Multi-product Firms and Competition: Fight or Flight? *

Luca Macedoni — Aarhus University†
Frederic Warzynski — Aarhus University‡
Rui Zhang — Sun Yat-Sen University§

September 27, 2023

Abstract

We propose a new model of multi-product firms in international trade, where firms select their product categories based on the categories' attractiveness and level of competition. Using Danish manufacturing data, we present two novel stylized facts that demonstrate the importance of product-specific characteristics in understanding firms' product choices. More attractive product categories also feature tougher competition, leading to the emergence of two sorting patterns: one in which only the most productive firms produce the most attractive products and another in which all firms produce the most attractive products. Our model can generate both sorting patterns depending on the value of a key preference parameter. Quantifying our model we find that product-specific differences in attractiveness and competition explain a quarter of the variation in sales. Furthermore, we find that the most attractive products tend to be produced by all firms, while the least attractive products are made only by the most productive firms.

Keywords: Multi-product firms, competition, product attractiveness, sorting.

JEL Code: F12, F14, L11 L25.

*We thank seminar participants at NOITS 2021 and CAED 2021.
†lmacedoni@econ.au.dk. Financial support from the Carlsberg Foundation is gratefully acknowledged.
‡fwa@econ.au.dk.
§zhangr359@mail.sysu.edu.cn.

1 Introduction

The growing availability of of firm-level data from multiple countries has spurred the development of an extensive body of literature that explores diverse aspects of firm-level reactions to demand and competitive shocks. For example, international competition resulting from trade liberalization can lead to changes in firm-level pricing, quality, labor demand, and scope (Bernard et al., 2011; Edmond et al., 2015; De Loecker et al., 2016; Piveteau and Smagghue, 2022; Fieler and Harrison, 2018). Often, shocks manifest at the product level, where shifts in consumer preferences may increase demand for a specific product, or alterations in product-specific tariffs or regulations may modify the competitive landscape for a singular product. Yet, the workhorse trade model of multi-product firms is ill-equipped to deal with these product-level shocks. Standard models cannot accommodate differences in shocks across core and non-core varieties of a firm as the implicit assumption of these models is that shocks are predominantly occurring at the firm-level (Bernard et al., 2011; Mayer et al., 2014, 2021). In this paper, we challenge this assumption and investigate the consequences of incorporating product-specific shocks on firms' product mix decisions and their responses to shifting competitive conditions.

Using Danish manufacturing data, we present two novel stylized facts demonstrating the significance of product-specific characteristics in understanding firm behavior. These product-specific characteristics include factors such as the number of firms producing a given product, or the total market size for that product. Although it is logical to assume that such factors play a role, we offer evidence to support our novel model, highlighting that approximating demand and competitive shocks using only firm-level characteristics does not adequately characterize the data pattern.

In our first stylized fact, we find that core competence and firm-product characteristics only partially explain sales variations across the products within a firm. In fact, we find that core products typically face less competition and higher demand than non-core products, suggesting product-specific attributes significantly impact a firm's product mix decisions. Additionally, we discover that product-specific shocks have an explanatory power equivalent to firm-specific shocks in influencing domestic sales at the firm-product level, challenging the assumption that firm-specific shocks dominate variations in firm-product sales.

Motivated by these stylized facts, we build a multi-country general equilibrium model of multi-product firms, which are heterogeneous in their productivity. Our model diverges from standard multi-product firm models, such as those of Bernard et al. (2011) and Mayer et al. (2014), by including a discrete set of products of which any firm can offer a unique variety. These products can be thought of as product categories or nests. This structure aligns with real-world data where firms produce multiple product codes and different firms manufacture varieties under the same code. For example, a product could be toasters, with several firms each producing their distinct varieties. Another product could be mobile phones, with the same or a different set of firms each creating a unique version.

In this framework, the key factor is how firms are sorted into different product categories - which firms produce a variety of a specific product? For instance, consider the toaster and mobile phone products. Hitachi, Toshiba, Mitsubishi, Panasonic, and Sharp were all active in both product categories, offering their unique varieties. While they continue to produce toaster varieties, they have all exited the mobile phone product category.[1] Our model provides an explanation for this sorting pattern, where numerous firms produce toasters and fewer firms produce mobile phones, even though the latter has a larger market demand.

We assume that product categories differ in their attractiveness, which is modeled as a demand shifter. For example, if the mobile phone category is more attractive than toasters, it implies that, given the same level of competition, firms can generate higher profits by producing mobile phone varieties rather than toaster varieties. This attractiveness can be attributed to differences in demand characteristics (such as a higher willingness to pay for a smartphone compared to a toaster) or unmodeled supply characteristics like R&D fixed costs. Firms are drawn to products with higher attractiveness, which in turn leads to increased competition in those categories. Consequently, a firm's profitability in a product depends on the extent to which competition intensifies with product attractiveness. There are two possible sorting patterns. First, if greater attractiveness results in stiffer competition, only the most productive firms will produce the most attractive products (e.g., mobile phones), and all firms will produce the least attractive products (e.g., toasters). Second, if competition does not escalate significantly as attractiveness increases, all

[1] See https://www.ft.com/content/f2876863-6f59-487e-8e9e-95ce6f82b0b6.

firms will produce the most attractive products, while only the most productive firms will manufacture the least attractive ones.

Our model can generate both sorting patterns because it employs a general demand function. Demand is generated by the Generalized Translated Power (GTP) preferences proposed by Bertoletti and Etro (2017). These preferences encompass various common types used in the literature, such as indirectly additive Bertoletti et al. (2018), directly additive Melitz and Ottaviano (2008), or homothetic preferences. With different preferences, firms face varying demand elasticities and, specifically, the demand elasticity and markup fluctuate in response to the number of firms. When markups are highly sensitive to changes in competition, as observed with directly additive and homothetic preferences, only the most productive firms offer varieties of the most attractive products, while the least productive firms are confined to less attractive products.[2] Conversely, when markups are less responsive to competition, as in the case of indirectly additive preferences, the opposite sorting pattern emerges.

The sorting pattern that arises also relates to the sign of the effects of international trade or import competition on the number of firms active in a product. When only the most productive firms produce the most attractive product, then less openness causes the number of domestic firms to increase. The opposite occurs when all firms produce the most attractive products. In this sorting pattern, less open products tend to have fewer producers as firms respond to lower competition by reducing markups.

In our model, each firm's decision regarding which products to produce is solely determined by the product's attractiveness level, the endogenous competitive responses of firms, and the firm's productivity, which remains constant across products. Therefore, decisions across different products are independent within a firm. To incorporate firm-product level differences in sales, we use product-specific demand shocks, following the approach of Arkolakis et al. (2021). These shocks enable us to create variations in firm-product level sales, which could be influenced by several factors such as differences in marginal costs and product quality.

We carry out a quantification of our model to achieve two main objectives. Firstly, we aim to determine the significance of the novel aspects of our model. Secondly, we seek to

[2] This second sorting pattern is also presented in the model of firm locating in different cities by Nocke (2006). In fact, the largest cities tend to attract the largest number of firms, but because of the tougher competition, only the most productive firms can survive.

measure the (unobserved) product attractiveness and associate it with observable characteristics of product codes. Our model requires four key parameters to be calibrated. The first three parameters are commonly used in the literature. First, we calibrate the parameter that shapes the Pareto distribution of productivity by targeting the trade elasticity. Second, we calibrate the parameter controlling demand curvature to target the sales advantage of exporters compared to non-exporters. Lastly, we calibrate the standard deviation of the firm-product specific demand shock by matching the coefficient of variation of revenues.

A key contribution of this paper in the quantitative analysis is the calibration of the parameter that governs the sorting of firms into different products, which controls whether preferences are homothetic, indirectly additive, directly additive or in between these. We calibrate this parameter by estimating the effects of changes in trade openness (measured by the product-specific domestic expenditure share) on the number of firms selling a product. This is a critical prediction of the model discussed earlier. To identify the causal effect of a change in domestic expenditure share on the number of firms producing a product, we adopt an instrumental variable approach, where we use the total exports of countries other than Denmark as the instrument.

With our calibrated parameters, we assess the significance of allowing firms to sort into different products depending on their attractiveness and level of competition. Firstly, we quantify the relative importance of product characteristics and discover that accounting for differences in product-level attributes explains a quarter of the sales variance across firms. Secondly, we compare our model to one that does not incorporate heterogeneity in attractiveness and competition across products, i.e., the standard model. We find that neglecting the product-specific levels of attractiveness and competition, which influence firms' choices, results in an overestimation of the explanatory power of firm-specific shocks.

Our calibration exercise supports a sorting pattern where the most attractive products are produced by all firms, while only the largest firms produce the least attractive products. Utilizing our model's structure, we can infer a value for market attractiveness and relate it to product characteristics. Specifically, the most attractive products generally have the highest production values and the largest export and import values. However, when controlling for total production, export values are negatively related to attractiveness, as is export participation.

Related Literature. Our study contributes to the expanding field of research on multi-product firms, which has recently been summarized by Irlacher (2022). Previous research has primarily explored how companies modify the diversity and distribution of their products in reaction to alterations in foreign market access and trade costs (Feenstra and Ma, 2007; Baldwin and Gu, 2009; Eckel and Neary, 2010; Bernard et al., 2011; Qiu and Zhou, 2013; Dhingra, 2013; Mayer et al., 2014; Nocke and Yeaple, 2014; Eckel et al., 2015; Lopresti, 2016; Mayer et al., 2021; Macedoni, 2022; Macedoni and Xu, 2022; Boehm et al., 2022).[3] For instance, a pioneering study by Bernard et al. (2011) showed that multi-product firms tend to concentrate on their core products following trade liberalization. However, these studies generally assume that demand or competition changes impact all of a firm's products uniformly. They suggest that only initial conditions, such as differences in marginal production costs, decide which products are retained and which are discontinued. Contrary to this assumption, our paper aims to open the black box of multi-product production and allow firms to face product-level attractiveness and competition.

Our paper complements to the growing body of literature on how firms employ different strategies to escape competition. Fieler and Harrison (2018) investigates the impact of import competition on firms' product choices outside the conventional model of multi-product firms. While both our study and Fieler and Harrison (2018) investigate firms' responses to heightened competition, their model permits firms to venture into producing more diversified varieties in response to increased competition. In contrast, in our firms cannot launch new product categories. Instead, they decide which product categories to engage in based on competition intensity and product attractiveness. The escape competition mechanism is also explored by Piveteau and Smagghue (2022) in the context of quality upgrading. The authors propose a model where competition from low-cost exporters affects low-quality goods more than high-quality goods, causing firms to evade competition by enhancing their quality. Unlike our study, they do not consider the selection of firms into different products. Lim et al. (2022) also consider the interplay between

[3] here have also been several studies investigating how changes in exchange rates, deunionization, taxation, trade liberalization, and horizontal merge and acquisition impact a firm's product scope (Chatterjee et al., 2013; Freitag and Lein, 2023; Flach and Irlacher, 2018; Egger and Koch, 2012; Flach et al., 2021; Qiu and Yu, 2020; Chan et al., 2022).

market attractiveness (driven by scale in their case) and the subsequent rise in competition. While our study focuses on firms' selection of product categories, they examine how increased foreign market access influences firms' quality upgrading along the quality distribution. In their research, quality upgrading serves as a strategy for firms to sidestep future competition.[4]

The paper is organized as follows. Section 2 presents the stylized facts that motivate the theory of the paper. Section 3 illustrates the model. Section 4 describes our calibration strategy and the estimated parameters. Section 5 discusses the results of the quantification of the model using the calibrated parameters.

2 Stylized Facts

2.1 Data

We utilize data from Denmark Statistics (DST) to analyze the performance of firms and products from 2000 to 2015. The data is collected from two sources, namely, the production statistics (VARS) and the trade statistics (UHDI). VARS is a survey that focuses on manufacturing firms with a minimum of 10 employees. It provides details on the total sales value and quantity of each product manufactured by these firms, regardless of whether the product is sold domestically or exported. To align this dataset with the trade statistics, we designate each product as a Combined Nomenclature (CN) eight-digit code. The trade statistics provide information on the exports and imports of each product by destination. The products are reported according to the eight-digit CN code.

2.2 Rank within Firms

In conventional models of multi-product firms the within-firm ranking of sales across products only depend on the firm-product specific appeal (Bernard et al., 2011) or marginal costs (Eckel and Neary, 2010), and the product-specific attributes bear no relationship with the within-firm ranking. This is because all of a firm's products are assumed to face the same level of competition and share the same set of customers. In our first stylized

[4] The effects of competition on innovation at the firm level, including the escaping competition mechanism, was initially examined in the seminal work of Aghion et al. (2001) and Aghion et al. (2005).

fact, we test this prediction by evaluating the correlation between product-specific characteristics and within-firm ranking, such as the level of competition and market size that are product-specific, on the ranking of sales across the products of a firm.

