Empirical Productivity Distributions and International Trade

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Abstract

We use firm-level data for 15 countries and 13 manufacturing sectors to estimate firm-level productivity parameters and to establish representative country-sector-specific empirical productivity distributions. We use these distributions against the backdrop of multi-sector versions of the models of Eaton and Kortum (2002) and Melitz (2003) to quantify the role of technology in shaping international trade flows. We find that, on average, absolute advantage measured as productivity differences across countries within sectors explains 15% and 21% of the total variation in bilateral trade shares in the models of Eaton and Kortum (2002) and Melitz (2003), respectively. In contrast, on average, comparative advantage measured as productivity differences across sectors within countries explains 39% and 47% of the variation in trade flows in these two models. We also demonstrate that empirical productivity distributions entail quantitatively important micro-to-macro implications for marginal responses of trade flows to changes in trade costs, for gravity-type estimation of trade models, and for comparative statics isomorphism between the customarily parameterized models of international trade. We confirm the predictions of the two aforementioned models under empirical productivity distributions in the data.

Keywords: Empirical trade analysis; Productivity distributions; Technology; Quantitative trade analysis

JEL-codes: F1; F10; F12.

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1 Introduction

Since the work of David Ricardo, trade economists have considered relative productivity differences between countries and sectors to be one of the main reasons for international trade. In the past two decades, this insight has been reinforced with new quantitative models of trade that rely on distributional representations of productivity differences across countries, sectors, and firms as the ones by Eaton and Kortum (2002) and Melitz (2003). In view of this, it is surprising that there is relatively little evidence regarding the quantitative importance of technological differences in productivity distributions for explaining the patterns of international trade.[1] The present work aims at filling this gap.

In this paper, we adopt a micro-to-macro approach to evaluate the role of technology for trade.[2] We start with firm-level balance-sheet data for 15 countries and 13 manufacturing sectors and estimate firm-level productivity parameters. We then develop a method of combining such estimates to arrive at representative country-sector-specific empirical productivity distributions that are robust to possible selection biases due to an underrepresentation of small firms in the data. We feed these distributions into multi-sector versions of two canonical models of international trade based on perfect and monopolistic competition as in Eaton and Kortum (2002) and Melitz (2003), respectively, and develop novel quantitative algorithms to accommodate the calibration and solution of these models under arbitrary nonparametric productivity distributions. Finally, we use calibrated model versions to assess the role of technology for international trade in two dimensions. On the one hand, conditional on other fundamentals, distributional productivity differences constitute an important determinant of the level of international trade flows across countries. We refer to this dimension as the *first-order effect* of technology on trade. On the other hand, productivity distributions also determine how changes in economic fundamentals such as reductions in variable trade costs drive changes in exports and imports, which we refer to as the *gradient effect* of technology on trade.

With this agenda, we contribute to the literature in four ways. First, to the best of our knowledge, this is the first paper that studies how productivity differences shape trade through the lens of a micro-to-macro approach that does not rely on specific parametric assumptions about productivity distributions. We find that, on average, the distributional differences across countries within each sector, which we refer to as *modified absolute advantage*, explain 15% and 21% of the total variation in trade flows in the Eaton-Kortum and Melitz model, respectively. On the other hand, within-country sectoral distributional differences, referred to as *modified comparative advantage*, on average explain 39% and 47% of the total variation in the Eaton-Kortum and Melitz model, respectively. For many country pairs and sectors, modified absolute and modified comparative advantage constitute

[1] Prior to the heterogeneous firms literature, Leamer (1984) and Harrigan (1997) quantified the importance of productivity for trade at the sectoral and country levels. Our focus, however, is different as we highlight the role of production and supply by heterogeneous firms as in Eaton and Kortum (2002) and Melitz (2003).

[2] Throughout this paper, we examine differences in technology as differences in total factor productivity (see Eaton and Kortum, 2002). There are, however, further dimensions of technology that might differ across countries and impact trade, like for instance production function parameters (see Boehm and Oberfield, 2020). We focus on the former, while keeping the latter fixed.

the single most important driver of export and import flows. In a next step, we examine the second dimension of the relationship between technology and trade, the gradient effect, and find that using empirical productivity distributions leads to substantially stronger responses of both international trade and welfare to changes in fundamentals such as trade costs. We also find that under equivalent reductions in variable trade costs, the marginal responses are decreasing in absolute terms across trade quantiles and that they are largely heterogeneous between the Eaton-Kortum and Melitz models. We demonstrate the relevance of this pattern for the gravity equation and macro-based model isomorphism with respect to changes in trade costs between the Eaton-Kortum and Melitz models and confirm the predictions of the two models under empirical productivity distributions in the data.

Second, our methodological contributions include developing a method of formulating representative country-sector productivity distributions that account for a possibility of non-randomly missing observations of smaller firms, which may pose a problem for many data sets when considering a larger cross section of countries and sectors. We combine firm-level data with coarser sectoral statistics on the true number of all firms across employment-size classes per country and sector in an inverse-probability-weighted estimation procedure that corrects for a possible associated firm-size-related selection bias.[3] The proposed methodology reduces the selection bias by 98.7% in a Monte Carlo simulation exercise which we present. Moreover, we develop numerical calibration and solution methods that allow solving large-scale models with heterogeneous firms under arbitrary productivity distributions.

Third, we contrast our approach with macro-based approaches that infer productivity differences from sectoral or aggregate data on trade flows. Such approaches always parameterize productivity distributions under a single few-parameters family such as the Fréchet or Pareto distributions. We show that such parameterizations overly simplify and vastly understate the extent of empirical productivity differences across countries, sectors, and their impact in shaping patterns of trade. We propose a statistical test based on the Kolmogovor-Smirnov statistic to show that typical parameterizations do not only lead to a poor approximation of the empirical distributions but, more importantly, that it is statistically impossible to assign the empirical distributions to a single few-parameter family. For example, we estimate that at the 10%-significance level only 12% and 5% of 195 country-sector productivity distributions can be placed under the umbrellas of the Fréchet and Pareto distributions, respectively

This paper is related to several strands of the literature. We relate to the pre-(heterogeneous-)firms literature on the role of total factor productivity differences across countries as a source of comparative advantage and driver of international trade including Leamer (1984) and Harrigan (1997, 1999). Relative to these papers, we evaluate the role of technological differences through all moments of firm-level productivity distributions. Doing so accounts for micro-to-macro mechanisms

[3]The data on the number of firms across five different employment sizes are available on the sectoral level for a large number of countries via the *OECD Structural and Demographic Business Statistics*.

that are absent in frameworks where productivity is a country-sector parameter rather than a firm-level characteristic. We also relate to Chor (2010), Costinot et al. (2012), Levchenko and Zhang (2014), Lind and Ramondo (2018) that examine Ricardian productivity differences in the Eaton-Kortum framework and to di Giovanni and Levchenko (2013), Melitz and Redding (2014), Hsieh and Ossa (2016) who examine how productivity differences affect trade and welfare in the Melitz framework. In contrast to the latter work, we do not rely on specific parametric assumptions to model technology distributions but build on their empirical form.

We contribute to the recent advancements in the *from-micro-to-macro* quantitative literature that employs firm-level data to infer features of the underlying firm heterogeneity and uses this information together with aggregate data to inform counterfactual analyses at the macro level (Hsieh and Klenow, 2009; di Giovanni et al., 2011; Eaton et al., 2011; Corcos et al., 2012; Eaton et al., 2013; di Giovanni and Levchenko, 2013; Arkolakis, 2016). We highlight the role of micro-based productivity distributions for correctly predicting aggregate responses. We show that using micro-data is essential for representing aggregate productivity differences across countries and sectors and for quantifying their effects on macro outcomes such as trade flows and real income.

Finally, the paper also contributes to the debate about which parametric families should be used to model productivity distributions to match certain data moments (Head et al., 2014; Freund and Pierola, 2015; Bas et al., 2017; Nigai, 2017; Fernandes et al., 2018). However, instead of trying to find a parametric family that would fit data on certain countries or sectors, we ask a fundamentally different question: can we statistically assign observed productivity distributions to any single few-parameter family? To that end, we propose a series of statistical tests that rely on the Kolmogorov-Smirnov statistic and show that neither of the seven most common fat-tailed distributions (including Fréchet, Pareto, and Log-normal) can subsume the empirical distributions under its umbrella. Hence, there is little hope for successful parsimonious parameterizations of empirical firm-productivity distributions. Moreover, we show that this is relevant both quantitatively as well as qualitatively. Customary parameterizations tend to understate the role of productivity differences for trade by constraining the dimensions in which technology can vary across countries, sectors, and firms which has important implications for the level and direction of trade flows, but also for marginal responses of trade flows to changes in trade costs, for the isomorphism between models in predicting the welfare gains from trade, and for gravity-type parameter estimation.

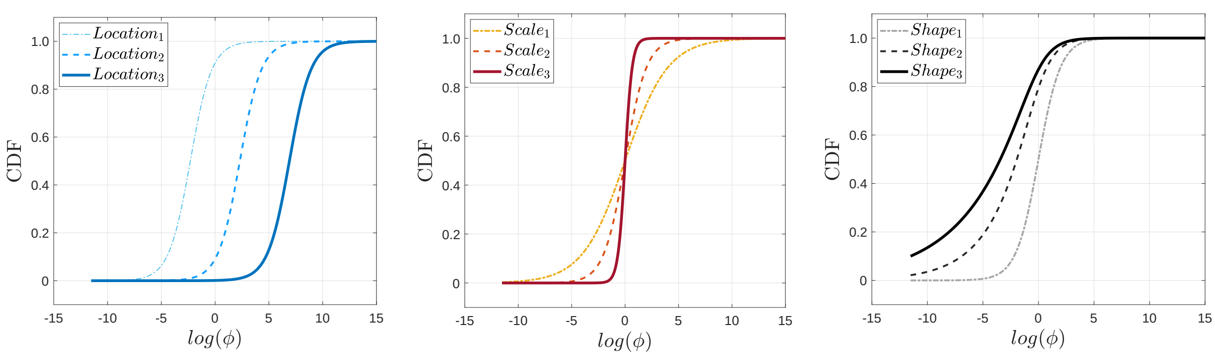
The remainder of the paper is organized as follows. In Section 2, we use a stylized two-country model to illustrate and develop intuition on why and how different moments of productivity distributions govern trade outcomes. In Section 3, we describe the procedure for estimating representative country-sector-specific productivity distributions from firm-level data. We examine dimensions in which the estimated empirical distributions differ across countries and sectors and whether they can be placed under the umbrella of a single few-parameter distribution family in Section 4. In Section 5, we the describe two multi-sector and multi-country general equilibrium models of trade based on perfect and monopolistic competition to be used in the subsequent counterfactual analysis. Section

6 describes the calibration procedure. We conduct a series of counterfactual experiments that quantify the role of technology for international trade and analyze micro-to-macro implications of using empirical productivity distributions in Section 7. In Section 8, we provide empirical support for the theoretical patterns uncovered in the previous sections. The last section offers a brief conclusion.

2 An illustration: The two country case

For illustration purposes, in this section we use simple two-country single-sector versions of the models of Eaton and Kortum (2002) and Melitz (2003) to provide intuition on how different moments of productivity distributions shape patterns of international trade. For that, we examine how both the first-order and the gradient effect matter for international trade in a stylized setting. We start with the first-order effect that determines how trade is shaped by differences in productivity across countries. We provide details on the multi-country and multi-sector versions of the two models used in the quantitative analysis in Section 5. However, here it suffices to use simpler, highly stylized model versions. Further details on the parameterization of the two-country model can be found in the Appendix.

Figure 1: Dimensions of productivity differences



Consider a world of two symmetric countries i and j and let $F_i(\phi)$ and $F_j(\phi)$ denote the cumulative distribution functions (CDFs) of firms' productivity ϕ in i and j, respectively. Though in reality there are infinitely many ways in which $F_i(\phi)$ and $F_j(\phi)$ may vary, it is instructive to discipline such differences and consider three particular dimensions of CDFs: location, scale, and shape.

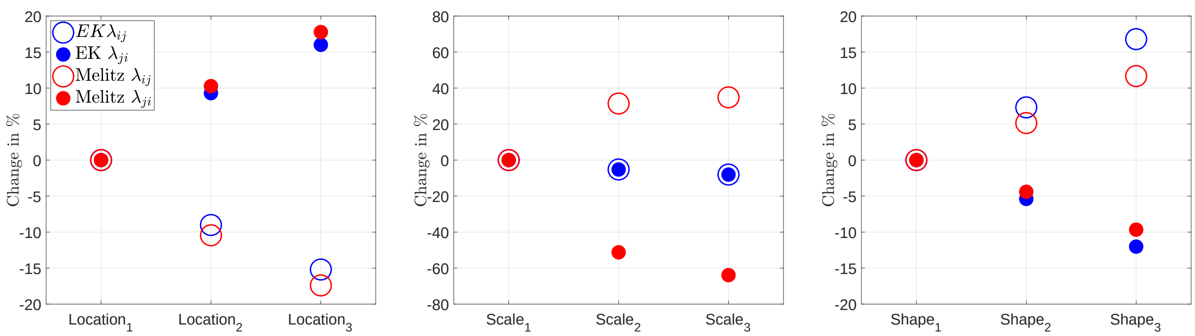
We illustrate how CDFs vary under changing location, scale, and shape in the left, middle, and right panels of Figure 1, respectively. In the overwhelming majority of earlier work, productivity distributions are allowed to differ across countries only in one of the three dimensions that we consider in Figure 1.[4] As we will show in Section 4, empirically there are many dimensions of

[4] This includes using the Fréchet distribution in different versions of the Eaton-Kortum model and the Pareto distribution in different versions of the Melitz model. In these two distributions, only scale parameters are allowed to vary across countries (within sectors) subject to a common shape parameter. Several papers employ alternative

heterogeneity in productivity distributions across countries and sectors that extend even beyond the three dimensions we consider in this section.

The main purpose here is to provide intuition and look at qualitative differences across productivity distributions and models and (some of) their economic consequences. We start with illustrating how location, scale, and shape alone govern patterns of international trade flows. At the outset, country i and country j are completely symmetric in all economic primitives. The productivity distributions are such that $F_i(\phi) = F_j(\phi)$ and are characterized by $\{Location_1, Scale_1, Shape_1\}$ as in Figure 1. We then change one dimension of $F_j(\phi)$ at a time by varying its location, scale, and shape, respectively, and illustrate how international trade flows between i and j respond. In this regard, we examine two trade outcomes – the import share of j from i, λ_{ij}, and the import share of i from j, λ_{ji} – in both the Eaton-Kortum and Melitz frameworks.

Figure 2: Dimensions of productivity differences and their impact on imports and exports in two canonical models of international trade



We report the results in Figure 2. In the left panel, we illustrate how λ_{ij} and λ_{ji} respond to a rightward shift of the entire CDF of j, $F_j(\phi)$. The intuition behind this shift is that all potential producers in j become more productive. This means that in both the Eaton-Kortum and the Melitz model exports of j to i will increase in terms of i's expenditures, whereas imports of j from i will decrease in terms of j's expenditures, which is confirmed in the left panel.

In the middle panel, we change the $F_j(\phi)$ by changing its scale as in Figure 1. As the density around the mode in $F_j(\phi)$ is becoming more pronounced, we see qualitative differences in the implications between the Eaton-Kortum and Melitz models. While in the Eaton-Kortum model increasing the scale of the productivity distribution leads to reductions in both λ_{ij} and λ_{ji}, in the Melitz model λ_{ij} increases and λ_{ji} decreases. The intuition behind these results is as follows. In relative terms, as $F_j(\phi)$ is becoming more centered, the differences between $F_i(\phi)$ and $F_j(\phi)$ become pronounced in both the left and right tails. Subject to positive international trade costs, this means that the mass of firms in j that are relatively more productive than in i in the right tail is shrinking but at the same time in the left tail the reverse is true. This simultaneous change leads to reductions in both

parametric distributions (Bas et al., 2017; Nigai, 2017; Fernandes et al., 2018). However, they assume that the distributions in all countries follow the same parametric family.

λ_{ij} and λ_{ji} in the Eaton-Kortum model. On the other hand, in the Melitz model, the moments of the distribution in the vicinity of the benchmark exporting cutoff play a relatively more important role due to selection effects. In this example – akin to typical data – this cutoff productivity is located in the right tail of the distribution (recall that relatively few and above-average productive firms export to a particular market). As the right tail becomes relatively more pronounced in i, λ_{ij} is increasing and λ_{ji} is decreasing. While we are well aware of the fact that the exact qualitative and quantitative deviations of the Melitz and Eaton-Kortum models in this illustration depend on the choice of parameters in this example, it is interesting to see that the aggregate implications of the two models – often considered to be isomorphic in the literature – can differ even in a very simplistic setting.

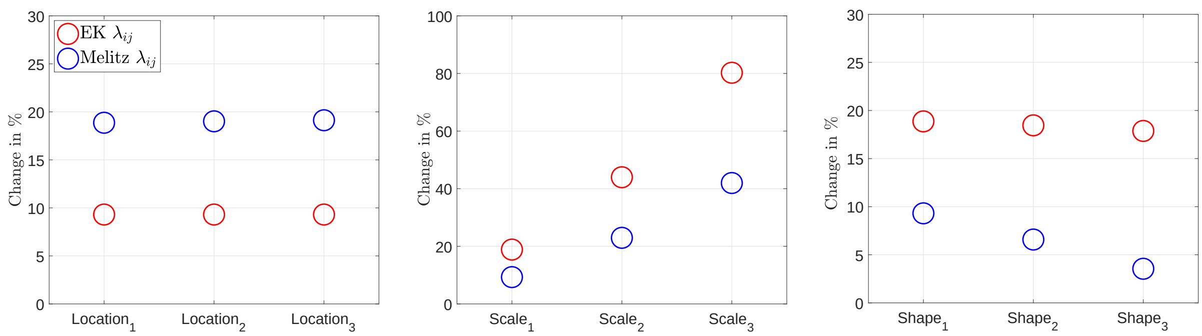
In the right panel of Figure 2, we plot changes in trade shares that exclusively result from changing the shape of the productivity distribution of j as in Figure 1. This change effectively shifts the weight from relatively more productive firms in the right tail of the distribution to relatively less productive ones in the left tail. In line with the intuition, this leads to a reduction in λ_{ji} and an increase in λ_{ij} in both the Eaton-Kortum and Melitz models.

The results in the three panels suggest that every dimension of heterogeneity in productivity distributions has potentially important implications for shaping international trade flows. Making any parametric assumptions a priori reduces the dimensionality of such heterogeneity. Quantifications of the role of technology for international trade, when they rely on stylized parametric representations of productivity distributions, run at risk of being biased, and the bias depends on whether one relies on an Eaton-Kortum-type or a Melitz-type model. We tackle this challenge by using empirical productivity distributions based on firm-level data.

In order to shed light on the second dimension of the interplay between technology and trade, we illustrate how different productivity distributions in the outset impact the trade-creation potential of trade-cost reductions. For that, we return to the outset of two symmetric countries with the same CDFs, $F_i(\phi) = F_j(\phi)$. Now we vary location, scale, and shape of the two distributions while keeping them symmetric and show the marginal effect of a 20% decrease in international trade costs on trade shares in Figure 3. Because countries are symmetric in every regard in the outset, for brevity we report changes in λ_{ij} only.

