Earnings Inequality in Production Networks*

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Abstract

How do production networks affect earnings inequality? We develop a model of heterogeneous firms that hire labor in imperfectly competitive markets and purchase intermediate goods from connected suppliers in their production network. Firms combine labor and intermediates to produce output which is sold to final consumers (common to all firms) and upstream to customers in their network. The model forges a direct link between production networks and wage determination. We show that several key parameters mediate the impact of networks on earnings – the firm-specific labor supply elasticity, the elasticity of substitution between labor and intermediates and the price elasticity of demand. Using the model, we establish identification of these parameters using a similar set of assumptions that follow existing approaches in the literature (Lamadon et al. (2019) and Bernard et al. (2019)). To estimate these parameters, we link population-level matched employee-employer data with firm-to-firm transaction VAT data from Chile. Our empirical estimates indicate that firms face upward-sloping labor supply curves and that labor and intermediates are complements in production. We use the estimated structural model to quantify the importance of production network heterogeneity for earnings inequality by simulating a counterfactual equilibrium under a random production network. We find that network heterogeneity accounts for 12 percent of log earnings variance in Chile.

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1 Introduction

A growing literature in labor economics has highlighted the importance of firms as a determinant of earnings inequality. This is based on a standard variance decomposition of earnings inequality into a firm component and worker component. Although there is some debate about the magnitude of the firm component, most studies find that firms matter either through the variance of the firm component and/or the correlation between the firm component and the worker component (see, for example, Card et al. (2013), Lamadon et al. (2019), Bonhomme et al. (2019)).

One channel through which firms may matter is market power in wage setting. Market power in labor markets can arise as a result of concentration (Berger et al. (2019), Jarosch et al. (2019)), differentiated jobs (Sorkin (2018), Card et al. (2018), Lamadon et al. (2019), Chan et al. (2019)), or search frictions (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Taber and Vejlin (2018)). Most models of imperfect competition share the common feature that wages at the firm are a markdown below the value of the marginal product. The markdown is a function of a the firm-specific labor supply elasticity which is constant when firms are atomistic or variable when there are a finite number of firms and hence strategic interactions between them.[1]. These models all have the implication that heterogeneity across firms due to technology differences and/or differences in demand leads to heterogeneity in wages and hence earnings inequality.

At the same time, a growing literature in macro and international trade has recently highlighted the role of production networks in shaping, amplifying or dampening disparities between firms. On one hand, production networks can be important in understanding the microfoundations of heterogeneity between firms as shown in Oberfield (2018) and Bernard et al. (2019). In particular, which type of buyers and suppliers the firm has, influences the size of firms through the creation of heterogeneous input costs and heterogeneous demand shifters. On the other hand, production networks can be relevant for propagating idiosyncratic shocks (Caliendo et al.,

[1]Several papers have estimated the firm-specific labor supply elasticity. See Staiger et al. (2010), Azar et al. (2019), Kline et al. (2019), Lamadon et al. (2019), Dube et al. (2019), and Kroft et al. (2019).

2017; Baqaee and Farhi, 2019a,b; Lim, 2019; Huneeus, 2019). This literature has shown how geography, dynamics and misallocation can generate propagation of idiosyncratic shocks through production networks. In particular, the literature shows how the economic setup and idiosyncratic shocks interact with upstream and downstream linkages to either amplify or dampen heterogeneity of outcomes across firms.

This paper seeks to bridge these two literatures. The first contribution of this paper is to develop a new model of imperfect competition in labor markets where firms purchase intermediate goods from suppliers in their network and sell their output to final consumers (common to all firms) and to other firms in their customer networks. We derive the equilibrium wage function in this model, which is a function of the standard markdown (depending on the labor supply elasticity facing the firm) and the marginal revenue product of labor (MRPL). The MRPL, in turn, is the product of worker productivity (which is permitted to vary across firms) and a term which is common to all workers at the firm.

The production network affects wages through this common term in two ways. First, there is a scale effect which operates directly through the firm's network demand and acts to increase wages. This effect is increasing in the demand elasticity. Second, there is an effect which operates through the unit cost of materials facing the firm. On one hand, a lower cost of materials leads to an increase in materials used which leads to more worker output, holding constant labor, and hence higher wages. On the other hand, the firm may optimally reduce (increase) labor if labor and materials are substitutes (complements) and this would act to further decrease (increase) wages. Hence, the net effect of the input cost on wages can be positive or negative, depending critically on how substitutable labor and materials are relative to how substitutable products are with each other. Note that critical for production networks to matter is the firm faces an upward-sloping labor supply curve. If labor supply was perfectly elastic, differences in network productivity would not matter for wages; firms would take the market wages as given and hire workers until the MRPL was equal to the market wage.

The advantage of modeling the formal connection between MRPL and the production network is that it allows one to consider how production networks matter for wage determination.

Within the theoretical framework, we derive comparative static results for how firm-level productivity shocks propagate through the network and affect wages at all other firms. We show that the strength of these effects can be measured to a first-order approximation by network observables, such as the shares of sales and expenditures accounted for by firms in each others' input and output markets. The results also highlight that the fundamental parameters governing these linkages are the labor supply elasticity, the demand elasticity, and the elasticity of substitution between labor and materials.

The second contribution of this paper is to develop a strategy for identification and estimation of these structural parameters. Central to our analysis are two key structural equations, one for log worker wage and another for the ratio of intermediate spending to total labor costs. The wage equation depends on a time-invariant firm fixed effect, a worker-firm interaction, time-varying worker characteristics and the firm's wage bill. The pass-through of "shocks" in the aggregate wage bill of a firm to the worker-level wage identifies the labor supply elasticity, similar to Lamadon et al. (2019).[2] The equation for the ratio of intermediate spending to total labor costs depends on relative factor prices, with the coefficient proportional to the substitutability between labor and intermediate goods. In order to estimate this equation, one has to estimate the firm component of wages and the unit price of intermediate goods. To empirically implement these equations, we follow the strategies proposed by Bonhomme et al. (2019) (labor) and Bernard et al. (2019) (intermediate goods). We make use of administrative employee-employer matched data and firm-to-firm transaction data for the full population of Chile. Our estimates of the wage equation indicate that firm fixed effects account for roughly 10 percent of the variance of log earnings across workers. We estimate a firm-specific labor supply elasticity around 6-7. In order to estimate the elasticity of substitution between materials and labor, we estimate intermediate input costs using a two-way reduced-form fixed effects regression based on firm-to-firm sales. We recover unbiased estimates of buyer and seller fixed effects as well as relationship-specific productivity residuals and establish the mapping between these fixed effects and the input costs. Our estimates indicate that the elasticity of substitution between labor and intermediates is 0.95

[2] A key difference with Lamadon et al. (2019) is that they use value-added shocks to identify the labor supply elasticity. We show that with intermediate goods, value-added shocks do not identify the labor supply elasticity.

implying that workers and suppliers are complements.

With these estimates in hand, we then use the model to quantify the importance of heterogeneity in the production network for earnings inequality in the labor market. To do so, we perform a simple counterfactual: we compare the equilibrium under the empirically-observed network with the equilibrium that obtains under a random network (in which all buyer-seller pairs are equally likely to match), holding constant the number of matches in the network. We find that eliminating heterogeneity in the production network reduces log earnings variance across workers by 12.0% and reduces the Gini coefficient of worker earnings by 6% (from 0.52 to 0.49).

2 Model

2.1 Labor market

2.1.1 Workers

Workers in the economy are heterogeneous in ability a with the set of worker abilities denoted by $A \subset \mathbb{R}_+^d$.[3] For each ability type $a \in A$, there is a continuum of workers of exogenous measure $L_t(a)$ at time t. We assume that workers have idiosyncratic preferences for working at different firms, where the set of firms is also exogenous and denoted by Ω. Specifically, the log utility that a worker m with ability a_{mt} obtains from working at firm i at time t is given by:

$$\log u_{mit} = \log w_{it}(a_{mt}) + \log g_i(a_{mt}) + \log \tau_t + \beta^{-1} \log \epsilon_{mit} \tag{2.1}$$

where $w_{it}(a)$ denotes the wage paid by firm i to workers of ability a, $g_i(a)$ denotes the value that workers of ability a derive from amenities of firm i, τ_t is a lump-sum income transfer received by each worker independent of ability or employer, and ϵ_{mit} is an idiosyncratic preference shock with β an inverse measure of the shock dispersion.

[3] The theoretical results that we will establish in this section do not require restrictions on the dimension d of the worker ability space. However, in the empirical estimation discussed in section 4, we will follow Lamadon et al. (2019) and assume $d = 2$ with worker ability comprised of a permanent component $\bar{a}$ and a time-varying component $\hat{a}$.

There are several important features of this utility specification. First, we model amenities in order to rationalize heterogeneity in compensating differentials across firms for workers of a given ability. This will be quantitatively important in matching the observed firm size-wage premium, for example. Second, lump-sum transfers τ_t will account for firm profits in the economy. Third, we assume that worker abilities are perfectly observable by firms but that firms cannot condition wages on the idiosyncratic preference shocks ϵ_{mit}. This will imply the existence of inframarginal workers at every firm who enjoy positive rents from their employment. Fourth, we assume for analytic tractability that the distribution of idiosyncratic preference shocks is characterized as follows.

ASSUMPTION **2.1.** *The distribution across workers of idiosyncratic worker preference shocks, $\epsilon_{mt} \equiv \{\epsilon_{mit}\}_{i \in \Omega}$, in every period is a logit distribution with cumulative distribution function:*

$$F(\epsilon_{mt}) = exp\left[-\left(\sum_{i \in \Omega} e^{-\frac{\epsilon_{mit}}{\rho}}\right)^{\rho}\right] \tag{2.2}$$

where $\rho \in (0, 1]$.

Note that Assumption 2.1 imposes structure on the *cross-sectional* distribution of worker preference shocks but does not otherwise restrict the *time-series* properties of these shocks for a given worker. Also, note that the parameter ρ controls the correlation of preference shocks across firms: as ρ approaches zero, workers view all firms as perfect substitutes, whereas as ρ approaches one, preference shocks across firms become independent random variables.

Assumption 2.1 implies that the probability that worker m chooses to work at firm i at time t is given by:

$$\mathrm{P}\left[i(m,t) = i\right] = \left[\frac{g_i(a_{mt})\, w_{it}(a_{mt})}{I_t(a_{mt})}\right]^{\gamma} \tag{2.3}$$

where $i(m,t)$ denotes the firm chosen by worker m at time t, $\gamma \equiv \beta/\rho$ is a composite parameter that depends on both the preference shock dispersion and correlation of these shocks across

firms, and $I_t(a_{mt})$ is a wage index given by:

$$I_t(a_{mt}) \equiv \left[\sum_{j\in\Omega} \left[g_j(a_{mt})\, w_{jt}(a_{mt})\right]^{\gamma}\right]^{\frac{1}{\gamma}} \tag{2.4}$$

Note that $I_t(a_{mt})$ captures the extent of labor market competition for workers of ability a_{mt}. Hence, we refer to this as the *competition index* for ability a_{mt} workers.

2.1.2 Firm-level labor supply

Given that the set of workers of ability a is continuous, the worker-choice probabilities (2.3) imply that the total supply of workers of ability a for firm i can be written as:

$$L_{it}(a) = \kappa_{it}(a)\, w_{it}(a)^{\gamma} \tag{2.5}$$

where $\kappa_{it}(a)$ is a firm-specific labor supply shifter:

$$\kappa_{it}(a) \equiv \bar{\kappa}_t(a)\, g_i(a)^{\gamma} \tag{2.6}$$

and $\bar{\kappa}_t(a)$ is the component of this shifter that is common to all firms:

$$\bar{\kappa}_t(a) \equiv \frac{L_t(a)}{I_t(a)^{\gamma}} \tag{2.7}$$

Now, we assume that the cardinality of the set of firms Ω is large enough such that each firm views itself as atomistic in the labor market. Specifically, in choosing wages for workers of any ability $a \in A$, each firm views the competition index $I_t(a)$ as invariant to the firm's choices. Hence, equation (2.5) implies that every firm behaves as though it faces an upward-sloping labor supply curve with a constant elasticity γ. Furthermore, this elasticity is common to all firms. In addition, note from (2.6) that conditional on wages, the supply of workers of a given ability only differs across firms due to differences in the value of firm amenities.

2.2 Final demand

We assume that every worker has identical preferences over the consumption of all goods produced by firms in the economy that are described as follows.

ASSUMPTION **2.2.** *The utility that worker m obtains from final consumption is:*

$$v_{mt} = \left[\sum_{i\in\Omega} (x_{mit})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2.8}$$

where x_{mit} is worker m's consumption of firm i's output and $\sigma > 1$ denotes the elasticity of substitution across varieties.

We take the CES price index of final consumption as the numeraire. In addition, we assume that each worker owns a common share of a mutual fund that rebates profits earned by all firms to workers, where the income derived from this share at time t is equal to the lump-sum transfer τ_t in (2.1). This rationalizes the worker utility specification described in section 2.1.1.[4] Furthermore, (2.8) implies that aggregate final demand for firm $i's$ output can be written as:

$$x_{Fit} = \Delta_{Ft} p_{Fit}^{-\sigma} \tag{2.9}$$

where p_{Fit} is the price of firm i's output for final sales and Δ_{Ft} is aggregate consumer income (labor income plus profits):

$$\Delta_{Ft} = \sum_{a\in A}\sum_{i\in\Omega} w_{it}(a) L_{it}(a) + \sum_{i\in\Omega} \pi_{it} \tag{2.10}$$

with π_{it} denoting profit earned by firm i. Note that Δ_{Ft} is also equivalent to aggregate value-added in the economy.

[4]Note that workers derive utility from three sources: consumption, firm amenities, and idiosyncratic tastes for employment at different firms. Hence, total utility for a worker m is $u_{mt} = v_{mt} h_{mt} \epsilon_{mt}$, where $u_{mt} \equiv u_{mi(m,t)t}$, $h_{mt} \equiv h_{i(m,t)}(a_{mt})$, and $\epsilon_{mt} \equiv \epsilon_{mi(m,t)t}$.

2.3 Production technology

Firms produce output by combining labor with materials (intermediate inputs). Combining one worker of ability a with $m_{it}(a)$ units of materials at firm i produces $f[\phi_{it}(a), m_{it}(a)]$ units of output, where $\phi_{it} : A \to \mathbb{R}_+$ maps worker ability into productivity. We assume that the properties of the worker-level production function f are as follows.

ASSUMPTION **2.3.** *The worker-level production function f:*

1. *is increasing in both arguments ($f_\phi > 0$ and $f_m > 0$);*
2. *is concave in materials per worker ($f_{mm} < 0$);*
3. *is homogeneous of degree one.*

Note also that we allow ϕ_{it} to vary by firm, which will capture potential worker-firm complementarities in productivity. Total output of firm i is then given by:

$$X_{it} = T_{it} \int_A L_{it}(a) f[\phi_{it}(a), m_{it}(a)] da \tag{2.11}$$

where T_{it} denotes TFP of firm i.

To model firm-to-firm trade, we assume that firms are potentially heterogeneous in their buyer-seller connections with other firms. We denote the set of firm i's customers and suppliers at date t by $\Omega_{it}^C \subset \Omega$ and $\Omega_{it}^S \subset \Omega$ respectively. Note that we do not restrict the network to be bipartite: firms can simultaneously be buyers and sellers. Materials for firm i are then produced by combining inputs from all of its suppliers as follows.