We compute domestic sales per firm-product in each year by subtracting the value of exports reported in UHDI from the value of total sales reported in VARS. We denote as Rankmf the ranking of a firm's product sales in a particular product category m. A Rankmf value of 1 indicates that product m is the top-selling or core product of firm f, with the highest sales in that category. As sales decrease for other products, the Rank$_{mf}$ value increases. We consider the following regression:

$$\begin{aligned}\text{Rank}_{mft} = {} & \beta_1 \text{Log \# Firms}_{mt} + \beta_2 \text{Log Domestic Sales}_{mt} + \\ & + \beta_3 \text{Market Share}_{mft} + \beta_4 \text{Log Import Competition}_{mt} FE_{ft} + \epsilon_{mft}\end{aligned} \quad (1)$$

where Log # Firms$_{mt}$ is the log of the number of Danish firms included in the production statistics with positive domestic sales in product m and year t, Log Domestic Sales$_{mt}$ is the aggregate level of domestic sales of Danish firms in product m and year t, Market Share$_{mft}$ is computed as the ratio of product-firm mf domestic sales over total domestic sales (and multiplied by 100), and Log Import Competition$_{mt}$ is the log of total imports of product m and year t minus the log of the total sales value (which includes both domestic sales and exports) of product m and year t. We control for firm unobservable characteristics with firm-year fixed effects FE_{ft} to make sure that the comparison is between products within a firm-year combination. ϵ_{mft} is the error term.

Table 1: Within Firm Ranking

	Dependent Variable: Within firm rank of product sales			
	(1)	(2)	(3)	(4)
Log Number of Firms	-8.765***	28.207***	12.169***	11.920***
	(0.287)	(0.380)	(0.385)	(0.396)
Log Domestic Sales		-19.630***	-24.066***	-23.763***
		(0.146)	(0.143)	(0.182)
Product Market Share			-1.038***	-1.038***
			(0.009)	(0.009)
Log Import Competition				0.426***
				(0.158)
Firm FE	Yes	Yes	Yes	Yes
R^2	0.69	0.74	0.77	0.77
# Obs.	114454	114454	114454	114454

Results from OLS of (1). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Table 1 presents the results of our analysis, where a negative coefficient indicates a correlation between the variable and a higher ranking within a firm (since the core product has a ranking of one). We find a negative correlation between the number of firms and product ranking, which suggests that the core products of firms typically face more competition. However, after controlling for total domestic sales, the coefficient on the number of firms turns positive, indicating a case of omitted variable bias. Our results show that the core varieties of a firm tend to have a large market size, as measured by total domestic sales, and fewer competitors. Furthermore, we find that a larger market share within a product also leads to a higher product ranking within a firm. Finally, we observe that larger imports are associated with a lower ranking. Results are robust to fixing a year: see Table 8 in the appendix.

While our results are intuitive, they are often overlooked in traditional frameworks of multi-product firms, which assume that the size of the market and the level of competition are uniform across all products. Our first stylized fact challenges this assumption, as we find that firms flee from product-specific competition from other Danish firms and from foreign firms and instead chase product-specific market size and market share.

2.3 Firm-product sales decomposition

Our study also investigates the contribution of firm-year and product-year shocks to the evolution of firm sales. To achieve this, we begin by regressing the log domestic sales of a firm f in product m and year t on a firm-product fixed effect. We record the residual ϵ_{mft}, which is the log of domestic sales adjusted for the average sales of firm f in product m:

$$\text{Log Domestic Sales}_{fmt} = FE_{fm} + \epsilon_{mft} \tag{2}$$

To investigate the contribution of firm-year and product-year shocks to the variance of the adjusted sales ϵ_{mft}, we use the variance decomposition approach utilized by Hottman et al. (2016) and Bernard et al. (2021). We estimate the following regression:

$$\epsilon_{mft} = FE_{ft} + FE_{mt} + \nu_{fmt} \tag{3}$$

Next, we regress the two estimated fixed effects FE_{ft} and FE_{mt} on ϵ_{mft} without a constant to conduct a variance decomposition. Using OLS properties, the coefficient of a regression of the estimated fixed effect on ϵ_{mft} without a constant represents the percentage of the variance of ϵ_{mft} explained by that fixed effect. In a workhorse model of multi-product firms, all products face the same demand and competition, so the contribution of the FE_{mt} should be zero. Our results, shown in Table 2, reveal that both the firm-year and product-year fixed effects have an equal impact, as they both explain around 25% of the variance in firm-product sales, respectively. This indicates that product-specific shocks, such as changes in demand and competition, are just as crucial as common shocks occurring within the firm, such as firm-level productivity shocks. The remaining 50% is explained by shocks which are idiosyncratic to the particular firm-product fm.

Table 2: Variance Decomposition

	(Firm-Year Shocks)	(Product-Year Shocks)
Variance of Firm-Product Log Sales	0.247***	0.246***
	(0.002)	(0.002)
# Obs.	83236	83236

Results from variance decomposition of (3). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

In a similar vein, we investigate the extent to which the level of import competition varies across firms and, within firms, across products. To achieve this, we compute the average growth rate of imports of the products produced by each firm and its corresponding standard deviation. We then calculate the standard deviation of the average firm-level import growth, as well as the average of the standard deviations of import growth within firms. Our findings reveal that the average standard deviation of import growth within firms is 0.35, on average, across all years. Conversely, the standard deviation of import growth across firms is slightly larger, averaging 0.4 (see Table 9 in the appendix). These results suggest that the dispersion in import competition within a firm is just as critical as the dispersion in import competition across firms, which is typically the focus of most studies in the literature (Bernard et al., 2011; Mayer et al., 2014).

3 Model

There are I countries, which are indexed by i for origin and j for destination. In each country, there are L_j consumers with per capita income y_j. Consumers enjoy the consumption of varieties of M differentiated products. Each product, or product category, is indexed by $m = 1, ..., M$. A continuum of multi-product firms can produce a variety ω for each product m. To draw a parallel with our empirical analysis, product m is a CN 8-digit code, and the varieties ω within a given product m are the different varieties produced by firms within the same CN 8-digit code.

Products differ in an exogenous product attractiveness, denoted by a_{mj}, which is modeled as a demand shifter common for all the varieties of a given product m. The parameter a_{mj} captures any difference in products that make some more attractive than others, either because of higher appeal, higher value added, or lower fixed costs of product development. Furthermore, products differ in the endogenous level of competition, which is a function of the degree of product attractiveness. Higher product attractiveness a_{mj} promotes introduction of new varieties by firms. However, this also leads to tougher competition which may lead to tougher selection, depending on the extent of competition. This latter channel is going to be governed by a key parameter that we will estimate in the data.

As in a standard Melitz (2003) model, there is a mass of firms J_i that paid a fixed cost of entry f_E in domestic labor units and discovered their marginal cost draw c. Marginal costs are drawn from a Pareto distribution with shape parameter θ and shift parameter b_i. Each firm can produce in any of the product category $m = 1, ..., M$.

The only driver of selection of firms into different products is the attractiveness a_{mj} and competition in these products. This assumption may seem in stark contrast with the literature that assumes the presence of a core competence (Eckel and Neary, 2010; Mayer et al., 2014; Arkolakis et al., 2021). However, this is done to maintain the selection of firms into different products as tractable as possible. Introducing a core competence that varies across firms would introduce a sizeable complication since, the order in which firms enter products would then become firm-specific. Note that this problem does not arise in the literature aforementioned, since competition and attractiveness are identical across products. To address this issue, we introduce demand-shocks that affect firm-product level sales and are realized upon consumption. The introduction of these demand shocks is important in the quantitative exercise since they capture any firm-product specific characteristics that affects sales independently of attractiveness and competition, which includes the core competence.

There is an iceberg trade cost $\tau_{mij} \geq 1$ of exporting from i to j which is specific to product m. Workers are the only factor of production and gain a wage w_i. Free entry drives expected profits to zero and, as a result, per capita income and wages are equal.

3.1 Consumer's Demand

Demand in each market m in country j originates from the Generalized Translated Power (GTP) preferences proposed by Bertoletti and Etro (2020):[5] The utility function equals:

$$U_{mj} = \int_{\Omega_{mj}} \delta_{mj}(\omega) \left(a_{mj}\xi_{mj}q_{mj}(\omega) - \frac{(\xi_{mj}q_{mj}(\omega))^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) d\omega + \frac{\xi_{mj}^{-\eta} - 1}{\eta} \tag{4}$$

where $q_{mj}(\omega)$ represents the quantity of variety ω in product m and country j. $\delta_{mj}(\omega)$ is a demand shock which is specific to the variety ω. We follow (Arkolakis et al., 2021) and assume that such a shock is *i.i.d.*, is realized upon consumption, and its expected value is one. We denote with σ the standard deviation of the demand shock. $a_{mj} > 0$ is a demand shifter that is common for any varieties of product m and captures the attractiveness or appeal of product m, as described above. $\gamma > 0$ is a parameter that controls the curvature of the demand. ξ_{mj} is a quantity aggregator that is implicitly defined as:

$$\xi_{mj}^{-\eta} = \int \delta_{mj}(\omega) \left(a_{mj}\xi_{mj}q_{mj}(\omega) - (\xi_{mj}q_{mj}(\omega))^{1+\frac{1}{\gamma}} \right) d\omega \tag{5}$$

The parameter $\eta \in [-1, \infty]$ is crucial in our analysis. In particular, it controls the elasticity of markups with respect to the degree of competition. As a result, it controls the sorting of firms of different productivity into different products. That these selection effects are controlled by η is a key advantage of these preferences, which allows for different sorting patterns. In other words, depending on the value of η, our model can feature an allocation in which only the most efficient firms are able to sell in the largest markets or an opposite allocation where all active firms sell to the largest markets. The most common preferences used in the literature fix the elasticity of markups with respect to competition and, as a result, only allow for one type of sorting. These common preferences are nested in our framework. In fact, for $\eta = -1$, preferences are indirectly additive (IA) as

[5] Fally (2018) describes the regularity conditions for these preferences. Macedoni and Weinberger (2022) uses these preferences to study how regulations reduce misallocation in Chile.

described by (Bertoletti et al., 2018). For $\eta = 0$, preferences become homothetic with a single aggregator. For $\eta \to \infty$, preferences become directly additive (DA), and generalize the preferences used by Melitz and Ottaviano (2008).

We assume that consumers spend a constant share α of their income on each product m, so that $\alpha = 1/M$. This is a stark assumption that allows us to link product attractiveness of a product to the selection forces for that product and no other product. In other words, given this assumption, whether the most productive firms produce a variety of a certain product only depends on the attractiveness and level of competition for that product and not from the attractiveness and competition of other products.[6]

The consumer's budget constraint is:

$$\int_{\Omega_{mj}} p_{mj}(\omega) q_{mj}(\omega) d\omega \leq \alpha y_j$$

Solving the consumers' problem yields the following inverse demand function:

$$p_{mj}(\omega) = \alpha y_j \xi_{mj}^{1+\eta} \delta_{mj}(\omega) \left[a_{mj} - \left(\xi_{mj} q_{mj}(\omega) \right)^{\frac{1}{\gamma}} \right] \tag{6}$$

where $p_{mj}(\omega)$ is the price of the variety ω.[7]

3.2 Firm's Problem

Let us now turn to the firm's problem. As the demand shocks $\delta_{mj}(\omega)$ are realized upon consumption, and are *i.i.d.*, they do not affect entry and production decisions of firms. This formulation is also present in Arkolakis et al. (2021), and it implies that firms decide in which product categories to introduce a variety and what prices to charge before the realization of the shocks. Thus, the scope and pricing decisions of firms are sunk. Upon consumption, the demand shocks are realized and firms revenues and profits will be affected. In our paper, these variety-specific demand shocks capture any other determinant of firm scope aside from market size and competition, such as firms core competences

[6] Given this assumption, the model can also be used to identify firm selection into different broadly defined markets m. This assumption is also equivalent to assuming that there are different sets of consumers purchasing each product. Finally, note that using a Cobb-Douglas aggregation of U_{mj} would generate general equilibrium effects that nullify the selection forces we are attempting at modeling.

[7] If $\eta \to \infty$, $\xi_{mj} = 1$ and, therefore, $p_{mj}(\omega) = y_j \delta_{mj}(\omega) \left[a_{mj} - q_{mj}(\omega)^{\frac{1}{\gamma}} \right]$. If also $\gamma = 1$, the demand becomes identical to the linear demand of Melitz and Ottaviano (2008) without the aggregator.

(Eckel and Neary, 2010; Arkolakis et al., 2021) and flexibility (Macedoni and Xu, 2022).