In the left panel of Figure 3, we show how the response of trade shares depends on the location of the productivity distributions. As illustrated, the marginal effect is largely independent of the location of the two distributions. A homogenous response as here is akin to the setting typically considered in the literature. In the middle panel, we illustrate how the reduction in trade costs would affect international trade shares under different scales of the productivity distributions. Both models suggest that the responses are generally larger when using $Scale_2$ and $Scale_3$ relative to $Scale_1$. This is intuitive because $Scale_2$ and $Scale_3$ increase the mass of firms in the center of the distributions such that a reduction in trade costs leads to a relatively larger mass of new exporters in the considered experiment. The same intuition is confirmed in the right panel of Figure 3, where we report how the shape of the distributions affects the magnitude of the marginal effects of trade liberalization. Changing $Shape_1$ to $Shape_2$ and $Shape_3$ increases the mass of firms in the left tail and reduces it in the right tail. Hence, a marginal reduction in trade costs would have a relatively lower number of new exporters, which leads to a less pronounced response. The intuition that the mass of firms in the neighbourhood of equilibrium quantities is decisive for the size of the gradient effect of technology on trade remains key in the more sophisticated and fully calibrated model in Section 7. This concept is also key to understanding heterogeneous responses of trade flows to trade cost changes observed in data.

Figure 3: Dimensions of productivity differences and their impact on responses of trade to changes in trade costs in two canonical models of international trade



Though the results presented in Figures 2 and 3 rely on simple two-country versions of the Eaton-Kortum and Melitz models, they illustrate that differences in productivity distributions across countries have three important effects on trade flows. First, such differences shape trade flows directly. Second, they affect how differences in other economic fundamentals such as trade costs influence patterns of international trade. Third, while models based upon Eaton and Kortum (2002) and Melitz (2003) can be calibrated and matched using the very same underlying data, marginal changes in fundamentals can have substantially different implications both quantitatively as well as qualitatively in these two models. The results also indicate that evaluating this encompassing role of technology quantitatively based on parametric assumptions about productivity distributions is dangerous, as such assumptions a priori constrain the dimensions and degrees in which technology can differ across countries, sectors, and firms. Hence, a quantification of the role of technology for trade requires using unconstrained empirical firm-level distributions. In the next section, we propose and describe a procedure which can obtain representative firm distributions for several economies.

3 Estimating representative country-sector productivity distributions

3.1 Empirical strategy to estimate firm-level productivities

In this section, we describe the estimation of productivity distributions. The estimation of firm-level productivity produces estimates of ϕ that will constitute the input for the nonparametric estimation of country-sector-level productivity distributions. In estimating firm-level productivity we closely follow the approach advanced by Ackerberg et al. (2015) and de Loecker and Warzynski (2012).

Since we make use of the time dimension for identifying firm-level productivity, which we explain in detail in what follows, let us write the production function of a firm f using a time index t and, for the ease of exposition, let us suppress country and sector indices. Assuming a fixed proportion of materials m_{ft} is used to produce a unit of output, we can specify the production function in terms of value added q_{ft} as a function of physical labor input l_{ft} and capital input k_{ft} which is scaled by total-factor productivity ϕ_{ft}:

$$q_{ft} = \phi_{ft} f\left(\ln(l_{ft}), \ln(k_{ft}), \right). \tag{1}$$

As we do not observe physical units, we consider deflated value added $\ln(\tilde{q}_{ft}) = \ln(q_{ft}) + \epsilon_{ft}$ to be a valid proxy thereof, as is customary in the firm-level productivity-estimation literature.[5] Further specifying production to be Cobb-Douglas and using ϵ_{ft} to denote a disturbance term that captures any measurement error of the left-hand-side variable, the empirical counterpart of the production function is given by:

$$\ln(q_{ft}) = \ln(\phi_{ft}) + \beta_l \ln(l_{ft}) + \beta_k \ln(k_{ft}) + \epsilon_{ft}. \tag{2}$$

In order to recover unobserved productivity ϕ_{ft}, the production-function parameters have to be consistently estimated. Since unobserved productivity is potentially correlated with input choices, we rely on the so-called proxy method of productivity estimation (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015). In this paper, we rely on the material inputs to proxy for unobserved productivity shocks. Specifically, we assume that material input choice can be described as:

$$\ln(m_{ft}) = m_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(\phi_{ft}), \mathbf{z}_{ft}\right), \tag{3}$$

where $\mathbf{z}_{ft}$ denotes additional variables that might affect optimal input demand. In our context we control for such factors by 4-digit-sector-time fixed effects. This specification considers capital and labor input to be state variables, which are determined prior to t and affect optimal intermediate-

[5] We deflate value added using sector-country-time-specific deflators (see, e.g., Amiti and Konings, 2007 and de Loecker and Warzynski, 2012).

input demand. Assuming monotonicity of (3) in productivity, one can specify productivity as the inverse of this function:

$$\ln(\phi_{ft}) = h_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}), \mathbf{z}_{ft}\right). \tag{4}$$

The parameters of the production function are recovered in a two-step procedure. First, we run a first-stage model that allows for all variables to enter in a flexible way without identifying specific production-function parameters at this stage:

$$\ln(q_{ft}) = \nu_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}), \mathbf{z}_{ft}\right) + \epsilon_{ft}. \tag{5}$$

With these estimates at hand, we can obtain expected added value as $\hat{\nu}_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}), \mathbf{z}_{ft}\right) = \hat{h}_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}), \mathbf{z}_{ft}\right) + \hat{\beta}_l \ln(l_{ft}) + \hat{\beta}_k \ln(k_{ft})$ and $\hat{\epsilon}_{ft}$. Assuming a first-order Markov process for the evolution of productivity allows us to specify the following:

$$\ln(\phi_{ft}) = g_t(\ln \phi_{ft-1}) + \xi_{ft}, \tag{6}$$

where we can identify all production-function parameters in a second-stage model. To do so, we back out productivity as a function of β_l and β_k:

$$\ln(\phi_{ft}) = \hat{\nu}_t\left(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}), \mathbf{z_{ft}}\right) - \beta_l \ln(l_{ft}) - \beta_k \ln(k_{ft}) \tag{7}$$

and use then both $\ln(\phi_{ft})$ and $\ln(\phi_{ft-1})$ in (6) to obtain estimates for β_l and β_k using the moments

$$E\left(\xi_{ft}(\beta_l, \beta_k)\begin{pmatrix} \ln(l_{ft}) \\ \ln(k_{ft}) \end{pmatrix}\right) = 0. \tag{8}$$

Since capital and labor are assumed to be state variables and decided upon one period ahead, they should not be correlated with ξ_{ft}. Hence, we assume that labor and capital input are not adjusted following contemporaneous shocks in productivity.[6]

3.2 Data and estimation

The main data sources are two. We use firm-level balance-sheet data as provided by Bureau van Dijk's ORBIS database complemented by balance-sheet data for Chinese firms as provided by the Chinese Statistical Office. Apart from these data we rely on sector-country-level deflators taken from the World Input-Output database (WIOD) to obtain measures of real values for all variables

[6] As noted by Ackerberg et al. (2015) this assumption leads to more precise estimates compared to using $\ln(l_{f,t-1})$ as instrument which would relax the assumption of labor input being predetermined.

measured in monetary terms.[7] In order to ensure comparability across countries, we adjust all values using sector-country-level estimates of purchasing power parity obtained from the GGDC Productivity Level Database. For the main specification, we use an unbalanced panel of 2,990,398 firm-year observations over the years 2000-2012 in $I = 15$ different countries and $S = 13$ 2-digit ISIC manufacturing sectors. The variables of interest are defined as follows: (i) l_{ft} is the average head count of the labor force of firm f in year t, (ii) k_{ft} is the deflated stock of fixed assets, (iii) m_{ft} are deflated intermediate input expenses, and (iv) q_{ft} is deflated value added which is calculated as the difference between operating revenues and intermediate-input expenses. Descriptive statistics for each country are presented in Table 1.

Table 1: Descriptive statistics (medians) of main inputs in the production function (in mn. Euros).

Country	m_{ft} **Materials**	l_{ft} **Employment**	k_{ft} **Capital**	q_{ft} **Value Added**	**Firm-year obs.**
BGR	0.17	22	0.18	0.26	48,363
CHN	4.38	130	1.42	1.24	746,851
CZE	0.77	23	0.22	0.68	86,039
DEU	9.86	89	2.93	11.39	51,676
ESP	0.31	8	0.21	0.37	426,966
EST	0.14	7	0.05	0.14	20,058
FIN	0.39	9	0.23	0.70	45,712
FRA	0.24	8	0.13	0.61	276,597
HUN	3.37	74	1.84	3.06	12,863
ITA	1.00	13	0.50	1.35	464,537
KOR	3.06	17	1.12	0.67	223,174
PRT	0.12	7	0.08	0.20	163,857
ROM	0.06	7	0.04	0.07	264,847
SVN	0.31	7	0.30	0.39	32,476
SWE	0.23	5	0.08	0.39	126,382

In the main specification, we estimate separate production functions for each of the 15 countries and 13 sectors.[8] We approximate the flexible function $\nu_t(\cdot)$ of the first stage in (5) by including all linear interaction terms of the variables $(\ln(l_{ft}), \ln(k_{ft}), \ln(m_{ft}))$ and allowing for 4-digit-sector-year shifters.[9] In the second stage, we specify the evolution of productivity over time as in (6) to allow lagged productivity to enter with a linear, a quadratic, and a cubic term and additionally include 4-digit-sector-year shifters. Descriptive statistics of the estimated input coefficients across WIOD-sector classifications are presented in Table 2. Using the estimates from both stages, we can calculate productivity from equation (7).[10]

[7]We extrapolate missing years in the WIOD data using GDP data as provided by the World Development Indicators.

[8]We exclude sector ISIC 3.1 23 (Coke and refined petroleum) as this sector contains too few firms to allow for an estimation of productivity distributions.

[9]The 4-digit-sector-year shifters control for, e.g., sector-year-specific demand/supply shocks.

[10]We also consider two other alternative productivity estimates: (i) real value added per worker and (ii) total-factor productivity estimates that are obtained using sector-specific production-function parameters as opposed to sector-country-specific ones.

Table 2: Descriptive statistics (medians) of estimated input coefficients in the production function across sectors.

	Sector	WIOD classification	β_l	β_k
1	Food, beverages, tobacco	15t16	0.86	0.24
2	Textiles	17t18	0.83	0.16
3	Leather	19	0.82	0.15
4	Wood and cork	20	0.84	0.17
5	Pulp and paper	21t22	0.81	0.17
6	Chemicals	24	0.72	0.24
7	Rubber and plastics	25	0.83	0.18
8	Other non-metallic mineral	26	0.78	0.21
9	Basic metals	27t28	0.84	0.16
10	Machinery n.e.c.	29	0.84	0.14
11	Electrical and optical equipment	30t33	0.83	0.15
12	Transport equipment	34t35	0.84	0.16
13	Manufacturing n.e.c.	36t37	0.84	0.16

3.3 Accounting for selectivity

A closer look at the estimation sample presented in Table 1 reveals that the firms present in the data are selectively large, especially when considering countries with relatively poorer coverage as measured by the number of firms in the sample relative to country size. As the productivity distribution of large firms may not be representative of the distribution in the overall economy, we take the problem of sample selection into account and propose a method to correct for it when estimating empirical productivity distributions. Specifically, we resort to inverse-probability-weighted estimation of the underlying distribution using information on the true size distributions of firms in countries and sectors (see Wooldridge, 2007; Wang, 2008).

Let us consider the entire population of firms in the year 2008 and sector s of country i.[11] We refer to the firms that are missing in the productivity estimation sample by $M_f = 1$ and by $M_f = 0$ to the ones for which we have a productivity estimate at hand. Let us further assume that, conditional on the size of a firm, the probability to be missing in the data is random after conditioning on some firm characteristics. Specifically, we explicitly allow for the fact that the probability for a firm to be missing in the estimation sample might be different for firms of different size and, hence, productivity. As long as for any firm with the *same* number of employees the probability to be missing in the data is not related to the level of productivity, the following holds:

$$Pr(M_f = 0|\phi_f, l_f) = Pr(M_f = 0|l_f) \equiv \Delta_f(l_f). \tag{9}$$

This condition guarantees that the probability of a firm to be non-missing in the estimation sample is a function of its employment level only. If we knew the true number of firms existing for each

[11]Note that the subsequent analysis is conducted at the country-sector level, but we skip the respective indices in the following and focus on a generic country and sector. Moreover, we drop the time index here, as we consider only a cross-section of data for the year 2008 to estimate the detailed moments of productivity distributions across countries and sectors.

level of employment, $N^{True}(l_f)$, we could easily calculate the probability that an observation is non-missing conditional on its level of employment:

$$\Delta_f(l_f) = \frac{N^{Sample}(l_f)}{N^{True}(l_f)}, \tag{10}$$

where $N^{Sample}(l_f)$ denotes the number of firms with employment level l_f observed in the estimation sample. While we do not observe $N^{True}(l_f)$ directly, we know the distribution of firms across employment-size bins as well as the total number of firms for any sector and any country.[12]

Specifically, the *OECD Structural and Demographic Business Statistics* (SBS) database provides information on the distribution across five size classes for all countries in our data except for China.[13] These size classes are (i) firms with 1-9 employees, (ii) firms with 10-19 employees, (iii) firms with 20-49 employees, (iv) firms with 50-249 employees, and (v) firms with at least 250 employees. Using these data on firm-size distributions in a histogram with different bin widths, we fit a piecewise-linear function to obtain an estimate of the true relative frequency of firms at any level of employment. To do so, we take the midpoint of each histogram bar and connect it via a linear piecewise function to the midpoints of neighbouring bars, respectively. With these estimates at hand and the true total number of firms in a country and sector, we obtain an estimate of $\widehat{N}^{True}(l_f)$ that allows us to calculate $\widehat{\Delta}_f(l_f)$ as follows:

$$\widehat{\Delta}_f(l_f) = \frac{N^{Sample}(l_f)}{\widehat{N}^{True}(l_f)}. \tag{11}$$

We can now obtain a consistent estimate of the nonparametric distribution of firms under the maintained assumptions:

$$\widehat{f}(\phi) = \frac{1}{nh_n} \sum_{f=1}^{n} \frac{(1 - M_f)}{\widehat{\Delta}_f(l_f)} K\left(\frac{\phi - \phi_f}{h_n}\right), \tag{12}$$

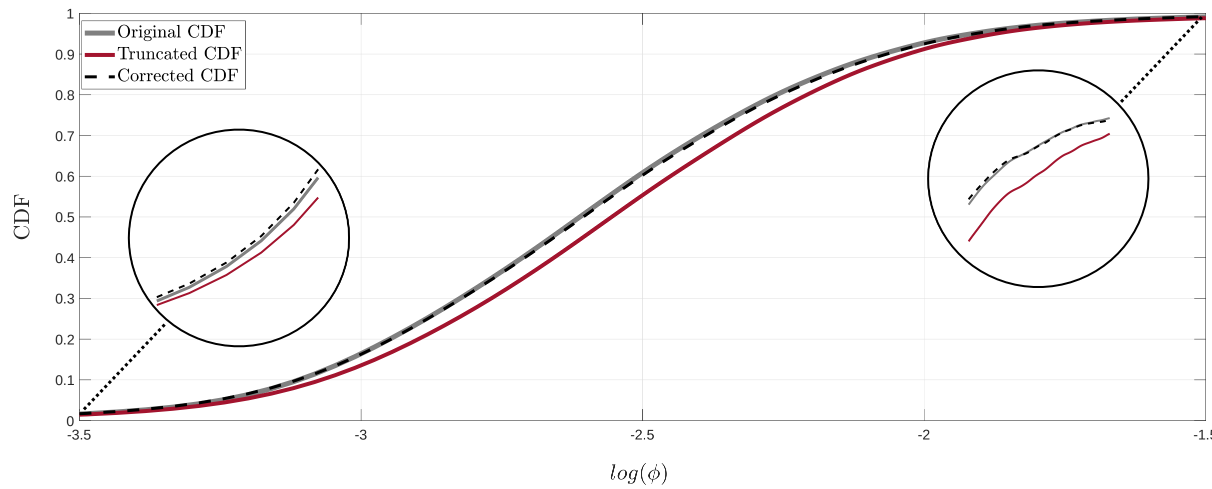
where h_n denotes the bandwidth, n is the true number of firms in a country's sector and $K(\cdot)$ is a kernel function. We use a second-order Epanechnikov kernel adapted to weighted kernel density estimation following Wang and Wang (2007) and determine the optimal bandwidth by likelihood cross validation (see Li and Racine, 2006).

Let us assess how well the latter approach works using a concrete example. For this, we take the estimation sample of Spain which provides one of the best firm coverages in the sample. Let us consider this sample as the true (untruncated) data and calculate the size distribution using five size classes as defined in the SBS data. Then we randomly discard 80% of the observations among

[12] Later on, we will use the fact that $\sum_{(l_f)} N^{True}(l_f)$ within a country and sector is equal to the total measure of firms in that country and sector, N_i^s.

[13] A similar data set based on two size classes is available for China from the Chinese Statistical Office. The two size classes comprise firms above and below 300 employees, respectively.

Figure 4: Monte-Carlo exercise to illustrate the methodology to account for selectivity



the 80% smallest firms. In total, we end up with a sample consisting of 36% of the original data with the average size of the remaining firms being bigger than in the outset. In the next step, we estimate the inverse-probability weight as described above using five size classes that are informed by the original data but provide only as much information as the data provided by the OECD SBS data, and estimate an inverse-probability-weighted kernel density. Figure 4 depicts the original (”true”) distribution, the truncated distribution, and the predicted distribution using the selected data together with the proposed methodology. Comparing the predicted distribution to the *true* distribution shows that the updating mechanism performs very well. In particular, by artificially dropping observations from the distribution we introduce a positive sum of squared deviations between the actual and truncated CDFs. Our correction procedure allows to reduce the sum of squared deviations by 98.7%.[14]

4 Empirical productivity distributions

4.1 Distribution heterogeneity between sectors and countries

We present estimated CDFs for all $15 \cdot 13 = 195$ country-sector productivity distributions in the left panel of Figure 5. By way of comparison, we observe a high level of heterogeneity of the CDFs across countries and sectors. This heterogeneity stems from large differences in location, scale, and shape of the distributions as well as all other dimensions. Hence, the results of our analysis in Section 2, which showed how such differences in distributions shape trade flows, are indeed relevant for the real data.

To formally test and measure the magnitude of deviations between two CDFs we use the

[14]Upon artificial truncation, the sum of squared deviations in terms of CDF points is 0.0947, which decreases to 0.0012 when using the proposed correction.

Kolmogorov-Smirnov (KS) statistic. This statistic is the maximum absolute distance of two CDFs, $F_A(\phi)$ and $F_B(\phi)$: $KS_{AB} = \sup_\phi |F_A(\phi) - F_B(\phi)|$, where $\sup_\phi$ denotes the supremum and $F_A(\cdot)$ and $F_B(\cdot)$ are two generic CDFs. We illustrate this measure in the right panel of Figure 5 where we plot two country-sector distributions and KS statistics between them. The KS statistic is useful because it allows us to test whether any two distributions, $F_i^s(\phi)$ and $F_j^{s\prime}(\phi)$, are statistically different from each other or not. We conduct the test for $195 \cdot (195-1)/2 = 18,915$ pairs of which a large number are statistically different from each other. With the data at hand, the null hypothesis of $F_i^s(\phi) = F_j^{s\prime}(\phi)$ is rejected at the 10% significance level for more than 87% of the pairs.