ASSUMPTION **2.4.** *Production of materials by firm i is given by:*

$$M_{it} = \left[\sum_{j \in \Omega_{it}^S} \alpha_{ijt}^{\frac{1}{\sigma}} (x_{ijt})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2.12}$$

where x_{ijt} denotes the quantity of inputs purchased by i from j, α_{ijt} is an idiosyncratic relationship-specific productivity shock and, $\sigma > 1$ denotes the elasticity of substitution across inputs.

As is standard in the literature, we assume the same elasticity of substitution across varieties in production as in final demand.[5] Note that the total allocation of materials to a firm's workers must be equal to total materials production by the firm:

$$\int_A m_{it}(a) L_{it}(a) da = M_{it} \tag{2.13}$$

2.4 Output market structure and profit maximization

We assume a market structure of monopolistic competition in output markets: each firm in the economy chooses a price of its output for each of its customers taking as given the prices set by all other firms, but does not internalize the effect of its choices on aggregate outcomes in either the labor market or output market. In choosing output prices, firms also take as given the labor supply function (2.5), the production technologies (2.11)-(2.12), and the final demand function (2.9).

To simplify exposition of the firm profit-maximization problem, we first note that since each buying firm takes its suppliers' prices as given, demand by firm i for inputs from firm j takes the standard form implied by the CES production technology (2.12):

$$x_{ijt} = \Delta_{it} \alpha_{ijt} p_{ijt}^{-\sigma} \tag{2.14}$$

where p_{ijt} is the price charged by seller j to buyer i, Δ_{it} is a firm-specific intermediate demand shifter that we refer to as *network demand*, given by:

$$\Delta_{it} = M_{it} (Z_{it})^{\sigma} \tag{2.15}$$

and Z_{it} is the unit cost of materials for firm i:

$$Z_{it} = \left[\sum_{j \in \Omega_{it}^S} \alpha_{ijt} \Phi_{jt} dj \right]^{\frac{1}{1-\sigma}} \tag{2.16}$$

[5] This simplifies the firm's profit maximization problem as it ensures that total (final plus intermediate) demand for a firm's output has a constant price elasticity.

Here, we have defined Φ_{it} as the *network productivity* of firm i, an inverse measure of the firm's price:

$$\Phi_{it} \equiv p_{it}^{1-\sigma} \tag{2.17}$$

Next, note that the upward-sloping labor supply curves faced by each firm imply that marginal costs of production are increasing in output. Hence, a firm's decisions to supply each of its customers are inherently interlinked: lowering prices to increase demand from one customer increases marginal cost and hence affects the choice of prices charged to another customer. However, even though firms are able to charge different prices to different customers, the following result establishes - perhaps somewhat surprisingly - that it is never optimal for them to do so.[6]

PROPOSITION **1.** *The profit-maximizing prices charged by any firm i to its customers (including final consumers) do not vary across customers:*

$$p_{jit} = p_{it},\ \forall j \in \Omega_{it}^{C} \cup \{F\}$$

Intuitively, each firm maximizes its profits by choosing prices such that marginal revenue from each customer is equal to marginal cost. From (2.9) and (2.14), demand from each customer features a constant and common price elasticity of $-\sigma$. Hence, marginal revenue is proportional to price. Furthermore, even though marginal cost is increasing, it depends only on total output of the firm and hence is common across customers. As a result, each firm optimally chooses to charge a common price to each of its customers in equilibrium.

With this result, total demand for firm i's output is then given from (2.9) and (2.14) as:

$$X_{it} = D_{it} p_{it}^{-\sigma} \tag{2.18}$$

where D_{it} is a demand shifter for the firm given by the sum of final demand (common to all

[6] All proofs are relegated to Section A of the appendix.

firms) and the network demands of the firm's customers:

$$D_{it} = \Delta_{Ft} + \sum_{j \in \Omega_{it}^{C}} \Delta_{jt} \alpha_{jit} \tag{2.19}$$

As with the labor market, we assume that each firm i behaves atomistically in each of its customers' input markets, taking the demand shifters $\{\Delta_{jt}\}_{j \in \Omega_{it}^{C} \cup \{F\}}$ as given when making choices.[7]

Finally, we can now write the profit-maximization problem for firm i concisely as a choice over its production inputs:

$$\pi_{it} = \max_{\{w_{it}(a), m_{it}(a)\}_{a \in A}} \left\{ D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} - \int_{A} w_{it}(a) L_{it}(a) \, da - Z_{it} \int_{A} L_{it}(a) m_{it}(a) \, da \right\} \tag{2.20}$$

$$\text{s.t. } X_{it} = T_{it} \int_{A} L_{it}(a) f\left[\phi_{it}(a), m_{it}(a)\right] da \tag{2.21}$$

$$L_{it}(a) = \kappa_{it}(a) w_{it}(a)^{\gamma} \tag{2.22}$$

The first-order conditions for this problem can be expressed as:

$$w_{it}(a) = \eta MRPL_{it}(a) \tag{2.23}$$

$$Z_{it} = MRPM_{it}(a) \tag{2.24}$$

where we define $\eta \equiv \frac{\gamma}{1+\gamma}$ for brevity. $MRPL_{it}(a)$ denotes the marginal revenue product of workers of ability a and $MRPM_{it}(a)$ denotes the marginal revenue product of materials allocated to workers of ability a.[8] These are in turn given by:

$$MRPL_{it}(a) = \frac{1}{\mu} p_{it} T_{it} \phi_{it}(a) f_{\phi}\left[\phi_{it}(a), m_{it}(a)\right] \tag{2.25}$$

$$MRPM_{it}(a) = \frac{1}{\mu} p_{it} T_{it} f_{m}\left[\phi_{it}(a), m_{it}(a)\right] \tag{2.26}$$

where we define $\mu \equiv \frac{\sigma}{\sigma-1}$ for brevity.

[7] This assumption is less restrictive when the cardinality of a firm's customer set is large.

[8] The second order conditions for the firm's profit maximization problem are satisfied under assumption 2.3.

Equation (2.23) hence states the familiar result that wages are a constant markdown $\eta \in (0,1)$ over the marginal revenue products of the respective worker types, while equation (2.24) states that the unit price of materials is equal to the marginal revenue product of materials. Furthermore, since the price of materials is invariant with respect to worker ability, equation (2.24) implies that the marginal revenue product of materials must be equalized across worker ability types. Equation (2.26) then implies[9] that materials are allocated to workers in proportion to their productivity:

$$m_{it}(a) = \nu_{it}\phi_{it}(a) \tag{2.27}$$

where ν_{it} is an endogenous constant of proportionality that can be interpreted as materials per efficiency unit of labor.

From equations (2.23) and (2.25), wages are then given by:

$$w_{it}(a) = \eta\phi_{it}(a) W_{it} \tag{2.28}$$

where W_{it} is the component of wages that is common to all workers employed at firm i:

$$W_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} f_{\phi}(1, \nu_{it}) \tag{2.29}$$

We hence refer to W_{it} as the *firm-level wage*. Similarly, the first-order condition for materials can be expressed as:

$$Z_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} f_{m}(1, \nu_{it}) \tag{2.30}$$

while equilibrium output for firm i is given by (2.11) as:

$$X_{it} = \eta^{\gamma} W_{it}^{\gamma} T_{it} f(1, \nu_{it}) \bar{\phi}_{it} \tag{2.31}$$

Here, we have defined $\bar{\phi}_{it}$ as:

$$\bar{\phi}_{it} \equiv \int_A \kappa_{it}(a) \phi_{it}(a)^{1+\gamma} da \tag{2.32}$$

[9] Recall that x is homogeneous of degree one and hence x_m is homogeneous of degree zero.

which is endogenous only through the labor supply shifters $\kappa_{it}(a)$. Hence, we refer to $\bar{\phi}_{it}$ as *labor productivity supply* for firm i.

Finally, we note that total labor and material costs for firm i are respectively:

$$E_{it}^{L} = \eta^{1+\gamma} \bar{\phi}_{it} W_{it}^{1+\gamma} \tag{2.33}$$

$$E_{it}^{M} = \eta^{\gamma} \nu_{it} Z_{it} \bar{\phi}_{it} W_{it}^{\gamma} \tag{2.34}$$

while sales from firm j to firm i can be written concisely in terms of the buyer's network demand, the seller's network productivity, and relationship-specific productivity:

$$R_{ijt} = \Delta_{it} \Phi_{jt} \alpha_{ijt} \tag{2.35}$$

Total sales for firm i can be expressed similarly as:

$$R_{it} = \Delta_{it} \Phi_{it} \tag{2.36}$$

2.5 Equilibrium definition and solution approach

We can now define an equilibrium of the model as follows.

Definition **1.** *An equilibrium of the model at time t is a list of values and functions for: (i) aggregate income, Δ_{Ft}; (ii) labor market competition indices, $I_t(\cdot)$; (iii) firm labor supply shifters, $\kappa_{it}(\cdot)$; (iv) network demands, Δ_{it}; (v) network productivities, Φ_{it}; (vi) demand shifters, D_{it}; (vii) material costs Z_{it}; (viii) material production, M_{it}; (ix) labor productivity supplies, $\bar{\phi}_{it}$; (x) output prices, p_{it}; (xi) output, X_{it}; (xii) firm-level wages W_{it}; (xiii) materials per efficiency unit of labor, ν_{it}; (iv) worker-level wages, $w_{it}(\cdot)$; (xv) materials per worker, $m_{it}(\cdot)$; and (xvi) employment, $L_{it}(\cdot)$, all of which satisfy equations (2.4), (2.5), (2.6) (2.10), (2.13), (2.15), (2.16), (2.17), (2.18), (2.19), (2.20), (2.27), (2.28), (2.29), (2.30), (2.31), and (2.32).*

A detailed computational algorithm that solves for an equilibrium of the model is provided in Section B of the appendix.

3 Equilibrium Analysis

We now study the model's equilibrium properties in more detail to sharpen intuition regarding the key economic mechanisms in the model.

3.1 Wages

3.1.1 Between-firm inequality: firm-level wages

We begin by establishing several comparative static results for firm wages, W_{it}, in order to better understand the determinants of between-firm earnings inequality.

Upstream, downstream, and firm effects. First, we examine how firm-level wages are affected by conditions at the firm. To do so, we study equations (2.29)-(2.31), which define a system in firm wages, output, and materials, $\{W_{it}, X_{it}, \nu_{it}\}$, given values for the firm's demand shifter, material cost, TFP, and labor productivity supply, $\left\{D_{it}, Z_{it}, T_{it}, \bar{\phi}_{it}\right\}$. The following proposition summarizes the main comparative static results for firm wages in this system.

PROPOSITION **2.** *Let* $\epsilon \equiv \left[\frac{\log\left(f_\phi / f_m\right)}{d \log \nu}\right]^{-1}$ *denote the elasticity of substitution between labor and materials. In response to marginal changes in* $\left\{D_{it}, Z_{it}, T_{it}, \bar{\phi}_{it}\right\}$*, the firm-level wage,* W_{it}*, is:*

(a) strictly increasing in D_{it}*;*

(b) strictly increasing in T_{it}*;*

(c) strictly decreasing in Z_{it} *if* $\sigma > \epsilon$*, strictly increasing in* Z_{it} *if* $\sigma < \epsilon$*, and independent of* Z_{it} *if* $\sigma = \epsilon$*; and*

(d) strictly decreasing in $\bar{\phi}_{it}$*.*

These effects on firm-level wages can be understood in terms of two economic forces: scale and substitution effects.

First, an increase in the firm's demand shifter, D_{it}, or TFP, T_{it}, leads to an upward shift of its isorevenue curve, allowing it to achieve higher scale holding constant its input mix of labor

and materials. Since firms face upward sloping labor supply curves, the increase in employment generated by these scale effects leads to higher wages at the firm.

Second, lower material costs Z_{it} also generate positive scale effects for the firm, which tends to increase wages. However, a reduction in material costs generates an additional substitution effect, as firms respond by moving along their isorevenue curve and changing the relative input of labor to materials in production. Intuitively, the strength of the scale effect is mediated by the elasticity of substitution across goods, σ: if products are more substitutable, the same reduction in input cost allows the firm to achieve a larger increase in sales since demand is more sensitive to price. The strength of the substitution effect, on the other hand, is naturally determined by the elasticity of substitution between labor and materials, ϵ: as ϵ decreases from 1 to 0 (labor and materials are complements), the same reduction in material cost induces the firm to hire *more* labor, whereas as ϵ increases from 1 to ∞ (labor and materials are substitutes), the same reduction in material cost induces the firm to hire *less* labor. Hence the net effect of changes in input cost Z_{it} on firm-level wages depends on how the scale effect (σ) compares with the substitution effect (ϵ).

Finally, an increase in labor productivity supply, $\bar{\phi}_{it}$, allows the firm to reduce the physical quantity of workers that it hires while maintaining the same scale and the same ratio of labor efficiency units to material inputs. Hence, the effect of an increase in $\bar{\phi}_{it}$ on firm-level wages is unambiguously negative.

Note that Proposition 2 implies two important predictions about the interaction between production network linkages and firm wages. First, increases in demand D_{it} resulting from shocks downstream in a supply chain always have positive effects on wages of firms upstream. Second, decreases in input cost Z_{it} arising from shocks upstream in a supply chain can have positive, negative, or neutral effects on wages of firms downstream, where the key determinant of these effects is how substitutable goods are with one another (σ) relative to how substitutable workers are with materials (ϵ).

Accounting for network effects. Proposition 2 characterizes how firm-level wages respond to changes in exogenous TFP, T_{it}, as well as changes in *endogenous* demand and supply variables,

$\{D_{it}, Z_{it}\}$. We now characterize how firm-level wages respond to exogenous shocks to TFP, accounting for the effect of these shocks on $\{D_{it}, Z_{it}\}$ through the production network. We will restrict attention to shocks that do not have general equilibrium effects in order to focus on other economic mechanisms of interest. Specifically, we will consider TFP shocks that do not affect aggregate income and the competition indices for each worker ability type, which we denote jointly by:

$$\Theta_t^{GE} \equiv \{\Delta_{Ft}, I_t(\cdot)\} \tag{3.1}$$

In what follows, we denote by $\hat{Y}_{it} \equiv d \log Y_{it}$ the marginal log change in a firm-level variable Y_{it} and by $\hat{Y}_t \equiv \left\{\hat{Y}_{it}\right\}_{i \in \Omega}$ the vector of these log changes for all firms. We also define the following observables. First, let Σ_t^C and Σ_t^S denote the output and input share matrices respectively, which are $|\Omega| \times |\Omega|$ matrices with (i,j)-elements given by:

$$\Sigma_{ijt}^C \equiv \frac{R_{jit}}{\sum_{k \in \Omega_{it}^C \cup \{F\}} R_{kit}} \tag{3.2}$$

$$\Sigma_{ijt}^S \equiv \frac{R_{ijt}}{\sum_{k \in \Omega_{it}^S} R_{ikt}} \tag{3.3}$$

In other words, Σ_{ijt}^C is the share of firm i's total sales accounted for by firm j, while Σ_{ijt}^S is the share of firm i's input purchases that are accounted for by firm j. Next, we define the labor share vector Λ_t, which is a $|\Omega| \times 1$ vector with i^{th}-element equal to the share of labor in total costs for firm i:

$$\Lambda_{it} \equiv \frac{E_{it}^L}{E_{it}^L + E_{it}^M} \tag{3.4}$$

Case 1: no intermediates. We begin by characterizing a special case of the model in which intermediate inputs are not used in production. The following proposition characterizes the effects of firm-level TFP shocks on firm-level wages in this setting.

