International Patenting Strategies with Heterogeneous Firms

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Abstract

This paper analyzes how firms decide where to patent by constructing a patent decision model into a heterogeneous firm model of trade with endogenous rival entry. In the model, innovating firms compete with rival firms on price, where rivals force the innovating firm to reduce markups and lower the innovating firm's probability of obtaining monopolistic profits. Patenting allows the innovating firm to reduce the number of rival firms by increasing their fixed overhead costs, thereby providing higher expected profits and increased markups from reduced competition. Countries with higher states of technology, more competition and better patent protection receive a greater number of patents. Similarly, industries that are more substitutable and have lower variability in their labor efficiency tend to patent more frequently. Using a generalized framework of the model, I am able to estimate market-based measures of country-level patent protection, which when compared with other IP indices, suggests that not enough international patenting is taking place. Finally, I confirm the predictions of the model using a newly available technology-to-industry concordance on bilateral patent flows and show that firms are increasingly sensitive to foreign IP protection. Countries that choose to maximize their IP protection can increase the number of foreign patents by almost 10%.

Keywords: Patents, international trade, heterogeneous firms, endogenous markups, intellectual property, imperfect competition

JEL Classification: F12, F29, O34, L11

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1 Introduction

Most worldwide innovation is concentrated among a handful of countries, with the majority of global innovation taking place in the U.S., Japan, China and Europe. Countries outside of this group increasingly rely on the international diffusion of these new technologies for technological change and productivity growth (Eaton and Kortum (1996a)). How this technology diffuses across international borders is still debateable, with trade and foreign direct investment (FDI) each playing prominent roles (Eaton and Kortum 1996b; Archaya and Keller 2009; Branstetter 2006). Much work has been done in assessing which national policies have most directly affected the dynamics of technology diffusion with trade policy and intellectual property rights (IPR) having received considerable attention (e.g. Grossman and Helpman 1991; Ethier and Markusen 1996; Branstetter et al. 2006; Keller 2004; Archaya and Keller 2009; Falvey et al. 2006).

In finding ways to narrow the current technological divide and promote international technology diffusion, patents offer a promising solution. Nearly all patentable innovations undergo the patenting process (Dernis et al. (2001)). International patents are often a precondition for collaborative technology transfer. Only after a firm holds a patent right in a country is it likely to actively share proprietary technology either through joint ventures with an already established company, granting licenses for production or relocating production to that country. For example, a rigorous analysis of multinational firms and their affiliates concludes that strengthening patent protection in the affiliate country increases subsequent patent applications, R&D investment, and technology transfer (Branstetter et al. (2006)). While a developing country can choose to use a patented invention without the authorization of the inventor, many developing countries will struggle to successfully leverage the invention without the interest and cooperation of the patent holder. For such countries, the patenting decision can be an important precursor for the transfer of cutting-edge technology, which can subsequently spark the long-run diffusion of technology and knowledge.

Firms consider many different strategies when it comes to patenting, with two of the most important factors being cost and timing (Livne 2006; Schneiderman 2007). In their survey of U.S. manufacturing firms, Cohen et al. (2000) find that firms patent mainly to prevent imitation and counterfeiting, but also for reasons such as patent blocking, negotiations with other firms, the prevention of lawsuits and competition. The cost of patenting can pile up very quickly, with filing fees, agent fees and translation fees bringing the total application cost to more than $10,000 per application in several countries (Source: WIPO). In addition, there are also transaction costs, interaction costs with licensing professionals and knowledge costs of exposing ideas to potential imitators. On the other hand, the benefits of patenting give the firm additional market power, which allows them to charge higher markups and receive higher profits (Horstmann et al. 1985; Owen-Smith and Powell 2001).

Unfortunately, our understanding of how firms decide whether and where to seek patent protection abroad is poor. This paper seeks to remedy this gap in the literature by incorporating a patenting decision component into a heterogeneous firm model of trade (similar to Helpman et al. (2004) (HMY)) with imperfect competition (similar to de Blas and Russ (2011) (DBR)). In the model, innovating firms compete with rival firms on price (Bertrand competition). The number of rivals and their productivities depend on the innovating firm's own productivity so that more productive innovating firms face a greater number of more competitive rivals. This creates greater incentives for them to patent, which acts as a way to reduce the number of rivals by increasing their overhead costs as rivals must now work around the innovation. The benefit of this model is that it manages to maintain the producer-level facts regarding the behavior and composition of exporting firms and multi-nationals that were reconciled in HMY and Bernard et al. (2003) (BEJK), while allowing for cross-country and cross-industry differences to determine the flows of international patenting. These cross-country and cross-industry differences play a key role in deciding spacial patenting outcomes.

These spatial patterns of patenting have important implications for development and are the topic of much debate regarding international technology transfer. The role of patent rights figured prominently in the original negotiations of the Trade-Related Aspects of Intellectual Property Rights Agreement (TRIPS) in the Uruguay Round of GATT in 1994. Throughout these negotiations developing countries expressed concern that stronger IPR would only benefit wealthy countries that had already developed strong innovation capacity. Wealthy countries therefore agreed to a provision to provide incentives for firms to transfer technology to developing countries and enable them to build a viable technological base (Article 66.2 of the TRIPS Agreement). Implicit in this provision is the hope that offering stronger patent protection to foreign innovators might increase the flow of patents and thereby speed up the process of technology transfer. Where firms choose to patent is therefore central in this debate.

This model begins to address some of the concerns in this debate by providing a specific framework to analyze market-based outcomes of international patenting patterns. Using a unique database of patent families (i.e. the set of patent applications in different countries that relate to a single parent invention) compiled by the World Intellectual Property Organization (WIPO) and the EPO, I am able to use the model to back out market-based measures of country patent protection (IPR). The IPR estimates are compared to previous measures of IPR from Park (2008). Initial evidence suggests that based on patent law interpretation, not enough foreign patenting is taking place given the parameter values. I then use these new measures of IPR and test the predictions of the model by estimating the determinants of bilateral patent flows to 28 destination countries between the years 1996 to 2005. The data confirms several predictions of the model and can explain cross-country variations in patenting to a much higher degree than previous models.

The contributions of this paper to the literature are several. The first contribution is that this paper describes a new version of a heterogeneous model of trade by incorporating endogenous entry of rivals. It is most similar in structure to the model described in DBR with the exception that rival entry will be determined by the innovating firm's productive capability. This has important implications on both the number and composition of rivals, as well as markups. In addition to this new framework, this model also overlays a patenting decision component onto the model, that is different from previous models, like Eaton and Kortum (1996b) (EK). EK incorporates patenting into a quality ladders model similar to Grossman and Helpman (1991). In the EK model, the firm decision to patent depends on the hazard rate of imitation and obsolescence of the invention. In this model, the decision to patent depends on the number of competitors and probability of imitation. Thus, if there are no rivals (either foreign or domestic), there is no need to patent. Finally, this paper is also the first to obtain market-based measures of patent protection, meaning that it uses actual patent flows to back out the IPR measures. Previous measures of IPR have been constructed using subjective determinants, such as enforcement of IPR, coverage and membership in international agreements. The market-based measures serve as an alternative measure of IPR that focus strictly on what firms consider to be most relevant in deciding whether/where to patent. The next section defines the model and outlines the process to calculate a numerical equilibrium. Section 3 describes the properties of the equilibrium using simulations and parameter estimates obtained from BEJK and Fieler (2007). Section 4 describes the empirical portion and constructs market-based measures of IPRs using non-linear least squares (NLLS). This is followed by the conclusion.

2 Model

The core elements of the model are based on Helpman et al. (2004) (HMY) and de Blas and Russ (2011) (DBR).

2.1 Demand

Assume that that there are $i = 1, ..., I$ countries where each country has the ability to produce $k = 1, ..., K$ different goods or industries. I assume only one factor of production, labor L_i, which is perfectly mobile across industries but not countries and paid wage w_i. Each good k is comprised of an infinite number of varieties, which will be indexed by $\omega \in \Omega$.

In each country, preferences are given by a representative consumer with a two-tier utility function. The upper-tier utility function is Cobb-Douglas where the share of expenditure on varieties from industry k in country i are given by α_i^k where $0 \leq \alpha_i^k \leq 1$. The lower-tier utility function is CES with elasticity of substitution σ^k between varieties. Thus, in any country i, the total expenditure on variety ω of good k will be given by:

$$x_i^k(\omega) = \left(\frac{p_i^k(\omega)}{P_i^k}\right)^{1-\sigma^k} \alpha_i^k w_i L_i \tag{1}$$

where P_i^k is the CES price index[1]. Given these assumptions, the consumer price index in country i is given by $P_i = \prod_{k=1}^{K} \left(p_i^k\right)^{\alpha_i^k}$.

2.2 Production and Innovation

Labor is the only factor used in production and is assumed to be perfectly mobile across types and goods, but immobile across countries. I denote $z_i^k(\omega)$ to be the measure of productivity of variety ω in industry k. I assume that there are two types of firms in the world economy: i.) Innovating firms who pay a one-time fixed cost of innovation I_i^k that allows them to draw their productivity parameter z from an unbounded distribution and ii.) Imitating or rival firms who do not pay an entry fee but are bounded in their productivity draws by the innovating firms' productive capability.

Both types of firms draw their productivity $z_i^k(\omega)$ from a Fréchet (inverted Weibull) distribution $F_i^k(z)$ with positive support. For innovating firms, the extreme value form $F_i^k(z)$ represents the best surviving idea available to produce variety ω (see Eaton and Kortum (2009), Chapter 4). The Fréchet distribution will be governed by two separate parameters: a country-industry specific technology parameter T_i^k which will govern the mean of the distribution and an industry specific shape parameter $\theta^k > 1$ that determines

[1] Given by $P_i^k = \left(\sum_{\omega\prime \in \Omega} p_i^k(\omega\prime)^{1-\sigma^k}\right)^{1/\left(1-\sigma^k\right)}$

the heterogeneity of efficiency levels. The distribution for the innovating firms is given by

$$F_i^{Ik}(z|T_i^k, \theta^k) = \Pr\left[z_i^k \leq z\right] = e^{-T_i^k z^{-\theta^k}} \tag{2}$$

A higher T_i^k implies higher technology and greater productivity on average, while a higher θ^k means lower variability in labor efficiencies so that producers are more homogeneous. In order to guarantee the existence of a well-defined CES price index P_j^k, I assume that the elasticity of substitution $\sigma_i^k < 1 + \theta^k$.

Once each innovating firm draws this parameter z, they decide whether to pay a per-period fixed cost to enter the market and sell their good, f_{ij}^k. I assume constant returns to scale as well as iceberg trade costs, denoted as d_{ij}^k which represents the cost of shipping from country i to country j. All trade costs are positive ($d_{ij}^k \geq 1$)and I assume that trade barriers obey the triangle inequality so that $d_{ij}^k \leq d_{in}^k d_{nj}^k$ for all i, j and n[2]. Given CES demand, the optimal price for the innovating firm will be to charge a CES/monopolistic markup. Without any rivals and with the exception of different productivity distributions, the equilibrium and properties of the equilibrium are similar to the results obtained in Melitz (2003), Chaney (2008) and Helpman et al. (2004). For the future sections, I drop the superscript k and assume that the following holds for each industry type $k = 1, ..., K$.

2.3 Production and Imitation

Assume that for each new variety ω in each market j, the innovating firm faces some number r_j of rivals or imitators who compete with the firm on price (Bertrand) as in Eaton and Kortum (2002) (BEJK) and de Blas and Russ (2011) (DBR). Unlike BEJK and DBR however, the number of rivals and possible imitators in each country is endogenously determined by the productivity parameter of the innovating firm[3] To do this, the rival firms' marginal cost distributions will be bounded by the marginal costs of the innovating firm, so they are never more efficient at producing variety ω than the firm who invented it. Since the firms compete on price, this implies that the rival firms will never make positive profits unless the innovating firm

[2] Equivalently, one can include an FDI component into the decision so that the marginal cost will be $c_{ij,FDI}^{Ik}(\omega) = \frac{w_j}{z_i^{Ik}}$ albeit with a higher initial fixed cost (see HMY). However, since this does not play a factor in determining whether the firm patents or not, I only look at the patenting decision from the exporter's perspective.

[3] As DBR note, BEJK assumes that the number of rivals for any given product is a random variable determined by a Poisson distribution. This assumption allows the number of rivals to cancel out in the analysis (see Eaton and Kortum (2009), Chapter 4). On the other hand, DBR assume that the number of rivals is determined solely by the free-entry condition, and is therefore unaffected by the productivity of the innovating firm.

is forced to exit. However, rivals do not pay a fixed cost of entry and are able to enter and exit at any given time. In other words, they simply act as a "credible threat" to the innovating firm and as a mechanism to ensure that the innovating firm does not charge a dubious markup.