Figure 5: Illustration of CDFs and Kolmogorov-Smirnov statistic.

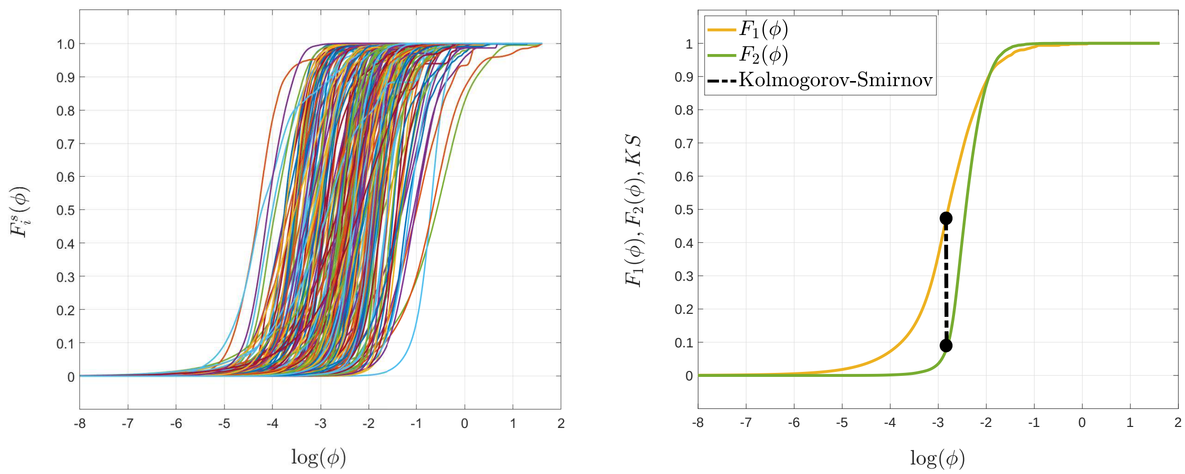
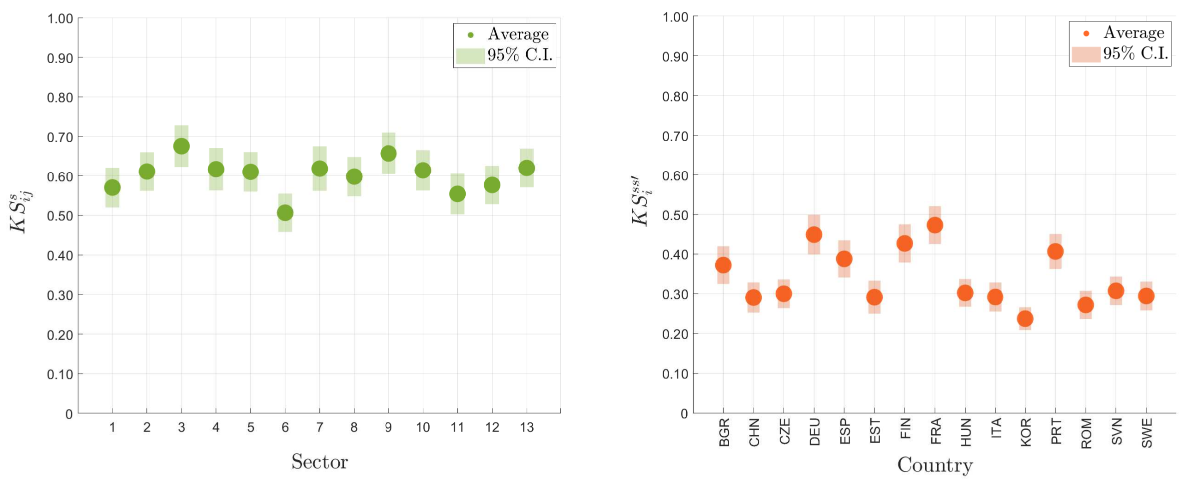


Figure 6: Illustration of CDFs and Kolmogorov-Smirnov statistic.



The heterogeneity stems from distributional differences across countries as well as sectors. To examine which one of the two dimensions is relatively more important, we calculate KS statistics across countries within each sector, KS_{ij}^s, and across sectors within each country, $KS_i^{ss\prime}$. We plot the former and the latter along with the 95% confidence bands in Figure 6. The numbering of sectors corresponds to the list of sectors described in Table 2. The left panel of the figure suggests that, on average, KS_{ij}^{s} amounts to about 0.60 and is quite stable across sectors. On the other hand, within-country differences in productivity distributions across sectors are relatively smaller with an average value of $KS_{i}^{ss\prime}$ of 0.34. However, the variation of KS statistics across different countries is relatively larger than that across sectors.

4.2 Empirical productivity distributions versus customary parameterizations

In this section, we conduct a series of statistical tests aimed at determining whether the empirical distributions that we have calculated can be parameterized under a single parametric family. Some earlier work compared how a specific parametric distribution model fitted the data in various dimensions usually using special cases of a single country or sector.[15] The goal here is not to choose some or the best-fitting customary parametric model among its alternatives but rather to establish a general upper bound for the potential success of such parameterization approaches.

We consider seven customary fat-tailed distributions that have been used to model firm-level heterogeneity and list them along with their CDFs in Table 3. We allow all parameters of each distribution to vary across countries and sectors but constrain them to be of the same family. Typically, quantitative trade theory allows only a subset of parameters to vary across countries and sectors. In a subset of our tests, we allow for a higher flexibility relative to the common assumptions by allowing all estimated productivity distribution parameters to vary across countries and sectors. For each parametric family, we estimate country-sector-specific parameters using a QQ-estimator as in Nigai (2017). The estimator minimizes the sum of squared differences between (log) observed quantiles of productivity and their predictions as follows:

$$\Theta = \min_{\Theta} \left\{ \sum_{q} \left(QE(q) - QP(q|\Theta)\right)^2 \right\}, \tag{13}$$

where $QE(q)$ is the log of the empirical quantile q of productivity and $QP(q|\Theta)$ is its parametric prediction conditional on the parameter vector Θ. In this equation and as specified in Table 3 for each parametric distribution family Ω, Θ is the vector of country-sector location parameter b_i^s and sector or even country-sector shape parameters θ_i^s for a generic country and sector. Once we obtain an estimate thereof which we denote by $\widehat{\Theta}$, we use it to calculate $\widehat{F}_i^s(\phi|\Omega)$ for each parametric family Ω in Table 3. We then use the respective associated KS-statistic to test whether the parametric prediction $\widehat{F}_i^s(\phi|\Omega)$ is statistically different from the empirical $F_i^s(\phi)$ or not. We report the share of distributions that are statistically different from the empirical distribution under each parametric family in Table 3.

For each pairwise comparison (with one pair referring to a sector, country, and parametric family versus the empirical analogue) of CDFs, we conduct a KS test under the null that $H_0 : F_i^s(\phi) = \widehat{F}_i^s(\phi|\Omega)$ at the 5% and 10% significance levels. First, we constrain the shape parameter to be

[15] For example, see di Giovanni et al. (2011), Corcos et al. (2012), Eaton et al. (2013), Head et al. (2014), Freund and Pierola (2015), Bas et al. (2017), and Fernandes et al. (2018).

Table 3: Common parametric distributions

Family (Ω)	CDF ($F_i^s(\phi \| \Omega)$)	Share of H_0 not rejected			
		$\Theta = \{b_i^s, \theta^s\}$		$\Theta = \{b_i^s, \theta_i^s\}$	
		at 5%	at 10%	at 5%	at 10%
Fréchet	$\exp\left(-b\phi^{-\theta}\right)$	0.09	0.12	0.14	0.16
Gumbel	$\exp\left(-\exp\left(-(\phi - b)/\theta\right)\right)$	0.11	0.12	0.56	0.58
Log-Logistic	$(1 + (\phi/b)^{-\theta})^{-1}$	0.29	0.33	0.63	0.68
Log-Normal	$\Phi\left((\ln\phi - b)/\theta\right)$	0.30	0.33	0.45	0.53
Pareto	1 - $b^{\theta}\phi^{-\theta}$	0.04	0.05	0.04	0.05
Power	$(\phi/b)^{\theta}$	0.05	0.06	0.11	0.12
Weibull	$1 - \exp\left(-(\phi/b)^{\theta}\right)$	0.20	0.22	0.50	0.52

Notes: We conducted a total of 195 tests for 15 countries and 13 sectors. The null hypothesis is $H_0 : F_i^s(\phi) = \widehat{F}_i^s(\phi|\Omega)$. We conduct tests at 5% and 10% significance levels and calculate the share of tests for which we cannot reject the null under each parametric family.

sector-specific with θ^s, whereas the location parameter b_i^s is unconstrained and can vary across both countries and sectors. This is the most common approach taken in quantitative trade theory.[16] We report the share of KS tests for which we cannot reject the null in Columns 3 and 4 in Table 3 under this set of assumptions for each parametric family Ω. At the 5% significance level, for 30% of the country-sector-specific productivity distributions we cannot reject that they belong to the Log-normal family, which is the best-performing family in this exercise. The other 70%, however, are statistically different from the Log-normal distribution. The share of not-rejected pairwise KS tests goes up slightly when we increase the significance level to 10% in the next column, but the overall result carries through. Second, we relax the constraint on the shape parameter and let it be country-sector specific, too, with θ_i^s. The corresponding results are reported in the next two columns. Now, in the best-case scenario one uses a Log-logistic distribution, and we cannot reject placing 63% and 68% of the empirical distributions under this distribution family at the 5% and 10% significance levels, respectively. However, even if one uses the relatively flexible parametric approach, which to the best of our knowledge has not been used in the literature, at least about one-third of the total number of country-sector-specific distributions are statistically different from a single parametric distribution family.

The results in this section suggest that there is little hope in finding a parametric family with a small number of parameters that would be able to simultaneously match empirical productivity distributions in multiple sectors and countries. Using the Fréchet distribution to parameterize

[16]For example, see Eaton and Kortum (2002), Arkolakis et al. (2008), Chaney (2008), and Costinot et al. (2012)

productivity distributions in models based on Eaton and Kortum (2002) and the Pareto distribution in models based on Melitz (2003) – the most commonly used parametrizations in the literature – is not appropriate for at least 86% and 95% of the country-sector distributions, respectively. Hence, assessing the role of technology for economic outcome or quantifying the effects of a change in fundamentals on trade outcomes is likely problematic unless one uses empirical productivity distributions. In the next section, we describe how to implement empirical technology distributions of the estimated kind in Eaton-Kortum and Melitz-type models and how technology shapes patterns of international trade.

5 Two prominent models of international trade

In what follows, we use the estimated empirical country-sector-specific CDFs, $F_i^s(\phi)$, in the two canonical models of international trade which embed heterogeneity in productivity at the level of firms, namely the ones of Eaton and Kortum (2002) and Melitz (2003). We consider multi-country and multi-sector versions of these two models with input-output linkages. Although these two models differ substantially in terms of the underlying micro-foundations and economic assumptions, they both emphasize the role of firm-level heterogeneity in productivity levels for trade outcomes, and they are natural frameworks for assessing how differences in $F_i^s(\phi)$ shape patterns of international trade. We start with presenting a set of aggregate conditions which are common to both models and highlight the relevant differences in the subsections that follow.

5.1 General setup

We assume that each country $i = 1, ..., I$ is endowed with a measure L_i of equipped labor as in Alvarez and Lucas (2007) which is mobile across sectors in i. Production in each sector $s = 1, ..., S$ requires labor and intermediate inputs. Workers have the following upper-tier utility function that incorporates consumption from all sectors of the economy s according to a Cobb-Douglas utility function as follows:

$$U_i(C_i^s) = \prod_{s=1}^{S} (C_i^s)^{\alpha_i^s} \text{ , where } \sum_{s=1}^{S} \alpha_i^s = 1, \tag{14}$$

which leads to the following expression for total final expenditure on C_i^s:

$$P_i^s C_i^s = \alpha_i^s L_i w_i. \tag{15}$$

We assume that C_i^s is a CES aggregate from all goods available for consumption (produced domestically and imported) in each sector s and P_i^s is the accompanying CES price index. Each CES aggregate combines varieties available in i according to the sector-specific elasticity of substitution σ^s. Let us use ϕ to index firms by the respective productivity level. The production function is

Cobb-Douglas and combines labor and materials in the following way:

$$q_i^s(\phi) = \phi\, \ell_i^s(\phi)^{\beta_i^s} m_i^s(\phi)^{1-\beta_i^s} \text{ , where } \beta_i^s \in (0,1). \tag{16}$$

We use $\ell^s(\phi)$ and $m_i^s(\phi)$ to denote labor and materials used for production by firm ϕ and by β_i^s and $1-\beta_i^s$ their share in production, respectively. The material input is comprised of the output of all sectors with sector-specific intensities. Let $\gamma_i^{k,s}$ be the intensity of sector-k output used in the production of sector s, then m_i^s is defined as:

$$m_i^s = \prod_{k=1}^{S} (C_i^k)^{\gamma_i^{k,s}} \text{ , where } \sum_{k=1}^{S} \gamma_i^{k,s} = 1. \tag{17}$$

Given cost minimization, the cost of the input bundle for firm ϕ required to produce one unit of output can be written as:

$$c_i^s = a_i^s w_i^{\beta_i^s} (r_i^s)^{1-\beta_i^s} \text{ , where } a_i^s = (\beta_i^s)^{-\beta_i^s} (1-\beta_i^s)^{\beta_i^s - 1}, \tag{18}$$

where w_i denotes the price of labor in i and the price of the material input, r_i^s, is defined as:

$$r_i^s = \prod_{k=1}^{S} \left(\frac{P_i^s}{\gamma_i^{k,s}} \right)^{\gamma_i^{k,s}} . \tag{19}$$

Let λ_{ij}^s denote the share of j's total expenditure in sector s on goods from country i. Total expenditures, Y_i^s, can be specified as the sum of final demand by consumers and intermediate demand by firms:

$$Y_i^s = \alpha_i^s (L_i w_i) + \sum_{k=1}^{S} \Gamma(\gamma_i^{k,s}) \sum_{j=1}^{I} \lambda_{ij}^s Y_j^s, \tag{20}$$

where $\Gamma(\gamma_i^{k,s})$ is a model-specific linear function of $\gamma_i^{k,s}$ specified separately for the two models in the subsequent two subsections. Finally, given the expression for total demand for goods from sector s in country i, we can specify the trade-balance condition as:

$$\sum_{s=1}^{S} \sum_{j=1}^{I} \lambda_{ji}^s Y_i^s = \sum_{s=1}^{S} \sum_{j=1}^{I} \lambda_{ij}^s Y_j^s . \tag{21}$$

Note that we have not specified the expressions for $\Gamma(\gamma_i^{k,s})$, P_i^s and λ_{ij}^s. This is because these two variables are characterized by different sets of model-specific equations. We discuss them next.

5.2 Perfect competition: Eaton and Kortum (2002)

In Eaton and Kortum (2002), producers are perfectly competitive. In each country and sector potential producers draw their productivity from the country-sector-specific productivity distribution $F_i^s(\phi)$ for each variety ϕ.[17] Producers from country i can supply internationally subject to iceberg-type trade costs d_{ij}^s such that the price offered by the producer of ϕ in sector s from country i to customers in country j is

$$p_{ij}^s(\phi) = \left\{ \frac{c_i^s d_{ij}^s}{\phi} \right\}. \tag{22}$$

Given the CDF of the productivity distribution, $F_i^s(\phi)$, the CDF of the distribution of prices that country i presents to country j in sector s is

$$F_{ij}^s(p) = 1 - F_i^s\left(\frac{c_i^s d_{ij}^s}{p} \right). \tag{23}$$

Customers in country j will purchase ϕ if its producer offers the lowest cost gross of trade costs such that the actual price of ϕ in country i is

$$p_i^s(\phi) = \min_k \left\{ \frac{c_k^s d_{kj}^s}{\phi} \right\}. \tag{24}$$

The share of goods that country j purchases from country i in sector s can be specified as the probability that i is the minimum-cost supplier in that sector, or

$$\lambda_{ij}^s = \Pr\left(p_{ij}^s(\phi) \leq \min_{k \neq i} \left\{ p_{kj}^s(\phi) \right\} \right) = \int_0^1 \prod_{k \neq i} \left(1 - F_{kj}^s(p)\right) dF_{ij}^s(p). \tag{25}$$

We also know the distribution of prices in sector s and importing country j:

$$F_j^s(p) = 1 - \prod_k \left(1 - F_{kj}^s(p)\right). \tag{26}$$

This means that the aggregate price in sector s in country i can be calculated as

$$P_i^s = \left(\int_0^1 p^{1-\sigma^s} dF_i^s(p) \right)^{\frac{1}{1-\sigma}}. \tag{27}$$

Finally, to close the model we need to specify that in the Eaton-Kortum model $\Gamma(\gamma_i^{k,s}) = \gamma_i^{k,s}$.

Note that we need strong assumptions about $F_i^s(\phi)$ to further simplify the expressions in (25) and (27). In particular, if $\phi \sim$ Fréchet(T_i^s, θ^s), then λ_{ij}^s and P_i^s can be expressed as

$$\lambda_{ij}^s = \frac{T_i^s (c_i^s d_{ij}^s)^{-\theta^s}}{\sum_k T_k^s (c_k^s d_{kj}^s)^{-\theta^s}} \qquad \text{and} \qquad P_i^s = B^s \left(\sum_k T_k^s (c_k^s d_{ki}^s)^{-\theta^s} \right)^{-\frac{1}{\theta^s}}, \tag{28}$$

[17] It is assumed that there is a unit measure of varieties ϕ in each sector.

where $B^s = \Gamma\left(\frac{\theta^s+1-\sigma^s}{\theta^s}\right)^{(1/(1-\sigma^s))}$ is a constant and $\Gamma(\cdot)$ denotes the Gamma function. However, as we have demonstrated in the previous sections, the empirical productivity distributions cannot be well approximated by the Fréchet distribution, so that the expressions in (28) do not generally hold. Note that this implies that the functional form of trade flows in the Eaton-Kortum model cannot generally be represented by a standard log-linear specification of the gravity equation of international trade.