Profits of a firm with cost draw c from i producing a variety in product m of country j are given by:

$$\pi_{mij}(c) = L_{mj}\alpha y_j \xi_{mj}^{1+\eta} \left[a_{mj} - \left(\xi_{mj} q_{mij}(c)\right)^{\frac{1}{\gamma}} \right] q_{mij}(c) - L_{mj}\tau_{mij} w_i c q_{mij}(c) \tag{7}$$

The first order condition is:

$$\alpha y_j \xi_{mj}^{1+\eta} \left[a_{mj} - \frac{1+\gamma}{\gamma} \left(\xi_{mj} q_{mij}(c)\right)^{\frac{1}{\gamma}} \right] - \tau_{mij} w_i c = 0$$

The cutoff firm with marginal cost from i to j, c^*_{mij}, is indifferent between selling to market m or not. To find c^*_{mij}, we set $q_{mij}(c^*_{mij}) = 0$ in the first order condition, which yields:

$$c^*_{mij} = \frac{\alpha a_{mj} y_j \xi_{mj}^{1+\eta}}{\tau_{mij} w_i} \tag{8}$$

Thus, the cutoff, which controls the sorting of firms into the various markets, increases with the demand shifter a_{mj} and declines with the iceberg trade cost. The cutoff also depends on ξ_{mj}, which is a general equilibrium object that controls the endogenous response of competition to the demand shifter. The parameter η controls the relationship between the cutoff and ξ_{mj}. In fact, let us hold constant the origin-destination ij pair and compare the cutoff between markets m and k:

$$\frac{c^*_{mij}}{c^*_{kij}} = \frac{a_{mj}\xi_{mj}^{1+\eta}\tau_{kij}}{a_{kj}\xi_{kj}^{1+\eta}\tau_{mij}} \tag{9}$$

The ratio between cutoffs, which determines the sorting of firms in the two markets, depends on the ratio of the market demand shifters, the ratio between the general equilibrium object ξ_{mj}, and the relative trade costs. Holding constant the market destination mj pair, the export cost cutoff can be expressed as a function of the domestic cost cutoff in the following way:

$$\frac{c^*_{mij}}{c^*_{mjj}} = \frac{\tau_{mjj} w_j}{\tau_{mij} w_i} \tag{10}$$

Using the cutoff definition, we can write the performance variables of a firm as:

$$q_{mij}(c) = \left(\frac{\gamma}{1+\gamma}\right)^{\gamma} \frac{a_{mj}^{\gamma}}{\xi_{mj}(c_{mij}^{*})^{\gamma}}(c_{mij}^{*}-c)^{\gamma} \tag{11}$$

$$p_{mij}(c) = \frac{w_i \tau_{mij}}{1+\gamma}(c_{mij}^{*}+\gamma c) \tag{12}$$

$$r_{mij}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c_{mij}^{*})^{\gamma}}(c_{mij}^{*}-c)^{\gamma}(c_{mij}^{*}+\gamma c) \tag{13}$$

$$\pi_{mij}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c_{mij}^{*})^{\gamma}}(c_{mij}^{*}-c)^{1+\gamma} \tag{14}$$

where $r_{mij}(c)$ are firm c revenues in market m and destination j. Letting $n_{ij}(c)$ denote the firm c product scope, the scope equals to:

$$n_{ij}(c) = \sum_{m=1}^{N} \mathbb{1}_{c \leq c_{mij}^{*}} \tag{15}$$

Notice that the demand shock $\delta_{mj}(c)$ faced by firm c across product-destinations mj affects the realized revenues and profits. In fact, realized revenues equal $\tilde{r}_{mij}(c) = \delta_{mj}(c) r_{mij}$ and realized profits equal $\tilde{\pi}_{mij}(c) = \delta_{mj}(c)\pi_{mij}$.

The parameter η significantly influences how firms sort into various products. To better understand the role of this parameter, let us examine the effects of a market's "remoteness" on the cost cutoff of selling product in that market. To do this, we calculate the elasticity of c_{mjj}^{*} with respect to the "multilateral resistance" term of market mj, $\Phi_{mj}^{-\theta} = \sum_i J_i (b_i \tau_{mij} w_i)^{-\theta}$, which relates negatively to the "remoteness" of market mj:[8]

$$\frac{d \ln c_{mjj}^{*}}{d \ln \Phi_{mj}^{-\theta}} = \frac{1}{\frac{1}{1+\eta} - (1+\theta)}. \tag{16}$$

The value of η dictates the sign of the elasticity. On one hand, a reduction in the remoteness of the market raises the overall demand for product m and tends to increase the cost cutoff c_{mjj}^{*}, with the size of this positive effect increasing in $\frac{1}{1+\eta}$. On the other hand, a reduction in the remoteness intensifies competition in that market and results in tougher selection, which tends to decrease the cost cutoff c_{mjj}^{*}, with the size of this negative effect

[8] For the derivation, see equations (43) and (44) in the appendix.

increasing in $1+\theta$. The net effect of Φ_{mj}^{θ} on c_{mjj}^{*} thus depends on the relative magnitudes of $\frac{1}{1+\eta}$ and $1+\theta$. In general, if $\eta > -\frac{\theta}{\theta+1}$, the elasticity is negative as it is typically assumed, meaning that a less remote market is associated with tougher selection, a smaller number of active firms, a larger average quantity sold per firm, and a lower average price. However, if $-1 < \eta < -\frac{\theta}{\theta+1}$, then a less remote market instead features a less selective environment, a larger number of active firms, a smaller average quantity per firm, and a higher average price.

For Indirectly Additive preferences where $\eta = -1$, the cost cutoff is unaffected by changes in the remoteness. For Directly Additive preferences where $\eta \to \infty$ and homothetic preferences, the cost cutoff decreases as remoteness decreases, with the most substantial decline occurring under homothetic preferences.

We can also calculate the elasticity of firm-product-level markup, represented as $\frac{p_{mjj}(c)}{c}$, for domestic firms with respect to the inverse of the remoteness measure of that market:

$$\frac{d \ln \frac{p_{mjj}(c)}{c}}{d \ln \Phi_{mj}^{-\theta}} = \frac{1}{c_{mjj}^{*} + \gamma c} \times \frac{1}{\frac{1}{1+\eta} - (1+\theta)}, \tag{17}$$

whose sign also depends on the relative magnitudes of $\frac{1}{1+\eta}$ and $1+\theta$.[9]

3.3 Sorting into Products

By exploiting the gravity formulation of the model, we can represent the equilibrium using a system of equations that uses a parsimonious set of parameters and data. The system of equations that characterizes the equilibrium is similar to most of the literature of international trade with heterogeneous firms. For this reason, we leave the derivations of the equilibrium to the appendix, and focus in this section on the key innovation of the model, which is how firms sort into different product markets.

Let us focus on the sorting of firms from j in their domestic economy j. Solving the

[9] Manova and Zhang (2012) and Harrigan et al. (2015) both document that within the same product, individual exporters charge higher prices at less remote markets, indicating that $-1 < \eta < -\frac{\theta}{\theta+1}$. The estimated parameters in the later section is indeed consistent with this finding.

model, we can express the relative cutoff (9) $\frac{c^*_{mjj}}{c^*_{kjj}}$ as a function of:

$$\frac{c^*_{mjj}}{c^*_{kjj}} = \left(\frac{\lambda_{mjj}}{\lambda_{kjj}}\right)^{-\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{a_{mj}}{a_{kj}}\right)^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}} \tag{18}$$

where λ_{mjj} is the domestic expenditure share of product m in destination j.[10] The higher the ratio, the larger the number of firms that produce a variety of product m. If the ratio is greater than one, more firms produce product m than product k.

Similar to the effect of remoteness, the market cutoff can increase or decrease (with different elasticities) with the product attractiveness depending on the parameter η. In particular, consider the derivative of the cutoff with respect to the product attractiveness, holding constant the relative trade shares $\frac{\lambda_{mjj}}{\lambda_{kjj}}$,

$$\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(a_{mj}/a_{kj})} > 0 \qquad \text{if } -1 < \eta < -\frac{\theta}{\theta+1} \tag{19}$$

$$\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(a_{mj}/a_{kj})} < 0 \qquad \eta > -\frac{\theta}{\theta+1} \tag{20}$$

When the cutoff is increasing with product attractiveness, it implies that a more attractive product market facilitates more active firms, albeit the intensified competition. As a result, all firms are attracted by products with higher attractiveness.[11] As a result, both large and small firms would sell the most attractive products, and only the most productive firms sell the least attractive products. This is, for instance, the case of Indirectly Additive preferences, in which $\eta = -1$ is within the range for which the elasticity of the cutoff with respect to willingness to pay is positive. When $\eta = -1$, the elasticity of $\frac{c^*_{mjj}}{c^*_{kjj}}$ with respect to $\frac{a_{mj}}{a_{kj}}$ converges to 1, so the market with the largest number of competitors also has the highest markups.

In contrast, when the cutoff is decreasing with the product attractiveness, the opposite sorting occurs, meaning that the intense competition at the most attractive mar-

[10] We use the relationship that $\Phi^{-\theta}_{mj} = \frac{J_j(b_j\tau_{mjj}w_j)^\theta}{\lambda_{mjj}}$. The ratio of cutoffs has a third component $\left(\frac{\tau_{kjj}}{\tau_{mjj}}\right)$ which is normalized to one by assuming that $\tau_{mjj} = \tau_{kjj} = 1$.

[11] The correct lower bound for η such that $\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(a_{mj}/a_{kj})} > 0$. However, we assume that $\eta > -1$ and, therefore, $-1 - \frac{1}{\gamma}$ is outside the allowed values for η.

kets dominates and only the most productive firms can survive in these markets. For instance, this is the case under Directly Additive ($\eta \to \infty$) and homothetic preferences ($\eta = 0$). In those cases, markups respond negatively to the decrease in remoteness. As firms are attracted by high-attractiveness products, the higher level of competition will reduce markups. The reduction in markups will force less productive firms to exit. The resulting sorting is similar to that outlined by Nocke (2006), in which the product with the largest attractiveness features the strongest level of competition and, thus, only the most productive firms are active in such product category. Products with lower attractiveness feature less competition, and thus less productive firms are also able to participate.

We can also study the relationship between the relative cutoff and the relative trade openness of a country:

$$\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(\lambda_{mjj}/\lambda_{kjj})} < 0 \qquad \text{if } -1 < \eta < -\frac{\theta}{\theta+1} \tag{21}$$

$$\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(\lambda_{mjj}/\lambda_{kjj})} > 0 \qquad \eta > -\frac{\theta}{\theta+1} \tag{22}$$

Note that $\lambda_{mjj}/\lambda_{kjj} = (\Phi_{mj}/\Phi_{kj})^{\theta}$, so we can interpret an increase in $\lambda_{mjj}/\lambda_{kjj}$ as an increase in the remoteness of product market m relative to product market k. If the country j becomes relatively remote in the product m (that is, the relative share of domestic expenditure λ_{mjj} increases), the number of domestic firms that sell the product m increases only if $\eta > -\frac{\theta}{\theta+1}$. That is, in the case where only the most productive firms produce the most attractive products.

4 Calibration

In this section, we describe the calibration procedure for the four key parameters of the model: the shape parameter of the productivity distribution θ, the demand curvature γ, the standard deviation of the demand shock σ, and the parameter that controls the sorting patterns η. To estimate the first three parameters, we use an exactly identified strategy, while for the parameter η, we adopt an instrumental variable (IV) approach. We present the calibration procedure in two parts. First, we calibrate θ, γ, and σ using an exactly identified strategy. Then, we calibrate the parameter η.

4.1 Exactly Identified Parameters: θ, γ, and σ

4.1.1 Moments

Shape Parameter of the Distribution of Firms' Productivity θ. To calibrate θ, we target the trade elasticity, i.e., the elasticity of trade flows with respect to trade costs τ. We use the gravity equation to derive the trade elasticity, which is precisely equal to the value of θ:[12]

$$\lambda_{mij} = \frac{L_i(b_i w_i \tau_{mij})^{-\theta}}{\sum_{v=1}^{I} L_v(b_v w_v \tau_{mvj})^{-\theta}} \quad \text{for } i, j = 1, ..., I \tag{23}$$

This moment is widely used in the literature (Arkolakis et al., 2012, 2019; Bertoletti et al., 2018). For instance, Arkolakis et al. (2012) and Arkolakis et al. (2019) use the trade elasticity as the moment to keep constant across models in welfare comparisons. In the literature, the absolute value of the trade elasticity has been estimated to be between 4 and 8, as reported in studies such as Eaton and Kortum (2002); Simonovska and Waugh (2014); Head and Mayer (2014). However, recent estimates that rely on time-series data and assume trade costs can be product and country-pair specific report even smaller estimates, ranging from 1.75 to 2.25, as highlighted in Boehm et al. (2020). In our baseline results, we assume a trade elasticity of 4. In an extension, we also consider a trade elasticity of 2 to assess the robustness of our calibration.

Demand Curvature Parameter γ. To capture the demand curvature parameter γ, we use another widely used moment in the literature, namely the exporter sales advantage, as in Bernard et al. (2003); Jung et al. (2019); Bertoletti et al. (2018). The exporter sales advantage is defined as the ratio of the average domestic sales of exporters to the average domestic sales of non-exporters. The parameter γ controls the difference in sales values between small and large firms by modulating the demand elasticity at different quantities. A higher γ value amplifies the differences between large and small firms, leading to an increase in the exporters' sales advantage.