PROPOSITION **3.** *Suppose that intermediates are not used in production. Then firm-level wages are given by:*

$$W_{it} = \left(\frac{\Delta_{Ft} T_{it}^{\sigma-1}}{\mu^\sigma \eta^\gamma \bar{\phi}_{it}}\right)^{\frac{1}{\gamma+\sigma}} \tag{3.5}$$

In response to marginal changes in TFP that do not affect Θ_t^{GE}, firm wages respond as follows:

$$\hat{W}_t = \frac{\sigma - 1}{\gamma + \sigma} \hat{T}_t \tag{3.6}$$

There are three important takeaways from Proposition 3.6. First, there is positive pass-through of firm TFP shocks to wages, which is a corollary of Proposition 2. Second, the degree of pass-through is increasing in the elasticity of substitution across goods, σ, and decreasing in the labor supply elasticity, γ. Pass-through is increasing in σ because a larger value of σ implies that the marginal revenue product of labor is more sensitive to changes in TFP, whereas pass-through is decreasing in γ because a larger value of γ implies that firms have less market power in labor markets.[10] Third, TFP shocks at one firm cannot affect wages at another firm except through general equilibrium.

Case 2: no upstream effects. Next, we consider a case of the model in which intermediates are used in production, but in which only downstream effects of TFP shocks on wages are operative. As shown in Proposition 2, upstream shocks that affect firm input costs Z_{it} have no effect on wages if $\sigma = \epsilon$.

PROPOSITION **4.** *Suppose that $\sigma = \epsilon$ globally. Then in response to marginal changes in TFP that do not affect Θ_t^{GE}, firm wages respond as follows:*

$$\hat{W}_t = \frac{\sigma - 1}{\gamma + \sigma} \left(\mathbb{I} - \Sigma_t^c\right)^{-1} \hat{T}_t \tag{3.7}$$

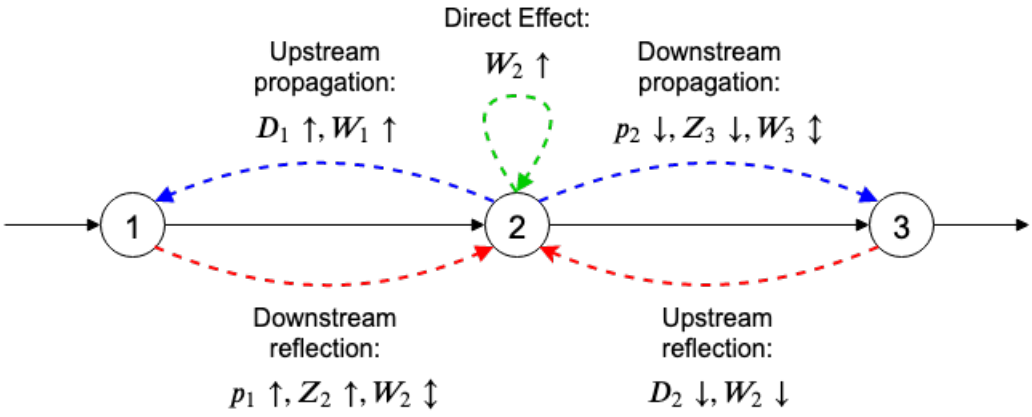
where $\mathbb{I}$ is the $|\Omega|$-dimensional identity matrix.

Equation (3.7) has a simple interpretation. First, note that the matrix $\left(\mathbb{I} - \Sigma_t^c\right)^{-1}$ can be written in terms of the following geometric series:

$$\left(\mathbb{I} - \Sigma_t^c\right)^{-1} = \mathbb{I} + \Sigma_t^c + \left(\Sigma_t^c\right)^2 + \cdots \tag{3.8}$$

[10] Equation (3.6) is identical to the pass-through of firm TFP shocks to wages in the Lamadon et al. (2019) model, which does not have intermediate inputs.

Figure 1: Example of interaction between upstream and downstream effects of TFP on wages



Hence, the (i, j)-element of this matrix summarizes the share of firm j in firm i's sales both directly and indirectly through the production network. Equation (3.7) thus implies that the pass-through from TFP shocks at firm j to changes in wages at firm i depends on how important j is as a buyer downstream of all the supply chains that i participates in, where buyer importance can be measured through observable sales shares Σ^c_{ijt}. Interestingly, in this case indirect buyer importance depends *only* on powers of the sales share matrix and does not otherwise decay with the degree of separation in the supply chain linkage between firms.

General case. Finally, we characterize the general case in which $\sigma \neq \epsilon$ and both downstream and upstream effects are operative. In this case, there are complex interactions between the upstream and downstream effects of TFP shocks on firm wages. To illustrate, consider three firms that are linked in a supply chain as shown in Figure 1 and suppose that there is a positive shock to TFP for the firm in the middle of the sub-chain ($\hat{T}_2 > 0$). This shock has several effects on wages in the supply chain.

First, as shown in Proposition 2, the direct effect of the shock is an increase in wages at the firm hit by the shock ($\hat{W}_2 > 0$). Second, there is upstream propagation of the shock to firm 1: it leads to higher demand for firm 1's output ($\hat{D}_1 > 0$) due to an increase in scale for firm 2, which increases wages at firm 1 ($\hat{W}_1 > 0$). Third, there is downstream *reflection* of the shock back from firm 1 to firm 2: due to increasing marginal costs, higher demand for firm 1 raises

its price of output ($\hat{p}_1 > 0$), which increases firm 2's input cost ($\hat{Z}_2 > 0$) and has non-neutral effects on wages at firm 2 if $\sigma \neq \epsilon$ ($\hat{W}_2 \gtrless 0$). Fourth, there is downstream propagation of the shock to firm 3: it reduces the price of firm 2's output ($\hat{p}_2 < 0$), which reduces firm 3's input cost ($\hat{Z}_3 < 0$) and has non-neutral effects on wages at firm 3 if $\sigma \neq \epsilon$ ($\hat{W}_3 \gtrless 0$). Finally, there is upstream *reflection* of the shock back from firm 3 to firm 2: changes in input cost and wages at firm 3 lead to lower demand for firm 2 ($\hat{D}_2 < 0$) which leads to lower wages at firm 2 ($\hat{W}_2 < 0$).

As is evident from this simple example, predicting the effects of TFP shocks on firm wages can be complicated in general. However, the following proposition establishes that the cross-elasticities of firm wages with respect to firm TFPs are completely determined by a small set of model parameters and observables.

PROPOSITION **5.** *In response to marginal changes in TFP that do not affect Θ_t^{GE}, firm wages respond as follows:*

$$\hat{W}_t = \Psi\left(\gamma, \sigma, \epsilon, \Sigma_t^C, \Sigma_t^S, \Lambda_t\right) \hat{T}_t \tag{3.9}$$

where the matrix of cross-elasticities Ψ depends only on the model parameters $\{\gamma, \sigma, \epsilon\}$ and the observables $\left\{\Sigma_t^C, \Sigma_t^S, \Lambda_t\right\}$.

Hence, Proposition 5 implies that if one observes sales shares Σ_t^C, input shares Σ_t^S, and labor shares Λ_t, then predicting the first-order effects of TFP shocks on wages in the production network only requires knowledge of the parameters $\{\gamma, \sigma, \epsilon\}$.[11] In particular, one does not require knowledge of the relationship-specific productivities $\{\alpha_{ijt}\}$ or other features of the worker-level production function f.

3.1.2 Within-firm inequality and worker sorting

What determines the sorting of workers across firms? Consider a firm i and two worker ability types $\{a', a\}$. Note from (2.28) that the relative wage paid by this firm to workers of the two

[11] For a general worker-level production function f, the elasticity of substitution between workers and materials may depend on the inputs chosen and hence may vary by firm, in which case ϵ should be interpreted as an $|\Omega| \times 1$ vector of elasticities. In the estimation below, we impose a restriction on the functional form of f that ensures ϵ is constant across all firms.

ability types is:

$$\frac{w_{it}(a')}{w_{it}(a)} = \frac{\phi_{it}(a')}{\phi_{it}(a)} \tag{3.10}$$

Since labor productivity functions are exogenous, this implies that relative wages between different worker ability types within a firm are also exogenous. Nonetheless, it is possible for the *composition* of workers within a firm to vary endogenously, which then gives rise to endogenous within-firm wage dispersion.

To see this, note from (2.5), (2.6), (2.7), and (2.28) that relative employment of workers of two ability types a' and a is:

$$\frac{L_{it}(a')}{L_{it}(a)} = \underbrace{\frac{L_t(a')}{L_t(a)}}_{\text{supply}} \times \underbrace{\left[\frac{I_t(a')}{I_t(a)}\right]^{-\gamma}}_{\text{competition}} \times \underbrace{\left[\frac{g_i(a')}{g_i(a)}\right]}_{\text{amenities}} \times \underbrace{\left[\frac{\phi_{it}(a')}{\phi_{it}(a)}\right]^{\gamma}}_{\text{productivity}} \tag{3.11}$$

Relative labor supply, amenity values, and labor productivities are exogenous. However, relative labor market competition for the two worker ability types, captured by the relative competition indices, is potentially endogenous. Specifically, from (2.4) and (2.28), we can express the relative competition indices as:

$$\left[\frac{I_t(a')}{I_t(a)}\right]^{\gamma} = \frac{\sum_{i\in\Omega}\left[g_i(a')\,\phi_i(a')\,W_i\right]^{\gamma}}{\sum_{i\in\Omega}\left[g_i(a)\,\phi_i(a)\,W_i\right]^{\gamma}} \tag{3.12}$$

Now, observe from (3.12) that if amenity values do not vary across firms ($g_i(a) = g(a)$ $\forall a \in A$) and there are no worker-firm complementarities ($\phi_i(a) = \phi(a)$ $\forall a \in A$) then the firm-level wages in (3.12) cancel out and relative competition is exogenous. With heterogeneous amenity values and worker-firm complementarities, however, relative competition depends on the distribution of firm-level wages across firms.

To illustrate this, consider a simple example in which the economy consists of two firms, 1 and 2. For brevity, let $h_i(a) \equiv g_i(a)\,\phi_i(a)$ denote the composite of amenities and labor productivity. Then relative competition for two worker types a' and a is:

$$\left[\frac{I_t(a')}{I_t(a)}\right]^{\gamma} = \frac{\left[h_1(a')\,W_1\right]^{\gamma} + \left[h_2(a')\,W_2\right]^{\gamma}}{\left[h_1(a)\,W_1\right]^{\gamma} + \left[h_2(a)\,W_2\right]^{\gamma}} \tag{3.13}$$

It is straightforward to show that the right-hand side of (3.13) is strictly increasing in the relative firm wage W_2/W_1 if and only if the following condition is satisfied:

$$\frac{h_2(a')}{h_2(a)} > \frac{h_1(a')}{h_1(a)} \tag{3.14}$$

Intuitively, if the comparative advantage (in terms of amenities and labor productivity) of a'-ability workers versus a-ability workers is higher at firm 2 than firm 1, then any shock to the economy that makes firm 2 "larger" than firm 1 in the labor market (in the sense that W_2 increases relative to W_1) will increase relative competition for a'-ability workers relative to a-ability workers.

What is the effect of an increase in W_2/W_1 on the within-firm earnings distribution in this example? Suppose for concreteness that $W_2 > W_1$ and $\phi_i(a') > \phi_i(a)$ for all $i \in \Omega$, so that a'-ability workers are more productive than a-ability workers in all firms and hence earn higher wages. Then (3.14) implies that high-ability workers have a comparative advantage in high wage firms. Now, note from (3.11) that an increase in relative labor market competition reduces relative employment within *all firms*. This implies that an increase in between-firm wage dispersion *reduces* within-firm employment dispersion if the comparative advantage condition holds. Since relative wages are exogenous, this implies a reduction in within-firm wage dispersion as well.

3.2 Aggregate Value-added

Next, we consider the effects of firm-level TFP shocks on aggregate value-added, Δ_{Ft}. Recall that we take the final consumption price index as the numeraire and hence Δ_{Ft} is equivalent to aggregate welfare from consumption. Here, we define a matrix of Domar weights S_t^R, which is a $|\Omega| \times |\Omega|$ matrix with (i,j) element equal to firm j's total sales as a share of aggregate value-added:

$$S_{ijt}^R = s_{jt}^R \equiv \frac{R_{jt}}{\Delta_{Ft}} \tag{3.15}$$

In addition, we define a matrix of labor market shares S_t^L, which is a $|\Omega| \times |\Omega|$ matrix with (i, j) element equal to firm j's share of employment aggregating across all worker ability types:

$$S_{ijt}^L = s_{jt}^L \equiv \frac{\int_A L_{jt}(a)\, da}{\int_A L_t(a)\, da} \tag{3.16}$$

The following proposition then establishes the first-order effects of TFP shocks on aggregate value-added in a special case of the model without intermediates and without heterogeneous sorting of workers across firms.

PROPOSITION **6.** *Suppose that intermediates are not used in production. Suppose also that amenities and labor productivities do not vary across firms, such that* $g_i(a) = g(a)$ *and* $\phi_{it}(a) = \phi_t(a)$ *for all* $a \in A$*. Then the first-order effect of changes in TFP on aggregate value-added,* Δ_{Ft}*, is:*

$$\hat{\Delta}_{Ft} = \left[\omega S_t^R + (1-\omega) S_t^L\right] \hat{T}_t \tag{3.17}$$

where $\omega \equiv \frac{\sigma(\gamma+1)}{\gamma+\sigma}$*. In addition, the Domar weight and total employment share for firm* i *are given respectively by:*

$$s_{it}^R = \frac{(T_{it})^{\frac{\gamma+1}{\gamma+\sigma}(\sigma-1)}}{\sum_{j\in\Omega}(T_{jt})^{\frac{\gamma+1}{\gamma+\sigma}(\sigma-1)}} \tag{3.18}$$

$$s_{it}^L = \frac{(T_{it})^{\frac{\gamma}{\gamma+\sigma}(\sigma-1)}}{\sum_{j\in\Omega}(T_{jt})^{\frac{\gamma}{\gamma+\sigma}(\sigma-1)}} \tag{3.19}$$

Proposition 6 shows that the introduction of imperfect competition in labor markets alone breaks the classic Hulten (1978) result, in the sense that firm-level Domar weights are no longer sufficient statistics for the first-order effects of firm-level TFP shocks on aggregate value-added. In particular, predicting these aggregate effects also requires knowledge of firm employment shares as well as the composite parameter $\omega \equiv \frac{\sigma(\gamma+1)}{\gamma+\sigma}$. Furthermore, if there is heterogeneous sorting of workers across firms (due to either heterogeneous amenities through g_i or worker-firm complementarities through ϕ_i), then one also needs to know firm employment shares *for each worker ability*. Finally, note that in the limit as $\gamma \to \infty$, firm shares of value-added and

employment are identical, hence we recover the Hulten (1978) result under perfectly competitive labor markets.