I denote the rival distribution in country j as z_j^R. Each of the rivals have constant returns to scale and their marginal costs are given by $c_{ij}^R(\omega) = \frac{w_j}{z_j^R(\omega)}$. The rivals face the same demand functions as their counterparts, so that the profit function is similar to the innovating firm's profit function, with the exception that each rival pays a different per-period fixed overhead cost f_j^R to actually enter into the market. Due to this per-period fixed cost, there exists a non-zero cutoff cost parameter $\tilde{c}_{ij}^R$ that governs whether the rival has the low-cost necessary to compete and serve as a credible threat. This cutoff condition $\tilde{c}_{ij}^R$ is determined by assuming monopolistic pricing and setting the profit equal to zero so that $\tilde{c}_{ij}^R$ corresponds to the productivity threshold sufficient to cover the fixed per-period overhead costs.

$$\tilde{c}_{ij}^R = \left[\frac{(\sigma)^\sigma}{(\sigma-1)^{\sigma-1}} \frac{f_j^R}{\alpha_j w_j L_j} \right]^{\frac{1}{\sigma-1}} P_j \tag{3}$$

Rivals who draw a cost parameter $c \leq c_{ij}^R$ are permanent entrants and remain as credible threats in the market for as long as the innovating firm competes. Rivals who draw $c > c_{ij}^R$ never enter and are therefore not deemed credible. Next, each rival draws their cost parameters c from a similar shape distribution as the innovating firm (i.e. Fréchet), but their support is truncated by the marginal costs of the innovating firm which will be denoted as c_{ij}^I. The CDF of the rivals' cost function in country j is[4]

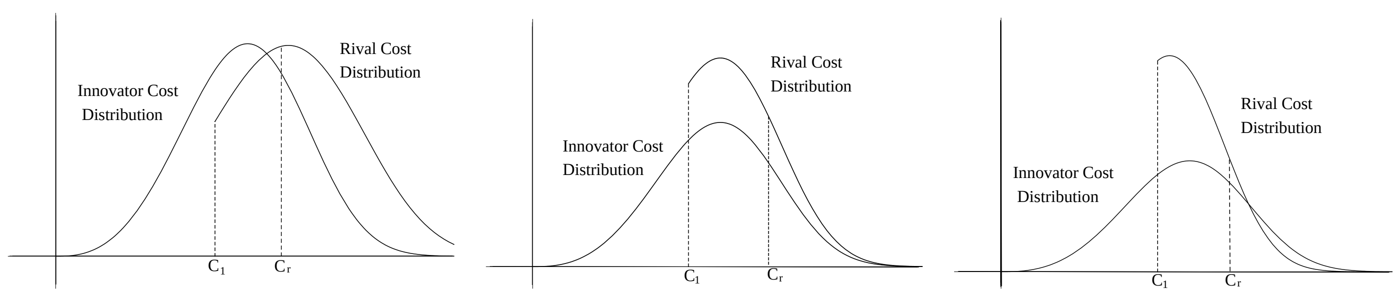
$$G_j^R(c|c_{ij}^I, T_j, \theta) = 1 - e^{-T_j w_j^{-\theta}\left(c^\theta - (c_{ij}^I)^\theta\right)} \tag{4}$$

A depiction of the rival and innovating firms' distributions is given below in Figure 1. In each chart, the leftmost c represents the cost parameter for the innovating firm and shows a left-truncation of the rivals' cost distribution. The rightmost c is the equivalent cost parameter for the cutoff condition for rival entry (given as the inverse of z_j^R, so that the area in between the two lines is the ex-ante probability of successful entry

[4]Formula for a left-truncated Weibull distribution can be found on pages 134-135 in Rinne (2009). Note that the corresponding productivity CDF for the rivals is given by

$$F_j^R\left(z|z_i^I, T_j, \theta\right) = e^{-T_j\left(z^{-\theta} - \left(\frac{w_j}{w_i d_{ij}}\right)^{-\theta}(z_i^I)^{-\theta}\right)}$$

where z_i^I is the productivity parameter of the innovating firm by the rivals in country j. The figure depicts three separate charts that are differentiated by the technology levels in the destination country. The state of technology in each country will play an important role in determining both the number of rivals, as well as the efficiency of the rivals. In figure 2a, the technology for the innovating firm's country is higher than the country of the rival firms. Figure 2a shows that the innovating firm from country i will not only face relatively fewer rivals, but also those rivals have lower average productivity than the innovating firm. In figure 2c, the opposite occurs. The country of the rival firms has higher technology and the innovating firm not only faces more competition, but each competitor will have cost parameters that are closer to the innovating firm's cost parameter.



2a. $T_i > T_j$ 2b. $T_i = T_j$ 2c. $T_i < T_j$

Figure 1 - Cost Distribution $G(c)$ of Innovating (T_i) and Rival (T_j) Firms

When the cost parameter associated with the rival entry condition $\tilde{c}_{ij}^R$ is less than the marginal cost of the innovating firm c_{ij}^I, then there will be zero rivals and the innovating firm will be guaranteed monopolistic profits. However, for the purposes of this paper, assume that the marginal cost of the innovating firm is always less than the rival entry cost condition so that there always exists some nonzero number of rivals. The area under the curve gives the ex-ante probability of successful entry by a rival and is given by the following formula

$$\lambda_{ij}\left(\tilde{c}_{ij}^R | c_{ij}^I\right) = \int_{c_{ij}^I}^{\tilde{c}_{ij}^R} g_j^R\left(c | c_{ij}^I, T_j \theta\right) dc = 1 - e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^\theta - \left(c_{ij}^I\right)^\theta\right]} \qquad (5)$$

Given this function, when c_{ij}^I decreases, the ex-ante probability of successful entry by the rivals increases, so that more productive innovators are more likely to face more rivals. The intuition behind is that more profitable firms will face a higher number of entrants than less profitable firms[5]. Assuming that the number

[5]This is supported in the literature by Luttmer (2007) who claims that there are stronger incentives for entry by imitators when the incumbent firm is larger and more profitable. This is also supported in Costinot et al. (2012) who model "follower"

of potential (ex-ante) rivals in each country is proportional to market size $Y_j^R = w_j L_j$, the total number of credible rivals $r_{ij}(\omega)$ competing in each variety is:

$$r_{ij}(\omega) = \lambda_{ij} Y_j^R = Y_j^R \left[1 - e^{-T_j w_j^{-\theta}\left[(\tilde{c}_{ij}^R)^\theta - (c_{ij}^I)^\theta\right]}\right] \tag{6}$$

Before moving to the next section, it bears going through a couple of different properties of the rivals and their production capabilities[6].

Result 1: The number of rivals $r_{ij}(\omega)$ and their average efficiency increases as the state of technology T_j increases.

Result 2: The number of rivals $r_{ij}(\omega)$ increases with the cutoff condition of rival entry $\tilde{c}_{ij}^R$.

Result 3: The number of rivals $r_{ij}(\omega)$ and their productivity increases with the productivity parameter of the innovating firm.

To summarize these results, more productive innovating firms not only face more rivals, but these rivals are also more productive on average. Innovating firms can reduce the number of rivals they face by increasing the cutoff condition for rival entry. These results help set-up some of the key properties to be uncovered in the simulations. As I move forward, I will only consider the case where $c_{ij}^I > \tilde{c}_{ij}^R$ (i.e. at least one rival exists).

2.4 The Distributions of Markups, Prices and Profits

The rivals in each country j are there to ensure that the innovating firm does not charge an unfair markup. As in BEJK and DBR, I assume that the two types of firms will compete in price (Bertrand). The innovating firm only needs to compete against the low-cost rival firm, since all of the other rivals will be unable to match their costs. I denote the marginal cost function of the low-cost rival in country j as c_{ij}^{R*}. Because the firms compete with Bertrand competition, the price will be determined as the minimum of the low cost rival's cost function and the CES/monopolistic price of the innovating firm. I denote the price under the Bertrand competition scenario as p_{ij}^B, while the price in the monopoly scenario is written as p_{ij}^M. Under this

firms in a similar manner where they do not exceed the productive capabilities of the innovating firm.

[6]Proofs can be found in the Appendix

scenario, prices are

$$p_{ij}(\omega) = \min\left\{p_{ij}^{B} = c_{ij}^{R*}, p_{ij}^{M} = \overline{m}c_{ij}^{I}\right\} \tag{7}$$

With markup

$$m_{ij}(\omega) = \min\left\{m_{ij}^{B} = \frac{c_{ij}^{R*}}{c_{ij}^{I}}, \overline{m} = \frac{\sigma}{\sigma-1}\right\}$$

Where $\overline{m}$ is the Dixit-Stiglitz CES markup. This price leads to the following possible profit outcomes for the innovating firm in country j:

$$\pi_{ij}^{I}(\omega) = \begin{cases} \pi_{ij}^{B} = \left(\frac{m_{ij}^{B}c_{ij}^{I}}{P_j}\right)^{1-\sigma}\left(\frac{m_{ij}^{B}-1}{m_{ij}^{B}}\right)Y_j - f_{ij} & \frac{c_{ij}^{R*}}{c_{ij}^{I}} \leq \frac{\sigma}{\sigma-1} \\ \pi_{ij}^{M} = \left(\frac{\overline{m}c_{ij}^{I}}{P_j}\right)^{1-\sigma}\left(\frac{\overline{m}-1}{\overline{m}}\right)Y_j - f_{ij} & \frac{c_{ij}^{R*}}{c_{ij}^{I}} > \frac{\sigma}{\sigma-1} \end{cases} \tag{8}$$

The price, markup and firm profits are all determined by the cost ratio of the low-cost rival and innovating firm. If the cost function of the low cost rival is greater than the monopolistic price, than the innovating firm will be able to charge a monopolistic price and obtain monopolistic profits. However, if the cost function of the low-cost rival is lower than the monopolistic price, then the innovating firm obtains Bertrand profits.

In order to determine when the low-cost rival's cost functions is greater than or less than the monopolistic prices, I need to define the distribution of this cost ratio and define the CDF of the low-cost imitator $G_{ij}^{R*}(c_{ij}^{R*})$.[7]
and combine it with the cost distribution of the innovating firm to obtain the markup distribution[8]. The PDF of the cost ratio (and subsequently, the markup under Bertrand competition) is given by

[7] I use the formulation from Rinne (2009) on pages 224 and 237 which provides the CDF for the first order statistic for Weibull distributions and gives me

$$G_{ij}^{R*}(c_{ij}^{R*}|c_{ij}^{I}, r_{ij}, T_j, \theta) = 1 - e^{-r_{ij}T_j w_j^{-\theta}\left[\left(c_{ij}^{R*}\right)^{\theta} - \left(c_{ij}^{I}\right)^{\theta}\right]}$$

[8] See Appendix section A.2 for derivation

$$h(\frac{c_{ij}^{R*}}{c_{ij}^{I}}) = h(m_{ij}^{B}) = \begin{cases} \frac{r_{ij}T_iT_j\theta(w_iw_jd_{ij})^{\theta}\left(m_{ij}^{B}\right)^{\theta-1}}{\left[r_{ij}T_j(w_id_{ij})^{\theta}\left(\left(m_{ij}^{B}\right)^{\theta}-1\right)+T_iw_j^{\theta}\right]^2} & \text{for } 1 \leq m_{ij}^{B} \leq \overline{m} \\ \int_{\overline{m}}^{\infty} \frac{r_{ij}T_iT_j\theta(w_iw_jd_{ij})^{\theta}\left(m_{ij}^{B}\right)^{\theta-1}}{\left[r_{ij}T_j(w_id_{ij})^{\theta}\left(\left(m_{ij}^{B}\right)^{\theta}-1\right)+T_iw_j^{\theta}\right]^2} dm_{ij}^{B} & \text{for } m_{ij}^{B} = \overline{m} \\ 0 & \text{for } m_{ij}^{B} > \overline{m} \end{cases} \tag{9}$$

With a mass point at $\overline{m}$. Notice that the distribution of the markup is entirely independent of the marginal costs drawn by the innovating firm and low-cost rival. Also, in the symmetric case with no trade costs and $r_{ij} = 1$ (i.e. there is one other rival), I get $h(m_{ij}^{B}) = \theta\left(m_{ij}^{B}\right)^{-\theta-1}$ which is identical to the Pareto density for markups obtained in BEJK. Figure 2 shows the distribution of $h(m_{ij}^{B})$ for varying levels of r_{ij}.

