (2016) for an empirical test of the Home Market Effect. See Devereux et al. (1996) for a fiscal multiplier model analysis in the context of increasing returns to scale.

[20] See Adao et al. (2017) for a general framework of this.

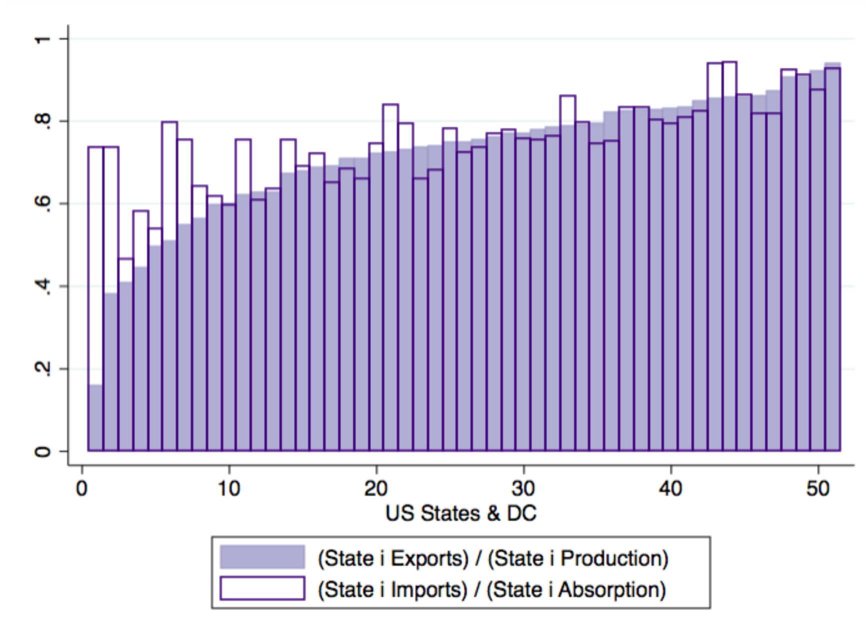


Figure 1: Measures of regional openness through trade

Notes. Manufacturing only. Absorption ≡ Production + Imports - Exports.
Datasource. Commodity Transportation Survery (1977).

describes how changes in aggregate GDP, $\Delta y_{agg,t}$ varies with changes in the distribution of government spending $\{\Delta g_{it}\}_i$

$$\frac{\Delta y_{agg,t}}{y_{agg,t}} = \sum_{j=1}^{N} \beta_j \frac{\Delta g_{jt}}{y_{agg,t}} + \varepsilon_t, \qquad t \in \{1, ..., T\} \tag{1}$$

where i is location, t is time, and β_j is the aggregate fiscal multiplier from spending in location j. Identifying the vector β would answer my research question – with it I will know how the fiscal multiplier depends on the distribution of spending. Though, this will rarely be feasible as panels are normally short: $T \lesssim N$, the number of observations, which is equal to the number of time periods T, is less than the number of parameters, which is equal to the of locations N. Therefore, in practice one would need to parameterize β to be a function of the relevant geographical characteristics of each location, such as

$$\beta_j = \alpha_1 + \sum_{k=1}^{K} w_{jk} \alpha_{2k}$$

where w_j is a $K \times 1$ vector of geographical characteristics (such as state import share and state export share), and, importantly $K \ll N$. Equation (1) becomes

$$\frac{\Delta y_{agg,t}}{y_{agg,t}} = \alpha_1 \frac{\Delta g_{agg,t}}{y_{agg,t}} + \sum_{k=1}^{K} \alpha_{2k} \sum_{j=1}^{N} w_{jk} \frac{\Delta g_{jt}}{g_{jt}} + \varepsilon_t, \qquad t \in \{1, ..., T\} \tag{2}$$

Knowledge of the vector α answers my research question: with it I will know how concentrating spending in locations with greater trade openness, for example, affect the aggregate fiscal multiplier.

There are two main practical challenges that make an approach based on (2) infeasible. The first is the (non-)existence of a datasource with high T that is available at a granular geographic aggregation. Consider US military procurement data, which is a common source used in the literature.[21] Records in the US at the state-level only go back to the 1950s, and only at the annual frequency. This gives a sample size of about size 70. This is problematic.

The second challenge is endogeneity. One requires a full-rank set of $K + 1$ instruments that are valid at the national level. A classic instrument in the aggregate fiscal multiplier literature is a narrative news shock,[22] but this can only identify one parameter. Furthermore the Bartik-style approaches of the local fiscal multiplier literature are not valid as a time fixed effect cannot be included.

Thus, an alternative methodology is required. This is the subject of the rest of this paper.

3 Theory

In section 3.1, I present the theoretical framework used to model the dependence of the fiscal multiplier on geography.[23] In section 3.2, I derive the fiscal multiplier object in this framework, and in section 3.3, I discuss its spatial implications. Finally, in section 3.4, I prove some theoretical results on amplification isomorphisms and a sufficient statistic result.

3.1 Theoretical Framework

The nation consists of locations $i \in \{1, ..., N\}$ and is static. In each location there is a demand equation and supply equation, trade between all locations, a federal government, and market clearing. The equilibrium can be summarized by the following conditions.

[21] See the seminal work of Nakamura and Steinsson (2014).

[22] See Ramey and Zubairy (2018).

[23] This is largely based on the *universal gravity* framework of Allen et al. (2014), extended to include a government sector.

3.1.1 Demand

X_{ij} is the sum of private and public nominal (dollar) demand from location j for products produced in location i and is described by the following equation

$$X_{ij} = \underbrace{\left(\frac{p_{ij}}{P_j}\right)^{1-\phi} E_j}_{private} + \underbrace{G_i^{transfer} \cdot \mathbb{1}[i=j]}_{public} \tag{3}$$

where $\mathbb{1}[i=j]$ is indicator that is 1 if $i=j$ and 0 otherwise. Private demand is described by a constant elasticity of substitution relation where E_j is total private expenditure by from location j, p_{ij} is the consumption price faced by j for products produced in i, and $P_j \equiv \left(\sum_i p_{ij}^{1-\phi}\right)^{\frac{1}{1-\phi}}$ is the ideal price index in j. Public demand for goods produced in i is given by $G_i^{transfer}$ and is described in section 3.1.3. I refer to the unobservable parameter ϕ as the *demand elasticity*, and it is equal to the elasticity of substitution between goods produced in different locations.

3.1.2 Supply

q_i is the quantity of goods produced in location i and is described by the following equation

$$q_i = A_i \left(\frac{p_i}{P_i}\right)^{\psi} \tag{4}$$

p_i is the production price in i, and A_i is an exogenous productivity shifter. I refer to the unobservable parameter ψ as the *supply elasticity*, and it is equal to the elasticity of output with respect to the production price, holding the price index fixed.

3.1.3 Government

Each location receives a real transfer $g_i^{transfer}$ by the federal government, which is related to nominal transfers through the aggregate price index, $\mathcal{P}$, by

$$g_i^{transfer} \equiv \frac{G_i^{transfer}}{\mathcal{P}} \tag{5}$$

where nominal transfers are constrained to sum to zero

$$0 = \sum_i G_i^{transfers} \tag{6}$$

The exogenous shocks I consider are arbitrary real spending by each location, g_i, which I define by

$$g_i^{transfers} \equiv g_i - by_i \tag{7}$$

where $b \in \mathbb{R}$ is endogenous. This has a very natural interpretation. g_i is an arbitrary distribution of government spending. The spending is financed through transfers from each state proportionally to their GDP, equal to by_i (b takes its equilibrium value so that $g^{transfers}$ satisfies the governments budget constraint, equation (6)). The net transfer each state therefore receives is $g_i^{transfers} = g_i - by_i$.[24]

3.1.4 Geography

Trade in the product market is subject to iceberg trade costs, τ_{ij}, (with the normalization $\tau_{ii} \equiv 1$) linking the production price and consumption price as follows

$$p_{ij} = \tau_{ij} p_i \tag{8}$$

Intuitively, to consume 1 product, τ_{ij} must be shipped and $\tau_{ij} - 1$ "melts" along the way.

3.1.5 Product Market Clearing

Total (nominal) production in location i, $Y_i \equiv p_i q_i$, is equal to total demand from all locations

$$Y_i = \sum_j X_{ij} \tag{9}$$

Real production, or equivalently real GDP,[25] is given by

$$y_i \equiv \frac{Y_i}{\mathcal{P}}$$

3.1.6 Trade (Im)balance

All income from production in i goes to private consumption from i, allowing for an exogenous expenditure-to-output ratio, ξ_i, up to an endogenous location-invariant scalar, Ξ. That is,

[24] This is done for two reasons. 1) The first is that in the empirical section, government spending is not necessarily constrained to be transfers. 2) The second is that $\{g_i^{transfers}\}_i$ is not a valid set of exogenous shocks. Due to equation (5), $0 = \sum_i G_i^{transfers} = \sum_i \mathcal{P} g^{transfers}$, transfers in at least one location must be determined endogenously. A simple example is choosing $\forall i: \ g_i^{transfers} > 0$; for equation (5) to be satisfied, it must be that $\mathcal{P} = 0$. This cannot be an equilibrium. Thus, $g_i^{transfers}$ in at least one location must be determined endogenously so that (5) is satisfied.

[25] With intermediate inputs, GDP and production are not equal, but with intermediate inputs as a constant share of output, GDP and production are proportional. As my structural analysis is on log-differences, the constant of proportionality drops out. This is nested in this framework (see Allen et al. (2014)).

$$\xi_i \Xi = E_i / Y_i$$

The endogenous scalar takes its equilibrium value so that trade is balanced on aggregate

$$0 = \sum_i TB_i = \sum_i Y_i (1 - \xi_i \Xi) \tag{10}$$

where $TB_i \equiv Y_i - E_i$ is the trade balance.[26]

3.1.7 Price Normalization

The equilibrium is indeterminate up to scale of prices, therefore I set the normalization for some $i = u$[27]

$$p_u \equiv 1 \tag{11}$$

3.1.8 Aggregate Price Index

Nominal aggregate GDP, $Y_{agg} \equiv \sum_i Y_i$ is related to real aggregate GDP, y_{agg}, through the aggregate price index, $\mathcal{P}$, by

$$Y_{agg} \equiv \mathcal{P} y_{agg}$$

I define the aggregate price index only in differential terms, by the following[28]

$$\mathrm{d} \ln \mathcal{P} \equiv \sum_i \frac{Y_i}{Y_{agg}} \mathrm{d} \ln P_i \tag{12}$$

3.1.9 Equilibrium

Equilibrium is attained in this framework when equations (3), (4, (6), (8), (9), (10), (11) hold.

[26] The endogenous scalar is required because TB is endogenous and therefore equation (10) is not guaranteed to hold in the perturbed equation, even though it holds identically in the initial equilibrium (i.e. $\Xi = 1$). See appendix A.1 for details.

[27] All theoretical results relevant for my empirical analysis are in real terms, therefore the choosing of $u \in \{1, ..., N\}$ is inconsequential. Nonetheless, inclusion of this condition is vital for solving the model in order to derive the fiscal multiplier.

[28] Formally, defining an aggregate price index requires some notion of an aggregate welfare function. However, rather than specifying this, I choose the price index instead to imply an intuitive form of aggregate welfare (or real aggregate GDP). That is, a weighted average of purchasing power across all states: $\mathrm{d} \ln y_{agg} = \sum_i \frac{Y_i}{Y_{agg}} \mathrm{d} \ln \frac{Y_i}{P_i}$. The definition of $\mathcal{P}$ in equation (12) implies this form.

3.2 The Fiscal Multiplier in a Spatial Framework

3.2.1 Cross-Location Local Fiscal Multiplier

To derive the fiscal multiplier in this framework, I start by combining the seven equilibrium conditions of section 3.1, log-linearize about the an equilibrium with transfers set to zero,[29] and solve for the change in real output in each location as a function of real government spending in each location, holding fixed all other exogenous variables of the model. The result is the following

$$\frac{\mathrm{d}y_i}{y_i} = \sum_{j\in\{1,\ldots,N\}} \Lambda_{ij}^{transfers} \frac{\mathrm{d}g_j}{y_j} \tag{13}$$

where $\mathrm{d}y_i$ is the infinitesimal change in real output y_i, $\mathrm{d}g_j$ is the infinitesimal change in real government expenditure, g_i, and the matrix Λ is a function of all endogenous and exogenous variables of the model,

$$\Lambda^{transfers} : \Omega^{all} \to \mathbb{R}^{N^2}$$

where $\Omega^{all} \equiv \{\{y_i, p_i, \{A_i\}_i, \{\tau_{ij}\}_{ij}, \phi, \psi\}$.[30] See appendix A.1 for the functional form of $\Lambda^{transfers}$ and all associated derivations.

Intuitively, the object $y_i \Lambda_{ij}^{transfers} y_j^{-1}$ gives the dollar change in i output caused by a \$1 change in j government expenditure, where the expenditure is funded by transfers from all locations proportional to their output.[31] Note that this is the natural cross-location extension of the familiar local fiscal multiplier, which in this framework is given by $y_i \Lambda_{ii}^{transfers} y_i^{-1} \equiv \Lambda_{ii}^{transfers}$, the dollar change in output in i due to \$1 spending in i.

3.2.2 Aggregate Effects of Localized Spending

The fiscal multiplier object of typical interest in macroeconomics relates how a change in spending affects real aggregate output[32]

[29]This is an assumption, and rules out the dependence of the fiscal multiplier on the initial distribution of government spending. This is motivated by an Occam's Razor argument: no model can feasibly account for the initial incidence across space of government spending and taxation that is realistically observed. I choose to set the initial to zero as this is simple, transparent, and reflects the fact the initial distribution of g_i is a higher order feature. Fiscal multipliers describe changes in g_i, and I am interested in their dependence on the initial geography of trade.

[30]Note that $\{g_i\}_i, b$ are not in the set because I'm expanding about an initial equilibrium with $\forall i : g_i = 0$ (this implies $b = 0$ at the initial equilibrium).

[31]The theoretical framework easily accommodates different transfer rules. For example, all spending being funded entirely by transfers from California.

[32]Follows from normalizing nominal aggregate GDP $Y_{agg} \equiv \sum_i Y_i$ by $\mathcal{P}$.

$$y_{agg} = \sum_i y_i \tag{14}$$

The relation in my framework is derived simply by inserting equation (13) into equation (14), giving

$$\mathrm{d}y_{agg} = \underbrace{\sum_{i \in \{1,\dots,N\}, j \in \{1,\dots,N\}} y_i \Lambda_{ij} y_j^{-1}}_{(\dagger)} \underbrace{\mathrm{d}g_j}_{(\dagger\dagger)} \tag{15}$$

This is the inner product of two objects: (†), the change in aggregate output due to location j receiving a transfer; and (††), the transfer received by j. The key implication is that even if the change in spending across all locations is zero-sum (only transfers are considered), the effect on aggregate output needn't be zero, given that (†) isn't zero.

To determine the implications of this for the aggregate fiscal multiplier, aggregate government spending must be allowed to vary, otherwise the aggregate fiscal multiplier is ill-defined.[33]. I constrain aggregate expenditure to be zero in my theoretical framework. However, as I only consider the effects on output up to first order, a change in real aggregate expenditure, $g_{agg} = \sum_j g_j$,[34] that is spent is spent proportional across states according to their GDP, is going to enter equation (15) additively and linearly.

First, looking at how this changes the relation at the local level, equation (13). Denote the effect of $\frac{\mathrm{d}g_{agg}}{y_{agg}}$ on $\frac{\mathrm{d}y_i}{y_i}$ by $\Lambda_i^{aggregate}$, then

$$\frac{\mathrm{d}y_i}{y_i} = \underbrace{\sum_{j \in \{1,\dots,N\}} \Lambda_{ij}^{transfers} \frac{\mathrm{d}g_j}{y_j}}_{(*)} + \underbrace{\Lambda_i^{aggregate} \frac{\mathrm{d}g_{agg}}{y_{agg}}}_{(**)} \tag{16}$$

What this equation shows is that the effect of a change in aggregate government spending on GDP in location i can be decomposed into two terms: $(**)$ the effect due to an increase in aggregate spending that is distributed proportionally, such that the change in local spending equals the change in the local tax burden; $(*)$ the differential effect due to spending actually being concentrated in some regions, such that some locations receiving spending beyond their local tax burden (i.e. a transfer). It is $(*)$ that is absent from single-location models (as local and aggregate are the same, and aggregate spending must equal the aggregate tax burden) and generates a novel *spatial mechanism* for fiscal multipliers. Distinctively, as only transfers are considered in $(*)$, this mechanism is independent of how the spending is financed.

[33]Ramey (2011) briefly considers a multiplier due to transfers and concludes its value must be zero or infinity.

[34]Following from normalizing nominal aggregate spending $G_{agg} \equiv \sum_i G_i$ by $\mathcal{P}$.

Second, at the aggregate level, equation (15), this becomes

$$\frac{\mathrm{d}y_{agg}}{\mathrm{d}g_{agg}} = \underbrace{\sum_{i\in\{1,\ldots,N\},j\in\{1,\ldots,N\}} y_i \Lambda_{ij}^{transfers} y_j^{-1} \frac{\mathrm{d}g_j}{\mathrm{d}g_{agg}}}_{(+)} + \underbrace{\sum_i \Lambda_i^{aggregate} \frac{y_i}{y_{agg}}}_{(++)} \tag{17}$$

This is the aggregate fiscal multiplier. Analogous to above, the term (++) is the effect on aggregate GDP due to spending that is distributed proportionally; and the term (+) is the differential effect due to spending actually being concentrated in some regions.

What this equation shows is that, once we go beyond a single-location model, the aggregate fiscal multiplier depends on the distribution of location (described by the vector $\left\{\frac{\mathrm{d}g_j}{\mathrm{d}g_{agg}}\right\}_j$). The term (+) captures the component of the aggregate fiscal multiplier that is due to the spatial mechanism.

3.3 Discussion of the Spatial Mechanism

The spatial mechanism of the fiscal multiplier described in the preceding section is best understood by making an analogy to the standard fiscal multiplier models. In order for the fiscal multiplier to be non-zero, two components are required. 1) A market must experience a net change in demand. 2) An amplification effect in output is exhibited when a net demand change is experienced.