4 Estimation

We now turn towards estimating the key parameters of the model. We discuss the estimation in four steps. First, we describe the data we use. Second, we impose additional assumptions to connect the model described in Section 2 to the data. Third, we outline the estimation strategy. We end this section by presenting the estimation results and fit of the model to data.

4.1 Data

We use three administrative datasets from tax records that firms report to the Internal Revenue Service from Chile (SII, for its acronym in Spanish). These datasets cover the entire formal private sector in Chile. First, we use a matched employer-employee dataset that has annual and monthly earnings from each job that a worker has for the 2005-2018 period.[12] General descriptive statistics of this dataset can be found in Aldunate et al. (2019). Second, we use a firm-to-firm dataset that is provided due to the value-added tax (VAT) for the 2005-2010 period. In this dataset, each firm reports the full list of intermediate buyers and suppliers each year, together with the value of the flow of the transaction for each pair reported. General descriptive statistics of this dataset can be found in Huneeus (2019). Finally, we use an administrative dataset that has a set of characteristics of the balance sheet of firms for the 2005-2018 period. This dataset includes, for example, total sales of the firm, the main industry, location of the headquarter, investment. All these datasets can be easily merged by using the unique tax ID that firms have.[13] Table 1 presents the size of the different samples of these datasets we use for different sections of the estimation and Table 2 presents basic descriptive statistics of the dataset aggregated at the firm-level.

[12] Note that the measure of labor income that this dataset has includes wages and benefits.

[13] Note that all the tax forms are reported at the headquarter level, so we do no have plant level information. Also, it might be that a firm has several tax IDs. We currently do not have ownership characteristics to consolidate the information at the firm level.

Table 1: Sample Sizes

Panel A: Firm-to-Firm Dataset	Links		Sellers		Buyers	
Sample	Unique	Observation-Years	Unique	Observation-Years	Unique	Observation-Years
Full	16,638,308	31,577,022	209,178	658,613	315,835	1,069,759
Estimation	16,548,039	31,384,524	163,584	505,695	306,366	1,030,175
Estimation/Full (%)	99	99	78	77	97	96

Panel B: Employer-Employee Dataset	Workers		Firms	
Sample	Unique	Observation-Years	Unique	Observation-Years
Full	8,641,570	59,000,000	563,578	2,704,564
Prime Age Male	4,183,202	29,896,655	460,769	2,091,107
Prime Male Stayers Only	341,769	3,073,829	3,108	34,188
Stayers/Prime Age Male (%)	8	10	1	2

Panel C: Firm Dataset	Firms	
Sample	Unique	Observation-Years
Full	536,602	2,704,601
Estimation	54,292	101,178
Estimation/Full (%)	10	4

Notes: This table presents basic descriptive statistics of the size of the different samples used throughout the paper. Panel A presents statistics of the sample size of the firm-to-firm transaction dataset. These statistics are presented for the full sample and the sample used in estimating the two-way fixed effect model from Equation (4.11). For each of these samples, we document the unique number of firm-to-firm links, the total number of links-year observations, the unique number of suppliers and buyers, and the total number of buyers-year and suppliers-year observations. Panel B presents statistics of the sample size of the employer-employee dataset. These statistics are presented for the full sample, the sample of prime age (25-60) male workers which is the main sample we use for estimating the two-way fixed effect model from Equation (4.7), and the sample of balanced panel of firms and prime age male workers that are stayers in those firms throughout the period of analysis. For each of these samples, we document the unique number of workers, the total number of worker-year observations, and the unique number of firms and the total number of firm-year observations. Finally, Panel C presents statistics of the sample size of the firm dataset. These statistics are presented for the full sample and the sample used for estimation, e.g., Equation (4.8).

Table 2: Basic Statistics Between Firms

Moments	Value Added	Employment	Average Wage	SD of Within Firm Wages	Labor Share	Number Buyers	Number Suppliers
Average	630592	31	604	0.6	0.5	12059	60
P10	16123	1	227	0.2	0.1	228	9
P50	118514	7	412	0.6	0.3	4052	27
P90	1266813	66	1209	1.0	0.8	37491	135
SD	1799710	80	535	0.3	0.6	20361	101

Notes: This table presents basic statistics of the dataset used in the paper. In particular, it presents information of firm characteristics coming from the three administrative datasets: the firm-to-firm dataset, the employer-employee and the firm dataset. The statistics are presented at the firm-level and the sample used in the one of Estimation from Panel C of Table 1.

4.2 Additional Assumptions

Before describing the estimation strategy in detail, we impose three sets of additional assumptions in order to connect the model described in Section 2 to the data described in section 4.1: (i) functional form assumptions; (ii) orthogonality conditions; and (iii) a steady-state assumption.

4.2.1 Parametric assumptions

Worker-level production function. First, we impose structure on the worker-level production function f.

ASSUMPTION **4.1.** *The production function f takes the following CES form:*

$$f(\phi, m) = \left[\lambda^{\frac{1}{\epsilon}} \phi^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda)^{\frac{1}{\epsilon}} m^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{4.1}$$

where $\lambda \in [0,1]$ and $\epsilon \in (0,\infty)$.

If $\lambda = 1$, output is produced using labor alone and the model simplifies to a version of the model studied in Lamadon et al. (2019), which does not include intermediate inputs. With this specification, the elasticity of substitution between labor and materials, ϵ, is constant. We allow this elasticity to take on any value: if $\epsilon > 1$, labor and intermediates are gross substitutes; if $\epsilon \in (0,1)$, labor and intermediates are gross complements; and if $\epsilon = 1$, production takes a Cobb-Douglas form.

Worker ability. To estimate the model using worker-level data, we follow Lamadon et al. (2019) and assume that worker ability is characterized as follows.

ASSUMPTION **4.2.** *The ability of worker m at time t, a_{mt}, is comprised of a permanent (time-invariant) component $\bar{a}_m$ and a transient (time-varying) component $\hat{a}_{mt}$. The transient component of worker ability is given by:*

$$\log \hat{a}_{mt} = \beta' \chi_{mt} + \xi_{mt} \tag{4.2}$$

where χ_{mt} is a vector of year and age effects. The stochastic process of the worker ability innovation ξ_{mt} is iid across workers.

The separation of worker ability into permanent and transient components will be important for allowing worker-firm complementarities in the estimation below. To perform model counterfactuals, we will also need to impose structure on the cross-sectional distribution of worker abilities in the economy.

ASSUMPTION **4.3.** *The distribution across workers of log permanent and transient worker ability in period t is a multivariate normal distribution with zero mean and covariance matrix $\Sigma_{a,t}$.*

Labor productivity. We next assume that worker ability maps into labor productivity as follows.

ASSUMPTION **4.4.** *The productivity function ϕ_{it} is time-invariant and takes the following form:*

$$\phi_{it}\left(a_{mt}\right) = \phi_i\left(a_{mt}\right) = \bar{a}_m^{\theta_i}\hat{a}_{mt} \tag{4.3}$$

where θ_i is an exogenous parameter.

The returns to permanent worker ability are hence allowed to vary by firm through θ_i, but the transient component of worker ability affects all firms in the same way. This will be important for two reasons. First, it guarantees that interaction terms between worker and firm effects in worker-level regressions that we estimate below are time-invariant. Second, it implies that innovations to the transient ability of a worker do not induce differences in the *relative* productivity of the worker across firms and hence do not generate sorting on this dimension of ability.

Firm amenities. As with labor productivity above, we also impose a restriction on the dependence of firm amenity values on worker ability.

ASSUMPTION **4.5.** *The firm amenity function depends only on the permanent component of worker ability and takes the following form:*

$$g_i\left(a_{mt}\right) = \bar{a}_m^{\delta_i} \tag{4.4}$$

where δ_i is an exogenous parameter.

This restriction is necessary for the same reasons as Assumption 4.4 above.

Relationship-specific productivity. Next, we assume that relationship-specific productivity can be decomposed as follows.

ASSUMPTION **4.6.** *Log relationship-specific productivity between buyer i and seller j is given by:*

$$\log \alpha_{ijt} = \log \alpha_{it} + \log \alpha_{jt} + \log \tilde{\alpha}_{ijt} \tag{4.5}$$

The relationship productivity residual $\log \tilde{\alpha}_{ijt}$ is a normal random variable with zero mean and standard deviation $\sigma_{\alpha,t}$, is iid across buyer-seller pairs within a period, and is independent across time within a buyer-seller pair.

Following Bernard et al. (2019), we refer to the firm-specific component of relationship productivity, α_{it}, as *relationship capability*. As discussed in Bernard et al. (2019), allowing for two dimensions of firm heterogeneity (TFP and relationship capability) is important for fitting key moments in production network data, especially the fact that firms with more customers have higher sales but lower sales per customer, which is a feature of the Chilean firm-to-firm transactions data as well.

Distribution of firm types. Under the above assumptions, firms are heterogeneous in two dimensions: TFP T_{it} and relationship capability α_{it}. We henceforth refer to this pair of firm characteristics as *firm type*, denoted by $\chi_{it} \equiv \{T_{it}, \alpha_{it}\}$. We now impose structure on the cross-sectional distribution of firm types in the economy.

ASSUMPTION **4.7.** *The distribution across firms of log firm type,* $\log \chi_{it}$*, in period* t *is a normal distribution with mean vector* $m_{\chi,t}$ *and covariance matrix* $\Sigma_{\chi,t}$*.*

The production network. To connect the observed production network with the network Ω_{it}^S in the model, we assume that matching between firms depends only on the exogenous firm types of the buyer and seller.

ASSUMPTION **4.8.** *The probability that buyer* i *matches with seller* j *in the production network at date* t *is given by* $m_t(\chi_{it}, \chi_{jt})$ *for some matching function* m_t*.*

4.2.2 Orthogonality conditions

Next, we impose an assumption about the orthogonality of worker-level and firm-level shocks that will be important for identification.

ASSUMPTION **4.9.** *Idiosyncratic worker preference shocks,* ϵ_{mit}*, worker ability shocks,* ξ_{mt}*, and firm types,* χ_{it}*, follow independent Markov processes.*

4.2.3 Steady-state

Finally, we assume that the data is characterized by a steady-state of the model in which the general equilibrium terms Θ^{GE} in (3.1) do not vary over time.

ASSUMPTION **4.10.** *Aggregate income,* Δ_{Ft}*, and the labor market competition indices,* $I_t(\cdot)$*, are time invariant.*

Importantly, Assumption 4.10 allows us to treat labor productivity supplies $\bar{\phi}_{it}$ for each firm as time-invariant as well, so that these are absorbed by firm fixed effects in the regression specifications discussed below.

4.3 Estimation Strategy

The exogenous parameters of the model that we estimate can now be summarized as: (i) the labor supply elasticity, γ;[14] (ii) the elasticity of substitution between labor and materials, ϵ;

[14]Recall that $\gamma \equiv \beta/\rho$. We will not require separate identification of the idiosyncratic preference shock dispersion β and the correlation of shocks across firms ρ.

(iii) the standard deviation of the log relationship productivity residual, $\sigma_{\alpha,t}$; (iv) the mean and covariance matrix of the firm type distribution, $\{m_{\chi,t}, \Sigma_{\chi,t}\}$; (v) the matching function m_t; (vi) the elasticity of substitution between firm products, σ; (vii) the weight on labor in the worker-level production function, λ; (viii) the worker-firm productivity parameters, θ_i; (ix) the covariance matrix of the worker ability distribution, $\Sigma_{a,t}$; and (x) the firm amenity parameters, δ_i.

The outline of our estimation strategy is as follows. First, we estimate $\{\gamma, \epsilon, \sigma_{\alpha,t}\}$ using reduced-form regressions that are derived from the model. Second, we back out firm types $\chi = \{T, \alpha\}$ from the data using the structure of the model and use these to calibrate the moments of the firm type distribution, $\{m_\chi, \Sigma_\chi\}$, as well as the matching function, m_t. Third, all remaining parameters are estimated via a simulated method of moments (SMM) algorithm since the model does not provide closed form solutions to estimate or calibrate these parameters directly.

4.3.1 Labor Supply Elasticity

To estimate the labor supply elasticity γ, first note that under Assumptions 4.2 and 4.4, the log productivity of worker m at firm $i = i(m,t)$ at time t is given by:

$$\log \phi_{mi(m,t)t} = \theta_{i(m,t)} \log \bar{a}_m + \beta' \chi_{mt} + \xi_{mt} \tag{4.6}$$

Combining this with (2.28) and (2.33), we can then express worker-level wages as:

$$\log w_{mt} = \underbrace{\theta_{i(m,t)} \log \bar{a}_m}_{\text{worker-firm interaction}} \underbrace{- \frac{1}{1+\gamma} \log \bar{\phi}_i}_{\text{firm FE}} + \underbrace{\beta' \chi_{mt}}_{\text{worker characteristics}} \underbrace{+ \frac{1}{1+\gamma} \log E^L_{i(m,t)t}}_{\text{firm shocks}} + \xi_{mt} \tag{4.7}$$

Note that the pass-through of changes in the aggregate wage bill of a firm to worker-level wages identifies the coefficient $\frac{1}{1+\gamma}$ and hence the labor supply elasticity γ. Intuitively, γ controls the extent of imperfect competition in the labor market and hence mediates the pass-through of firm-level shocks to wages.

Equation (4.7) resembles the pass-through equation estimated by Lamadon et al. (2019).

An important difference, however, is that in the LMS model, the wage bill E^L is a constant fraction of value-added for any given firm. Hence, the pass-through coefficient $\frac{1}{1+\gamma}$ in (4.7) can be estimated using value-added data. In contrast, with both imperfect competition in output markets and intermediate inputs in our model, E^L is no longer proportional to firm value-added. Hence, estimation of $\frac{1}{1+\gamma}$ will require data on the wage bill instead of value-added.[15]

Furthermore, note that if θ_i is common to all firms, equation (4.7) fits within the Abowd et al. (1999) framework. If instead there are worker-firm complementarities and θ_i differs across firms, then equation (4.7) can be estimated via the procedure in Bonhomme et al. (2019).

We hence estimate the labor supply elasticity, γ, by implementing the reduced-form regression implied by (4.7). Under Assumptions 4.4 and 4.5, workers sort to firms based only on the permanent component of worker ability. Hence, innovations to transient worker ability, ξ_{mt}, are independent of the type of the worker's firm, $\chi_{i(m,t)t}$, which determines firm-level equilibrium variables such as the wage bill, $E^L_{i(m,t)t}$. Furthermore, under Assumption 4.9, the stochastic processes for transient worker ability innovations and firm types are independent. Hence, estimation of (4.7) using ordinary least squares (OLS) yields an unbiased estimate of $\frac{1}{1+\gamma}$.

The sample used for estimating γ is with a balanced panel of firms and workers. That is, we use firms operating every year of the sample and workers that stay in the firm throughout the period of analysis. After applying this filter, we exclude the first and last year of employment spells of each worker. Table 1 presents descriptive measures of the size of this sample of stayers relative to the full sample used in the remaining analysis.