5.3 Monopolistic competition: Melitz (2003)

Firms are monopolistically competitive and maximize the usual total profit function comprised of all profits in potential markets j by charging corresponding prices $p_{ij}^s(\phi)$. The profit function for market j is

$$\pi_{ij}^s(\phi) = \max_{p_{ij}^s(\phi)} \left\{ \left(p_{ij}^s(\phi)^{1-\sigma^s} - \frac{c_i^s}{\phi} p_{ij}^s(\phi)^{-\sigma^s} \right) (P_j^s)^{\sigma^s-1} Y_j^s - w_i f_{ij}^s \right\}, \tag{29}$$

where f_{ij}^s is a sector-specific cost of exporting paid in the units of domestic labor. Differentiating equation (29) with respect to the price yields the usual constant-mark-up expression for profit-maximizing prices:

$$p_{ij}^s(\phi) = \frac{\sigma^s}{\sigma^s-1} \frac{c_i^s \tau_{ij}^s}{\phi}, \tag{30}$$

where τ_{ij}^s is a measure of sector-specific iceberg trade costs between countries i and j. Next, we plug back the expression for prices into the profit function to obtain:

$$\pi_{ij}^s(\phi) = \frac{1}{\sigma^s} \left(\frac{\sigma^s}{\sigma^s-1} \frac{c_i^s}{\phi} \right)^{1-\sigma^s} (P_j^s)^{\sigma^s-1} Y_j^s - w_i f_{ij}^s. \tag{31}$$

The zero-profit condition pins down the value of productivity of the marginal exporter:

$$(\phi_{ij}^{s*})^{1-\sigma^s} \quad = \quad \frac{(P_j^s)^{\sigma^s-1} Y_j^s}{\sigma^s w_i f_{ij}^s} \left(\frac{\sigma^s}{\sigma^s-1} c_i^s \tau_{ij}^s \right)^{1-\sigma^s}. \tag{32}$$

Let N_i^s denote the number of potential entrants in country i and sector s. Note that we follow Chaney (2008) and assume that N_i^s is fixed. This is also sufficient to avoid potential corner solutions in multi-sector models with monopolistic competition as described in Kucheryavyy et al. (2016). We can calculate the CES price index as

$$P_j^s = \left(\sum_{i=1}^{I} N_i^s \left(\frac{\sigma^s}{\sigma^s-1} c_i^s \tau_{ij}^s \right)^{1-\sigma^s} \int_{F_i^s(\phi_{ij}^{s*})}^{1} \phi^{\sigma^s-1} dF_i^s(\phi) \right)^{\frac{1}{1-\sigma^s}}. \tag{33}$$

Similarly, we can derive the expression for trade shares between i and j in sector s:

$$\lambda_{ij}^s = N_i^s (P_j^s)^{\sigma^s-1} \left(\frac{\sigma^s}{\sigma^s-1} c_i^s \tau_{ij}^s \right)^{1-\sigma^s} \int_{F_i^s(\phi_{ij}^{s*})}^{1} \phi^{\sigma^s-1} dF_i^s(\phi). \tag{34}$$

Finally, we specify that $\Gamma(\gamma_i^{k,s}) = \dfrac{(\sigma^s - 1)\gamma_i^{k,s}}{\sigma^s}$.

If we assume that $\phi \sim \text{Pareto}(b_i^s, \vartheta^s)$, then the expressions for λ_{ij}^s and P_i^s can be simplified to:

$$\lambda_{ij}^s = \frac{N_i^s (c_i^s \tau_{ij}^s)^{1-\sigma^s} (b_i^s)^{\vartheta^s} (\phi_{ij}^*)^{(\sigma^s-1)-\vartheta^s}}{\sum_k N_k^s (c_k^s \tau_{kj}^s)^{1-\sigma^s} (b_k^s)^{\vartheta^s} (\phi_{kj}^*)^{(\sigma^s-1)-\vartheta^s}} \text{ and } P_i^s = D^s \left(\sum_k N_k^s (c_k^s \tau_{ki}^s)^{1-\sigma^s} (b_k^s)^{\vartheta^s} (\phi_{ki}^*)^{(\sigma^s-1)-\vartheta^s} \right)^{\frac{1}{1-\sigma^s}}.$$

where $D^s = \left(\frac{\vartheta^s}{\vartheta^s - (\sigma^s - 1)}\right)^{\frac{1}{1-\sigma^s}} \frac{\sigma^s}{\sigma^s - 1}$ is a constant. Again, as demonstrated in Section 4, ϕ does not generally follow a Pareto distribution, and then the previous set of equations does not hold.

6 Calibration

In this section, we describe the calibration procedure and data sources. The data sources used to estimate firm-level productivity parameters, ϕ, are described in Section 3. Given the availability of the productivity data, we calibrate the two models to 15 countries (plus one residual country capturing the rest of the world) and 13 sectors (plus one residual sector). The benchmark year for all data is 2008. The algorithms developed in this paper are suitable for solving large-scale general equilibrium models with arbitrary firm-level productivity distributions. The distributions are generally made usable for a quantitative analysis by quantizing them on a fine sector-country-specific grid. We provide further details on the solution algorithm in the Appendix.

In our calibration as well as the counterfactual exercises, we contrast the results obtained under the empirical productivity distributions to the two default parametric families that are used to parameterize distributions in the Eaton-Kortum and Melitz models. In the former model we use the Fréchet and in the latter the Pareto distribution, respectively. While comparing trade outcomes between the two is not our primary goal, results obtained under the single-family parameterizations serve two purposes: they demonstrate severe limitations imposed by the customary parameterizations and they provide familiar benchmark levels.

6.1 Data sources

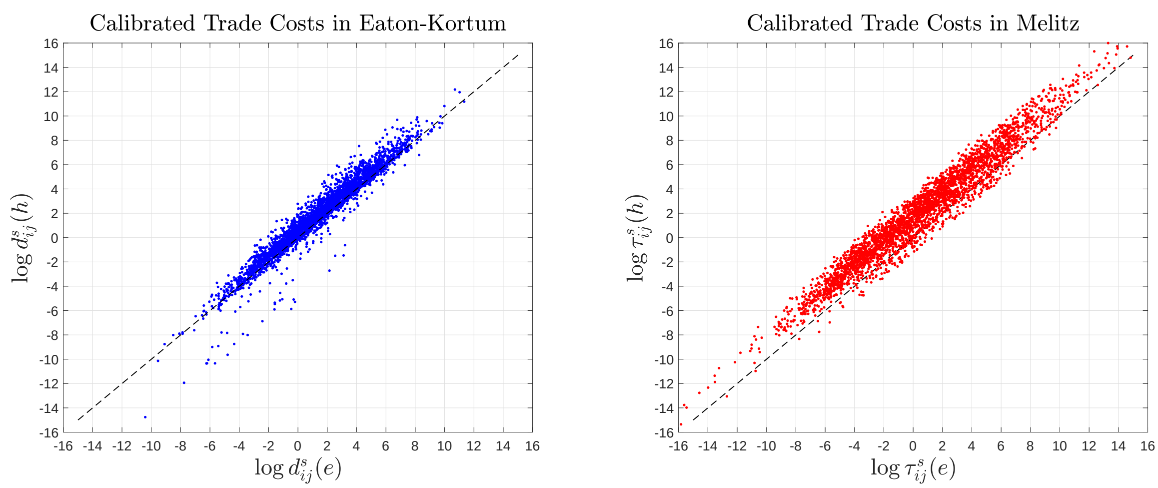
Parameters common to the models of Eaton and Kortum (2002) and Melitz (2003) are $\{\alpha_i^s, \beta_i^s, \gamma_i^{k,s}\}$. These parameters are calculated using data from the World Input-Output Database as Cobb-Douglas consumption shares, value added shares, and input-output shares, respectively. In the calibration procedure, we also use data on trade flows in 2008 from the World Input-Output Database and nominal GDP per capita in 2008 from the World Development Indicators.

In addition, for the calibration of the Melitz model we use data on the number of firms described in Section 3. We also use data on the number of exporters in each sector and the number of exporters by partner country from the *OECD Trade by Enterprise Characteristics Database*. Together with the data on the total number of firms in each sector, we calculate the share of exporters in each sector for each country pair as a product of the probability to export and the probability to export to each destination in that sector.[18]

6.2 Calibration procedure

In the multi-sector version of the model of Eaton and Kortum (2002), we calibrate the I levels of L_i and the I^2S levels of d_{ij}^s. We choose the respective values so as to fit I observations on nominal GDP per capita, w_i, and I^2S observations on trade shares, λ_{ij}^s, subject to the general-equilibrium constraints in Section 5. We calculate two versions of d_{ij}^s. We use $d_{ij}^s(e)$ and $d_{ij}^s(h)$ to denote trade costs calibrated given the empirical CDFs, $F_i^s(\phi)$, and their Fréchet approximations, respectively. Given that we target and match the same data moments with the two distributions, differences between the non-parametric and parametric productivity distributions inevitably result in differences in calibrated trade costs. We illustrate them in the left panel of Figure 7. On average, trade costs calibrated under the assumption of the Fréchet distribution are 40% higher relative to those calibrated when using the empirical CDFs.

Figure 7: CALIBRATED TRADE COSTS



We calibrate L_i and τ_{ij}^s in the multi-sector version of Melitz (2003) using the same data. Again, calibrated variable trade costs depend on whether we use the empirical CDFs, $F_i^s(\phi)$, or their Pareto approximations. We use $\tau_{ij}^s(e)$ and $\tau_{ij}^s(h)$ to denote the former and the latter, respectively. They are plotted in the right panel of Figure 7. On average, trade costs calibrated under the non-parametric CDFs are higher than those under the Pareto approximation by 145%. In addition, the Melitz model also requires us to calibrate I^2S values of fixed market-entry costs f_{ij}^s. For identifying the latter, we use the I^2S observations on sectoral export shares together with the general-equilibrium structure of the underlying model.

[18]Whenever observations on probability to export were missing, we imputed them using a log-linear regression with the number of firms in that sector in origin and destination, and (log) distance between them as predictors. We also imputed missing instances of probability to export to each destination using a log-linear regression with export share in total output to that market and sector, and total number of firms. For residual country and residual sector, we impute the variables using averages for the rest of our sample.

With regard to the calibration, the following consideration is important. First, the Eaton-Kortum and Melitz models rely on different structures and mechanisms at work. For this reason, when looking on trade and other data through the lens of these models, the same data on outcome will lead to different fundamentals – such as variable trade and fixed market-access costs. Hence, the same outcome is generated by different economic primitives. Within each class of models, a similar conclusion is reached when comparing fundamentals calibrated under empirical productivity distributions to those obtained under their parametric counterparts. Given limited degrees of freedom and the fact that we match the same observable data, differences between empirical and parametric CDFs lead to different sets of fundamentals, as mentioned above.[19] In other words, when requiring to fit the same outcome data, the fundamental parameters will not be identical neither within distributional forms between model types nor between distributional forms and within model type.

However, we should note that the key results presented here are qualitatively robust and carry through when using a single set of fundamentals to inform both the non-parametric and parametric model versions of the underlying models (then, however, as said, the trade-share and factor-cost vectors inevitably differ between the nonparametric and parametric model versions).

7 Quantifying the impact of technology on international trade outcomes

In this section, we make use of the data on empirical productivity distributions to understand the role of technology for trade. We start by quantitatively assessing the importance of the first-order effect defined as the impact of technological differences between countries as well as across sectors on trade outcomes. In the second part of the section, we asses the importance of the gradient effect by examining how productivity distributions shape the effect of changes in fundamentals on changes in trade and welfare. Finally, we discuss micro-to-macro implications of using empirical productivity distributions for model choice and the perceived isomorphism with respect to changes in trade costs between the Eaton-Kortum and Melitz models.

7.1 The first-order effect of technology on international trade

What drives differences in trade patterns across countries and sectors? How much does technology contribute to explaining the observed λ_{ij}^s? While these questions are at the heart of international trade, little is known about the role of technology in canonical trade models with heterogeneous firms. This is due to the fact that the customary approach towards technology in most of the quantitative work in international economics is what we call a *macro*-based approach, which is unable to answer the above questions for two reasons.

[19] We should emphasize that one of the following two cases hold, if the empirical and the parametrically approximated CDFs of firm productivity generally differ: the same trade shares in the two models must rest on different vectors of fundamentals, or the same vectors of fundamentals will produce different vectors of trade shares.

First, the *macro*-representation of technology usually relies on two parameters only, $\{T_i^s, \theta^s\}$ in the Eaton-Kortum model and $\{b_i^s, \vartheta^s\}$ in the Melitz model, where country(-sector) differences in average technology are determined as a residual quantity and the dispersion of technology is estimated as a parameter on ad-valorem trade or production costs. As we have seen in Section 3, this approach drastically reduces the dimensionality of empirical productivity distributions, which may lead to a stark mis-characterization of how the CDF of productivity, $F_i^s(\phi)$, shapes trade patterns. Second, due to the highly non-linear nature of trade models, quantifying the share of the variance in λ_{ij}^s which is explained by the empirical nonparametric $F_i^s(\phi)$ using traditional estimation techniques is not possible.

We tackle these two challenges as follows. First, instead of relying on parameterizations we use observed distributions of firm-level productivity. Second, we quantify the role of technology for explaining trade flows by exploring a series of counterfactual experiments in open-economy general equilibrium specifically designed for the purpose of gauging the importance of technology for outcome in the relevant models.

In quantitative general-equilibrium models, the observed level of trade shares λ_{ij}^s is the outcome of an interplay between a set of economic fundamentals and the economic equilibrium conditions that are inherent to the model at hand. We can think of the two considered models as mappings that translate inputs in the form of a set of economic fundamentals into output in the form of endogenous variables including counterfactual trade shares, $\lambda_{ij}^{s\prime}$. In order to quantify the role of total factor productivity differences in shaping observed trade shares, we will evaluate counterfactual scenarios where we eliminate the differences in productivity distributions across countries or sectors while keeping all other economic fundamentals as in the baseline equilibrium pertaining to data of the year 2008. Let us use $\mathbb{F} = \{\mathbb{D}, \mathbb{O}\}$ to denote all fundamentals, where $\mathbb{D}$ contains all parameters describing the moments of productivity distributions and $\mathbb{O}$ contains all other economic primitives of a considered trade model. We consider two counterfactual scenarios: $\mathbb{F}_{\mathbb{D}'} = \{\mathbb{D}', \mathbb{O}\}$ and $\mathbb{F}_{\mathbb{O}'} = \{\mathbb{D}, \mathbb{O}'\}$, where $\mathbb{D}'$ and $\mathbb{O}'$ denote counterfactual productivity distributions and counterfactual values of all other economic fundamentals, respectively. For each scenario, we define changes in counterfactual trade shares relative to their benchmark values:

$$\Delta\lambda_{ij}^s(\mathbb{F}_{\mathbb{K}}) = \log\left(\lambda_{ij}^{s\prime}(\mathbb{F}_{\mathbb{K}})\right) - \log\left(\lambda_{ij}^s(\mathbb{F})\right) \quad \text{for} \quad \mathbb{K} = \mathbb{D}', \mathbb{O}' \ , \tag{35}$$

where $\lambda_{ij}^s(\mathbb{F})$ are trade flows in the benchmark equilibrium. We then calculate the share of sum of squares attributed to eliminating differences in productivity-related fundamentals in $\mathbb{D}$ relative to eliminating differences in all other factors as follows:

$$R(\mathbb{F}_{\mathbb{D}'}) = \frac{SSQ\left[\Delta\lambda_{ij}^s(\mathbb{F}_{\mathbb{D}'})\right]}{SSQ\left[\Delta\lambda_{ij}^s(\mathbb{F}_{\mathbb{D}'})\right] + SSQ\left[\Delta\lambda_{ij}^s(\mathbb{F}_{\mathbb{O}'})\right]}, \tag{36}$$

where $SSQ[\cdot]$ is the sum of squares of all international trade flows such that $i \neq j$.[20] The measure $R(\mathbb{F}_{\mathbb{D}'})$ is naturally bounded between 0 and 1 and can be interpreted as a pseudo-R^2. By eliminating asymmetries in the two counterfactual scenarios, $\mathbb{F}_{\mathbb{D}'}$ and $\mathbb{F}_{\mathbb{O}'}$, in specific ways, we are not only able to quantify the relative importance of technology relative to other factors but also examine different dimensions of technological differences and their roles in shaping trade flows. In particular, technological differences shape trade through their interplay across sectors within countries as well as through their interplay within sectors across countries. As we have mentioned before, we broadly follow David Ricardo's ideas in defining the former as *modified absolute advantage* and the latter as *modified comparative advantage.*

As demonstrated in Section 4, the two dimensions of technological differences matter empirically, as productivity distributions are heterogeneous both across sectors and even more so across countries. To emphasize these two distinct margins of productivity heterogeneity, we conduct two distinct series of counterfactual experiments. First, we eliminate differences in productivity distributions for each sector s across different countries but not across sectors such that $KS_{ij}^s \to 0$ for all i and j. This exercise eliminates *modified absolute advantage.* Second, we eliminate differences in sectoral productivity distributions within each country such that $KS_i^{ss'} \to 0$ for all s and s', which eliminates *modified comparative advantage.* We quantify the response of trade flows to these counterfactual changes in technology next. Naturally, the concepts of absolute and comparative advantages are not as clear-cut in our context due to the presence of multiple frictions and asymmetries in fundamentals. However, they are in the spirit of the original notions and capture the main intuition.

7.1.1 Absolute advantage: Eliminating productivity differences across countries

In the first series of counterfactual experiments, we eliminate country differences in $F_i^s(\phi)$ denoted by scenario $\mathbb{F}_{\mathbb{D}'}$ and relate the outcome of this experiment to the counterfactual scenario $\mathbb{F}_{\mathbb{O}'}$, where we eliminate differences in all other factors $\{L_i, \alpha_i^s, \beta_i^s, \gamma_i^{s,k}, d_{ij}^s, \tau_{ij}^s, f_{ij}^s, N_i^s\}$. We endow each country i with the corresponding target country's fundamentals. To make sure that our results do not depend on the chosen benchmark, we conduct I counterfactual experiments choosing each country in the sample as the target once at a time. This eliminates differences across countries in each sector but not across sectors within countries. Let v denote the target country all others converge to, then productivity convergence implies for the counterfactual productivity distribution (indicated by a prime) that:

$$\mathbb{F}_{\mathbb{D}'} \Leftrightarrow F_i^s(\phi)' = F_v^s(\phi) \text{ for all } i, s. \tag{37}$$

Convergence in all other factors implies:

$$\mathbb{F}_{\mathbb{O}'} \Leftrightarrow \{L_i', \alpha_i^{s\prime}, \beta_i^{s\prime}, \gamma_i^{s,k\prime}, d_{ij}^{s\prime}, \tau_{ij}^{s\prime}, f_{ij}^{s\prime}, N_i^{s\prime}\} = \{L_v, \alpha_v^s, \beta_v^s, \gamma_v^{s,k}, d_{vj}^s, \tau_{vj}^s, f_{vj}^s, N_v^s\} \text{ for all } i, s. \tag{38}$$

[20]Our results are robust to including intra-trade shares, λ_{ii}^s, in the analysis. However, we choose not to include them because we concentrate on explaining international patterns.