1) Net change in market demand. Typically in macroeconomic models of the fiscal multiplier the government taxes (contemporaneously or in deficit) households and spends the revenue on firm output. Ceteris paribus, the taxation reduces private demand for output by \$1 (say), and the spending increases public demand for output by \$1. The fiscal multiplier is the description of the effect on output from the *combination* of these actions.[35] Holding everything else fixed, the change in total demand is \$0, and therefore, regardless of any amplification mechanisms being present, the fiscal multiplier is identically zero, as equilibrium output will not respond as the demand curve has not shifted.

This illustrates that, when ceteris paribus is lifted, there must be a mechanism initiated from either the taxation or spending that causes the change in total demand to be non-zero. In standard macroeconomic fiscal multiplier models, this manifests through an income effect on labor supply. When households are taxed (contemporaneously or in deficit), their lifetime income is reduced. This creates a negative income effect which, assuming leisure is a normal

[35] Although the separate effect of each may be of interest, separate identification is impossible due to accounting: the dollars spent by the government must come from somewhere.

good, causes labor supply to increase. Therefore, private demand decreases by less than \$1, while public demand still increases by \$1 as before, creating positive net increase in total demand.

Distinctively, this change in net demand is entirely dependent on the behavioral response of agents to taxation. This is where the spatial mechanism diverges. Acknowledging that spending by the government is geographically segmented, then spending can be simultaneously increased in one region while decreased in another, so that total expenditure, and therefore taxation, is unchanged. The result is a net increase in total market demand in the first region. It's public demand increases by \$1, due to the transfer. It's private demand decreases as the second region is now importing less (as it is poorer due to financing the spending but receiving none of it), but this reduction is by less than \$1, as the second region reduces spending from all regions, not just the first. The novelty here is that the net demand change is now entirely dependent on the behavioral response of agents to the spending (that is, adjustments in imports).

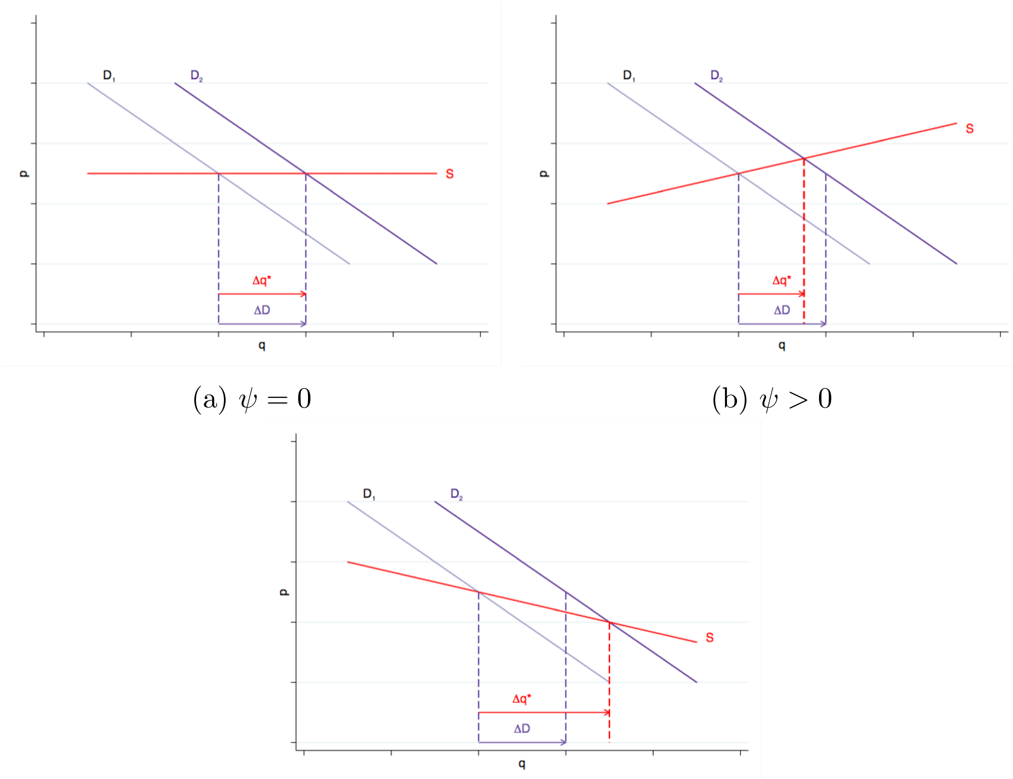
This generates a positive demand shift in the first region, but of course is accompanied by a negative demand shift in the second. In order for there to be a net effect on aggregate output (and create a non-zero fiscal multiplier), there must be sufficient geographic asymmetry between financing and beneficiary locations. A transfer between symmetric locations will cancel out.

2) Amplification Effect. Given a net change in market demand, the demand curve shifts out. This shift is, as described above, net of taxation or crowded-out imports, and therefore will be less than \$1, and possibly much less. How are fiscal multipliers close to or even greater than one therefore possible? Some form of amplification mechanism is required.

In standard macroeconomic models, this is predicated on mark-ups that countercyclical.[36] Intuitively, first consider the case where, under fixed mark-ups, output increases one-for-one with the demand shift. The fiscal multiplier here will be small, as the demand shift, and therefore the resulting output increase, is less than the government spending increase (and the fiscal multiplier equals $= \Delta Y/\Delta G$). If the mark-up is countercyclical, then, as output increases, the mark-up decreases. Private demand therefore increases as purchasing power has increased (mark-up is inversely related to the real wage), thus further increasing output.[37]

[36] See Nekarda and Ramey (2013).

[37] This is the basis of the New-Keynesian model of the fiscal multiplier, and in modern models is generated by imposing sticky prices, and can generate fiscal multipliers greater than one (see Christiano et al. (2011)). The Neoclassical model of the fiscal multiplier does not have such a amplification mechanism, but instead can generate larger fiscal multipliers by having strong income effects so that the net change in market demand is not small (see Aiyagari et al. (1992)). Of course, here, the fiscal multiplier is bounded above by one.



(a) $\psi = 0$

(b) $\psi > 0$

(c) $\psi < 0$

Figure 2: Stylized response of equilibrium output to demand shock for various ψ.
Notes. 2a: the output response is equal to the size of the demand shock. 2b: the output response is less than the size the of the demand shock. 2c: the output response is greater than the size the of the demand shock.

The result is a amplification of the increase in equilibrium output, raising the value of the fiscal multiplier.

My framework naturally allows me to generalize this amplification effect beyond countercyclical mark-ups. The starting point is understanding precisely what constitutes a amplification mechanism. Given a shift in the demand curve, equilibrium output can increase by more than one-for-one only if the supply curve is downward-sloping (see figure 2). In my framework, the slope of the supply curve is controlled by the supply elasticity, ψ, with a downward-sloping curve generated by $\psi < 0$.

A downward-sloping supply curve may go against priors, but it's important to realize this is a general equilibrium supply curve (notice the absence of wages in the supply equation (4)), and therefore combines, for example, the optimal pricing conditions from production and optimal labor supply (resulting in wages being substituted out of the supply relation). Below in section 3.4, I prove that a downward-sloping supply can be generated from, for example, countercyclical mark-ups, increasing returns to scale, and labor-consumption complementaries.[38] These are the same features that generate large multipliers in canonical fiscal multiplier models in the literature.[39]

A Static Framework for Fiscal Multipliers? A final note is in order regarding the absence of dynamics from my framework. The canonical fiscal multiplier models are rich along the temporal dimension, but extremely limited along the spatial dimension (most often absent). My approach takes the opposite approach, being very rich spatially but static. The primary limitation is that I can say nothing about dynamics, and all my inference is regarding the contemporaneous *impact multiplier*. However, I'm nonetheless able to account for common amplification mechanisms, as illustrated above, and importantly, able to do so tractably. This creates a complementary framework to the macroeconomic literature, that is able to acutely probe the spatial implications for the fiscal multiplier, both theoretically, as described here, and empirically, in section 4.

3.4 Theoretical Results

3.4.1 Amplification Isomorphisms

Some form of amplification mechanism is key in modern-day New Keynesian macroeconomic models of the fiscal multiplier. Here I show formally that some common microfoundations from the literature underpinning these — countercyclical mark-ups, labor-consumption complementarities, and scale economics — are isomorphic in the canonical International Trade framework that I have extended above. Precisely, regardless of the microfoundation considered, each leads to the same set of equilibrium conditions in section 3.[40]

To do so, I add two additional assumptions to my framework describing the microfoundations, then a proposition proving the isomorphisms.

Assumption 1 (Microfoundation: Households)**.** *A representative household in each location that maximizes utility*

$$\max_{\{c_{ji}\}_{j\in\{1,\dots,N\}},L_i} v_i \frac{c_i^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{L_i^{1+\nu^{-1}}}{1+\nu^{-1}}, \qquad c_i \equiv \left(\sum_j c_{ji}^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$

[38]On the other hand, when $\psi > 0$ (figure 2b), supply is upward-sloping, and the change in equilibrium output is less than the demand shift, leading to small multipliers. This is reflective of decreasing returns to scale in production, or inelastic labor supply; anything that makes production more costly at greater scales.

[39]See Devereux et al. (1996) for scale economies. See Nakamura and Steinsson (2014) for a countercyclical mark-up (arising from sticky-prices) and labor-consumption complementarities. Although a workhouse in modern-day macroeconomic models, the empirical evidence on mark-up countercyclicality is varied. See Bils (1987) for supportive evidence and Nekarda and Ramey (2013) for evidence against.

[40]More detail and derivations are provided in appendix A.4.

subject to the budget constraint

$$E_i \equiv \sum_j p_{ji} c_{ji} = w_i L_i$$

where $v_i \equiv L_i^\theta$ but is not internalized by the consumer when choosing labor supply.

Assumption 2 (Microfoundation: Firms)**.** *A representative firm in each location maximizes profit*

$$\max_{q_i, l_i} p_i q_i - w_i l_i \qquad q_i \equiv A_i l_i$$

The production technology is $q_i = A_i l_i$ with external economies of scale $A_i \equiv \bar{A}_i l_i^\chi$. The mark-up, by definition, is $\mu_i \equiv \frac{p_i}{w_i/A_i}$. I assume that the mark-up is a constant elasticity function of scale, $\mu_i = q_i^\xi$. Free entry is imposed implying zero profit.

Proposition 1 (Amplification Isomorphisms)**.** *Under assumptions 1 and 2, the supply elasticity ψ of equation (4) is equal to*

$$\psi = \frac{1 - \sigma^{-1}}{\frac{1+\nu^{-1}-\theta}{1+\chi} + \xi - (1 - \sigma^{-1})} \tag{18}$$

Proof. *See appendix A.4.*

$\psi < 0$ is required for my International Trade framework to exhibit amplification. Proposition 1 shows that $\psi < 0$ can be generated by many combinations of underlying micro-elasticities $(\sigma, \nu, \theta, \chi, \xi)$, each combination therefore being isomorphic in my framework. Particular ranges of each of these correspond to various amplification mechanisms common in the macroeconomic literature. Table 1 gives a description of each of these parameters, and in what range amplification is present.

Because each of these amplification mechanisms is isomorphic in my framework, this allows me to be agnostic about what exactly the underlying mechanism operating is.[41] However, given my estimated value of ψ later in the paper, I can use equation (18) to infer what values of these micro-elasticities are required to be theoretically consistent, and get insight on the plausibility of my estimate. I undertake this exercise in section 4.4 at my estimated value of ψ.

[41] Typical macroeconomic models of the fiscal multiplier have rich dynamics which preclude expression of these mechanisms via a single parameter, such as ψ in my framework.

Parameter	Name	Generate $\psi < 0$	Description
σ	Intertemporal Elasticity of Substitution	$\sigma < 1$	Income effect on labor dominates substitution effect
ν	Frisch Elasticity of Labor Supply	—	Not possible. $\nu \gg 1$ accommodates*
χ	Elasticity of (production technology) productivity with respect to labor	$\chi > 0$	Increasing returns to scale
ξ	Elasticity of the mark-up with respect to output	$\xi < 0$	Countercyclical mark-up (sticky prices)
θ	Labor-Consumption complementarities, $\propto \partial^2$utility$/\partial L \partial c$	$\theta > 0$	Edgeworth Complements

Table 1: Microfoundations theoretically consistent with a fiscal multiplier amplification effect, $\psi < 0$

Notes. ν:* without any of the other parameter inequalities being satisfied, an arbitrary Frisch Elasticity is unable to generate $\psi < 0$; however, with one of the other inequalities satisfied, a large Frisch Elasticity makes ψ even more negative.

3.4.2 Sufficient Statistics

I report here a theoretical result that will be key for identification in the empirical section of the paper.

Proposition 2 (Sufficient Statistics)**.** *The cross-location effects of transfers on locations' outputs admit the following functional dependence on observables and unobservables*

$$\Lambda^{transfers} : (s^{Im}, s^{Ex}, \phi, \psi) :\longrightarrow \mathbb{R}^{N^2}$$

where $s^{Im}_{ij} \equiv \frac{X_{ij}}{E_j}$ *is the import share and* $s^{Ex}_{ij} \equiv \frac{X_{ij}}{Y_i}$ *is the export share.*

Proof: see appendix A.2.

What this proposition says is conditional on observable import shares and export shares, the unobservable demand ϕ and supply ψ elasticities are sufficient statistics for identification of the full matrix $\Lambda^{transfers}$. That is, even though $\Lambda^{transfers}$ directly depends on all the variables and parameters of the model, Ω^{all}; knowledge of Ω^{all} in it's entirety is not necessary for identification.[42]

[42]This relates to a large literature in trade on *Exact-Hat Algebra*, with seminal work by Dekle et al. (2008). My result specifically closely follows the work of Allen et al. (2014).

Intuitively, proposition 2 allows me to effectively change the identification problem from directly identifying $\Lambda_{ij}^{transfers}$ for all i, j pairs, to instead identifying ϕ, ψ which indirectly identify $\Lambda^{transfers}$. That is, I've reduced the number of parameters to identify down from N^2 to 2.

4 Empirical Framework

Directly identifying the dependence of the aggregate fiscal multiplier on state geography is not realistic, however it is for the local fiscal multiplier. As implied by proposition 2, my structural framework provides a mapping between the two given observable trade flows and the unobservable parameters (ϕ, ψ). Therefore, I first identify (ϕ, ψ) using the local fiscal multiplier moments, and then recover the implied values of the aggregate fiscal multiplier given these.

Section 4.1 details my identification strategy. Section 4.2 provides the data. Section 4.3 the estimation and results. Section 4.4 for a discussion of the results.

4.1 Identification of (ϕ, ψ)

I first explain in section 4.1.1 a baseline to identifying local fiscal multipliers as popularized by Nakamura and Steinsson (2014). In section 4.1.3 I extend this methodology to identification conditional on geography, and detail how I use this to identify (ϕ, ψ).

4.1.1 Local fiscal multiplier unconditional on geography: a literature baseline

The local fiscal multiplier describes the change in real state GDP given a change in government real spending in that state, $\frac{\partial y_{it}}{\partial g_{it}}$. Making reference to the seminal work of Nakamura and Steinsson (2014), a standard empirical specification for identifying local fiscal multipliers in the literature is

$$\frac{\Delta y_{it}}{y_{it}} = \beta \frac{\Delta g_{it}}{y_{it}} + \alpha_i + \gamma_t + \varepsilon_{it}, \qquad i \in \{1, ..., N\},\ t \in \{1, ..., T\} \tag{19}$$

where i indexes state, t indexes year, Δ indicates two year time differences,[43] and α_i, γ_t are fixed effects.[44] Government spending is endogenous, therefore the regression is estimated

[43] $\frac{\Delta y_{it}}{y_{it}} \equiv \frac{y_{it+2} - y_{it}}{y_{it}}$, $\frac{\Delta g_{it}}{y_{it}} \equiv \frac{g_{it+2} - g_{it}}{y_{it}}$

[44] This specification omits spillover effects. A complete account of spillovers necessitates the following specification $\frac{\Delta y_{it}}{y_{it}} = \sum_{j=1}^{N} \beta_{ij} \frac{\Delta g_{jt}}{y_{jt}} + \alpha_i + \gamma_t + \varepsilon_{it}$, which is impractical due to $T \lesssim N$. Some recent papers do partially account for spillovers by including for example, closest neighbors (Serrato and Wingender (2016)) or major trading partner (Dupor and Guerrero (2017)). However, I follow Nakamura and Steinsson (2014) and use equation (19).

by 2SLS using a Bartik-style instrument for government expenditure

$$z_{it}^{unconditional} \equiv s_i^{Bartik} \cdot \frac{\Delta g_{agg,t}}{y_{agg,t}} \tag{20}$$

which is the inner product of a state-specific, time-invariant share, s^{Bartik}, with the change in national government expenditure. s_i^{Bartik} is a measure of how sensitive a state's expenditure is to changes in national government expenditure. The identifying assumption in the Bartik research design is that the *differential* sensitivity across states, $s_i - \frac{1}{N}\sum_j s_j$, is exogenous with respect to state-specific GDP shocks. It is essential therefore that equation (19) includes a time fixed effect so that differential variation across states is used.[45]

In the preferred estimates of Nakamura and Steinsson (2014), the s^{Bartik} are the coefficient estimates from running the following regression separately for each i by OLS

$$\frac{g_{it} - g_{it-2}}{y_{it-2}} = s_i \cdot \frac{g_{agg,t} - g_{agg,t-2}}{y_{agg,t-2}} + \alpha_i + \gamma_t + \epsilon_{it}, \quad t \in \{1, ..., T\} \tag{21}$$

The validity of this design in my empirical application is discussed in section 4.3.1.

4.1.2 Local fiscal multipliers conditional on geography

I extend the analysis reviewed above from the literature to allow for heterogeneity in the local fiscal multiplier due to geography, specifically arising from the import share of a state, s_i^{Im}. That is, the following empirical specification

$$\frac{\Delta y_{it}}{y_{it}} = \beta_1^{non-structural} \frac{\Delta g_{it}}{y_{it}} + \beta_2^{non-structural} \frac{\Delta g_{it}}{y_{it}} \cdot s_i^{Im} + \alpha_i + \gamma_t + \varepsilon_{it} \tag{22}$$

As I show and explain in section 4.3.2, the state import share is chosen as it is an important determinant of local fiscal multipliers, whereas other statistics of local economic geography are not. The non-structural local fiscal multiplier by state implied by specification (22) is

$$LFM_i^{non-structural} \equiv \beta_1^{non-structural} + \beta_2^{non-structural} s_i^{Im} \tag{23}$$

I identify equation (22) by 2SLS using the Bartik instrument of section 4.1.1 along with its interaction with the state import share

[45]Note this is the reason why the Bartik research design cannot be used to identify the aggregate effects of government expenditure: if equation (19) is ran with aggregate GDP on the lefthand-side, $\frac{\Delta y_{agg,t}}{y_{agg,t}}$, a time-fixed effect soaks up all the variation and β is not identified.

$$z_{it} \equiv \begin{pmatrix} s_i^{Bartik} \cdot \frac{\Delta g_{agg,t}}{y_{agg,t}} \\ s_i^{Im} \cdot s_i^{Bartik} \cdot \frac{\Delta g_{agg,t}}{y_{agg,t}} \end{pmatrix}$$

The identified $(\beta_1^{non-structural}, \beta_2^{non-structural})$ provide two moments which I match with their structural analogues in order to identify the structural parameters (ϕ, ψ).