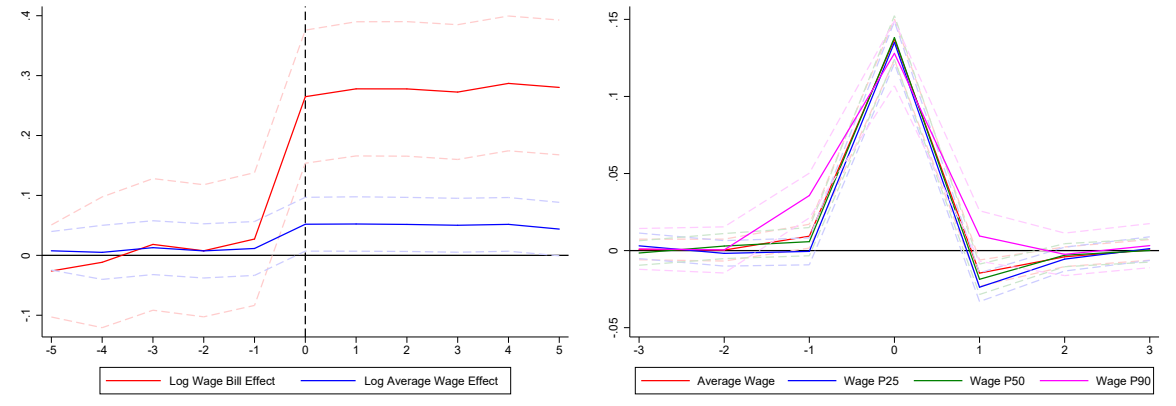
We present evidence of this regression in two different strategies. First, we follow Lamadon et al. (2019), and show this elasticity as a result from a difference-in-difference approach (DiD). For this, we follow a three step procedure. First, for each year, we order firms according to log changes of the wage bill of the firm. Second, we identify the treatment when firms have log changes of their wage bill above the median of log changes of wage bill across firms. Finally, we plot difference in wage bill of treated and control firms both at each year ($t = 0$) and years before ($t < 0$) and after ($t > 0$). We perform this step for each calendar year. We perform the

[15] In ongoing work, we also estimate (4.7 using value-added to determine the extent of the bias arising from misspecification.

same exercise for other characteristics of the firm, such as moments of the earnings distribution within the firm.

Results are presented in Panel A of Figure 2. By construction, the treatment and control groups differ in the wage bill from period $t = -1$ to $t = 0$. On average, firms in the treatment group face an increase of 30 log points growth in their wage bill relative to firms in the control group. The effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Figure 2 also shows the effect on the average earnings of firms. On average, firms in the treatment group face an increase of 4 log points of their average earnings relative to firms in the control group. Once again, the effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Finally, firms in the treatment and control group do not experience statistically significant differences up to 5 years before the treatment, for both the wage bill and the average earnings. Through the lens of a DiD design, these results imply a pass-through rate of firms shocks of around 0.13 (= 0.04/0.30). From equation (4.7), this implies $\hat{\gamma} = 6.7$.

Figure 2: Pass-Through of Firm Shocks to Workers Earnings



A. DiD Strategy **B.** Continuous Strategy

Notes: This figure presents the results of the two strategies used for estimating γ. Panel A shows the evidence of the difference-in-difference strategy outlined in Section 4.3.1. Panel B shows the evidence of the continuous strategy also outlined in Section 4.3.1.

Our results are in line with previous evidence of the pass-through of firms shocks to workers earnings. For example, Lamadon et al. (2019) find that the pass-through elasticity is 0.15 and

Card et al. (2018) find an elasticity in the range of 0.10-0.15. Note that these estimates often refer to different sources of variation. For example, Lamadon et al. (2019) use value added as the firm characteristic where the shock comes from, whereas Card et al. (2018) use value added per worker. We add another variety to these strategies: the wage bill. In a model with intermediate inputs such as ours, the relevant measure for firm shocks is the wage bill, rather than value added or value added per worker. Intuitively, value added includes variation in the profits of the firm (and in a more general model, potentially variation in other inputs such as capital), and thus does not correctly capture the margin of variation that is relevant for workers. Actually, we show in Appendix C we replicate the same exercise as in Panel A of Figure 2 but using value added to define the treatment, rather than the wage bill. We find that by doing this, the pass-through elasticity is an order of magnitude lower (0.013 rather than our preferred estimate of 0.13). This difference might be driven by the evolution of capital of these firms.

The second strategy that we follow to estimate γ is to run the regression from equation (4.7) directly with OLS. To test for pre-trends and the dynamics of the pass-through of firm shocks, we write the specification from equation (4.7) in first differences:

$$\Delta \log w_{mt} = \frac{1}{1+\gamma} \Delta \log E^L_{i(m,t)t} + \Delta \xi_{mt} \tag{4.8}$$

The results are presented in Panel B of Figure 2. We conclude three facts from this figure. First, the effect of wage bill changes on impact are around 0.13, which confirms the result from the DiD strategy. Second, the figure shows both no clear pre-trends and also that the effect is transitory in changes, which means it is permanent in levels. Finally, one concern about using the wage bill instead of value added, is that there might be a mechanical relationship between the wage bill and average earnings driven by outliers. To control for this, we implement Equation (4.8) also with other moments of the earnings distribution within the firm, such as the percentile 25, 50 and 90. The results of these alternative elasticities are presented in Panel B of Figure 2. We find that there are no significant differences across these groups and furthermore, there are no significant difference with the behavior of the average earnings within the firm.[16] This

[16] An alternative test to the concern of a mechanical relationship between the wage bill and the wages of a

lack of heterogeneity in the pass-through of firm shocks is consistent with the evidence found in Card et al. (2018).

4.3.2 Elasticity of Substitution between Labor and Materials

To estimate the elasticity of substitution between labor and materials, ϵ, we first note from equations (2.29), (2.30), (2.33), and (2.34) that the ratio of intermediate spending to total labor costs depends on relative factor prices:

$$\log\left(\frac{E_{it}^M}{E_{it}^L}\right) = \log\left[\frac{1}{\eta}\left(\frac{1-\lambda}{\lambda}\right)\right] + (\epsilon - 1)\log\frac{W_{it}}{Z_{it}} + \psi_{it} \tag{4.9}$$

We treat the residual ψ_{it} as measurement error and hence estimate (4.9) via OLS to recover ϵ. However, note that in order to implement this regression, we first require measures of firm-level wages, W_{it}, and intermediate input costs, Z_{it}. Since we do not observe these measures directly in the data, we estimate them by running two-way reduced-form fixed effects regressions implied by the model for both the labor market and intermediate input markets.

On the labor market side, our theory implies a two-way fixed effect model as in Bonhomme et al. (2019), presented in Equation (4.7). As such, we estimate the firm fixed effect by following the strategy proposed by Bonhomme et al. (2019). In particular, we estimate the firm effect in a two-step procedure. First, we cluster firms into K groups by using moments of the earnings distribution of workers within the firm. Second, we use those K groups as the relevant firm identifier and run OLS on Equation (4.7). We implement this strategy with $K = 10$.

particular group within the firm is to follow a leave-one-out strategy. For example, when looking at the pass-through of firm shocks on the median wage, leave out the wage of the median workers from the wage bill used as the measure of firm shock. Our results are robust to this strategy.

Table 3: Wage Variance Decomposition: BLM vs AKM

	Model	
	BLM	AKM
Share explained by:	(1)	(2)
1. Worker Effects (%)	55	55
2. Firm Effects (%)	11	17
3. Sorting (%)	23	18
4. Residual (%)	11	10
Sorting Correlation	0.08	0.06

Notes: This table presents the results from estimating the two-way fixed effect model of wages outlined in Section 4.3.2. The results of the estimation are presented in terms of a variance decomposition of log earnings outlined in Lamadon et al. (2019). Column 1 presents the results from the estimation strategy proposed in Bonhomme et al. (2019), whereas Column 2 presents the results from the estimation strategy proposed in Abowd et al. (1999).

We present results from this estimation by reporting the standard decomposition of the variance of log earnings in Table 3. We find that the firm fixed effect accounts for 11% of the variance of log earnings between workers. This is after controlling for the limited mobility bias that firm fixed effects estimates have when using the empirical strategy from Abowd et al. (1999).[17] With an estimate of the firm fixed effect, we can recover W_{it} by using the following relationship from our model:

$$\log W_{it} = \frac{1}{1+\gamma} \log \bar{\phi}_i + \frac{1}{1+\gamma} \log E^L_{i(m,t)t} \tag{4.10}$$

To recover Z_{it}, we follow a three-step procedure. First, we write log sales from firm j to firm i using equations (2.35) and (4.5) as:

[17] For reference to the literature, we also document in Table 3 the results from the decomposition coming from estimates according to the empirical strategy of Abowd et al. (1999). Using that strategy, we find that firm fixed effects accounts for 17% of the variance of log earnings between workers. Going from the model of Abowd et al. (1999) to Bonhomme et al. (2019), there is redistribution between the share accounted for by firm fixed effects and the share accounted for by sorting. The model from Bonhomme et al. (2019) has a higher share on sorting and a lower share on the role of firm fixed effects. This is qualitatively consistent with evidence from the US (Lamadon et al., 2019). Another point worth noting is that, relative to the US, worker fixed effects account for a significantly smaller share of the variance of log earnings. In the US, Lamadon et al. (2019) show that it accounts for 75% whereas in Chile it accounts for 55%.

$$\log R_{ijt} = \gamma \log \eta + \underbrace{\log \tilde{\Delta}_{it}}_{\text{buyer effect}} + \underbrace{\log \tilde{\Phi}_{jt}}_{\text{seller effect}} + \log \tilde{\alpha}_{ijt} \tag{4.11}$$

where

$$\tilde{\Delta}_{it} \equiv \Delta_{it}\alpha_{it} \tag{4.12}$$

$$\tilde{\Phi}_{it} \equiv \Phi_{it}\alpha_{it} \tag{4.13}$$

Under Assumption 4.6, the assignment of buyers to sellers is exogenous with respect to $\tilde{\alpha}_{ijt}$. Hence, as discussed in Bernard et al. (2019), estimation of equation (4.11) using a two-way fixed effects estimator delivers unbiased estimates of the buyer and seller fixed effects, $\left\{\tilde{\Delta}_{it}, \tilde{\Phi}_{it}\right\}$, as well as the relationship-specific productivity residuals, $\log \tilde{\alpha}_{ijt}$, and their standard deviation, $\sigma_{\alpha,t}$.[18]

Note that to estimate (4.11), firms must have multiple connections. To identify seller fixed effects, each seller needs to have at least two buyers. Similarly, to identify buyer fixed effects, each buyer needs to have at least two sellers. In the data, some firms have either one supplier or one seller. Hence, we implement the aforementioned restriction using an iterative approach known as "avalanching". Specifically, we first drop firms with one supplier or seller. Doing this may result in additional firms that have one supplier or seller, hence in the next step, we drop these firms as well. We continue this process until firms are no longer dropped from the sample. Table 1 presents descriptive statistics of the effect of this sample restriction. The algorithm takes three iterations to converge in practice and reduces the sample size of firm-to-firm linkages from a total of 31.6 million transactions to 31.4 million transactions, that is, a reduction of 1% of transactions. Hence, the avalanching algorithm has little impact on our sample size.[19]

Second, we recover firm relationship capabilities, α_{it}. Note from (2.9) and (2.14) that the

[18] Note that equation (4.11) can be estimated using cross-sectional data. This is in contrast with two-way fixed effect models of earnings such as (4.7), which are identified based on workers moving between firms. The difference in intermediate input markets is that matching occurs many-to-many: each seller can have several buyers at once and each buyer can have several sellers.

[19] Bernard et al. (2019) report that avalanching also eliminates around 1% of firm-to-firm links in the production network for Belgium.

share of a firm's total sales that come from the network (i.e. excluding final sales) is:

$$s_{it}^F = \frac{\sum_{j \in \Omega_{it}^C} \Delta_{jt} \alpha_{jit}}{\Delta_{Ft} + \sum_{j \in \Omega_{it}^C} \Delta_{jt} \alpha_{jit}} \tag{4.14}$$

$$= \frac{\alpha_{it} \sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\alpha}_{jit}}{\Delta_{Ft} + \alpha_{it} \sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\alpha}_{jit}} \tag{4.15}$$

Solving for α_{it}, we obtain:

$$\alpha_{it} = \Delta_{Ft} \left(\frac{s_{it}^F}{1 - s_{it}^F} \right) \frac{1}{\sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\alpha}_{jit}} \tag{4.16}$$

Since s_{it}^F is observable and we can construct $\sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \tilde{\alpha}_{jit}$ from estimates of (4.11), we can identify α_{it} up to a normalizing constant.[20] Given α_{it}, we can then parse the buyer and seller fixed effect estimates $\left\{ \tilde{\Delta}_{it}, \tilde{\Phi}_{it} \right\}$ to obtain estimates of the network characteristics $\{\Delta_{it}, \Phi_{it}\}$. We also construct relationship-specific productivity from (4.5).

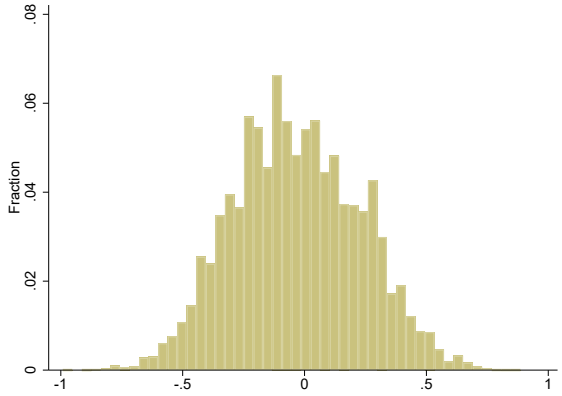
Third, given estimates of Φ_{it} and α_{ijt}, we then construct firm-level input costs using equation (2.16) as $Z_{it}^{1-\sigma} = \sum_{j \in \Omega_i^S} \Phi_{jt} \alpha_{ijt}$.

Figure 3 plots the distribution of the firm fixed effect of wages, the buyer and seller fixed effect, $\log W_{it}$ and $\log Z_{it}$. Three features of these distributions are worth mentioning. First, all these variables are well-behaved and follow a normal distribution. Second, the adjusted R^2 from Equation (4.11) is 45% which highlights the role of buyer-supplier match quality for firm-to-firm transactions.[21] Finally, we find a negative correlation between the seller and the buyer fixed effect. This happens because firms that have more and better inputs, can afford a lower production costs, which implies a lower marginal cost and therefore becoming more attractive to buyers and a greater capacity of reaching more buyers, which implies more sales.

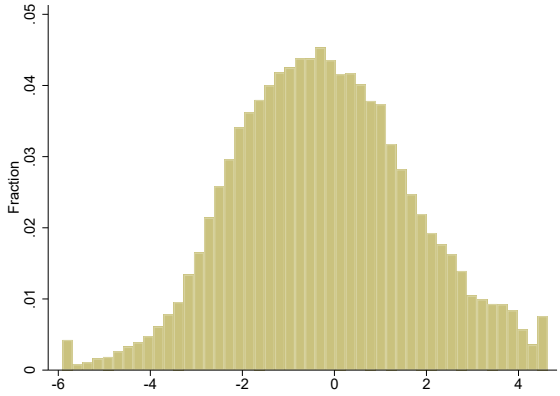
[20] The intuition here is that a higher value of α_{it} increases sales only within the network but not to final consumers. Hence, after controlling for total final expenditure Δ_{Ft} and characteristics of a firm's customers within the network through $\sum_{j \in \Omega_{it}^C} \tilde{\Delta}_{jt} \alpha_{jit}$, variation in s_{it}^F is informative about α_{it}.

[21] The R^2 in our data is similar to the one found in Bernard et al. (2019), which is 38%.