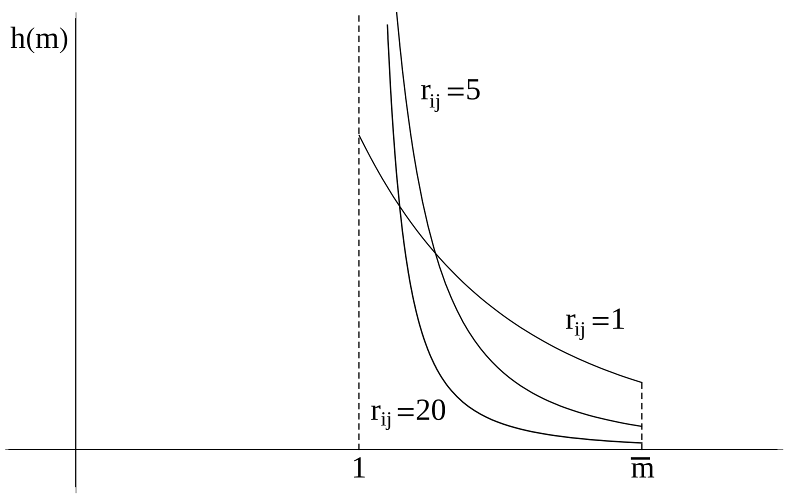


Figure 2: Density of the Markup

Integrating $h(m_{ij})$ over the values from $\overline{m}$ to ∞ gives the probability that the innovating firm in country i charges the monopolistic markup in country j and achieves profit π_{ij}^{M}. I denote this probability as ϕ_{ij}.

$$\phi_{ij} = \Pr\left[m_{ij}^{B} \geq \overline{m}\right] = \int_{\overline{m}}^{\infty} h(m_{ij}^{B})dm_{ij}^{B} = \frac{T_iw_j^{\theta}}{r_{ij}T_j\left(w_id_{ij}\right)^{\theta}\left(\overline{m}^{\theta}-1\right)+T_iw_j^{\theta}} \tag{10}$$

Having defined when the markup is Bertrand and when the markup will be CES, I can now rewrite the innovating firm's profit equation as

$$\begin{aligned} \mathrm{E}\left[\pi_{ij}^{I}(\omega)\right] &= \phi_{ij}\pi_{ij}^{M} + (1-\phi_{ij})\,\mathrm{E}\left[\pi_{ij}^{B}\right] \\ &= \left(\frac{c_{ij}^{I}}{P_j}\right)^{1-\sigma}\alpha_j w_j L_j\left[\phi_{ij}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} + (1-\phi_{ij})\left(\bar{m}_{ij}^{B}\right)^{-\sigma}\left(\bar{m}_{ij}^{B}-1\right)\right] - f_{ij} \end{aligned} \tag{11}$$

where $\bar{m}_{ij}^{B} = \mathrm{E}\left[m_{ij}^{B}|m_{ij}^{B} \leq \overline{m}\right]$ is the expected value of the markup when it is less than the CES markup[9]. This leads to the next set of results.

Result 4: The probability the innovating firm charges the CES markup in country j is decreasing in contestability r_{ij}[10].

Result 5: The probability the innovating firm charges the CES markup in country j increases as the cutoff condition for rival entry decreases and decreases as the innovating firm becomes more productive (lower costs)[11].

Result 6: The innovating firm's expected profit $\mathrm{E}\left[\pi_{ij}^{I}(\omega)\right]$ is decreasing in contestability r_{ij}.

All that is left to define are the prices, which lead to our final result. The expected price $p_{ij}^{1-\sigma}(\omega)$ is:

$$\mathrm{E}\left[p_{ij}^{1-\sigma}\right] = \phi_{ij}\overline{m}^{1-\sigma}\mathrm{E}\left[\left(c_{ij}^{I}\right)^{1-\sigma}\right] + (1-\phi_{ij})\,\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right] \tag{12}$$

Result 7: The price of variety ω charged to consumers in country j is decreasing in contestability r_{ij}

[9]Specifically,

$$\begin{aligned} \bar{m}_{ij}^{B} &= \int_{1}^{\overline{m}} m_{ij}^{B}h(m_{ij}^{B})dm_{ij}^{B} = \int_{1}^{\overline{m}} \frac{r_{ij}T_iT_j\theta\left(w_iw_jd_{ij}\right)^{\theta}\left(m_{ij}^{B}\right)^{\theta}}{\left[r_{ij}T_j\left(w_id_{ij}\right)^{\theta}\left(m_{ij}^{\theta}-1\right)+T_iw_j^{\theta}\right]^2}dm_{ij}^{B} \\ &= \frac{T_iw_n^{\theta}\theta}{r_{ij}T_j\left(w_id_{ij}\right)^{\theta}(\theta-1)}\left({}_2F_1\left(2,\frac{\theta-1}{\theta},\frac{2\theta-1}{\theta},\frac{r_{ij}T_j\left(w_id_{ij}\right)^{\theta}-T_iw_j^{\theta}}{r_{ij}T_j\left(w_id_{ij}\right)^{\theta}}\right) - \overline{m}^{1-\theta}\,{}_2F_1\left(2,\frac{\theta-1}{\theta},\frac{2\theta-1}{\theta},\frac{r_{ij}T_j\left(w_id_{ij}\right)^{\theta}-T_iw_j^{\theta}}{r_{ij}T_j\left(w_id_{ij}\overline{m}\right)^{\theta}}\right)\right) \end{aligned}$$

Where ${}_2F_1()$ are hypergeometric functions

[10]This result is similar to the findings in DBR who similarly show that lower markups occur with increased contestability.

[11]It may seem counterintuitive that more productive firms are less likely to be monopolists, but note that the expected markup for the innovating firm increases with their productivity so that they are still guaranteed more profits than low productivity firms.

To sum up the results, the number of rivals negatively effects the innovating firm's expected profits, so that holding the innovating firm's productivity constant, they will want to reduce the number of rivals. Note that despite the increased contestability, innovating firms with higher productivities still receive larger profits due to CRS and capturing a larger market share. This completes the set-up for the first stage of the model. The next section looks at when the innovating firm decides to patent.

2.5 The Decision to Patent

Up until this point, the innovating firm's operating profits depends on the number of rivals which is dependent on the firm's productivity draw and rival cutoff condition. I now introduce patenting as a mechanism that allows the innovating firm to reduce the number of rivals they face in any given market j. When firms patent, they pay a fixed cost f_j^P and in return, rival firms will have to pay an overhead cost of $f_{j,pat}^R$ where $f_{j,pat}^R \geq f_{j,not}^R$. One way to interpret this is that patenting causes the rival firms to pay an additional fixed cost, such as legal fees, in order to produce around the patent. The better the patent protection, the greater the cost to produce around the patent. This higher cost $f_{j,pat}^R$ decreases the threshold cost condition $\tilde{c}_{ij}^R$ for rival firms, which by Result 2, reduces the number of rivals. This reduction in rivals has a doubly positive effect on firm profits as it not only increases the probability for the innovating firm to charge their optimal markup, but also increases the expected markup should the firm operate in Bertrand competition. The expected profits from patenting are:

$$\mathrm{E}\left[\pi_{ij,pat}^I(\omega)\right] = \phi_{ij,pat}\pi_{ij}^M + (1-\phi_{ij,pat})\,\mathrm{E}\left[\pi_{ij,pat}^B\right] - f_{ij} - f_j^P \tag{13}$$

The firm will patent if the benefits, measured as $\pi_{ij,pat}^I(\omega) - \pi_{ij,not}^I(\omega)$ are greater than the cost f_j^P. Figure 3 plots the patenting decision and shows that when productivity $z > z^P$ the firm will elect to patent.

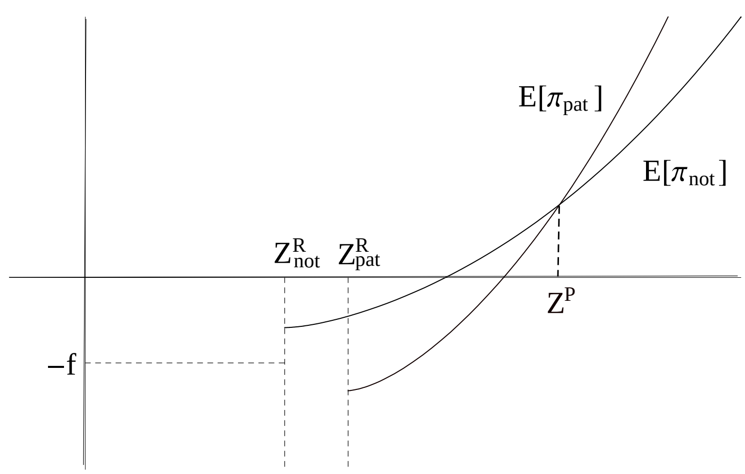


Figure 3: Innovating Firm's Expected Profit from Patenting

Analytically, this patenting cutoff condition, c_{ij}^{P}=[12].

$$\left(\frac{f_j^P}{\alpha_j w_j L_j}\right)^{\frac{1}{1-\sigma}} P_j \left\{\phi_{ij,pat}\left[V\left(\overline{m}\right)-V\left(\bar{m}_{ij,pat}^B\right)\right]-\phi_{ij,not}\left[V\left(\overline{m}\right)-V\left(\bar{m}_{ij,not}^B\right)\right]+V\left(\bar{m}_{ij,pat}^B\right)-V\left(\bar{m}_{ij,not}^B\right)\right\}^{\frac{1}{\sigma-1}} \quad (14)$$

Where $V(x) = x^{-\sigma}(x-1)$. Therefore, whenever $c \leq c_{ij}^P$ innovating firms will elect to patent. I assume that when $c = c_{ij}^P$, the firm still elects to patent.

Next I define the market-entry condition. Since not all exporting firms or firms who commit to FDI elect to patent, it must be the case that the patenting cutoff condition c_{ij}^P is less than the market entry condition c_{ij}^E. Hence, the market entry condition will be determined by setting the profits in the non-patenting scenario to zero. This leads to:

$$c_{ij}^E = \left(\frac{f_{ij}}{\alpha_j w_j L_j}\right)^{\frac{1}{\sigma-1}} P_j \left[\phi_{ij,not} V\left(\overline{m}\right) + \left(1-\phi_{in,not}\right) V\left(m_{ij,not}^B\right)\right]^{\frac{1}{\sigma-1}} \quad (15)$$

So that when $c \leq c_{ij}^E$, the firm will enter into market j, otherwise they will immediately exit. To ensure that not all firms elect to patent, I require that the cost of patenting be high enough where:

$$f_j^P \geq f_{ij}\left[\frac{\phi_{ij,pat}\left(V\left(\overline{m}\right)-V\left(m_{ij,pat}^B\right)\right)+V\left(m_{ij,pat}^B\right)}{\phi_{ij,not}\left(V\left(\overline{m}\right)-V\left(m_{ij,not}^B\right)\right)+V\left(m_{ij,not}^B\right)}\right] \quad (16)$$

This condition guarantees that not all firms will patent and that the entry condition is determined by Equation (12). Note that this also guarantees the country j's patent protection will have no effect on the firm's decision to enter into a foreign market, thereby preserving the properties uncovered in the new, new trade theory models. I am then left with two types of firms in every market: Non-patenting firms where $c_{ij}^E \geq c \geq c_{ij}^P$ and patenting firms where $c_{ij}^P \geq c$.

The proportion of patenting firms from country i to country j will be based on the distributions $\frac{G(c_{ij}^P)}{G(c_{ij}^E)}$

[12]See Appendix section A.3 for derivation

so that the expected price for variety ω supplied by a firm from country i in country j is

$$\begin{aligned}\mathrm{E}\left[p_{ij}^{1-\sigma}\right] = \frac{G(c_{ij}^{P})}{G(c_{ij}^{E})}\left(\phi_{ij,pat}\overline{m}^{1-\sigma}\mathrm{E}\left[\left(c_{ij}^{I}\right)^{1-\sigma}\right] + (1-\phi_{ij,pat})\,\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right]\right) \\ + \left(1-\frac{G(c_{ij}^{P})}{G(c_{ij}^{E})}\right)\left(\phi_{ij,not}\overline{m}^{1-\sigma}\mathrm{E}\left[\left(c_{ij}^{I}\right)^{1-\sigma}\right] + (1-\phi_{ij,not})\,\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right]\right)\end{aligned} \tag{17}$$

Due to the fact that no closed form solution exists for the expected markup in the Bertrand competition, scenario, there is no closed form solution for the partial equilibrium. However, the process to obtain a numerical solution to the equilibrium is relatively straightforward and can be calculated through an iterative process that eventually converges on the fixed point equilibrium. One needs only to define all of the non-patenting exogenous parameters of the model (T, θ, σ, L_j^R, d and fixed costs) and pin down wages by including a tradeable nonmanufactured good that serves as the numeraire. From there, one can derive an initial productivity threshold (defined as c_{ij}^E) for entry into each market, which will yield a nonzero number of rivals. From there, the expected markup and adjusted profits emerge, which adjust the productivity thresholds. One then repeats this process until c_{ij}^E converges. Once the entry condition is defined, simply incorporate the patenting parameters and repeat. The process can be easily done using mathematical software such as MATLAB or Mathematica.