Our model provides a closed-form expression for the exporters' sales advantage in

[12] Our model assumes that there is a common trade elasticity across all products, which is a reasonable assumption for a single-sector model. This assumption is widely used in the literature, as evidenced by studies such as Arkolakis et al. (2012, 2019); Bertoletti et al. (2018).

each product m, allowing us to estimate the value of γ:

$$M_{mj}^{sadv} = \frac{\left[\left(\frac{c^*_{mjj}}{c^*_{ex}}\right)^{\theta} - 1\right]\left[B\left(\frac{c^*_{ex}}{c^*_{mjj}};\theta;\gamma+1\right) + \gamma B\left(\frac{c^*_{ex}}{c^*_{mjj}};\theta+1;\gamma+1\right)\right]}{B(\theta,\gamma+1) + \gamma B(\theta+1,\gamma+1) - B\left(\frac{c^*_{ex}}{c^*_{mjj}};\theta;\gamma+1\right) - \gamma B\left(\frac{c^*_{ex}}{c^*_{mjj}};\theta+1;\gamma+1\right)} \tag{24}$$

where $c^*_{ex} = \max_{v \neq j} c^*_{mjj}$ is the highest export cutoff, which is the cutoff to export to the destination that is most easily reachable. $B(z,h)$ is the Euler Beta Function and $B(u;z,h)$ is the incomplete Euler Beta Function. Detailed derivations for the expression can be found in the appendix.

The cutoff ratio $\frac{c^*_{mjj}}{c^*_{ex}}$ in (24) can be computed given θ and the value for export participation: $\left(\frac{c^*_{mjj}}{c^*_{ex}}\right)^{\theta} = N_{mjj}/N^{ex}_{mj}$, where N^{ex}_{mj} is the number of exporters. To compute the export sales advantage we proceed as follows.

First, for each CN 8-digit good and year, we calculate the firm-product level domestic sales by taking the difference between the total production value reported in the production statistics and the export value reported in the customs data.[13] Next, we compute aggregate domestic sales at the firm level and calculate the average domestic sales of both exporters and non-exporters for each year. Finally, to calculate the export participation, we divide the total number of exporters in our production survey by the total number of firms in the survey.

For our baseline year, 2000, the exporter sales advantage in our model equals 2.37, while the export participation is 0.55. These values are different than those documented for the US, as reported in Bernard et al. (2003): the exporter sales advantage is smaller in our case, while the export participation is larger. One possible reason for this difference could be that our production data only includes firms with at least 10 employees, which leads to an overestimation of the export participation (as most small firms do not export) and an underestimation of the exporter sales advantage. We observe that the value of the sales advantage remains relatively stable in the first decade of the 2000s, around 2.5-2.6, but from 2009 onwards, it increases and takes on values greater than 3.5. The export

[13] We discard any firm-products with a negative value of domestic sales, which can arise due to carry along trade.

participation also increases over time, rising from 0.55 in 2000 to 0.62 in 2015, as presented in Table 10. To examine the robustness of our results, we also consider the moments documented for the US, where the exporter sales advantage is estimated to be 4.8 and the export participation is 0.18.

Standard Deviation of the Firm-Product Shock σ. To calibrate σ, we target the coefficient of variation of domestic sales in a market. This coefficient of variation is constant across products and is defined as the ratio between the standard deviation of sales across firms and the average sales. Our assumption of i.i.d shocks and a Pareto distribution of unit costs allows us to derive the following closed-form expression:

$$CV_{mj} = \frac{\left(T_3(1+\sigma) - \theta T_1^2\right)^{0.5}}{\theta^{0.5} T_1} \tag{25}$$

where

$$\begin{aligned} T_1 &= B(\theta, \gamma+1) + \gamma B(\theta+1, \gamma+1) \\ T_3 &= B(\theta, 2\gamma+1) + 2\gamma B(\theta+1, 2\gamma+1) + \gamma^2 B(\theta+2, 2\gamma+1) \end{aligned}$$

Given θ and γ, the coefficient of variation is positively related to the value of σ. Using our production survey data, we calculate the coefficient of variation of domestic sales for each CN 8-digit code. We then compute the simple average across all products, which is equal to 1.69 for the year 2000. We find that the coefficient of variation tends to increase over time, with values rising to around 1.8-1.9 in later years (see Table 10). Previous research has estimated the standard deviation of ex-post demand shocks using an overidentified approach, which employs moments from the sales distribution and the scope distribution across firms, as documented in Arkolakis et al. (2021) and Macedoni and Xu (2022). Using the coefficient of variation to estimate σ is a faster approach and yields parameter estimates that are consistent with the literature.

4.1.2 Results

Table 3 reports the results of the calibration using data from 2000. The value of the trade elasticity parameter θ is 4, which is consistent with the literature. The demand curvature parameter γ takes on a value of 0.59, which is smaller than previous estimates from the literature (Bertoletti et al., 2018; Macedoni and Weinberger, 2022) due to the relatively low exporter sales advantage value. The standard deviation of the demand shock σ is estimated to be 2.18, which is in line with previous estimates using Brazilian data (Arkolakis et al., 2021) and Chinese data (Macedoni and Xu, 2022). Overall, our calibration results are broadly consistent with previous estimates from the literature.

Table 3: Moments and Parameters

Moments	
Trade Elasticity	4
Exporters Sales Advantage	2.37
Export Participation	0.55
Coefficient of Variation of Sales	1.69
Parameters	
θ	4
γ	0.59
σ	2.18

Moments computed using data from the year 2000.

As a robustness check, we conduct the calibration using the moments computed for each year between 2000 and 2015. The results are presented in Table 11 in the appendix. We observe that a higher exporter sales advantage is associated with higher values of γ, with estimates between 0.82 and 1.02 from 2009 to 2015. The standard deviation of the demand shock ranges between 1.89 and 2.53 across the years. Changes in the value of σ are mainly driven by variations in the value of γ, as higher values of γ tend to increase the standard deviation of revenues (as it magnifies productivity differences across firms). Thus, the portion of the coefficient of variation that is left to be explained by σ becomes smaller. Overall, our calibration results remain robust across the years.

As an additional robustness check, we consider the case of a lower trade elasticity, equal to 2, and report the results in Table 12 in the appendix. The resulting estimate of γ is higher than the baseline case, while the value of σ barely changes. A lower value of θ is associated with a lower coefficient of variation and a lower exporter sales advantage, but the increase in γ offsets these two effects.

In another robustness check, we consider the case of using the US exporter sales advantage and the US export participation values, and report the results in Table 13 in the appendix. In this case, the estimate of γ is significantly higher, at 1.6, while the value of σ is lower, at 0.86.

4.2 The Sorting Parameter: η

4.2.1 Moment

We exploit a novel prediction of our theoretical model to calibrate the parameter η that governs the sorting pattern in different product markets. Combining (18) with the Pareto distribution of marginal cost c, we can rewrite the relative number of firms producing product m as

$$\frac{N_{mjj}}{N_{kjj}} = \left(\frac{\lambda_{mjj}}{\lambda_{kjj}}\right)^{\frac{\theta(1+\eta)}{\theta+\eta+\theta\eta}} \left(\frac{a_{mj}}{a_{kj}}\right)^{-\frac{\theta[1+\gamma(\eta+1)]}{\theta+\eta+\theta\eta}}. \tag{26}$$

The relative number of product-m producers depends on the relative domestic expenditure share and the relative product attractivenss of product m. Inspecting the elasticity of (N_{mjj}/N_{kjj}) with respect to $(\lambda_{mjj}/\lambda_{kjj})$, we find that the sign of the elasticity is also informative about the sorting pattern:

$$\frac{\partial \ln\left(N_{mjj}/N_{kjj}\right)}{\partial \ln\left(\lambda_{mjj}/\lambda_{kjj}\right)} = \frac{\theta(1+\eta)}{\theta+\eta+\theta\eta} < 0 \qquad -1 < \eta < -\frac{\theta}{\theta+1} \tag{27}$$

$$\frac{\partial \ln\left(N_{mjj}/N_{kjj}\right)}{\partial \ln\left(\lambda_{mjj}/\lambda_{kjj}\right)} = \frac{\theta(1+\eta)}{\theta+\eta+\theta\eta} > 0 \qquad \eta > -\frac{\theta}{\theta+1}. \tag{28}$$

As the domestic expenditure share of product m increases so that the product-level import competition becomes less intense, the number of domestic firms producing product m decreases when $-1 < \eta < -\frac{\theta}{\theta+1}$. In this case, the reduced trade openness leads to a lower quantity aggregator and an overall decline in the demand for product m. Hence, the selection becomes more stringent and fewer domestic firms producing m remain active. In contrast, when $\eta > -\frac{\theta}{\theta+1}$, the reduced trade openness raises the quantity aggregator and the overall demand for product m. Therefore, less productive firms can also survive at this product market.

Fixing the country j, we can further write the number of active firms at product market m as follow

$$\ln N_{mjj} = \left(\frac{\theta(1+\eta)}{\theta+\eta+\theta\eta}\right) \ln \lambda_{mjj} - \left(\frac{\theta[1+\gamma(\eta+1)]}{\theta+\eta+\theta\eta}\right) \ln a_{mj} + \Gamma_j, \tag{29}$$

where the parameter Γ_j is a collection of variables that only depend on j.

Let j be Denmark, so we can ignore the j subscript. We compute the number of active Danish firms that sell product m in year t, N_{mt}, and Denmark's domestic expenditure share of product m in year t, λ_{mt}. Taking the first-difference form of (29), we obtain the key regression specification:

$$\Delta \ln N_{mt} = \kappa \Delta \ln \lambda_{mt} + \epsilon_{mt}, \tag{30}$$

where $\kappa = \frac{\theta(1+\eta)}{\theta+\eta+\theta\eta}$ and the error term $\epsilon_{mt} = -\left(\frac{\theta[1+\gamma(\eta+1)]}{\theta+\eta+\theta\eta}\right) \Delta \ln a_{mt} + \Delta\Gamma_t$ contains the overtime changes in product attractiveness a_{mt} in Denmark and other variables Γ_t that are specific to Denmark but (assumed to be) constant across different products.

Ideally, if we can consistently estimate the value of κ in (30), we can compute the value of the sorting parameter η given the calibrated value of θ. In fact,

$$\eta = \frac{\theta(\frac{1}{\kappa} - 1)}{1 - \theta(\frac{1}{\kappa} - 1)} \tag{31}$$

4.2.2 Identification

In practice, estimating (30) using OLS may yield an inconsistent estimate of κ because of omitted variable bias. For instance, an increase in the product attractiveness of m in Denmark in the error term, $\Delta \ln a_{mt} > 0$, may be correlated with the change in the productivity or the competition environment of producing m in Denmark, which directly affects Denmark's domestic expenditure share of m, $\Delta \ln \lambda_{mt}$. To tackle this endogeneity issue, we adopt an instrumental variable approach to estimate κ, making sure that the variation of $\Delta \ln \lambda_{mt}$ used in the estimation is plausibly orthogonal to $\Delta \ln a_{mt}$ and other product-specific factors in Denmark.[14]

For identification, we assume that the product-level supply-side determinants of trade in other exporting countries are not correlated with the product-level supply and demand conditions in Denmark. We then construct the following instrument for $\Delta \ln \lambda_{mt}$

[14] Appendix C.2 describes how we calculate λ_{mt} using the Danish data.

by purging the supply-side factors in other countries that affect λ_{mt}:

$$\Delta \ln \tilde{\lambda}_{mt} = -\ln \left(\lambda_{m\text{DNK},t-1} + (1 - \lambda_{m\text{DNK},t-1}) \sum_{i \neq \text{DNK}} Imp\_Share_{mi,t-1} \times \frac{EX^{\text{high-income}}_{mi,t}}{EX^{\text{high-income}}_{mi,t-1}} \right). \tag{32}$$

In equation (32), $\lambda_{m\text{DNK},t-1}$ is Denmark's domestic expenditure share of product m in year $t-1$, and $Imp\_Share_{mi,t-1}$ is the share of imports from i on product m over total imports to Denmark for product m. These trade shares at $t-1$ are considered pre-determined for the subsequent changes from $t-1$ to t. The variables $EX^{\text{high-income}}_{mi,t}$ and $EX^{\text{high-income}}_{mi,t-1}$ are country i's total exports of product m to a set of other high-income countries except for Denmark in years t and $t-1$, respectively. The ratio $(EX^{\text{high-income}}_{mi,t} / EX^{\text{high-income}}_{mi,t-1})$ aims to capture the product-level export growth driven by pure supply-side factors of country i (e.g., productivity growth), and thus is presumably orthogonal to the change in product attractiveness $\Delta \ln a_{mt}$. Autor et al. (2013) and Hummels et al. (2014) use similar assumptions and strategies to construct instruments for increases in import competition and offshoring that are orthogonal to the demand-side factors, respectively. Intuitively, an increase in country i's supply-side export capability at product m should result in an increase in rising export from i to Denmark, and hence a decrease in Denmark's domestic expenditure share of product m, λ_{mt}. Therefore, the use of instrument $\Delta \ln \tilde{\lambda}_{mt}$ should consistently recover the parameter of interest, κ.

For robustness, we experiment with a few different choices of the set of other high-income countries when constructing the instrument. The first set follows Autor et al. (2013) and contains Australia, Finland, Germany, Japan, New Zealand, Spain, Switzerland, USA. We refer to this country set as the "ADH set". The second set is the EU member states before the 2004 EU enlargement, including Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden. We refer to this country set as the "EU set". To be consistent with Autor et al. (2013), we focus on a ten-year period from 1997 to 2007, which is characterized with major trade shocks.

4.2.3 Results

We report the estimation results of (30) in Table 4. To mitigate any potential time-varying confounding factors common to all products, we also control for year fixed effects in

Columns 2, 4 and 6. Columns 1-2 present the ordinary least squares (OLS) estimation result. The OLS estimator yields an estimated κ of almost zero.

However, the OLS estimator may be subject to omitted variable bias, so we use $\Delta \ln \tilde{\lambda}_{mt}$ as the instrument for $\Delta \ln \lambda_{mt}$ to obtain a more consistent estimate of κ. Columns 3-6 present the IV regression results using the "ADH set" and the "EU set" to construct our instrument. The Kleibergen–Paap (K-P) LM statistics are all statistically significant at the 1% level and the K-P F statistics are all larger than 10, so our IVs are not subject to under-identification or weak instrument. The IV estimator yields an estimate of κ ranging from -0.096 to -0.161. Therefore, an increase in domestic expenditure share by 1% drives down the number of domestic firms by about $0.1 - 0.16\%$, indicating an environment in which a larger number of firms survive in a product market with higher product attractiveness, rather than the opposite.

Table 4: IV Regression: Baseline

	Dependent Variable: Log change in number of firms					
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	IV	IV	IV	IV
Log change in domestic expenditure share	0.003	0.004	-0.115*	-0.161***	-0.096*	-0.153**
	(0.004)	(0.004)	(0.069)	(0.074)	(0.054)	(0.061)
Year FE	No	Yes	No	Yes	No	Yes
Country set for IV	.	.	ADH	ADH	EU	EU
Kleibergen-Paap LM stat.	.	.	12.819***	13.506***	13.785***	14.818***
Kleibergen-Paap F stat.	.	.	14.650	15.501	15.658	16.869
# Obs.	24,377	24,377	13,845	13,845	13,870	13,870

Estimation results of (30). Standard errors clustered at the CN 6-digit level are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels.

Our estimated elasticities of the number of firms with respect to the domestic trade share range between 0.003 (OLS specification) to -0.161 (one of the IV specifications). Despite the change in sign, these elasticities give a similar value of η between -1 and -0.96 (which corresponds to indirectly additive preferences) and a positive sorting of firms into products that have higher attractiveness despite the tougher competition.

5 Quantification

Using the calibrated parameters discussed in the previous section, this section presents the results of two quantification exercises. First, we aim to quantify the importance of heterogeneous product attractiveness for firm-product-level sales. Our model predicts that firm-sales within a certain product category are determined by two crucial factors: the product's attractiveness (which determines the endogenous level of competition) and the firm-product specific demand shock that accounts for any other factor that is not related to product attractiveness or competition.

Second, we infer the level of product attractiveness for each CN 8-digit good in our sample and explore its correlations with observable product-level characteristics. This analysis will help us understand how product features affect product attractiveness and can potentially provide insights into strategies that firms can use to improve their sales.

5.1 The Importance of Cutoff in Explaining Sales Variations

To measure the impact of heterogeneity in product attractiveness and competition on firm-level sales, we undertake two exercises. Firstly, we use the revenue predictions from our model and conduct a variance decomposition analysis to determine the proportion of total variance that each component of firm-level sales explains. This model-based decomposition provides insight into the relative importance of product attractiveness and endogenous competition on firm sales.

Secondly, we compare our model to a standard model used in the literature that does not account for product heterogeneity. By quantifying the differences between the two models, we can assess the importance of incorporating product heterogeneity into the analysis. This exercise will help us understand how much of the variance in firm sales can be explained by product heterogeneity and the potential limitations of using a model that ignores this heterogeneity.

5.1.1 Decomposing Firm-Level Sales

We can rewrite the revenue equation for a firm with unit cost c producing product m and selling it from j to j (as shown in equation (13) in the model section) as follows:

$$\ln(r(c))_m = FE_m + \ln\left[(c_m^* - c)^\gamma (c_m^* + \gamma c)\right] + \ln \delta_m(c) \tag{33}$$

where FE_m is a product fixed effect, and we drop origin and destination subscript since we only focus on domestic sales of Danish firms. By using data generated by the model, as we describe below, such a regression generates an R^2 of one. To assess the significance of the second component in (33), which represents the heterogeneity in cutoffs across products, we estimate the following regression:

$$\ln(r(c))_m = FE_m + \ln \delta_m(c) \tag{34}$$

We can then compare the R^2 of this regression to the R^2 of equation (33). The difference between the two R^2 values indicates the explanatory power of the term $\ln\left[(c_m^* - c)^\gamma (c_m^* + \gamma c)\right]$.

Apart from assessing the importance of heterogeneity in product attractiveness and competition, we are also interested in quantifying the significance of the demand shock term $\delta_m(c)$. In our model, the demand shock term captures all factors that may cause a firm to have different sales across products, aside from the product-specific attractiveness and competition (such as differences in productivity). To achieve this, we estimate the following regression:

$$\ln(r(c))_m = FE_m \tag{35}$$

We can calculate the quantitative significance of the demand shock by comparing the R^2 value of equation (34) to that of equation (35).

Simulation Algorithm. To conduct the counterfactual analysis, we need to simulate the sales of firms. Here is how we proceed. To ensure that we have the maximum number of observations available for our algorithm, we simulate the firms' sales conditional on them being active in the most popular market R, i.e., the market with the largest number of active firms N_R. We can normalize the market cutoff c_R^* for this market to one, without loss of generality.

Next, we derive the cost cutoffs for all other markets as follows:

$$c_m^* = \left(\frac{N_m}{N_R}\right)^{\frac{1}{\theta}}$$

where we use the result that $N_m = J(c_m^*/b)^\theta$.

For each Danish CN 2-digit industry, we consider the 20 most successful products based on the number of firms producing them, denoted by $m = 1, ..., 20$.[15] We then compute $\frac{N_m}{N_R}$ for each each product in each industry and take the average across industries.

[15] We consider only CN 8-digit products with at least 5 firms.

Using data on the average relative number of firms in each CN 8-digit code (which is equivalent to the subscript m), we can determine the cost cutoffs c_m^* for each market.

We simulate the draws of marginal cost c_f for 10,000 hypothetical firms. Subscript f denotes a firm level variable in our simulation. We first draw realizations of the uniform distribution u_f. Then, we compute the marginal cost draws as:

$$c_f = u_f^{\frac{1}{\theta}}$$

We compute the revenues of the firm f in market m as:

$$\tilde{r}_{fm} = (c_m^* - c_f)^\gamma (c_m^* + \gamma c_f)\delta_{fm}$$

where δ_{fm} is drawn from a log-normal distribution for each firm-product combination fm with mean one and standard deviation σ. To simplify our analysis, we can drop the multiplicative component of revenues from our model, as it will be captured by the product fixed effect. This assumption does reduce the explanatory power of the product-specific fixed effects, but that is not the main focus of our counterfactual analysis

Results. After simulating the revenues for our hypothetical firms, we can estimate the regression equations (33), (34), and (35). Results are in Table 5. The difference in R^2 between equation (33) and equation (34) is 26%. This indicates that the presence of cutoffs that are product-specific and depend on the product-specific levels of competition and market attractiveness can explain 26% of the variance in sales across firms. The remaining variance is largely due to the demand shocks, which account for 72% of the variance (i.e., in R^2 between (34) and (35)). We can achieve similar results using the parameter estimated under a lower trade elasticity assumption. Moreover, assuming a higher sales advantage (using moments for the US) yields the largest explanatory power for the heterogeneity in cutoffs of 82%.

We also replicate our analysis using the demeaned revenues per firm as the dependent variable (see Table 14 in the appendix). Specifically, we first regress the logarithm of firm revenues on firm fixed effects and then use the residual of this regression as the dependent variable in equations (33), (34), and (35). We find that the results are similar to those obtained using the original revenues as the dependent variable. The explanatory power of the product-level cutoffs ranges from 20-50%, while that of the demand shock ranges from 20-70%.

Table 5: Variance of Sales Across Firms

	Product-Level Cutoffs	Demand Shock
Baseline Estimates	26	72
Low Trade Elasticity	28	64
Higher Sales Advantage	82	16

The first column reports the difference in R^2 between (33) and (34). The second column reports the difference in R^2 between (34) and (35). Different rows use different parameters for the simulation of firm sales: the Baseline Estimates are reported in Table 3, the Low Trade Elasticity estimates are reported in Table 12, and the Higher Sales Advantage estimates are reported in Table 13. All values are multiplied by 100.

5.1.2 Model Comparison

For our second exercise, we consider a variance decomposition of the following regression:

$$\ln(r)_{cm} = FE_m + FE_c + \epsilon_{cm} \tag{36}$$

where FE_m is a product fixed effect, FE_c is a firm fixed effect, and ϵ_{cm} is the residual. In order to decompose the variance of log revenues across different components, we follow the approach used by Hottman et al. (2016) and Bernard et al. (2021). Specifically, we regress the estimated fixed effects on the logarithm of revenues without a constant, and use the resulting coefficient as a measure of the percentage of the variance in log revenues explained by the fixed effects.[16]

We consider two values for the logarithm of revenues. First, we use the logarithm of revenues generated by our baseline model. In this case, the variance in the residual captures both the demand shock and the interaction between the firm fixed effect and product fixed effect due to cutoff heterogeneity across products. Second, we consider the logarithm of revenues generated by a model in which the cutoffs across products are identical (and equal to one). In this case, the variance in the residual is due to the demand shock only. We use the same simulation algorithm used in the previous section. Results are in Table 6.

[16] To make full use of the Stata command *reghdfe*, we first regress $\ln(r)_{cm}$ on a constant, and then apply the decomposition outlined. This initial step is required to fully account for the variance with the fixed effects outlined: Without this initial step, part of the variance would be captured by the constant of the regression.

Table 6: Variance Decomposition - Baseline Parameters

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.076***	0.201***	0.723***
	(0.001)	(0.002)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000***	0.305***	0.695***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the baseline parameters are reported in Table 3.

Our results show that relative to the model with no cutoff heterogeneity, our baseline model features a larger explanatory power of the firm fixed effect and a smaller explanatory power of the residual and the product fixed effect. This suggests that when we examine the determinants of firm-product revenues and ignore the product-specific levels of attractiveness and competition that affect firm's choices, we tend to overestimate the explanatory power of firms and underestimate the explanatory power of the residual and of product characteristics.

We find similar results when we use alternative parameters to simulate revenues. Specifically, we use a lower trade elasticity and a higher sales advantage moment. In both cases, our results are consistent with those obtained using the baseline parameter values. However, we find that the differences in explanatory power between the models with and without cutoff heterogeneity are magnified when we use a higher value for the sales advantage moment (see Tables 15 and 16 in the appendix).

5.2 Product Attractiveness

Now, we can move on to inferring the level of attractiveness for each specific product, denoted as a_{mjt}, where we add a subscript t for year relative to the expression in the model. In this analysis, we will be specifically examining the product attractiveness of CN 8-digit codes in Denmark. By using equation (18), we can express the relative product attractiveness of product m compared to k as follows:

$$\frac{a_{mjt}}{a_{kjt}} = \left(\frac{\lambda_{mjjt}}{\lambda_{kjjt}}\right)^{-\frac{1+\eta}{1+\gamma(\eta+1)}} \left(\frac{c^*_{mjjt}}{c^*_{kjjt}}\right)^{-\frac{\theta+\eta+\theta\eta}{1+\gamma(\eta+1)}} \tag{37}$$

We define product k for each CN 2-digit sector and year as the product that has the highest number of firms producing a variety of that particular product. Using this definition, we can calculate the relative cutoffs as follows:

$$\frac{c^*_{mjjt}}{c^*_{kjjt}} = \left(\frac{N_{mjt}}{N_{kjt}}\right)^{\frac{1}{\theta}} \tag{38}$$

To compute the domestic expenditure shares λ_{mjjt}, we follow the same procedure outlined in Section 4.2, but this time we apply it to CN 8-digit products. With the estimated values for the parameters η, γ, and θ, we can then calculate the relative product attractiveness. Our baseline values are $\gamma = 0.59$ and $\theta = 4$. We test two different values for η: the estimated value of -1 using OLS, and the value of -0.97 estimated using the IV strategy. As there is not a significant difference between the two estimates, it is not surprising that the results do not vary significantly.

Next, we regress the log of $\frac{a_{mj}}{a_{kj}}$ on product-year specific variables. These variables include the log of total production and total domestic sales, the log of the number of firms, the log of exports and imports, and the export participation rate. Export participation rate is defined as the ratio of the number of firms exporting product m relative to the total number of firms producing product m. We also include sector-year fixed effects, where each sector is defined by a CN 2-digit code.

The results of our regression analysis are presented in Tables 7 and 17. We observe that the most attractive markets are typically characterized by the largest total production, total domestic sales, and number of firms. Export levels also tend to be higher in the most attractive markets, although this effect becomes less significant when we control for total production. Moreover, we find that export participation rates tend to be lower in more attractive markets. This result is likely driven by selection bias, as the larger number of firms in the most attractive markets means that a smaller proportion of them are engaged in exporting. Finally, we observe that more attractive markets also tend to have higher levels of imports.

Our model explicitly links product attractiveness to competition, and the estimated value of η suggests that the increase in competition resulting from higher product attractiveness is insufficient to drive out the least productive firms from the most attractive products. Moreover, our results also suggest that product attractiveness is linked to other product characteristics, such as the level of internationalization.

Table 7: Product Attractiveness and Market Characteristics

	Dependent Variable: Product Attractiveness							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Total Production	0.054***					0.066***	0.045***	
	(0.000)					(0.001)	(0.001)	
Log Domestic Sales		0.052***						
		(0.000)						
Log # Firms			0.254***					
			(0.000)					
Log Exports				0.030***		-0.006***		
				(0.000)		(0.001)		
Log Imports					0.047***		0.020***	
					(0.001)		(0.001)	
Export Participation								-0.058***
								(0.003)
CN2-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.71	0.71	1.00	0.61	0.65	0.69	0.72	0.57
# Obs.	29763	29763	29763	23733	29460	23733	29460	29763

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the OLS estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

6 Conclusions

We presented a novel model of multi-product firms in international trade that takes into account product-specific characteristics and their impact on firms' product mix decisions. Using Danish manufacturing data, we provide empirical evidence supporting the importance of product-specific factors in understanding firm behavior. Our model diverges from standard multi-product firm models by incorporating a discrete set of products, with firms offering unique varieties within each product category. We use a general demand function that allows for different sorting patterns of firms into product categories based on their attractiveness and level of competition. Our quantitative analysis demonstrates the significance of accounting for product-level attributes and the sorting patterns that emerge as a result. We find that incorporating product-specific levels of attractiveness and competition explains a quarter of the sales variance across firms. Moreover, we find that the most attractive products tend to be produced by all firms, while the least attractive products are made only by the most productive firms.

Our model can be used for alternative applications in which the characteristics of a certain market generate endogenous levels of competition that affect the sorting of firms into these markets. For instance, the model can be applied in the context of economic geography in which firms sort in different regions and the most attractive regions feature the toughest competition. Our model also provides a foundation for analyzing the effects of trade policy changes on firm product-mix, particularly in the context of product-specific tariffs and regulations.

References

Aghion, P., Bloom, N., Blundell, R., Griffith, R., Howitt, P., 2005. Competition and Innovation: an Inverted-U Relationship*. The Quarterly Journal of Economics 120, 701–728.

Aghion, P., Harris, C., Howitt, P., Vickers, J., 2001. Competition, imitation and growth with step-by-step innovation. The Review of Economic Studies 68, 467–492.

Arkolakis, C., Costinot, A., Donaldson, D., Rodrigues-Clare, A., 2019. The elusive pro-competitive effects of trade. Review of Economic Studies 86, 46–80.

Arkolakis, C., Costinot, A., Rodriguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94–130.

Arkolakis, C., Ganapati, S., Muendler, M.A., 2021. The extensive margin of exporting products: A firm-level analysis. American Economic Journal: Macroeconomics 13, 182–245.

Autor, D.H., Dorn, D., Hanson, G.H., 2013. The china syndrome: Local labor market effects of import competition in the united states. American Economic Review 103, 2121–2168.

Baldwin, J., Gu, W., 2009. The Impact of Trade on Plant Scale, Production-Run Length and Diversification. In NBER Chapters: Producer Dynamics: New Evidence from Micro Data , 557–592.

Bernard, A.B., Dhyne, E., Magerman, G., Manova, K., Moxnes, A., 2021. The Origins of Firm Heterogeneity: A Production Network Approach. Forthcoming Journal of Political Economy .

Bernard, A.B., Eaton, J., Jensen, J.B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93, 1268–1290.

Bernard, A.B., Redding, S.J., Schott, P.K., 2011. Multiproduct Firms and Trade Liberalization. The Quarterly Journal of Economics 126, 1271–1318.

Bertoletti, P., Etro, F., 2017. Monopolistic competition when income matters. Economic Journal In Press.

Bertoletti, P., Etro, F., 2020. Monopolistic competition with generalized additively separable preferences. Oxford Economic Papers 73, 927–952.

Bertoletti, P., Etro, F., Simonovska, I., 2018. International trade with indirect additivity. American Economic Journal: Micro 10, 1–59.

Boehm, C., Levchenko, A.A., Pandalai-Nayar, N., 2020. The Long and Short (Run) of Trade Elasticities. CEPR Discussion Papers.

Boehm, J., Dhingra, S., Morrow, J., 2022. The comparative advantage of firms. Journal of Political Economy 130, 3025–3100.

Chan, J.M., Irlacher, M., Koch, M., 2022. Multiproduct mergers and the product mix in domestic and foreign markets .

Chatterjee, A., Dix-Carneiro, R., Vichyanond, J., 2013. Multi-product firms and exchange rate fluctuations. American Economic Journal: Economic Policy 5, 77–110.

De Loecker, J., Goldberg, P.K., Khandelwal, A.K., Pavcnik, N., 2016. Prices, markups, and trade reform. Econometrica 84, 445–510.

Dhingra, S., 2013. Trading Away Wide Brands for Cheap Brands. American Economic Review 103, 2554–84.

Eaton, J., Kortum, S., 2002. Technology, geography, and trade. Econometrica 70, 1741–1779.

Eckel, C., Iacovone, L., Javorcik, B., Neary, J.P., 2015. Multi-product firms at home and away: Cost- versus quality-based competence. Journal of International Economics 95, 216–232.

Eckel, C., Neary, J.P., 2010. Multi-Product Firms and Flexible Manufacturing in the Global Economy. Review of Economic Studies 77, 188–217.

Edmond, C., Midrigan, V., Xu, D.Y., 2015. Competition, markups, and the gains from international trade. American Economic Review 105, 3183–3221.

Egger, H., Koch, M., 2012. Labour unions and multi-product firms in closed and open economies. Canadian Journal of Economics 45, 1456–1479.

Fally, T., 2018. Integrability and Generalized Separability. NBER Working Paper .

Feenstra, R., Ma, H., 2007. Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition. in E. Helpman, D. Marin and T. Verdier, eds., The Organization of Firms in a Global Economy, Harvard University Press .

Fieler, A.C., Harrison, A., 2018. Escaping Import Competition in China. Working Paper 24527. National Bureau of Economic Research.

Flach, L., Irlacher, M., 2018. Product versus process: Innovation strategies of multiproduct firms. American Economic Journal: Microeconomics 10, 236–77.

Flach, L., Irlacher, M., Unger, F., 2021. Corporate taxes and multi-product exporters: Theory and evidence from trade dynamics. Journal of International Economics 132.

Freitag, A., Lein, S.M., 2023. Endogenous product adjustment and exchange rate pass-through. Journal of International Economics 140, 103706.

Harrigan, J., Ma, X., Shlychkov, V., 2015. Export prices of us firms. Journal of International Economics 97, 100–111.

Head, K., Mayer, T., 2014. Chapter 3 - gravity equations: Workhorse,toolkit, and cookbook, in: Gopinath, G., Helpman, E., Rogoff, K. (Eds.), Handbook of International Economics. Elsevier. volume 4 of *Handbook of International Economics*, pp. 131–195.

Hottman, C., Redding, S.J., Weinstein, D.E., 2016. Quantifying the sources of firm heterogeneity. The Quarterly Journal of Economics 131, 1291—1364.

Hummels, D., Jørgensen, R., Munch, J., Xiang, C., 2014. The wage effects of offshoring: Evidence from danish matched worker-firm data. American Economic Review 104, 1597–1629.

Irlacher, M., 2022. Multi-Product Firms in International Economics. CESifo Working Paper Series 9589. CESifo.

Jung, J.W., Simonovska, I., Weinberger, A., 2019. Exporter heterogeneity and price discrimination: A quantitative view. Journal of International Economics 116, 103–124.

Lim, K., Trefler, D., Yu, M., 2022. Looking Backward, Innovating Forward: A Theory of Competitive Cascades. NBER Working Papers 30455. National Bureau of Economic Research, Inc.

Lopresti, J., 2016. Multiproduct firms and product scope adjustment in trade. Journal of International Economics 100, 160–173.

Macedoni, L., 2022. Large multiproduct exporters across rich and poor countries: Theory and evidence. Journal of Development Economics 156, 102835.

Macedoni, L., Weinberger, A., 2022. Quality heterogeneity and misallocation: The welfare benefits of raising your standards. Journal of International Economics 134, 103544.

Macedoni, L., Xu, M.J., 2022. Flexibility And Productivity: Toward The Understanding Of Firm Heterogeneity. International Economic Review 63, 1055–1108.

Manova, K., Zhang, Z., 2012. Export prices across firms and destinations. The Quarterly Journal of Economics 127, 379–436.

Mayer, T., Melitz, M.J., Ottaviano, G.I.P., 2014. Market Size, Competition, and the Product Mix of Exporters. American Economic Review 104, 495–536.

Mayer, T., Melitz, M.J., Ottaviano, G.I.P., 2021. Product Mix and Firm Productivity Responses to Trade Competition. The Review of Economics and Statistics 103, 874–891.

Melitz, M.J., 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. Econometrica 71, 1695–1725.

Melitz, M.J., Ottaviano, G.I.P., 2008. Market Size, Trade, and Productivity. Review of Economic Studies 75, 295–316.

Nocke, V., 2006. A gap for me: entrepreneurs and entry. Journal of the European Economic Association 4, 929–956.

Nocke, V., Yeaple, S., 2014. Globalization and multiproduct firms. International Economic Review 55, 993–1018.

Piveteau, P., Smagghue, G., 2022. Foreign Competition along the Quality Ladder. Conditionally Accepted at Economic Journal .

Qiu, L., Zhou, W., 2013. Multiproduct firms and scope adjustment in globalization. Journal of International Economics 91, 142–153.

Qiu, L.D., Yu, M., 2020. Export scope, managerial efficiency, and trade liberalization: Evidence from chinese firms. Journal of Economic Behavior & Organization 177, 71–90.

Simonovska, I., Waugh, M., 2014. The elasticity of trade: Estimates and evidence. Journal of International Economics 92, 34–50.

A Stylized Facts

Table 8: Within Firm Ranking - Year=2000

	Dependent Variable: Within firm rank of product sales			
(lr)2-5	(1)	(2)	(3)	(4)
Log Number of Firms	-7.105***	16.059***	7.258***	6.794***
	(0.742)	(1.061)	(1.084)	(1.097)
Log Domestic Sales		-13.537***	-16.649***	-15.859***
		(0.466)	(0.466)	(0.551)
Product Market Share			-0.688***	-0.690***
			(0.029)	(0.029)
Log Import Competition				1.203***
				(0.446)
Firm FE	Yes	Yes	Yes	Yes
R^2	0.73	0.76	0.78	0.78
# Obs.	8696	8696	8696	8696

Results from OLS of (1). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Table 9: Standard Deviation of Imports

	Standard Deviation of Import Growth	
	Within Firms	Across Firms
2000	0.39	0.42
2001	0.36	0.36
2002	0.43	0.40
2003	0.34	0.33
2004	0.33	0.35
2005	0.30	0.36
2006	0.33	0.44
2007	0.32	0.43
2008	0.38	0.39
2009	0.39	0.45
2010	0.35	0.39
2011	0.34	0.40
2012	0.34	0.41
2013	0.32	0.40
2014	0.32	0.45
2015	0.33	0.36
Average	0.35	0.40

The first column reports the average standard deviation of within-firm import growth rate while the second column reports the standard deviation of the average import growth rate within firms.

B Model

B.1 Equilibrium

Average revenues conditional on firms being active in product-origin-destination mij are computed as follows:

$$\begin{aligned}
\bar{r}_{mij} &= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^\gamma}{\xi_{mj}(c_{mij}^*)^\gamma}\int_0^{c_{mij}^*}(c_{mij}^*-c)^\gamma(c_{mij}^*+\gamma c)\frac{\theta c^{\theta-1}}{(c_{mij}^*)^\theta}dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^\gamma c_{mij}^*}{\xi_{mj}}\int_0^{c_{mij}^*}(1-c/c_{mij}^*)^\gamma(1+\gamma c/c_{mij}^*)\frac{\theta c^{\theta-1}}{(c_{mij}^*)^\theta}dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^\gamma c_{mij}^*}{\xi_{mj}}\int_0^1(1-t)^\gamma(1+\gamma t)\theta t^{\theta-1}dt = \\
&= \left(\frac{\tilde{T}_1\theta\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^\gamma c_{mij}^*}{\xi_{mj}}
\end{aligned}$$

where we used the change of variable technique ($t = c/c_{mij}^*$) and where

$$\tilde{T}_1 = \int_0^1 (1-t)^\gamma(1+\gamma t)t^{\theta-1} = \frac{\Gamma(\theta+2)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}$$

Aggregate revenues in market-origin-destination mij are then given by:

$$R_{mij} = J_i\left(\frac{c_{mij}^*}{b_i}\right)^\theta \bar{r}_{mij} = \left(\frac{\tilde{T}_1\theta\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right)\frac{J_i L_j w_i \tau_{mij} a_{mj}^\gamma (c_{mij}^*)^{\theta+1}}{\xi_{mj} b_i^\theta} \tag{39}$$

We can define a the expenditure share of country i in country j for product m as follows:

$$\lambda_{mij} = \frac{R_{mij}}{\sum_{v=1}^I R_{mvj}} = \frac{J_i(b_i\tau_{mij}w_i)^{-\theta}}{\sum_{v=1}^I J_v(b_v\tau_{mvj}w_v)^{-\theta}} \tag{40}$$

Let us consider the aggregate revenues in product-destination mj R_{mj}. Using (10) and

(8), we obtain:

$$\begin{aligned}
R_{mj} = \sum_{i=1}^{I} R_{mij} &= \left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j a_{mj}^{\gamma}}{\xi_{mj}} \sum_{i=1}^{I} J_i b_i^{-\theta} w_i \tau_{mij} (c_{mij}^{*})^{\theta+1} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j a_{mj}^{\gamma} (c_{mjj}^{*} w_j \tau_{mjj})^{\theta+1}}{\xi_{mj}} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j a_{mj}^{\gamma} (a_{mj} \alpha y_j \xi_{mj}^{1+\eta})^{\theta+1}}{\xi_{mj}} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) L_j a_{mj}^{\theta+\gamma+1} (\alpha y_j)^{\theta+1} \xi_{mj}^{(1+\eta)(\theta+1)-1} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta}
\end{aligned}$$

By market clearing $R_{mj} = L_j \alpha y_j$. Thus, the market aggregate ξ_{mj} is given by:

$$\xi_{mj} = \left[\left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) a_{mj}^{\theta+\gamma+1} (\alpha y_j)^{\theta} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta}\right]^{\frac{1}{1-(1+\eta)(\theta+1)}} \tag{41}$$

Substituting (41) into (8) yields the expression for the market cutoff, in which we use the fact that $\Phi_{mj}^{-\theta} = \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta}$:

$$c_{mij}^{*} = \left[\left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \Phi_{mj}^{-\theta}\right]^{-\frac{1+\eta}{\theta+\eta+\theta\eta}} a_{mj}^{-\frac{1+\gamma(\eta+1)}{\theta+\eta+\theta\eta}} (\alpha y_j)^{\frac{\eta}{\theta+\eta+\theta\eta}} (w_i \tau_{mij})^{-1} \tag{42}$$

Let $i = j$ and consider the cutoff of product m in the domestic market of j,

$$c_{mjj}^{*} = \left[\left(\frac{\tilde{T}_1 \theta \gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \Phi_{mj}^{-\theta}\right]^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} a_{mj}^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}} (\alpha y_j)^{\frac{\eta}{\theta+\eta+\theta\eta}} (w_j)^{-1} \tag{43}$$

The parameter $\Phi_{mj}^{-\theta}$ is the "multilateral resistance" term. A higher level of $\Phi_{mj}^{-\theta}$ means that country j is less "remote" to the suppliers of product m around the world, and thus should generate a higher overall demand for product m.

We find two opposite effects of $\Phi_{mj}^{-\theta}$ and $a_{mj}^{\gamma+\frac{1}{1+\eta}}$ on c_{mjj}^{*} by inspecting (43). On one hand, an increase in $\Phi_{mj}^{-\theta}$ or $a_{mj}^{\gamma+\frac{1}{1+\eta}}$ raises the overall demand for product m and tends to increase the cost cutoff c_{mjj}^{*} and the number of active firms. This effect is increasing in $\frac{1}{1+\eta}$. On the other hand, an increase in $\Phi_{mj}^{-\theta}$ or $a_{mj}^{\gamma+\frac{1}{1+\eta}}$ intensifies competition and results in tougher selection in the market, which tends to decrease the cost cutoff c^*_{mjj} and the number of active firms. This size of this effect is increasing in $1+\theta$. The net effect depends on the relative magnitude of $1/(1+\eta)$ and $-(1+\theta)$. If $-1 < \eta < -\frac{\theta}{1+\theta}$, the positive effect dominates the negative one, and an increase in $\Phi^{-\theta}_{mj}$ results in a less selective market with a larger number of active firms, a smaller average firm size measured by quantity, and a higher average price, so less productive firms can survive in such a market. Otherwise, the negative effect dominates and an increase in $\Phi^{-\theta}_{mj}$ or $a_{mj}^{\gamma+\frac{1}{1+\eta}}$ results in a tougher market environment with a smaller number of active firms, a larger average firm size, and a lower average price, so only the most productive firms can survive in this market.

So the elasticity of the domestic cutoff with respect to the multilateral resistance is:

$$\frac{d\ln c^*_{mjj}}{d\ln \Phi^{-\theta}_{mj}} = \frac{1}{\frac{1}{1+\eta} - (1+\theta)}, \tag{44}$$

while the elasticity of the domestic cutoff with respect to the product attractiveness is:

$$\frac{d\ln c^*_{mjj}}{d\ln a_{mj}} = \frac{\gamma + \frac{1}{1+\eta}}{\frac{1}{1+\eta} - (1+\theta)}. \tag{45}$$

Therefore, the ratio of cutoffs from i to j across different markets, not only depends on the demand shifters, but also on the market specific trade costs and on the market specific trade shares:

$$\frac{c^*_{mij}}{c^*_{kij}} = \left(\frac{\lambda_{mij}}{\lambda_{kij}}\right)^{\frac{1+\eta}{\theta+\eta+\theta\eta}} \left(\frac{a_{mj}}{a_{kj}}\right)^{-\frac{1+\gamma(\eta+1)}{\theta+\eta+\theta\eta}} \left(\frac{\tau_{kij}}{\tau_{mij}}\right)$$

Total Revenues from i to j are given by:

$$R_{ij} = \sum_{m=1}^{M} R_{mij} = \left(\frac{\tilde{T}_1\theta\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{(c^*_{kjj}\tau_{kjj}w_j)^{\theta+1}}{(c^*_{kjj}\tau_{kjj})^{\frac{1}{\eta+1}}} J_i(w_i b_i)^{-\theta} \sum_{m=1}^{M} L_j a_{mj}^{\gamma+\frac{1}{\eta+a}} \tau_{mij}^{-\theta}$$

The aggregate trade share is then defined as:

$$\lambda_{ij} = \frac{J_i(w_i b_i)^{-\theta} \sum_{m=1}^{M} L_j a_{mj}^{\gamma+\frac{1}{\eta+a}} \tau_{mij}^{-\theta}}{\sum_{v=1}^{I} J_v(w_v b_v)^{-\theta} \sum_{m=1}^{M} L_j a_{mj}^{\gamma+\frac{1}{\eta+a}} \tau_{mvj}^{-\theta}}$$

Let us now consider the free entry condition. First, let us derive average profits of active firms from i to j in product m:

$$\begin{aligned}
\bar{\pi}_{mij}(c) &= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mij})^\gamma} \int_0^{c^*_{mij}} (c^*_{mij} - c)^{1+\gamma} \frac{\theta c^{\theta-1}}{(c^*_{mij})^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} c^*_{mij}}{\xi_{mj}} \int_0^1 (1-t)^{1+\gamma} \theta t^{\theta-1} dt = \\
&= \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} c^*_{mij}}{\xi_{mj}}
\end{aligned}$$

where we used the same change of variable technique we used before and where

$$\tilde{T}_2 = \int_0^1 (1-t)^{1+\gamma} t^{\theta-1} dt = \frac{\Gamma(\theta+1)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}$$

The expected profits of producing for market-origin-destination mij are given by:

$$E[\pi_{mij}] = \left(\frac{c^*_{mij}}{b_i}\right)^\theta \bar{\pi}_{mij} = \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} (c^*_{mij})^{\theta+1}}{b_i^\theta \xi_{mj}}$$

By using the definition of aggregate revenues in a market-origin-destination (39), we obtain:

$$E[\pi_{mij}] = \frac{R_{mij} \tilde{T}_2}{J_i \tilde{T}_1} = \frac{R_{mij}}{J_i(\theta+1)}$$

We find the mass of entrants by using the zero profit condition as follows:

$$\begin{aligned}
\sum_{j=1}^{I}\sum_{m=1}^{M} E[\pi_{mij}] &= f_E w_i \\
\sum_{j=1}^{I}\sum_{m=1}^{M} \frac{R_{mij}}{J_i(\theta+1)} &= f_E w_i \\
\sum_{j=1}^{I} \frac{R_{ij}}{J_i(\theta+1)} &= f_E w_i \\
\sum_{j=1}^{I} \frac{R_{ji}}{J_i(\theta+1)} &= f_E w_i \\
J_i &= \frac{L_i}{f_E(\theta+1)}
\end{aligned}$$

where we used the trade balance condition $\sum_{j=1}^{I} R_{ij} = \sum_{j=1}^{I} R_{ji}$, market clearing condition $\sum_{j=1}^{I} R_{ji} = L_i y_i$, and we set $y_i = w_i$ by the zero profit condition.

This implies that the equilibrium cutoff is:

$$c^*_{mjj} = \alpha^{\frac{\eta}{\theta+\eta+\theta\eta}} \left(\frac{\tilde{T}_1 \theta L_j (b_j)^{-\theta} \gamma^{\gamma}}{f_E(\theta+1)(1+\gamma)^{1+\gamma}} \lambda^{-1}_{mjj} \right)^{-\frac{1+\eta}{\theta+\eta+\theta\eta}} a_{mj}^{-\frac{1+\gamma(\eta+1)}{\theta+\eta+\theta\eta}} \tau_{mjj}^{-\frac{\eta}{\theta+\eta+\theta\eta}} \tag{46}$$

Notice that the elasticity of the domestic cutoff with respect to the mass of entrants (44) is identical to the elasticity of the domestic cutoff with respect to the size of the country L_j.

Furthermore, notice that the number of firms producing a variety in product m from i to j equals:

$$N_{mij} = J_i \left(\frac{c^*_{mij}}{b_i} \right)^{\theta} = \frac{L_i}{f_E(\theta+1)} \left(\frac{c^*_{mij}}{b_i} \right)^{\theta} \tag{47}$$

Finally, changes in trade costs affect the domestic market cutoff c^*_{mjj} through the change in the domestic trade share:

$$\hat{c}^*_{mjj} = \hat{\lambda}_{mjj}^{\frac{1+\eta}{\theta+\eta+\theta\eta}} \tag{48}$$

C Calibration

C.1 Derivations

Demand Curvature Parameter γ. Recall that firm c revenues in the domestic economy of market m equal:

$$r_{mjj}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_j \tau_{jj} a_{mj}^{\gamma}}{\xi_{mj} (c_{mjj}^{*})^{\gamma}} (c_{mjj}^{*} - c)^{\gamma} (c_{mjj}^{*} + \gamma c) \delta_{mjj}(c) \tag{49}$$

Revenues can be re-written as:

$$r_{mjj}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^{\gamma} c_{mjj}^{*}}{\xi_{mj}} \left(1 - \frac{c}{c_{mjj}^{*}}\right)^{\gamma} \left(1 + \gamma \frac{c}{c_{mjj}^{*}}\right) \delta_{mjj}(c)$$

The average revenues equal (note that the shock δ is *i.i.d.* with mean one)

$$\begin{aligned}
\bar{r} &= \int_0^{c_{mjj}^{*}} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c_{mjj}^{*})^{\theta}} dc = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^{\gamma} c_{mjj}^{*}}{\xi_{mj}} \int_0^{c_{mjj}^{*}} \left(1 - \frac{c}{c_{mjj}^{*}}\right)^{\gamma} \left(1 + \gamma \frac{c}{c_{mjj}^{*}}\right) \frac{\theta c^{\theta-1}}{(c_{mjj}^{*})^{\theta}} dc
\end{aligned}$$

We apply the change of variable technique and substitute $t = c/c_{ex}^{*}$.

$$\begin{aligned}
\bar{r} &= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^{\gamma} c_{mjj}^{*}}{\xi_{mj}} \theta \int_0^1 (1-t)^{\gamma} (1+\gamma t)\, t^{\theta-1} dt = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^{\gamma} c_{mjj}^{*}}{\xi_{mj}} \theta T_1
\end{aligned}$$

where T_1 is a constant that equals:[17]

$$\begin{aligned}
T_1 &= \int_0^1 (1-t)^{\gamma} (1+\gamma t)\, t^{\theta-1} dt = \\
&= \int_0^1 (1-t)^{\gamma} t^{\theta-1} dt + \gamma \int_0^1 (1-t)^{\gamma} t^{\theta} dt \\
&= B(\theta, \gamma+1) + \gamma B(\theta+1, \gamma+1)
\end{aligned}$$

[17] $T_1 = \tilde{T}_1$, but is written in a different way here for convenience.

where $B(z,h)$ is the Euler Beta Function:[18]

$$B(z,h) = \int_0^1 (1-t)^{h-1} t^{z-1} dt$$

Let $c^*_{ex} = \max_{v \neq j}\{c^*_{mjv}\}$ denote the cutoff for exporters: the smallest cost-cutoff across destination markets. The average revenues for exporters equal:

$$\begin{aligned}
\bar{r}_{ex} &= \int_0^{c^*_{ex}} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c^*_{ex})^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a^\gamma_{mj} c^*_{mjj}}{\xi_{mj}} \int_0^{c^*_{ex}} \left(1 - \frac{c}{c^*_{mjj}}\right)^\gamma \left(1 + \gamma \frac{c}{c^*_{mjj}}\right) \frac{\theta c^{\theta-1}}{(c^*_{ex})^\theta} dc \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a^\gamma_{mj} (c^*_{mjj})^{\theta+1}}{\xi_{mj} (c^*_{ex})^\theta} \theta \int_0^{\frac{c^*_{ex}}{c^*_{mjj}}} (1-t)^\gamma (1+\gamma t) t^{\theta-1} dt = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a^\gamma_{mj} (c^*_{mjj})^{\theta+1}}{\xi_{mj} (c^*_{ex})^\theta} \theta T_2
\end{aligned}$$

where we applied the change of variable techique and where T_2 is a constant that equals:

$$\begin{aligned}
T_2 &= \int_0^{\frac{c^*_{ex}}{c^*_{mjj}}} (1-t)^\gamma (1+\gamma t) t^{\theta-1} dt = \\
&= \int_0^{\frac{c^*_{ex}}{c^*_{mjj}}} (1-t)^\gamma t^{\theta-1} dt + \gamma \int_0^{\frac{c^*_{ex}}{c^*_{mjj}}} (1-t)^\gamma t^\theta dt = \\
&= B\left(\frac{c^*_{ex}}{c^*_{mjj}}; \theta; \gamma+1\right) + \gamma B\left(\frac{c^*_{ex}}{c^*_{mjj}}; \theta+1; \gamma+1\right)
\end{aligned}$$

where $B(u:z,h)$ is the incomplete Euler Beta function:[19]

$$B(u;z,h) = \int_0^u (1-t)^{h-1} t^{z-1} dt$$

[18] In Matlab, this is computed with the function *beta(z,h)*.
[19] In Matlab, this is computed as *betainc(u,z,h)*beta(z,h)*.

Let us now consider the average revenues of non-exporters:

$$
\begin{aligned}
\bar{r}_{dom} &= \int_{c_{ex}^*}^{c_{mjj}^*} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^\theta - (c_{ex}^*)^\theta} dc = \\
&= \int_{c_{ex}^*}^{c_{mjj}^*} \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma c_{mjj}^*}{\xi_{mj}} \left(1 - \frac{c}{c_{mjj}^*}\right)^\gamma \left(1 + \gamma \frac{c}{c_{mjj}^*}\right) \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^\theta - (c_{ex}^*)^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \int_{\frac{c_{ex}^*}{c_{mjj}^*}}^{1} (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \left[\int_0^1 (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt\right] - \\
&\left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \left[\int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt\right] = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta (T_1 - T_2)
\end{aligned}
$$

The sales advantage moment M_{mj}^{sadv} equals:

$$
M_{mj}^{sadv} = \frac{\bar{r}_{ex}}{\bar{r}_{dom}} = \left[\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta - 1\right] \frac{T_2}{T_1 - T_2} = \tag{50}
$$

$$
= \left[\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta - 1\right] \frac{B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta; \gamma+1\right) + \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta+1; \gamma+1\right)}{B(\theta, \gamma+1) + \gamma B(\theta+1, \gamma+1) - B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta; \gamma+1\right) - \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta+1; \gamma+1\right)} \tag{51}
$$

where $\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta = N_{mjj}/N_{ex}$, where $N_e x$ is the number of exporters. The ratio is the reciprocal of export participation (i.e., the ratio of exporters to the total number of firms).

Standard Deviation of the Firm-Product Shock σ. Let us re-write revenues in the following way:

$$
r_{mjj}(c, \delta) = f(c)\delta \tag{52}
$$

where

$$f(c) = \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_{mj} w_i \tau_{jj} a_{mj}^\gamma c_{mjj}^*}{\xi_{mj}} \left(1 - \frac{c}{c_{mjj}^*}\right)^\gamma \left(1 + \gamma \frac{c}{c_{mjj}^*}\right) = r_{0mj} \left(1 - \frac{c}{c_{mjj}^*}\right)^\gamma \left(1 + \gamma \frac{c}{c_{mjj}^*}\right) \tag{53}$$

The variance of the product of two random variables equals:

$$Var(XY) = Var(X)Var(Y) + Var(X)E(Y)^2 + Var(Y)E(X)^2 \tag{54}$$

Hence, the variance of firms' revenues equals

$$Var(f(c)\delta) = Var(f(c))Var(\delta) + Var(f(c))E(\delta)^2 + Var(\delta)E(f(c))^2 \tag{55}$$

since the two variables are independent. The variance of δ is σ and the expected value of δ is 1. The expected value of $f(c)$ equals:

$$E(f(c)) = r_{0mj}\theta T_1 \tag{56}$$

We now have to compute the variance of $f(c)$. First, we compute the expected value of the square of $f(c)$.

$$\begin{aligned}
E(f(c)^2) &= \int_0^{c_{mjj}^*} f(c)^2 \frac{\theta c^{\theta-1}}{(c_{ex}^*)^\theta} dc = \\
&= r_{0mj}^2 \int_0^{c_{mjj}^*} \left(1 - \frac{c}{c_{mjj}^*}\right)^{2\gamma} \left(1 + \gamma \frac{c}{c_{mjj}^*}\right)^2 \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^\theta} dc = \\
&= r_{0mj}^2 \int_0^1 (1-t)^{2\gamma} (1+\gamma t)^2 \theta t^{\theta-1} dt = \\
&= r_{0mj}^2 \int_0^1 (1-t)^{2\gamma} \left(1 + 2\gamma t + \gamma^2 t^2\right) \theta t^{\theta-1} dt = \\
&= r_{0mj}^2 \theta \left[B(\theta, 2\gamma+1) + 2\gamma B(\theta+1, 2\gamma+1) + \gamma^2 B(\theta+2, 2\gamma+1)\right] = \\
&= r_{0mj}^2 \theta T_3
\end{aligned}$$

Hence, the variance of $f(c)$ equals:

$$Var(f(c)) = E(f(c)^2) - E(f(c))^2 = r_{0mj}^2 \theta \left(T_3 - \theta T_1^2\right) \tag{57}$$

We then obtain that:

$$
\begin{aligned}
Var(f(c)\delta) &= r_{0mj}^2\theta\left(T_3-\theta T_1^2\right)\sigma + r_{0mj}^2\theta\left(T_3-\theta T_1^2\right)+\sigma r_{0mj}^2\theta^2T_1^2 = \\
&= r_{0mj}^2\theta\left(T_3(1+\sigma)-\theta T_1^2\right)
\end{aligned}
$$

The coefficient of variation is given by:

$$
CV_{mj}=\frac{Var(f(c)\delta)^{0.5}}{E(f(c)\delta}=\frac{\theta^{0.5}\left(T_3(1+\sigma)-\theta T_1^2\right)^{0.5}}{\theta T_1} \tag{58}
$$

C.2 Measuring the Domestic Expenditure Share

The formula to calculate the domestic expenditure share in Denmark for product m and year t (23) is:

$$
\lambda_{mt}=\frac{Output_{mt}}{Output_{mt}+Imports_{mt}-Exports_{mt}} \tag{59}
$$

where $Output_{mt}$ is the value of manufacturing production in Denmark for product m at time t, $Imports_{mt}$ is the value of imports to Denmark, and $Exports_{mt}$ is the value of exports from Denmark. As our instrumental variable relies on export data at the HS 6-digit level from BACI, we define a product as a HS 6-digit code, which is analogous to the CN 6-digit classification.

While data on total imports and exports is available, data on the value of manufacturing output at the product level is accessible through the dataset VARS. As mentioned in the main text, VARS is a survey that covers all manufacturing firms that have at least 10 employees. Consequently, using the value of output from VARS can introduce bias in the calculation of domestic expenditure shares due to the under-representation of smaller firms. This underestimation of manufacturing output implies that the denominator of (59) can be negative. Given that we take logarithms in our calculations, this underestimation also results in a reduction in the number of observations.

To address this issue, we employ the following modified formula for the domestic expenditure share:

$$
\lambda_{mt}=\frac{\frac{Output_{mt}}{Exports_{mt}}}{\frac{Output_{mt}}{Exports_{mt}}+\frac{Imports_{mt}}{Exports_{mt}}-1} \tag{60}
$$

where $\frac{Output_{mt}}{Exports_{mt}}$ represents the ratio of output production to exports and is calculated solely using the exports of manufacturing firms included in VARS. Conversely, $\frac{Imports_{mt}}{Exports_{mt}}$ represents the ratio of imports to exports and is computed using aggregate imports and exports from the Danish customs data (UHDI). This method implicitly assumes that the ratio of output to exports for the firms sampled in VARS is representative of the ratio for the entire population of firms, which is a necessary approximation under the given circumstances. Using this approach, we significantly reduce the number of product-years where the numerator in the domestic expenditure share formula is negative.

C.3 Results

Table 10: Moments Across Years

Year	Trade Elast.	Sales Adv.	Export Part.	CV of Sales
2000	4	2.37	0.55	1.69
2001	4	2.46	0.55	1.75
2002	4	2.63	0.56	1.77
2003	4	2.57	0.58	1.84
2004	4	2.68	0.57	1.78
2005	4	2.99	0.57	1.90
2006	4	2.64	0.57	1.85
2007	4	2.59	0.59	1.73
2008	4	1.99	0.58	1.69
2009	4	3.69	0.56	1.78
2010	4	4.32	0.57	1.84
2011	4	3.52	0.59	1.80
2012	4	3.87	0.63	1.77
2013	4	3.67	0.62	1.86
2014	4	3.59	0.62	1.89
2015	4	3.68	0.62	1.87

Moments computed across years.

Table 11: Parameters Across Years

Year	θ	γ	σ
2000	4	0.59	2.18
2001	4	0.62	2.29
2002	4	0.66	2.30
2003	4	0.63	2.53
2004	4	0.66	2.31
2005	4	0.74	2.52
2006	4	0.65	2.53
2007	4	0.63	2.22
2008	4	0.45	2.38
2009	4	0.91	1.92
2010	4	1.02	1.89
2011	4	0.84	2.08
2012	4	0.87	1.95
2013	4	0.84	2.24
2014	4	0.82	2.35
2015	4	0.84	2.27

Parameters estimated using the moments shown in Table 10

Table 12: Moments and Parameters - Low Trade Elasticity

Moments	
Trade Elasticity	2
Exporters Sales Advantage	2.37
Export Participation	0.55
Coefficient of Variation of Sales	1.69
Parameters	
θ	2
γ	0.72
σ	2.21

Results from estimating the parameters using a lower trade elasticity. All other moments are as in (10).

Table 13: Moments and Parameters - US Moments

Moments	
Trade Elasticity	4
Exporters Sales Advantage	4.80
Export Participation	0.18
Coefficient of Variation of Sales	1.69
Parameters	
θ	4
γ	1.60
σ	0.86

Results from estimating the parameters using the exporters sales advantage and the export participation from Bernard et al. [2003]. All other moments are as in (10).

D Quantification

Table 14: Variance of Sales Within Firms

	Product-Level Cutoffs	Demand Shock
Baseline Estimates	20	70
Low Trade Elasticity	24	60
Higher Sales Advantage	50	20

The first column reports the difference in R^2 between (33) and (34). The second column reports the difference in R^2 between (34) and (35). The dependent variables in these regression is the residual of a regression of the log of firm revenues on firm fixed effects. Different rows use different parameters for the simulation of firm sales: the Baseline Estimates are reported in Table 3, the Low Trade Elasticity estimates are reported in Table 12, and the Higher Sales Advantage estimates are reported in Table 13. All values are multiplied by 100.

Table 15: Variance Decomposition - Low Trade Elasticity

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.177***	0.155***	0.668***
	(0.002)	(0.002)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000***	0.338***	0.662***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the parameters that are reported in Table 12.

Table 16: Variance Decomposition - US Moments

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.149***	0.482***	0.368***
	(0.002)	(0.003)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000	0.850***	0.150***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the parameters that are reported in Table 13.

Table 17: Product Attractiveness and Market Characteristics

	Dependent Variable: Product Attractiveness							
(lr)2-9	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Total Production	0.046***					0.059***	0.040***	
	(0.000)					(0.001)	(0.000)	
Log Domestic Sales		0.045***						
		(0.000)						
Log # Firms			0.206***					
			(0.000)					
Log Exports				0.024***		-0.008***		
				(0.000)		(0.000)		
Log Imports					0.037***		0.013***	
					(0.000)		(0.000)	
Export Participation								-0.056***
								(0.003)
CN2-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.72	0.72	0.99	0.60	0.64	0.70	0.72	0.57
# Obs.	29763	29763	29763	23733	29460	23733	29460	29763

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the IV estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.