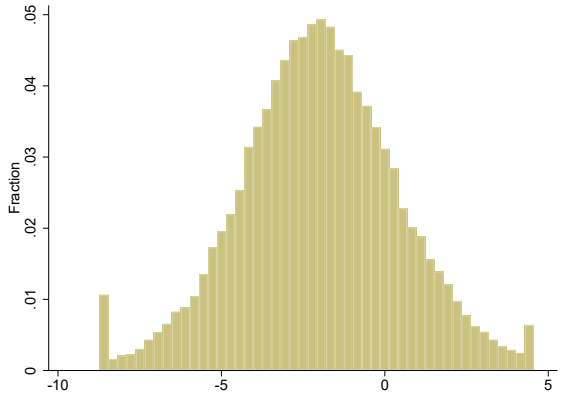
Figure 3: Distribution of Firm Estimates



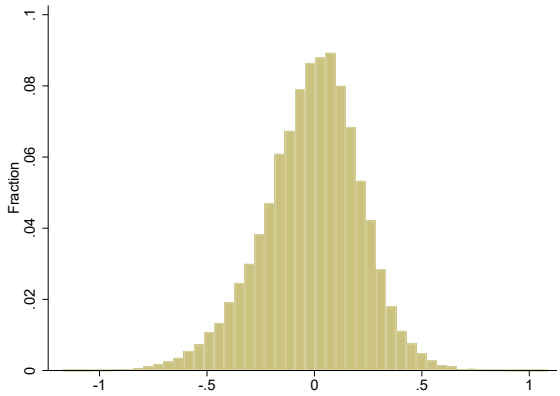
A. Firm Earnings Fixed Effect



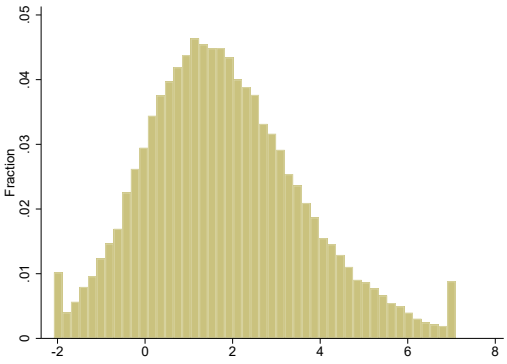
B. Seller Effects, $\log \Psi_{it}$



C. Buyer Effects, $\log \Delta_{it}$



D. $\log W_{it}$



E. $\log Z_{it}$

Notes: This figure presents the distributions of different characteristics of firms that we estimate. Panel A plots the firm earnings fixed effect estimated from Equation (4.7). Panel B and C plots seller and buyer fixed effects estimates of $\log \Psi_{it}$ and $\log \Delta_{it}$, respectively. The estimation strategy of these estimates is outlined in Section 4.3.2. Panel D and E plots estimates of $\log W_{it}$ and $\log Z_{it}$, respectively, which are described in Section 4.3.2.

With estimates of W_{it} and Z_{it} in hand, we can now estimate equation (4.9) via OLS. Note that a typical challenge encountered in estimating substitution elasticities by regressing relative expenditures on relative prices as in (4.9) is that the residuals of such regressions often contain factor-augmenting productivities.[22]

To illustrate this issue, suppose that instead of estimating the firm wage W_{it}, we use the average earnings of the firm as a regressor in (4.9). From equation (2.28), the average wage at firm i is:

$$\bar{w}_{it} = \eta \hat{\phi}_{it} W_{it} \tag{4.17}$$

where $\hat{\phi}_{it}$ is a measure of aggregate labor productivity at firm i:

$$\hat{\phi}_{it} \equiv \frac{\int_A \kappa_{it}(a)\, \phi_{it}(a)^{1+\gamma}\, da}{\int_A \kappa_{it}(a)\, \phi_{it}(a)^{\gamma}\, da} \tag{4.18}$$

Hence, we can rewrite the factor choice equation (4.9) as:

$$\log\left(E_{it}^M / E_{it}^L\right) = \log\left[\frac{1}{\eta^\epsilon}\left(\frac{1-\lambda}{\lambda}\right)\right] + (\epsilon - 1)\log\frac{\bar{w}_{it}}{Z_{it}} - (\epsilon - 1)\log\hat{\phi}_{it} \tag{4.19}$$

Notice that if we were to estimate Equation (4.19) using average wages $\bar{w}_{it}$ instead of the firm wage component W_{it}, the labor productivity term $\hat{\phi}_{it}$ would lead to a downardly-biased estimate of ϵ as it is positively correlated with the average wage and the coefficient in front of the residual is negative.

To highlight the strength of our estimation strategy, we also estimate (4.19) using average firm wages according to Equation (4.19). Table 4 presents the results. We find that $\hat{\epsilon} - 1 = -0.039$ and thus $\hat{\epsilon} = 0.96$, which means that workers and suppliers are complements in our model. If instead, we ran the version of Equation (4.19), we find that $\hat{\epsilon}^B - 1 = -0.099$, which is around 2.5 times the unbiased estimate. This biased estimates leads to $\hat{\epsilon} = 0.90$, which is significantly lower than our preferred estimate. Note that this result confirms the bias predicted by the theory.

[22] For example, see León-Ledesma et al. (2010).

Table 4: Estimation of Elasticity of Substitution between Materials and Labor (ϵ)

	$\log E^M/E^L$	
	(1)	(2)
$\log W/Z$	-0.039***	
	(.009)	
$\log \bar{w}/Z$		-0.099***
		(.007)
Adjusted R^2	0.207	0.211
Number of Observations	79,794	79,794

Notes: This table presents the results of the estimation of ϵ outlined in Section 4.3.2. Column 1 and 2 presents the estimates according to Equation (4.9) and Equation (4.19), respectively.

4.3.3 Firm types

Recall that estimates of relationship capabilities for each firm are obtained from (4.16) in the process of estimating ϵ. To estimate the remaining dimension of firm heterogeneity - TFP, T_{it} - we proceed as follows.

Using equations (2.17), (2.18), (2.29), and (2.31), we can express TFP as:

$$T_{it}^{\epsilon-1} = \frac{\mu^\epsilon}{1-\lambda}\left(\frac{\Delta_{it}}{D_{it}}\right)\left(\frac{\Phi_{it}}{Z_{it}^{1-\sigma}}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} \tag{4.20}$$

Now using (2.16) and (2.19), we write this as:

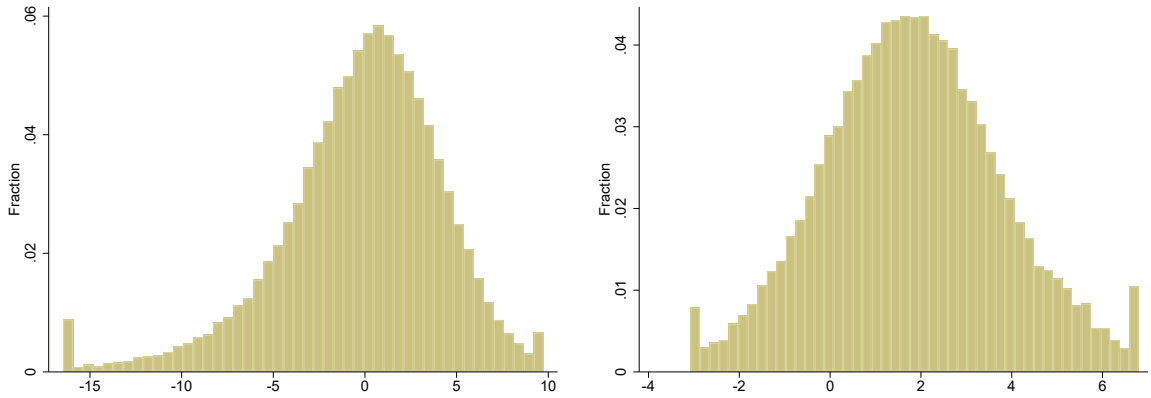
$$T_{it}^{\epsilon-1} = \frac{\mu^\epsilon}{1-\lambda} s_{it}^{network}\left(\frac{\Delta_{it}}{\sum_{j\in\Omega_{it}^C}\Delta_{jt}\alpha_{jit}}\right)\left(\frac{\Phi_{it}}{\sum_{j\in\Omega_{it}^S}\alpha_{ijt}\Phi_{jt}dj}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} \tag{4.21}$$

Hence, given estimates of ϵ from (4.9) and $\{\Delta_{it}, \Phi_{it}, \alpha_{ijt}\}$ from (4.11) and (4.16), we can estimate TFP at the firm-level up to a normalizing constant. Intuitively, we infer variation in a firm's TFP from variation in its network demands and productivities, $\{\Delta_{it}, \Phi_{it}\}$, after controlling for the network demands and productivities of its customers and suppliers respectively through $\sum_{j\in\Omega_{it}^C}\Delta_{jt}\alpha_{jit}$ and $\sum_{j\in\Omega_{it}^S}\alpha_{ijt}\Phi_{jt}dj$.

Figure 4 shows histograms of our estimates of log TFPs and relationship capabilities. As is evident, the distributions are well-approximated by normal distributions. From this, we obtain

estimates of the mean and covariance matrix of log firm types, $\{m_\chi, \Sigma_\chi\}$. For the covariance matrix, we use only the relative variance of log TFP to log relationship capability, $\sigma_T^2/\sigma_\alpha^2$ and the correlation between the two variables, $\rho_{T\alpha}$. We estimate the level of log TFP variance in the SMM procedure below in order to match the firm size distribution observed in the data.

Figure 4: Distribution of Productivity: Firms and Links



A. $\log T_{it}$ **B.** $\log \alpha_{it}$

Notes: This figure presents the estimates of firm-level TFP, $\log T_{it}$, and firm-level relationship capability, $\log \alpha_{it}$. The estimation of these objects is outlined in Section (4.3.3).

4.3.4 Matching function

To estimate the matching function m_t described in Assumption 4.8, we first bin firms by percentiles of firm type $\{T, a\}$ and measure empirically the fraction of firm pairs within each bin that match. We treat this as an estimate of the matching function.

In practice, we discretize the TFP and relationship capability space into 20 grid points each. This implies $20^4 = 160,000$ bins of the matching function. This illustrates the need for a large dataset such as ours: with between 200m to 400m firm-to-firm transactions per year, we have an average of between 12 to 25 matches per bin even with such a large number of bins.

4.3.5 Remaining Parameters: SMM

The remaining parameters of the model to be estimated are: (i) the elasticity of substitution between firm products, σ; (ii) the weight on labor in the worker-level production function, λ;

(iii) the variance of log TFP, σ_T^2; (iv) the covariance matrix of the worker ability distribution, $\Sigma_{a,t}$; (v) the worker-firm productivity parameters, θ_i; and (vi) the firm amenity parameters, δ_i. We estimate these via a simulated method of moments approach.

Although the model does not admit closed-form solutions that allow direct estimation or calibration of these parameters, they are intuitively connected to key moments in the data.

First, the elasticity of substitution between products, σ, controls the extent of imperfect competition in output markets. Consequently, σ has a strong influence on firm profits. With this parameter in mind, we include as a targeted SMM moment the share of labor in aggregate value-added.

Second, the weight on labor in the worker-level production function, λ, naturally has a strong influence on the relative input of labor versus materials at each firm. With this parameter in mind, we include as a targeted SMM moment the ratio of value-added to gross output. Note that as λ approaches 1, for example, intermediates are not used in production and hence the ratio of value-added to gross output approaches 1.

Third, the variance of log TFP, σ_T^2, controls the degree of firm heterogeneity in the model.[23] With this parameter in mind, we include as a targeted SMM moment the standard deviation of log firm sales.

Fourth, the covariance matrix of the worker ability distribution, $\Sigma_{a,t}$, is tightly linked to the dispersion of earnings across workers. To simplify the estimation, we assume that the permanent and transient components of worker ability have a common variance σ_a^2 and are perfectly correlated.[24] With this parameter in mind, we include as a targeted SMM moment the standard deviation of log earnings.

Finally, the worker-firm productivity parameters, θ_i, and amenity parameters, δ_i, determine the sorting of workers across firms. Hence, these parameters play a key role in determining the share of worker earnings variance that is accounted for by within-firm wage variance versus

[23] Recall from Section 4.3.3 that we fix the relative variance of log TFP and log relationship capability as well as the correlation of the two varaibles given our estimtates of $\{T, a\}$ for every firm.

[24] Relaxing this is work in progress.

between-firm wage variance. We parameterize these as functions of firm TFP:[25]

$$\theta_i = T_i^{\tau_\theta} \tag{4.22}$$

$$\delta_i = T_i^{\tau_\delta} \tag{4.23}$$

With the parameters $\{\tau_\theta, \tau_\delta\}$ in mind, we include as a targeted SMM moment the between-firm share of log wage variance, computed as in Song et al. (2019). In the current estimation, we set $\tau_\delta = 0$ and estimate only τ_θ.[26]

4.4 Estimation Results and Model Fit

Table 5 shows our estimated parameter values. There are several points of note. First, we estimate a labor supply elasticity of $\gamma = 5.3$, which is close to (but slightly higher than) typical estimates in the literature.[27] Second, we estimate an elasticity of substitution between labor and materials of $\epsilon = 0.95$, implying that labor and materials are complements in production. Third, we estimate a product substitution elasticity of $\sigma = 4.96$, implying output market markups over average costs of around 25%. Note also that our estimates imply $\sigma > \epsilon$, which from Proposition 2 implies that reductions in material input costs have *positive* effects on wages. Fourth, we estimate a strong negative correlation between firm TFP and relationship capability of -0.79. This is in line with the findings of Bernard et al. (2019), who also estimate TFP and relationship capability to be negatively correlated in Belgian production network data. Finally, we estimate $\tau_\theta = 0.78$, indicating positive worker-firm complementarities in production.

[25] This approach is work in progress.

[26] This is work in progress. Note that although both θ_i and δ_i control worker-firm sorting, only θ_i has a direct influence on earnings conditional on sorting. Hence, τ_θ and τ_δ can be separately identified by also targeting the correlation between log firm sales and log average wages within the firm.

[27] For example, see the references cited in footnote 1.

Table 5: Parameter Estimates

Description	Parameter	Value
labor supply elasticity	γ	5.30
labor-materials substitution elasticity	ϵ	0.95
product substitution elasticity	σ	4.96
weight on labor in production	λ	0.19
mean of log TFP	m_T	0.08
mean of log relationship capability	m_α	1.68
variance of log TFP	σ_T^2	0.65
variance of log relationship capabiltiy	σ_α^2	0.32
correlation(log TFP, log relationship	$\rho_{T\alpha}$	-0.79
variance of log a	σ_a^2	0.19
worker-firm production complementarity	τ_θ	0.78

Notes: Estimates are based on 2005 data.

Table 6 shows the fit of the estimated model to several key moments in the data. There are several important takeaways. First, although we target only the standard deviation of log earnings, the model matches well with other moments of the log earnings distribution. This is reflective of the fact that the distribution is well-approximated by a log-normal parametric form. The implied Gini coefficient of 0.52 for earnings is close to but slightly lower than the Gini coefficient of 0.61 observed in the data. Second, the model fits fairly well the positive correlation between firm size and degree: firms with larger sales have more customers and suppliers, both in the model and data. Third, we over-predict the firm size premium on log earnings and on the dispersion of with-firm log earnings in the current estimation.[28] Finally, the model slightly under-predicts the dispersion of firm-to-firm sales. In addition, the model predicts robustly negative matching patterns between firms, measured in terms of both sales and average wages.