4.1.3 Structural analogue of the local fiscal multipliers

To derive the structural analogue of equation (22), I start by taking an empirical analogue of equation (16), giving

$$\frac{\Delta y_{it}}{y_{it}} = \sum_{j \in \{1,\dots,N\}} \Lambda_{ij}^{transfers} \frac{\Delta g_{jt}}{y_{jt}} + \Lambda_i^{aggregate} \frac{\Delta g_{agg,t}}{y_{agg,t}} + \tilde{\alpha}_i + \tilde{\gamma}_t + \tilde{\varepsilon}_{it} \tag{24}$$

where, in doing the transformation, I have added a time index, replaced the differentials by two-year time differences and added fixed effects and an error term.[46][47] Next, I make the following assumption

Assumption 3 (Homogeneous Aggregate Incidence)**.** *The incidence on location i from a change in aggregate government spending is constant up to some residual ν_{it}*

$$\Lambda_i^{aggregate} \frac{\Delta g_{agg,t}}{y_{agg,t}} = \bar{\Lambda}^{aggregate} \frac{\Delta g_{agg,t}}{y_{agg,t}} + \nu_{it}$$

where the residual ν_{it} is uncorrelated with $\{\frac{dg_{it}}{y_{it}}\}$.

This assumption is already widely imposed in the literature, though rarely made explicit. $\Lambda_i^{aggregate}$ is the effect of aggregate policy on local GDP. It is indexed by location i as, even though the aggregate policy is the same in all locations, the transmission of this policy to local GDP is heterogeneous, as locations themselves are heterogeneous (in their trade exposure, for example). Research designs typically impose this transmission to be homogeneous, so that it may be absorbed into a time fixed effect.[48] Assumption 3 formalizes this.

Imposing this assumption on equation (24), and subsuming the spillover terms into the error term yields[49]

[46] Λ is assumed fixed across time. This is primarily due to data limitations: trade flows are not available for most of my timeframe.

[47] The error term is a function of contemporaneous changes in the other exogenous variables of the model (such as productivity, A and trade costs τ). For exposition, these were held fixed when log-linearizing. See appendix A.1.1 for the full expression where these are permitted to vary.

[48] See e.g. Nakamura and Steinsson (2014): in their abstract, they say "...'differences out' these [aggregate] effects because monetary and tax policies are uniform across the nation."

[49] The same approximation made in equation (22); see footnote 44.

$$\frac{\Delta y_{it}}{y_{it}} = \Lambda_{ii}^{transfers} \frac{\Delta g_{it}}{y_{it}} + \alpha_i + \gamma_t + \varepsilon_{it} \tag{25}$$

where

$$\alpha_i + \gamma_t + \varepsilon_{it} \equiv \sum_{j \neq i} \Lambda_{ij}^{transfers} \frac{\Delta g_{jt}}{y_{jt}} + \tilde{\alpha}_i + \tilde{\gamma}_t + \bar{\Lambda}^{aggregate} \frac{\Delta g_{agg,t}}{y_{agg,t}} + \tilde{\varepsilon}_{it} + \nu_{it}$$

Thus, the analogous local fiscal multiplier implied by my structural framework is

$$LFM_i^{structural} \equiv \Lambda_{ii}^{transfers}$$

Importantly, even though the local fiscal multiplier is a measure of arbitrary government spending – that is, not restricted to transfers – under assumption 3, the local fiscal multiplier identified is the same whether one restricts to transfers or not. This is a powerful result. And one which relies on an assumption 3, which is widely imposed. This result allows me in my theoretical framework of section 3 to be agnostic about how the spending is financed (for example, whether through contemporaneous taxation or deficit-spending, and where the nominal incidence of the taxation falls), and what the behavioral response to the taxation is.[50] This can be seen because, as described in the theory section 3.2.2, all behavioral response to taxation is contained within $\Lambda^{aggregate}$. As this term is absorbed into the time fixed effect, I needn't make any further assumption on its form.

4.1.4 Matching moments: structural to the non-structural

Conditional on observable trade flows, for a given value of (ϕ, ψ), my structural framework implies a value for the local fiscal multiplier by equation (23). Then, using this to run the following regression by OLS

$$LFM_i^{structural} = \beta_1^{structural} + \beta_2^{structural} s_i^{Im} + \varepsilon_{it} \tag{26}$$

I identify $(\beta_1^{structural}, \beta_2^{structural})$ as a function of (ϕ, ψ), the structural analogues of the non-structural moments identified in equation (22). The true (ϕ^*, ψ^*) are the values such that the structural moments match the non-structural moments

[50] This is particularly helpful in my context of government procurement. The federal government finances these contracts in deficit. Does the taxpayer adjust their behavior to the increased future tax burden resulting from these contracts? Seems like the effect will be small, but then how should it be incorporated in the model? I am able to avoid this question altogether.

$$\begin{pmatrix} \beta_1^{structural} \\ \beta_2^{structural} \end{pmatrix} = \begin{pmatrix} \beta_1^{non-structural} \\ \beta_2^{non-structural} \end{pmatrix} \tag{27}$$

However, equation (27) does not uniquely identify (ϕ^*, ψ^*). Therefore, I chose (ϕ^*, ψ^*) that satisfy the following

$$\max_{\phi,\psi} \frac{|\beta_1^{structural}|}{se(\beta_1^{structural})} + \frac{|\beta_2^{structural}|}{se(\beta_2^{structural})}, \qquad s.t. \qquad \begin{pmatrix} \beta_1^{structural} \\ \beta_2^{structural} \end{pmatrix} = \begin{pmatrix} \beta_1^{non-structural} \\ \beta_2^{non-structural} \end{pmatrix} \tag{28}$$

This is motivated as follows. The multiplicity arises because $\Lambda^{transfers}$ in certain regions across (ϕ, ψ)-space is not well-behaved and becomes chaotic at high N. For a given $(\beta_1^{non-structural}, \beta_2^{non-structural})$, there are therefore many (ϕ, ψ) in these regions that will satisfy (27). I want to rule these values of (ϕ, ψ) out. Equation (28) achieves this by minimizing (a weighted combination of) the inverse of the standard errors of $(\beta_1^{structural}, \beta_2^{structural})$. In the chaotic regions, the dependence of local effect of spending, $\Lambda_{ii}^{transfers}$, on state import shares is highly variable, therefore leading to high standard errors in equation (26). Hence, choosing the low standard error $(\beta_1^{structural}, \beta_2^{structural})$ moves us away from these undesirable values of (ϕ, ψ).

4.2 Data

The data used are listed below with their sources and description.

4.2.1 Interstate Trade Flows

The trade flow data for $\{X_{ij}\}_{i\in\{1,\dots,N\},j\in\{1,\dots,N\}}$ comes from the Commodity Transportation Survey (CTS), which provides statistics on the volume and characteristics of commodity shipments by manufacturing establishments in the United States.[51] The survey began in 1963, though only began publishing statistics on the value of shipments (as opposed to just shipment weight) and with shipments disaggregated to the state-state level[52] (as opposed to just at the Census region-region level) in 1977. In this year, the survey took a stratified probability sample (across each of the 456 manufacturing SIC industries) of 19,500 manufacturing establishments from the 1977 Census of Manufactures' universe of manufacturing establishments (approximately 350,000).[53] Respondents were asked to report the "net selling

[51] For details, see https://archive.org/details/1977censusoftran03unse/page/n0. Table 1 presents the data used in this paper. Appendix B describes the sample design.

[52] All 50 states and DC.

[53] 16,000 establishments responded.

value" (after discounts and allowances, exclusive of freight charges and excise taxes) on a sample of their shipments. All shipments were within the scope of the sampling procedure, except classified defense materials, which were excluded. Exports are included with the destination listed as the US port of export, meaning they are indistinguishable from domestic shipments.

The import and export shares are constructed as follows

$$s_{ij}^{Ex} \equiv \frac{X_{ij}}{\sum_k X_{ik}}, \quad s_{ij}^{Im} \equiv \frac{X_{ij}}{\sum_k X_{kj}}$$

4.2.2 Military Expenditure

The government spending data for $\{G_{it}\}_{i\in\{1,\dots,N\},t\in\{1,\dots,T\}}$ comes from the DD-350 military procurement forms;[54] these report contracts for goods and services between the private sector and the military services agencies of the US Department of Defense with a value of $\$10,000$ or more from fiscal year 1965 to 1984, and of a value of $\$25,000$ or more from fiscal year 1983 to 2006. The forms document everything from tank wheels to aircraft carriers, form catering to military factory repairs. The forms present data by principal place of performance: manufacturing contracts are attributed to the state where the product was processed and assembled; construction and service contracts are attributed to the state where the construction or the service was performed.[55]

I aggregate the contracts to state and calendar year for 1966-2006.[56]

4.2.3 Other Economic Data

Other economic data come from standard sources.

State GDP in the variables $\frac{\Delta y_{it}}{y_{it-2}}$, and denominator of $\frac{\Delta g_{it}}{y_{it-2}}$ come from the BEA at the state-calendar year level.

[54]For the electronic database and additional details, see Record Group 330: Records of the Office of the Secretary of Defense at https://www.archives.gov/research/electronic-records/reference-report/federal-contracts.

[55]Note that the trade flow dataset only covers manufacturing, whereas GDP and military expenditure cover all sectors. Data on only manufacturing GDP for the time period is feasible, but the military data does not provide industry codes per contract during my time period. See footnote 66 for the implication of this.

[56]An important concern is the extent of interstate subcontracting, the presence of which means the reported location of spending is not the actual location of spending. Nakamura and Steinsson (2014) present evidence showing these concerns are minimal.

Price Index used to deflate GDP y and spending g is the national CPI taken from the BLS.

4.3 Estimation of (ϕ, ψ)

4.3.1 Exogenous Variation in Government Spending: Bartik Instrument

My source of government spending is federal defense procurement, as detailed in the data section 4.2. However, the allocation of government contracts is notoriously political, therefore regional spending, Δg_{it}, cannot be assumed exogenous. I allow for this by using an instrumental variable strategy.

Following the strategy used in Nakamura and Steinsson (2014), I construct a Bartik-style instrument, dating back to Bartik (1991). This strategy is based on two characteristics of military spending: first, national military spending is dominated by geopolitical events; second, given a rise in national expenditure, there is a differential increase in some states — such as California — relative to others — such as Illinois — consistently across the sample timeframe (see figure 3). The identification assumption is that the US did not embark on military buildups — such as those associated with the Vietnam War — to differentially benefit those states that consistently receive more of the spending (California) relative to those that receive less (Illinois). A typical endogeneity concern with military spending is that the US went to war to benefit the domestic economy. With this Bartik-style identification strategy, this would not invalidate the identification assumption. What would be problematic, is if the US went to war to benefit California relative to Illinois. This violation seems less plausible.

The Bartik instrument is defined as in equation (20), with the shares constructed from the regression in equation (21). The predicted values for $\frac{g_{it}-g_{it-2}}{y_{it-2}}$ from this regression are scaled versions of changes in national spending, allowing for heterogeneous sensitivity by state. For the identification assumption to be valid, the state shares s_i must be exogenous with respect to changes in state GDP in i across time.[57] It may well be the case that states that receive greater shares of government spending (higher s_i) exhibit greater growth in state GDP, which would violate the identification assumption. This is addressed by the inclusion of state fixed effect; the assumption being that the share of spending s_i can be correlated with the state GDP growth rate, but not correlated with changes in the growth rate. The inclusion of the time fixed effect controls for any confounding variation from aggregate shocks or policy (e.g. monetary policy) that is correlated with national spending and regional GDP; it is the time fixed effect that allows for the California-Illinois intuition above.[58] Mathematically, the state

[57]See Goldsmith-Pinkham et al. (2017) for details.

[58]Subtly, this also requires s_i to be uncorrelated with state i exposure to aggregate shocks i.e. the marginal

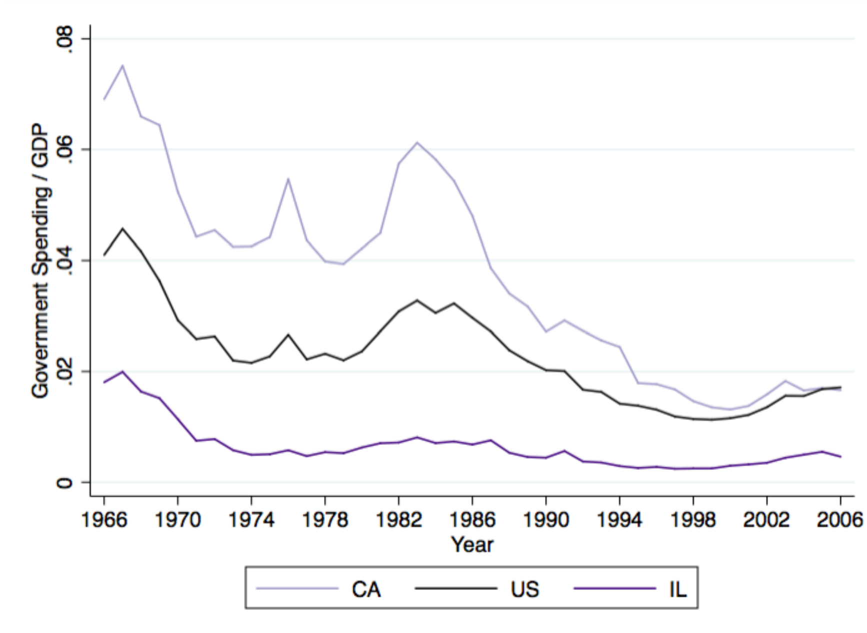


Figure 3: Prime Military Contract Spending as a Fraction of State GDP

and time fixed effects achieve these by demeaning the regressor (Δg) and regressand (Δy) along the cross-section and time dimension respectively.

4.3.2 Results

Table 2 presents the non-structural moment estimates of equation (22). Columns (1) and (3) display the results contrasting OLS to 2SLS. As can be seen, the OLS estimate of the main term in row one is severely biased downwards, as would be expected due to endogeneity: the government is likely to spend more money in states that are suffering economically, therefore leading to a negative correlation between state GDP and state spending, hence negatively biasing the OLS.

The estimates of the interaction term in row two have large standard errors, therefore I weight population in columns (2) and (4), showing the OLS and 2SLS results respectively. Following the prescription of Solon et al. (2015), the motivation for this is due to the squared predicted residuals of equation (22) exhibiting a robust dependence on population, as can be seen in table 4. The dependence remains even when winsorizing at the 95% level, and when considering the relation in logs.

The coefficient values are expected and intuitive. $\beta_1^{non-structural}$ is the local fiscal multiplier, and a value of about 1 to 2 is consistent with the results in literature.[59] $\beta_2^{non-structural}$

local effect in i from an aggregate shock, $\Lambda_i^{aggregate}$. Assumption 3 rules this out.

[59] See Nakamura and Steinsson (2014).

describes the dependence of the local fiscal multiplier on the state import share. A negative value is expected as there is a lot of leakage from spending in a state that is very open to imports: the GDP gains disperse to other states.[60]

Columns (5) and (6) include an additional geographic characteristic in equation (22): state export share. As can be seen, this is not an important determinant of the local fiscal multiplier. This motivates why I choose only to match $(\beta_1^{non-structural}, \beta_2^{non-structural})$ in the strategy of section 4.1.4.

The shaded cells in table 2 contain the values which I use for the subsequent moment matching with the structural model. The estimated results of this exercise for the structural demand and supply elasticities, (ϕ, ψ), are displayed in table 3. The demand elasticity is the more precisely estimated parameter out of the two, and is approximately 1. The supply elasticity is more noisy, though significantly negative. A discussion of their values is given in section 4.4.

Confidence intervals are computed from the standard errors of the $(\beta_1^{non-structural}, \beta_2^{non-structural})$ estimates in table 2. To do this, I appeal to asymptotics and assume the true $(\beta_1^{non-structural}, \beta_2^{non-structural})$ are distributed normally with mean and covariance matrix equal to the mean and covariance of my estimated values $(\hat{\beta}_1^{non-structural}, \hat{\beta}_2^{non-structural})$. Using this distribution, I simulate M values of $(\beta_1^{non-structural}, \beta_2^{non-structural})$, and for each value conduct the moment matching exercise and find the associated (ϕ, ψ). This gives me a distribution of (ϕ, ψ) of size M. I calculate the percentiles of this distribution for inference: the 95% confidence interval is shown in brackets in table 2.[61]

4.3.3 Validation

To confirm the performance of the structural estimation procedure in section 4.1.4, I regress the estimated $(\beta_1^{structural}, \beta_2^{structural})$ on the geographic characteristics – equation (26). The coefficients here should be the same as the non-structural coefficients in table 2 column (4), as these are what the procedure has tried to match them to. The results are in table 5 column (1). As can be seen, they are very close. This is reassuring that the procedure is running as expected.

I calculate the confidence intervals as follows. As explained in the inference of section 4.3.2, I have M simulated values of (ϕ, ψ). For each of these, I can calculate the associated

[60]This is consistent with analysis of fiscal multipliers at the national level and their dependence on international trade exposure. See Auerbach and Gorodnichenko (2013).

[61]Appendix table 10 displays the full joint density of this distribution. Note this distribution is not normal, and is the reason why I provide confidence intervals in table 2, rather than standard errors. It is the confidence intervals that are relevant for inference; standard errors are informative about the confidence intervals for normal distributions, but in general, are not.