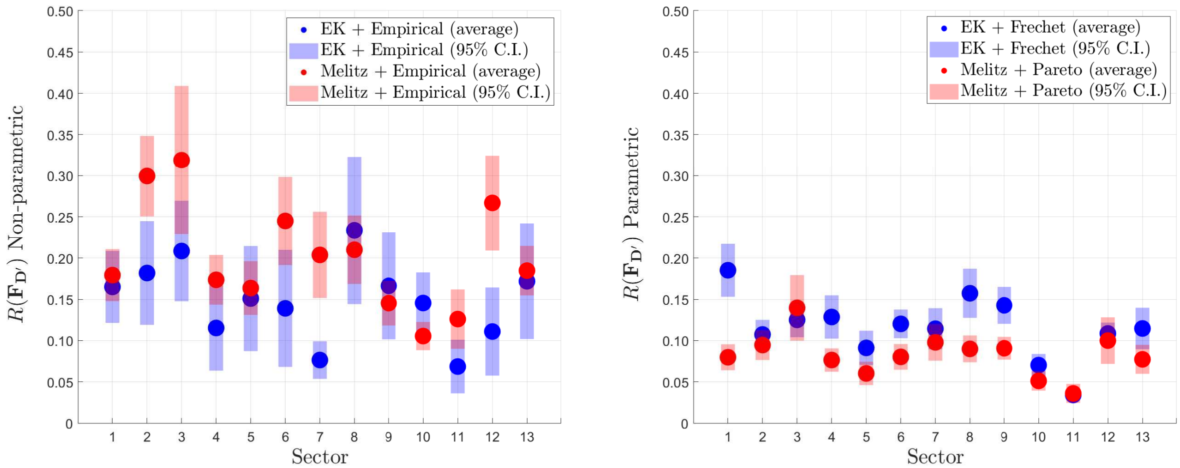
We calculate counterfactual trade flows under each scenario and the associated $R(\mathbb{F}_{\mathbb{D}'})$ for models based on both Eaton and Kortum (2002) and Melitz (2003). In total, we conduct I experiments and calculate $\mathbb{F}_{\mathbb{D}'}$ for S sectors such that there are IS counterfactual values of $\mathbb{F}_{\mathbb{D}'}$. We report summary statistics of the counterfactual results across all experiments and sectors in Table 4. On average, differences in productivity distributions explain 15% and 21% in the Eaton-Kortum and Melitz model, respectively. However, the role of technology is underemphasized by customary parameterization as productivity differences under Fréchet and Pareto explain 12% and 8% of the total variation in trade flows in the two models. The results also have a relatively higher variation under empirical productivity distributions.

Table 4: Summary Statistics (Absolute Advantage)

	Average	**Std. Dev.**	**Min.**	**Median**	**Max.**
EK + Empirical	0.15	0.13	0.00	0.11	0.66
Melitz + Empirical	0.21	0.11	0.03	0.20	0.64
EK + Fréchet	0.12	0.06	0.02	0.11	0.28
Melitz + Pareto	0.08	0.05	0.01	0.08	0.30

The results in Table 4 suggest that there is a substantial variation in the role of modified absolute advantage across sectors and experiments. We next turn to decomposing this variation across sectors and countries to study heterogeneous effects of modified absolute advantage on international trade. We start with examining how the role of modified absolute advantage varies across sectors. For that, we use IS results on $R(\mathbb{F}_{\mathbb{D}'})$ and calculate the average along with 95% confidence bands for each sector s across I experiments and summarize the associated findings in Figure 8. In the left panel, we report results obtained under the empirical productivity distributions. The average contribution of modified absolute advantage in explaining trade shares varies across sectors in intervals of (7%, 23%) and (11%, 32%) in the Eaton-Kortum and Melitz models, respectively. According to the Eaton-Kortum model, the three sectors with the highest $R(\mathbb{F}_{\mathbb{D}'})$ are $\{$*Textiles* 0.18 (0.13), *Leather* 0.21 (0.12), *Non-metallic minerals* 0.23 (0.18)$\}$, where we list averages next to the sector name and standard deviations in parentheses. In the Melitz model, the three sectors with the highest $R(\mathbb{F}_{\mathbb{D}'})$ are $\{$*Transport equipment* 0.27 (0.10), *Textiles* 0.30 (0.09), *Leather* 0.32 (0.16)$\}$.

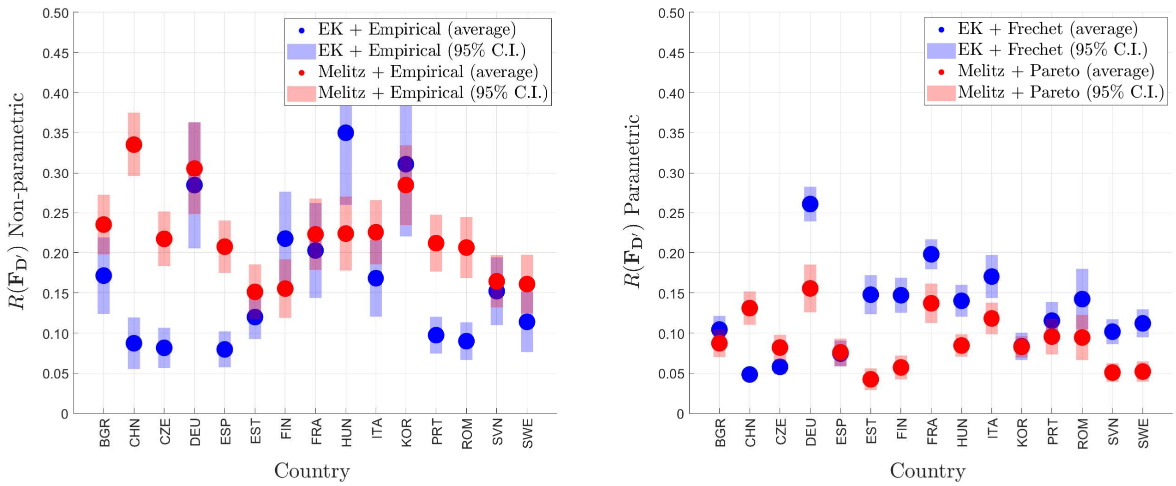
Figure 8: Eliminating modified absolute advantage (sectoral heterogeneity)



For comparison, we present the results of the same experiment but obtained using parameterized productivity distributions in the right panel. The respective average values of $R(\mathbb{F}_{\mathbb{D}'})$ within each sector vary in the intervals of (3%, 19%) and (4%, 14%) in the Eaton-Kortum and Melitz models, respectively. The contribution of technology across sectors is significantly lower relative to the results calculated using the empirical (nonparametric) productivity distributions suggesting that customary parametrizations of productivity distributions tend to bias the role of technology for trade downwards but also in certain instances lead to qualitative differences. In the Eaton-Kortum and Melitz models the three sectors with the highest average $R(\mathbb{F}_{\mathbb{D}'})$ are {*Basic metals* 0.14 (0.05), *Other non-metallic mineral* 0.16 (0.06), *Food, beverages, tobacco* 0.19 (0.06)} and {*Rubber and plastics* 0.10 (0.05), *Transport equipment* 0.10 (0.06), *Leather* 0.14 (0.08)}, respectively, when using the parameterized productivity distributions.

Next, we examine how modified absolute advantage shapes exports and imports differentially across countries. For that, we pool benchmark and counterfactual trade shares on the export and import side for each country i in all sectors S and calculate $R(\mathbb{F}_{\mathbb{D}'})$ as in equation (36) for each of the I experiments. We calculate average values of that statistic along with the 95% confidence bands for each country i across all experiments in Figure 9. The results suggest a high level of heterogeneity in $R(\mathbb{F}_{\mathbb{D}'})$ across countries. The average of $R(\mathbb{F}_{\mathbb{D}'})$s varies across countries in the intervals of (8%, 35%) and (15%, 34%) in the Eaton-Kortum and Melitz models, respectively. According to the Eaton-Kortum model, modified absolute advantage matters most for exports and imports for *Germany* 0.28 (0.16), *Korea* 0.31 (0.18), and *Hungary* 0.35 (0.18). For these three economies, roughly one-third of the variance in international trade shares can be explained by cross-country differences in productivity distributions alone. In the Melitz model, the three countries with the highest $R(\mathbb{F}_{\mathbb{D}'})$ with the following averages and standard deviations are *Korea* 0.28 (0.10), *Germany* 0.31 (0.11), and *China* 0.34 (0.08).

Figure 9: Eliminating modified absolute advantage (country heterogeneity)



For comparison purposes, we once more report the results of the same experiment calculated under the parameterized distributions in the right panel of Figure 9. There are several important differences to the nonparametric outcome. First, the calculated values of $R(\mathbb{F}_{\mathbb{D}'})$ are quantitatively significantly lower for the Eaton-Kortum and Melitz models. The average of $R(\mathbb{F}_{\mathbb{D}'})$s varies in the intervals of (5%, 26%) and (4%, 16%), respectively. The variation within countries is also significantly smaller which again points to an understated role of technology for explaining trade due to the customary parameterizations of technology. According to the Eaton-Kortum and Melitz models, the three top countries with the largest impact of modified absolute advantage are *Italy* 0.17 (0.05), *France* 0.20 (0.04), *Germany* 0.26 (0.04) and *China* 0.13 (0.04), *France* 0.14 (0.05), and *Germany* 0.16 (0.06), respectively.

7.1.2 Comparative advantage: Eliminating productivity differences across sectors within countries

To understand the second dimension of how technological differences determine trade flows, we eliminate productivity differences across all sectors within a country but not across countries. As mentioned above, we refer to such within-country differences as *modified comparative advantage*. Again we conduct two series of experiments, where we first eliminate differences across $F_i^s(\phi)$ in each country i such that $KS_i^{ss\prime} \to 0$. Then, we contrast the results with those obtained when we eliminate all other sectoral asymmetries where we endow each sector s in country i with the corresponding fundamentals observed in the benchmark sector (including bilateral trade and fixed costs). We conduct S counterfactual experiments choosing each sector in the sample alternatively as a target. Let ℓ denote the target sector, then eliminating productivity differences across sectors means:

$$\mathbb{F}_{\mathbb{D}'} \Leftrightarrow F_i^s(\phi)' = F_i^{\ell}(\phi) \text{ for all } i, s. \tag{39}$$

Sectoral convergence in other factors is defined as follows:

$$\mathbb{F}_{\mathbb{O}'} \Leftrightarrow \{\sigma^{s\prime}, \alpha_i^{s\prime}, \beta_i^{s\prime}, \gamma_i^{k,s\prime}, d_{ij}^{s\prime}, \tau_{ij}^{s\prime}, f_{ij}^{s\prime}, N_i^{s\prime}\} = \{\sigma^{\ell}, \alpha_i^{\ell}, \beta_i^{\ell}, \gamma_i^{k,\ell}, d_{ij}^{\ell}, \tau_{ij}^{\ell}, f_{ij}^{\ell}, N_i^{\ell}\} \text{ for all } i, s. \tag{40}$$

As before, we calculate counterfactual values of trade shares for each convergence scenario and report summary statistics calculated accross all experiments and sectors in Table 5.[21] There is a total of S experiments and we calculate $R(\mathbb{F}_{\mathbb{D}'})$ for each sector, which leads to S^2 values of $R(\mathbb{F}_{\mathbb{D}'})$. On average, modified comparative advantage explains 39% and 46% of the total variation in trade flows in the Eaton-Kortum and Melitz model, respectively, suggesting a substantially more pronounced role for modified comparative advantage as compared to modified absolute advantage. Customary parameterizations lead to a substantial understatment of the role of technology, which is almost halved in the Eaton-Kortum model under the Fréchet parameterization and is more than three times lower in the Melitz model under the Pareto parameterization.

Table 5: Summary Statistics (Comparative Advantage)

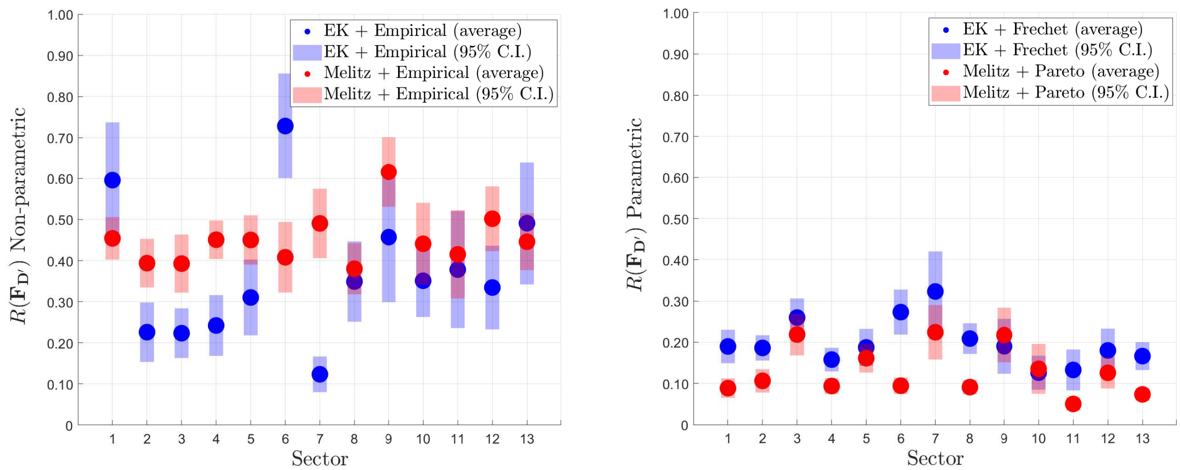
	Average	**Std. Dev.**	**Min.**	**Median**	**Max.**
EK + Empirical	0.39	0.18	0.06	0.39	0.80
Melitz + Empirical	0.46	0.11	0.20	0.47	0.77
EK + Fréchet	0.21	0.08	0.04	0.20	0.49
Melitz + Pareto	0.15	0.06	0.03	0.14	0.33

As before, we quantitatively assess the heterogeneity in the role of modified comparative advantage across sectors. For that, we calculate counterfactual values of trade shares for each convergence scenario and report sector- and country-specific average values of $R(\mathbb{F}_{\mathbb{D}'})$ across S experiments along with the 95% confidence bounds. In Figure 10, we report how modified comparative advantage shapes sectoral trade shares under the empirical productivity distributions. The results are not only stronger relative to the ones associated with modified absolute advantage but also more heterogeneous – in particular in the Eaton-Kortum model. The average of $R(\mathbb{F}_{\mathbb{D}'})$ varies across sectors in the intervals of (12%, 62%) and (38%, 62%) in the Eaton-Kortum and Melitz model, respectively. Recall that *Leather* and *Textiles* were the two sectors where modified absolute advantage mattered most for explaining international trade. Modified comparative advantage, on the other hand, matters the least for these two sectors as, on average, it explains only 0.22 (0.12) and 0.39 (0.13) in the Eaton-Kortum model and 0.23 (0.14) and 0.39 (0.11) in the Melitz model, respectively. The three sectors where modified comparative advantage matters most are {*Manufacturing n.e.c.* 0.49 (0.28), *Food, beverages, tobacco* 0.60 (0.27), *Chemicals* 0.73 (0.24)} and {*Rubber and plastics* 0.49 (0.16), *Transport equipment* 0.50 (0.15), *Basic metals* 0.62 (0.16)} in the Eaton-Kortum and Melitz models, respectively.

In the right panel of Figure 10, we report the results of the experiments obtained using param-

[21] Note that for parametric models we cannot always let $\sigma^{s\prime} \to \sigma^{\ell}$ due to the constraint that $\sigma^s - 1 < \theta^s$. To avoid this, we let $\sigma^{s\prime}$ converge to σ^1 which adheres to the constraint. Our results do not depend on this.

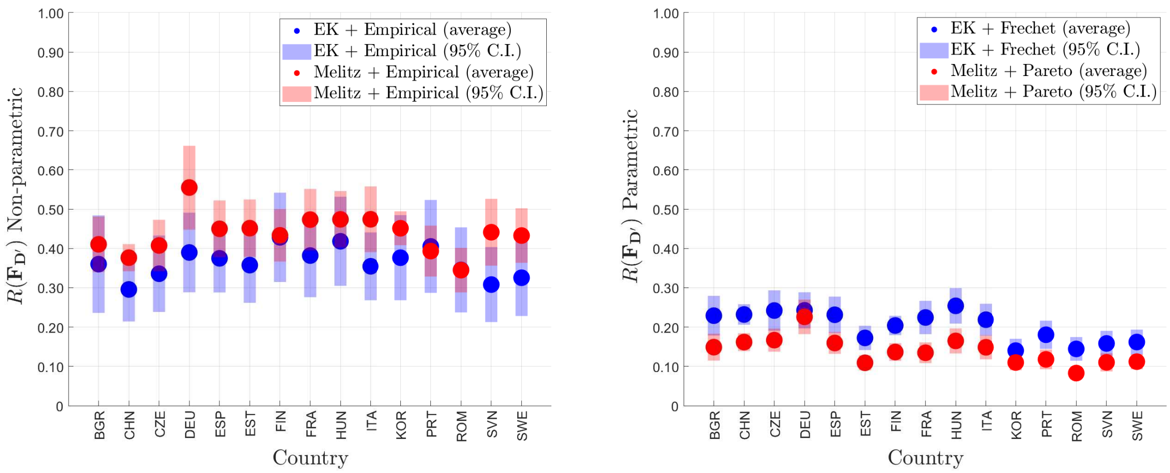
Figure 10: Eliminating Modified Comparative Advantage (sectoral heterogeneity)



eterized versions of the productivity distributions. The role of technology is apparently vastly downward biased by the customary parameterizations across almost all sectors with sector averages in the Eaton-Kortum and Melitz frameworks being in the intervals (13%, 32%) and (5%, 22%), respectively. Once again, there are also important qualitative differences to the results based on the empirical distributions, as the parameterized distributions predict that {*Leather* 0.26 (0.09), *Chemicals* 0.27 (0.10), *Rubber and plastics* 0.32 (0.19)} and {*Basic metals* 0.22 (0.13), *Leather* 0.22 (0.10), *Rubber and plastics* 0.22 (0.13)} are the sectors where modified comparative advantage matters most in the Eaton-Kortum and Melitz model, respectively.

Finally, we examine how modified comparative advantage shapes exports and imports differently for each country. We again calculate $R(\mathbb{F}_{\mathbb{D}'})$ using benchmark and counterfactual trade shares on the export and import sides for each country pooled across all sectors for each experiment. We plot the results based on the empirical (nonparametric) productivity distributions in the left panel. They suggest substantially smaller heterogeneity across countries relative to sectors. The average $R(\mathbb{F}_{\mathbb{D}'})$ lies in the interval (30%, 43%) and (34%, 55%) in the Eaton-Kortum and Melitz models, respectively. In the right panel of Figure 11, we report the results obtained under the parameterized distributions. The results again indicate that the parameterizations induce a substantial downward-bias in the importance of technology for trade. Across countries, the results vary in the intervals (14%, 25%) and (8%, 23%) in the Eaton-Kortum and Melitz models, respectively. Due to a substantially lower heterogeneity across sectors in the parametric version, the variance within each country is also much lower under parameterized productivity distributions compared to nonparametric ones.

Figure 11: Eliminating modified comparative advantage (country heterogeneity)



Interestingly, comparing the role of technology – both in terms of modified absolute advantage as well as in terms of modified comparative advantage – across the empirical and parameterized versions of the experiments reveals that, on average, technology plays a more pronounced role in the Melitz model than in the Eaton-Kortum model. While the role of technology is generally substantially constrained under customary parametrizations, the constraints are more severe in the Melitz model under the Pareto parameterization.

7.2 The gradient effect of technology on international trade

So far, we have emphasized the role of productivity differences in shaping patterns of international trade levels across countries. We now turn to the discussion of how differences in productivity distributions govern changes in trade outcomes in response to changes in variable trade costs. This margin of the role of technology is at the heart of quantitative trade theory as it determines how changes in policy and non-policy trade barriers affect trade and welfare. We refer to this margin as the gradient effect of technology on trade. As before, we contrast the results derived under the empirical productivity distributions and those derived under their parameterized versions in the Eaton-Kortum and Melitz models.

We start with examining how changes in variable trade costs affect trade shares in each of the two models considered in this work. In the Eaton-Kortum model this relationship is described in equation (25) and the effect of a change in trade costs on trade shares is governed by the marginal effect of trade costs on the distribution of goods prices from i in j which in turn is driven by the slope of the respective CDFs about productivity at equilibrium quantities. Hence, to understand the effect of trade costs on trade shares in the Eaton-Kortum model, we have to examine the relationship between the slope of the respective CDFs describing productivity in i, s and variable production costs for goods produced in i and shipped to j, $c_{ij}^s \equiv c_i^s d_{ij}^s$, which we summarize by the relevant statistic $W_i^s(c_{ij}^s)$:

$$W_i^s(c_{ij}^s) = \int_0^1 K_i^s d\left(1 - F_i^s\left(c_{ij}^s/p\right)\right), \tag{41}$$

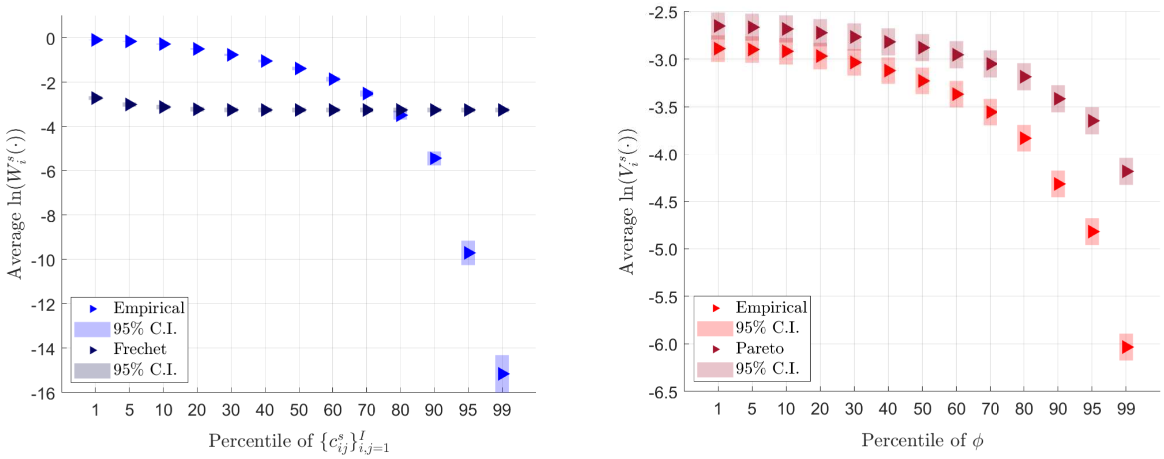
where $K_i^s = \prod_{k \neq i}\Big(1 - F_{kj}^s(p)\Big)$ is a function of the productivity CDFs of all countries but i in sector s. In order to define a meaningful support to plot this relationship we take the sector-specific distribution of c_{ij}^s across country pairs in our data and determine 13 quantiles ranging from the 1^{st} to the 99^{th} percentile for each sector. Then, we evaluate $\ln(W_i^s(c_{ij}^s))$ for all s and i at each of those quantiles and compute the average value across all countries and sectors in our sample to understand how the marginal response to changes in trade costs varies along the support of $\ln(W_i^s(c_{ij}^s))$. Note that the face value at which $\ln(W_i^s(c_{ij}^s))$ is evaluated at each quantile is the same across countries within a sector. In the left panel of Figure 12, we plot the shape of the average $\ln(W_i^s(c_{ij}^s))$ when using the nonparametric productivity distributions versus their Fréchet counterparts, respectively. Along with the average locus of $\ln(W_i^s(c_{ij}^s))$, we plot 95% confidence intervals. The figure illustrates that for the nonparametric productivity distributions the sensitivity of $\ln(W_i^s(c_{ij}^s))$ to changes in c_{ij}^s is substantially higher at higher quantiles of c_{ij}^s as compared to lower ones. And it illustrates that the slope of the relationship is considerably flatter with the Fréchet parameterization.

In the Melitz model, the effect of trade costs on trade shares is described by equation (34) and is composed of the direct effect of τ_{ij}^s on λ_{ij}^s (intensive margin) and the effect through ϕ_{ij}^s (extensive margin). As the intensive-margin effect is constant and identical for the parametric and nonparametric versions, we focus on the extensive-margin effect that is a function of the slope of the productivity CDFs at the respective equilibrium quantities and can be described by the following relevant statistic:

$$V_i^s(\phi_{ij}^{s*}) = \int_{F_i^s(\phi_{ij}^{s*})}^1 \phi^{\sigma^s - 1} dF_i^s(\phi). \tag{42}$$

To plot a meaningful average locus of this relationship across countries and sectors, we define a set of quantiles of interest and obtain the sector-country specific values of ϕ at these quantiles. Then, we evaluate $\ln(V_i^s(\phi_{ij}^{s*}))$ at each quantile and compute an average across all sectors s and countries i over these quantiles. Note that the face value of the quantile at which the function is evaluated is different across i and s. Analogously to the Eaton-Kortum model, we plot the average locus of $\ln(V_i^s(\phi_{ij}^{s*}))$ across percentiles of ϕ to illustrate the generic relationship between them in the data. The result is plotted in the right panel of Figure 12.

Figure 12: EMPIRICAL MARGINAL RESPONSES OF $W_i^s(\cdot)$ AND $V_i^s(\cdot)$



Again, we see that the relationship is much flatter in the lower quantiles of ϕ_q than in the higher ones, and in the Pareto-parameterized model it is much flatter in the upper quantiles than in the nonparametric (empirical) quantitative model. Recall that the higher quantiles of ϕ tend to characterize the part of the CDF support relevant for the exporting productivity cutoffs according to most data around the world. Hence, the figure clearly illustrates that there is a considerable sensitivity to changes in productivity in that part of the CDF, where it is highly relevant for the extensive margin of exporting. In other words, using a parameterized Pareto-Melitz model will likely downward-bias effects of fundamentals on equilibrium outcomes, in particular, to the extent that they depend on extensive export-margin adjustments.

A general conclusion from this analysis in conjunction with insights from earlier work is the following. First, in most countries on the globe exporting is a relatively rare activity across the firms in a country and sector (see Bernard et al., 2007). Through the lens of the Eaton-Kortum model, one would conclude that most firms in a country and sector are rarely successful in competing at global markets and computing their export propensity over longer time intervals leads to a big mass of firms with tiny export shares and propensities. This is consistent with our baseline analysis using the Melitz model, where the average share of firms in i exporting to $j \neq i$ is 0.3% and even the 99th percentile of a distribution of these shares (F_{ij}^s) is below 2.5%. All of this clearly indicates that what is the relevant part of the productivity distribution when it comes to quantifying responses to changes in trade costs is the right and not the left part of productivity distributions, and this is where the relevance of productivity for general-equilibrium outcome is much stronger when using distributions that are informed by details in the empirical data (i.e., nonparametric distributions) rather than customarily parameterized versions thereof.

7.2.1 Marginal responses of trade shares to changes in variable trade costs

Next, we calculate general-equilibrium-consistent predictions about the marginal responses of trade shares to changes in trade costs via counterfactual simulations. For that, we conduct $(I-1)^2S$ experiments in each of the two models. In each experiment, we reduce the variable trade costs for each importer by 20%. For the ease of exposition, from now on let us use τ^s_{ij} to denote variable trade costs in both models. Then, in each experiment the counterfactual trade costs for each importer i and exporter j in sector s are calculated as follows:

$$\tau^{s\prime}_{ji} = 0.8\tau^s_{ji}. \tag{43}$$

Given these counterfactual trade costs we calculate marginal changes in trade shares as the log difference between counterfactual and benchmark trade shares $\Delta\lambda^s_{ij}(\tau^s_{ij}) = \log(\lambda^s_{ij}(\tau^{s'}_{ij})) - \log(\lambda^s_{ij}(\tau^s_{ij}))$. First, we pool the $\Delta\lambda^s_{ij}(\tau^s_{ij})$ across all countries and sectors and plot them against the benchmark λ^s_{ij}. We form quantiles of the latter, which we call λ_q, and plot the average of $\Delta\lambda^s_{ij}(\tau^s_{ij})$ for any quantile λ_q in the left panel of Figure 13. The marginal responses clearly vary across quantiles and models, consistent with the intuition formed after inspection of Figure 12. They are decreasing in absolute value as we move to higher quantiles of λ^s_{ij}. Hence, an equivalent reduction in trade costs tends to lead to larger increases in λ^s_{ij} for country pairs and sectors with smaller trade shares in the outset. These are exactly the country pairs where baseline equilibrium values of $W^s_i(\cdot)$ and $V^s_i(\cdot)$ are to the right end of the abscissa in Figure 12 and marginal changes are substantially larger than further to the left.

For comparison purposes, we plot the results of the same experiment for the parameterized model versions as well. Here, the results indicate that the customary parameterizations of the Eaton-Kortum and Melitz models significantly dampen the heterogeneity in the marginal responses of trade shares across quantiles of λ^s_{ij} in the outset. Note that the very modest effect in the parametric version is a consequence of the constant direct relationship between trade shares and trade costs (in logs) implied both by the Fréchet-parameterized Eaton-Kortum model and the Pareto-parameterized Melitz model. The only source of heterogeneity across quantiles in Figure 13 stems from general-equilibrium effects.

Figure 13: Marginal responses of trade shares to changes in trade costs and outset-trade-share quantile

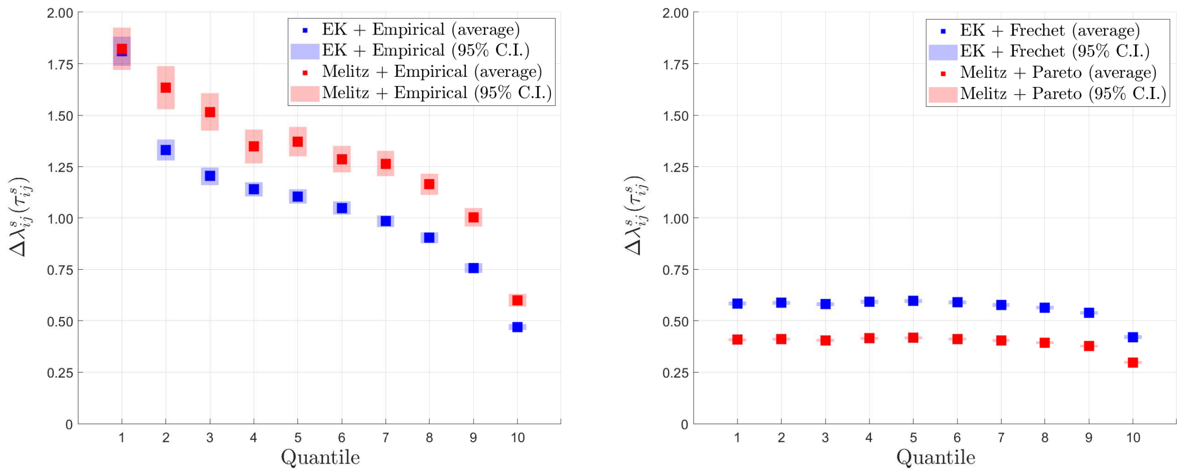
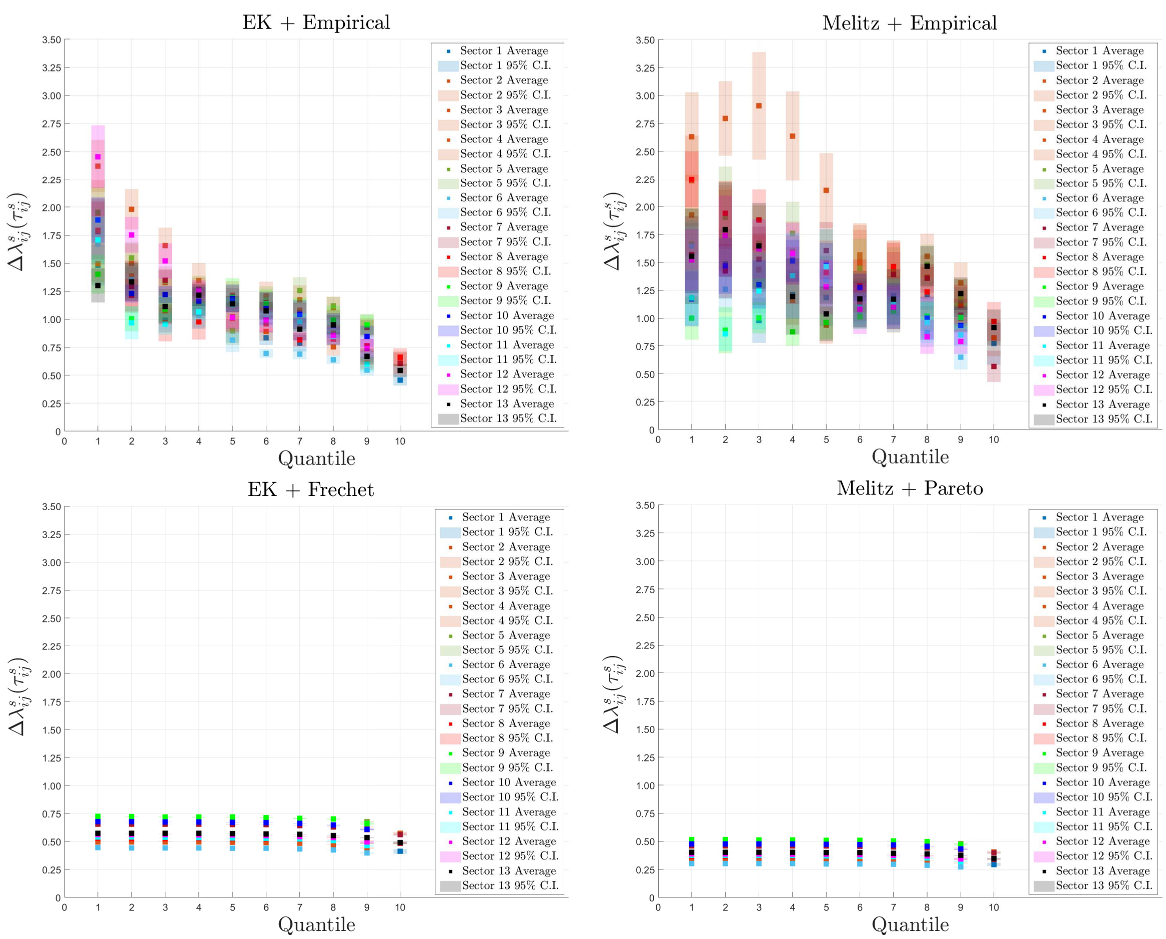


Figure 14: Marginal responses of trade shares to changes in trade costs by sector and outset-trade-share quantile



Next, we calculate average responses within each trade-share quantile by sector and plot them in the upper two panels of Figure 14. Again, the results calculated using the empirical (nonparametric) productivity distributions suggest strong dependence of the magnitude of average responses on the sectoral trade-share quantile in the outset. There is a large heterogeneity across and within sectors but the overall pattern suggests that on average sectoral marginal responses in absolute value are decreasing for higher trade-share quantiles in the outset. For comparison, we plot the results based on the parameterized productivity distributions in the lower two panels of Figure 14. Due to different shape parameters in the Fréchet and Pareto distributions across sectors, the magnitudes of the response differ slightly across sectors but less so across quantiles.

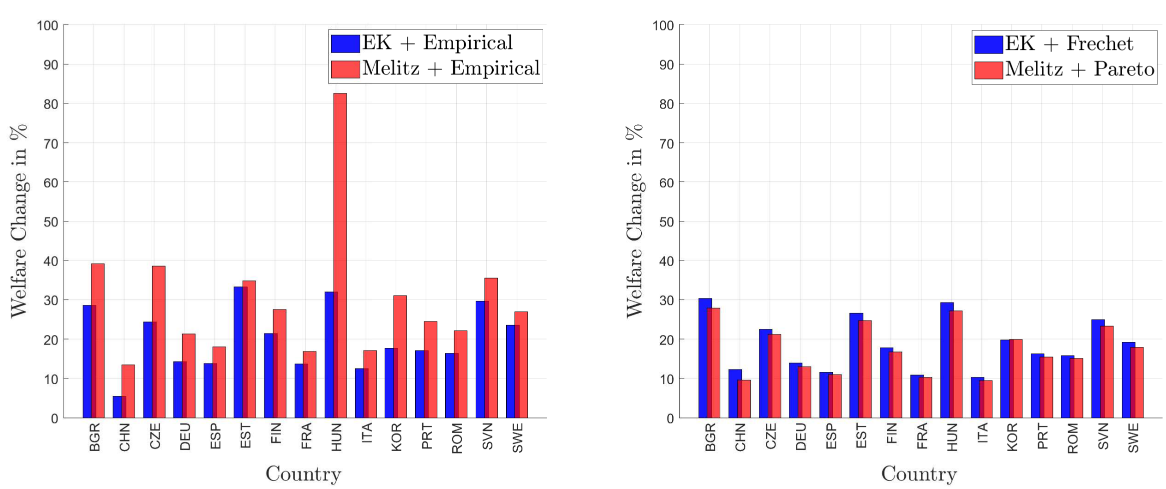
7.2.2 Welfare changes in response to changes variable trade costs

Using empirical rather than parameterized productivity distributions also has implications for the welfare gains from trade resulting from the two canonical trade models that we consider. Ultimately, the differences in marginal effects of reductions in trade barriers described above translate into reductions in price indices and changes in overall costs and wages. We calculate counterfactual gains from trade from a 20% cost reduction in τ_{ij}^s for all $i \neq j$ as follows:

$$\text{Welfare Change}_i = 100\% \times \left(\frac{w_i\prime}{\prod_{k=1}^{S} (P_i^k\prime)^{\alpha_i^k}} \frac{\prod_{k=1}^{S} (P_i^k)^{\alpha_i^k}}{w_i} - 1 \right), \tag{44}$$

where w_i' and $P_i^{k\prime}$ are counterfactual wages and aggregate price indices, respectively. We plot counterfactual changes in welfare calculated under the empirical productivity distributions for the countries at hand and the calibration for 2008 in the left panel of Figure 15. Across the board, the welfare gains from trade are higher in the Melitz than in the Eaton-Kortum model with the data at hand. On average, the gains between the models differ by 9.7 percentage points. The standard deviation of the differences between the two models is 11.9 percentage points.

Figure 15: Welfare Gains from Trade

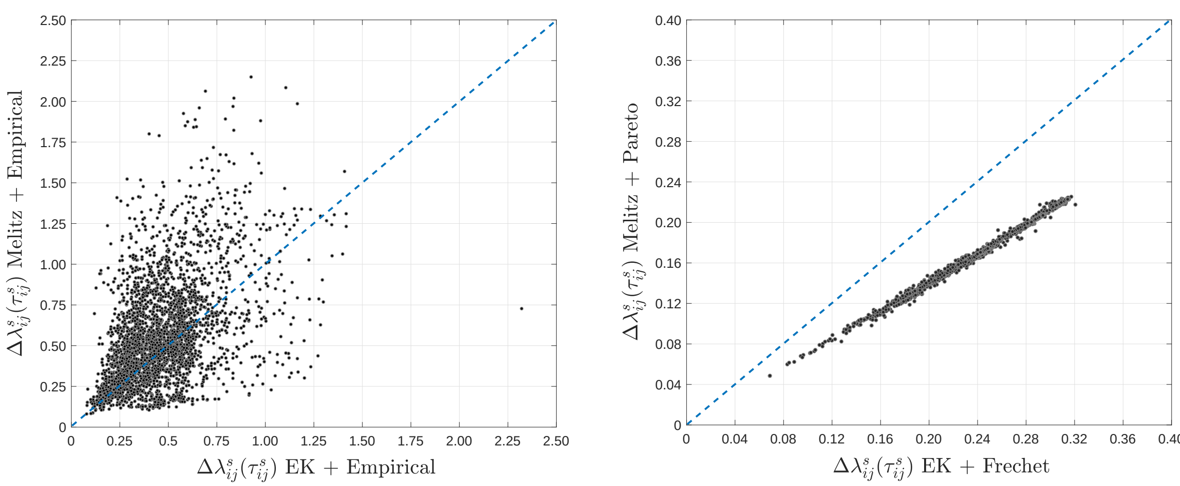


We plot the results of the same experiment calculated under the Fréchet parameterization of the productivity distributions in the Eaton-Kortum model and the Pareto parameterization in the Melitz model in the right panel of Figure 15. Again, the results are very close to each other due to the underlying isomorphism that comes in effect under the respective parameterizations. On average, the gains are larger in the Eaton-Kortum model by 1.2 percentage points with a standard deviation of 0.8. Here the differences again mainly stem from differences in the shape parameters of the two parameterized distributions. We report the estimates of the shape parameters in the Appendix.

7.2.3 Implications for model choice

Under customary macro-level assumptions, restrictive standard parameterizations of productivity distributions lead to quantitatively similar or even identical relationships between trade outcomes in response to changes in trade costs, e.g., intra-trade share and welfare changes, as demonstrated in Arkolakis et al. (2012). This isomorphism across different models of international trade makes the micro foundations irrelevant to a certain degree such that Fréchet- and Pareto-parameterized versions of the Eaton-Kortum and Melitz models are often used interchangeably.[22] This is in spite of the fact that these two models feature fundamentally different competition types and selection effects. In this section, we show that the isomorphism across different models of trade is partly an artifact of specific parameterizations of the productivity distributions and that under empirical productivity distributions there are vastly different micro-to-macro implications between the Eaton and Kortum (2002) and Melitz (2003) models of trade.

Figure 16: Marginal responses in the nonparametric Eaton-Kortum vs. Melitz models



We start with examining the differences in the marginal responses of trade to a 20% reduction in international trade costs across the two models. Using the same notation as in the previous section, we plot the results under the empirical productivity distributions for all covered country-pairs and

[22] Melitz and Redding (2015) discuss how using a bounded Pareto instead of an unbounded Pareto distribution breaks this isomorphism between the Melitz and other models.

sectors in our data in the left panel of Figure 16. The figure suggests that the two models have largely different quantitative predictions about how trade flows would change in response to the counterfactual trade liberalization. The correlation coefficient between $\Delta\lambda_{ij}^{s}(\tau_{ij}^{s})$ calculated in the nonparametric Eaton-Kortum and Melitz models is 0.5. On average, the marginal response in the latter model is 0.54, which is higher than 0.45 in the former one. We contrast these results with the ones under the customary parameterized model versions of the productivity distributions in the right panel of Figure 16. The differences in the predictions of the two models under the empirical and parameterized distributions are obviously stark as the correlation between the predictions in the Eaton-Kortum and Melitz models is unity due to the aforementioned isomorphism under the parameterized distributions. There are also important qualitative differences as the parameterized versions imply a relatively more acute response in the Eaton-Kortum than in the Melitz model. Note that the quantitative differences between the parameterized versions stem from the shape parameters of the Fréchet and Pareto distributions that are different across models in this paper because they are estimated from micro data as opposed to the common practice of using macro data and in particular using trade-share responses to trade-cost changes. We report these parameters in the Appendix.

8 A look at the data: Suggestive evidence for the importance of micro-to-macro implications in macro data

In the analysis of the previous section, we focused on counterfactual predictions of calibrated models, while highlighting micro-to-macro implications of using empirical productivity distributions for trade shares and welfare relative to those under commonly parameterized model versions. One important insight was that there was a much stronger heterogeneity of the responses in trade shares in terms of the level of trade shares in the outset in the quantitative models based on the empirical productivity distributions of firms than with the parameterized distributions.

We present two types of analysis. First, we consider patterns of (tariff-change-scaled) changes in trade shares in the data between 1999 and 2008. In addition to the data used in the calibration for 2008, we use data for trade flows in 1999 together with import-tariff rates for our sample of countries and sectors. We then compute scaled changes in trade shares from 1999 to 2008 for sector s and country pair ij as follows:

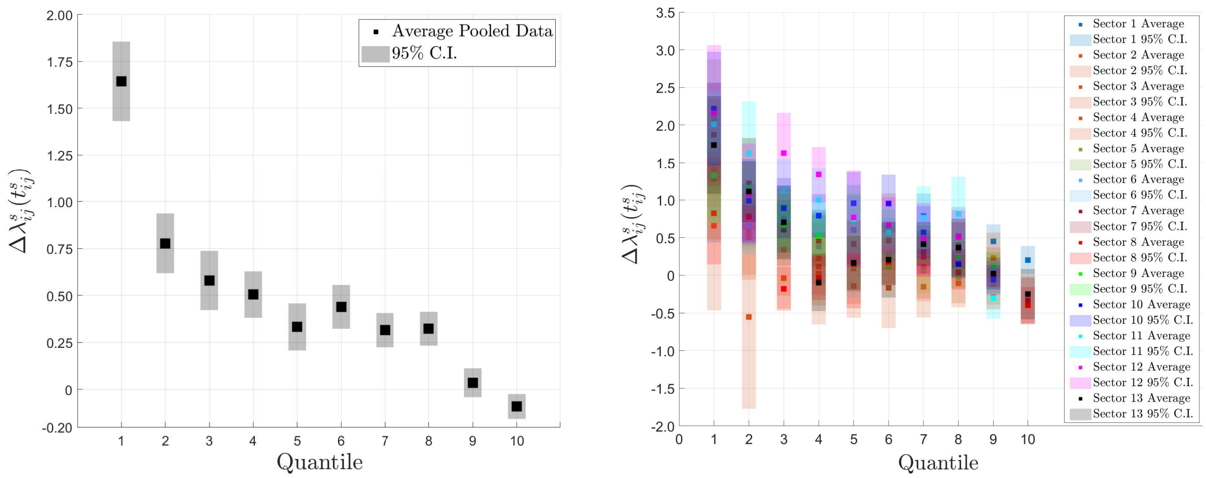
$$\Delta\lambda_{ij}^{s}(t_{ij}^{s}) = \log\left(\frac{\lambda_{ij}^{s}(2008)}{\lambda_{ij}^{s}(1999)}\Big/\frac{t_{ij}^{s}(2008)}{t_{ij}^{s}(1999)}\right) \quad \text{for each } s, \tag{45}$$

where t_{ij}^{s} denotes the average tariff rate applied by country j on imports of h from i.[23] We plot scaled changes $\Delta\lambda_{ij}^{s}(t_{ij}^{s})$ against quantiles of the trade-share levels in 1999 in Figure 17. The left and

[23] We aggregate product-level tariff lines to unweighted sector-level average import tariffs using data from the World Integrated Trade Solutions (WITS).

right panels present tariff-scaled changes in trade shares when pooling the data across all products and when pooling them within 13 sectors, respectively.

Figure 17: Normalized unconditional changes in trade shares in response to tariff changes (data)



An inspection of Figure 17 suggests the following insights. There is a high degree of dependence of the tariff-normalized changes in trade shares between 1999 and 2008 on the trade-share levels in 1999 and the degree of variation and pattern in this relationship is similar to the results that we saw in the counterfactual analysis of marginal trade-cost effects in Figures 13 and 14. This holds true for the data pooled across all sectors as well as within sectors.[24] Clearly, looking at unconditional data patterns is not a formal test, but we interpret the similarities in patterns of correlations between trade-share changes and outset-trade-share levels as reassuring.

To make more formal comparisons between the data and the models' predictions, we ran counterfactual experiments by using the full-fledged general-equilibrium models when imposing tariff changes as observed between 1999 and 2008 while keeping all other primitives as in the 2008 benchmark. We then categorize changes based on quantiles of the model-implied trade shares in 1999 to produce model-consistent data as presented in Figure 17 for each of the four versions of the model. We regress average changes in each quantile of trade shares observed in the data on the corresponding changes predicted by the Eaton-Kortum and Melitz models when using empirical productivity distributions as well as their parameterized versions. We report the explained variation of each of these four regressions (R^2) and present it in Table 6.

The results in Table 6 suggest that the predictions of the Eaton-Kortum and Melitz models under the empirical productivity distributions significantly outperform the predictions under the parameterized distributions both when we use data pooled across all sectors as well as within the average sector. We can conclude that both models considered in this work predict very strong heterogeneous responses of trade outcomes to changes in trade costs when informed by empirical productivity dis-

[24] Note that the change in trade costs was homogeneous in Figures 13 and 14, while it is heterogeneous here.

Table 6: Data vs. Models' Predictions

	Pooled Average	**Average across sectors**
EK + Empirical	0.90	0.58
Melitz + Empirical	0.90	0.65
EK + Frechet	0.70	0.50
Melitz + Pareto	0.70	0.47

tributions which stands in stark contrast to models using customary parameterized productivity distributions which predict much more homogeneous responses.

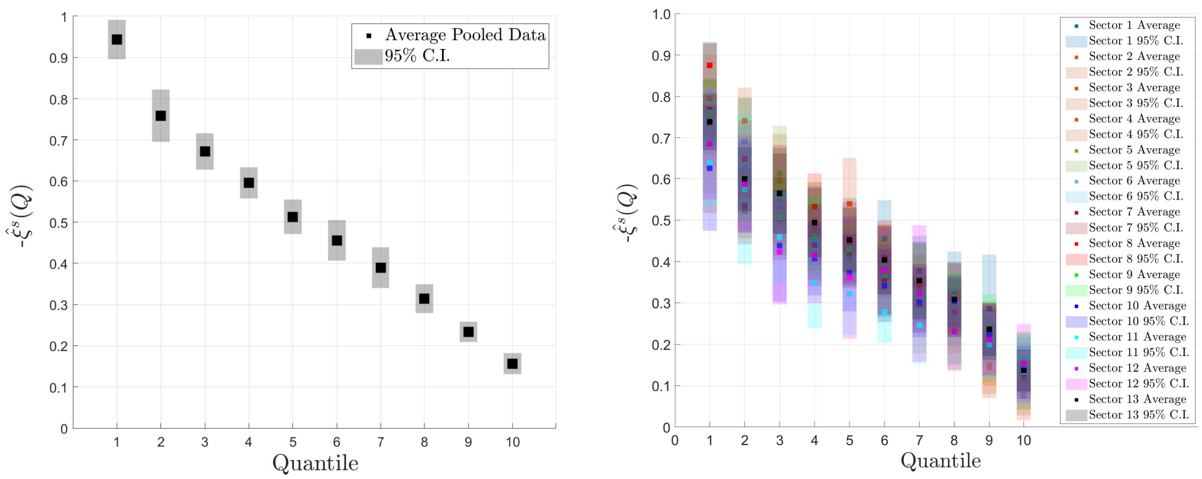
In a next step, we evaluate the models' predictions using, perhaps, the most popular empirical tool in international trade – the gravity equation. In particular, both models predict that under the empirical productivity distributions the marginal responses of trade shares to changes in bilateral trade barriers are higher for country pairs where initial trade shares are lower. We assess how these predictions square with the data by using the gravity equation, allowing for quantile-specific distance coefficients in two ways. First, we run a standard version of the gravity equation while allowing the coefficient on the (log) bilateral distance variable to vary across quantiles of the trade data. As classifying quantiles on λ_{ij}^s may lead to endogeneity problems, we use lagged data on trade shares (from 1999) to form quantiles, Q, while we use data from 2008 as our dependent variable. We then run the following regression:

$$\log \lambda_{ij}^s(2008) = \sum_Q \mathbb{Q}(Q)\xi^s(Q) \ln \text{distance}_{ij} + Z_{ij}^s \Lambda + \epsilon_{ij}^s \text{ for each } s, \tag{46}$$

where Z_{ij}^s is a vector of controls that consists of exporter and importer fixed effects and ϵ_{ij}^s is the disturbance term. Also, Q denotes the data quantile and $\mathbb{Q}(Q)$ is an indicator function, which is unity whenever $\lambda_{ij}^s(1999) \in Q$ and zero otherwise. We report the estimates of $\xi^s(Q)$ in Figure 18. Consistent with the predictions of the two models under non-parametric productivity distributions, the elasticity of trade shares to bilateral distance is decreasing (in absolute terms) across quantiles. This holds both when we estimate the gravity equation using pooled data across sectors (left panel) and when we run it by sector (right panel). Note that the results in Figure 18 can also be potentially explained by a non-CES structure of demand as in Carrère et al. (2020). We, however, emphasize that the pattern of a decreasing elasticity of trade shares to bilateral distance across trade-share quantiles observed in the data is generated by using empirical productivity distributions even under CES preferences.

In order to check how well the reduced-form gravity estimates are aligned with the predictions from our model, we relate the model-implied responses from a 20% reduction in trade costs to an equivalent response in the reduced form (which corresponds to $-20\hat{\xi}^s(Q)$). The resulting measures of explained variation (R^2) in Table 7 suggest that the reduced-form estimates of the gravity equation align better with the predictions of the Eaton-Kortum and Melitz models under the empirical productivity distributions compared to their parameterized counterparts.

Figure 18: Estimates of distance elasticities across trade-share quantiles



Alternatively, we use a quantile estimator where quantile thresholds are determined in estimation as opposed to lagged values of λ_{ij}^s. For computational reasons, we exclude the exporter-sector and importer-sector fixed effects here and focus on the unconditional quantile-specific correlation between log distance and λ_{ij}^s. We use the following quantile estimator for each sector:

$$\hat{\xi}^s(Q) = \underset{\xi^s \in \mathbb{R}^k}{\operatorname{argmin}} \sum_{ij} \rho^s(Q) \left[\ln(\lambda_{ij}^s) - \xi^s \ln(\text{distance}_{ij})\right] \quad \text{for each} \;\; s, \tag{47}$$

where $\rho^s(Q)$ is a linear "check function". We obtain quantile-specific distance coefficients for each sector s. As explained above, we calculate the R^2 between the predictions of the two models regarding the response of trade shares to a 20% reduction in variable trade costs both under empirical productivity distributions as well as under their parametric counterparts and the reduced-form prediction corresponding to $-20\hat{\xi}^s(Q)$. We report the results in the lower panel of Table 7. Again, the correlations between the models' predictions and the estimated distance coefficients are bigger under empirical productivity distributions than those based on customary Fréchet and Pareto

Table 7: Gravity coefficients vs. models' predictions

Estimator	**Model**	**Pooled Average**	**Average across sectors**
OLS	EK + Empirical	0.94	0.85
	Melitz + Empirical	0.90	0.59
	EK + Fréchet	0.41	0.48
	Melitz + Pareto	0.41	0.49
Quantile	EK + Empirical	0.91	0.61
	Melitz + Empirical	0.85	0.37
	EK + Fréchet	0.32	0.27
	Melitz + Pareto	0.32	0.27

parameterizations. While both regressions in (46) and (47) admittedly have certain limitations, we find it reassuring that the corresponding results are consistent with each other and that they both suggest that the predictions under the empirical productivity distributions align relatively better with the data.

9 Conclusion

This paper quantifies the role of distributional productivity differences as drivers of international trade flows. It employs firm-level productivity parameters to formulate empirical country-sector-specific productivity distributions and uses them to inform two canonical models of international trade that feature heterogeneous firms. We find that depending on the underlying model at hand, absolute productivity differences across countries on average explain 15%-21% of the variation in international trade shares, whereas relative productivity differences across sectors on average account for 39-47%. This suggests that technological differences play a quantitatively important role in shaping trade patterns and often constitute the most important driver of trade.

We also find that using empirical productivity distributions leads to a substantially larger heterogeneity of marginal responses of trade shares to changes in variables trade costs, which has important implications for the predictions of the welfare gains from trade, the macro-based model isomorphism with respect to changes in trade costs, and the gravity equation. We confirm the predictions obtained under empirical productivity distributions using unconditional data on tariffs and trade flows as well as the coefficients from the gravity equation.

This paper also highlights the importance of paying attention to the empirical distributions of firms in data when informing quantitative models. We detect large deviations of such empirical distributions across 13 sectors and 15 countries from a host of customary functional forms. We use Kolmogorov-Smirnov statistics and reject the hypothesis that country-sector-specific productivity distributions can be parameterized under a single parametric family. We show that attempts to parameterize the distributions under the Fréchet and Pareto families, which dominate current quantitative work, understate the role of productivity differences for trade outcomes.

We examined the cross-sectional variation in productivity distributions across countries in different sectors while treating them as given and exogenous. It would be potentially fruitful to use our estimates to examine factors such as institutions that shape country-sector-specific productivity distributions. We leave this for future research.

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Appendix A: Two-country models of trade

In Section 2, we used two-country versions of the Eaton-Kortum and Melitz models. Here, we provide details on the fundamentals that we used in these two models. The notation follows the main text. We solve the Eaton-Kortum model using the following equations:

$$\begin{aligned}
F_{ij}(p) &= 1 - F_i\left(\frac{w_i \tau_{ij}}{p}\right) \\
\lambda_{ij} &= \int_0^1 \prod_{k \neq i} \left(1 - F_{kj}(p)\right) dF_{ij}(p) \\
P_i &= \int_0^1 p dF_i(p) \\
w_i &= \sum_j \lambda_{ij} (L_j w_j)/L_i.
\end{aligned}$$

The Melitz model is described using the following set of equations:

$$\begin{aligned}
(\phi_{ij}^*)^{1-\sigma} &= \frac{(P_j)^{\sigma-1}(L_j w_j)}{\sigma w_i f_{ij}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}^s\right)^{1-\sigma} \\
P_j^{1-\sigma} &= \sum_{i=1}^{I} N_i \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} \int_{F_i(\phi_{ij}^*)}^1 \phi^{\sigma-1} dF_i(\phi) \\
\lambda_{ij} &= N_i (P_j)^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} \int_{F_i(\phi_{ij}^*)}^1 \phi^{\sigma^s-1} dF_i(\phi) \\
w_i &= \sum_j \lambda_{ij} (L_j w_j)/L_i.
\end{aligned}$$

The primitives that we use are described in Table 8:

Table 8: Model Primitives

L_1	L_2	τ_{11}	τ_{12}	τ_{21}	τ_{22}	f_{11}	f_{12}	f_{21}	f_{22}	N_1	N_2	σ
100	100	1	2	2	1	0.1	1	1	0.1	100	100	2

To illustrate different dimensions of productivity distributions and how they can differ across countries, we use Richards' curve for generalized logistic functions in Richards (1959) to generate CDFs according to the following equation:

$$F(z) = \frac{1}{\left(1 + \{Location\} e^{-\{Scale\} \ln(z)}\right)^{-\frac{1}{\{Shape\}}}}.$$

The advantage of this approach is that the three parameters independently govern the three margins of CDFs location, scale, and shape. In the outset, we fix all parameters to unity.

Appendix B: Alternative measures of firm productivity

The main analysis of this paper relies on productivity estimates that are based on the estimation of country-sector-specific production functions following the approach by Ackerberg et al. (2015). The conclusions of this paper remain however valid if we use alternative measures of productivity. We show this by employing two different approaches to obtain productivity estimates. First, we constrain production functions to have the same Cobb-Douglas production function parameters across countries. Second, we alternatively take a widely-used ad-hoc measure of productivity which is value added per worker. In order to show that the underlying patterns of productivity distributions across countries and sectors our analysis is based on also hold under these alternative estimation procedures, we correlate each of the alternative levels of productivity with our baseline measure across sectors and countries for each quantile of the distributions. We present two alternative correlation measures, the standard Pearson correlation coefficient as well as Spearman's rank correlation coefficient for both alternative measures of productivity in Figure 19. The figure confirms that the alternative measures produce a very comparable ranking of productivity across countries and sectors in each quantile of the distribution with correlation coefficients being largely within the interval of 0.85-0.90. A second analysis demonstrating that the identified patterns across distributions – which eventually drive the main results of the paper – are largely comparable for alternative distributions, is presented in Figures 20 and 21. The patterns of Kolmogorov-Smirnov statistics across country-sector-specific distributions resembles the pattern presented in Figure 6 in the main text.

Figure 19: Correlation of alternative measures of productivity across countries and sectors for each quantile

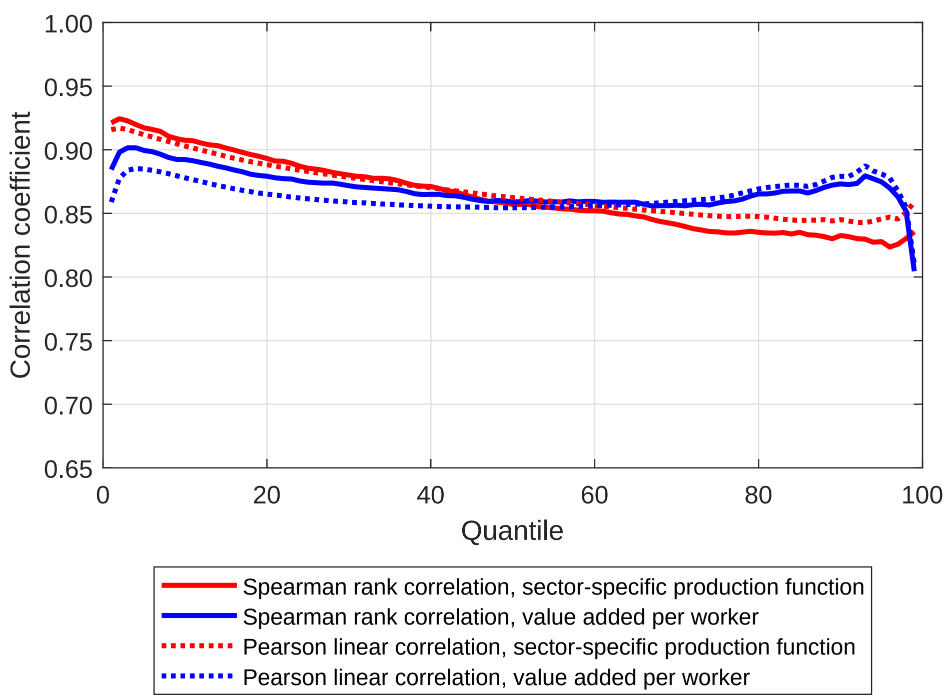


Figure 20: Illustration of CDFs and Kolmogorov-Smirnov statistics – Estimation based on sector-specific production functions

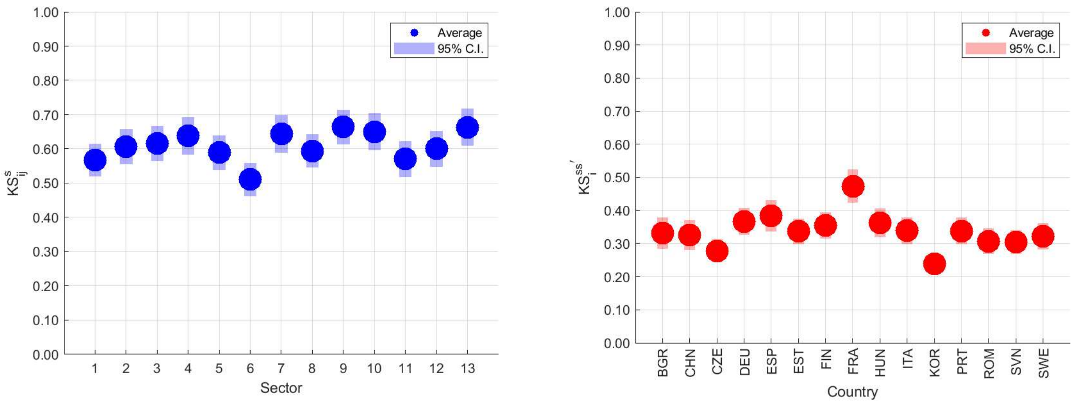
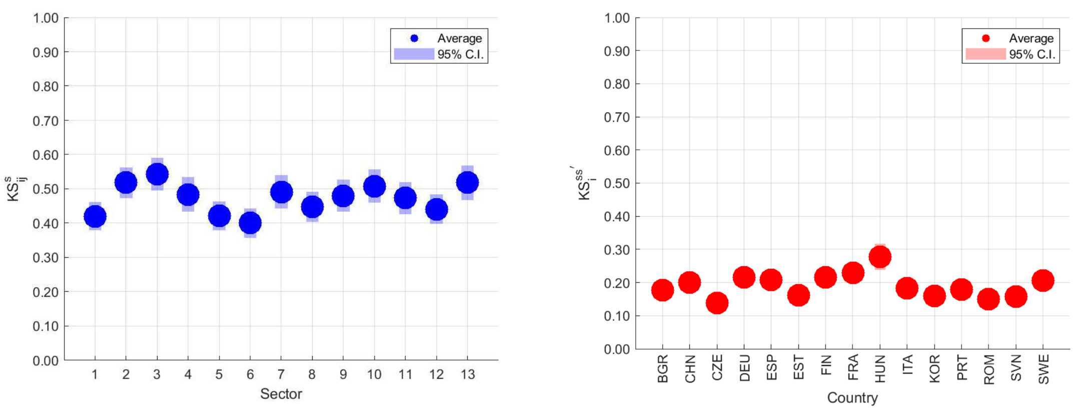


Figure 21: Illustration of CDFs and Kolmogorov-Smirnov statistics – Estimation based on value added per worker



Appendix C: Quantitative solution methods and calibration

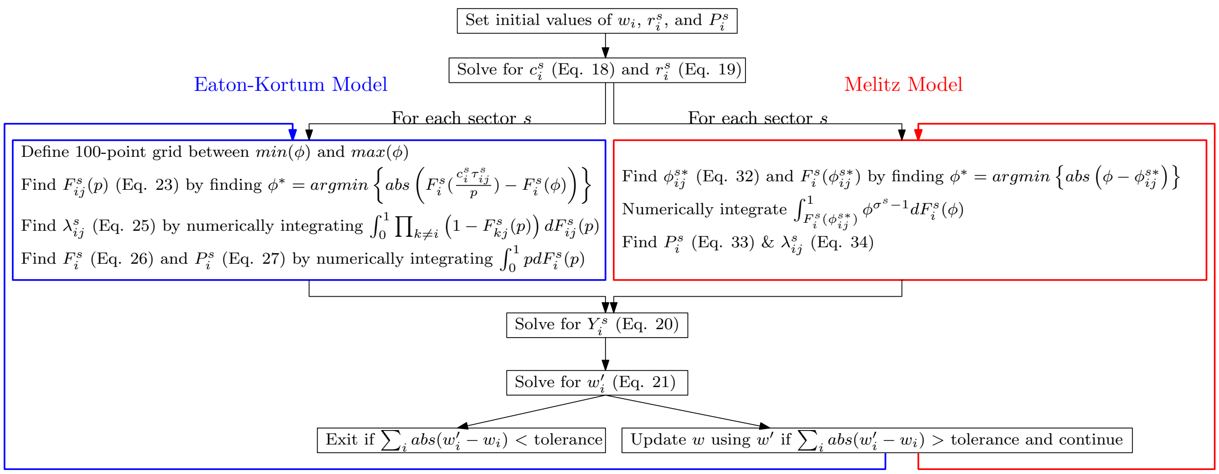
In this Appendix, we describe the solution algorithms that we use to calibrate and solve the Eaton-Kortum and Melitz models. The key difference between the techniques used with the parametric versus the non-parametric distributions is the treatment of ϕ in $F_i^s(\phi)$. In the former case, ϕ is are viewed as continuous argument of $F_i^s(\phi)$, and the CDF and its truncations can be readily calculated analytically.

We focus our attention on the case, where functional form of $F_i^s(\phi)$ is not known. There, the empirical CDFs are represented on a model-specific line with K segments, where each average level $\phi(K)$ corresponds to a specific value of $F_i^s(\phi(K))$. We choose the number of segments K in the Eaton-Kortum and Melitz models to be 10,000 and 110,010, respectively. These numbers were chosen to minimize computational time subject to the constraint that the discretized versions of the distributions lead to the same quantitative outcome as their continuous counterparts, when using the Fréchet and Pareto parameterizations. We conclude that the grid is sufficiently fine as long as adding more grid points does not change the outcomes and all endogenous variables are identical in two equilibria.

A computational challenge that discretization of $F_i^s(\phi)$ entails is calculating integrals in Equations (25), (27), (33), and (34). For that, we use the MATLAB functions *trapz* and *cumtrapz* that are based on trapezoidal numerical integration. Given that we make sure that the numerical grid is sufficiently populated, the accuracy of these numerical functions is very high such that the values of the integrals computed numerically is negligibly small relative to the analytical counterparts with the customary parameterizations.

The overall solution algorithms are based on contraction mapping and are sketched out in Figure 22. As the two models share common equations as given in Section 5.1 and otherwise involve the model-specific equations in Sections 5.2 and 5.3, we are able to present the algorithms in a single scheme. Clearly, the models are always solved separately.

Figure 22: Sketch of the solution algorithms



Appendix D: Estimated parameters across parametric distribution families

Figures 9 and 10 present summary statistics of the estimated parameters when fitting the micro data of estimated productivity to various parametric families using QQ estimation. For each family, we present the estimates averaged over sectors across countries (see Table 9) and averaged over countries across sectors (see Table 9). We present estimates based on a flexible estimation that allows both parameters to vary at the country-sector level in the first row and a more constrained estimation where shape parameters (θ_i^s) are only sector-specific in the second row, respectively. Note that, even though b_i^s is country-sector specific under both estimation procedures the estimated parameters b_i^s that guarantee a best fit might be different under the two procedures.

Table 9: PARAMETER ESTIMATES FOR COMMON PARAMETRIC DISTRIBUTION FUNCTIONS – AVERAGES OVER SECTORS ACROSS COUNTRIES.

Family	BGR	CHN	CZE	DEU	ESP	EST	FIN	FRA	HUN	ITA	KOR	PRT	ROM	SVN	SWE
$b_i^s\|_{\theta_i^s}$ and $b_i^s\|_{\theta^s}$															
Fréchet	0.00	0.00	0.01	0.06	0.00	0.00	0.01	0.01	0.03	0.01	0.02	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.04	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Gumbel	0.03	0.06	0.06	0.24	0.07	0.03	0.12	0.18	0.09	0.16	0.10	0.03	0.02	0.08	0.09
	0.01	0.06	0.05	0.32	0.05	0.02	0.13	0.18	0.12	0.17	0.11	0.01	-0.00	0.07	0.08
Log Logistic	0.03	0.07	0.06	0.29	0.08	0.04	0.14	0.20	0.11	0.18	0.11	0.04	0.03	0.09	0.10
	0.03	0.07	0.06	0.29	0.08	0.04	0.14	0.20	0.11	0.18	0.11	0.04	0.03	0.09	0.10
Log Normal	-3.44	-2.66	-2.80	-1.31	-2.58	-3.31	-2.06	-1.64	-2.27	-1.73	-2.23	-3.27	-3.64	-2.44	-2.29
	-3.44	-2.66	-2.80	-1.31	-2.58	-3.31	-2.06	-1.64	-2.27	-1.73	-2.23	-3.27	-3.64	-2.44	-2.29
Pareto	0.02	0.04	0.03	0.15	0.05	0.02	0.08	0.14	0.04	0.11	0.05	0.02	0.02	0.05	0.07
	0.02	0.04	.04	0.17	0.05	0.02	0.08	0.12	0.06	0.11	0.06	0.02	0.02	0.05	0.06
Power	0.08	0.14	0.13	0.57	0.12	0.08	0.24	0.29	0.27	0.29	0.24	0.06	0.04	0.14	0.15
	0.06	0.12	0.11	0.50	0.13	0.07	0.23	0.34	0.18	0.30	0.19	0.07	0.04	0.15	0.17
Weibull	0.05	0.10	0.09	0.39	0.09	0.05	0.17	0.23	0.16	0.22	0.15	0.05	0.03	0.11	0.12
	0.04	0.09	0.08	0.37	0.10	0.05	0.17	0.25	0.14	0.22	0.14	0.05	0.03	0.11	0.13
θ_i^s and θ^s															
Fréchet	1.69	1.96	1.85	2.45	3.49	2.17	2.77	4.28	1.52	2.95	1.79	2.99	2.76	2.87	3.48
	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60
Gumbel	-0.03	-0.04	-0.03	-0.20	-0.02	-0.02	-0.06	-0.06	-0.10	-0.06	-0.07	-0.02	-0.01	-0.03	-0.03
	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
Log Logistic	2.51	2.87	2.78	3.58	5.10	3.23	4.12	6.26	2.26	4.38	2.66	4.38	4.04	4.23	5.18
	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84
Log Normal	0.75	0.64	0.66	0.56	0.36	0.60	0.47	0.32	0.85	0.42	0.69	0.43	0.46	0.45	0.36
	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
Pareto	1.20	1.40	1.28	1.75	2.50	1.52	1.95	3.06	1.06	2.08	1.27	2.13	1.96	2.04	2.45
	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Power	1.31	1.47	1.50	1.84	2.60	1.72	2.17	3.20	1.20	2.30	1.39	2.25	2.08	2.19	2.73
	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Weibull	1.78	2.01	2.00	2.51	3.57	2.31	2.93	4.38	1.61	3.11	1.88	3.07	2.84	2.98	3.68
	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71	2.71

Notes: We estimated parameters based on QQ estimation as detailed in section 4.1. Here, we present averages over sectors across countries. For each family and parameter, the first row corresponds to a flexible estimation approach where both b_i^s and θ_i^s are allowed to vary across countries and sectors. In the second row, θ^s is held constant across countries within sectors.

Table 10: Parameter estimates for common parametric distributions – averages over countries across sectors.

Family	15t16	17t18	19	20	21t22	24	25	26	27t28	29	30t33	34t35	36t37
$b_i^s\|_{\theta_i^s}$ and $b_i^s\|_{\theta^s}$													
Fréchet	0.01	0.03	0.02	0.00	0.01	0.03	0.00	0.01	0.00	0.00	0.01	0.01	0.01
	0.01	0.01	0.01	0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Gumbel	0.08	0.11	0.13	0.07	0.09	0.11	0.09	0.10	0.08	0.09	0.09	0.08	0.07
	0.08	0.11	0.13	0.07	0.09	0.11	0.09	0.10	0.08	0.09	0.09	0.08	0.07
Log Logistic	0.09	0.13	0.15	0.08	0.10	0.14	0.10	0.12	0.09	0.10	0.10	0.09	0.08
	0.09	0.13	0.15	0.08	0.10	0.14	0.10	0.12	0.09	0.10	0.10	0.09	0.08
Log Normal	-2.63	-2.41	-2.29	-2.75	-2.52	-2.24	-2.48	-2.40	-2.59	-2.46	-2.48	-2.60	-2.80
	-2.63	-2.41	-2.29	-2.75	-2.52	-2.24	-2.48	-2.40	-2.59	-2.46	-2.48	-2.60	-2.80
Pareto	0.05	0.06	0.08	0.05	0.06	0.06	0.06	0.07	0.06	0.06	0.06	0.05	0.04
	0.05	0.07	0.09	0.05	0.06	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.04
Power	0.18	0.27	0.27	0.12	0.16	0.29	0.16	0.21	0.14	0.16	0.18	0.16	0.14
	0.17	0.23	0.25	0.12	0.15	0.27	0.16	0.21	0.14	0.16	0.17	0.15	0.13
Weibull	0.12	0.18	0.19	0.09	0.12	0.19	0.12	0.15	0.11	0.12	0.13	0.12	0.10
	0.12	0.17	0.19	0.09	0.12	0.18	0.12	0.15	0.11	0.12	0.13	0.11	0.10
θ_i^s and θ^s													
Fréchet	2.22	2.18	2.51	2.91	3.20	1.89	2.93	2.31	3.24	3.03	2.39	2.46	2.54
	2.22	2.18	2.51	2.91	3.20	1.89	2.93	2.31	3.24	3.03	2.39	2.46	2.54
Gumbel	-0.05	-0.09	-0.07	-0.03	-0.04	-0.12	-0.03	-0.06	-0.03	-0.05	-0.04	-0.04	-0.04
	-0.05	-0.09	-0.07	-0.03	-0.04	-0.12	-0.03	-0.06	-0.03	-0.05	-0.04	-0.04	-0.04
Log Logistic	3.22	3.24	3.72	4.31	4.71	2.78	4.36	3.38	4.77	4.47	3.55	3.65	3.74
	3.22	3.24	3.72	4.31	4.71	2.78	4.36	3.38	4.77	4.47	3.55	3.65	3.74
Log Normal	0.59	0.60	0.57	0.45	0.44	0.69	0.45	0.58	0.43	0.46	0.56	0.59	0.55
	0.59	0.60	0.57	0.45	0.44	0.69	0.45	0.58	0.43	0.46	0.56	0.59	0.55
Pareto	1.60	1.53	1.77	2.06	2.27	1.35	2.06	1.65	2.30	2.14	1.68	1.74	1.80
	1.60	1.53	1.77	2.06	2.27	1.35	2.06	1.65	2.30	2.14	1.68	1.74	1.80
Power	1.62	1.71	1.95	2.25	2.44	1.43	2.30	1.73	2.47	2.33	1.87	1.91	1.93
	1.62	1.71	1.95	2.25	2.44	1.43	2.30	1.73	2.47	2.33	1.87	1.91	1.93
Weibull	2.24	2.31	2.64	3.05	3.32	1.95	3.10	2.37	3.36	3.16	2.52	2.59	2.63
	2.24	2.31	2.64	3.05	3.32	1.95	3.10	2.37	3.36	3.16	2.52	2.59	2.63

Notes: We estimated parameters based on QQ estimation as detailed in section 4.1. Here, we present averages over countries across sectors. For each family and parameter, the first row corresponds to a flexible estimation approach where both b_i^s and θ_i^s are allowed to vary across countries and sectors. In the second row, θ^s is held constant across countries within sectors.