[28]This is likely a result of setting $\delta_i = 0$ in the current estimation. For example, allowing for heterogeneous amenities across firms will reduce the firm size wage premium if larger firms are also estimated to have better amenities.

For example, the correlation between the log average wage of a firm and the log of average wages among the firm's suppliers is -.18. This is at odds with the data, which exhibits positive assortative matching on wages.

Table 6: Model Fit

	Model	Data	Targeted
aggregate variables			
VA to GO ratio	.28	.28	y
labor share of VA	.43	.43	y
between-firm share of var(log wage)	.48	.48	y
worker variables			
sd(log wages)	.96	.96	y
90/10 wage ratio	13.14	13.11	
75/25 wage ratio	4.16	3.38	
90/50 wage ratio	4.41	3.99	
50/10 wage ratio	2.98	3.28	
wage Gini	.52	.61	
firm variables			
sd (log sales)	1.53	1.53	y
sd (log VA)	1.53	1.53	
sd (log wage bill)	1.54	1.53	
corr(log sales, log #cus)	.54	.69	
corr(log sales, log #sup)	.60	.41	
corr(log sales, log avg wage)	.94	.55	
corr(log sales, sd(log avg wage))	.72	.41	
firm-to-firm variables			
std(log f2f sales)	1.74	2.13	
corr(log sales, log avg cus sales)	-.29	.10	
corr(log sales, log avg sup sales)	-.46	-.06	
corr(log avg wage, log avg sup wage)	-.18	.12	
corr(log avg wage, log avg cus wage)	-.06	.25	

Notes: Empirical moments are from 2005. Model moments are based on parameters estimated from 2005 data. Targeted column indicates if moment was targeted in simulated method of moments algorithm. For firm-to-firm variables, customer and supplier averages are weighted based on sales and expenditure shares respectively.

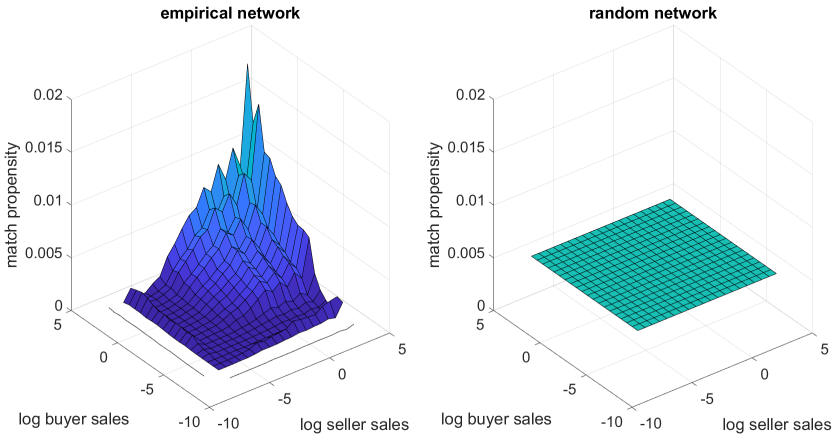


Figure 5: The empirical network versus a random network in terms of log firm sales

5 Counterfactuals

How important is heterogeneity in the production network for earnings inequality? To quantify this, we perform a simple counterfactual exercise: we solve for the model's equilibrium under the empirically-observed network and compare this to the equilibrium that would obtain under a random network with the same density. This change in the network structure is illustrated in Figure 5. Comparisons across the two equilibria then allow us to quantify the contribution of heterogeneous sorting of buyers and sellers in the production network to earnings inequality in the labor market.

Figures 6 and 7 show the distribution of log worker wages w_{mt} and log firm wages W_{it} under the empirical and random networks. The variance of log worker wages is 12.0% lower under the random network as compared with the empirical network, while the variance of log firm wages W_{it} is 29.9% lower. Figure 8 shows the Lorenz curves for worker earnings under the empirical and random networks. Note that the Lorenz curve for the empirical network lies everywhere below the curve for the random network, indicating a more unequal wage distribution with network heterogeneity. The Gini coefficient of worker earnings falls from 0.52 under the empirical network to 0.49 under the random network.

In sum, we find that eliminating heterogeneity in production network linkages reduces earnings inequality by a non-trivial amount. In ongoing work, we explore in more detail the mech-

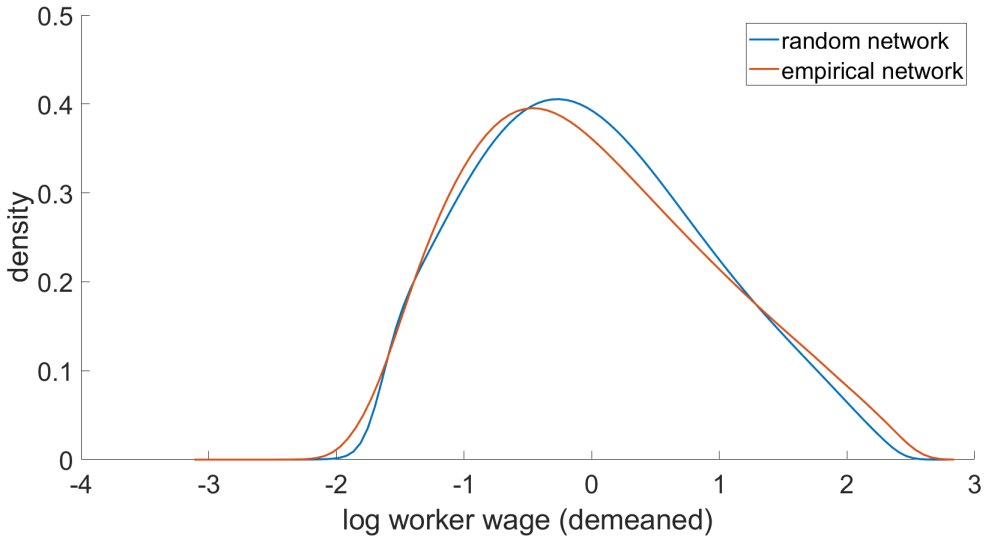


Figure 6: Worker wage distribution under empirical and random networks

anisms through which this occurs, as well as the sensitivity of these results to key structural parameters such as the labor supply elasticity.

6 Conclusion

This paper considers the impact of networks on inequality. Our findings indicate that networks are very important for understanding inequality. There are several directions for future research. First, it might be useful to consider how labor markets affect production networks. Such an approach would require endogenizing the network. Second, firms' reliance on production networks may give rise to outsourcing. Although several studies have considered the earnings impact of outsourcing (e.g., **?**), no study has been able to track worker flows in a dataset that jointly contains firms' links in the production network. We are currently exploring this question in Chile. Finally, there are several policy questions that are worth revisiting given the linked earnings-VAT data. In ongoing work, we are exploring the welfare consequences of a VAT reform using our model of production networks. Our objective is to derive the efficiency costs and incidence effects of a VAT using a sufficient statistics approach that can be empirically implemented with our dataset.

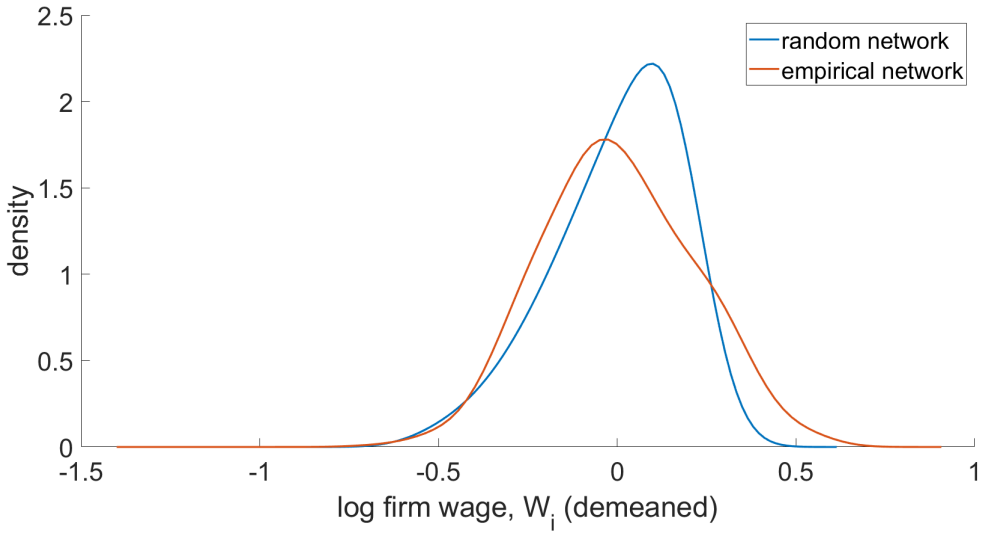


Figure 7: Firm wage distribution under empirical and random networks

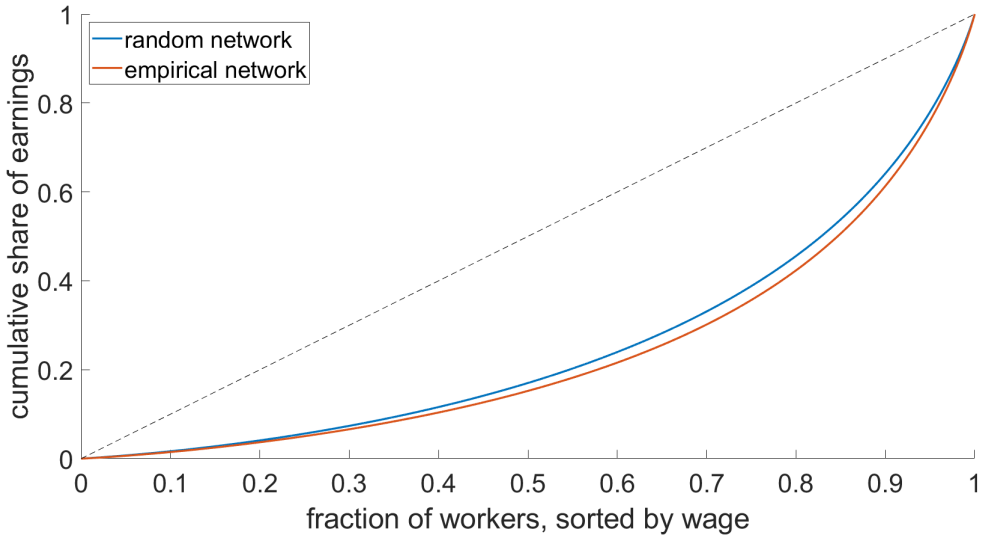


Figure 8: Firm wage distribution under empirical and random networks

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A Proofs of Propositions

A.1 Proof of Proposition 1

Omitting time subscripts for brevity, the profit-maximization problem for a firm i can be written generally as:

$$\max_{\{p_{ji}\}_{j\in\Omega_i^c\cup\{F\}}} \left\{ \sum_{j\in\Omega_i^c\cup\{F\}_i} p_{ji}x_{ji} - C\left[X_i|l_i\left(\cdot\right),Z_i\right] \right\} \tag{A.1}$$

$$\text{s.t. } x_{ji} = \Delta_j \alpha_{ji} p_{ji}^{-\sigma} \tag{A.2}$$

$$X_i = \sum_{j\in\Omega_i^c\cup\{F\}} x_{ji} \tag{A.3}$$

where $\alpha_{Fi} = 1$. Here, $C\left[X_i|l_i\left(\cdot\right),Z_i\right]$ denotes the total cost of producing X_i units of output given the labor supply functions $l_i\left(\cdot\right)$ and material input cost Z_i. The latter depends on the prices charged by suppliers of firm i, which firm i takes as given in the problem above. Importantly, the total production cost for firm i depends only on total output of the firm X_i and not on how this output is allocated to each customer.

The first-order condition for the profit-maximization problem with respect to p_{ji} is then:

$$\left(1-\sigma\right)\Delta_j\alpha_{ji}p_{ji}^{-\sigma} = -\sigma C'\left[X_i|l_i\left(\cdot\right),Z_i\right]\Delta_j\alpha_{ji}p_{ji}^{-\sigma-1} \tag{A.4}$$

Solving for the optimal price yields:

$$p_{ji} = \frac{\sigma}{\sigma-1}C'\left[X_i|l_i\left(\cdot\right),Z_i\right] \tag{A.5}$$

Note that the right-hand side of (A.5) does not vary by customer j. Hence, the optimal prices set by firm i do not vary by customer and are equal to the standard CES markup over the firm's marginal cost. The existence of imperfect competition in the labor market implies that marginal cost is not constant, but this does not break the standard CES markup result.

A.2 Proof of Proposition 2

Totally differentiating (2.29)-(2.31) for a given firm, we obtain:

$$\hat{W} + \frac{1}{\sigma}\hat{X} - \left(\frac{f_{\phi m}\nu}{f_\phi}\right)\hat{\nu} = \frac{1}{\sigma}\hat{D} + \hat{T} \tag{A.6}$$

$$\frac{1}{\sigma}\hat{X} - \left(\frac{f_{mm}\nu}{f_m}\right)\hat{\nu} = \frac{1}{\sigma}\hat{D} + \hat{T} - \hat{Z} \tag{A.7}$$

$$-\gamma\hat{W} + \hat{X} - \left(\frac{f_m v}{f}\right)\hat{\nu} = \hat{T} + \hat{\bar{\phi}} \tag{A.8}$$

where we omit firm and time subscripts for brevity and all derivatives of f are evaluated at $\{\phi, m\} = \{1, \nu\}$. Solving for $\left\{\hat{W}, \hat{X}, \hat{\nu}\right\}$:

$$\hat{W} = \Gamma\hat{D} + (\sigma - 1)\,\Gamma\hat{T} + (\epsilon - \sigma)\,\Gamma\hat{Z} - \Gamma\hat{\hat{\phi}} \tag{A.9}$$

$$\hat{X} = (\gamma + \epsilon\varepsilon_m)\,\Gamma\hat{D} + \sigma\,(\gamma + \epsilon\epsilon_m + 1 - \varepsilon_m)\,\Gamma\hat{T} - \sigma\,(\gamma + \epsilon)\,\varepsilon_m\Gamma\hat{Z} + \sigma\,(1 - \varepsilon_m)\,\Gamma\hat{\hat{\phi}} \tag{A.10}$$

$$\hat{v} = \epsilon\Gamma\hat{D} + \epsilon\,(\sigma - 1)\,\Gamma\hat{T} - \epsilon\,(\gamma + \sigma)\,\Gamma\hat{Z} - \epsilon\Gamma\hat{\hat{\phi}} \tag{A.11}$$

where $\varepsilon_m \equiv \frac{f_m \nu}{f}$ denotes the elasticity of f with respect to materials, $\epsilon \equiv \left(\frac{f_{\phi m}\nu}{f_\phi} - \frac{f_{mm}v}{f_m}\right)^{-1}$ denotes the elasticity of substitution between labor and materials, and $\Gamma \equiv [\gamma + \sigma + (\epsilon - \sigma)\,\varepsilon_m]^{-1}$. We have used the result that $\frac{f_{mm}m}{f_m} = -\frac{1}{\epsilon}(1 - \varepsilon_m)$ for a function f that is homogeneous of degree one. Examining coefficients on the right-hand sides of (A.9)-(A.11) and noting that $\epsilon > 0$ and $\varepsilon_m \in (0, 1)$ under Assumption 2.3 then gives the desired results.