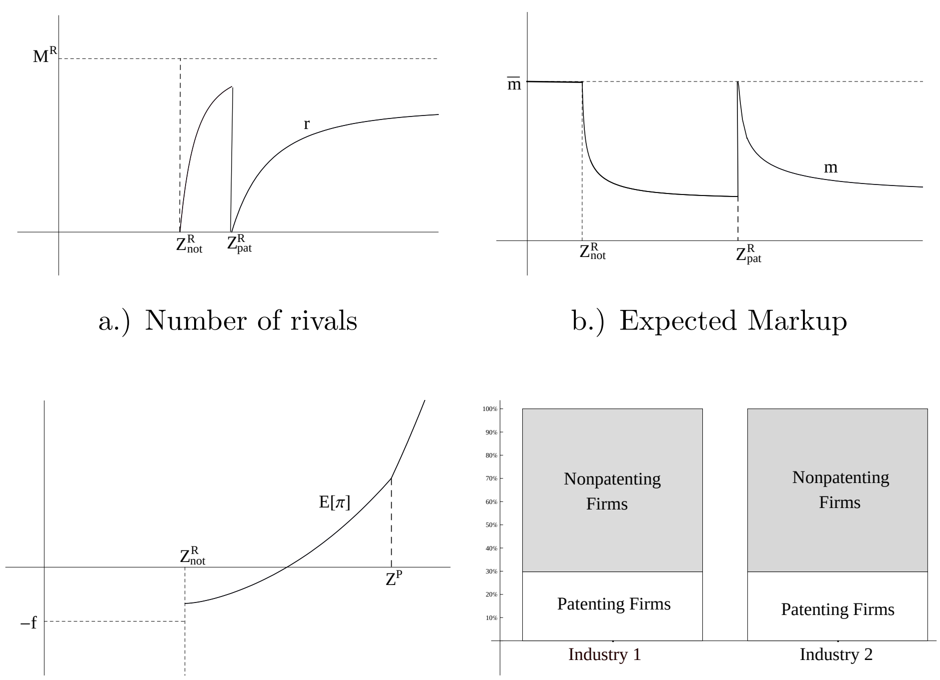
3 Properties of the Model

In order to uncover some of the properties of the model in equilibrium, this section runs several simulations with different parameter estimates to predict when firms will patent and how country and industry differences will impact the patenting decision. I start by first looking at the closed economy simulation with industry-level differences. I then analyze the open economy scenario that accounts for country-level differences.

3.1 Closed Economy with Different Industry Parameters

The simulations depict the behavior of rival firms, expected markups and profits for different parameter values of z drawn by the innovating firm. The first chart depicts the number of rivals the innovating firm faces with varying levels of z, while the second chart shows the innovating firm's expected markup. The third chart depicts the innovating firm's expected profits from patenting, while the final chart looks at the proportion of innovating firms that patent. Each simulation calculates the patenting cutoff condition z^P and assumes that when $z \geq z^P$ that the innovating firm always elects to patent.

For the first run of simulations, I look at industry differences within a closed economy (i.e. infinite trade costs) to analyze how different industry parameters impact patenting. For the baseline estimate, I employ symmetry between both industries and set $\sigma = 5$ and $\theta = 8.28$, as obtained from BEJK and Eaton and Kortum (2002), along with 20 potential rivals in the economy (as used in Atkeson and Burstein (2008) and DBR). I set the technology parameter T to be one. I also assume that patenting doubles the fixed overhead costs for rival firms. Parameter estimates for other variables such as the market size, number of innovating firms and wages do not matter for the simulations. Figure 4 plots the baseline results below.



a.) Number of rivals

b.) Expected Markup

c.) Firm Profits

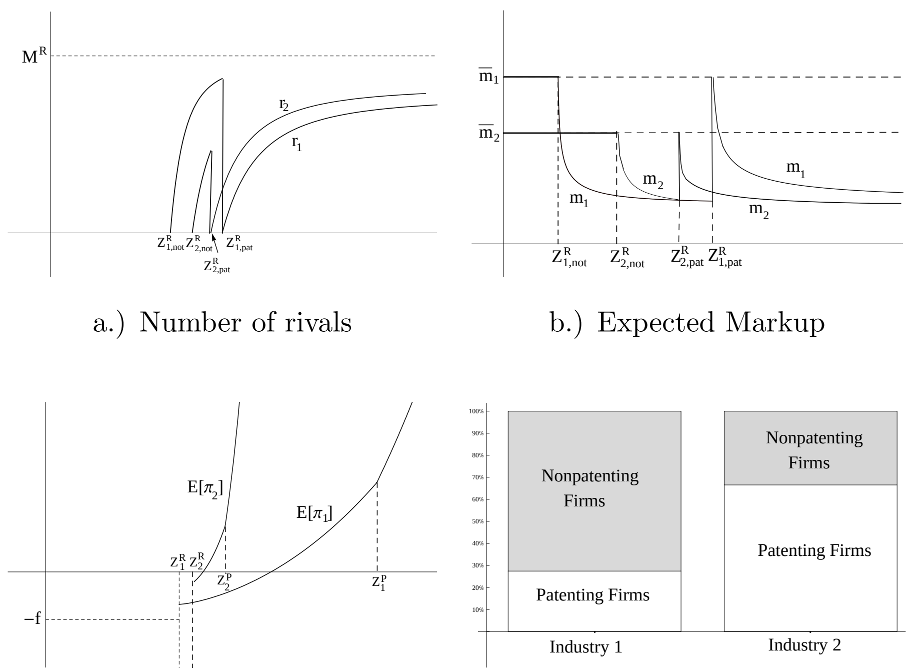
d.) Composition of Firms

Figure 4: Closed Economy Baseline Example

In the baseline scenario, roughly 30% of firms in each industry patent. Figure 6a displays the number of rivals the innovating firm faces and shows a steep drop-off immediately following the patenting cutoff condition z^R_{pat}. This is because the potential profits from entering are too small to justify the increased fixed costs as a result from patenting. However, as the efficiency parameter continues to increases and potential profits for the innovating firm get higher, more and more rival firms will be willing to pay this increased cost. Figure 4b displays the markup the innovating firm can expect to charge with increasing z. Initially, they charge the CES markup, until rival entry begins at z^R_{not}. From that point, the expected markup decreases as the innovating firm faces increased competition. At z^R_{pat}, the innovating firm can once again charge the CES markup since there are no rivals, but as their efficiency increases, their expected markup will continue its downward trend. Finally, Figure 4c displays the expected profits accrued by the innovating firm as z increases. There is a kink at z^P where the innovating firm decides to patent and the profit equation becomes more steep. The next sections shows how industry differences will impact the firm decision to patent.

3.1.1 Differences in Elasticity of Substitution

For the first scenario, I assume differences in the elasticity of substitution σ. Specifically, I assume that the elasticity of substitution for industry two is increased to 8, while keeping the elasticity the same in industry one (i.e. $\sigma_1 = 5$). A higher elasticity of substitution for industry two implies that price differences within this industry will have larger effects and that rivals will have a smaller range of profitable outcomes. Therefore, one would expect fewer potential rivals to enter, but increased competition between the innovating firm and rivals who do enter. Figure 5 shows the results.



a.) Number of rivals b.) Expected Markup

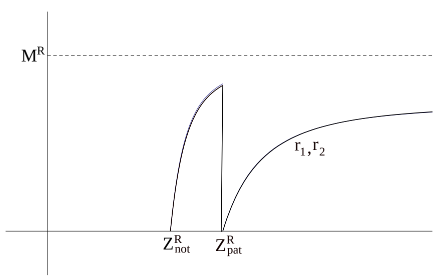
c.) Firm Profits d.) Composition of Firms

Figure 5: Closed Economy - Difference in Elasticity $\sigma_2 > \sigma_1$

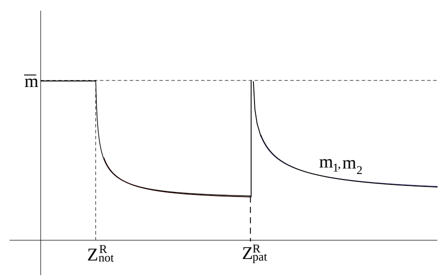
As expected, fewer rivals decide to enter into the market, and there is a stark difference in the allocable markups allowed by firms in each industry. Figure 7 also shows that a much higher proportion of firms in industry two decide to patent when compared to industry one. Nearly twice as many firms patent, which is expected given that patenting will have a stronger impact in industry two, than in industry one. The next section looks at differences in the heterogeneity of labor efficiencies θ.

3.1.2 Differences in the Variance of Technologies

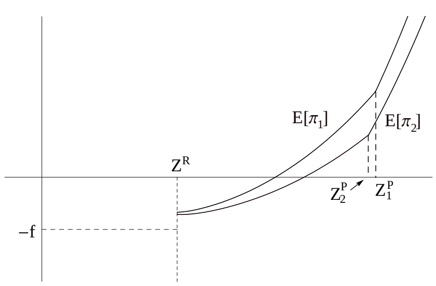
In this scenario, the two industries' elasticities return to the baseline values and the parameter θ will vary. The estimate for θ in industry two increases to 12.86, which was one of the alternative specifications used in Fieler (2007). With the higher θ, there is less variability in labor efficiencies, which means less dispersion in the productivities of rivals, and consequently more competition for the innovating firm since the all firms will be clumped closer together. Therefore, the expectation is that more firms in industry two will patent. Below are the results.



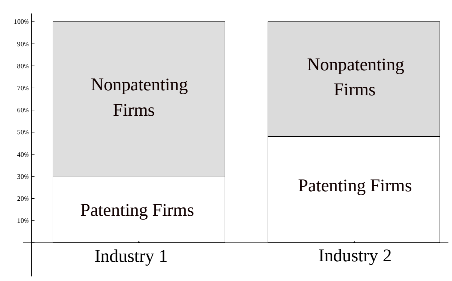
a.) Number of rivals



b.) Expected Markup



c.) Firm Profits



d.) Composition of Firms

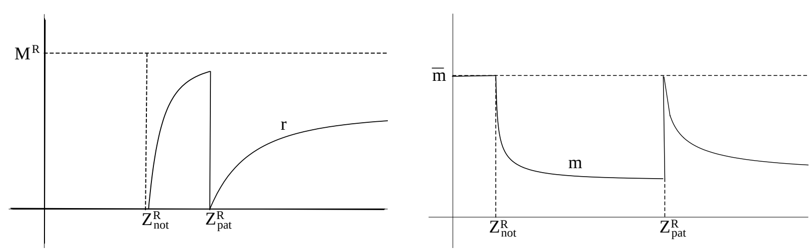
Figure 6: Closed Economy - Difference in Variability of Technologies $\theta_2 > \theta_1$

As expected, a greater proportion of firms in industry two patent. The change in labor efficiency dispersion has only minor effects on the number of rivals and the expected markup. However, as mentioned earlier, these minor changes have compounding effects on the expected profits of the innovating firm, leading more firms to patent.

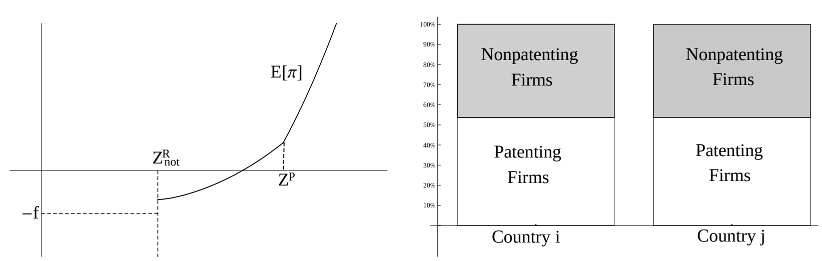
Based on these simulations, industry differences play a key role in determining which types of firms patent. The model predicts that firms in industries with high elasticities and lower variability in labor efficiencies will have a higher proportion of firms who patent. Exploiting these industry differences will be one of the tools used in the empirical portion of the paper. I now turn to the open-economy scenario to look at how country-level differences for the same industry can influence the patterns of patenting.

3.2 Open Economy

This section simulates results in a single-industry open economy scenario where countries can trade and exchange patents. For the baseline scenario, I again assume symmetry and look at firm and rival behavior when an innovating firm from country i operates in country j and vice versa. I use the same parameter estimates as the baseline scenario in the closed economy where the technology levels in both countries are one, $\sigma = 5$, $\theta = 8.28$ and the number of potential rivals in each economy is 20. I also introduce trade costs of 10% and once again assume that patenting doubles the fixed overhead cost of rival firms. Note that because of the addition of trade costs, innovating firms will require a higher z to break-even. Since the patenting cutoff condition essentially remains unchanged, this means that a higher proportion of exporting firms will patent than domestic-only firms in the closed economy. Below is the baseline scenario for the open economy.



a.) Number of rivals b.) Expected Markup



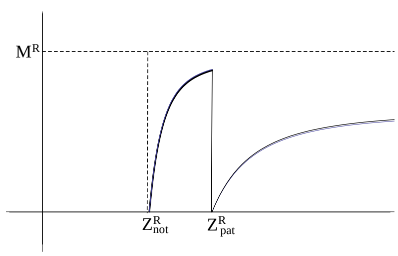
c.) Firm Profits d.) Composition of Firms

Figure 7: Open Economy - Baseline Example

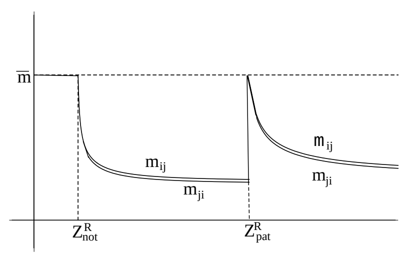
In the baseline scenario, a little more than 50% of country i firms patent in country j and vice versa. There is also the same pattern of rivals, markups and profits that is found in the baseline scenario of the closed economy. I now introduce some country level heterogeneity and examine how this impacts each the firms within each industry's decision to patent.

3.2.1 Differences in Technology

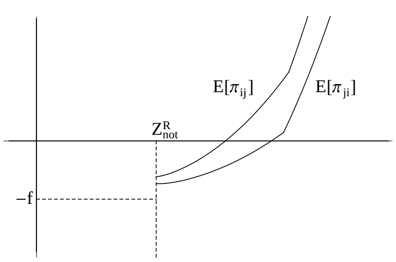
For the first simulation in the open economy, I assume that the two countries have different technology levels where $T_i > T_j$ so that country i is more technologically advanced than country j. This means that all firms in country i (both innovating and rival firms) draw their productivities from a distribution with higher average efficiency levels. As Figure 1 and Result 1 show, this will have big implications on the composition of competitors that firms from both countries face. Innovating firms from country j who export to country i will face more rivals whose efficiency levels are more likely to be close to the innovating firm's. On the other hand, innovating firms from country i who operate in country j will face fewer actual rivals, whose productive capabilities will be relatively lower. Therefore, one would expect that more firms from country j to patent in country i, than vice versa. The results are below.



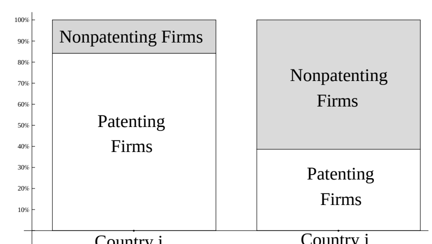
a.) Number of rivals



b.) Expected Markup



c.) Firm Profits



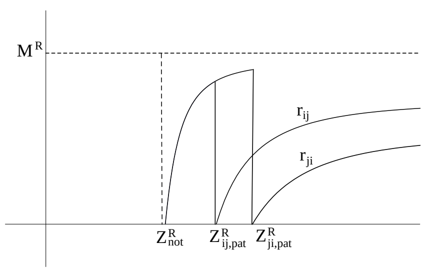
d.) Composition of Firms

Figure 8: Open-Economy Results where $T_i > T_j$

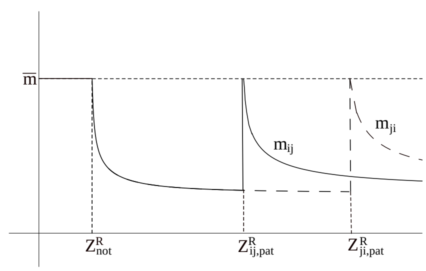
The results are quite dramatic. Almost all firms from country j elect to patent in country i, while a lower proportion of firms from country i patent in country j. The differences in technology have only modest effects on the number of rivals each firm faces, but more pronounced effects on the the expected markup. The next scenario looks at what happens when countries differ in the amount of protection a patent provides.

3.2.2 Differences in Patent Protection

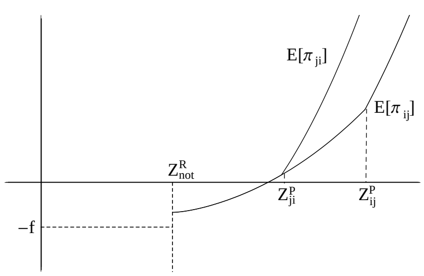
In this section, I revert back to the baseline scenario and set technology levels equal. However, I now assume that the patent protection in country i is greater than the patent protection in country j so that rivals in country i have to pay a higher overhead cost to invent around the patent than the rivals in country j ($f^R_{i,pat} > f^R_{j,pat}$). In this case, I assume that patenting triples the fixed overhead cost that rivals must pay in country i, while leaving patent protection the same in country j. This should have no impact on the number of rivals before patenting, nor should it affect the number of firms who choose to export or pre-patenting profits. However, the returns to patenting are much higher in country i so that more firms from country j would be expected to patent in country i. The results from the simulation are below.



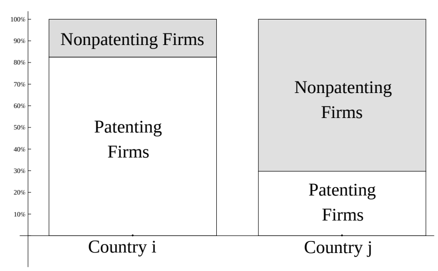
a.) Number of rivals



b.) Expected Markup



c.) Firm Profits



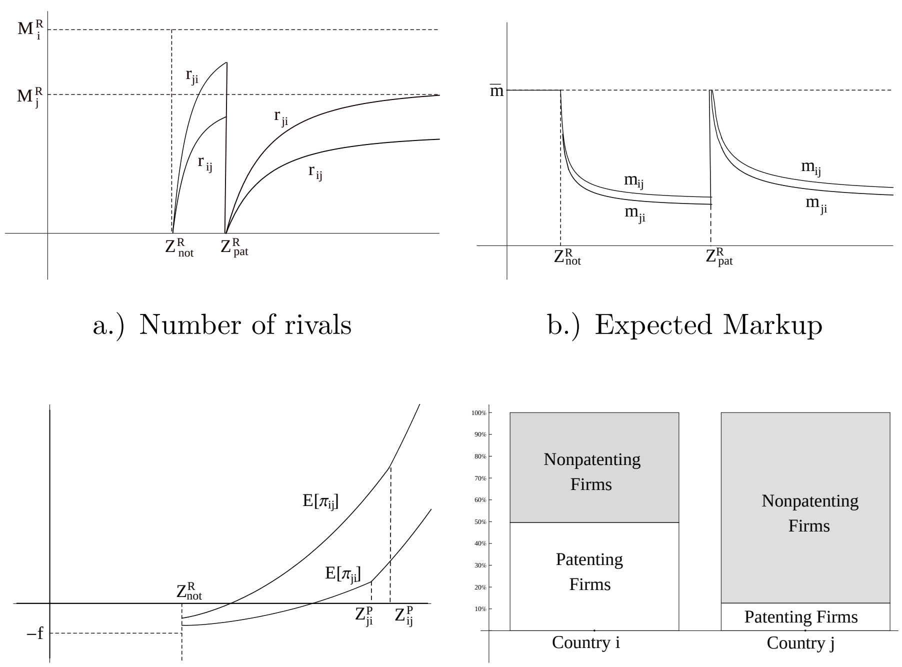
d.) Composition of Firms

Figure 9: Open-Economy Results where $f^R_{i,pat} > f^R_{j,pat}$

Unsurprisingly, a much larger percentage of firms from country j operating in country i elect to patent. We can conclude that countries with a greater degree of patent protection can expect both a greater proportion of firms to patent there, as well as an increase in the absolute number of firms who patent. The last simulation looks at differences in the number of potential rivals M^R.

3.2.3 Differences in Number of Potential Rivals

In this scenario, I set the parameters equal between the two countries but assume that the mass of potential rivals in country i is greater than the mass of potential rivals in country j[13]. Note that this does not impact either of the rival firms' or innovating firm's entry conditions, but only leads to an absolute increase in the number of rivals, which will reduce the expected mark-up, as well as profits for those firms operating in country i. Specifically, I keep r_{ij} at 20, but reduce r_{ji} to 15. Below are the results.



a.) Number of rivals b.) Expected Markup

c.) Firm Profits d.) Composition of Firms

Figure 10: Open-Economy Results where $M_i^R > M_n^R$

As expected, the decrease in rival competition in country j leads to firms patent much less frequently in country j.

To conclude, the simulations tell us that countries that are technologically advanced, have good patent protection and lots of competition from rivals experience a higher proportion of patenting firms. Similarly,

[13]Equivalently, since rivals are proportional to the market size, this is stating that the market size for country i is greater than country j

industries with high elasticities of substitution and less variance in labor efficiencies will have a higher proportion of patenting firms. These properties provide testable implications that can be verified using patent data.

4 Data and Estimation Strategy

The model predicts that country-level and industry-level differences all play a role in determining which firms patent and where. The objective of the empirical section is to apply actual patent data to the model and back out some of these determinants. I start by first deriving country-level measures of patent protection. I use destination fixed effects and normalize the measure of patent protection to be the reduction in the number of actual rivals that innovating firms face so that if the number of rivals an innovating firm faces after patenting decreases by 100%, then the country has an IPR rating of 1 (100%). Unlike other Ricardian trade models, this specification does not incorporate any trade flows. It merely considers market size, distance, wages and technology states as being the key determinants of whether firms patent.

Using the formulas for productivity distributions and substituting destination country fixed effects, I run a non-linear least squares estimation on the following measure of bilateral patent flows M_{ij} (where M_i is the number of domestic patents taken out in country i):

$$\ln M_{ij} = \ln M_i - K_j T_i X_j^{\frac{\theta}{\sigma-1}} w_i^{-\theta} \left(\frac{T_i w_j^{\theta}}{\delta_j T_n w_i^{\theta} \left(\bar{m}^{\theta} - 1\right) + T_i w_j^{\theta}} - \frac{T_i w_j^{\theta}}{T_j w_i^{\theta} \left(\bar{m}^{\theta} - 1\right) + T_i w_j^{\theta}} \right)^{\frac{\theta}{\sigma-1}}$$

where K_j is a destination country-time fixed effect[14] and δ_j is the country-level measure of IPR (defined as $\delta_j = r_{j,pat}/r_{j,not}$). In order to have simplified the expression, I assumed trade costs of 5% for all bilateral pairs. The other parameters of the estimating equation come from the following:

[14]Defined as $K_j = \left(\frac{f_j^P}{A - \left(m_{ij}^B\right)^{1-\sigma} + \left(m_{ij}^B\right)^{-\sigma}} \right)^{\frac{1}{\sigma-1}} P_j^{\theta}$

Table 1: Parameter Values and Data Used

Parameter	Values/Sources
M_i^I, M_{ij}^P	PATSAT
σ	5
θ	8.28
T_i	Fieler (2007)
X_j, w_i	World Bank

For bilateral patent flows, I use a special subset of the PATSTAT database compiled by WIPO and the EPO. The subset consists of all patent families, or the patents for a single invention applied for over multiple jurisdictions. This patent family database is comprehensive and measures bilateral patent flows for more than 64 destination countries between the years 1996 and 2005. This sample reduces to 28 when I remove countries with too few nonzero observations and member countries in the EPO[15]. The reason for doing this is that multinational firms can apply for a single patent through the EPO and receive blanket IPR coverage across all of it's member countries. Therefore, the country-level estimates of IPR for member countries of the EPO are going to be significantly underestimated since much fewer firms opt for the single-country scenario. There are other regional patent agreements that exist that are similar to the EPO such as ARIPO (African Regional Intellectual Property Organization), but no member countries of these regional agreements were found in the data.

Finally, I compare my results to another country-level IPR index provided by Park (2008) whose values are normalized to 1. This comparison is useful for a couple of reasons. The first is that it provides a benchmark for the IPR estimates from NLLS to compare against. The second purpose is that because index compiled by Park is based on the interpretation of the patent law and environment for the country[16], while the IPR figures derived in this paper are based on the actual outcomes. I will argue that the differences between the two measures can point to whether countries are patenting to little or too much abroad. This

[15]Member countries that were excluded from the data are: Austria (AUT), Belgium (BEL), Bulgaria (BGR), Switzerland (CHE), Cyprus (CYP), Czech Republic (CZE), Germany (DEU), Denmark (DNK), Estonia (EST), Spain (ESP), Finland (FIN), France (FRA), Great Britain (GBR), Greece (GRC), Hungary (HUN), Ireland (IRL), Iceland (ISL), Italy (ITA), Lithuania (LTU), Luxembourg (LUX), Latvia (LVA), Netherlands (NLD), Norway (NOR), Portugal (PRT), Sweden (SWE), Slovenia (SVN), Slovakia (SVK), Turkey (TUR)

[16]Specifically, the values are compiled using five separate criteria: coverage, membership in international treaties, duration of protection, enforcement and restrictions

is particularly helpful since many of the countries surveyed in this estimate are developing countries.

Table 2: Estimates of Country IPR Protection (NLLS)

Country	Country Code	Estimated IPR (NLLS)	IPR (Park)	Difference
Argentina	ARG	0.332	0.758	-0.426
Australia	AUS	0.584	0.833	-0.249
Bosnia And Herzegovina	BIH	0.419	N/A	N/A
Brazil	BRA	0.412	0.648	-0.236
Canada	CAN	0.495	0.924	-0.429
China	CHN	0.554	0.653	-0.099
Costa Rica	CRI	0.394	0.537	-0.143
Algeria	DZA	0.686	0.604	0.082
Egypt	EGY	0.272	0.426	-0.154
Hong Kong	HKG	0.286	0.736	-0.45
Croatia	HRV	0.701	N/A	N/A
Indonesia	IDN	0.614	0.486	0.128
India	IND	0.595	0.514	0.081
Israel	ISR	0.255	0.797	-0.542
Japan	JPN	0.817	0.927	-0.11
Korea, Republic Of South	KOR	0.505	0.833	-0.328
Morocco	MAR	0.593	0.602	-0.009
Mexico	MEX	0.400	0.733	-0.333
New Zealand	NZL	0.733	0.802	-0.069
Philippines	PHL	0.368	0.766	-0.398
Poland	POL	0.661	0.790	-0.129
Russian Federation	RUS	0.683	0.730	-0.047
Singapore	SGP	0.524	0.812	-0.288
Tajikistan	TJK	0.718	N/A	N/A
Ukraine	UKR	0.668	0.735	-0.067
Uruguay	URY	0.524	0.626	-0.102
United States	USA	0.961	0.975	-0.014
South Africa	ZAF	0.643	0.826	-0.183
Mean		0.550	0.723	-0.173
Correlation				0.358

The results show that for most countries, the estimates for IPR based on actual patent flows is lower than the measures used in Park (2008). Figure 10 below plots the results with the IPR measures from Park (2008) and a 45-degree line to show how many of the new results tend to systematically underestimate Park's IPR measures.

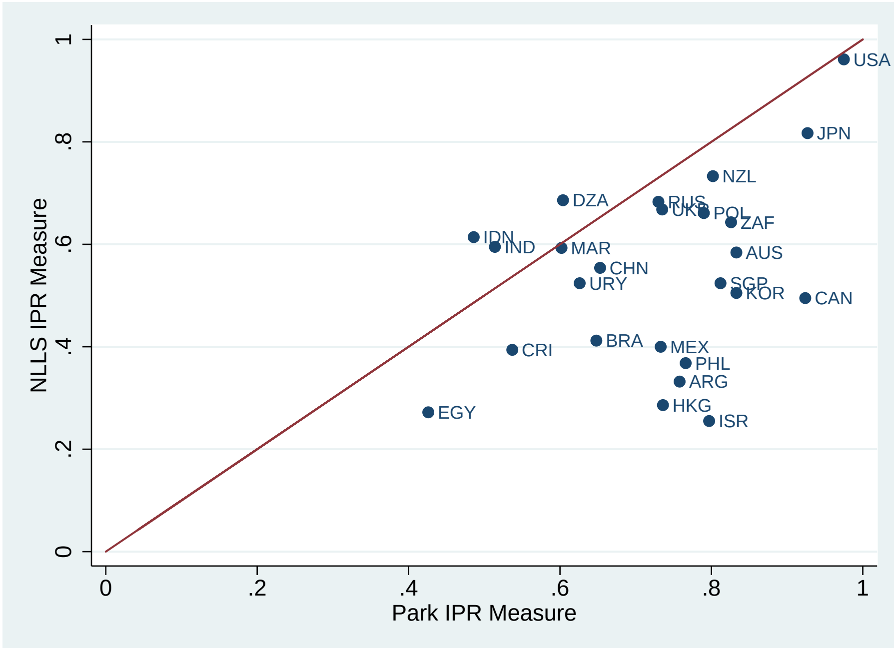


Figure 11: IPR Estimate Comparison

This graph highlights some suggestive evidence that not enough international patenting is taking place abroad, since otherwise the market-based IPR measures would not systematically underestimate the Park index. It may be the case that firms are purposefully withholding patents for some unknown reason. This has potential implications for whether Article 66.2 of the TRIPS agreement, which states that wealthy countries should provide incentives for firms to transfer technology to developing countries. There has been vigorous debate as to whether or not wealthy countries are fulfilling their end of the bargain. While patents alone do not constitute specific technology transfer, patenting is typically seen as a precursor for future diffusion through other investments. Given that the specification is generalized and does not take into account industry-specific factors or bilateral trade agreements, there is still more work to be done to investigate whether or not this is actually the case. Nevertheless, the model provides a framework for future analysis of this type which can provide additional insight as to how IP should be measured abroad.

4.1 Reduced Form Estimation

Next, I want to test whether the patent data itself holds to some of the predictions of the model, mainly looking at whether the country-specific and industry-specific factors identified in the model influence patenting in the way predicted. The model has numerous predictions regarding the proportion of patents flowing to countries based on their market size, technology base, IP protection, trade costs and other industry factors such as elasticity substitution and labor variability. Currently, there exists data to test almost all of the predictions, with the exception of the number of rivals (which may be imputed from the market size of the destination country) and labor variability (industry fixed effects are used instead)[17]. To run the test, I use a logit regression where the dependent variable is the proportion of patents by origin country-industry flowing to the destination country. I also incorporate a newly available industry-technology crosswalk from Lybbert and Zolas (2014) which allows me to incorporate industry-specific attributes to patent flows. The basic reduced form of the estimating equation is:

$$\frac{PAT_{ijkt}}{PAT_{ikt}} = \alpha + \delta_1 \ln(GDP_{jt}) + \delta_2 \ln(IPP_{jt}) + \delta_3 TECH_{jt} + \delta TRADE_{ij} + \delta_5 EPO_{jt} + \delta_4 \sigma_k + \alpha_t + \alpha_k + \varepsilon_{ijkt} \quad (18)$$

Where i is the origin country, j is the destination country, k is the industry (as measured by 4-digit SITC Rev. 2) and t is the year. For GDP, IPR and $TECH$, I use the same data as in the previous exercise and use the Park IPR index since the coverage is better. The variable $TRADE$ includes the numerous trade costs between countries i and j such as distance, whether they share a border, language dummies and trade agreements. I also include a dummy variable for whether the destination country is a member of the EPO since countries are more likely to patent through the EPO than individual member nations. I include industry measures such as the elasticity of substitution, σ, which is gathered from Broda and Weinstein (2006). In addition to this industry-specific measure, I also include the Rauch classification (Rauch 1999) which classifies goods whether they are priced on an organized exchange, referenced priced or are differentiated products. Finally, I include year fixed-effects, as well as two-digit industry fixed effects.

For the patent data, I use the same PATSTAT database of patent family flows and expand the number of destination countries to the full sample. The patent flows are organized by the International Patent Classification (IPC) system, which classifies technologies. To convert this classification into industry classifications, I use the technology-industry concordance from Lybbert and Zolas (2014) which converts 4-digit IPCs into 4-digit Standard International Trade Classification (SITC) Revision 2. This concordance uses keyword extraction algorithms to read through patents, collect the relevant keywords and then match them with the industry descriptions. Once the patents are concorded, I am left with country-industry-year patent totals and flows. My initial frame began with 131 countries, 1189 four-digit industries over the period 1996-2005, resulting in more than 200 million possible observations. However, due to the fact that very few countries patent in all industries (or at all), my unbalanced panel contains roughly 18 million observations. I also run the regression using a "balanced" panel which I define as having nonzero values of PAT_{ikt} for all

[17]It may be possible to estimate these missing variables with a structural estimation, but that will be left for a future paper

of the years in the sample. There are approximately 11 million observations in the balanced panel. Table 3 below shows the results from the unbalanced and balanced estimation.

Table 3: Logit Regression of Proportions of Patents Flowing to Each Country

Dependent Variable: Proportion of Country-Industry Patent Flows

Explanatory Variable	(1)	Unbalanced (2)	(3)	(4)	Balanced (5)	(6)
log Destination GDP	0.634***	0.670***	0.775***	0.706***	0.735***	0.816***
	(0.000806)	(0.000829)	(0.000936)	(0.000927)	(0.000949)	(0.00104)
Destination IPR	0.318***	0.250***	0.349***	0.362***	0.301***	0.380***
	(0.00212)	(0.00222)	(0.00237)	(0.00230)	(0.00240)	(0.00254)
log Destination Tech.	0.0198***	0.0182***	0.0218***	0.0228***	0.0207***	0.0245***
	(0.000149)	(0.000158)	(0.000167)	(0.000161)	(0.000168)	(0.000177)
log Distance	-0.276***	0.0184***	-0.163***	-0.251***	-0.000442	-0.152***
	(0.00118)	(0.00153)	(0.00192)	(0.00133)	(0.00177)	(0.00212)
Border Dummy	1.417***	0.805***	0.656***	1.209***	0.653***	0.611***
	(0.00393)	(0.00475)	(0.00617)	(0.00462)	(0.00541)	(0.00680)
EPO Member	-0.743***	-0.728***	-0.917***	-0.833***	-0.783***	-0.979***
Dummy	(0.00291)	(0.00312)	(0.00350)	(0.00345)	(0.00361)	(0.00389)
Sigma	0.0144***	0.0155***	0.0213***	0.0186***	0.0198***	0.0143***
	(0.00140)	(0.00143)	(0.00154)	(0.00159)	(0.00161)	(0.00170)
Common Language		0.400***	0.180***		0.426***	0.136***
Dummy		(0.00302)	(0.00353)		(0.00342)	(0.00385)
Trade Agreement		0.796***	0.354***		0.628***	0.380***
Dummy		(0.00321)	(0.00409)		(0.00385)	(0.00453)
Differentiated Good		0.0693***	0.0737***		0.0878***	0.123***
Dummy		(0.00585)	(0.00639)		(0.00663)	(0.00711)
Reference Priced		-0.0242***	-0.0659***		0.0355***	0.0288***
Dummy		(0.00541)	(0.00593)		(0.00614)	(0.00660)
Organized Exchange Dummy	Dropped	Dropped	Dropped	Dropped	Dropped	Dropped
log Origin GDP			0.473***			0.412***
			(0.000881)			(0.000992)
log Origin Tech.			0.0670***			0.0675***
			(0.000174)			(0.000201)
2-digit Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Constant	-16.62***	-20.04***	-33.16***	-19.68***	-22.08***	-36.95***
	(0.0311)	(0.0335)	(0.0460)	(0.0472)	(0.0441)	(0.0645)
Observations	18709711	18546196	17138773	11634504	11522934	10981974
Pseudo R-squared	0.301	0.286	0.402	0.336	0.321	0.400

Robust standard errors are in parentheses.

Regression disturbance terms are clustered at the destination country level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results confirm all of the predictions of the model. Namely that larger, more technologically sophisticated markets attract a greater proportion of patents. At the industry-level, we also see that products that are more elastic are also more likely to be patented. In addition, other trade factors play a significant role in the proportion of patents mainly due to increased trade flows. I find that both distance, border dummies, language and trade agreements play a significant role in determining which patents get sent where.

Run IV Using Trade Flows with Gravity Specification

The regression is also quite accurate given the low resolution of the industry fixed effects and lack of country, country-industry pair or country-pair fixed effects. To get a sense of the actual impact of these coefficients, I calculate the marginal effect of each variable (i.e. elasticity) based on the coefficients from column (3) and column (6). These results are found below:

Table 4: Estimated Elasticities of Coefficients

Explanatory Variable	Unbalanced (1)	Balanced (2)
Destination GDP	0.0368***	0.0510***
	0.000043	0.0000593
Destination IPR	0.0102***	0.0106***
	0.000069527	0.000070878
Destination Technology	0.0010***	0.0015***
	0.00000794	0.0000111
Distance	-0.007***	-0.009***
	0.0000912	0.0001328
EPO Dummy	-0.043***	-0.061***
	0.0001623	0.0002351
Language	0.0085***	0.0084***
	0.0001676	0.0002403
Trade Agreement	0.0168***	0.0237***
	0.0001945	0.0002831
Sigma	0.0010***	0.0008***
	0.000073	0.0001063

Robust standard errors are in parentheses.

Regression disturbance terms are clustered at the destination country level

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The elasticities indicate that patenting is most sensitive to the destination country's GDP. This may be due to a variety of reasons, the most obvious being the larger market size to sell the goods and increased competition. Unfortunately, it is not possible to measure the number of competitors each firm faces in other countries, but it seems clear that competition, whether domestic or foreign, plays a significant role in determining patenting outcomes. Among the continuous variables, IP protection appears to be the next most sensitive factor determining international patenting, which is unsurprising. The technology level is also significant, but less sensitive. Among the dummy variables, being a member of the EPO will lead to a 4-6% drop in the proportion of patents entering individual member countries. Signing a trade agreement leads to a 1-2% increase in the exchange of patents between the agreeing countries. Finally, sharing a language only leads to a modest increase in patent exchange. To map the full effect, I include charts showing the predicted patenting propensity based on log destination GDP, log technology, IPR and log sigma.

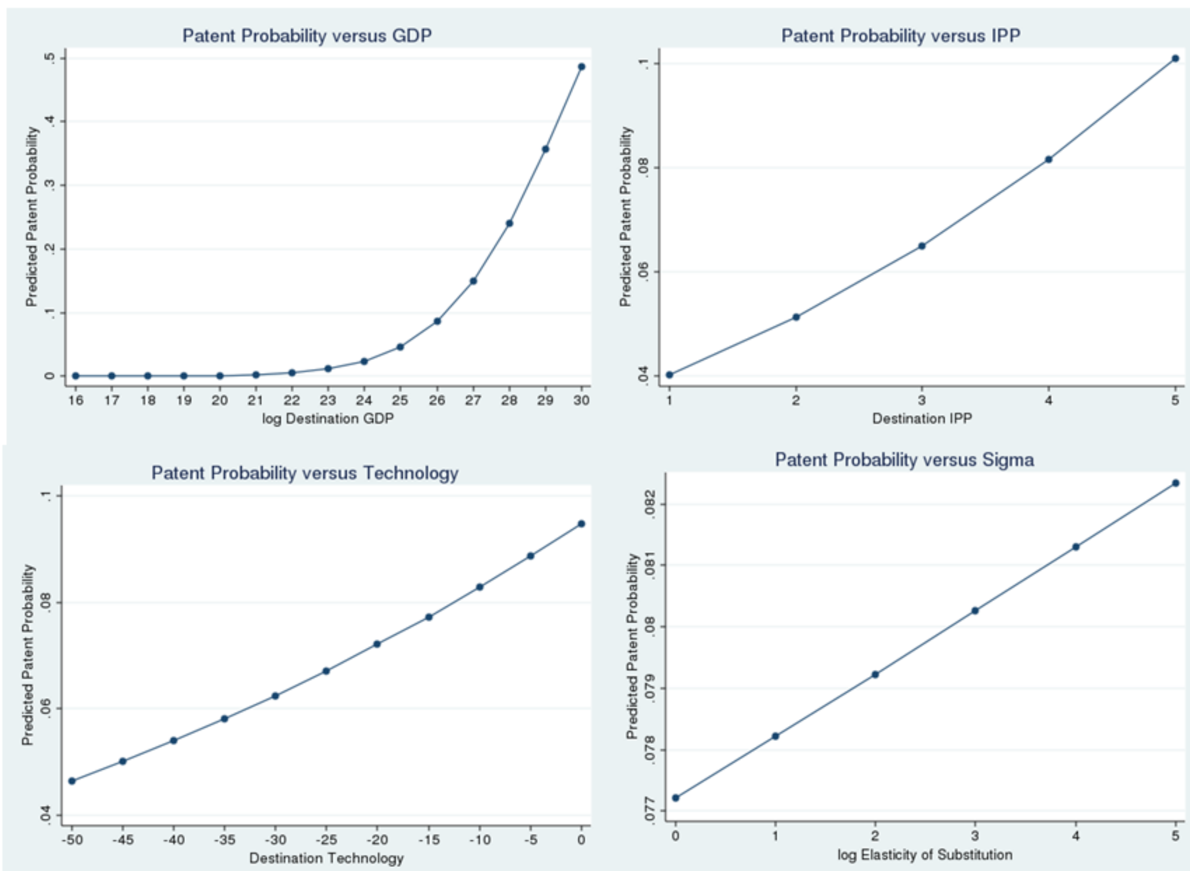


Figure 11: Patenting Propensity

Figure 11 shows several interesting properties. Namely, destination GDP appears to not influence patenting until the destination GDP becomes sufficiently large. From that point, it appears that the destination country becomes a "core" country where patenting essentially becomes automatic. As for the other variables, the benefits to increasing them appear to be linear. Technology appears to plays a large role, with the most technologically sophisticated countries attaining a nearly 10% increase in patenting over the least technologically sophisticated. Destination IPR also plays an important role with the highest levels of intellectual property protection leading to a 10% increase in patenting. Finally, we see that the elasticity of substitution also is important, but has only minor effects.

As an additional exercise, I also wanted to map whether the conditions to patent abroad have changed or become more/less sensitive over time. To do this study, I run the same logit regression for each individual year from 1996 to 2005 and then chart the values of the coefficients for each variable below. Note that all of the variables are significant at the 0.1% confidence and the flat red line in the charts is the coefficient from column (3) in Table 3.

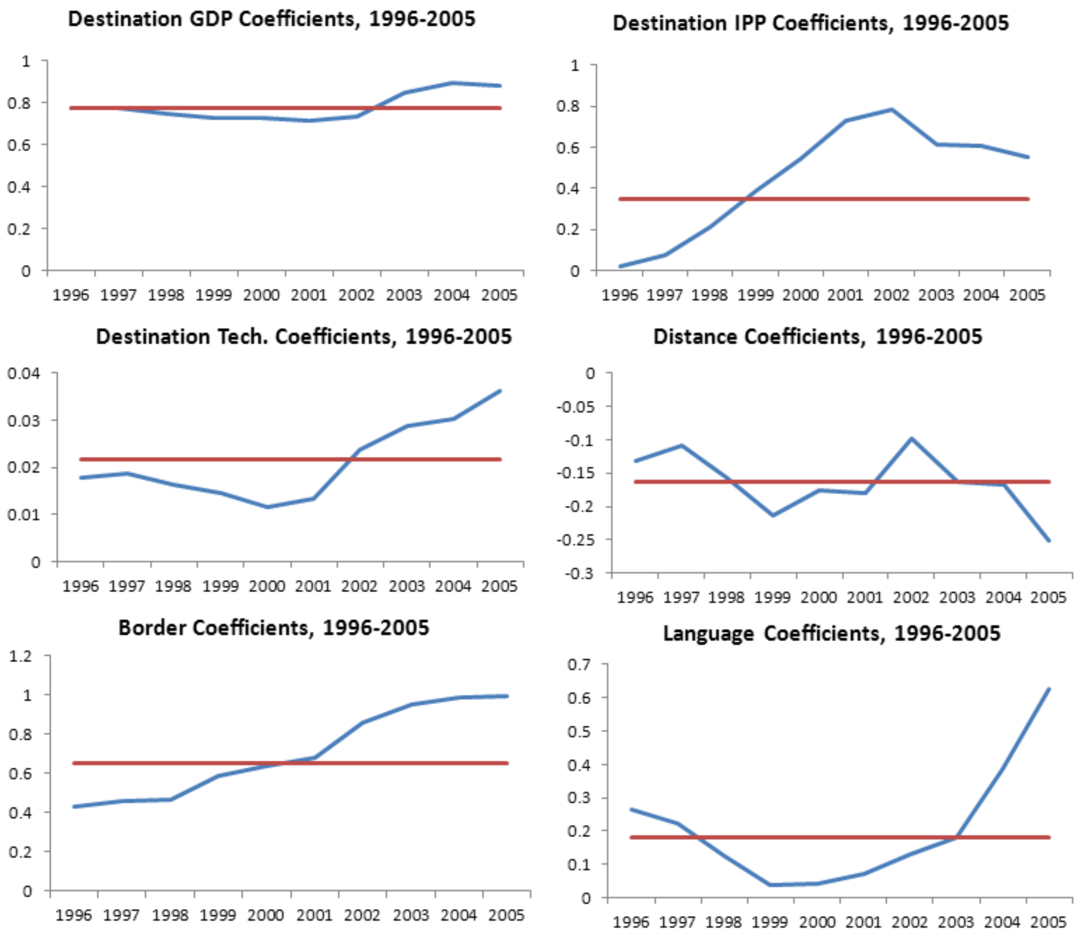


Figure 12: Change in Coefficients over Time

The figure indicates that the destination country's IPR measure has taken an increasingly significant role in determining patenting outcomes beginning in 1999. This implies that companies are placing more and more importance on the intellectual property environment of a country, while still considering other factors more or less equally. Firms are more sensitive to the IP environment which means that as countries continue to improve their IPR, then we can be reasonably confident that firms from abroad will respond positively to these improvements. On the other hand, this also implies that countries that do not make the necessary improvements can expect to see fewer and fewer patents coming their way. Technological sophistication is also playing a more important role in determining patent flows. This may be a result of the desire for increased specialization. Finally, we see that border effects and language are also becoming somewhat more relevant in today's decision to patent.

5 Conclusion

The goal of this paper was to better understand how multinational firms decide whether and where to seek international patent protection. These decisions are shown to have critical implications for future investment, technology diffusion and economic growth, especially for developing countries who linger outside of the patent core. This paper proposes a new type of patenting decision model that borrows elements from the heterogeneous firm trade literature and can explain significant portions of spatial patenting patterns. The model explains why countries with higher levels of technology, better patent protection and more competition are able to solicit a greater number of patents. Using a generalized version of the patenting cutoff condition, I was able to compile IPR measures for almost 30 countries of various size and income over the ten-year period from 1996-2005 using patent family data and parameter estimates from previous trade models. These IPR measures take into account the actual patent flows to each country and when matched with alternative IPR indices, strongly suggest that there is not enough patent transfer taking place, which has important implications for Article 66.2 of the TRIPs agreement.

In addition, a logit regression testing the model's properties was run using country-industry patent flows occurring between the years 1996 to 2005. The estimation confirmed the model's predictions and also provided measures for the effect of each factor on a firm's propensity to patent. Firms consider the destination's market size to be the most important factor in determining whether or not to patent, followed by the country's IP environment and technological sophistication. In terms of policy, countries with the highest level of IPR can expect to attract 10% more patents than countries who do not value IP protection. Also, of interest, it appears that the destination country's IP environment is taking an increasingly important role in the decision to patent abroad, which implies that as countries continue to make improvements to their IP, firms are responding accordingly.

Although the model is described in full detail, several properties of the model remain unknown. One of the more interesting aspects that has yet to be explored are the welfare effects that arise from strengthening IPRs. Increased patent protection has been shown to increase the expected profits of innovating firms, but it is not clear whether that leads to more potential entrants/varieties or what the negative effects it has on consumers who must now pay higher markups. It may be the case that the gain in welfare from the availability of new varieties outweighs the welfare loss from higher prices, which is the argument put forth by rich countries in the TRIPS agreement. Analyzing this question will help in addressing whether Article 66.2 of the TRIPS agreement has had a positive or negative impact on developing countries who were forced to make improvements to their IPRs. Another related property to explore would be the impact of trade liberalization on patenting and welfare.

Other possible extensions to the model include allowing foreign entry of rivals and incorporating an innovation component. Under the current framework, all of the potential rivals are local. Given the assumptions on the productivity constraints of rivals, it makes little sense to include foreign rivals since they would have to pay for the additional trade costs, making it unlikely that they would ever become the low-cost rival. On the other hand, by making the number of potential rivals in each country proportional to market size, I leave open the possibility of foreign entry (similarly based on market size). Including foreign rivals would add robustness to the model since in many cases, multinational firms use patents as a deterrence and blocking device for outside competitors trying to gain access to a particular market. Second, although the model includes innovating firms, there is no decision variable for innovation. It is certainly possible to include this component, since the profits for innovating firms are well-defined and it would be interesting to see how rivals, patent protection and country variables impact this decision.

Outside of the theoretical extensions, many empirical extensions can be made. A more robust parameterization of the model may be possible using the country-industry level data. For instance, it may be possible to obtain country technology measures for different industries using the patent data, which would then be used to analyze the effects of investments in key industries and follow these investments over time. This would be a similar-type analysis to Shikher (2004), but while using patent data instead of trade and expand into more industries and include more developing countries.

Modeling international patent flows is an important step in understanding the process of technology diffusion and the transfer of knowledge abroad. The policy implications provided by the model are suggestive and rather broad. Improving country-level technological ability is rather difficult and requires a multi-pronged approach with investments in many different sectors of the economy. Similarly, improvement in the competitive environment also requires coordination among a number of different sectors. Many developing countries have improved upon their intellectual property protection and this has shown to be increasingly effective, but it is not clear whether firms will continue to respond positively to these changes. There also appears to be a trade-off between intellectual property rights and developing industrial capacity (Falvey et al. (2006)). Nevertheless, the model provides a testable framework for international patenting decisions and may lead to more policy in the future for developing more effective IPR regimes.

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A Mathematical Appendix

A.1 Proofs

Result 1: The number of rivals $r_{ij}(\omega)$ and their average efficiency increases as the state of technology T_j increases.

Proof: First, I show that the number of rivals increases as with T_j:

$$\frac{\partial r_{ij}}{\partial T_j} = w_j^{-\theta} Y_j^R \left[\left(\tilde{c}_{ij}^R\right)^{\theta^k} - \left(c_{ij}^I\right)^{\theta} \right] e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta}\right]} > 0$$

Next, I show that the productivity of the rivals increases with technology T_j. I do this by showing that for any given cost parameter $c_{ij}^I \leq c'$, the probability that c is less than or equal to c' is increasing with T_j:

$$\frac{\partial \Pr\left[c \leq c' | c_{ij}^I\right]}{\partial T_j} = w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right] e^{-T_j^k w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta}\right]} > 0$$

■

Result 2: The number of rivals $r_{ij}(\omega)$ increases with the cutoff condition of rival entry $\tilde{c}_{ij}^R$.

Proof:

$$\frac{\partial r_i j}{\partial \tilde{c}_{ij}^R} = Y_j^R T_j w_j^{-\theta} \theta \left(\tilde{c}_{ij}^R\right)^{\theta - 1} e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta}\right]} > 0$$

■

Result 3: The number of rivals $r_{ij}(\omega)$ and their productivity increases with the productivity parameter of the innovating firm.

Proof: I start by first showing that the number of rivals increases as the cost parameter for the innovating firm declines:

$$\frac{\partial r_{ij}}{\partial c_{ij}^I} = -Y_j^R T_j w_j^{-\theta} \theta \left(c_{ij}^I\right)^{\theta - 1} e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta}\right]} < 0$$

So that as c_{ij}^I declines (i.e. the innovating firm is more productive), the number of rivals increases. the Next, I show that for any given productivity $c_{ij}^I \leq c'$, the probability that c is less than or equal to c' is decreasing with c_{ij}^I, meaning that as c_{ij}^I decreases, it is more likely for c to be less than c':

$$\frac{\partial \Pr\left[z \geq z' | z^I\right]}{\partial z^I} = -T_j w_j^{-\theta} \theta \left(c_{ij}^I\right)^{\theta - 1} e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta}\right]} < 0$$

So that the distribution of rival costs when the innovating firm has very low c^I_{ij}, first-order stochastically dominates the distribution with higher c^I_{ij} implying that the expected value of the rivals' productivity is increasing in z^I_{ij}. ■

Result 4: The probability the innovating firm charges the CES markup in country j is decreasing in contestability r_{ij}.

Proof:

$$\frac{\partial \phi_{ij}}{\partial r_{ij}} = \frac{-T_i T_j^{\theta} \left(w_i d_{ij} w_j\right)^{\theta} \left(\overline{m}^{\theta} - 1\right)}{\left[r_{ij} T_j \left(w_i d_{ij}\right)^{\theta} \left(\overline{m}^{\theta} - 1\right) + T_i w_j^{\theta}\right]^2} < 0$$

■

Result 5: The probability the innovating firm charges the CES markup in country j increases as the cutoff condition for rival entry decreases and decreases as the innovating firm becomes more productive (lower costs).

Proof:

$$\frac{\partial \phi_{ij}}{\partial \tilde{c}^R_{ij}} = \underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)} \underbrace{\frac{\partial r_{ij}}{\partial \tilde{c}^R_{ij}}}_{(-)} > 0 \qquad \text{and} \qquad \frac{\partial \phi_{ij}}{\partial c^I_{ij}} = \underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)} \underbrace{\frac{\partial r_{ij}}{\partial c^E_{ij}}}_{(+)} < 0$$

■

Result 6: The innovating firm's expected profit $\mathrm{E}\left[\pi^I_{ij}(\omega)\right]$ is decreasing in contestability r_{ij}.

Proof:

$$\frac{\partial \pi^I_{ij}}{\partial r_{ij}} = \left(\frac{c^I_{ij}}{P_j}\right)^{1-\sigma} \alpha_j w_j Lj \left[\underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} + \underbrace{\frac{\partial \left(1-\phi_{ij}\right)}{\partial r_{ij}}}_{(+)} \underbrace{\frac{\partial \left(\left(\bar{m}^B_{ij}\right)^{1-\sigma}\left(1-\left(\bar{m}^B_{ij}\right)^{-1}\right)\right)}{\partial r_{ij}}}_{??}\right]$$

So that the sign is going to depend on how the expected markup under Bertrand competition $\bar{m}^B_{ij}$ changes with r_{ij}. I show that for any given markup $1 \leq m' \leq \overline{m}$, the probability that m is great than or equal to

m' decreases as r_{ij} increases.

$$\frac{\partial \Pr\left[m \geq m'\right]}{\partial r_{ij}} = \frac{-T_i T_j^{\theta}\left(w_i d_{ij} w_j\right)^{\theta}\left(\left(m'\right)^{\theta}-1\right)}{\left[r_{ij} T_j\left(w_i d_{ij}\right)^{\theta}\left(\left(m'\right)^{\theta}-1\right)+T_i w_j^{\theta}\right]^2} < 0$$

This implies that markup m_{ij}^B with a small amount of rivals r_{ij} first-order stochastically dominates m_{ij} with a high number of rivals r_{ij}, so that $\frac{\partial\left(\left(\bar{m}_{ij}^B\right)^{1-\sigma}\left(1-\left(\bar{m}_{ij}^B\right)^{-1}\right)\right)}{\partial r_{ij}} < 0$, which means that $\frac{\partial \pi_{ij}^I}{\partial r_{ij}} < 0$, thus completing the proof. ■

Result 7: The price of variety ω charged to consumers in country j is decreasing in contestability r_{ij} From the price definition (Equation 9), I first compute the moment $1-\sigma$ for the expected marginal costs:

$$\mathrm{E}\left[\left(c_{ij}^I\right)^{1-\sigma}\right] = \int_0^{\infty}\left(c_{ij}^I\right)^{1-\sigma} g_{ij}\left(c_{ij}^I\right) dc_{ij}^I = \left(T_i\left(w_i d_{ij}\right)^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)$$

And[18]

$$\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right] = \int_{c_{ij}^I}^{\infty}\left(c_{ij}^{R*}\right)^{1-\sigma} g_{ij}^{R*}\left(c_{ij}^{R*}\right) dc_{ij}^{R*} = e^{r_{ij}\frac{T_j}{T_i}\left(\frac{w_i d_{ij}}{w_j}\right)^{\theta}}\left(r_{ij} T_j w_j^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}, r_{ij}\frac{T_j}{T_i}\left(\frac{w_i d_{ij}}{w_j}\right)^{\theta}\right)$$

From this, the proof is relatively straightforward.

Proof:

$$\frac{\partial p_{ij}}{\partial r_{ij}} = \left(\underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)}\overline{m} + \underbrace{\underbrace{\frac{\partial\left(1-\phi_{ij}\right)}{\partial r_{ij}}}_{(+)}\underbrace{\frac{\partial \bar{m}_{ij}^B}{\partial r_{ij}}}_{(-)}}_{(-)}\right) c_{ij}^I < 0$$

[18]Note that after integrating $\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right]$, I have

$$\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right] = \int_{c_{ij}^I}^{\infty}\left(c_{ij}^{R*}\right)^{1-\sigma} g_{ij}^{R*}\left(c_{ij}^{R*}\right) dc_{ij}^{R*} = e^{r_{ij} T_j w_j^{-\theta}\mathrm{E}\left[\left(c_{ij}^I\right)^{\theta}\right]}\left(r_{ij} T_j w_j^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}, r_{ij} T_j w_j^{-\theta}\mathrm{E}\left[\left(c_{ij}^I\right)^{\theta}\right]\right)$$

Next, I substitute the expected value $\mathrm{E}\left[\left(c_{ij}^I\right)^{\theta}\right]$ which is

$$\mathrm{E}\left[\left(c_{ij}^I\right)^{\theta}\right] = \int_0^{\infty}\left(c_{ij}^I\right)^{\theta} g_{ij}\left(c_{ij}^I\right) dc_{ij}^I = \frac{\left(w_i d_{ij}\right)^{\theta}}{T_i}$$

to complete the formula.

■

A.2 Derivation of Markup Distribution

To derive the distribution of the markup m_{in} requires me to look at the distribution of the ratio c_{1n}/c^I_{in}. To calculate this, I will use the methodology in Nadarajah (2010) who use the following Lemma from Prudnikov et al. (1986)

Lemma 1 (Equation (2.3.1.13), Prudnikov et al. (1986), Vol. 1) For $\gamma > 1$, $a > 0$ and $s > 0$

$$\int_0^\infty x^{\gamma-1} e^{\left(-sx-ax^k\right)} dx = I(\gamma, a, k, s)$$

Where

$$I(\gamma, a, k, s) = \begin{cases} \sum_{j=0}^{q-1} \frac{(-a)^n}{j! s^{\gamma+kn}} \Gamma\left(\gamma + kj\right) {}_{p+1}F_q\left(1, \Delta\left(\rho, \gamma + kj\right); \Delta\left(q, 1+j\right); (-1)^q z\right) & 0 < k < 1 \\ \sum_{h=0}^{p-1} \frac{(-s)^h}{kh! a^{(\gamma+h)/k}} \Gamma\left(\frac{\gamma+h}{k}\right) {}_{q+1}F_p\left(1, \Delta\left(q, \frac{\gamma+h}{k}\right); \Delta\left(p, 1+h\right); \frac{(-1)^p}{z}\right) & k > 1 \\ \frac{\Gamma(\gamma)}{(a+s)^\gamma} & k = 1 \end{cases}$$

Where $k = p/q$ and $z = (p^p a^q)/(s^p q^q)$ and $\Delta(v, a) = (a/v, (a+1)/v, ..., (a+v-1)/v)$.

Given the following distributions for c^I_{ij} and c^{R*}_{ij}

$$G^I_{ij}(c^I_{ij}) = 1 - e^{-T_i(w_i d_{ij})^{-\theta}\left(c^I_{ij}\right)^\theta} \qquad \text{and} \qquad G^{R*}_{ij}(c^{R*}_{ij}) = 1 - e^{-r_j T_j w_j^{-\theta}\left(\left(c^{R*}_{ij}\right)^\theta - \left(c^I_{ij}\right)^\theta\right)}$$

Then I can write the CDF of $m_{ij} = c_{ij}^{R*}/c_{ij}^{I}$ as:

$$
\begin{aligned}
H(m_{ij}) &= \int_0^\infty G_{ij}^{R*}(c_{ij}^I m_{ij}) g_{ij}^I(c_{ij}^I) dc_{ij}^I \\
&= \int_0^\infty \left[1 - e^{-r_j T_j w_j^{-\theta}\left(\left(c_{ij}^I m_{ij}\right)^\theta - \left(c_{ij}^I\right)^\theta\right)}\right] \theta T_i \left(w_i d_{ij}\right)^{-\theta} \left(c_{ij}^I\right)^{\theta-1} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} \left[1 - e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta}\right] e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= \int_0^\infty g^I(c_{ij}^I) dc_{ij}^I - \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= 1 - \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I
\end{aligned}
$$

I do the following substitution

$$
\begin{aligned}
x &= r_j T_j w_j^{-\theta} \left(m_{ij}^\theta - 1\right) \left(c_{ij}^I\right)^\theta \\
a &= \frac{T_i \left(w_i d_{ij}\right)^{-\theta}}{r_j T_j w_j^{-\theta} \left(m_{ij}^\theta - 1\right)}
\end{aligned}
$$

I can now rewrite my equation above as

$$
H(m_{ij}) = 1 - a \int_0^\infty e^{-x} e^{-ax} dx
$$

I can now apply Lemma 1 where $\gamma = 1$, $k = 1$ and $s = 1$ so that the CDF for markup m is

$$
H(m_{ij}) = 1 - aI(1, a, 1, 1) = 1 - \frac{a\Gamma(1)}{a+1} = 1 - \frac{a}{a+1} = \frac{1}{a+1} = 1 - \frac{T_i w_j^\theta}{r_j T_j \left(w_i d_{ij}\right)^\theta \left(m_{ij}^\theta - 1\right) + T_i w_j^\theta}
$$

With PDF

$$
h(m_{ij}) = \frac{r_j T_i T_j \theta \left(w_i w_j d_{ij}\right)^\theta m_{ij}^{\theta-1}}{\left[r_j T_j \left(w_i d_{ij}\right)^\theta \left(m_{ij}^\theta - 1\right) + T_i w_j^\theta\right]^2}
$$