Parameter Identified	Regressor	Dependent variable: $\frac{\Delta y_{it}}{y_{it}}$ (1)	(2)	(3)	(4)	(5)	(6)
$\beta_1^{non-structural}$	$\frac{\Delta g_{it}}{y_{it}}$	0.24	0.24	1.56***	1.01***	1.55***	0.99***
		(0.17)	(0.18)	(0.45)	(0.25)	(0.43)	(0.27)
$\beta_2^{non-structural}$	$\frac{\Delta g_{it}}{y_{it}} \cdot s_i^{Im}$	-4.74**	-2.14	-7.42*	-4.63***	-8.06	-5.60*
		(1.96)	(1.29)	(3.81)	(0.62)	(6.16)	(3.24)
$\beta_3^{non-structural}$	$\frac{\Delta g_{it}}{y_{it}} \cdot s_i^{Ex}$					0.58	0.86
						(3.56)	(2.77)
Estimator		OLS	OLS	2SLS	2SLS	2SLS	2SLS
First-stage F		-	-	3×10^{16}	6454	3.5×10^{15}	4463
Weights		none	pop	none	pop	none	pop
Observations		1989	1989	1989	1989	1989	1989

Cluster (i) robust standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Local fiscal multiplier dependence on geography: non-structural estimation, equation (22).

Notes. Shaded values are those used for the structural estimation.

	(1)
ϕ	0.97
	[0.95, 1.08]
ψ	-1.43
	[-2.29,-1.11]

95% coverage confidence intervals shown in brackets.

Table 3: Estimation of structural parameters, equation (28).

Notes. Standard errors computed using method described in section 4.3.2 with $M = 1000$. (ϕ, ψ) is searched for on a grid of increment 0.005 in the domains $\phi \in [0.5, 1.5]$ and $\psi \in [-3, -0.5]$. Domains chosen from successive narrowing of initial coarser but wider grids.

	Dependent variable: $\hat{\varepsilon}_{it}^2$ (1)	$\hat{\varepsilon}_{it}^2$ (2)	$\ln(\hat{\varepsilon}_{it}^2)$ (3)
$population_{it}^{-1}$	6255***	555***	
	(1793)	(99)	
$\ln(population_{it}^{-1})$			0.40***
			(0.06)
Winsorized	no	< 95%	no
Observations	1989	1989	1989

Robust standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Residual analysis of equation (22): heteroskedasticity due to population.

Parameter Identified	Regressor	Dependent variable: $LFM_i^{structural}$ (1)	(2)
$\beta_1^{structural}$	constant	1.14	1.15^{+++}
		[0.37,1.59]	[0.37,1.59]
$\beta_2^{structural}$	s_i^{Im}	-3.94	-2.64^{+}
		[-9.27,-2.40]	[-8.45,1.98]
$\beta_3^{structural}$	s_i^{Ex}		-1.28
			[-4.64,0.52]
Estimator		OLS	OLS
Observations		51	51

95% confidence intervals shown in brackets.

One-sided test away from zero for (2): $^{+}$ $p < 0.1$, $^{++}$ $p < 0.05$, $^{+++}$ $p < 0.01$

Table 5: Local fiscal multiplier dependence on geography: structural estimation, equation (26).

$LFM^{structural}$, and get the associated $\beta^{structural}$ vector by running the regression (26). This gives me a distribution (size M) of the $\beta^{structural}$ vector. I calculate the percentiles of this distribution, and the 95% confidence intervals are shown in brackets in table 2.

To provide evidence that my theoretical framework is an appropriate model of the real world, I see how my structural results perform on moments that were not matched. Specifically, the dependence of the local fiscal multiplier on state export share; this is column (2) in table 5. The corresponding non-structural moments are in table 2 column (6). Reassuringly, both the non-structural and structural estimates reject positive dependence of the local fiscal multiplier on state import share, and both cannot reject zero dependence on state export share.

4.4 Discussion

In this section I discuss the implied economics by the values of the supply and demand elasticities.

4.4.1 Supply Elasticity

The negative estimated value of $\psi = -1.43$ implies that supply is downward sloping. This may seem to go against expectation, but it is important to acknowledge that the supply equation (4) is a general equilibrium relation, combining, for example, both the partial equilibrium production optimal pricing equation and optimal labor supply.

In fact, a negative supply elasticity is actually expected in order to generate a multiplier mechanism that underlies the fiscal multiplier. As explained in section 3.2.2 and illustrated by figure 2. Only when $\psi < 0$ is the equilibrium change in output greater than the size of the demand shock (figure 2c); that is, only negative ψ can generate the fiscal multiplier magnification effect. In canonical macroeconomic fiscal multiplier models, the analogous supply equation is highly complex due to dynamics and as a result does not exhibit an analogous constant elasticity of supply. Therefore this finding of a negative equilibrium supply elasticity is likely obscured in those models, but nonetheless present.

In order to understand the economic forces generating $\psi = -1.43$, I revisit the micro-structure underlying ψ presented in equation (18)

$$\psi = \frac{1-\sigma^{-1}}{\frac{1+\nu^{-1}-\theta}{1+\chi} + \xi - (1-\sigma^{-1})}$$

with the parameter interpretations detailed in table 1. Any mechanism that implies prices decreases at greater scales of output will push ψ towards being negative. The three present here are increasing returns to scale in production (production is more efficient at greater scales, meaning price is less) and is generated by $\chi > 0$; countercyclical mark-up, (modeled in the the macroeconomics literature by sticky prices: the mark-up decreases at greater scales as wages adjust up, in response to moving up the labor supply curve, but prices are slow to adjust)[62] and is reflective of $\xi < 0$; and labor-consumption complementarities (laborers consume even more as they work more meaning equilibrium output increases by even more, leading to a multiplication effect)[63] and is reflective of $\theta > 0$.

Can $\psi = -1.43$ be rationalized by plausible magnitudes of these underlying channels? I investigate this in table 6, where I present various combinations of $(\sigma, \nu, \chi, \xi, \theta)$ that generate $\psi = -1.43$. A challenge is that the value of these parameters are very much context dependent, and even then there is not always a strong consensus in the literature.[64] Nonetheless, table 6 shows that $\psi = -1.43$ is easily able to be matched by values of the underlying parameters within the literature's range. Note that in absence of these three mechanisms, as illustrated by row f), $\psi = -1.43$ is not able to be rationalized, even with income effects dominating on labor.[65]

[62] See Christiano et al. (2011). Paraphrasing the authors: since prices are sticky, price over marginal cost falls after a rise in demand. This fall in the mark-up induces an outward shift in the labor demand curve.

[63] Intuitively, at larger scales, households are working more, therefore consume more due to the complementarity, therefore increase the scale of output even more. The complementarity can, for example, represent the extra consumption on food away from home, clothing, gas, and the like that often arises in the context of work.

[64] For σ, the literature has found values close to zero and greater than one (for a summary, see Nakamura and Steinsson (2014), page 775), For ν, in microeconomic contexts it is about 0.5, in macroeconomic contexts, can be 1 or higher (Nakamura and Steinsson (2014)); for females ($\nu = 2.2$) it is much higher than males ($\nu = 0.3$) (see Greenwood et al. (1988), page 412). For χ, the Urban economics literature suggests a value in the range 0.3 to 0.6.

[65] Of course, there could be a mechanism unconsidered in this paper that rationalizes $\psi = -1.43$ without

	σ	ν	χ	ξ	θ	ψ
a)	1.13	1	0.45	-1	0.5	-1.43
b)	1.13	0.5	0.45	-1	1.50	-1.43
c)	1.13	1	0.45	0	1.95	-1.43
d)	1.13	1	0	-1	0.97	-1.43
e)	1.21	1	0.45	-1.34	0	-1.43
f)	0.1	1	0	0	0	-0.82

Table 6: Rationalizing $\psi = -1.43$ using various microfoundations.

Notes. a) is the baseline. I consider changes keeping $\sigma = 1.13$ (either around this or close to zero in literature) and $\chi = 0.45$ (somewhat the consensus in literature) fixed.
b): ν, θ are substitutable in their effect on ψ. For example, a micro-level ($\nu = 0.5$) labor supply can be rationalized with high $L - c$ complementarities ($\theta = 1.5$)
c): Invariable mark-ups ($\xi = 0$) can be accommodated if L-c complementarities are higher ($\theta = 1.95$ rather than $\theta = 0.5$)
d): No scale economies ($\chi = 0$) can be accommodated if L-c complementarities are higher ($\theta = 0.97$ rather than $\theta = 0.5$)
e): no $L - c$ complementarity ($\theta = 0$) can be accommodated if the mark-ups have even greater intensity of countercyclicality. ($\xi = -1.34$ rather than $\xi = -1$)
f) $\psi < -1$ cannot be rationalized without any of these three channels, even with income effects dominating on labor ($\sigma < 1$)

4.4.2 Demand Elasticity

Recall ϕ is the elasticity of substitution between products produced in different locations. The estimated value of $\phi = 0.97$ implies that the substitutability between products from different locations is low, with $\phi = 1$ not being rejected. Economically, this means that, given a relative price increase in location i^*, consumers do consume a lower quantity of goods from i^*, but the substitution is low enough that their relative expenditure on goods from i^* is roughly unchanged (the price increase offsets the quantity decrease).

Consistent with my empirical setting, there are at least three explanations for this finding. The first derives from the industrial composition of states. ϕ is the interstate elasticity of substitution; this is an average of the constitutent interindustry elasticity of substitutions, weighted by their state shares. The military contracts in my analysis are heavily represented in both manufacturing and services (see figure 4),[66] and the literature is suggestive of a complementarity elasticity of substitution between manufacturing and service industries.[67]

requiring increasing returns to scale, a countercyclical mark-up, or labor-consumption complementarities.

[66] Note that my data on trade flows is restricted to manufacturing. The implication of only using manufacturing trade flows is that I have measurement error in the trade flows; effectively, I am assuming that the trade flows of non-manufacturing follow the same pattern as manufacturing.

[67] See e.g. Comin et al. (2015) in the context of structural change.

Therefore, the implied interstate elasticity of substitution is a weighted average of elasticities that are greater than and less than one. The finding of ϕ close to one therefore is not too surprising.

The second derives from industrial specialization across locations, as this can further decrease the substitutability of state product aggregates.[68] Intuitively, even if your preferences are highly substitutable across industries, if the industries you consume from are concentrated in a single state, a price increase will cause you to substitute to another industry in the same location, rather than to a different location. Thus leading to an attenuated interstate elasticity of substitution. Figure 5 graphs the share of military expenditure in manufacturing by state.[69] The share varies widely from 2% in Wyoming to 85% in Missouri, offering evidence in support of the specialization mechanism.[70]

The third explanation is due to the duration of shocks.[71] If it is costly to switch products, it is immediate that the elasticity of substitution is going to be smaller for shocks that are of shorter duration, and evidence of this has been provided in the literature.[72] The military procurement shocks are certainly of transitory nature, therefore it is certainly reasonable to consider ϕ as a short-run estimate.[73]

5 Inference on the Fiscal Multiplier

Given the previous section's estimates of ψ, ϕ, the object $\Lambda^{transfers}$ can now be identified, therefore allowing me to quantify the component of the fiscal multiplier due to the spatial mechanism. In section 5.1, I quantify the implied multipliers by state. In section 5.2, I analyze the dependence of the aggregate fiscal multiplier on geography. In section 5.3, I investigate the role of the structural demand and supply elasticity in determining the multiplier.

[68] I show this in appendix A.5 using a multi-sector nested CES model.

[69] The more recent datasource on USAspending has industry codes per contract.

[70] A story based on specialization is interesting though beyond the scope of this paper as my model is only informative about the movement of goods at the region-aggregate level. Such direction provides an avenue for future research.

[71] Estimates using gravity specifications in the trade literature tend to find elasticities in the range of 3 to 7 (see e.g. Eaton and Kortum (2002)), but these are long-run elasticities as the variation — distance — does not change over the short-run. Estimates using tariff changes can produce longer or shorter run estimates depending on the policy setting. See Fajgelbaum et al. (2019) for elasticities estimated to be less than 1.

[72] For short- and long-run elasticity estimates, see Alessandria and Choi (2019) for Armington elasticities, and Bentzen and Engsted (1993) for in the energy industry. Both estimate the elasticity of substitution to be less than one in the short run and greater than one in the long-run.

[73] This is in contrast to, for example, identification based on gravity regressions in the trade literature (see e.g. Allen et al. (2014)). The identifying variation here is usually distance between origin and destinations of trade flows. Distance does not vary much over time and therefore the identified elasticities correspond to long-run estimates.

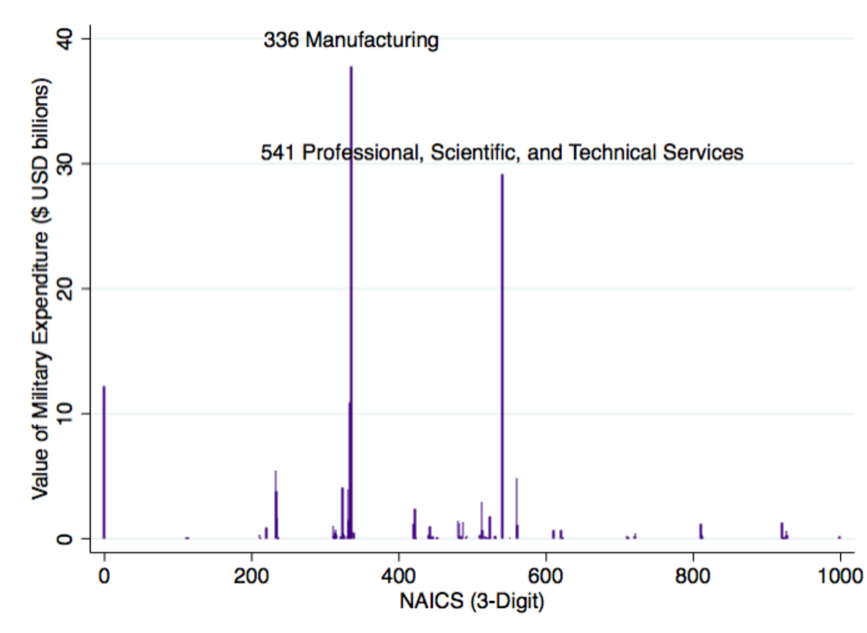


Figure 4: Industry composition of Defense Procurement.

Notes: Year 2001. Across 50 US States + DC.
Datasource: USAspending.

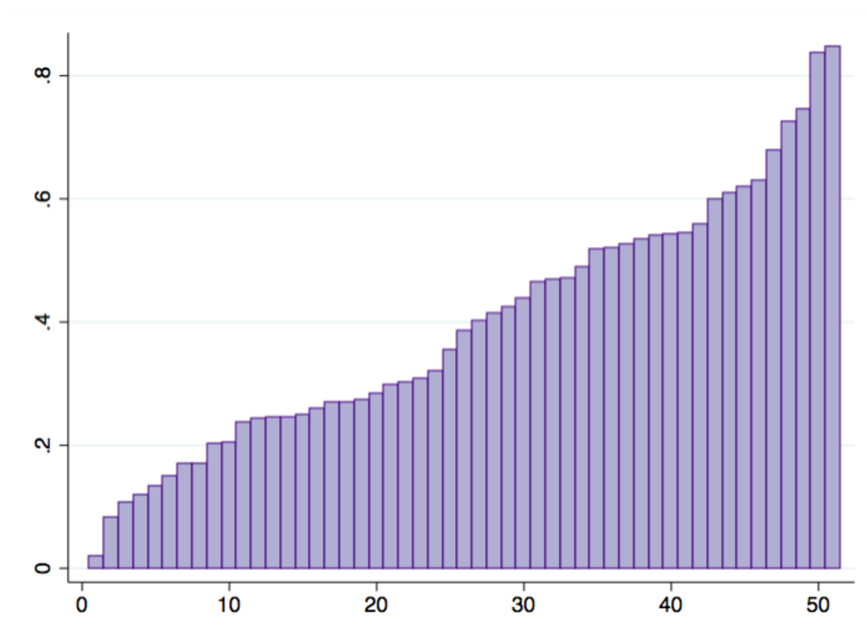


Figure 5: Share of Military Expenditure in Manufacturing, by state.

Notes: Year 2001. 50 states + DC. x-axis: states ordered by manufacturing share.
Datasource: USAspending.

5.1 Quantifying the Spatial Mechanism

Figure 6 graphically illustrates the heterogeneity by state in the fiscal multiplier at the estimated values of (ϕ, ψ). I consider two objects, the local effect LFM_i in figure 6a and aggregate effect AFM_i in figure 6b; the darker the state is shaded, the greater the change in GDP. Due to my structural framework of section 4.1 exploiting transfers between states for identification, both of these objects are relative measures. Precisely, the interpretations (and formulae) are

$$\begin{aligned} LFM_i &\equiv \Lambda_{ii}(\hat{\phi}, \hat{\psi}, \Omega^{observables}) \\ &\equiv \Delta(\text{GDP})_i \text{ from spending \$1 in state } i \\ &\quad - \\ &\quad \Delta(\text{GDP})_i \text{ from spending \$1 proportionally by GDP}_j \text{ across all } j \end{aligned}$$

$$\begin{aligned} AFM_i &\equiv \sum_j y_j \Lambda_{ji}(\hat{\phi}, \hat{\psi}, \Omega^{observables}) y_i^{-1} \\ &\equiv \Delta(\text{national GDP}) \text{ from spending \$1 in state } i \\ &\quad - \\ &\quad \Delta(\text{national GDP}) \text{ from spending \$1 proportionally by GDP}_j \text{ across all } j \end{aligned}$$

That is, the level of the aggregate fiscal multiplier is not identified, but this is no problem as I am only interested in the heterogeneity across states, which the relative measure is still informative about.

For the local effect (figure 6a), the local fiscal multiplier at the median is 1.02. That is, at the average, a state receiving an increase in government spending by \$1, that is financed by a transfer from all states proportionally to their GDP, will increase the receiving state's GDP by \$1.02. The large magnitude is a consequence of the value of supply elasticity. It is estimated to be negative, and therefore implies a magnification effect due to spending (see section 3.2.2). Note that a negative supply elasticity doesn't necessitate a local multiplier greater than one: this is because even though the state receives a transfer of \$1, demand from other states falls as they are financing the transfer.

For the aggregate effect (figure 6b), the aggregate fiscal multiplier at the median is 0.15, with close to half of the states exhibiting a positive multiplier, and the other negative. This is by construction due to the symmetry of the operation: if transferring from state i to j generates a positive change in national GDP, then a transfer from j to i must generate a negative change, as it's exactly the same operation in reverse. Note too that this implies the average is also close to zero by construction: the aggregate fiscal multiplier for a state involves summing the effect of transfers to it from all other states; taking the average across all states involves summing the effect of transfers for all pairs of states and in both directions, therefore canceling out.[74] A more informative statistic about the magnitude therefore may be the median of the absolute values, which is equal to 0.28. This says that, keeping aggregate expenditure fixed, a \$1 transfer between states on average affects national GDP in some direction by \$0.28.

How can these numbers be understood in the broader, more conventional, context of fiscal multipliers, without restricting to transfers, and allowing aggregate spending to change? Consider the median state, Delaware, with fiscal multiplier 0.28. The implication is that if a government increases aggregate spending by \$1, and spends all of this dollar in Delaware, the conventional fiscal multiplier will be 0.28 greater than if the government distributed this spending across all states, proportionally to their GDP.

The important take-away from these maps is less about the magnitudes of the local and aggregate fiscal multipliers, but the geographic heterogeneity. I find it to be very heterogeneous with the length of the interquartile range, $length(IQR)$, across states to be 0.37 in the local fiscal multiplier, and of 0.44 in the aggregate fiscal multiplier.[75] If the multiplier was homogeneous across all states, then the geography of spending would be inconsequential. As explained in section 3.3, the spatial mechanism arises due to geographic asymmetry between the financing and beneficiary states. My estimates quantify the effect of geographic asymmetry due to the internal geography of trade. Section 5.2 below discusses how we can understand this implied relation between geography and fiscal multipliers.

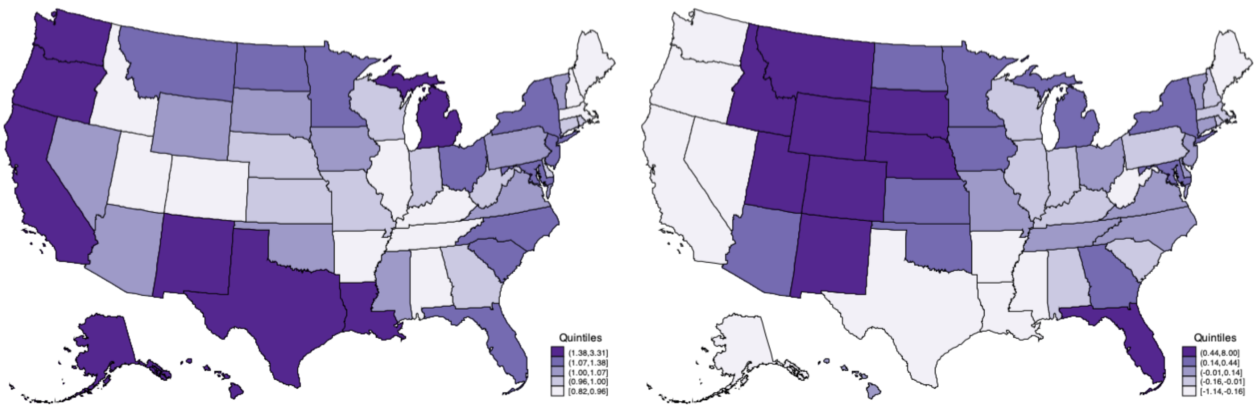
5.1.1 Sensitivity Analysis

The estimated (ϕ, ψ) are subject to measurement error, and therefore $\Lambda^{transfers}$ and the resulting fiscal multiplier estimates in figure 6 are subject to error too. To tractably get a sense of this, I look at the error in the IQR length, as this statistic is what captures the heterogeneity across states, and it is the heterogeneity that is of interest. Figure 7 displays the distribution of this across the simulated (ϕ, ψ) for both the local and aggregate fiscal multipliers.

For the aggregate fiscal multiplier, in sub-figure 7b, the 10^{th} percentile is 0.13. This implies that at 0.1 significance, heterogeneity in the aggregate fiscal multiplier of an IQR length less than 0.13 can be rejected. This is big, and on an order of magnitude close

[74]The average is precisely zero if the average is weighted by state GDP. This is because the transfers are proportional to each state's GDP.

[75]I report the length of interquartile range rather than the standard deviation as I have the quantiles of the distribution, therefore needn't use a normal approximation.



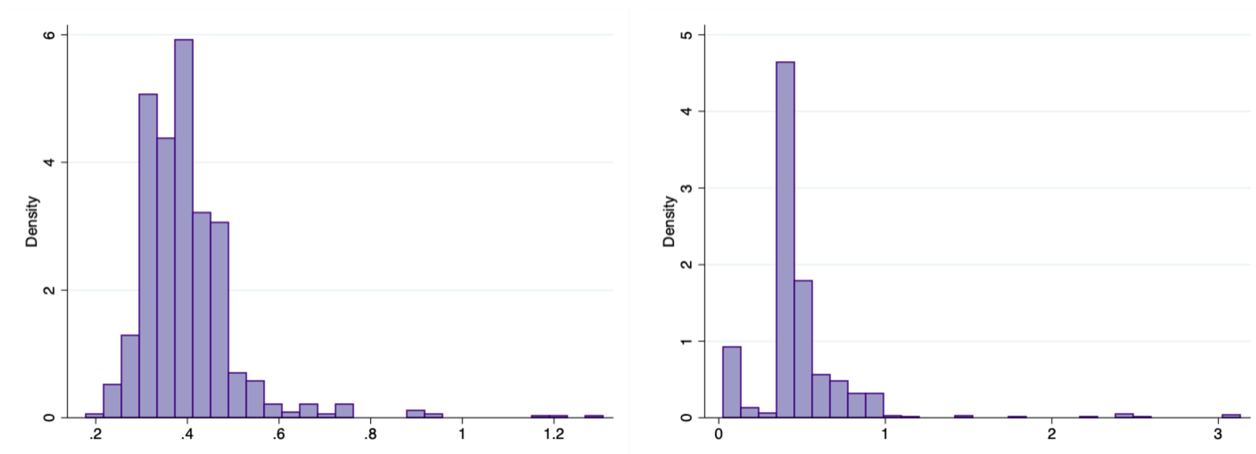
(a) $\{LFM_i\}_{i\in\{1,\ldots,N\}}$

Standard deviation across states, $length(IQR[i : LFM_i]) = 0.37$

(b) $\{AFM_i\}_{i\in\{1,\ldots,N\}}$

Standard deviation across states, $length(IQR[i : AFM_i]) = 0.44$

Figure 6: Local and Aggregate Fiscal Multipliers by state, at the estimated $(\phi, \psi) = (0.97, -1.43)$



(a) Distribution of $length(IQR[i : LFM_i])$

(b) Distribution of $length(IQR[i : AFM_i])$

(Distribution truncated at 95% percentile.)

Figure 7: Error in the Local and Aggregate Fiscal Multipliers due to error in the estimated (ϕ, ψ)

to the median absolute value of the aggregate fiscal multiplier (0.28). That is, states are significantly heterogeneous in the transmission of government expenditure to national GDP.

5.2 Dependence on Geography

The quantitative results in the above section indicate that the multiplier exhibits large heterogeneity arising due to the internal geography of trade. Precisely how the fiscal multiplier depends on the geography is complex relation, depending on the entire network of interstate trade flows, with each state being a node in a 51×51 network. To distill some helpful regularities and get some insight into the mechanism, I investigate the correlation with state-specific trade statistics, for example with the following regression

$$AFM_i^{structural} = \theta_1^{structural} + \theta_2^{structural} s_i^{Im} + \theta_3^{structural} s_i^{Ex} + \varepsilon_i \tag{29}$$

where s_i^{Im} and s_i^{Ex} are the import and export shares of state i. Table 7 presents the regression results. In column (1), the results from regression (29) are displayed; in column (2) the regressor is the trade balance of a state, TB_i, which is defined as state production minus state absorption.

Table 7 column (1) reveals a strong positive correlation with state import share, and a negative correlation with state export share. This inverse dependence on import vs export share suggests a negative dependence on the state trade balance, which column (2) confirms.

The trade balance also provides a convenient, single state-specific statistic summarizing the geography of the state. This analysis implies that increasing the trade balance of a state by 5 percentage points decreases the aggregate fiscal multiplier from spending in that state by 0.63. This magnitude is meaningfully big, as the consensus from the literature on average aggregate fiscal multipliers is in the range 0.5 to 1.

From these results follows the key result of the paper. When it comes to fiscal multipliers, the geography of spending matters a lot. The spatial mechanism – the result of distinct financing and spending locations of asymmetric geography – can generate large variation in the aggregate fiscal multiplier. A *geography-dependent* fiscal multiplier can characterized with greater magnitudes expected in states that run a trade deficit.

5.2.1 Mechanism

So, what is going on? To understand the qualitative dependence of the aggregate fiscal multiplier on geography, it's instructive to first ask why isn't the multiplier zero? After all, the experiment here is a transfer between states. This increases GDP in the beneficiary state, and decreases GDP in the financing state. Shouldn't the two cancel out leading to net zero effect on aggregate? This is the case if trade is balanced between states. However when trade is not balanced — the state import and export shares become decoupled — this is no longer the case and a non-zero effect on aggregate will result. Note that this is the empirically relevant case for US states: trade is not balanced for any state.

This can be understood as follows. Consider state i^* receiving a transfer. Then, initially, its GDP directly increases from the spending, while all other states GDP initial decrease due

	(1)	(2)
constant	0.11	0.12
	[0.06]	[0.06]
s_i^{Im}	13.76	
	[0.01]	
s_i^{Ex}	-9.15	
	[0.05]	
Trade Balance$_i$		-12.6
		[0.05]
Observations	51	51

p-value for $|\beta| > 0$ shown in brackets.

Table 7: Aggregate fiscal multiplier dependence on geography: structural estimation, equation (29).

Notes. Column (1) is equation (29); column (2) is AFM regressed on the state trade balance.

to receiving less spending.[76] This has two effects. First, this GDP increase in i^* increases income and therefore consumption from i^*. Due to trade, consumers and firms in i^* do not just increase consumption of goods produced locally, but from all states. This creates positive spillovers, raising GDP in all other states. The strength of these positive spillovers are controlled by state i^* *import share*. Aggregate GDP is a function of all states GDP, therefore leading to a positive dependence of the aggregate fiscal multiplier on state import share.

Second, because of the amplification mechanism, arising due to the negative supply elasticity ψ, real wage in i^* increases, therefore further increasing expenditure by i^*, thus furthering increasing the real wage; a positive feedback loop ensues. The strength of this feedback is determined by the amount of i^* expenditure that is spent on i^* products, as it is an outward shift in i^* production that moves us down the negatively-sloped i^* supply curve. The proportion of expenditure spent locally is controlled by the state's *export share*: a smaller share implies a greater proportion spent locally, and therefore a greater amplification effect. Thus, a negative dependence of the aggregate fiscal multiplier on state export share

5.3 The Role of the Supply and Demand Elasticity

Key to the mechanics of the model are the value of the supply and demand elasticity. Here I discuss how their value affects the value of the fiscal multiplier.

[76] Of course, it is all general equilibrium and everything happens simultaneously, but for exposition it is helpful to consider the process in the described order.

Supply Elasticity. Figures 8a and 8b illustrate the dependence on ψ of the local fiscal fiscal multipliers, $\Lambda_{ii}^{transfers}$, and the spillover effects, $\Lambda_{i\neq j}^{transfers} + \frac{y_j}{y_{agg}}$ from transfers, respectively.[77] I graph the effects at the estimated supply elasticity, $\psi = -1.43$, along with two counterfactual supply elasticities, $\psi \in \{-2, -1.3\}$. All three are at the estimated $\phi = 0.97$.

As ψ becomes more negative, the supply curve becomes more intensely downward-sloped. A spending shock therefore decreases local prices to a greater extent, and the output response is magnified to a greater extent. Figure 8a demonstrates this: as ψ becomes more negative, the distribution $\{\Lambda_{ii}^{transfers}\}_i$ shifts right: the effect on local GDP from a transfer is greater.

A similar result is seen in the spillovers, though less pronounced. As ψ becomes more negative, GDP in other locations expands more too.

Demand Elasticity. Figures 9a and 9b present the analogous analysis to above, except now varying the demand elasticity, ϕ. I graph the effects at the estimated demand elasticity, $\phi = 0.97$, along with two counterfactual demand elasticities, $\psi \in \{0.92, 1.02\}$. All three are at the estimated $\psi = -1.43$.

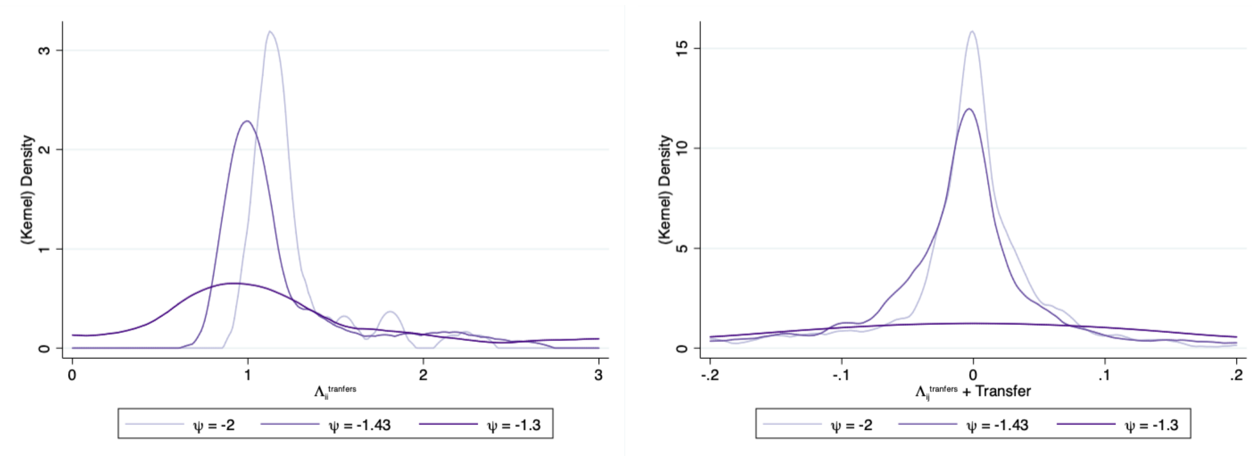
As ϕ becomes more positive, the degree of substitutability between products from different states becomes greater. In response to a spending increase in j, j GDP increases and j prices relatively decrease (as supply is downward-sloping). This makes j products more attractive to consumers and firms in other states. The more substitutable the goods are, the greater the substitution towards the j goods, and the greater the increase in j GDP. Figure 9a demonstrates this: as ϕ increases, the distribution $\{\Lambda_{ii}^{transfers}\}$ shifts to the right: the effect on local GDP from a transfer is greater.

The spillovers don't exhibit a clear directional dependence with the demand elasticity, though I present the graph for completeness.

6 Conclusion

In this paper, I reveal a spatial mechanism for generating the aggregate fiscal multiplier. Because government budgets balance nationally, the geographic distribution of a government spending stimulus is able to vary independently to the geographic distribution of the tax burden. Given asymmetric economic geography, the resulting wedge between local spending and local tax burdens can generate an increase in aggregate GDP. This effect is independent of the behavioral response to taxation, thereby providing a distinct mechanism to the

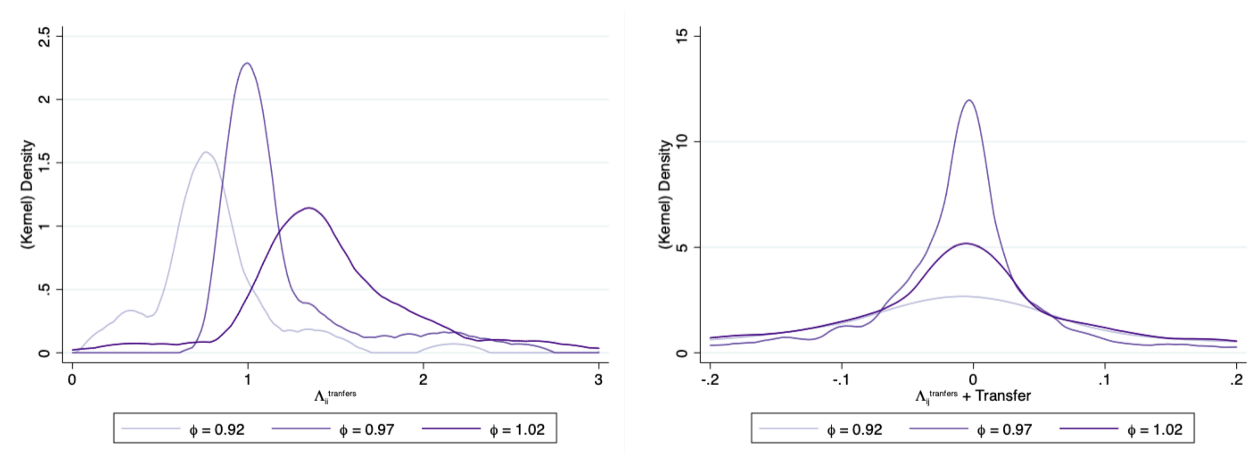
[77] Note that the mechanical decrease in GDP due to financing the transfer, $\frac{y_j}{y_{agg}}$, is included in the measure of $\Lambda^{transfers}$. As this is not normally considered part of the spillover effect, I correct for this by adding the transfer back on.



(a) Local Fiscal Multipliers, $\forall\, i : \Lambda_{ii}^{transfers}$ (b) Spillovers, $\forall\, i, j : i \neq j : \Lambda_{ij}^{transfers} + \frac{y_j}{y_{agg}}$

Figure 8: ψ and the magnitude of $\Lambda^{transfers}$

Notes. At estimated value of ϕ.



(a) Local Fiscal Multipliers, $\forall\, i : \Lambda_{ii}^{transfers}$ (b) Spillovers, $\forall\, i, j : i \neq j : \Lambda_{ij}^{transfers} + \frac{y_j}{y_{agg}}$

Figure 9: ϕ and the magnitude of $\Lambda^{transfers}$

Notes. At estimated value of ψ.

canonical New Keynesian and Neoclassical models.

Geographic heterogeneity is a key component of the mechanism. I focus on the internal geography of trade and quantify this in the context of states in the US. However, directly identifying the dependence of the aggregate fiscal multiplier on internal geography is not feasible. Therefore, I exploit that local fiscal multipliers suffer less severe identification issues, and use a general equilibrium model of trade to map these non-structurally identified local moments to the structurally-implied aggregate fiscal multiplier.

Using models from the International Trade literature to characterize fiscal multipliers is unique in the literature, crucially allowing me to bring in spatial richness. Importantly, in justifying this application, I prove that the standard fiscal multiplier amplification mechanisms in the macroeconomic literature are isomorphic in this framework.

I estimate the model using US Federal Defense Procurement in the late 20^{th} century, and apply Bartik instruments for exogeneity. My structural framework requires observable trade flows for this period, yet this pre-dates standard sources for the US. Therefore I digitize a new dataset on interstate trade from the US National Archives.

I find large heterogeneity by state in the aggregate fiscal multiplier, and I'm able to distill qualitative dependencies on the internal geography of trade. Greater magnitudes are expected in states that are more open to imports from other states, and less open to exports to other states.

This is the key result of this paper. The spatial mechanism is of meaningful importance, yet largely abstracted from in the literature. Just like the advancement of state-dependent fiscal multipliers, my results suggest an analogy with *geography-dependent* fiscal multipliers: greater changes in aggregate GDP are expected when spending is concentrated in states that run a trade deficit with other states.

Appendices

A Theory

A.1 Deriving the Cross-Location Multiplier, equation (13)

The outline of the derivation is as follows. The first step is to construct the structural demand and supply equations of the economy. These are N-location analogues of textbook supply and demand equations; each are implicit functions of endogenous GDP y and prices p, and exogenous government spending shocks g. These structural equations are highly non-linear, therefore, next I log-linearize. I log-linearize about the equilibrium with observed trade flows in 1977, and transfers set to zero. The final step is to invert the log-linearized structural equations to yield the reduced form equations; each are explicit functions of GDP and prices as a function of government spending shocks. The relation between GDP and government spending is precisely the cross-location multiplier, equation (13). However, the inversion is non-trivial as the set of equations is not full rank due to price being invariant up to scale. I detail all this in the following.

Structural Equations Use the local labor markets equation (10), government spending equations (5), (7), and geography equation (8) to substitute out E_i, G_i and p_{ij}, respectively, in the demand equation (3)

$$X_{ij} = \left(\frac{\tau_{ij} p_i}{P_j}\right)^{1-\phi} Y_j \xi_j^{-1} \Xi^{-1} + \mathcal{P}(g_i - b y_i) \cdot \mathbb{1}[i = j]$$

Insert this equation into the product market clearing equation (9)

$$Y_i = \sum_j \left(\frac{\tau_{ij} p_i}{P_j}\right)^{1-\phi} Y_j \xi_j^{-1} \Xi^{-1} + \mathcal{P}(g_i - b y_i)$$

Convert to real GDP, insert the price index dependence $P_j = P_j(\boldsymbol{p}) \equiv \left(\sum_i (\tau_{ij} p_i)^{1-\phi}\right)^{\frac{1}{1-\phi}}$ and rearrange

$$D_i(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{\xi}, b, \Xi) \equiv \mathcal{P} y_i - \sum_j \left(\frac{\tau_{ij} p_i}{P_j(\boldsymbol{p})}\right)^{1-\phi} \mathcal{P} y_j \xi_j^{-1} \Xi^{-1} + \mathcal{P}(g_i - b y_i) = 0 \tag{30}$$

where bolded variables denote the vector across locations, $\boldsymbol{z} \equiv \{z_i\}_{i=1}^N$. $\boldsymbol{D}(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, b) = \boldsymbol{0}$ form the structural demand equations. Next, multiply the supply equation (4) by p_i, convert to real GDP, insert $P_j = P_j(\boldsymbol{p})$ and rearrange to give

$$S_i(\boldsymbol{y}, \boldsymbol{p}) \equiv \mathcal{P} y_i - A_i \frac{p_i}{\mathcal{P}} \left(\frac{p_i}{P_i(\boldsymbol{p})}\right)^{\psi} = 0 \tag{31}$$

$\boldsymbol{S}(\boldsymbol{y}, \boldsymbol{p}) = \boldsymbol{0}$ form the structural supply equations. Next, is the government budget constraint, equation (6)

$$B(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, b) \equiv \sum_i \mathcal{P}(g_i - b y_i) = 0$$

And finally, is the trade imbalance equation (10)

$$TB(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{\xi}, \Xi) \equiv \sum_i \mathcal{P} y_i (1 - \xi_i \Xi)$$

Together, the complete set of structural equations for the system are

$$\begin{aligned} \boldsymbol{D}(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{\xi}, b, \Xi) &= \boldsymbol{0} \\ \boldsymbol{S}(\boldsymbol{y}, \boldsymbol{p}) &= \boldsymbol{0} \\ B(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, b) &= 0 \\ TB(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{\xi}, \Xi) &= 0 \end{aligned} \tag{32}$$

Log-linearized Structural Equations. Log-linearizing equation (32) to a perturbation in $\boldsymbol{g}$ gives

$$\underbrace{\begin{pmatrix} \nabla_{\ln \boldsymbol{y}} \boldsymbol{D} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{D} & \nabla_b \boldsymbol{D} & \nabla_{\ln \Xi} \boldsymbol{D} \\ \nabla_{\ln \boldsymbol{y}} \boldsymbol{S} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{S} & \boldsymbol{0} & \nabla_{\ln \Xi} \boldsymbol{S} \\ \nabla_{\ln \boldsymbol{y}} B & \nabla_{\ln \boldsymbol{p}} B & \nabla_b B & \nabla_{\ln \Xi} B \\ \nabla_{\ln \boldsymbol{y}} TB & \nabla_{\ln \boldsymbol{p}} TB & \nabla_b TB & \nabla_{\ln \Xi} TB \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} \mathrm{d} \ln \boldsymbol{y} \\ \mathrm{d} \ln \boldsymbol{p} \\ \mathrm{d} b \\ \mathrm{d} \ln \Xi \end{pmatrix} = \underbrace{\begin{pmatrix} -(\nabla_{\boldsymbol{g}} \boldsymbol{D}) \mathbb{Y} \\ \mathcal{O} \\ -\nabla_{\boldsymbol{g}} B \\ \mathcal{O} \end{pmatrix}}_{\equiv \tilde{\Gamma}} \frac{\mathrm{d} \boldsymbol{g}}{\boldsymbol{y}} \tag{33}$$

where, I use the gradient notation $(\nabla_{\boldsymbol{x}} \boldsymbol{y})_{ij} \equiv \frac{\partial y_i}{\partial x_j}$; the diagonal matrix of nominal GDP $(\{\mathbb{Y}\}_{ij} \equiv Y_i \mathbb{1}[i=j]$; $(\boldsymbol{0})_i = 0$ for the zero vector of size $N \times 1$; $(\mathcal{O})_{ij} = 0$ is the square zero matrix of size $N \times N$; I use, with slight abuse of notation, $\left(\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}}\right)_i \equiv \frac{\mathrm{d} g_i}{y_i}$; and, finally, the matrices

$$\Gamma : (\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, b) \to \mathbb{R}^{(2N+2)\times(2N+2)}$$
$$\tilde{\Gamma} : (\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, b) \to \mathbb{R}^{(2N+2)\times N}$$

This step is done in detail in appendix section XX. Equation (42) separates the endogenous variables, on the left, from the exogenous variables, on the right. This is important for the inversion in the next step.

Reduced-Form Equations. Deriving the reduced-form equation requires inverting Γ. However, a matrix inversion is not possible as, due to price normalization, the maximum rank of Γ is $2N+1$; it is therefore rank deficient.[78] A pseudo-inverse must be taken instead and is done so as follows.

Before inverting, the price normalization equation (11) (which in changes implies $\mathrm{d} \ln p_u \equiv 0$) must be imposed in the structural set of equations (42). This is equivalent to dropping the $(N+u)^{th}$ column in Γ as each element in this column is multiplying zero

$$\Gamma_{\cdot, -(N+u)} \begin{pmatrix} \mathrm{d} \ln \boldsymbol{y} \\ \mathrm{d} \ln \boldsymbol{p}_{-u} \\ \mathrm{d} b \\ \mathrm{d} \ln \Xi \end{pmatrix} = \tilde{\Gamma} \frac{\mathrm{d} \boldsymbol{g}}{\boldsymbol{y}} \tag{34}$$

[78] Intuitively, if Γ were full rank, then equation (42) could be inverted and, for all $i \in \{1, ..., N\}$, the price changes $\mathrm{d} \ln p_i$ will be determined. But this cannot be possible as one of the prices is determined by the normalization condition 11; a condition which has not anywhere been imposed in the set of structural equations (42).

where $-n$ indicates that the n^{th} element is excluded.[79] However, these structural equations can still not be inverted as $\Gamma_{.,-(N+u)}$ is no longer square. The second step is to drop one of the structural equations in (34), which drops a row in $\Gamma_{.,-(N+u)}$, so that the matrix becomes square. I exclude the $(2N+1)^{th}$

$$\Gamma_{-(2N+1),-(N+u)}\begin{pmatrix}\mathrm{d}\ln\boldsymbol{y}\\ \mathrm{d}\ln\boldsymbol{p}_{-u}\\ \mathrm{d}b\\ \mathrm{d}\ln\Xi\end{pmatrix}=\tilde{\Gamma}_{-(2N+1),.}\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} \tag{35}$$

Even though the last equation is dropped and therefore not explicitly imposed when solving for the endogenous variables (i.e. when I soon invert), the equation is still satisfied - and therefore general equilibrium maintained - due to Walras' Law. That is, the following is true

$$\sum_i D_i((\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},\boldsymbol{\xi},b,\Xi)\equiv B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b)+TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi) \tag{36}$$

I show this formally in appendix A.1.2. Importantly, this holds whether $\{\boldsymbol{y},\boldsymbol{p},b,\Xi\}$ are general equilibrium values or not.[80] The implication is that when $D_i(\boldsymbol{y},\boldsymbol{p},b,\boldsymbol{g})=0$ holds for all $i\in\{1,...,N\}$, and $TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi)=0$ holds, equation (36) implies that $B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b)=0$ holds automatically. Thus, $\boldsymbol{S}(\boldsymbol{y},\boldsymbol{p})=\boldsymbol{0}$, $\boldsymbol{D}(\boldsymbol{y},\boldsymbol{p},b,\boldsymbol{g})=\boldsymbol{0}$ and $TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi)=0$ are sufficient for general equilibrium; $B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b)=0$, and therefore the $(2N+1)^{th}$ equation of (42), need not directly be imposed and can be dropped.

Inverting equation (35) yields the reduced-form equations

$$\begin{pmatrix}\mathrm{d}\ln\boldsymbol{y}\\ \mathrm{d}\ln\boldsymbol{p}_{-u}\\ \mathrm{d}b\ \mathrm{d}\ln Xi\end{pmatrix}=(\Gamma_{-(2N+1),-u})^{-1}\tilde{\Gamma}_{-(2N+1),.}\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} \tag{37}$$

The cross-location fiscal multiplier of equation (13) concerns the firs N rows of equation (37), with

$$\Lambda^{transfers}\equiv\left((\Gamma_{-(2N+1),-u})^{-1}\tilde{\Gamma}_{-(2N+1),.}\right)_{i,j},\quad i\in\{1,,,.N\},j\in\{1,,,.N\}$$

and where

[79] $\boldsymbol{p}_{-u}\equiv\{x_i\}_{i\in\{1,...,u-1,u+1,...,N\}}$; $\Gamma_{.,-(N+u)}\equiv\{M_{ij}\}_{i\in\{1,...,2N+1\},j\in\{1,...,N+u-1,N+u+1,...,2N+1\}}$.

[80] $\{\boldsymbol{y},\boldsymbol{p},b\}$ are general equilibrium values if equation (**??**) holds. i.e. $D_i(\boldsymbol{y},\boldsymbol{p},b,\boldsymbol{g})=0$ and $B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b)=0$ in addition to $\boldsymbol{S}(\boldsymbol{y},\boldsymbol{p})=\boldsymbol{0}$.

$$\Lambda^{transfers} : \Omega^{all} \to \mathbb{R}^{N^2}$$

with $\Omega^{all} \equiv \{\{y_i, p_i, b, \Xi, g_i, \xi_i, A_i\}_i, \{\tau_{ij}\}_{ij}, \phi, \psi\}$, that is, simply all variables and parameters of the model. No attempt has been made to simplify this dependence down — yet.

A.1.1 Log-Linearizing the structural equations

Here I in detail derive equation (42). In what follows, I use the notation: doublestroke font to denote trade share matrices: $\{\mathbb{S}^{Im}\}_{ij} \equiv s^{Im}_{ij}, \{\mathbb{S}^{Ex}\}_{ij} \equiv s^{Ex}_{ij}$; and diagonal matrices $\{\mathbb{G}^{transfers}\}_{ij} \equiv P_i g_i^{transfers} \mathbb{1}[i=j], \{\mathbb{Y}\}_{ij} \equiv Y_i \mathbb{1}[i=j], \{\mathbb{S}^G\}_{ij} \equiv s^G_i \mathbb{1}[i=j]$; for the trade cost matrix I use $\{\bar{\tau}\}_{ij} \equiv \tau_{ij}$, and for the vector of ones $\{\boldsymbol{e}\}_i \equiv 1$.

As it will be used extensively in the derivation, first the derivative of the price index

$$\mathrm{d}\ln P_i = \sum_j \left(\frac{p_{ji}}{P_j}\right)^{1-\phi} \mathrm{d}\ln p_{ji} = \sum_j s^{Im}_{ji} \mathrm{d}\ln(p_j \tau_{ji})$$

$$\iff \quad \mathrm{d}\ln \boldsymbol{P} = \mathbb{S}^{Im'} \mathrm{d}\boldsymbol{p} + (\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})' \boldsymbol{e} \tag{38}$$

where $\odot$ repesents the Hadamard product (element-wise multiplication). Now, to differentiating the structural equations.

Demand equation

$$\mathrm{d}D_i = Y_i \mathrm{d}\ln(P_i y_i) - \sum_j X_{ij} \{(1-\phi)\mathrm{d}\ln(\tau_{ij} p_i) + \phi \mathrm{d}\ln P_j + \mathrm{d}\ln y_j\} + \cdots$$

$$\cdots - G_i^{transfers} \mathrm{d}\ln P_i + b Y_i \mathrm{d}\ln y_i - Y_i \frac{\mathrm{d}g_i}{y_i} + Y_i \mathrm{d}b$$

$$= \sum_j \{(1+b) Y_j \mathbb{1}[i=j] - X_{ij}\} \mathrm{d}\ln y_j + \sum_j \left\{(Y_j - G_j^{transfers})\mathbb{1}[i=j] - \phi X_{ij}\right\} \mathrm{d}\ln P_j + \cdots$$

$$\cdots - \sum_j (1-\phi) X_{ij} \mathrm{d}\ln p_i - \sum_j (1-\phi) X_{ij} \mathrm{d}\ln \tau_{ij} - Y_i \frac{\mathrm{d}g_i}{y_i} + Y_i \mathrm{d}b$$

Rewriting in matrix form, using 38 and $\sum_j X_{ij} = Y_i - G_i^{transfers}$

$$\mathrm{d}\boldsymbol{D} = \{(1+b)\mathbb{Y} - \mathbb{X}\} \mathrm{d}\ln y + \left\{\mathbb{Y} - \mathbb{G}^{transfers} - \phi \mathbb{X}\right\} \left\{\mathbb{S}^{Im'} \mathrm{d}\ln \boldsymbol{p} + (\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})' \boldsymbol{e}\right\} + \cdots$$

$$\cdots - (1-\phi)(\mathbb{Y} - \mathbb{G}^{transfers})\mathrm{d}\ln \boldsymbol{p} - (1-\phi)(\mathbb{X} \odot \mathrm{d}\ln \bar{\tau})\boldsymbol{e} - \mathbb{Y}\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \boldsymbol{Y}\mathrm{d}b$$

In equilibrium, $b = b^* \equiv \frac{\sum_i P_i g_i}{\sum_i P_i y_i}$, so that the government's budget is balanced. Rearranging and collecting terms

$$\mathrm{d}\boldsymbol{D} = \underbrace{\{(1+b^*)\mathbb{Y} - \mathbb{X}\}}_{\nabla_{\ln \boldsymbol{y}}\boldsymbol{D}}\mathrm{d}\ln \boldsymbol{y} + \underbrace{\left[\left\{\mathbb{Y} - \mathbb{G}^{transfers} - \phi\mathbb{X}\right\}\mathbb{S}^{Im'} - (1-\phi)(\mathbb{Y} - \mathbb{G}^{transfers})\right]}_{\nabla_{\ln \boldsymbol{p}}\boldsymbol{D}}\mathrm{d}\ln \boldsymbol{p} + \cdots$$

$$\cdots + \underbrace{-\mathcal{I}}_{\nabla_{\boldsymbol{g}}\boldsymbol{D}}\mathbb{Y}\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \underbrace{\boldsymbol{Y}}_{\nabla_b\boldsymbol{D}}\mathrm{d}b + \cdots$$

$$\cdots + \underbrace{\left\{\mathbb{Y} - \mathbb{G}^{transfers} - \phi\mathbb{X}\right\}(\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})'\boldsymbol{e} - (1-\phi)(\mathbb{X} \odot \mathrm{d}\ln \bar{\tau})\boldsymbol{e}}_{(\nabla_\varepsilon \boldsymbol{D})\mathrm{d}\varepsilon} \tag{39}$$

Setting the transfers to zero at the expansion point, $\mathbb{G}^{transfers} = \mathbf{0}, b^* = 0$

$$\mathrm{d}\boldsymbol{D} = \underbrace{\{\mathbb{Y} - \mathbb{X}\}}_{\nabla_{\ln \boldsymbol{y}}\boldsymbol{D}}\mathrm{d}\ln \boldsymbol{y} + \underbrace{\left[\{\mathbb{Y} - \phi\mathbb{X}\}\mathbb{S}^{Im'} - (1-\phi)\mathbb{Y}\right]}_{\nabla_{\ln \boldsymbol{p}}\boldsymbol{D}}\mathrm{d}\ln \boldsymbol{p} + \cdots$$

$$\cdots + \underbrace{-\mathcal{I}}_{\nabla_{\boldsymbol{g}}\boldsymbol{D}}\mathbb{Y}\frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \underbrace{\boldsymbol{Y}}_{\nabla_b\boldsymbol{D}}\mathrm{d}b + \cdots$$

$$\cdots + \underbrace{\{\mathbb{Y} - \phi\mathbb{X}\}(\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})'\boldsymbol{e} - (1-\phi)(\mathbb{X} \odot \mathrm{d}\ln \bar{\tau})\boldsymbol{e}}_{(\nabla_\varepsilon \boldsymbol{D})\mathrm{d}\varepsilon} \tag{40}$$

Supply equation

$$\mathrm{d}S_i = Y_i \mathrm{d}\ln(P_i y_i) - Y_i\left\{\mathrm{d}\ln P_i + (1+\psi)(\mathrm{d}\ln p_i - \mathrm{d}\ln P_i) + \mathrm{d}\ln A_i\right\}$$

Converting to matrix notation

$$\mathrm{d}\boldsymbol{S} = \mathbb{Y}\mathrm{d}\ln \boldsymbol{y} - (1+\psi)\mathbb{Y}\mathrm{d}\ln \boldsymbol{p} + (1+\psi)\mathbb{Y}\mathrm{d}\ln \boldsymbol{P} - \mathbb{Y}\mathrm{d}\ln \boldsymbol{A}$$

and using 38

$$\mathrm{d}\boldsymbol{S} = \underbrace{\mathbb{Y}}_{\nabla_{\ln \boldsymbol{y}}\boldsymbol{S}}\mathrm{d}\ln \boldsymbol{y} + \underbrace{-(1+\psi)\mathbb{Y}\left(\mathcal{I} - \mathbb{S}^{Im'}\right)}_{\nabla_{\ln \boldsymbol{p}}\boldsymbol{S}}\mathrm{d}\ln \boldsymbol{p} + \underbrace{(1+\psi)\mathbb{Y}(\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})'\boldsymbol{e} - \mathbb{Y}\mathrm{d}\ln \boldsymbol{A}}_{(\nabla_\varepsilon \boldsymbol{S})\mathrm{d}\varepsilon} \tag{41}$$

Budget Constraint

$$\mathrm{d}B = \sum_i \left\{ G_i^{transfers} \mathrm{d}\ln P_i - Y_i \frac{\mathrm{d}g_i}{y_i} - bY_i \mathrm{d}\ln y_i - Y_i \mathrm{d}b \right\}$$

In matrix form (and $b = b^*$)

$$\mathrm{d}B = \underbrace{-b^* \boldsymbol{Y}'}_{\nabla_{\ln \boldsymbol{y}} B} \mathrm{d}\ln \boldsymbol{y} + \underbrace{\boldsymbol{e}' \mathbb{G}^{transfers} \mathbb{S}^{Im'}}_{\nabla_{\ln \boldsymbol{p}} B} \mathrm{d}\ln \boldsymbol{p} + \underbrace{-\boldsymbol{Y}'}_{\nabla_{\boldsymbol{g}} B} \frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \underbrace{-\boldsymbol{e}'\boldsymbol{Y}}_{\nabla_b B} \mathrm{d}b + \underbrace{\boldsymbol{e}' \mathbb{G}^{transfers} (\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})' \boldsymbol{e}}_{(\nabla_\varepsilon B)\mathrm{d}\varepsilon}$$

Setting the transfers to zero at the expansion point, $\mathbb{G}^{transfers} = \boldsymbol{0}, b^* = 0$

$$\mathrm{d}B = \underbrace{-\boldsymbol{Y}'}_{\nabla_{\boldsymbol{g}} B} \frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \underbrace{-\boldsymbol{e}'\boldsymbol{Y}}_{\nabla_b B} \mathrm{d}b$$

Linearized Structural Equations Equation (42) is derived by inserting the above into

$$\begin{pmatrix} \mathrm{d}\boldsymbol{D} \\ \mathrm{d}\boldsymbol{S} \\ \mathrm{d}B \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ 0 \end{pmatrix}$$

and rearranging to give

$$\underbrace{\begin{pmatrix} \nabla_{\ln \boldsymbol{y}} \boldsymbol{D} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{D} & \nabla_b \boldsymbol{D} \\ \nabla_{\ln \boldsymbol{y}} \boldsymbol{S} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{S} & \boldsymbol{0} \\ \nabla_{\ln \boldsymbol{y}} B & \nabla_{\ln \boldsymbol{p}} B & \nabla_b B \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} \mathrm{d}\ln \boldsymbol{y} \\ \mathrm{d}\ln \boldsymbol{p} \\ \mathrm{d}b \end{pmatrix} = \underbrace{\begin{pmatrix} -(\nabla_{\boldsymbol{g}} \boldsymbol{D})\mathbb{Y} & -\nabla_\varepsilon \boldsymbol{D} \\ \mathcal{O} & -\nabla_\varepsilon \boldsymbol{S} \\ -\nabla_{\boldsymbol{g}} B & -\nabla_\epsilon B \end{pmatrix}}_{\equiv \tilde{\Gamma}} \begin{pmatrix} \frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} \\ \mathrm{d}\varepsilon \end{pmatrix} \tag{42}$$

The $\mathrm{d}\varepsilon$ shows how the other exogenous shocks in the model $(\mathrm{d}\ln \boldsymbol{A}, \mathrm{d}\ln \bar{\tau})$ enter into the structural equations; I omit these for simplicity in the main text. These contribute towards the residual in the empirical framework.

A.1.2 Walras Law and equation (36)

Generally, Walras Law holds when all agents of the economy balance their budgets. The implication is that aggregate market clearing holds. Importantly, Walras Law does not require the allocation to be a general equilibrium; the first order conditions do not need to hold.[81] The consequence of Walras Law in my framework is

$$\sum_i D_i((\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},\boldsymbol{\xi},b,\Xi) \equiv B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b) + TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi)$$

This holds because budget balance (equations (9) and (10) in my framework) is imposed in the construction of $D_i(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},\boldsymbol{\xi},b,\Xi)$ (see section A.1). Formally, equation (36) can be shown by summing D_i over i

$$\begin{aligned}
D_i(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},\boldsymbol{\xi},b,\Xi) &\equiv Y_i - \sum_j \left(\frac{\tau_{ij}p_i}{P_j}\right)^{1-\phi} E_j + (G_i - bY_i) \\
\sum_i D_i(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b) &\equiv \sum_i Y_i - \sum_{ij} \left(\frac{\tau_{ij}p_i}{P_j}\right)^{1-\phi} E_j + \underbrace{\sum_i (G_i - bY_i)}_{\equiv B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b)} \\
&\equiv \sum_i Y_i - \sum_j \frac{\sum_i (\tau_{ij}p_i)^{1-\phi}}{P_j^{1-\phi}} E_j + B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b) \\
&\equiv \underbrace{\sum_i Y_i - \sum_j E_j}_{\equiv TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi)} + B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b) \\
&\equiv B(\boldsymbol{y},\boldsymbol{p},\boldsymbol{g},b) + TB(\boldsymbol{y},\boldsymbol{p},\boldsymbol{\xi},\Xi)
\end{aligned}$$

where in going from the third to the fourth line, the identity $P_j(\boldsymbol{p}) \equiv \sum_i (\tau_{ij}p_i)^{1-\phi}$ has been used.

□

A.2 Proof of Proposition 2 (Sufficient Statistics)

Given that the $(2N+1)^{th}$ equation is dropped from $\Gamma, \tilde{\Gamma}$ before inverting to form $\Lambda^{transfers}$, Proving that the first $2N$ structural equations depend only on s^{Im}, s^{Ex}, s^G is sufficient to showing that $\Lambda^{transfers}$ only depend on s^{Im}, s^{Ex}, s^G.

First, under the assumption $Y_i \neq 0$, define

[81] If $(\boldsymbol{y},\boldsymbol{p},b,\boldsymbol{g})$ constitute a general equilibrium, then $\forall i : D_i(\boldsymbol{y},\boldsymbol{p},b,\boldsymbol{g}) = 0$ (and $\forall i : S_i(\boldsymbol{y},\boldsymbol{p}) = 0$, though this does not enter for Walras Law). However, as can be seen, this needn't be constrained to for equation 36 to hold.

$$H \equiv \begin{pmatrix} \mathbb{Y}^{-1} & \mathcal{O} \\ \mathcal{O} & \mathbb{Y}^{-1} \end{pmatrix}$$

Then, the first $2N$ structural equations, given by

$$\begin{pmatrix} \mathrm{d}\boldsymbol{D} \\ \mathrm{d}\boldsymbol{S} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

can be transformed without loss of generality

$$H \begin{pmatrix} \mathrm{d}\boldsymbol{D} \\ \mathrm{d}\boldsymbol{S} \end{pmatrix} = \begin{pmatrix} \mathbb{Y}^{-1}\mathrm{d}\boldsymbol{D} \\ \mathbb{Y}^{-1}\mathrm{d}\boldsymbol{S} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{43}$$

Noting that $\mathbb{Y}^{-1}\mathbb{X} = \mathbb{S}^{Ex}$, then

$$\begin{aligned} \mathbb{Y}^{-1}\mathrm{d}\boldsymbol{D} = \left\{\mathcal{I} - \mathbb{S}^{Ex}\right\} \mathrm{d}\ln \boldsymbol{y} + \left[\left\{\mathcal{I} - \phi\mathbb{S}^{Ex}\right\} \mathbb{S}^{Im'} - (1-\phi)\mathcal{I}\right] \mathrm{d}\ln \boldsymbol{p} + \cdots \\ \cdots - \frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} + \boldsymbol{e}\mathrm{d}b + \left\{\mathcal{I} - \phi\mathbb{S}^{Ex}\right\} (\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})'\boldsymbol{e} - (1-\phi)(\mathbb{S}^{Ex} \odot \mathrm{d}\ln \bar{\tau})\boldsymbol{e} \\ \mathbb{Y}^{-1}\mathrm{d}\boldsymbol{S} = \mathrm{d}\ln \boldsymbol{y} - (1+\psi)\left(\mathcal{I} - \mathbb{S}^{Im'}\right) \mathrm{d}\ln \boldsymbol{p} + (1+\psi)(\mathbb{S}^{Im} \odot \mathrm{d}\ln \bar{\tau})'\boldsymbol{e} - \mathrm{d}\ln \boldsymbol{A} \end{aligned}$$

All the partial derivatives only depend on $\mathbb{S}^{Ex}, \mathbb{S}^{Im}, \phi, \psi$. In consequence $H\Gamma_{-(2N+1),-(N+u)}, H\tilde{\Gamma}_{-(2N+1)}$ also only depend on these variables. Now, using this and the definition of $\Lambda^{transfers}$

$$\begin{aligned} \Lambda^{transfers} &\equiv (\Gamma_{-(2N+1),-u})^{-1}\tilde{\Gamma}_{-(2N+1),.} \\ &\equiv (\Gamma_{-(2N+1),-u})^{-1}H^{-1}H\tilde{\Gamma}_{-(2N+1),.} \\ &\equiv (H\Gamma_{-(2N+1),-u})^{-1}H\tilde{\Gamma}_{-(2N+1),.} \end{aligned}$$

using the trivial property that $H^{-1}H = \mathcal{I}$ in going from line one to two. Thus $\Lambda^{transfers}$ only depends on $\{s_i^G\}_i, \{s_{ij}^{Ex}, s_{ij}^{Im}\}_{ij}$.

□

A.3 Proof of Proposition ?? (Aggregate Fiscal Multiplier and Internal Geography)

To facilitate the proof, rather than consider arbitrary $\{g_i\}_i$ with endogenous b as in the main text, I instead omit b and allow $\mathrm{d}g_2$ to be endogenous. The linearized structural equations become

$$\underbrace{\begin{pmatrix} \nabla_{\ln \boldsymbol{y}} \boldsymbol{S} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{S} \\ \nabla_{\ln \boldsymbol{y}} \boldsymbol{D} & \nabla_{\ln \boldsymbol{p}} \boldsymbol{D} \end{pmatrix}}_{\equiv \Gamma} \begin{pmatrix} \mathrm{d}\ln \boldsymbol{y} \\ \mathrm{d}\ln \boldsymbol{p} \end{pmatrix} = \begin{pmatrix} \mathcal{O} \\ -(\nabla_{\boldsymbol{g}} \boldsymbol{D})\mathbb{Y} \end{pmatrix} \frac{\mathrm{d}\boldsymbol{g}}{\boldsymbol{y}} \tag{44}$$

where I've written the supply equations first, then the demand equations. Given $N = 2$, this matrix is of size 4×4. I drop the 4th equation, normalize the $p_2 \equiv 1$, and invert. This gives the equilibrium change in output

$$\mathrm{d}\ln \boldsymbol{y} = \frac{1+\psi}{|\Gamma|} \begin{pmatrix} 1 - S_{11}^{Im} \\ -S_{12}^{Im} \end{pmatrix} \frac{\mathrm{d}g_1}{y_1}$$

where $|\Gamma|$ indicates the determinant of matrix Γ. Note that $\frac{\mathrm{d}g_2}{y_2}$ is absent as it is now endogenous. The change in aggregate GDP is given by

$$\frac{\mathrm{d}y_{agg}}{\mathrm{d}g_1} = \sum_i y_i \Lambda_{i1}^{transfers} y_1^{-1} = \frac{1+\psi}{|\Gamma|}\Omega$$

where I've defined

$$\Omega \equiv 1 - S_{11}^{Im} - \frac{y_1}{y_2} S_{12}^{Im}$$

To prove the propostion, I need to determine the sign, and gradient with respect to X_{11}, of the two objects $\Omega, |\Gamma|$

Lemma 1 (Properties of Ω)**.**

A.4 Isomorphisms

In this section I present some common microfoundations that isomorphic with the equilibrium in section 3.1.

A.4.1 A Simple Macroeconomic Baseline

Described here is a correspondence between the model in the main text and a stylized macroeconomic framework.

ECONOMY

Households A representative household in each location that maximizes utility

$$\max_{\{c_{ji}\}_{j\in\{1,\ldots,N\}},L_i} \frac{c_i^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{L_i^{1+\nu^{-1}}}{1+\nu^{-1}}, \qquad c_i \equiv \left(\sum_j c_{ji}^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$

subject to the budget constraint

$$\sum_j p_{ji}c_{ji} = w_i L_i \equiv E_i \tag{45}$$

Firms A representative firm in each location maximizes profit

$$\max_{q_i,l_i} p_i q_i - w_i l_i \qquad q_i \equiv A_i l_i$$

The production technology is $q_i = A_i l_i$ with external economies of scale $A_i \equiv \bar{A}_i l_i^{\chi}$. Free entry is imposed implying zero profit.

Government (As in main text.)

Geography (As in main text.)

Product Market Clearing (As in main text.)

Labor Market Clearing Labor demand, l_i equals labor supply, L_i, in all locations:

$$l_i = L_i$$

Price Normalization (As in main text.)

Correspondence with Structural Demand and supply equations

Demand Equation, (3). The first order conditions with respect to $\{c_{ij}\}_i$ of a household in location j leads to optimal disaggregate consumption

$$c_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{-\phi} c_j$$

combining with the budget constraint

$$P_j c_j = w_j L_j = E_j$$

gives

$$p_{ij}c_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{1-\phi} E_j$$

The sum of private and public demand give the demand equation

$$X_{ij} = p_{ij}c_{ij} + G_i^{transfers}\mathbb{1}[i=j] = \left(\frac{p_{ij}}{P_j}\right)^{1-\phi} E_j + G_i^{transfers}\mathbb{1}[i=j]$$

Supply Equation, (4). The first order conditions with respect to $\{c_i, L_i\}$ of a household in location i leads to

$$L_i^{1/\nu} = c^{-1/\sigma}\frac{w_i}{P_i} \tag{46}$$

combining with the budget constraint, $P_i c_i = w_i L_i$ to solve for optimal aggregate consumption

$$c_i = L_i \frac{w_i}{P_i} \tag{47}$$

into (46) to solve for optimal labor supply in terms of the real wage[82]

$$L_i^{1/\nu+1/\sigma} = \left(\frac{w_i}{P_i}\right)^{1-1/\sigma} \tag{48}$$

Next, the optimal pricing equation from the firm's problem is

$$p_i A_i = w_i$$

Combined with the productivity definition $A_i \equiv \bar{A}_i l_i^{\chi}$ and labor market clearing $L_i = l_i$ to write labor demand

$$p_i \bar{A}_i L_i^{\chi} = w_i \tag{49}$$

Using this to substitute out wages in equation (48)

$$L^{1/\nu+1/\sigma-\chi(1-1/\sigma)} = \bar{A}_i^{1-1/\sigma}\left(\frac{p_i}{P_i}\right)^{(1-/\sigma)} \tag{50}$$

[82]The analogous equation for consumption is $c_i^{1/\nu+1/\sigma} = \left(\frac{w_i}{P_i}\right)^{1+1/\nu}(1-t)^{\zeta+1/\nu}$.

Finally, using the production function $q_i = A_i L_i$ to rewrite L_i in terms of output

$$L_i = \bar{A}_i^{\frac{1}{1+\chi}} q_i^{\frac{1}{1+\chi}}$$

and inserting this into equation (50)

$$\left(q_i \bar{A}_i\right)^{\frac{1+1/\nu}{1+\chi} - (1-1/\sigma)} = \bar{A}_i^{1-1/\sigma} \left(\frac{p_i}{P_i}\right)^{(1-/\sigma)}$$

rearranging gives the supply equation for output q_i

$$q_i = \tilde{A}_i \left(\frac{p_i}{P_i}\right)^{\psi}$$

with

$$\psi \equiv \frac{1-\sigma^{-1}}{\frac{1+\nu^{-1}}{1+\chi} - (1-\sigma^{-1})}, \qquad \tilde{A}_i \equiv \bar{A}_i^{\psi} \tag{51}$$

substituting output for real GDP, using $q_i = P_i/p_i y_i$ gives the supply equation (**??**).

The dependence of ψ on structural parameters of the microfoundation indicate under which conditions $\psi < 0$ may arise. Looking at each parameter in turn

- $\sigma < 1$. This corresponds to the income effect in labor outweighing the substitution effect. In this case, even without scale economies, when the real wage falls, the labor supplied increases. This is what generates the magnification effect associated with negative ψ and fiscal multipliers.

- $\chi > 0$ indicates increasing returns to scale. As output rises, production becomes more efficient causing prices to fall. Consumption increase and therefore magnifying output.

- ν is the Frisch Elasticity of labor supply. This alone (i.e. without either $\chi > 0$ or $\sigma < 1$) cannot create $\psi < 0$. However, given either $\chi > 0$ or $\sigma < 1$, a greater ν makes $\psi < 0$ more likely. Intuitively, the more elastic labor supply is (the greater ν is), the more responsive output is to smaller changes in prices, thus leading to a greater magnification effect.

A.4.2 Variable Mark-Ups

An alternative microfoundation to generating $\psi < 0$ is with variable mark-ups. The definition of the mark-up is

$$\frac{p_i}{w_i/A_i} \equiv \mu_i \tag{52}$$

Consider $\mu \neq 1$ in the macroeconomic baseline microfoundation described in section A.4.1. Only the derivation of the supply equation is changes; the derivation of the demand equation is the same. With $\mu \neq 1$, profits are no longer zero. Denoting profits by Π, then

$$\Pi_i \equiv p_i q_i - w_i l_i = (\mu_i - 1) w_i l_i$$

Assuming these are transferred lump-sum to colocal consumers, then the budget constraint in equation (45) becomes

$$P_i c_i = w_i L_i + \Pi_i$$

With Π being lump-sum, the first-order conditions of the consumer are unaffected. In equilibrium, inserting Π into the budget constraint

$$P_i c_i = w_i L_i + \Pi_i = w_i L_i + (\mu_i - 1) w_i L_i = \mu_i w_i L_i$$

this is the analogy of equation (47). Using equation (46) to substitute out optimal c_i

$$L_i^{1/\sigma + 1/\nu} = \mu_i^{-1/\sigma} \left(\frac{w_i}{P_i}\right)^{1-1/\sigma}$$

Using the pricing relation, equation (52),

$$L_i^{1/\sigma + 1/\nu} = \mu_i^{-1} A_i^{1-1/\sigma} \left(\frac{p_i}{P_i}\right)^{1-1/\sigma}$$

Allowing for external scale economies, $A_i = \bar{A}_i L_i^{\chi}$

$$L_i^{1/\sigma + 1/\nu - \chi(1-1/\sigma)} = \mu_i^{-1} \bar{A}_i^{1-1/\sigma} \left(\frac{p_i}{P_i}\right)^{1-1/\sigma}$$

Using $q_i = \bar{A}_i L_i^{1+\chi}$

$$(\bar{A}_i^{-1} q_i)^{\frac{1/\sigma + 1/\nu - \chi(1-1/\sigma)}{1+\chi}} = \mu_i^{-1} \bar{A}_i^{1-1/\sigma} \left(\frac{p_i}{P_i}\right)^{1-1/\sigma}$$

Imposing that the mark-up be a function of scale,

$$\mu_i = q_i^{\xi} \tag{53}$$

for some $\xi \in \mathbb{R}$,[83] then

$$q_i = \bar{A}^{\frac{\frac{1+\nu^{-1}}{1+\chi}}{\frac{1+\nu^{-1}}{1+\chi}+\xi-(1-\sigma^{-1})}} \left(\frac{p_i}{P_i}\right)^{\frac{1-\sigma^{-1}}{\frac{1+\nu^{-1}}{1+\chi}+\xi-(1-\sigma^{-1})}}$$

That is, the microfoundation for the supply elasticity becomes

$$\psi \equiv \frac{1-\sigma^{-1}}{\frac{1+1/\nu}{1+\chi}+\xi-(1-\sigma^{-1})}$$

We are back in the baseline case of equation (51) when $\xi = 0$. Adding variable mark-ups to the microfoundation reveals an alternative channel generating $\psi < 0$: mark-ups being decreasing in scale, $\xi < 0$. In the derivation, I stipulated at a high-level that mark-ups are variable, equation (53). What underlying mechanism is consistent with $\xi < 0$? In fact a very common feature of macroeconomic models for fiscal multipliers: sticky prices. In those models, as scale increases, wages increase

A.4.3 Labor-Consumption Complementarity

Consider the modified utility

$$v(L)\frac{c^{1-1/\sigma}}{1-1/\sigma} - \frac{L^{1+1/\nu}}{1+1/\nu}$$

where $v(L) \equiv L^\theta$ but is not internalized by the consumer when choosing labor supply.[84][85] This specification is motivated by the concept of Edgeworth complements, which requires $\partial^2 u/\partial c\partial L > 0$.[86] Taking FOCs with respect to both labor and consumption and combining gives

$$\frac{L^{1/\nu-\theta}}{C^{-1/\sigma}} = \frac{w}{P}$$

Continuing the derivation of the supply equation yields

[83] Formally, this is where the optimal pricing decision of the firm enters. But rather than deriving it from profit maximizing behavior, I assume it at the top-level.

[84] An interpretation can be understood by making analogy to external economies of scale. The more that the population consumes, the greater the incentive for an individual to consume. For example, with brand status, people spend more on expensive brands, such as Apple products, the more that other people do. This shares similarity with Networks in Economics.

[85] In this example, $v(L)$ is not internalized in order to create the isomorphism. If $v(L)$ is internalized, then the functional form of the supply equation is different and is not isomorphic (there is also concern about the utility continuing to be concave for high θ).

[86] See Gliksberg (2010) for discussion of its empirical relevance; see Greenwood et al. (1988) and Nakamura and Steinsson (2014) for example applications of Edgeworth complements, particularly the latter in arguing its importance for the fiscal multiplier.

$$\psi = \frac{1-\sigma^{-1}}{\frac{1+\nu^{-1}-\theta}{1+\chi} - (1-\sigma^{-1})}$$

Therefore if $\theta > 1+\nu^{-1}$, then $\psi < 0$ is possible without requiring $\chi < 0$. The intuition is that as consumers work more after the demand shock, their marginal utility of consumption increases due to labor-consumption complementarities.[87] This increase means they consume even more and therefore magnify the increase in output i.e. exactly what a negative ψ describes.

A.4.4 Investment

It has been documented theoretically that including capital can amplify the magnification effect of government spending.[88] The basic intuition can be seen from the formula for GDP $Y = C + I + G$ (ignoring net exports). In the neoclassical model, an increase in ΔG crowds out $\Delta C \in [0, -\Delta G]$ with the decrease being more negative the greater the income effect on C. This bounds $\Delta Y/\Delta G \in [0,1]$ in a model without capital.

With capital, the multiplier can rise above 1. The reason is that the increase in labor supply induced by the spending pushes up the labor-capital ratio and therefore increases the MPK and thus the rental rate. This incentivizes greater investment $\Delta I > 0$. This additional positive term on the RHS on $\Delta Y = \Delta C + \Delta I + \Delta G$ is therefore able to break the $[0,1]$ bound on the fiscal multiplier.

To see this in a simple framework, consider a Cobb-Douglas production function, $q = k^\alpha L^{1-\alpha}$, and non-traded capital that depreciates at rate δ. In this setting, investment is a constant share of local income, $s \equiv \frac{\alpha\delta\beta}{1-(1-\delta)\beta}$, where β is the consumer discount factor.

This only multiplicatively affects the supply equilibrium relation and therefore does not change the relation in logs (unless there is a lump sum tax, then it will appear and magnifies the neoclassical income effect channel). The demand equation is affected as follows

$$X_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{1-\phi} P_j y_j (1-s) + \mathbb{1}[i=j](G_i + sP_i y_i)$$

[87]The complementarity can, for example, represent the extra consumption on food away from home, clothing, gas, and the like that often arises in the context of work.

[88]See e.g. Baxter and King (1993), specifically page 323 for a summary of the short-run and long-run mechanisms of capital's effect on the fiscal multiplier.

Giving

$$(1-s)P_i y_i = \sum_j \left(\frac{p_{ij}}{P_j}\right)^{1-\phi} P_j y_j (1-s) + G_i$$

or

$$P_i y_i = \sum_j \left(\frac{p_{ij}}{P_j}\right)^{1-\phi} P_j y_j + \frac{G_i}{1-s}$$

That is, a positive share spent on investment does magnify the fiscal multiplier due to transfers. However, it's form mathematically is distinct to the how ψ enters and therefore cannot microfound $\psi < 0$ using the standard function form for investment as outlined here.

A.5 Specialization Depressing the Regional Elasticity of Substitution

Here I show how the regional demand elasticity of substitution, ϕ, depends on the regional industry concentration, in addition to structural preference parameters. To do this, I consider a multi-industry extension of my single-industry model, and derive the effective ϕ. This parameter is a partial equilibrium concept and therefore I only need consider the demand side.

Consider a nested CES with the outer layer being an aggregation across industries $k \in \{1, ..., K\}$ and the inner layer being an aggregation across industry varieties, one from each region $i \in \{1, ..., N\}$. For simplicity I assume all regions produce in the same set of industries, and consumers in all regions have the same preferences. Consumption in region j is given by

$$C_j = \left[\sum_k C_{kj}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

where ω is the elasticity of substitution between industries. Consumption in region j of industry k goods from all regions

$$C_{kj} = \left[\sum_i C_{kij}^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}$$

where α is the elasticity of substitution within industries. Optimal consumption in region j of industry k goods from region i is

$$C_{kij} = \frac{1}{P_{kij}} \left(\frac{P_{kij}}{P_{kj}}\right)^{1-\alpha} \left(\frac{P_{kj}}{P_j}\right)^{1-\omega} P_j C_j$$

where the P_{kij} is the price faced in region i of industry k goods from region i. The aggregate price indexes are

$$P_{kj} = \left[\sum_i P_{kij}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$$

$$P_j = \left[\sum_k P_{kj}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

In order to relate to the object ϕ in the single-industry framework in the main text, I need the aggregate price $\ln P_{ij}$.: the cost faced by consumers in location i of a bundle of all industry goods in location j. Moreover, as everything regarding counterfactuals in the main text is in changes, I only need this price in changes, that is $\mathrm{d}\ln P_{ij}$. In the single industry framework, this is related to $\mathrm{d}\ln P_j$ by

$$\mathrm{d}\ln P_j = \sum_i s_{ij} \mathrm{d}\ln P_{ij}$$

where s_{ij} is the share of goods from i consumed in j. This is related to the many-industry model by $s_{ij} \equiv \frac{\sum_k P_{kij} C_{kij}}{P_j C_j}$. This can be written in terms of $\mathrm{d}\ln P_{kij}$ by noting the following

$$\mathrm{d}\ln P_j = \sum_k s_{kj} \mathrm{d}\ln P_{kj} = \sum_{ki} s_{kij} \mathrm{d}\ln P_{kij} = \sum_i s_{ij} \underbrace{\sum_k \frac{s_{kij}}{s_{ij}} \mathrm{d}\ln P_{kij}}_{\equiv \mathrm{d}\ln P_{ij}}$$

where the first equality can be derived by totally differentiating P_j, and $s_{kj} \equiv \frac{P_{kj} C_{kj}}{P_j C_j}$, $s_{kij} \equiv \frac{P_{kij} C_{kij}}{P_j C_j}$.. Thus comparing the above equations gives

$$\mathrm{d}\ln P_{ij} \equiv \sum_k \frac{s_{kij}}{s_{ij}} \mathrm{d}\ln P_{kij}$$

With this, the regional elasticity of substitution is

$$1 - \phi \equiv \frac{\partial \ln(s_{ij}/s_{i'j})}{\partial \ln(P_{ij}/P_{i'j})}$$

However this isn't well-defined; because preferences are not CES across locations, the response of the shares vary depending on which industry prices in i, i' are changing. I first consider the case where $\mathrm{d}\ln P_{k^*ij} \neq 0$, and all other price changes are zero, and then discuss how this result generalizes. Formally

$$1-\phi_{ij,k=k^*}\equiv\left.\frac{\partial\ln(s_{ij}/s_{i'j})}{\partial\ln(P_{ij}/P_{i'j})}\right|_{\forall k\neq k^*,i,j:\mathrm{d}\ln P_{kij}=0}$$

Differentiating the shares

$$\mathrm{d}\ln(s_{ij}/s_{i'j})=(1-\alpha)\sum_k\left(\frac{s_{kij}}{s_{ij}}\mathrm{d}\ln P_{kij}-\frac{s_{kij'}}{s_{i'j}}\mathrm{d}\ln P_{ki'j}\right)+(\alpha-\omega)\sum_k\left(\frac{s_{kij}}{s_{ij}}-\frac{s_{ki'j}}{s_{i'j}}\right)\mathrm{d}\ln P_{kj}$$

Using that only $\mathrm{d}\ln P_{k^*ij}\neq 0$ and noting $\mathrm{d}\ln P_{ij}=\frac{s_{k^*ij}}{s_{ij}}\mathrm{d}\ln P_{k^*ij}$ and

$$\mathrm{d}\ln P_{k^*j}=\sum_i\frac{s_{k^*ij}}{s_{k^*j}}\mathrm{d}\ln P_{k^*ij}=\frac{s_{k^*ij}}{s_{k^*j}}\mathrm{d}\ln P_{k^*ij}=\frac{s_{ij}}{s_{k^*j}}\mathrm{d}\ln P_{ij}$$

into the above

$$\mathrm{d}\ln(s_{ij}/s_{i'j})=(1-\alpha)\mathrm{d}\ln P_{ij}+(\alpha-\omega)\left(\frac{s_{k^*ij}}{s_{ij}}-\frac{s_{k^*i'j}}{s_{i'j}}\right)\frac{s_{ij}}{s_{k^*j}}\mathrm{d}\ln P_{ij}$$

Thus

$$\phi_{ij,k=k^*}-1=\alpha-1+\underbrace{(\omega-\alpha)\left(\frac{s_{k^*ij}}{s_{ij}}-\frac{s_{k^*i'j}}{s_{i'j}}\right)\frac{s_{ij}}{s_{k^*j}}}_{(\dagger)}\tag{54}$$

The difference to the single industry case is the addition of the term (†). Take the presumptive case where varieties within-industries are more substitutable than across-industries, $\sigma>\omega$. Then, (†) is negative, and therefore reduces the effective substitutability between regions if $\frac{s_{k^*ij}}{s_{ij}}>\frac{s_{k^*i'j}}{s_{i'j}}$. This inequality being satisfied means that for consumers in j, industry k^* forms a greater share of your consumption from i than it does from i'; that is, location i is relatively specialized in the production of industry k^* goods from the perspective of consumers in j. The greater this concentration, the more negative the term (†) is. Intuitively, if a lot of your consumption for an industry comes from a single location, and if your preferences are not strongly substitutable across industries, then you do not substitute away from that location as much when the price of those industry goods in that location rise.

For this mechanism to push ϕ to become complements, it must be that $\omega<1$ as $\left(\frac{s_{k^*ij}}{s_{ij}}-\frac{s_{k^*i'j}}{s_{i'j}}\right)\frac{s_{ij}}{s_{k^*j}}>1$ is not possible. Intuitively, if all varieties are substitutes including between industries, then the effective regional substitutability has to be substitutes too.

Generally, it there will not be only a single industry price changing in location i. Qualitatively, the same mechanism results. Consider the other extreme where all industry prices in location i shift equally $\forall k : \mathrm{d}\ln P_{kij} \equiv \mathrm{d}\ln p$. Denote the regional elasticity of substitution in this case

$$1 - \phi_{ij,\forall k} \equiv \left. \frac{\partial \ln(s_{ij}/s_{i'j})}{\partial \ln(P_{ij}/P_{i'j})} \right|_{\forall i'' \neq i,k,j : \mathrm{d}\ln P_{ki''j}=0}$$

In this case, the equivalent of equation (54) becomes

$$\phi_{ij,\forall k} - 1 = \alpha - 1 + \underbrace{(\omega - \alpha) \sum_k \left(\frac{s_{kij}}{s_{ij}} - \frac{s_{ki'j}}{s_{i'j}} \right) \frac{s_{kij}}{s_{kj}}}_{(\dagger\dagger)}$$

As in the case of only a single price shifting, the regional substitutability is reduced when ($\dagger\dagger$) is negative. and this occurs if region i is specialized e.g. consider the extreme example where $s_{kij} \neq 0$ only for $k = k^*$ then $(\dagger\dagger) = (\omega - \alpha) \left(\frac{s_{k^*ij}}{s_{ij}} - \frac{s_{k^*i'j}}{s_{i'j}} \right) \frac{s_{k^*ij}}{s_{k^*j}}$, which, like before, is negative if $\frac{s_{k^*ij}}{s_{ij}} > \frac{s_{k^*i'j}}{s_{i'j}}$; that is, if region i is relatively specialized in k^*.

For general price movements, the degree of substitutability will be depressed the more these price movements are concentrated in the specialized industries.

In summary, the effective regional elasticity of substitution can exhibit complementarity even though intraindustry goods are substitutes. A necessary condition is that interindustry goods do exhibit complementarity. This result of regional complementarity is more likely to occur when the exogenous shocks analyzed are concentrated within the specialized industry-locations.

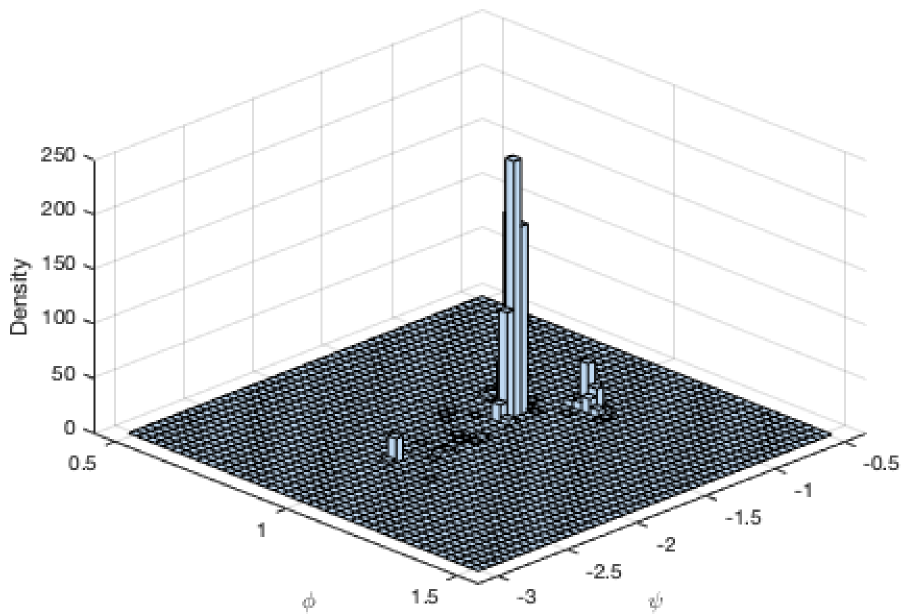


Figure 10: Distribution of (ϕ, ψ) from moment matching exercise. $M = 1000$ simulations.