A.3 Proof of Propositions 3, 4, and 5

First, we derive an expression for marginal changes in demand shifters, $\hat{D}$. Totally differentiating equation (2.19) gives:

$$\hat{D} = \Sigma^C\hat{\Delta} \tag{A.12}$$

where we have used the result that the share of firm j's sales accounted for by firm i can be expressed using (2.9), (2.19) and (2.35) as:

$$\Sigma^C_{ijt} \equiv \frac{R_{ijt}}{\sum_{k\in\Omega^C_{it}\cup\{F\}} R_{kit}} = \frac{\Delta_j}{D_i} \tag{A.13}$$

Recall also that we are assuming no changes in general equilibrium variables and hence $\hat{\Delta}_{Ft} = 0$. Totally differentiating (2.15) and using (2.34), we obtain:

$$\hat{\Delta} = \gamma\hat{W} + \sigma\hat{Z} + \hat{\nu} \tag{A.14}$$

Then, taking the ratio of the first-order conditions for the profit-maximization problem (2.29)-(2.30) and totally differentiating gives:

$$\hat{W} - \hat{Z} = \epsilon^{-1}\hat{\nu} \tag{A.15}$$

where here ϵ denotes a $|\Omega|\times|\Omega|$ diagonal matrix with i^{th}-diagonal element equal to the elasticity of substitution between labor and materials for firm i, $\epsilon_i \equiv \left[\frac{f_{\phi m}(1,\nu_i)\nu_i}{f_\phi(1,\nu_i)} - \frac{f_{mm}(1,\nu_i)v_i}{f_m(1,\nu_i)}\right]^{-1}$. Combining (A.12), (A.14), and (A.15), we then obtain the following expression for marginal changes in demand shifters:

$$\hat{D} = \Sigma^C\left[(\gamma + \epsilon)\,\hat{W} + (\sigma - \epsilon)\,\hat{Z}\right] \tag{A.16}$$

Next, we derive an expression for marginal changes in material costs, $\hat{Z}$. Totally differentiating equation (2.16) gives:

$$\hat{Z} = -\frac{1}{\sigma - 1}\Sigma^S\hat{\Phi} \tag{A.17}$$

where we have used the result that the share of firm i's input expenditures accounted for by firm j can be expressed using (2.16) and (2.35) as:

$$\Sigma^S_{ijt} \equiv \frac{R_{ijt}}{\sum_{k\in\Omega^S_{it}} R_{ikt}} = \frac{\alpha_{ijt}\Phi_{jt}}{Z_{it}^{1-\sigma}} \tag{A.18}$$

Then, from (2.17) and (2.18), we can express marginal changes in network productivities as:

$$\hat{\Phi} = \frac{\sigma-1}{\sigma}\left(\hat{X} - \hat{D}\right) \tag{A.19}$$

Hence, combining (A.17) and (A.19), we obtain the following expression for marginal changes in material costs:

$$\hat{Z} = \frac{1}{\sigma}\Sigma^S\left(\hat{D} - \hat{X}\right) \tag{A.20}$$

Now equations (A.9), (A.10), (A.16), and (A.20) define a system in $\left\{\hat{W}, \hat{X}, \hat{D}, \hat{Z}\right\}$ given changes in TFP, $\hat{T}$. Recall that we are assuming no changes in general equilibrium variables and hence $\hat{\bar{\phi}} = 0$. Solving this system yields the following expression for changes in wages as a function of changes in TFPs:

$$\hat{W} = \Theta_1\left[\Theta_2\left(\sigma-1\right) + \Theta_3\left(\gamma+1-\left(1-\epsilon\right)\epsilon_m\right)\right]\Gamma\hat{T} \tag{A.21}$$

where ϵ_m is a $|\Omega|\times|\Omega|$ diagonal matrix with i^{th}-diagonal element equal to $\epsilon_{m,i} \equiv \frac{f_m(1,\nu_i)\nu_i}{f(1,\nu_i)}$, Γ is a $|\Omega|\times|\Omega|$ diagonal matrix with i^{th}-diagonal element equal to $\left[\gamma+\sigma+\left(\epsilon_i-\sigma\right)\epsilon_{m,i}\right]^{-1}$, and we have defined the following $|\Omega|\times|\Omega|$ matrices:

$$\Theta_1 \equiv \left[I + \Theta_3\left(1-\epsilon_m\right)\Gamma\Sigma^C\left(\gamma+\epsilon\right)\right]^{-1} \tag{A.22}$$

$$\Theta_2 \equiv \left[I - \Gamma\Sigma^C\left(\gamma+\epsilon\right)\right]^{-1} \tag{A.23}$$

$$\Theta_3 \equiv \Theta_2\Gamma\left(I-\Sigma^C\right)\left(\sigma-\epsilon\right)\left[I - \Theta_4\left(1-\epsilon_m\right)\Gamma\Sigma^C\left(\sigma-\epsilon\right)\right]^{-1}\Theta_4 \tag{A.24}$$

$$\Theta_4 \equiv \left[I - \Sigma^S\left(\gamma+\epsilon\right)\epsilon_m\Gamma\right]^{-1}\Sigma^S \tag{A.25}$$

Now, first observe that (A.21)-(A.25) depend only on the model parameters $\{\gamma,\sigma,\epsilon\}$, the network shares $\left\{\Sigma^C,\Sigma^S\right\}$, and the worker-level production function elasticities ϵ_m. We now show that the elasticities ϵ_m depend only on the labor supply elasticity γ and labor shares Λ. From (2.33) and (2.34), we first express the share of labor in total cost for firm i as:

$$\Lambda_i = \frac{\eta}{\eta + \nu_i\left(Z_i/W_i\right)} \tag{A.26}$$

where recall $\eta \equiv \frac{\gamma}{1+\gamma}$. Then, from the first-order conditions (2.29) and (2.30), relative factor prices can be expressed as:

$$\frac{Z_i}{W_i} = \frac{f_m\left(1,\nu_i\right)}{f_\phi\left(1,\nu_i\right)} \tag{A.27}$$

Hence, combining (A.26) and (A.27) and using the result that $f = f_m\nu + f_\phi$ for a degree-one

homogeneous function f, we can solve for $\epsilon_{m,i}$ as:

$$\epsilon_{m,i} = \frac{\eta\left(1-\Lambda_i\right)}{\Lambda_i + \eta\left(1-\Lambda_i\right)} \tag{A.28}$$

Hence, the matrix of cross-elasticities in (A.21) depends only on model parameters $\{\gamma, \sigma, \epsilon\}$ and observables $\left\{\Sigma^C, \Sigma^S, \Lambda\right\}$, completing the proof of Proposition 5.

For the proof of Proposition 3, we set $\Sigma^C = \Sigma^S = 0$ and $\epsilon_m = 0$. In this case, $\Gamma = \frac{1}{\gamma+\sigma}$, $\Theta_1 = I$, $\Theta_2 = I$, $\Theta_3 = 0$, and $\Theta_4 = 0$. Hence from (A.21) we have:

$$\hat{W} = \frac{\sigma-1}{\gamma+\sigma}\hat{T} \tag{A.29}$$

as claimed. This in fact holds globally as can be seen by combining (2.29), (2.31), and (2.19), and setting $f_\phi\left(1,\nu\right) = 1$, $f_m\left(1,\nu\right) = 0$, and $f\left(1,\nu\right) = 1$. Hence, if in addition there are general equilibrium effects, then:

$$\hat{W} = \frac{\sigma-1}{\gamma+\sigma}\hat{T} + \frac{1}{\gamma+\sigma}\left(\hat{\Delta}_F - \hat{\bar{\phi}}\right)$$

For the proof of Proposition 4, we set $\sigma = \epsilon$. In this case, $\Gamma = \frac{1}{\gamma+\sigma}$, $\Theta_1 = I$, $\Theta_2 = \left(I - \Sigma^C\right)^{-1}$, and $\Theta_3 = 0$. Hence from (A.21) we have:

$$\hat{W} = \frac{\sigma-1}{\gamma+\sigma}\left(I-\Sigma^C\right)^{-1}\hat{T} \tag{A.30}$$

as claimed.

A.4 Proof of Proposition 6

If intermediates are not used in production, value-added for each firm is equivalent to total sales. Omitting time subscripts for brevity, the first-order change in aggregate value-added can then be written as:

$$\hat{\Delta}_F = \sum_{i\in\Omega} s_i^R \hat{R}_i \tag{A.31}$$

Total sales for firm i are given by (2.29) and (2.31) as:

$$R_i = \mu\eta^\gamma \bar{\phi}_i W_i^{1+\gamma} \tag{A.32}$$

Hence:

$$\hat{R}_i = \left(1+\gamma\right)\hat{W}_i + \hat{\bar{\phi}}_i \tag{A.33}$$

Next, from equation (3.5), we have:

$$\hat{W}_i = \frac{1}{\sigma+\gamma}\hat{\Delta}_F + \frac{\sigma-1}{\gamma+\sigma}\hat{T}_i - \frac{1}{\sigma+\gamma}\hat{\bar{\phi}}_i \tag{A.34}$$

In addition, if amenities and labor productivities do not vary across firms, then from (2.4), (2.6), (2.7), and (2.32), we have:

$$\bar{\phi}_i = \text{const.} \times \frac{1}{\sum_{j\in\Omega} W_j^{\gamma}} \tag{A.35}$$

Hence:

$$\hat{\bar{\phi}}_i = \hat{\bar{\phi}} = -\gamma \sum_{j\in\Omega} s_j^L \hat{W}_j \tag{A.36}$$

where s_j^L is firm $j's$ share of aggregate employment:

$$s_i^L \equiv \frac{W_i^{\gamma}}{\sum_{j\in\Omega} W_j^{\gamma}} \tag{A.37}$$

Combining (A.33), (A.34), and (A.36) and solving for $\hat{\Delta}_F$, we obtain:

$$\hat{\Delta}_F = \sum_{i\in\Omega} \left[\omega s_i^R + (1-\omega)\, s_i^L\right] \hat{T}_i \tag{A.38}$$

where $\omega \equiv \frac{\sigma(\gamma+1)}{\gamma+\sigma}$. Finally, note that in this special case of the model, firm Domar weights and employment shares are given by (3.5), (A.32), (A.35), and (A.37) as:

$$s_i^R = \frac{(T_i)^{\frac{\gamma+1}{\gamma+\sigma}(\sigma-1)}}{\sum_{j\in\Omega} (T_j)^{\frac{\gamma+1}{\gamma+\sigma}(\sigma-1)}} \tag{A.39}$$

$$s_i^L = \frac{(T_i)^{\frac{\gamma}{\gamma+\sigma}(\sigma-1)}}{\sum_{j\in\Omega} (T_j)^{\frac{\gamma}{\gamma+\sigma}(\sigma-1)}} \tag{A.40}$$

B Solution Algorithm

We solve numerically for an equilibrium of the model using the following solution algorithm.

1. Guess Δ_{Ft}.
   (a) Guess $\left\{\Delta_{it}, \Phi_{it}, \bar{\phi}_{it}\right\}_{i\in\Omega}$.
   (b) Compute $\{Z_{it}\}_{i\in\Omega}$ from (2.16) and $\{D_{it}\}_{i\in\Omega}$ from (2.19).
   (c) Solve for $\{W_{it}, \nu_{it}, X_{it}\}_{i\in\Omega}$ from (2.29), (2.30), and (2.31).
   (d) Compute new guesses of $\{\Delta_{it}\}_{i\in\Omega}$ from (2.15), $\{\Phi_{it}\}_{i\in\Omega}$ from (2.17), and $\left\{\bar{\phi}_{it}\right\}_{i\in\Omega}$ from (2.32) using (2.4), (2.6), and (2.7) .
   (e) Iterate on steps (a)-(d) until convergence.
2. Compute a new guess of Δ_{Ft} from (2.10), using (2.5), (2.28), and (2.20).
3. Iterate on steps 1-2 until convergence.

Note that step 1(c) involves numerical solution of a system in $\{W_{it}, \nu_{it}, X_{it}\}$ defined by:

$$W_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} \left(1 + \tilde{\nu}_{it}\right)^{\frac{1}{\epsilon-1}} \lambda^{\frac{1}{\epsilon-1}} \tag{B.1}$$

$$Z_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} \left(1 + \frac{1}{\tilde{\nu}_{it}}\right)^{\frac{1}{\epsilon-1}} (1-\lambda)^{\frac{1}{\epsilon-1}} \tag{B.2}$$

$$X_{it} = \eta^{\gamma} W_{it}^{\gamma} T_{it} \left(1 + \tilde{\nu}_{it}\right)^{\frac{\epsilon}{\epsilon-1}} \lambda^{\frac{1}{\epsilon-1}} \bar{\phi}_{it} \tag{B.3}$$

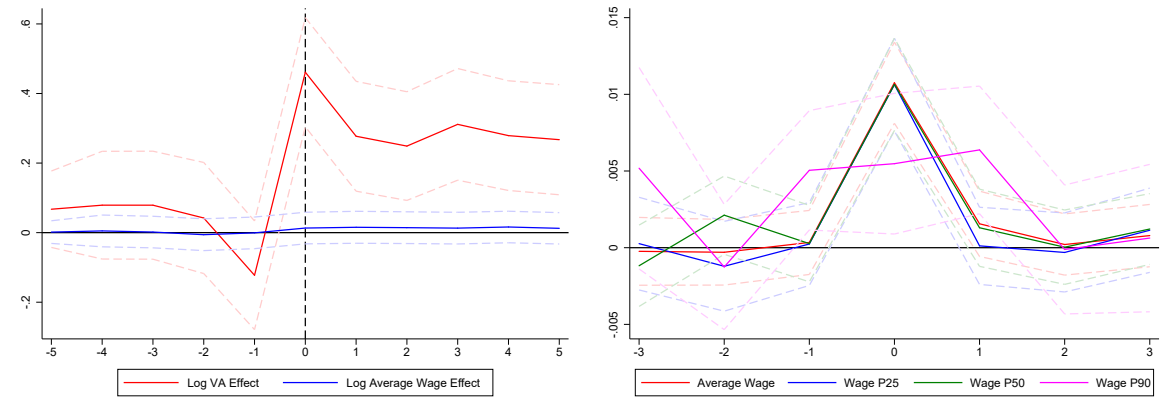
where $\tilde{\nu}_{it} \equiv \left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{\epsilon}} \nu_{it}^{\frac{\epsilon-1}{\epsilon}}$. This system can be reduced to one in firm wages alone:

$$W_{it}^{\gamma+\sigma} \left[1 + \frac{1-\lambda}{\lambda} \left(W_{it}/Z_{it}\right)^{\epsilon-1}\right]^{\frac{\epsilon-\sigma}{\epsilon-1}} \bar{\phi}_{it} = \frac{\lambda^{\frac{\sigma-1}{\epsilon-1}}}{\mu^{\sigma} \eta^{\gamma}} D_{it} T_{it}^{\sigma-1} \tag{B.4}$$

which has unique solution for W_{it} given $\left\{D_{it}, Z_{it}, \bar{\phi}_{it}, T_{it}\right\}$.

C Robustness to the Structural Estimation

Figure A.1: Pass-Through of Firm Shocks to Worker Earnings: Using Value Added



A. DiD Strategy **B.** Continuous Strategy

Notes: