Firm heterogeneity, misallocation, and trade

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1 Introduction

Firms vary greatly in size, measured productivity, and export performance even within narrowly defined industries. Only a few firms export even in the exporting industries, and those that do are larger and more productive.[1] Standard trade models with firm heterogeneity such as Melitz (2003) successfully explain these patterns through heterogeneous firm productivity – more productive firms can both grow larger and export to more destinations.

Productivity, however, is not the only source of firm heterogeneity, especially in developing countries such as China. A firm may grow large not because it is productive, but because it benefits from preferential policies or political connections. Such policies can take different forms, such as direct subsidies and preferential credit access. For example, reported subsidies accounted for more than a third of state-owned firms' total profits from 1998 to 2004 in China (Girma, Gong, Görg, and Yu (2009)). State-owned firms also enjoy easier access to credit – one estimate finds the average annual interest rate for state-owned firms is 1.6%, compared to 5.4% for private firms (Hong and Nong (2012)). Furthermore, preferential treatments may exist not only between private and state-owned firms but also among the private firms. Milhaupt and Zheng (2015) document that founders of 95 out of the top 100 private firms in China are currently or formerly a member of party-state organizations, suggesting that political connection contributes to the firm's success and size. Indeed, the authors claim "large firms in China...survive and prosper precisely because they have fostered connections to state power and have succeeded in obtaining state-generated rents." The claim is in stark contrast to the standard trade models that rationalize large firms with their high innate productivity.

Firm heterogeneity induced by such preferential policies and political connections leads to

[1] Bernard, Jensen, Redding, and Schott (2012) provide a review.

misallocation of resources across firms. The aggregate cost of misallocation can be substantial. Hsieh and Klenow (2009) estimates that China can improve its aggregate productivity by 30 to 50 percent by removing the misallocation to the U.S. level. Similarly, Brandt, Tombe, and Zhu (2013) estimates that the misallocation between state and non-state firms alone contributes to more than 10% loss in total productivity.

A channel of gains from trade predicted by the standard model is resource reallocation across firms induced by trade liberalization – when trade costs are lowered, more productive firms expand while less productive firms exit, leading to a welfare gain (Melitz and Redding (2015)). In the presence of misallocation, however, firms that benefit from lower trade costs need not necessarily be the high-productivity ones, so it is no longer clear whether the trade-induced reallocation contributes to the welfare gain. On the other hand, initial misallocation also presents a new margin of gains if trade can reduce the extent of misallocation.

In this paper, I answer two questions that follow from these observations. First, given the suggestive evidence that resources are misallocated across firms in China, can the standard trade model still explain the observed firm-level patterns? Second, how does accounting for firm-level misallocation change the gains from trade estimate?

To answer the first question, I examine three predictions made by the standard model. While the model's focus has been the difference between exporting and non-exporting firms, I highlight the within-exporter and within-non-exporter patterns. More precisely, I show that the model predicts: (i) revenue productivity is increasing in firm size among non-exporters, (ii) revenue productivity is constant conditional on firm size among non-exporters, and (iii) export intensity is non-decreasing in firm size among exporters. These predictions are not idiosyncratic to Melitz (2003) which assumes constant fixed cost and markup across firms. In particular, I show that the predictions hold under Arkolakis (2010) which allows fixed costs to be endogenously chosen by firms as well as under Melitz and Ottaviano (2008) which allows variable markups through linear demand.

Then I turn to firm-level data to show that each of these predictions fails to hold in the Chinese manufacturing industry. First, I show that revenue productivity is decreasing in firm size among non-exporters and this negative relationship is not driven by sample selection or measurement error. Second, I show that firm size accounts for only about 12% of the unconditional variance. Some of the remaining variances are explained by the state-ownership, suggesting measurement error alone does not explain the conditional variance. Third, I show export intensity is decreasing in firm size among exporters. A known feature of Chinese exporters is the high density of high export intensity firms.[2] While these high-intensity exporters contribute, the negative pattern remains even when high-intensity exporters are excluded.

I propose the firm-level misallocation as the missing ingredient that explains the gap between the standard model's predictions and the observed firm-level patterns in China. I follow the recent literature by introducing misallocation through exogenous, firm-specific subsidies that generate a "wedge" between the social and private marginal value of labor.[3] The main innovation of my misallocation model is the introduction of destination-specific subsidies which generate across-destination misallocation that is absent in the closed economy setting. While firm-specific subsidies generate misallocation of labor across firms, these destination-specific subsidies distort firms' decisions on export intensity, or how much to sell in the foreign market relative to the home market, and allow the model to flexibly capture the effects of policies that provide benefits contingent on export performance to a subset of firms.[4]

The misallocation model can reverse the predictions of the standard model and therefore explain the observed patterns. Across-firm misallocation generates a negative relationship between revenue productivity and size, as subsidized firms tend to become larger but also exhibit lower revenue productivity. A subsidy reduces the marginal cost of labor faced by the subsidized firms, leading to higher employment and lower price, but does not change the true productivity and hence lowers the revenue productivity. Furthermore, since both productivity and a subsidy affect a subsidized firm's revenue productivity, variance in subsidy rates conditional on productivity generates conditional revenue productivity variance observed in the data. Finally, destination-specific subsidies provide an explanation for the negative relationship between export intensity and size. When subsidies on domestic sales play a more dominant role in determining firm size than subsidies on export sales, large firms are also more likely to face incentives to sell relatively more in the home market and exhibit lower export intensity.

The fact that the standard model fails to capture every aspect of data does not necessarily imply that it cannot be useful for policy evaluation. Therefore, I ask the second question of whether and how the introduction of misallocation to the model affects the estimated gains from trade.

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[2] Lu (2010) and Defever and Riaño (2017) document unusually large share of high-intensity exporters in China.

[3] Hopenhayn (2014) discusses this modeling technique in reviewing the misallocation literature.

[4] Defever and Riaño (2017) discusses specific policies that would generate such distortions. For example, until 2008, a Chinese exporter located in the Free Trade Zone could receive a substantial tax break if it exported more than 70% of its output. Another prominent example is processing trade regime, which exempts tariffs on imports conditional on exporting the output.

Theoretically, the misallocation model can predict either higher or lower gains from trade relative to the standard model. While the across-destination misallocation dampens the gains from trade, the effect of across-firm misallocation is ambiguous.[5]

Given the theoretical ambiguity, I turn to the Chinese manufacturing firm-level census data to quantitatively assess the impact of modeling misallocation on the estimation of gains from trade. Suppose two researchers try to estimate the gains from trade using the same Chinese firm-level data, but one researcher uses the standard model while the other uses the misallocation model. How different are their estimates? I find that the percentage increase in the real consumption per capita from trade cost reduction is about 45% lower under the misallocation model and that both across-destination and across-firm misallocation contribute to the dampened gains from trade.

To reach this answer, I develop an estimation strategy that does not require a functional form assumption on the distribution of firm-level primitives. The majority of quantitative exercises in the literature assume Pareto distributed productivity, and the papers that relax the Pareto assumption instead impose an alternative distribution, such as lognormal or truncated Pareto.[6] In contrast, I devise a strategy that can estimate gains from trade by nonparametrically estimating relevant distributions. This matters, as the distributional assumption plays an important role in estimating the gains from trade.

Nonparametric estimation presents unique challenges. When the distribution of firm idiosyncrasies is determined by a small number of parameters, the distribution can be estimated from aggregate data. On the other hand, nonparametrically estimating the distribution in each country requires firm-level data for each country. Furthermore, selection-into-production implied by the model requires an extrapolation from the observed distribution to estimate the primitive distribution. Without a functional form assumption, this extrapolation is challenging.

I bypass these issues by focusing on counterfactual scenarios that involve a specific type of bilateral trade liberalization that maintains the ratio of the home and foreign real expenditures. All the effects of the foreign distribution on the gains from trade are summarized by this ratio, so by holding the ratio constant, I avoid estimating the foreign distribution. Furthermore, by focusing on liberalization, I do not need to estimate the distribution of unobserved firms. Ad-

[5] The theoretically ambiguous effect of misallocation on the gains from trade has been noted in many different settings. See, for example, Helpman and Krugman (1985), Epifani and Gancia (2011), Holmes, Hsu, and Lee (2014), and Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2018).

[6] Costinot and Rodríguez-Clare (2014) quantify the gains from trade under Pareto assumption. Bas, Mayer, and Thoenig (2017) considers lognormal, while Feentra (2018) considers bounded Pareto.

ditionally, I show that the welfare change can be estimated from a handful of one-dimensional functions derived from the joint distribution of the firm primitives. These functions can be easily estimated from the observables without needing to estimate the multidimensional joint distribution.

The result of the quantitative exercise shows that the estimated size of the gains from trade is about 45% lower under the model with misallocation than in the model without. To better understand the underlying mechanism, I perform additional counterfactual exercises. Taking the estimated misallocation model as given, I ask how much the welfare can be improved through reallocation under different levels of the variable trade cost. I consider two types of reallocation. First, I remove all distortions by equalizing both the domestic and export subsidies across all firms. Second, I remove only the export intensity distortions by equalizing domestic and export subsidies for each firm. The results show that the welfare gains from both the size and export intensity distortions are larger when the trade cost is lower. In other words, distortions have higher welfare costs when the country is more open to international trade. In the absence of the trade cost, the welfare cost of the export intensity distortions is 4% higher, and the welfare cost of total distortions is 9% higher than at the trade cost inferred from the data. These results suggest that both types of distortions play a quantitatively significant role in reducing the gains from trade.

My paper contributes to a growing literature that examines the effect of domestic misallocation on the patterns of and the gains from trade. Costa-Scottini (2018) develops an open economy model with firm-level misallocation that can explain the trade elasticity heterogeneity across countries. As the focus of the model is explaining the aggregate trade patterns, it employs a particular parameterization that shrinks more productive firms under misallocation. In contrast, my model focuses on the role of firm heterogeneity and imposes no specific relationship between firm productivity and misallocation, inferring the relationship from the firm-level data instead. Pulido (2018) considers the impact of firm-level misallocation on sector-level productivity and subsequent distortion in the country's sector-level comparative advantages. This differs from my model's focus on how the trade-induced reallocation interacts with existing misallocation. In fact, I show that under the distributional assumption of Pulido (2018), the size of the gains from trade is invariant to misallocation in a single sector model. Berthou, Chung, Manova, and Sandoz-Dit-Bragard (2018) directly examine the impact of trade liberalization on the firm-level patterns by considering the effect of trade shocks on both the aggregate productivity and the covariance between productivity and size and analyzing the results under a model of firm-level misallocation. This paper provides a complementary study that uses cross-sectional data to infer the nature of misallocation and the model to estimate the effects of trade shocks.

Papers discussed above focus on the across-firm misallocation. Studies suggest, however, that across-destination misallocation plays an important role in explaining the exporter pattern in China. Manova and Yu (2016) and Dai, Maitra, and Yu (2016) show that processing trade regime dampens the size and productivity exporter premia in China, while Defever and Riaño (2017) shows export-contingent subsidies can rationalize the large share of high-intensity exporters.[7] My misallocation model captures the distortionary effects of such policies in a general equilibrium framework and allows studying how such distortions interact with the trade liberalization.

The paper also contributes to the literature that finds evidence of across-firm misallocation in China. Hsieh and Klenow (2009) documents large dispersion of revenue productivity across firms within sectors while Dollar and Wei (2007) and Brandt et al. (2013) document evidence of factor misallocation between private and state-owned firms. As Bartelsman, Haltiwanger, and Scarpetta (2013) note, the presence of overhead costs can generate revenue productivity variance even in the absence of misallocation. By examining the productivity variance conditional on size, however, I find that overhead cost can explain only a part of the observed variance. The fact that revenue productivity and size have a negative relationship in China is consistent with the fact that less developed countries exhibit lower productivity-size covariance, as documented by Bartelsman et al. (2013). My misallocation model further provides a theoretical ground that links the negative relationship between revenue productivity and firm size to the presence of misallocation.

Lastly, the paper continues the effort of quantifying the gains from trade. Following Arkolakis, Costinot, and Rodríguez-Clare (2012), who show that the gains from trade can be estimated with only aggregate data in a large class of heterogeneous firms model including Melitz-Pareto, researchers have shown that firm-level distribution matters if not Pareto and that some of the predictions under Pareto fail to hold in data.[8] This paper provides a theoretical framework and empirical strategy to estimate the gains from trade without imposing a functional form assumption on the productivity distribution while also allowing non-productivity heterogeneity across firms.

[7] Lu (2010) shows that the standard model can explain the lack of exporter premia and the high intensity exporters when the selection into the domestic market is more competitive than the foreign market. This explanation, however, cannot rationalize the within-non-exporter patterns I document.

[8] See, for example, Melitz and Redding (2015), Feentra (2018), Fernandes, Klenow, Meleshchuk, Pierola, and Rodríguez-Clare (2015), and Bas et al. (2017).

2 Predictions of the standard model

I introduce what I refer to as the standard model characterized by CES utility, heterogeneous productivity, and fixed overhead costs. The model follows Melitz (2003) but allows asymmetric countries. I highlight three predictions of the model: revenue productivity is increasing in firm size, revenue productivity is constant conditional on firm size, and export intensity is increasing in firm size. These predictions hold under alternative models that allow variable markups and endogenous fixed costs.

2.1 The standard model environment

Consider the economy with N countries. Each country j has a mass L_j of consumers with CES utility over varieties of goods

$$U_j = \left(\int_{\Omega_j} q_j(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where Ω_j is the set of varieties consumed in country j, $q_j(\omega)$ is the quantity of variety ω consumed, and $\sigma > 1$ is the elasticity of substitution. Let E_j denote the aggregate expenditure of country j consumers and P_j the ideal price index.[9]

The market for the varieties is characterized by monopolistic competition. Each country i has mass M_i of firms. Each firm has a blueprint for a distinct variety with associated productivity $\varphi > 0$ drawn from a distribution $G_i(\varphi)$. In equilibrium, some firms may not operate at all.

Labor is the only factor of production and each consumer supplies a unit of labor inelastically to the domestic firms. Each firm can convert a unit of labor into φ units of output. Selling in destination j from country i requires fixed overhead cost $f_{ij} > 0$. Additionally, international trade incurs iceberg cost so that to deliver 1 unit to j, a firm in country i must ship $\tau_{ij} \geq 1$ units. In summary, to produce and deliver q units of its variety to destination j, a firm in country i with productivity φ requires $l_{ij}(q;\varphi)$ units of labor, where

$$l_{ij}(q;\varphi) = f_{ij} + \frac{\tau_{ij} q}{\varphi}. \tag{2}$$

Each firm with productivity φ in country i maximizes profit earned from sales to each desti-

[9] $P_j \equiv \left(\int_{\Omega_j} p_j(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}}$.

nation country j

$$\pi_{ij}(q;\varphi) = p_j(q)q - w_i l_{ij}(q;\varphi) \tag{3}$$

where w_i is the wage in country i and $p_j(q)$ is the inverse demand function of country j consumers. Let $q_{ij}(\varphi)$ denote the solution to the first order condition of maximizing $\pi_{ij}(q;\varphi)$ and let $p_{ij}(\varphi) \equiv p_j(q_{ij}(\varphi))$, $l_{ij}(\varphi) \equiv l_{ij}(q_{ij}(\varphi);\varphi)$, $r_{ij}(\varphi) \equiv p_{ij}(\varphi)q_{ij}(\varphi)$, and $\pi_{ij}(\varphi) \equiv \pi_{ij}(q_{ij}(\varphi);\varphi)$ denote the corresponding price, employment, revenue, and profit. The CES demand implies constant markup pricing

$$p_{ij}(\varphi) = \tilde{\sigma}\left(\frac{w_i\tau_{ij}}{\varphi}\right)$$

where $\tilde{\sigma} \equiv \frac{\sigma}{\sigma-1}$ denotes the markup. Furthermore, destination-specific overhead costs imply selection into exporting. Each firm in country i sells to destination j if and only if $\pi_{ij}(\varphi) \geq 0$. One can show the variable profit function $\pi_{ij}(\varphi) + w_i f_{ij}$ is strictly increasing in φ. Consequently, there exists unique value $\varphi_{ij} > 0$ such that a firm sells to destination j if and only if $\varphi \geq \varphi_{ij}$.

2.2 Among non-exporters, revenue productivity is increasing in size

The constant markup from CES and increasing returns to scale from the overhead cost imply that for all firms selling to destination j,

$$\log\left(\frac{r_{ij}(\varphi)}{l_{ij}(\varphi)}\right) = \underbrace{\log(\tilde{\sigma})}_{\text{markup}} + \underbrace{\log\left(1 - \frac{f_{ij}}{l_{ij}(\varphi)}\right)}_{\text{variable labor share}}. \tag{4}$$

Given that the marginal cost is constant, revenue productivity can be decomposed into markup and the share of variable cost in total cost. Under CES, markup is constant across all firms, while the increasing returns to scale implies the variable labor share is increasing in firm size. Together, the model predicts that firm's revenue productivity $\frac{r_{ij}(\varphi)}{l_{ij}(\varphi)}$ associated with a given destination is increasing in the firm's employment $l_{ij}(\varphi)$ for that destination.

Testing this prediction for each destination country is difficult, however, as destination-specific employment is unobserved in the data. Therefore, I focus on non-exporting firms, as their observed total employment is equal to $l_{ii}(\varphi)$.[10]

[10]The weighted average revenue productivity $\frac{r_i(\varphi)}{l_i(\varphi)} \equiv \frac{\sum_j \mathbb{1}(\varphi \geq \varphi_{ij}) r_{ij}(\varphi)}{\sum_j \mathbb{1}(\varphi \geq \varphi_{ij}) l_{ij}(\varphi)}$ is increasing in $l_i(\varphi)$ among the firms that sell to the same set of destination countries but not across all firms.

2.3 Among non-exporters, revenue productivity is constant conditional on size

The second prediction considers the variance of revenue productivity conditional on firm size. Equation (4) implies that conditional variance of revenue productivity is zero:

$$\operatorname{Var}\left[\log\left(\frac{r_{ij}(\varphi)}{l_{ij}(\varphi)}\right) \middle| \, l_{ij}(\varphi)\right] = 0. \tag{5}$$

Intuitively, firm employment captures its productivity, which is the only source of firm heterogeneity in the model. Consequently, all firms of the same employment behave identically.

In fact, firm's total employment $l_i(\varphi) \equiv \mathbb{1}(\varphi \geq \varphi_{ij}) l_{ij}(\varphi)$ has one-to-one relationship with its productivity, so that it predicts all firm-level outcomes, including export status. Even in developed countries, however, there is substantial size overlap between non-exporting and exporting firms, despite the model's prediction that all exporting firms are larger than all non-exporting firms.

Given this observation, I test a weaker prediction by considering the revenue productivity variance among only the non-exporting firms. This allows filtering out the variance stemming from export status heterogeneity conditional on firm size.

2.4 Among exporters, export intensity is non-decreasing in size

Finally, I turn to the export performance pattern. Consider the ratio of firm's export sales to domestic sales conditional on exporting:

$$\frac{\sum_{j \neq i} r_{ij}(\varphi)}{r_{ii}(\varphi)} = \sum_{j \neq i} \mathbb{1}(\varphi \geq \varphi_{ij}) \left(\frac{E_j P_j^{\sigma-1}}{E_i P_i^{\sigma-1}}\right) \tau_{ij}^{1-\sigma} \tag{6}$$

The model predicts no variation on the intensive margin of export ratio. In other words, all firms that sell to the same set of countries exhibit the same export intensity. The variation arises solely from the extensive margin, as more productive firms are able to cover the fixed overhead costs associated with greater number of destinations.

The direct consequence is that export intensity, which I define as the share of export sales in total sales, is non-decreasing in firm size measured by employment.

2.5 Predictions under alternative models

Admittedly, the standard Melitz model makes strong assumptions that result in constant markup and fixed overhead costs across firms. In response, researchers have proposed models that relax these assumptions. Do three predictions survive under these alternative models?

I answer this question under two alternative models. First, I consider Arkolakis (2010), which allows heterogeneous fixed costs, and show that all three predictions remain. Second, I consider citemelitz2008, which allows heterogeneous markups. Again, all three predictions remain if the firm size is measured on the output side rather than with the input side. The fact that predictions hold under these alternative models suggests that they are not idiosyncratic to the standard model.

Below, I discuss some of the intuitions behind the results from each model, leaving the details to Appendix A.

2.5.1 Endogenous fixed cost

In Arkolakis (2010), firms can choose how much to spend on advertising for each potential destinations. Marketing allows firms to reach a larger fraction of consumers but is subject to diminishing returns in each destination. Consequently, more productive firms choose to spend more on advertising, which translates to higher fixed (non-production) costs associated with larger firms. Nonetheless, it can be shown that the fixed cost share is still decreasing in firm size so that the revenue productivity is still increasing in firm size. This result relates to the Dorfman-Steiner theorem, which states that the advertising intensity is equal to the ratio of advertising elasticity to demand elasticity. Diminishing returns to advertising implies advertising elasticity is decreasing, while CES demand implies demand elasticity is constant. Subsequently, advertising intensity falls with firm size, and as the sales are proportionate to production cost, the share of advertisement costs also falls with firm size.

The model, like the standard model, features productivity as the only source of heterogeneity. Furthermore, firm size $l_{ij}(\varphi)$ is strictly increasing in productivity φ. Therefore, firm size perfectly predicts its revenue productivity so that no variance remains conditional on firm size.

Finally, endogenous market access implies that export intensity is increasing on both intensive and extensive margins, enforcing Prediction 3. Even within a destination, more productive and thus larger firms pay higher marketing costs and export more. Diminishing returns within the destination implies a marginal improvement in productivity leads firms to expand more in foreign markets than in the home market. Furthermore, the selection mechanism is still present, so the extensive margin also predicts a positive relationship between export intensity and size.

2.5.2 Variable markup

Next I consider Melitz and Ottaviano (2008). All three predictions still hold when firm size is measured in terms of revenue (or output) rather than cost (or employment).[11]

The key feature of the model is the linear demand for each variety so that the markup is no longer constant. The model also dispenses with fixed costs as the selection occurs through demand threshold. With constant returns to scale, revenue productivity is equal to the markup. Demand linearity implies that more productive firms, which sell more output, face more inelastic demand and charge higher markup. Consequently, there is a positive relationship between revenue productivity and sales, and Prediction 1 holds when firm size is measured with revenue rather than with employment.

Again, the model features productivity as the only source of heterogeneity, and firm size measured in revenue (or output) is monotonic in productivity. Therefore, Prediction 2 holds when firm size is measured with revenue.

Lastly, Prediction 3 also still holds when firm size is measured in revenue. Similar to Arkolakis (2010), the model predicts export intensity is increasing in productivity on both intensive and extensive margins. In this model, higher productivity firm will both charge higher relative markup and sell relative more output in the foreign market. Both channels contribute high-productivity firms to exhibit higher export intensity within a destination market. On the extensive margin, not all firms sell to a given destination since demand hits zero above a threshold price level so that firms below some productivity cutoff cannot make a profit in that market. As a result, more productive firms sell to a greater number of destinations.

3 Facts about Chinese manufacturing firms

Do three predictions of the standard model hold in the Chinese manufacturing industry? A short answer is, no. Using the firm-level survey data collected by China's National Bureau of

[11] The model predicts a quadratic relationship between firm employment and productivity. This, in turn, implies among the high-output firms, employment is decreasing in output, contradicting the pattern observed in data.

Statistics, I show that revenue productivity is decreasing in firm size, revenue productivity variance is large even when conditioned on firm size, and export intensity is decreasing in firm size.

3.1 Chinese manufacturing survey data

China's annual firm-level survey data set has been widely used in the literature and its features are documented by Brandt, van Biesebroeck, and Zhang (2014). Although the data is available starting 1998, I use the data from the year 2000, the year before China joins WTO. There was sufficient trade activity to capture the export patterns across firms, but also trade barriers were high such that the potential impact of trade liberalization remained large.

An important feature of the data is its above-scale sampling. The survey includes all state-owned firms and all private firms with sales exceeding 5 million yuan, which translates to roughly 600,000 USD using the year 2000 exchange rate.

The raw data includes 144,799 firms in the year 2000 of which 10,752 (7.4%) are dropped for reporting non-positive sales, value-added, wage payment, or capital, where the value-added is constructed as the total output minus the total intermediate input.

Firms are assigned to one of 446 categories of the 4-digit sector according to its primary product. I also use the firm's location information at the province level. Firm ownership is inferred from the registration type. I partition the ownership status into domestic private, state-owned, and foreign firms, which include joint ventures. In 2000, about 19% of firms are state-owned, and another 19% are foreign.

Since the model does not consider intermediate inputs, I map the firm's revenue in the model $r_i(\varphi) = \sum_j \mathbb{1}(\varphi > \varphi_{ij}) r_{ij}(\varphi)$ to firm's value added. Survey also reports firm's total export sales. Firm's export revenue $r_i(\varphi) - r_{ii}(\varphi)$ is constructed by assuming the value added content in export sales is the same as the total value added share in sales.

I translate firm's employment in the model $l_i(\varphi) \equiv \sum_j \mathbb{1}(\varphi \geq \varphi_{ij}) l_{ij}(\varphi)$ to the total labor compensation in the data. The aggregate wage payment share in value-added is 24%. Following Hsieh and Klenow (2009), I assume that the total labor compensation share in value-added is 50% and that the non-wage benefits are proportional to the wages.

When checking robustness, I use the Cobb-Douglas composite inputs as a measure of firm size, using the book value of the net fixed assets as the capital measure and estimating the capital share at the 2-digit sector level. The capital share is constructed as 1 minus the labor share within each sector, and the median value (across all firms) is 0.41.

Before testing the model predictions, I briefly discuss the extent of firm heterogeneity and the systematic difference between non-exporters and exporters in China. First, exporters are relatively rare, and export participation varies by firms even within sectors. Of 134,047 observed firms, 35,459 (36.5%) have positive reported exports. At the 2-digit level, the most export intensive sector is the educational and sports goods, which has 72% of firms exporting. The least export intensive sector, printing and recorded media, has 6% of firms exporting. Second, exporters are on average larger. Regressing log employment and log revenue on the export status with 4-digit sector fixed effect yields coefficient estimates of 1.08 and 0.90, respectively. Both estimates are significant at 1% level.

3.2 Among non-exporters, revenue productivity is decreasing in size

The standard model predicts that revenue productivity is increasing in firm size among non-exporters. Figure 1 plots the relationship between the average revenue productivity and size among the non-exporting firms. Both productivity and log employment are demeaned at the sector-level. The solid line shows the smoothed average log revenue productivity of private firms as a function of size while the dashed line shows that of SOEs. The band around each line shows 95% confidence interval. The histogram plots the frequency of non-exporting firms (both private and SOE) by log employment size. The plot drops top and bottom 1% of firms by size.

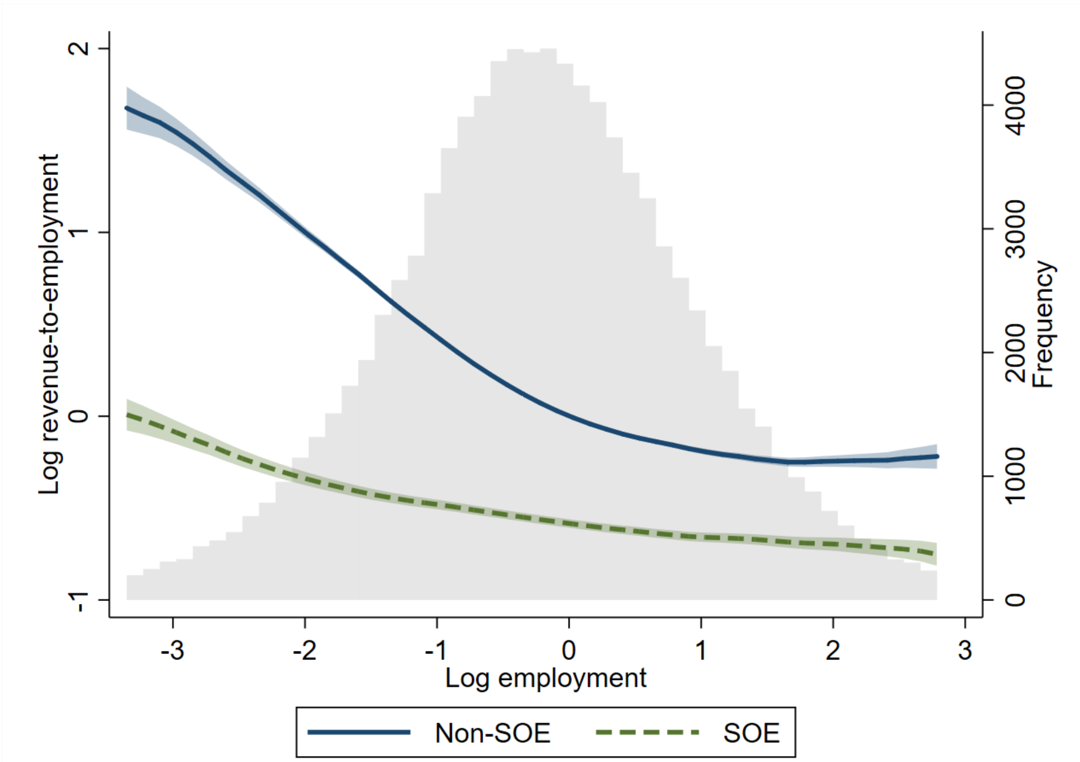
Contrary to Prediction 1, Figure 1 suggests that revenue productivity is nearly monotonically decreasing in firm size among the non-exporting firms. The pattern holds within private and state-owned firms and hence is not driven by the systematic difference between the two groups.

Table 1 provides formal statistical tests by reporting the linear regression results, where the dependent variable is log revenue productivity and the main regressor is firm size measured from the input side.

of revenue productivity on firm size measured Columns (1) to (4) report the results of OLS estimates with an increasingly larger set of controls. Column (1) shows the simple regression of revenue productivity on employment, while column (2) includes 4-digit sector fixed effects. Column (3) additionally includes indicators for state and foreign ownership. Column (4) allows different slopes for each ownership group.

As one would expect from Figure 1, the coefficient estimate on the firm size is significantly

Figure 1: Log revenue-to-employment ratio vs. log employment



Note: The figure shows the conditional mean of log revenue-to-employment ratio as a function of the log employment among the non-exporting firms estimated by kernel-weighted local polynomial regression. The solid line shows for private firms while the dashed line shows for state-owned firms. The bands show 95% confidence interval. The graph is overlaid with the histogram of non-exporting firms by the log employment. Firm revenue is measured by the output net of intermediate inputs while the employment is measured by total labor payment.

negative in all four specifications. Results in Column (4) show that among the state- and foreign-owned firms, the relationship between revenue productivity and firm size is weaker but still negative.

Columns (5) addresses the concern with the bias due to sampling. Since the survey targets firms with sales above the threshold, small firms would exhibit high revenue productivity due to the selection. Noting that this above-scale sampling applies only to the non-SOEs, Column (5) runs the regression with only the state-owned firms. While the magnitude of the coefficient is smaller, the sign remains significantly negative.

Column (6) addresses the concern with the bias due to measurement error on the firm size. Since firm size appears negatively in the dependent variable, measurement error in the size generates negative bias.[12] To check the negative relationship is not driven by such bias, I use the log number of employees as an instrument for the firm size. The underlying assumption is that the measurement error on the total wage compensation is independent of the measurement error on the total number of employees. Compared to the OLS result in Column (3), the IV regression result in Column (6) shows a smaller magnitude in the estimate of size coefficient, suggesting that the measurement error matters quantitatively. However, the estimate remains significantly negative under IV estimation, so the measurement error alone does not explain the negative relationship.

Finally, Column (7) uses the total factor productivity as the dependent variable and composite input as the firm size measure. If low-employment firms are more capital intensive, then the negative relationship between revenue-to-employment ratio and employment can be driven by the variation in capital intensity. Results in Column (7), however, refutes this conjecture by showing that the relationship between total factor productivity and composite input is also negative.

Does the negative relationship hold when size is measured from the output side? If we believe that $\mathrm{E}[\log(r/l) \mid \log l] = \alpha + \beta \log l$ and $\mathrm{E}[\log(r/l) \mid \log r] = \gamma + \delta \log r$, then $\delta = \frac{\beta}{1+\beta}$. Hence, δ is negative if and only if $-1 < \beta < 0$, which hold with the estimated values of β. For example, the coefficient estimate of $\beta = -0.197$ from column (6) implies $\delta = -0.245$. Regression result of using firm revenue as the size measure under the same specification of column (6) yields the firm size coefficient estimate of -0.218.

In summary, the standard model's prediction that the revenue productivity is increasing in firm size is rejected in the Chinese manufacturing data. There is, in fact, a negative relationship, robust to controlling for the sector, region, and ownership. The negative relationship is statistically significant even after the biases from the sample selection and measurement errors are addressed.

3.3 Among non-exporters, revenue productivity varies conditional on size

The standard model predicts that the revenue productivity variance is zero conditional on firm size. To test this prediction, I decompose the variance of log revenue productivity using

[12] More precisely, measure error implies that the estimated coefficient is a weighted average of the "true" coefficient and -1. Since the OLS estimate is greater than -1, the bias from the measurement error would be negative.

Table 1: Relationship between revenue productivity and firm size

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log size	-0.275	-0.276	-0.292	-0.391	-0.167	-0.197	-0.422
	(0.004)	(0.004)	(0.004)	(0.004)	(0.007)	(0.004)	(0.004)
$\mathbb{1}$(SOE)			-0.789	-2.538		-0.778	-1.819
			(0.010)	(0.055)		(0.010)	(0.056)
$\mathbb{1}$(Foreign)			0.133	-0.896		0.090	-0.989
			(0.012)	(0.096)		(0.012)	(0.085)
Log size × $\mathbb{1}$(SOE)				0.242			0.137
				(0.007)			(0.007)
Log size × $\mathbb{1}$(Foreign)				0.138			0.124
				(0.012)			(0.010)
Sector FE	No	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	No	No	Yes	Yes	Yes	Yes	Yes
Remark					SOE	IV	TF
N	98588	98588	98544	98544	22566	98544	95254
R^2	0.085	0.146	0.258	0.272	0.168	0.250	0.298

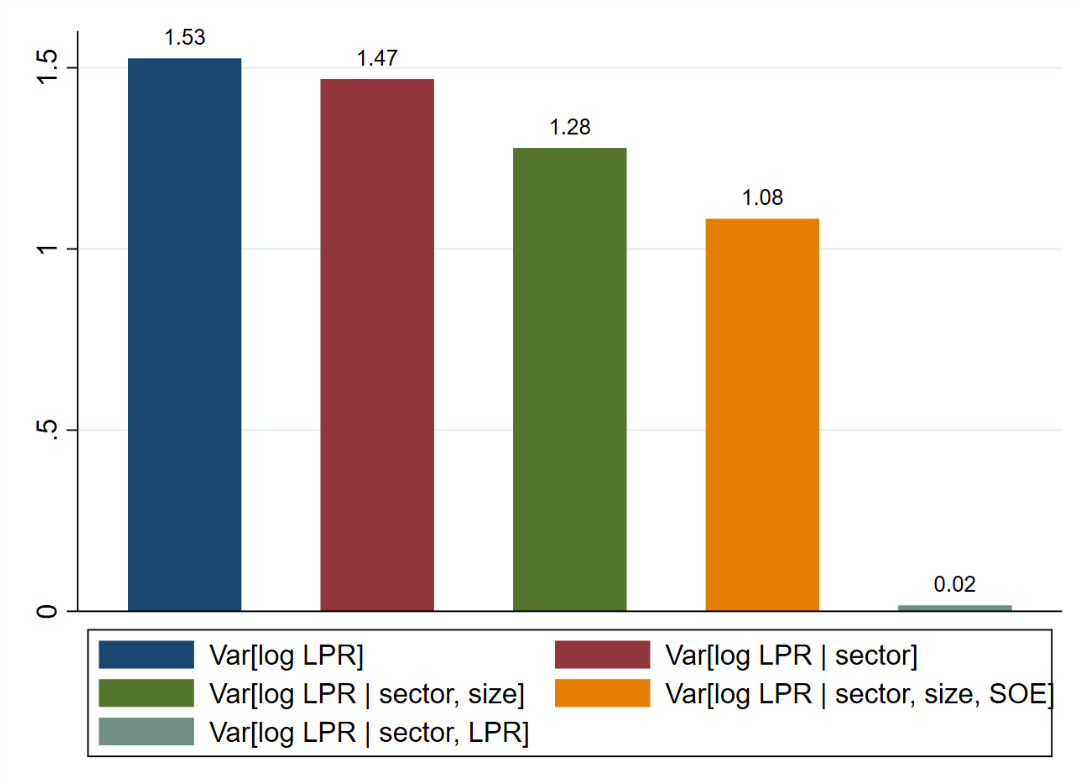
Note: The dependent variable is log revenue-to-employment for columns (1) to (6) and log TFPR in column (7). Log size refers to log employment in columns (1)-(6) and to log total factor in column (7). Revenue is measured as value-added, employment is measured as total labor compensation, and the total factor is expenditure share weighted sum of log employment and log capital. SOE refers to state-owned firms while Foreign refers to foreign-owned firms according to the registration code. Column (6) uses only the state-owned firms. Column (7) uses the log number of employees as the instrument for log size. Sector fixed-effect is at the 4-digit industry code level and region fixed effect is at the province level. Robust standard errors reported are in the parentheses.

the law of total variance:

$$\text{Var}[\log(r/l)] = \text{Var}\left[\text{E}[\log(r/l) \mid X]\right] + \text{E}\left[\text{Var}[\log(r/l) \mid X]\right].$$

This identity allows decomposing the total variance into the part explained by some variable X, the variance of the conditional mean, and the residual, the mean conditional variance.

Figure 2: Conditional variance of revenue productivity



Note: Each bar represents the variance of revenue productivity among non-exporting Chinese manufacturing firms. The first bar shows the unconditional variance, while the second bar shows the average variance conditional on 2-digit sector. The third bar shows the average variance conditional on both 2-digit sector and firm size bin within each sector. The fourth bar shows the average variance conditional on sector, size, and state ownership. The last bar shows the variance conditional on sector and revenue productivity bin within each sector.

Figure 2 summarizes the results of variance decomposition exercise. All variance calculations include only the non-exporting firms. The first bar shows the unconditional variance of log revenue productivity. Since the model assumes a single sector, I first decompose the variance by conditioning on the 2-digit sector. The (average) variance conditional on the sector is 3.9% lower than the unconditional variance.

The third bar represents the variance conditional on the sector and firm size, testing the prediction that the variance conditional on firm size is zero. To get this number, I divide firms into size centile bins within each sector, calculate the sample variance of each sector-size bin, and take the average weighted by the bin's firm count. The resulting value is 16.3% lower than the unconditional variance, showing that the firm size explains an additional 12.4% of the variance. Therefore, while firm size provides some predictive power on revenue productivity, much of the variance remains even within the firm size groups, contrary to Prediction 2.

A major concern with the result is that the remaining variance is due to measurement errors. To argue that the measurement error alone is unlikely to explain all the observed conditional variance, I calculate the variance conditional on the sector, firm size, and state ownership. The fourth bar of Figure 2 shows the result. To see the argument, suppose the observed firm revenues $\log \hat{r}$ contain independent measurement errors ε such that $\log \hat{r} = \log r + \varepsilon$, and that the measurement error accounts for all the observed conditional variance, so that $\text{Var}[\log(\hat{r}/l) \mid \log l] = \text{Var}[\varepsilon \mid \log l] = \text{Var}[\varepsilon]$. If ε is also independent of firm's ownership status, then the variance would remain the same when further conditioned on the ownership: $\text{Var}[\log(\hat{r}/l) \mid \log l] = \text{Var}[\log(\hat{r}/l) \mid \log l, \mathbb{1}(SOE)] = \text{Var}[\varepsilon]$.[13] The fourth bar in Figure 2 shows that this is not the case – state ownership provides additional 13.1% explanation for the revenue productivity variance, given sector and firm size.[14]

Since I estimate the conditional variance by grouping firms into discrete bins, within which firm size varies, the estimated variance would be positive even if the true conditional variance is zero. The last bar in Figure 2 provides a sense of how much variance stems from the discretization by showing the average variance of revenue productivity within sector and revenue productivity centile bins. The resulting value of 0.02 suggests that the contribution of discretization on the conditional variance is negligible.

3.4 Among exporters, export intensity is decreasing in size

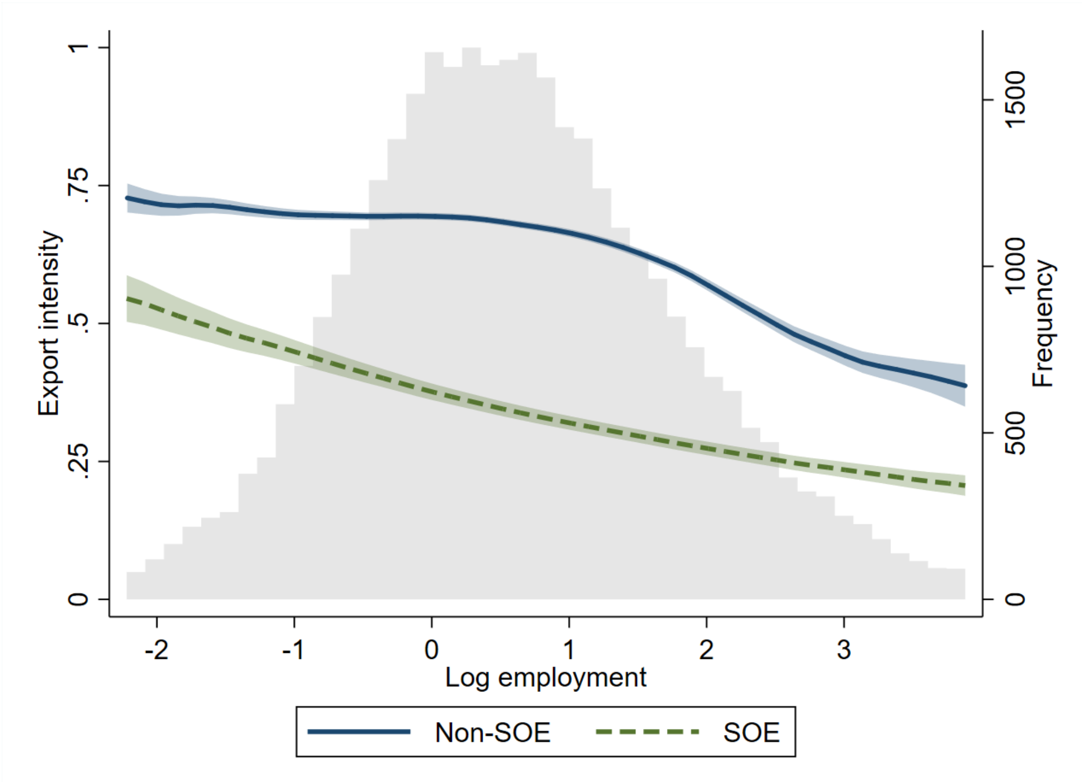
Finally, I test the prediction that the export intensity is increasing in firm size. Figure 3 plots the relationship between export intensity, measured as export sales over total sales, and

[13] The argument can be generalized to allow measurement errors on the log employment, as long as all the measurement errors are independent of state ownership.

[14] Finite sample leads to a mechanical reduction in variance by additional conditioning. To verify the variance reduction from ownership is statistically significant, I run a placebo test where a firm is randomly assigned to the state-ownership group based on a random permutation of the observed state-ownership vector. The test is repeated 1000 times. The average variance conditional on this placebo state ownership is 1.24, with a standard deviation value of 0.001. Hence, the mechanical decomposition leads to about 3% drop in the variance compared to 13% drop from conditioning on the observed ownership.

firm size measured as the log employment (total labor compensation). I truncated the log employment axis to drop the top and bottom 1% exporting firms. The solid line shows the smoothed average export intensity of private firms while the dashed line shows that of SOEs. The histogram in the background plots the density of the firms by log employment. Both the export intensity and log employment have been demeaned at the 4-digit sector level.

Figure 3: Export intensity vs. log employment



Note: The figure shows conditional mean of export-to-sales ratio as a function of the log employment among exporting firms estimated by kernel-weighted local polynomial regression. Each exporter's export intensity and log employment is demeaned at the sector level. The solid line shows for private exporters while the dashed line shows for state-owned firms. The band around each line shows 95% confidence interval. The graph is overlaid with the histogram of exporting firms by the log employment. The firm employment is measured by total labor payment.

Contrary to Prediction 3, Figure 3 suggests that the export intensity is decreasing in firm size among the exporting firms. The pattern holds within private and state-owned firms.

Table 2 provides formal statistical tests by reporting the linear regression results where the dependent variable is export intensity and the main regressor is firm size measured as log employment for columns (1) to (6) and as log composite input for column (7).

Table 2: Relationship between export intensity and firm size

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log size	-0.060	-0.036	-0.034	-0.051	-0.037	-0.020	-0.063
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)
$\mathbb{1}$(SOE)			-0.093	-0.144	-0.068	-0.057	-0.223
			(0.006)	(0.035)	(0.035)	(0.027)	(0.034)
$\mathbb{1}$(Foreign)			0.105	-0.208	-0.211	-0.094	-0.230
			(0.004)	(0.023)	(0.028)	(0.023)	(0.021)
Log size × $\mathbb{1}$(SOE)				0.007	0.003	0.005	0.017
				(0.004)	(0.004)	(0.003)	(0.003)
Log size × $\mathbb{1}$(Foreign)				0.038	0.036	0.016	0.040
				(0.003)	(0.003)	(0.003)	(0.002)
Sector FE	No	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	No	No	Yes	Yes	Yes	Yes	Yes
Remark					EI < 99%	EI < 70%	TF
N	35459	35459	35448	35448	24359	16754	35448
R^2	0.045	0.322	0.382	0.385	0.274	0.186	0.396

Note: The dependent variable is export intensity, measured as the share of export sales over total sales. The "main" sample includes non-exporting firms with sales greater than 5 million yuan. The "Exp. Int. < p" sample includes all the firms with export intensity less than p within the main sample. Sector fixed-effect is at 4-digit industry code level and region fixed effect is at the province level. Robust standard errors reported are in the parentheses.

Column (1) reports the simple linear regression result. One percent increase in firm size predicts 6 percentage point lower export intensity. Column (2) includes the 4-digit sector fixed effect. The magnitude of the coefficient drops from 0.060 to 0.036, suggesting part of the negative raw correlation is due to export intensive sectors having smaller firms. Nonetheless, the coefficient remains significantly negative.

Column (3) further includes region and ownership fixed effects. Stated owned firms on average have lower export intensity, while foreign-owned firms on average have higher export intensity. The coefficient estimate on the size, however, remains nearly unchanged.

Column (4) allows different slope across ownership groups. The negative relationship between export intensity and size is less steep among state-owned and especially foreign-owned firms. Nonetheless, the relationship remains negative within each group.

Columns (5) and (6) investigate whether the negative relationship is driven by high export intensity firms. Column (5) excludes firms with export intensity greater than 99%, while column (6) excludes firms with export intensity greater than 70%. Not surprisingly, the magnitude of the slope coefficient estimate drops, and among the foreign-owned firms, the slope is indistinguishable from zero. For other firms, however, the negative relationship remains statistically significant.

Finally, column (7) measures firm size with composite input as an additional robustness check. The relationship remains negative across all ownership groups.

In summary, the standard model's prediction that the export intensity is increasing in firm size is rejected.

4 Explaining the gap: model of misallocation in open economy

In previous sections, I have shown that the standard model which features productivity as the only source of firm heterogeneity fails to account for the firm-level patterns observed in the Chinese manufacturing industry. In this section, I introduce the misallocation model motivated by the evidence of distortionary policies and the political connections discussed in the introduction. The model generalizes the standard model by allowing firm-specific subsidies as an additional source of heterogeneity. These firm-specific subsidies create a wedge between a firm's social and private marginal values of labor, which in turn leads to misallocation of labor when subsidies vary across firms. I call this across-firm misallocation.

Many papers have used exogenous wedges as a modeling tool to study across-firm misallocation.[15] In an open-economy setting, however, there is another margin of misallocation that has received less attention associated with labor allocation across destinations. A given firm needs to allocate its labor across production for different potential destinations. In other words, a firm needs to decide the share of its total output shipped to each country. In my model, each firm faces potentially different subsidy rates based on these shares. This feature of the model allows what I call across-destination misallocation – distortions in each firm's labor allocation across sales destinations.

I remark that the market equilibrium of the standard model achieves optimal allocation as proved in Dhingra and Morrow (2018). Hence, any deviation in the resource allocation induced by the subsidies, in fact, results in misallocation. In the remaining section, I formally introduce the generalized model and show how it can explain the three facts. I conclude by discussing the plausibility of misallocation as the primary explanation.

4.1 The misallocation model environment

Consider the economy with the same preference (1) and technology (2) as the standard model. Firms are now endowed with an idiosyncratic vector of ad valorem subsidy rates $\eta = (\eta_1, \ldots, \eta_N)$, so that they are distinguished by their productivity and subsidy rates (φ, η).

The profit of a firm in country i earned from sales to destination j is

$$\pi_{ij}(q; \varphi, \eta) = (1 + \eta_j) p_j(q) q - w_i l_{ij}(q; \varphi),$$

and the profit maximizing price is

$$p_{ij}(\varphi, \eta) = \tilde{\sigma} \left(\frac{w_i \tau_{ij}}{\varphi} \right) \frac{1}{1 + \eta_j}.$$

A firm's variable profit $\pi_{ij}(\varphi, \eta) + w_i f_{ij}$ from sales to destination j is increasing in $z_j \equiv \varphi(1 + \eta_j)^{\tilde{\sigma}}$. Hence, there exists a unique value $z_{ij} > 0$ such that a firm sells to the destination j if and only if $z_j \geq z_{ij}$. I refer to z_j as the firm's destination j profitability and $z = (z_i, \ldots, z_N)$ as the firm's profitability vector. It can be also shown that firm's employment for sales to j is a

[15] See Hopenhayn (2014) for a review.

function of the associated profitability:

$$l_{ij}(\varphi,\eta) = l_{ij}(z_j) = f_{ij} + \left(\frac{E_j}{\tilde{\sigma} w_i}\right)\left(\frac{P_j}{\tilde{\sigma} w_i \tau_{ij}}\right)^{\sigma-1} z_j^{\sigma-1}.$$

4.2 Revenue productivity and size

In this section, I show how across-firm misallocation can generate negative relationship between revenue productivity and firm size. The revenue productivity of a firm associated with a particular destination can be expressed as

$$\log\left(\frac{r_{ij}(\varphi,\eta)}{l_{ij}(\varphi,\eta)}\right) = \underbrace{\log(\tilde{\sigma})}_{\text{markup}} + \underbrace{\log\left(1 - \frac{f_{ij}}{l_{ij}(\varphi,\eta)}\right)}_{\text{variable labor share}} - \underbrace{\log\left(1+\eta_j\right)}_{\text{subsidy}}. \tag{7}$$

In comparison to the expression (4) from the standard model, the only difference is that the revenue productivity now also depends on the subsidy rate. As the subsidy rate enters negatively, the model predicts negative relationship between revenue productivity and firm size when the (conditional mean) subsidy rate grows sufficiently fast in size.

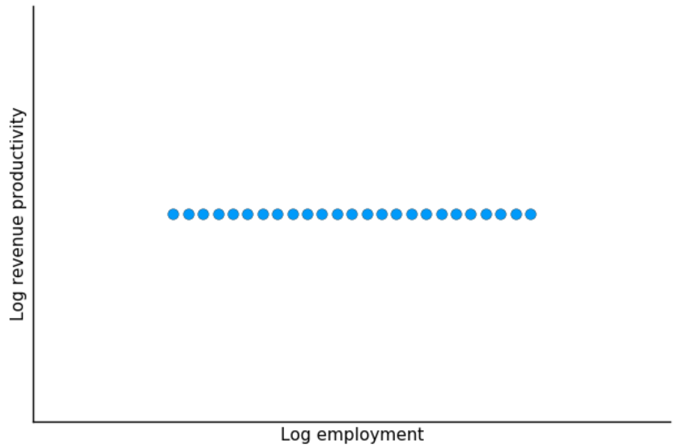
Intuitively, firms that are subsidized become larger, so one expects a positive relationship between subsidy rate and size, which in turn pushes the revenue productivity and size relationship toward the negative. More precisely, it can be shown that a firm's employment $l_{ij}(\varphi,\eta)$ associated with the destination j is strictly increasing in the corresponding profitability $z_j \equiv \varphi\eta_j^{\tilde{\sigma}}$. Hence, as long as φ and η_j are not too negatively correlated, the subsidy rate and the firm size would be positively related.

To gain an intuition of this result, consider a simple setting of a closed economy with a fixed number of firms and zero fixed overhead costs. Figure 4 illustrates the outcomes under this environment. Each marker in the figure represents a single firm, with x-coordinate representing log employment and y-coordinate representing log revenue productivity.

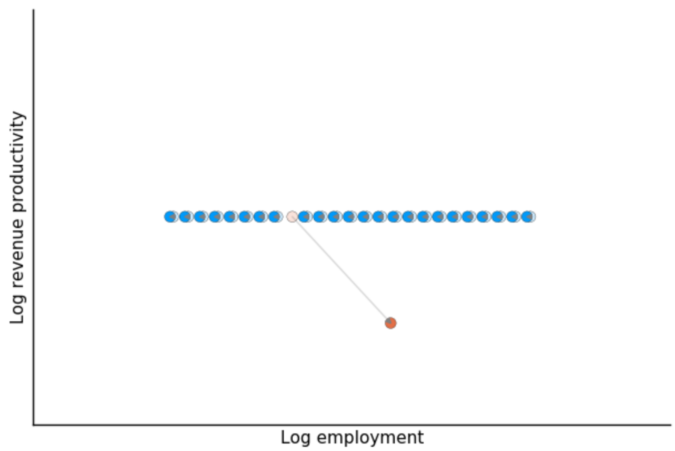
Panel (a) depicts the outcome without misallocation. Firms vary in their sizes, reflecting heterogeneity in productivity. With zero fixed overhead cost, however, revenue productivity is equalized across all firms in the absence of misallocation.

Panel (b) depicts the effect of the subsidy on a single firm, colored red in the figure. The subsidy incentivizes the firm to expand its employment l. The subsidy does not affect the firm's productivity and hence the average output $\frac{q}{l}$ remains the same. The firm's effective marginal cost, however, is now lower due to the ad valorem subsidy. Since markup remains

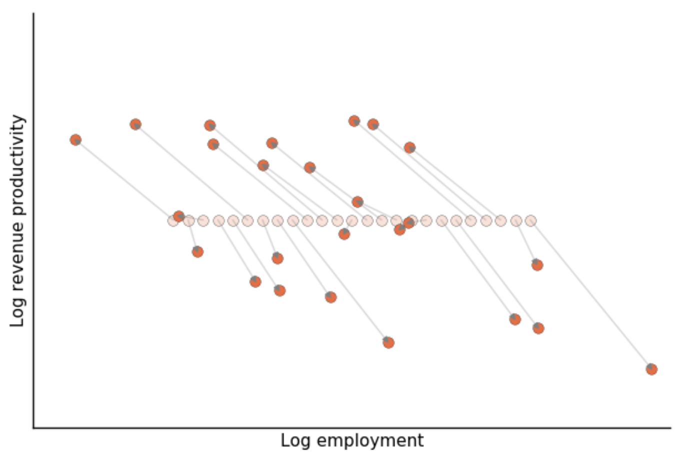
Figure 4: Illustration of the effects of firm-specific subsidies



(a) Equilibrium without misallocation



(b) Equilibrium after one firm is subsidized



(c) Equilibrium after random subsidies

constant, this reduction in marginal cost leads to a lower price, and consequently lower revenue productivity $\frac{pq}{l}$. Overall, the subsidized firm is larger but has exhibits lower revenue productivity. Since the labor supply is fixed, the expansion of the subsidized firm requires shrinkage of other firms, as illustrated by the leftward movement of all other firms in the figure.

Finally, Panel (c) illustrates the aggregate impact of firm-specific subsidies by depicting the outcome after each firm has received a random subsidy or tax. Implicitly, productivity and subsidy rate are independent, and as expected from the earlier discussion, revenue productivity and size exhibit a negative relationship. A firm can be large because either it is more productive or receives more subsidy. Higher productivity (in the absence of fixed overhead costs) does not affect the firm's revenue productivity, while higher subsidy lowers it. Therefore, on average, larger firms exhibit lower revenue productivity. An exception arises when the subsidy is highly negatively correlated with productivity. In this case, ex-ante small firms receive relatively large subsidies so that they exhibit low revenue productivity. If this selection effect is stronger than the expansionary effect of subsidy, the subsidies could generate a positive relationship between revenue productivity and size.

In the presence of fixed overhead costs, larger firms will also exhibit higher revenue productivity in the absence of misallocation. Hence, the effect of misallocation needs to be sufficiently large to generate a negative relationship.

4.3 Revenue productivity dispersion

Next, I show the misallocation can generate conditional variance of revenue productivity. From equation (7),

$$\mathrm{Var}\left[\log\left(\frac{r_{ij}(\varphi,\eta)}{l_{ij}(\varphi,\eta)}\right)\,\middle|\, l_{ij}(\varphi,\eta)\right] = \mathrm{Var}\left[\log\!\left(1+\eta_j\right) \mid l_{ij}(\varphi,\eta)\right].$$

Therefore, as long as subsidy rate varies conditional on size, which in turn can be reduced to the primitive condition $\mathrm{Var}[\eta_j \mid \varphi] > 0$, the model predicts positive conditional variance of revenue productivity.

The intuition of the result is straightforward. Firm size now depends on both productivity and subsidy rate. A low-productivity firm can be as large as a high- productivity firm if it receives a relatively larger subsidy. Since these two firms face different subsidies, they, in turn, exhibit different revenue productivity.

4.3.1 Export intensity and firm size

The across-destination misallocation helps to explain the negative relationship between export intensity and size. The export intensity can be expressed as

$$\frac{\sum_{j\neq i} r_{ij}(\varphi,\eta_j)}{r_{ii}(\varphi,\eta_i)} = \sum_{j\neq i} \mathbb{1}(z_j \geq z_{ij}) \left(\frac{E_j P_j^{\sigma-1}}{E_i P_i^{\sigma-1}}\right) \tau_{ij}^{1-\sigma} \left(\frac{1+\eta_j}{1+\eta_i}\right) \tag{8}$$

Compared to the equation (6) from the standard model, there are two differences. First, the intensive margin of export intensity now depends on the subsidy rate ratio $\tilde{\eta}_j \equiv \frac{1+\eta_j}{1+\eta_i}$. Second, the extensive margin depends on profitability rather than productivity. Since profitability distribution is destination-specific, the model no longer predicts hierarchical entry – firms that sell to the $k+1$st most popular destination do not necessarily all sell to the kth most popular destination.

Both differences can help to explain the negative relationship between export intensity and size. On the intensive margin, the export intensity is decreasing in size if $\tilde{\eta}_j$ is. Intuitively, this occurs when firm size variance, holding productivity, is driven by domestic subsidy more so than by export subsidy. A simple example is when η_j is the same across all firms for $j \neq i$, while η_i varies. In this case, $\tilde{\eta}_j \propto \frac{1}{1+\eta_i}$. Hence, as long as η_i is increasing in firm size (i.e. η_i and φ are not too negatively correlated), the intensive margin of export intensity would decrease in firm size.

To see the effect of misallocation on the extensive margin, consider a simple case of a low productivity firm that is receiving high subsidy rates on foreign sales but low subsidy (or tax) on the domestic sales. Such a firm would export to many destinations, aided by the associated subsidies, but can be still smaller than firms that export to fewer destinations (or do not export at all) due to low productivity and low domestic subsidy.

4.4 Is misallocation the right explanation?

In the previous section, I have shown that introducing across-firm and across-destination misallocation to the model helps to explain the three patterns observed in the data. It is not surprising, however, that a more general model can explain a broader set of observations. The three patterns documented in Section 2 are essentially the characteristics of the distribution of the three variables: total employment, domestic sales, and export sales. Therefore, one may expect that any such distribution can be rationalized with a model that features three primitives, such as the misallocation model with productivity, domestic subsidy, and export subsidy. This leads to the question: why is firm-level misallocation the appropriate way to explain the firm heterogeneity in China?

In this section, I answer the question in two ways. First, I show that a model that features exogenous firm-destination specific fixed cost cannot explain the data, despite having the same number of primitives as the observables. In particular, the model cannot match the observed conditional variance of revenue productivity documented in Section 3.3, illustrating that the mapping from the primitives to outcomes implied by the model is not necessarily surjective. Furthermore, compared to the misallocation model, the heterogeneous fixed cost model requires more stringent assumptions on the joint distribution to explain the negative relationships of performance and size. Second, I briefly review the literature suggesting that the standard model predictions hold better in developed countries. Studies show that both the firm revenue productivity and export intensity increase in firm size among the firms in developed countries. The patterns fail to hold in developing countries where the policy-induced misallocation is likely more prevalent.

4.4.1 Rejecting heterogeneous fixed cost model

Consider the economy with the same preference (1) and technology (2) as the standard model. Firms are endowed with an idiosyncratic vector of overhead fixed costs $f = (f_1, \ldots, f_N) \in \mathbb{R}^N_{++}$ along with productivity φ. Note that the model is different from Arkolakis (2010) considered in Section 2.5 in that the fixed costs are exogenously given to each firm.

Firms sell in market j if and only if they can make a non-negative profit by doing so. In Appendix B, I show that this selection leads to the following inequalities:

$$0 \leq \log\left(\frac{r_{ij}(\varphi, f)}{l_{ij}(\varphi, f)}\right) \leq \log \tilde{\sigma}.$$

Intuitively, conditional on making a profit from destination j, firm's fixed overhead cost share there, $\frac{f_j}{l_{ij}(\varphi, f)}$, cannot be too high. In the extreme case, if the fixed cost share is 1, then the firm is earning zero revenue but incurring the fixed cost, so it must be making a loss. The fixed cost share of zero-profit firm is $\frac{1}{\sigma}$, which in turn provides the upper bound. The revenue productivity can be still expressed as (4), so the upper bound on $\frac{1}{\sigma}$ translates to a lower bound of zero on the log revenue productivity. Similarly, the lower bound of zero on the fixed cost share provides the upper bound value of log revenue productivity.

Given this boundary, how well can this model explain the conditional variance of revenue productivity? I assign a fixed cost share to each observed non-exporting firm such that the resulting conditional revenue productivity variance is minimized while the boundaries on the fixed cost share are enforced. More precisely, for each non-exporting firm, I assign variable cost share

$$1 - \frac{f}{l} = \max\left\{\min\left\{\frac{1}{\gamma}\left(\frac{r}{l}\right), \frac{1}{\tilde{\sigma}}\right\}, 1\right\},$$

where r and l are the observed firm's revenue and employment. If equation (4) is taken literally, $\gamma = \tilde{\sigma}$. Instead, I let γ to vary across sector-size bins and choose to maximize the variance of the variable labor share within each bin. Since the boundary is less restrictive when σ value is lower (and hence $\tilde{\sigma}$ is higher), I choose conservatively low value of $\sigma = 3$.

The result of this exercise reveals that heterogeneous fixed cost has very limited explanatory power. With $\sigma = 3$, the conditional variance is reduced by only 3.3%. To gain intuition on why the heterogeneous fixed cost model generates very little revenue productivity variance, compare it to the misallocation model. In the misallocation model, firm size is a function of profitability – conditional on profitability, all firms have the same size. On the other hand, revenue productivity cannot be written as a function of profitability – conditional on profitability, revenue productivity is inverse proportional to the subsidy rate. Therefore, variance in subsidy rates translates to the variance of revenue productivity conditional on size. In the heterogeneous fixed cost model, the pattern is flipped. Revenue productivity is now a function of the firm profitability (a combination of productivity and fixed cost that characterizes firm profit), while the firm's employment depends on both profitability and the idiosyncratic fixed cost. In other words, variance in fixed cost translates to the variance of employment conditional on revenue productivity. Because the range on the revenue productivity is bounded, this "horizontal" variation provides a limited explanation for the revenue productivity variance in contrast to the "vertical" variation from the distortionary subsidies.

4.4.2 Patterns in other countries

If these patterns that deviate from the standard model predictions are indeed due to misallocation, then we expect they do not hold in developed countries that are a prior subject to less misallocation.

Comparing the revenue productivity and size relationship across countries can be difficult as this relationship is sensitive to the sampling procedure and the extent of measurement errors, both of which vary across countries. Bartelsman et al. (2013) uses harmonized firm-level data to compute the covariance between revenue productivity and size share in the U.S. and seven European countries. They find that the covariance, relative to the U.S., is lower in Western Europe and even lower in Central and Eastern Europe. Although the interpretation of this result requires caution as the covariances are computed using both exporting and non-exporting firms,[16] the negative link between country's development and the covariance suggest that misallocation is in fact the driving force behind the negative revenue productivity and size relationship observed in China.

Monteiro, Moreira, and Sousa (2013) provides a review of the literature on the export intensity and firm size. The countries examined by the reviewed papers include the U.S., Canada, Australia, Netherlands, UK, France, Italy, Norway, and Thailand. These studies find other positive or statistically insignificant relationship between firm size and export intensity, with the exception of Archarungroj and Hoshino (1998) which find a negative relationship in Thailand. In summary, the existing literature suggests a positive relationship between export intensity and size in developed countries.

5 Gains from trade under misallocation

So far, I have shown the gap between the standard model and the Chinese data can be reconciled by firm-level misallocation to the model. In the remainder of the paper, I explore the implications of the misallocation on the gains from trade. In this section, I show that theoretically, the gains from trade in the presence of misallocation can be larger or smaller than in the absence of misallocation. While the across-destination misallocation dampens the gains from trade, the reallocation effect of the trade can exacerbate or improve the resulting across-firm misallocation.

5.1 Equilibrium

The predictions discussed in Section 2 require only that consumer demands are driven from maximizing CES utility and that firms maximize profit given the technology. To discuss the aggregate welfare, however, it is necessary to introduce the equilibrium. I define equilibrium under the misallocation model environment, noting that the standard model is a special case where $\eta = (0, \dots, 0)$ for all firms. Appendix C provides details of derivations.

[16] The standard model predicts that conditional on true productivity, a firm exhibits lower revenue productivity as it exports to a larger set of countries due to the fixed cost associated with each destination. The revenue productivity and size relationship, therefore, depend on the country's extent of globalization, which in turn correlates with its development.

Zero cutoff profit Recall that firms in country i sell in destination j if and only if their relevant profitability z_j exceeds the cutoff value z_{ij}. The cutoff value, by definition, satisfies $\pi_{ij}(z_{ij}) = 0$. Under profit maximization, this condition can be expressed as

$$z_{ij} = \left(\frac{\sigma w_i f_{ij}}{E_j}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j}\right) \tag{9}$$

Free entry The mass of firms is determined by free entry condition. A potential entrant in country i can pay the sunk cost of $w_i f_i^E$ to receive a blueprint of a variety and draw its productivity and subsidy rates (φ, η) from a known joint distribution $G_i(\varphi, \eta)$. Note that one can derive the marginal distribution of profitability z_j from the joint distribution. Let $G_{ij}(z_{ij})$ denote this marginal distribution. The free entry condition asserts that the ex-ante expected profit net of entry cost is zero: $\sum_j \mathrm{E}_i \left[\pi_{ij}(z_j) \mathbb{1}(z_j \geq z_{ij})\right] = w_i f_i^E$. Here, $\mathrm{E}_i[\cdot]$ refers to the expectation with respect to country i distribution $G_i(\varphi, \eta)$. Expanding out yields the following expression for the free entry condition

$$f_i^E = \sum_j f_{ij} \left(H_{ij}(z_{ij}) - 1\right) S_{ij}(z_{ij}) \tag{10}$$

where

$$S_{ij}(z_{ij}) = \int \mathbb{1}(z_j \geq z_{ij})\, dG_{ij}(z_j)$$
$$H_{ij}(z_{ij}) = \int \left(\frac{z_j}{z_{ij}}\right)^{\sigma-1} \mathbb{1}(z_j \geq z_{ij}) \frac{dG_{ij}(z_j)}{S_{ij}(z_{ij})}$$

are the *survival* and *average-to-minimum* functions of the profitability, respectively.

Labor market clearing The labor market clears in the equilibrium. Since each consumer provides a unit labor inelastically, the total supply of labor in country i equals its mass of consumers L_i. The total demand for labor is the sum of labor used for production and the entry: $M_i \left(E_i\left[\sum_j l_{ij}(\varphi, \eta)\right] + f_i^E\right)$. After expansion, the labor market clearing condition can be expressed as

$$L_i = \sigma M_i \left(\sum_j f_{ij} H_{ij}(z_{ij}) S_{ij}(z_{ij})\right) \tag{11}$$

Price index To formally define equilibrium, it is useful to express the aggregate price index in terms of the supply-side variables. Let P_{ij} be the CES price index of country i goods sold in destination j. Then the aggregate price index P_j is in turn the CES aggregation of P_{ij}

over the origin countries i. Applying this yields the following expression for price index

$$P_j^{1-\sigma} = \sum_i \left(\frac{\tilde{\sigma} w_i \tau_{ij}}{z_{ij}}\right)^{1-\sigma} M_i K_{ij}(z_{ij}) S_{ij}(z_{ij}) \tag{12}$$

where

$$K_{ij}(z_{ij}) = \iint \left(\frac{\varphi(1+\eta_j)}{z_{ij}}\right)^{\sigma-1} \mathbb{1}(z_j \geq z_{ij}) \frac{dG_i(\varphi,\eta)}{S_{ij}(z_{ij})}. \tag{13}$$

For convenience, I refer to this function as the *distorted ratio* function. To be clear, however, the value of K_{ij} does not indicate the level of misallocation in the economy.

Aggregate expenditure Finally, I assume that all subsidies are financed through lump-sum tax on the domestic consumers.[17] This, together with free entry, implies that the aggregate expenditure equals to the aggregate pre-subsidy revenue. This equality can be expressed as

$$E_i = \sigma w_i M_i \sum_j f_{ij} K_{ij}(z_{ij}) S_{ij}(z_{ij}). \tag{14}$$

The equilibrium of the economy is the set of cutoffs $\{z_{ij}\}$, mass of entrants $\{M_i\}$, wages $\{w_i\}$, price indices $\{P_i\}$, and the aggregate expenditures $\{E_i\}$ such that equations (9), (10), (11), (12), and (14) are satisfied.

5.2 Welfare

Define the welfare as the real consumption per capita, $\frac{Q_i}{L_i}$, where $Q_i = \frac{E_i}{P_i}$ by the properties of CES aggregation. The cutoff condition (9) for z_{ii}, the labor market clearing (11), and the aggregate expenditure condition (14) imply

$$\frac{Q_i}{L_i} = \frac{1}{\tilde{\sigma}\tau_{ii}} \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \left(\frac{\sum_j f_{ij} K_{ij}(z_{ij}) S_{ij}(z_{ij})}{\sum_j f_{ij} H_{ij}(z_{ij}) S_{ij}(z_{ij})}\right)^{\tilde{\sigma}} z_{ii}. \tag{15}$$

Note that in the absence of misallocation, $K_{ij}(z) = H_{ij}(z)$ for all z and the welfare becomes proportional to that the domestic cutoff z_{ii}. As noted in Melitz, 2003, the domestic cutoff serves as sufficient statistics for welfare. In the presence of misallocation, however, this is no longer true. Conditional on the domestic cutoff z_{ii}, the welfare further depends on the export cutoffs $\{z_{ij}\}$ through the average-to-minimum functions H_{ij} and the distorted ratio functions K_{ij}.

[17] If firms are taxed on aggregate, then the tax revenue is lump-sum rebated to consumers.

Equation (15) proves to be useful for quantitative evaluation. Specifically, it relates the welfare in country i to only the cutoffs from there. This allows estimating the gains from trade using only the firm-level data from the country of interest, as illustrated in 6.2. The equation, however, does not elucidate how the misallocation interacts with the gains from trade because misallocation affects both the cutoffs $\{z_{ij}\}$ and the functions H_{ij} and K_{ij}.

To provide some intuition on how the gains from trade depend on the existing domestic misallocation, I consider a simplified model with two symmetric countries. Given symmetry, I drop the country index and use subindex $j = \{d, x\}$ to denote the destination market as domestic or foreign, respectively.

Suppose φ follows Pareto distribution with shape parameter $\theta > \sigma - 1$ and location parameter $\varphi^m > 0$. Firms either receive both domestic and export subsidies (η_d, η_x) with probability $\lambda \in (0,1)$ or do not receive any subsidy with probability $1-\lambda$. The level of subsidy conditional on receiving one is common across all subsidized firms, so that $\eta_j = \bar{\eta}_j$. The probability of receiving the subsidies is independent of firm's productivity φ.

Under these simplifying assumptions, the welfare can be expressed in terms of the primitives

$$\frac{Q}{L} = \beta \times \underbrace{\varphi^m}_{\text{productivity}} \times \underbrace{m_d^{\tilde{\sigma}} n_d^{-(\tilde{\sigma}-\frac{1}{\theta})}}_{\text{domestic misallocation}} \times \underbrace{\left(1+\chi\frac{m_x}{m_d}\right)^{\tilde{\sigma}}\left(1+\chi\frac{n_x}{n_d}\right)^{-(\tilde{\sigma}-\frac{1}{\theta})}}_{\text{trade}} \tag{16}$$

where β is a function of σ, θ, and f_d,[18] $\chi \equiv \left(\frac{f_x}{f_d}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}} \tau_x^{-\theta}$ is a measure of trade liberalization, and $m_j = (1-\lambda) + \lambda(1+\bar{\eta}_j)^{\theta\tilde{\sigma}-1}$ and $n_j = (1-\lambda) + \lambda(1+\bar{\eta}_j)^{\theta\tilde{\sigma}}$ are terms associated with misallocation.

Not surprisingly, welfare increases with better productivity distribution captured by $\bar{\varphi}$.[19] The domestic misallocation term is strictly decreasing in $\bar{\eta}_d$ for $\lambda \in (0,1)$. When $\lambda \in \{0,1\}$, there is no across-firm misallocation, and the domestic misallocation term reaches its maximum possible value of one. In particular, if all firms were subsidized ($\lambda = 1$), then there would be no welfare loss from the domestic misallocation term.

The welfare depends on the trade costs f_x and τ_x only through χ, which is decreasing in both. Hence, I define the gains from trade in this special case as the change in log welfare with

18

$$\beta = f_d^{-\frac{\theta-(\sigma-1)}{\theta(\sigma-1)}}(\sigma-1)\left(\frac{\theta}{\theta-(\sigma-1)}\right)^{\frac{\sigma}{\sigma-1}}\left(1+\frac{\theta(\sigma-1)}{\theta-(\sigma-1)}\right)^{-\frac{\theta\sigma-(\sigma-1)}{\theta(\sigma-1)}}.$$

19 Both mean and median of φ are proportional to $\bar{\varphi}$.

respect to χ:

$$GFT \equiv \frac{d\log(Q/L)}{d\chi}$$

To examine how the misallocation interacts with trade, I consider three special cases.

No across-destination misallocation First, suppose $\bar{\eta}_d = \bar{\eta}_x = \bar{\eta}$ so that there is no across-destination misallocation. The gains from trade in this case simplifies to

$$GFT = \frac{1}{\theta}\frac{1}{1+\chi}.$$

Note the absence of $\bar{\eta}$ in the expression: the the size of the gains from trade is unaffected by the extent of misallocation. By construction, there is no across-destination misallocation in this case. Furthermore, the extent of across-firm misallocation is unaffected by the trade liberalization due to the assumption of Pareto distributed productivity and independence of subsidy. Under this distributional assumption, the shape of the *ex-post* joint distribution is invariant to the cutoff. As a result, the across-firm misallocation does not depend on the trade cost.[20]

No domestic subsidy Second, suppose $\bar{\eta}_d = 0$ so that the subsidies are only on the export sales. In this case, the gains from trade can be approximated to

$$GFT \approx \frac{1}{\theta}n_x - \tilde{\sigma}(n_x - m_x) = \frac{1}{\theta}(1-\lambda) + \lambda(1+\bar{\eta}_x)^{\theta\tilde{\sigma}}\left(\frac{1}{\theta} - \tilde{\sigma}\frac{\bar{\eta}_x}{1+\bar{\eta}_x}\right)$$

when $\chi \approx 0$.[21] For $\lambda > 0$, the gains from trade liberalization is strictly decreasing in $\bar{\eta}_x$, reflecting that distortions created by export subsidy dampens the gains from trade. In autarky, the assumption $\bar{\eta}_d = 0$ implies that there is no welfare loss due to misallocation. Trade liberalization introduces misallocation of resources between domestic and export productions. More than optimal number of firms export and those that export will export too much due to the subsidy. Greater export opportunity exacerbates the extent of misallocation. In fact, for sufficiently large λ and $\bar{\eta}_x$, the gains from trade can be even negative.

No exports subsidy Finally, suppose $\bar{\eta}_x = 0$ so that the subsidies are only on the domestic

[20] In Appendix C, I show that this intuition holds in the more general case where η follows an arbitrary distribution.

[21] The fact that only a small fraction of firms export and that export sales is a small proportion of total sales suggests that this approximation is reasonable. Note that if $f_x > f_d$, χ is at most 1.

sales. The gains from trade can be approximated to

$$GFT \approx \frac{1}{m_d n_d}\left(\frac{1}{\theta}(1-\lambda)+\lambda(1+\bar{\eta})^{\theta\tilde{\sigma}-1}\left(\frac{1}{\theta}+\tilde{\sigma}\bar{\eta}_d\right)\right).$$

The effect of subsidy is more nuanced in this case. On one hand, trade liberalization has additional benefit of dampening the distortion created by domestic subsidies. As firms charge the same markup for their exported goods, greater share of export brings the resource allocation closer to the optimal. Hence, larger initial distortion can increase the gains from trade liberalization. On the other hand, higher domestic subsidy relative to export subsidy dampens the effect of trade cost reduction again by promoting inefficient allocation of resources between domestic and export productions.

To illustrate the second point more clearly, suppose $\lambda = 1$. [Graph over lambda?] As discussed earlier, there is no welfare loss from the domestic misallocation term in this case. Hence, the allocation-improving effect of trade is nullified. As such, the gains from trade liberalization, which simplifies to

$$(1+\bar{\eta}_x)^{-\theta\tilde{\sigma}}\left(\frac{1}{\theta}+\tilde{\sigma}\bar{\eta}_x\right),$$

is unambiguously decreasing in $\bar{\eta}_x$. However, it can be shown that for $\lambda \in (0,1)$, the gains from trade can be either increasing or decreasing in $\bar{\eta}_x$.

In summary, the model illustrates how the nature of misallocation interacts with trade liberalization in a complex manner. The joint distribution between productivity and the subsidy rate affects the gains from trade liberalization. More precisely, the welfare loss due to misallocation depends on the ex-post joint distribution which can change in response to trade shocks. The gains from trade liberalization can be therefore larger or smaller in the presence of distortionary policies.

The cases when $\bar{\eta}_d \neq \bar{\eta}_x$ show policies that discriminate exporters create additional margin of distortion. Such policies divert the resources away from the optimal allocation between domestically sold and exported goods. Hence, even when the state-owned firms do not receive subsidies on export sales, the between-firm allocation improvement can be outweighed by the within-firm misallocation.

6 Quantifying the gains from trade

In the previous section, I have shown that the effect of misallocation on the gains from trade is ambiguous. It is therefore unclear whether it is important to capture the misallocation if the goal is to estimate the gains from trade using the model. In this section, I show that for the Chinese manufacturing sector, accounting for the misallocation leads to roughly 45% lower estimate of gains from trade. To reach this conclusion, I estimate the gains from trade using the same firm-level data, once using the standard model and then again using the misallocation model.

As illustrated in the previous section, the distribution of the firm primitives has an important implication on the gains from trade. Therefore, I do not impose functional form assumptions on the distribution, departing from the previous works in the literature. Below, I discuss some of the challenges associated with nonparametric estimation and formally state the assumptions I make to overcome the challenges. In essence, the assumptions restrict the parameter space over which the counterfactual outcome can be estimated. Then I describe in detail the estimation procedure first for the standard model and then for the misallocation model, before turning to the estimation and counterfactual results.

6.1 Challenges of nonparametric estimation

This section discusses the assumptions I make for the estimation exercises.

First, I assume there are two countries, China and the rest of the world (ROW), as the survey data includes only the aggregate export value. To simplify the notation, I abbreviate China to d and the ROW to x for country-level variables. For bilateral variables, I abbreviate (China, China) with d and (China, ROW) with x. I normalize $\tau_d = 1$ and set the Chinese labor as the numeraire so that $w_d = 1$.

I make additional assumptions to overcome challenges that arise from nonparametrically estimating the distribution of the firm primitives.

The first challenge is that the firm-level distribution is often observed for only one country, but the gains from trade for that country depends on the firm heterogeneity of all of its trading partner countries. When distribution in each country is assumed to take certain parametric form, the country-level parameters can be estimated from the aggregate data. A notable example is a model that assumes firm productivity follows Pareto distribution with a common shape parameter across all countries. Under this assumption, the firm-level heterogeneity in each country can be summarized by a single parameter value, which can be estimated from the bilateral trade data. Even without the strong symmetry assumption, parameterization allows model estimation without firm-level data. In contrast, nonparametrically estimating firm heterogeneity requires firm-level data from each country.

To overcome this challenge, I restrict the counterfactual trade liberalization to be "bilateral" in a particular way.

Assumption 6.1 (Bilateral liberalization)**.** *Let T denote the set of all iceberg trade cost matrix τ considered for the counterfactual exercises, including the initial economy. Then for all $\tau \in T$,*

$$\left(\frac{E_d}{E_x}\right)\left(\frac{P_d}{P_x}\right)^{\sigma-1} = 1. \tag{17}$$

There are two components in Assumption 6.1. The first component assumes the aggregate "real" expenditure $EP^{\sigma-1}$ is the same in both countries. While this assumption is strong, the actual difference in the aggregate demands is absorbed by the estimated trade cost. In other words, deviation from this assumption leads to a bias in the estimated level of trade cost, but not the gains from trade.

The second component restricts the counterfactual exercises to the ones that maintain the ratio of the aggregate real expenditures in China and ROW. In practice, I estimate the welfare in China as a function of the export iceberg cost τ_x. By Assumption 6.1, the import cost is also changing, so that $\tau_{ROW,China}$ is a function of τ_x implicitly defined by (17). In this sense, I am considering bilateral trade liberalization that enforces the percentage change in the real expenditure is the same in both countries.

The second challenge is that the observed distribution does not correspond to the primitive ex-ante distribution because the model implies low profitability firms do not produce at all. Functional form assumption on the distribution essentially allows extrapolation of the left-tail of the distribution based on the observed distribution. The issue becomes particularly trivial when productivity is assumed to follow Pareto, due to the property that left-truncated Pareto is also Pareto with the same shape parameter. The shape parameter estimated from the observed data can be then applied to the ex-ante distribution.

Theh next assumption formalizes how the firm in a model can be mapped to an observed firm in the data.

Assumption 6.2 (Data generating process)**.** *Data is generated in the following manner. A*

Chinese firm draws its idiosyncrasy $(\varphi, \eta_d, \eta_x, \rho_d, \rho_x)$ *from a joint distribution* $G(\cdot)$*, where* $\varphi > 0$*,* $\eta_j \geq -1$*, and* $\rho_j \in [0,1]$*. Firm's profitability* z_j *for each destination* $j \in \{d, x\}$ *is defined as* $z_j \equiv \varphi(1+\eta_j)^{\tilde{\sigma}}$*. Firm's observation indicator* $\mathbb{1}_j$ *is defined as a random variable that is 1 if* $z_j \geq z_j^*$ *and Bernoulli with probability* ρ_j *otherwise. Each firm's observed domestic sales, export sales, and employment* (r_d, r_x, l) *is the following function of firm's idiosyncrasy*

$$\begin{bmatrix} r_d \\ r_x \\ l \end{bmatrix} = \begin{bmatrix} A_d \mathbb{1}_d (\varphi(1+\eta_d))^{\sigma-1} \\ A_x \mathbb{1}_x (\frac{\varphi(1+\eta_x)}{\tau_x})^{\sigma-1} \\ \sum_j \mathbb{1}_j \left(f_j + B_j \left(\frac{z_j}{\tau_j} \right)^{\sigma-1} \right) \end{bmatrix}, \tag{18}$$

where $A_j \equiv E_j P_j^{\sigma-1} \tilde{\sigma}^{1-\sigma}$ *and* $B_j \equiv \tilde{\sigma}^{-1} A_j$ *are common across firms. Firm is unobserved if* $\mathbb{1}_d = \mathbb{1}_x = 0$.

The gist of Assumption 6.2 is that a firm is observed if its profitability is above the cutoff and may or may not be observed if below the cutoff. This generalizes a more typical assumption that a firm is observed if and *only if* the profitability is above the cutoff, which is the special case of Assumption 6.2 where $\rho_j = 0$.

The main reason to allow this more relaxed assumption on observability is that without it, the model predicts sharp size cutoff such that the cutoff profitability is tied to the smallest observed firm. This sharp cutoff prediction almost certainly does not hold in the data – the smallest observed firm in the Chinese manufacturing survey has a single employee. The ideal way to address this issue would be to use the model that can rationalize small firms, for example by allowing heterogeneous fixed costs, to capture the equilibrium welfare effect of such firms. In practice, I cannot identify both firm-specific subsidy and fixed cost with the given data. As such, I add the noise to the extensive margin through idiosyncratic observability ρ_j.

Note that Assumption 6.2 is a statement of, rather than the solution to, the challenge of unobserved ex-ante distribution. Below the cutoff, the observed distribution still does not represent the true ex-ante distribution, and the assumption does not provide a way to correct the bias since ρ_j distribution is unknown. The solution is rather that I limit the counterfactual scenarios to the ones that increase the cutoff, namely the ones with lower trade costs.

6.2 Estimating the misallocation model

I turn to the estimation of gains from trade under the misallocation model. The general strategy is similar to the standard model, and I leave the detail to Appendix D. In this section, I highlight the main differences from the standard model estimation.

Unlike in the standard model, firm profitability in the misallocation model is not log-linear in firm revenue. Instead, profitability z_j is log-linear in firm's variable labor $v_j \equiv l_j - f_j$. As I only observe the total employment $l = l_d + l_x$, this presents two problems. First, I need to split the employment into domestic and export components. Second, I need to estimate the fixed cost to infer the variable labor.

To address the first problem, I use the following equality implied by the model:

$$\frac{v_x}{v_d} = \tau_x \left(\frac{r_x}{r_d}\right)^{\frac{\sigma}{\sigma-1}}.$$

This allows inferring the variable labor ratio from the revenue ratio given the values of σ and τ_x. I take $\sigma = 3$ as given, following the literature. Estimating τ_x from the average export intensity is now difficult as it is confounded by the average level of $\frac{1+\eta_x}{1+\eta_d}$. As such, I take the estimate of τ_x from the standard model as given, implicitly assuming the mean of $\log(1+\eta_x)$ is the same as the mean of $\log(1+\eta_d)$.

To address the second problem of estimating fixed costs, I employ a type of guess-and-verify method. Given the value of fixed costs (f_d, f_x), I can infer the distribution of the variable labors (v_d, v_x) and estimate the equilibrium cutoffs (z_d^*, z_x^*) by matching $\log \mathcal{M}_2 - \log \mathcal{M}_1$ and the zero profit condition. This allows to choose f_x that matches the observed ratio of exporting firms to non-exporting firms for a given value of f_d. Finally, the model predicts $f_d = \frac{v_d(z_d^*)}{\sigma-1}$, so given the estimated cutoffs and σ, I get a new fixed cost values f_d'. I choose f_d such that the distance between f_d and corresponding f_d' is minimized.

Welfare estimation further requires estimating the distorted ratio functions (K_d, K_x) defined in (13). Note that while the variable labor is proportional to the profitability, firm's revenue is proportional to $(\varphi(1+\eta_j))^{\sigma-1}$. Therefore, $K_j(z_j^*)$ function can be estimated by taking the mean of revenue r_j among the firms whose variable labor is above the cutoff $v_j(z_j^*)$.

The estimated value of the relative export overhead cost is $\frac{f_x}{f_d} = 0.41$. When estimating the standard model, the relative scarcity of exporters together with the relative scarcity of large firms requires high f_x. In contrast, the misallocation model allows separate profitability distribution among exporters and non-exporters, so the relatively large share of small exporters leads to a lower estimated value of f_x given the variable export cost is assumed to be the same as in the standard model.[22]

6.3 Results

Figure 5 shows the main results by plotting the gains from trade estimated under the standard model, shown with the solid line, and under the misallocation model, shown with the dashed line. The x-axis shows the value of $(\sigma-1)\log\tau_x$ starting from the estimated level and decreasing to zero, at which point there is no iceberg cost ($\tau_x = 1.0$). Note that by Assumption 6.1, there is an implicit change in the import cost that balances the real aggregate expenditures. The y-axis shows the percentage change in the welfare, measured as the real consumption, relative to the estimated level at the observation. Denoting the real consumption inferred from the observation as Q and the counterfactual real consumption as Q', the value equals to $100 \times \left(\frac{Q'}{Q} - 1\right)$.

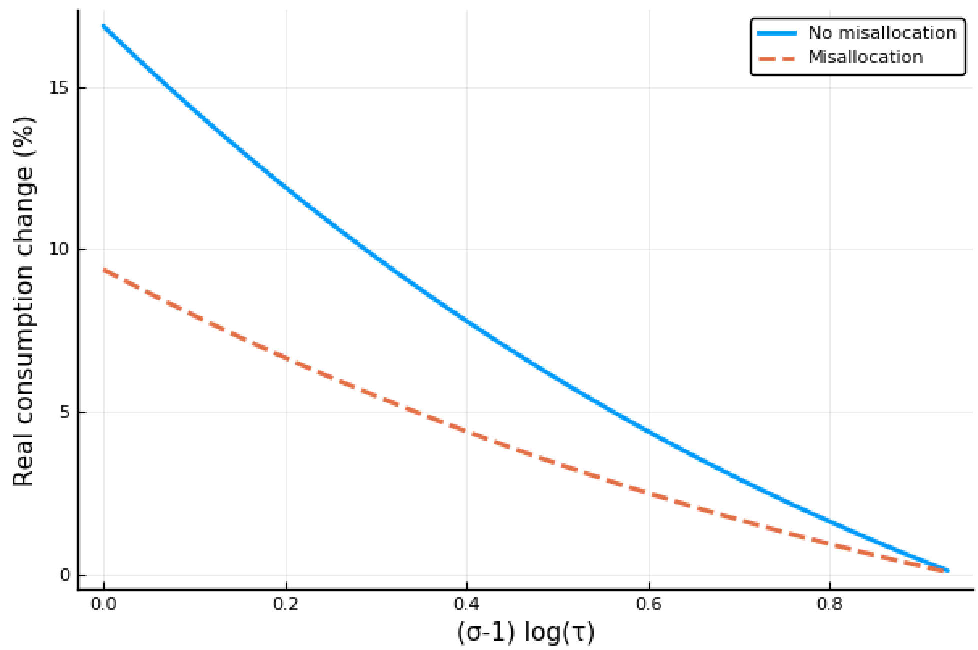
The figure shows that the estimated size of the gains from trade under the misallocation model is roughly 55-57% of the gains estimated under the standard model. Under the standard model, removing the iceberg cost leads to 17% estimated welfare gains while under the misallocation model, the same exercise leads to 9.4% estimated gains. The quantitatively large gap shows that accounting for the firm-level misallocation in the Chinese manufacturing sector matters not only for explaining the observed patterns but also for estimating the gains from trade.

There are two reasons why the estimates differ. First, the models have a different mapping from the observables to the model parameters, leading to different parameter estimations given the same data. Second, the models have different predictions on the gains from trade given the parameters. As both potentially matter, one cannot conclude from Figure 5 that trade liberalization leads to an allocative efficiency loss and therefore lower welfare gain.

To explore the impact of trade liberalization on allocative efficiency, I take the estimated misallocation model as given and consider how the extent of misallocation varies with trade cost. The extent of misallocation is measured by the welfare gains that occur from removing firm-specific subsidies. I consider two scenarios: removing all the subsidies so that the resulting allocation is socially optimal and removing export-contingent subsidies by setting each firm's export-specific subsidy rate $1+\eta_x$ to equal the domestic subsidy rate $1+\eta_d$.

[22] Alternatively, small exporters could have been explained through high relative export subsidy, which would lead to a lower estimate of τ_x and a higher estimate of f_x to match the moments. This explanation would imply a greater extent of across-destination misallocation.

Figure 5: Estimated gains from trade liberalization

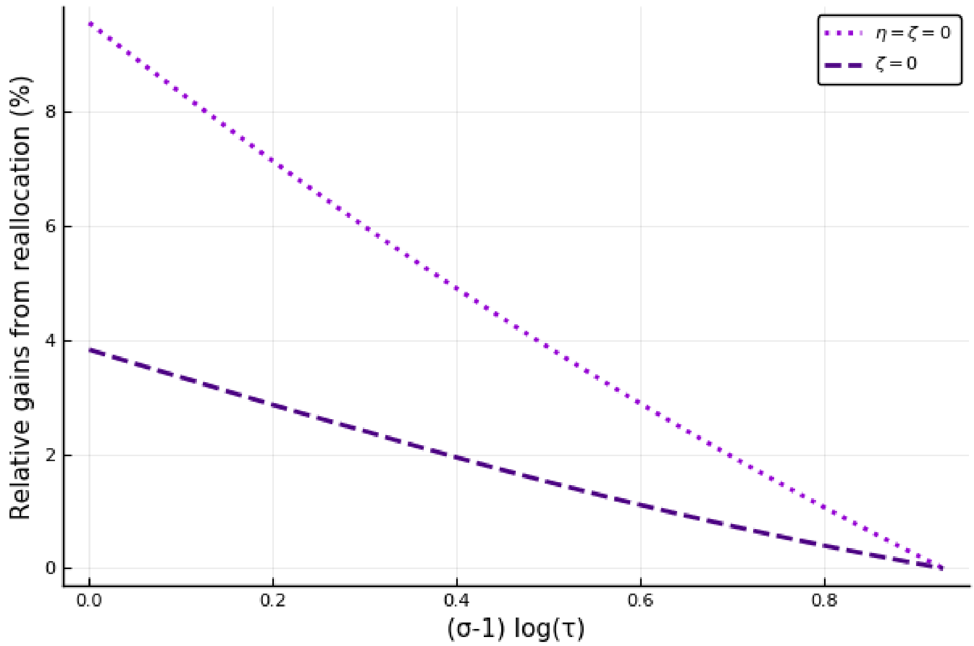


Note: The figure shows the estimated percentage change in Chinese welfare relative to the observed level, $100 \times (\frac{Q'}{Q} - 1)$. The x-axis is reversed scaled log export variable trade cost $(\sigma - 1)\tau_x$. The counterfactual involves implicit change in import cost as described in Assumption 6.1. The solid line shows the welfare change estimated under the standard model, while the dashed line shows the welfare change estimated under the misallocation model.

Performing this exercise requires separating the profitability into productivity and subsidy. To do so, I use the weighted ratio of revenue and variable cost to isolate the productivity for each firm, as described in Appendix D. Because the productivity can be identified only up to scalar, I cannot calculate the level of gains from reallocation. However, I can measure the counterfactual change in the reallocation gain to examine the impact of trade liberalization on the extent of misallocation.

Figure 6 shows the results of this exercise. The solid line shows relative gains from optimal reallocation, while the dashed line shows the gains from removing across-destination misallocation. The relative gains increase with trade liberalization, indicating that trade liberalization increases the cost of misallocation. The fact that the across-destination misallocation becomes more costly under lower trade cost is expected from the theory. The fact that the cost of total misallocation increases more than the cost of across-destination misallocation in trade liberalization suggests that the cost of across-firm misallocation is also becoming larger with globalization.

Figure 6: Relative gains from reallocation



Note: The figure shows the estimated gains from reallocation relative to the observed level at each trade cost level lower than the observed. The solid line shows the relative gains from removing all subsidies, while the dashed lien shows the relative gains from removing export-contingent subsidies.

The large share of high-export intensity exporters likely plays an important role in diminishing the gains from trade. These firms contribute to the high variance of export intensity, which translates to large across-destination misallocation. Furthermore, the fact that the high-intensity exporters tend to be smaller suggests that firms that expand from increased export opportunities tend to be less productive, exacerbating across-firm misallocation.

These results illustrate the importance of accounting for firm-level misallocation in China to accurately estimate the gains from trade. My estimation suggests that the estimated size of the gains from trade is nearly half as large when estimated under the misallocation model, and the welfare cost of misallocation becomes larger as trade cost falls. Discriminatory policies and the influence of political connections on firm performance prevent China from fully reaping the benefits of globalization.

7 Conclusion

Standard trade models explain the observed firm heterogeneity with productivity differences. While productivity heterogeneity together with increasing returns to scale successfully explains the within-sector export performance heterogeneity and exporter premia, it cannot fully explain the patterns observed among Chinese manufacturing firms. I propose state connections and policies as an important additional source of firm heterogeneity and show that accounting for state-generated misallocation can explain the patterns that the standard model cannot. Furthermore, doing so matters for estimating the gains from trade. Using a nonparametric estimation strategy, I find that the estimated gains from trade is about 45% lower under the misallocation model.

This paper focused on misallocation as a source of heterogeneity for China based on the systematic difference between private and state-owned firms, explicit policies that discriminate firms based on ownership, export performance, and location, and the anecdotal evidence that links political connections with firm's success. More broadly, however, the paper illustrates that understanding firm-level heterogeneity through microdata can have important implications for researchers who wish to estimate the gains from trade and understand the effects of trade liberalization.

Appendices

A Predictions under alternative models

In this appendix section, I show that the three predictions of the standard model considered in 2 hold under alternative models as well. First, I consider the model of Arkolakis, 2010 in which firms endogenously choose the fixed cost rationalized as a marketing cost. Second, I consider the model of Melitz and Ottaviano, 2008 in which firms endogenously choose different markups due to non-CES demand.

A.1 Endogenous fixed cost

Consider the model of Arkolakis, 2010, which modifies the standard model in the following ways. The fixed cost of market entry from i to j is no longer constant at $w_i f_{ij}$ for all firms in country i. Instead, each firm can choose to reach fraction n of consumers in each market. The associated cost is

$$f_{ij}(n) = \frac{L^{\alpha}}{\psi} \frac{1-(1-n)^{1-\beta}}{1-\beta},$$

where $\beta \geq 0$ determines the diminishing returns to advertisement and α governs the increasing returns to scale with respect to the market size. The optimal market j penetration for firm in country i with productivity φ is

$$n_{ij}(\varphi) = \max\left\{1 - \left(\frac{\varphi_{ij}}{\varphi}\right)^{\frac{\sigma-1}{\beta}}, 0\right\}.$$

It follows the destination-specific revenue productivity is given by

$$\log\left(\frac{r_{ij}(\varphi)}{l_{ij}(\varphi)}\right) = \log(\tilde{\sigma}) + \log\left(1 - \frac{f_{ij}(\varphi)}{c_{ij}(\varphi)}\right),$$

where

$$f_{ij}(\varphi) = \frac{f_{ij}}{1-\beta}\left(1 - \left(\frac{\varphi_{ij}}{\varphi}\right)^{\frac{\sigma-1}{\tilde{\beta}}}\right), \quad \tilde{\beta} \equiv \frac{\beta}{1-\beta}$$

is the cost of optimal amount of advertisement a firm of productivity φ chooses, and

$$c_{ij}(\varphi) = v_{ij}\left(1 - \left(\frac{\varphi_{ij}}{\varphi}\right)^{\frac{\sigma-1}{\beta}}\right)\left(\frac{\varphi_{ij}}{\varphi}\right)^{-(\sigma-1)} + f_{ij}(\varphi).$$

Here, $f_{ij} > 0$ and $v_{ij} > 0$ are terms common across all firms serving from i to j.[23] Define $x \equiv \left(\frac{\varphi_{ij}}{\varphi}\right)^{\sigma-1} \in (0,1]$. Note that both $f_{ij}(x)$ and $c_{ij}(x)$ are decreasing in x, so that they are increasing in φ.[24]

Prediction 1 More productive and larger firms also choose to spend more resources on marketing. Despite this, the advertising cost *share* still decreases in productivity and hence decreases in firm size. In other words, $\frac{f_{ij}(\varphi)}{c_{ij}(\varphi)}$ is decreasing in φ, or equivalently,

$$h(x) \equiv \frac{f_{ij}}{v_{ij}}(1-\beta)^{-1}\left(\frac{1-x^{\frac{1-\beta}{\beta}}}{1-x^{\frac{1}{\beta}}}\right)x$$

is increasing in x. Note that the derivative of $h(x)$ is

$$h'(x) = \frac{A}{1-\beta}\left(\beta x(1-x^{\frac{1}{\beta}}) - x^{\frac{1}{\beta}}(1-x)\right)$$

where $A \equiv \frac{f_{ij}}{v_{ij}}\left(\beta x\left((1-x^{\frac{1}{\beta}}\right)^2\right)^{-1} > 0$ for all $x > 0$. Suppose $\beta > 1$. Then $h'(x) > 0$ over $x \in (0,1)$ if and only if

$$\frac{1-x}{x} > \frac{1-x^b}{bx^b}, \quad b \equiv \frac{1}{\beta}.$$

To show this inequality holds, consider a function $\tilde{h}(x) = \frac{\frac{1-x}{x}}{\frac{1-x^b}{x^b}}$ for $x \neq 1$ and $\tilde{h}(1) = \frac{1}{b}$. Then $\tilde{h}(x)$ can be shown to be continuous and increasing for $b \in (0,1)$ over $x > 0$.[25] Hence $\tilde{h}(x) < \tilde{h}(1)$ for $x \in (0,1)$ and the inequality follows. The case for $\beta < 1$ can be shown analogously.

Prediction 2 Since revenue productivity is function of φ, the revenue productivity variance is zero conditional on true productivity. As argued, the cost $c_{ij}(\varphi)$ associated with sales to destination j is monotonically increasing in φ. Hence, among non-exporting firms, firm size $c_i(\varphi) = c_{ii}(\varphi)$ captures the firm's productivity. Therefore, revenue productivity variance is zero conditional on firm size among non-exporting firms.

[23] Following Arkolakis, 2010, $f_{ij} \equiv \frac{w_j^{\gamma} w_i^{1-\gamma} L_j^{\alpha}}{\psi}$ where γ is the share of destination country labor in marketing cost and ψ is the amount of advertisement a unit bundle of labor produces, and $v_{ij} = \frac{E_j}{\tilde{\sigma}}\left(\frac{\tilde{\sigma} w_i \tau_{ij} \varphi_{ij}}{P_j}\right)^{1-\sigma}$.

[24] $f'_{ij}(x) = -\frac{f_{ij}}{\beta} x^{\frac{1}{\beta}-1} < 0$, $c'_{ij}(x) = -\frac{v_{ij}}{x^2}\left(\beta^{-1} x^{\frac{1}{\beta}} + \left(1 - x^{\frac{1}{\beta}}\right)\right) + f'_{ij}(x) < 0$.

[25] Note the ratio of the derivatives in $\tilde{h}(x)$ is $\frac{1}{b} x^{1-b}$, which is increasing for $b \in (0,1)$. It follows from l'Hôpital's Monotone Rule that $\tilde{h}(x)$ is increasing in x.

Prediction 3 Finally, the export intensity of a firm in this model can be expressed as

$$\frac{\sum_{j\neq i} r_{ij}(\varphi)}{r_{ii}(\varphi)} = \sum_{j\neq i} \mathbb{1}(\varphi \geq \varphi_{ij}) \left(\frac{E_j P_j^{\sigma-1}}{E_i P_i^{\sigma-1}}\right) \underbrace{\left(\frac{1-\left(\frac{\varphi_{ij}}{\varphi}\right)^{\frac{\sigma-1}{\beta}}}{1-\left(\frac{\varphi_{ii}}{\varphi}\right)^{\frac{\sigma-1}{\beta}}}\right)}_{\equiv k_{ij}(\varphi)} \tau_{ij}^{1-\sigma}.$$

Compared to the simple model, there is now an intensive margin variation within destination across firms. The firm-specific term $k_{ij}(\varphi)$ can be shown to be increasing in φ given $\varphi_{ij} > \varphi_{ii}$. Intuitively, more productive firms are willing to incur larger advertisement cost to capture larger share of the market. Diminishing returns to advertisement implies that as firm becomes more productive, it captures relatively larger share of the foreign market, where it has relatively smaller market penetration, than in the home market. This generates increasing export share on the intensive margin.[26]

A.2 Endogenous markup

Consider the model of Melitz and Ottaviano, 2008. In this model, the consumer utility is such that the inverse demand for a variety is linear:

$$L_j p_j(q) = \alpha - \gamma q - \eta Q_j \quad \text{if } q > 0,$$

where L_j is the mass of consumers, $Q_j \equiv \int_{\Omega_j} q_j(\omega)\, d\omega$ is the aggregate consumption, and $\alpha, \gamma, \eta > 0$. Let $\bar{p}_j \equiv \frac{\alpha-\eta Q_j}{L_j}$ denote the price threshold above which the quantity demanded is zero. It is determined endogenously at the equilibrium but taken as given by each firm.

Firms still differ in productivity φ, but there is no fixed overhead cost. Profit maximization implies firm's optimal quantity and price associated with destination j satisfy

$$q_{ij}(\varphi) = \frac{L_j}{\gamma}\left(p_{ij}(\varphi) - \frac{w_i \tau_{ij}}{\varphi}\right),$$

where w_i is the wage in country i and τ_{ij} is the iceberg trade cost. Despite the absence of fixed overhead costs, the selection occurs through the threshold price $\bar{p}_j$. Let φ_{ij} denote the cutoff productivity such that $p_{ij}(\varphi_{ij}) = \bar{p}_j$. Then $\bar{p}_j = \frac{w_i \tau_{ij}}{\varphi_{ij}}$, and firm's revenues and costs

[26] [Chen and Sun 2017] extend the model by allowing the returns-to-advertisement parameter β to vary across countries and show that if $\beta_j < \beta_i$, then $k_{ij}(\varphi)$ is initially increasing but decreasing after some threshold value $\varphi^* > \varphi_{ij}$. Even in this scenario, the export intensity would have inverse-U relationship with firm size, contrary to the observed monotonic pattern.

can be expressed as

$$r_{ij}(\varphi) = \frac{L_j}{4\gamma} w_i \tau_{ij} \left(\varphi_{ij}^{-2} - \varphi^{-2} \right), \quad c_{ij}(\varphi) = \frac{L_j}{2\gamma} (w_i \tau_{ij})^2 \varphi^{-1} \left(\varphi_{ij}^{-1} - \varphi^{-1} \right).$$

Prediction 1 In the absence of fixed cost, the revenue productivity is equal to the (relative) markup. With linear demand, however, the demand elasticity is no longer constant, so the markup depends on the firm's productivity. Revenue productivity associated with domestic sales can be expressed as

$$\log\left(\frac{r_{ij}(\varphi)}{l_{ij}(\varphi)}\right) = -\log\left(2\tau_{ij}\right) + \log\left(\frac{\varphi}{\varphi_{ij}} + 1\right).$$

Clearly, revenue productivity is increasing in productivity φ, so that more productive firms exhibit higher revenue productivity. Due to the linearity of demand, more productive firms are able to charge higher markup.

In this model, firm's total cost is quadratic. Therefore, the relationship between average revenue productivity and total cost depends on the distribution of φ. However, firm's output and revenue is monotonically increasing in firm's productivity. Since both revenue productivity and revenue are monotonically increasing in productivity, they have positive relationship.

Prediction 2 Since revenue productivity is function of φ, the revenue productivity variance is zero conditional on productivity. Among the firms that have positive sales ($\varphi \geq \varphi_{ii}$), domestic sales $r_{ii}(\varphi)$ has one-to-one relationship with φ. Hence, among non-exporting firms, firm size measured in revenue $r_i(\varphi) = r_{ii}(\varphi)$ captures the firm's true productivity. Therefore, revenue productivity variance is zero conditional on firm revenue among non-exporting firms.

Prediction 3 Firm's export intensity is

$$\frac{\sum_{j\neq i} r_{ij}(\varphi)}{r_{ii}(\varphi)} = \sum_{j\neq i} \mathbb{1}(\varphi \geq \varphi_{ij}) \left(\frac{L_j}{L_i}\right) \underbrace{\left(\frac{\varphi_{ij}^{-2} - \varphi^{-2}}{\varphi_{ii}^{-2} - \varphi^{-2}}\right)}_{\equiv h_{ij}(\varphi)} \tau_{ij}.$$

The model features intensive margin variation within destination across firms, shown through the term $h_{ij}(\varphi)$. The sign of $h'_{ij}(\varphi)$ equals to the sign of $\varphi_{ii}^{-2} - \varphi_{ij}^{-2}$, which is positive as long as $\varphi_{ij} > \varphi_{ii}$. The inequality holds theoretically as $\varphi_{ij} = \tau_{ij}\varphi_{ii}$, and also empirically given the rarity of exporters. Hence, the model predicts that the export intensity is increasing in firm size, measured in terms of revenue, on both intensive and extensive margins.

B Exogenous fixed costs

I consider a model in which a firm draws idiosyncratic fixed overhead costs along with productivity and show that the production cost share is bounded.

Upon entry, a firm draws both the productivity φ and destination-specific fixed costs $f = (f_1, \ldots, f_N)$ from a joint distribution $G_i(\varphi, f_1, \ldots, f_N)$. The profit of a firm in country i earned from sales to destination j is

$$\pi_{ij}(q; \varphi, f) = p_j(q)q - w_i l_{ij}(q; \varphi, f)$$

where

$$l_{ij}(q; \varphi, f) = f_j + \frac{\tau_{ij} q}{\varphi}.$$

Revenue, cost, and profit at the critical point satisfy

$$\begin{aligned}
r_{ij}(\varphi, f) &= E_j \left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j} \right)^{1-\sigma} \varphi^{\sigma-1}, \\
c_{ij}(\varphi, f) &= w_i f_j + \tilde{\sigma}^{-1} r_{ij}(\varphi, f), \\
\pi_{ij}(\varphi, f) &= \frac{r_{ij}(\varphi, f)}{\sigma} - w_i f_j.
\end{aligned}$$

Define $\zeta_j \equiv \varphi f_j^{-\frac{1}{\sigma-1}}$. Then firm's revenue to overhead cost ratio $\frac{r_{ij}(\varphi, f)}{f_j}$ and profit to overhead cost ratio $\frac{\pi_{ij}(\varphi, f)}{f_j}$ are functions of ζ_j. Hence there exists some value $\zeta_{ij} > 0$ such that firms in i make non-negative profit from sales to j if and only if $\zeta_j \geq \zeta_{ij}$. Furthermore, firm's revenue to overhead cost ratio can be expressed as

$$\frac{r_{ij}(\varphi, f)}{f_j} = \sigma w_i \left(\frac{\zeta_j}{\zeta_{ij}} \right)^{\sigma-1}.$$

Using the expressions above, the log revenue productivity can be written as

$$\log\left(\frac{r_{ij}(\varphi, f)}{l_{ij}(\varphi, f)} \right) = \log(\tilde{\sigma}) + \log\left(1 - \frac{f_j}{l_{ij}(\varphi, f)} \right).$$

where
$$\frac{f_j}{l_{ij}(\varphi, f)} = \frac{1}{1+(\sigma-1)\left(\frac{\zeta_j}{\zeta_{ij}}\right)^{\sigma-1}}, \quad l_{ij}(\varphi, f) = f_j\left(1+(\sigma-1)\left(\frac{\zeta_j}{\zeta_{ij}}\right)^{\sigma-1}\right).$$

Conditional on non-negative profit, $\zeta_j \geq \zeta_{ij}$, the following inequalities hold:

$$0 \leq \log\left(\frac{r_{ij}(\varphi, f)}{l_{ij}(\varphi, f)}\right) \leq \log\tilde{\sigma}.$$

C Derivation of equilibrium conditions

This appendix section provides the derivations of the equilibrium conditions discussed in Section 5.1.

The profit of a firm in country i earned from sales to destination j is

$$\pi_{ij}(q;\varphi,\eta) = (1+\eta_j)p_j(q)q - w_i l_{ij}(q;\varphi)$$

where $l_{ij}(q;\varphi) = f_{ij} + \frac{\tau_{ij}q}{\varphi}$. Firm's price, pre-subsidy revenue, post-subsidy revenue, cost, and profit at the critical point are

$$\begin{aligned}
p_{ij}(\varphi,\eta) &= \frac{\tilde{\sigma} w_i \tau_{ij}}{\varphi(1+\eta_j)}, \\
r_{ij}(\varphi,\eta) &= E_j\left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j}\right)^{1-\sigma} \varphi^{\sigma-1}(1+\eta_j)^{\sigma-1}, \\
\tilde{r}_{ij}(\varphi,\eta) &= (1+\eta_j) r_{ij}(\varphi,\eta), \\
c_{ij}(\varphi,\eta) &= w_i f_{ij} + \tilde{\sigma}^{-1}\tilde{r}_{ij}(\varphi,\eta), \\
\pi_{ij}(\varphi,\eta) &= \sigma^{-1}\tilde{r}_{ij}(\varphi,\eta) - w_i f_{ij}.
\end{aligned}$$

Define $z_j \equiv \varphi\eta_j^{\tilde{\sigma}}$. Then firm's post-subsidy revenue $\tilde{r}_{ij}$ and profit π_{ij} are continuous and increasing functions of z_j:

$$\begin{aligned}
\tilde{r}_{ij}(z_j) &= E_j\left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j}\right)^{1-\sigma} z_j^{\sigma-1}, \\
\pi_{ij}(z_j) &= \left(\frac{E_j}{\sigma}\right)\left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j}\right)^{1-\sigma} z_j^{\sigma-1} - w_i f_{ij}.
\end{aligned}$$

Hence there exists some value $z_{ij} > 0$ such that firms in i make non-negative profit from sales to j if and only if $z_j \geq z_{ij}$. By continuity, the cutoff value satisfies $\pi_{ij}(z_{ij}) = 0$. Expanding the equality yields the zero cutoff profit condition

$$z_{ij} = \left(\frac{\sigma w_i f_{ij}}{E_j}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tilde{\sigma} w_i \tau_{ij}}{P_j}\right).$$

The free entry condition requires ex-ante expected profit in country i is equal to the entry cost $w_i f_i^E$. The post-subsidy revenue earned by the zero-profit firms is $\tilde{r}_{ij}(z_{ij}) = \sigma w_i f_{ij}$, so the post-subsidy revenue and profit can be expressed in terms of the cutoff as follows:

$$\tilde{r}_{ij}(z_j) = \left(\frac{z_j}{z_{ij}}\right)^{\sigma-1} \sigma w_i f_{ij},$$
$$\pi_{ij}(z_j) = w_i f_{ij} \left(\left(\frac{z_j}{z_{ij}}\right)^{\sigma-1} - 1\right).$$

The ex-ante expected post-subsidy revenue from sales to destination j is

$$\begin{aligned} \mathrm{E}[\tilde{r}_{ij}(z_j)] &= \int_{z_{ij}}^{\infty} \tilde{r}_{ij}(z_j)\, dG_{ij}(z_j) \\ &= \sigma w_i f_{ij} \underbrace{\left(\int_{z_{ij}}^{\infty} \left(\frac{z_j}{z_{ij}}\right)^{\sigma-1} \frac{dG_{ij}(z_j)}{1 - G_{ij}(z_{ij})}\right)}_{\equiv H_{ij}(z_{ij})} \underbrace{\big(1 - G_{ij}(z_{ij})\big)}_{\equiv S_{ij}(z_{ij})}, \end{aligned}$$

where $G_{ij}(z_j)$ is the (derived) distribution of z_j in country i. The expected profit from sales to destination j is therefore

$$\mathrm{E}[\pi_{ij}(z_j)] = \sigma^{-1}\mathrm{E}[\tilde{r}_{ij}(z_j)] - w_i f_{ij} S_{ij}(z_{ij}) = w_i f_{ij} \big(H_{ij}(z_{ij}) - 1\big) S_{ij}(z_{ij}).$$

Summing over the destinations and equating to the entry cost $w_i f_i^E$ yields the free entry condition:

$$f_i^E = \sum_j f_{ij} \big(H_{ij}(z_{ij}) - 1\big) S_{ij}(z_{ij}).$$

The labor market clearing condition requires total labor used by firms in country i equals to the exogenous supply, L_i. The ex-ante expected labor cost is

$$\mathrm{E}[c_{ij}(\varphi,\eta)] = w_i f_{ij} S_{ij}(z_{ij}) + \tilde{\sigma}^{1-\sigma}\mathrm{E}[\tilde{r}_{ij}(\varphi,\eta)] = w_i f_{ij} S_{ij}(z_{ij}) \big(1 + (\sigma-1) H_{ij}(z_{ij})\big).$$

Total labor used for production in country i is therefore

$$L_i^p = M_i w_i^{-1} \sum_j \mathrm{E}[c_{ij}(\varphi,\eta)] = M_i \sum_j f_{ij} \left(1+(\sigma-1)H_{ij}(z_{ij})\right) S_{ij}(z_{ij}),$$

where M_i is the mass of total entrants. There is further labor demanded for entry, which in aggregate equals to $L_i^e = M_i f_i^E = M_i \sum_j f_{ij} \left(H_{ij}(z_{ij}) - 1\right) S_{ij}(z_{ij})$ from the entry condition.

Combined, the labor market clearing condition is

$$L_i = L_i^p + L_i^e = \sigma M_i \sum_j f_{ij} H_{ij}(z_{ij}) S_{ij}(z_{ij}).$$

By the property of CES aggregation, the consumer price index P_j in country j satisfies

$$P_j^{1-\sigma} = \sum_i P_{ij}^{1-\sigma}, \quad P_{ij}^{1-\sigma} = M_i \iint p_{ij}(\varphi,\eta)^{1-\sigma} \mathbb{1}(z_j \geq z_{ij})\, dG_i(\varphi,\eta).$$

Given the profit maximizing price,

$$\begin{aligned} P_{ij}^{1-\sigma} &= M_i \left(\tilde{\sigma} w_i \tau_{ij}\right)^{1-\sigma} \iint \left(\varphi(1+\eta_j)\right)^{\sigma-1} \mathbb{1}(z_j \geq z_{ij})\, dG_i(\varphi,\eta) \\ &= M_i \left(\frac{\tilde{\sigma} w_i \tau_{ij}}{z_{ij}}\right)^{1-\sigma} \underbrace{\left(\iint \left(\frac{\varphi(1+\eta_j)}{z_{ij}}\right)^{\sigma-1} \mathbb{1}(z_j \geq z_{ij}) \frac{dG_i(\varphi,\eta)}{S_{ij}(z_{ij})}\right)}_{\equiv K_{ij}(z_{ij})} S_{ij}(z_{ij}). \end{aligned}$$

Summing over the origins yields the expression for $P_j^{1-\sigma}$.

The subsidies are financed through lump-sum tax on domestic consumers. Let T_i denote the aggregate subsidies provided to the firms in country i:

$$T_i = \sum_j T_{ij}, \quad T_{ij} = M_i \left(\mathrm{E}[\tilde{r}_{ij}(z_j)] - \mathrm{E}[r_{ij}(\varphi,\eta_j)]\right).$$

The free entry condition implies that the aggregate post-subsidy revenue earned by firms in country i equals to the aggregate labor compensation $w_i L_i$:

$$w_i L_i = M_i \sum_j \mathrm{E}[\tilde{r}_{ij}(z_j)].$$

The aggregate expenditure of country i consumers is therefore equal to the aggregate pre-subsidy revenue:

$$E_i = w_i L_i - T_i = M_i \sum_j \mathrm{E}[r_{ij}(\varphi,\eta)] = \sigma w_i M_i \sum_j f_{ij} K_{ij}(z_{ij}) S_{ij}(z_{ij}).$$

D Gains from trade under simplified setting

This appendix section derives the gains from trade expression under a specific assumption on the distribution of $(\varphi, \eta_d, \eta_x)$.

Consider an economy with two symmetric countries. The marginal distribution of φ is Pareto with shape $\theta > \sigma - 1$ and location $\varphi_m > 0$. The subsidy pair (η_d, η_x) is independent of φ and takes one of two possible values: $(\bar{\eta}_d, \bar{\eta}_x)$ with probability $\lambda \in [0,1]$ and $(0,0)$ with probability $1 - \lambda$.

Given this distributional assumption, the survival, average-to-minimum, and the distorted ratio functions can be expressed as follows:

$$\begin{aligned}
S_j(z_*) &= \Pr\big[z_j > z_*\big] \\
&= \Pr\big[\varphi(1+\eta_j)^{\tilde{\sigma}} > z_* \mid \eta_j = \bar{\eta}_j\big]\lambda + \Pr\big[\varphi(1+\eta_j)^{\tilde{\sigma}} > z_* \mid \eta_j = 0\big](1-\lambda) \\
&= \left(\frac{\varphi_m}{z_*}\right)^{\theta} \underbrace{\Big(\lambda(1+\bar{\eta}_j)^{\theta\tilde{\sigma}} + (1-\lambda)\Big)}_{\equiv n_j}, \\
H_j(z_*) &= z_*^{-(\sigma-1)} \mathrm{E}[z_j^{\sigma-1} \mid z_j \geq z_*] \\
&= z_*^{-(\sigma-1)} \Big(\mathrm{E}[z_j^{\sigma-1} \mid z_j \geq z_* \cap \eta_j = \bar{\eta}_j] \Pr\big[\eta_j = \bar{\eta}_j\big] + \mathrm{E}[z_j^{\sigma-1} \mid z_j \geq z_* \cap \eta_j = 0] \Pr\big[\eta_j = 0\big]\Big) \\
&= \frac{\theta}{\theta - (\sigma-1)}, \\
K_j(z_*) &= z_*^{-(\sigma-1)} \mathrm{E}[\varphi^{\sigma-1} \eta_j^{\sigma-1} \mid z_j \geq z_*] \\
&= \frac{\theta}{\theta - (\sigma-1)} \underbrace{\big(\lambda(1+\bar{\eta}_j)^{-1} + (1-\lambda)\big)}_{\equiv m_j}.
\end{aligned}$$

Given the symmetry, the cutoff profitability levels can be determined from the zero cutoff condition and the free entry condition:

$$z_x = \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \tau_x z_d,$$

$$\left(\frac{\theta-(\sigma-1)}{\sigma-1}\right)\frac{f^E}{f_d}\varphi_m^{-\theta} = n_d z_d^{-\theta} + \frac{f_x}{f_d} n_x z_x^{-\theta}.$$

Solving the system of equations yields the domestic cutoff

$$z_d = \left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)^{\frac{1}{\theta}} \left(\frac{f_d}{f^E}\right)^{\frac{1}{\theta}} \varphi_m n_d^{\frac{1}{\theta}} \left(1+\chi\frac{n_x}{n_d}\right)^{\frac{1}{\theta}},$$

where $\chi \equiv \left(\frac{f_x}{f_d}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}} \tau_x^{-\theta}$.

Finally, using the welfare expression yields

$$\frac{Q}{L} = \frac{1}{\tilde{\sigma}}\left(\frac{1}{\sigma f_d}\right)^{\frac{1}{\sigma-1}} \left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)^{\frac{1}{\theta}} \left(\frac{f_d}{f^E}\right)^{\frac{1}{\theta}} \times \varphi_m \times n_d^{\frac{1}{\theta}} m_d^{\tilde{\sigma}} \times \left(1+\chi\frac{n_x m_x}{n_d m_d}\right)^{\tilde{\sigma}} \left(1+\chi\frac{n_x}{n_d}\right)^{-(\tilde{\sigma}-\frac{1}{\theta})}.$$

E Estimation steps for the misallocation model

E.1 Reducing equilibrium conditions

From the zero cutoff profit condition, the ratio of cutoffs from an origin country is

$$\left(\frac{z_x}{z_d}\right)^{\sigma-1} = \left(\frac{f_x}{f_d}\right)\tau_x^{\sigma-1}\left(\frac{E_d}{E_X}\right)\left(\frac{P_d}{P_x}\right)^{\sigma-1} = \left(\frac{f_x}{f_d}\right)\tau_x^{\sigma-1},$$

where the second equality follows from Assumption [1].

Let $\tilde{z}_j \equiv \mathbb{1}_j z_j$ from the Assumption [2] and let $\tilde{S}_j$ denote the survival function of $\tilde{z}_j$ conditional on observation. Then

$$\tilde{S}_j(z^*) = \Pr\left[\tilde{z}_j \geq z^* \mid \mathbb{1}_j = 1\right] = \frac{\Pr\left[\tilde{z}_j \geq z^* \cap \mathbb{1}_j = 1\right]}{\Pr\left[\mathbb{1}_j = 1\right]} = \frac{\Pr\left[z_j \geq z^*\right]}{\Pr\left[\mathbb{1}_j = 1\right]} \quad \forall z^* \geq z_j^*$$

where the last equality holds because $\mathbb{1}_j = 1$ with certainty when $z_j \geq z_j^*$. Letting S_j denote the (unconditional) survival function of z_j and $\mathscr{S}_j \equiv \mathrm{E}[\mathbb{1}_j]$, so that

$$S_j(z^*) = \tilde{S}_j(z^*)\mathscr{S}_j.$$

Similarly, let $\tilde{H}_j$ denote the average-to-minimum function of $\tilde{z}_j$ conditional on observation. Then for all $z^* \geq z_j^*$,

$$\tilde{H}_j(z^*) = \iint \left(\frac{\tilde{z}_j}{z^*}\right)^{\sigma-1} \mathbb{1}(\tilde{z}_j \geq z^*) \frac{d\tilde{G}_j(\tilde{z}_j \mid \mathbb{1}_j = 1)}{\tilde{S}_j(z^*)} = \iint \left(\frac{z_j}{z^*}\right)^{\sigma-1} \mathbb{1}(z_j \geq z^*) \frac{dG_j(\tilde{z}_j)}{S_j(z^*)} = H_j(z^*).$$

where $\tilde{G}_j$ denotes the distribution of $\tilde{z}_j$ conditional on observation $\mathbb{1}_j = 1$. The free entry condition can be therefore expressed as

$$\frac{f_i^E}{f_d \mathscr{S}_d} = \left(\tilde{H}_d(z_d^*) - 1\right) \tilde{S}_d(z_d^*) + \frac{f_x}{f_d} \frac{\mathscr{S}_x}{\mathscr{S}_d} \left(\tilde{H}_x(z_x^*) - 1\right) \tilde{S}_x(z_x^*).$$

E.2 Translating observables to primitives

Let $v_j(\varphi, \eta) \equiv l_j(\varphi, \eta) - f_j$ denote the labor used for production associated with sales to destination j. Then

$$\log v_j = \log B_j + (\sigma - 1)\log z_j - (\sigma - 1)\log \tau_j$$

where $B_j > 0$ is common across firms.

Suppose the values of σ, τ_x, f_d and f_x were known. Then v_d and v_x can be inferred from the data (r_d, r_x, l) using the relationship

$$\frac{v_x}{v_d} = \tau \left(\frac{r_x}{r_d}\right)^{\tilde{\sigma}}.$$

More precisely,

$$v_d = \begin{cases} l - f_d & \text{if } r_x = 0 \\ 0 & \text{if } r_d = 0 \\ \frac{l - f_d - f_x}{\tau_x \left(\frac{r_x}{r_d}\right)^{\tilde{\sigma}} + 1} & \text{if } r_x > 0 \text{ and } r_d > 0 \end{cases}, \quad v_x = \begin{cases} 0 & \text{if } r_x = 0 \\ l - f_x & \text{if } r_d = 0 \\ (l - f_d - f_x) - v_d & \text{if } r_x > 0 \text{ and } r_d > 0 \end{cases}.$$

Let g_{v_j} denote the density function of $\log v_j$ and let S_{v_j} and H_{v_j} associated survival and average-to-minimum functions. Then

$$\begin{aligned} S_{v_j}(\log v_j(z^*)) &= \Pr\left[\log v_j \geq \log v_j(z^*)\right] = S_j(z^*), \\ H_{v_j}(\log v_j(z^*)) &= \int_{\log v_j(z^*)}^{\infty} \exp\left(\log v_j - \log v_j(z^*)\right) \frac{g_{v_j}(\log v_j)}{S_{v_j}(\log v_j(z^*))} \, d\log v_j = H_j(z^*). \end{aligned}$$

The zero profit cutoff and free entry conditions can be then expressed in terms of $\log v_j$ at the cutoff profitability:

$$\log v_x(z_x^*) - \log v_d(z_d^*) = \log\left(\frac{f_x}{f_d}\right),$$

$$\frac{f^E}{f_d \mathscr{S}_d} = \left(\tilde{H}_{v_d}(\log v_d(z_d^*)) - 1\right) \tilde{S}_{v_d}(\log v_d(z_d^*)) + \frac{f_x}{f_d} \frac{\mathscr{S}_x}{\mathscr{S}_d} \left(\tilde{H}_{v_x}(\log v_x(z_x^*)) - 1\right) \tilde{S}_{v_x}(\log v_x(z_x^*)).$$

For the zero cutoff profit condition, I used $B_d = B_x$ from Assumption [1].

E.3 Gains from trade

The welfare expression (15) derived under the misallocation additionally depends on the distorted ratio functions K_j. The distorted ratio conditional on observation is equal to the unconditional above the equilibrium cutoff:

$$\tilde{K}_j(z^*) = K_j(z^*) \quad \forall z^* \geq z_j^*.$$

Consider a function $\tilde{K}_{v_j}$ that takes a threshold production labor size and evaluates the mean of the sales to conditional on the production labor size being above the given threshold. Then this function is proportional to $\tilde{K}_j$ evaluated at the profitability z^* associated with the threshold $v_j(z^*)$.

$$\begin{aligned}
\tilde{K}_{v_j}(\log v_j(z^*)) &\equiv \mathrm{E}\left[\exp\left(\log r_j - \log v_j(z^*)\right) \,\middle|\, \log v_j \geq \log v_j(z^*)\right] \\
&= \tilde{\sigma} \iint \left(\frac{\varphi(1+\eta_j)}{z^*}\right)^{\sigma-1} \mathbb{1}(z_j \geq z^*) \frac{g_v(\log v_j)}{S_{v_j}(\log v_j(z^*))} \, d\log v_j \\
&= \tilde{\sigma} \tilde{K}_j(z^*).
\end{aligned}$$

Evaluating the gains from trade then requires the functions $\tilde{S}_{v_j}$, $\tilde{H}_{v_j}$, and $\tilde{K}_{v_j}$, the elasticity of substitution σ, and the log variable labors evaluated at the observed and counterfactual cutoffs $(\log v_d(z_d^*), \log v_x(z_x^*))$.

E.4 Estimating conditions

The sample equivalent of $\frac{\mathscr{S}_x}{\mathscr{S}_d}$ is the number of observed firms with $r_x > 0$ over the number of observed firms with $r_d > 0$. Let $\mathscr{M}_1$ denote this ratio. If the observed ratio of exporters to non-exporters is equal to the theoretical ratio of exporters to non-exporters, then

$$\mathcal{M}_1 = \frac{\Pr\left[z_x \geq z_x^*\right]}{\Pr\left[z_d \geq z_d^*\right]}.$$

This condition is equivalent to

$$\log \tilde{S}_{v_x}(\log v_x(z_x^*)) - \log \tilde{S}_{v_d}(\log v_d(z_d^*)) = 0. \tag{19}$$

Let $\mathcal{M}_2$ denote the observed ratio of aggregate exports to aggregate domestic sales. If the observed ratio is equal to the corresponding theoretical ratio, the following condition holds:

$$\mathcal{M}_2 = \frac{\mathrm{E}[r_x(z_x)]}{\mathrm{E}[r_d(z_d)]} = \frac{f_x}{f_d}\frac{K_x(z_x^*)}{K_d(z_d^*)}\frac{S_x(z_x^*)}{S_d(z_d^*)} = \frac{f_x}{f_d}\frac{\tilde{K}_{v_x}(\log v_x(z_x^*))}{\tilde{K}_{v_d}(\log v_d(z_d^*))}\mathcal{M}_1, \tag{20}$$

where the last equality uses (19).

E.5 Estimating steps

To construct $\{(v_d, v_x)\}$ for each observed firm, it is necessary to know σ, τ_x, f_d, and f_x. Of these, I take σ and τ_x as given, using $\sigma = 3$ and $\tau_x = 1.60$.

1. **Estimating (z_d^*, z_x^*):** Given data $\{(r_d, r_x, v_d, v_x)\}$, estimate the cutoff values z_d^* and z_x^* as follows. First, estimate the kernel density of $\log v_j$ and numerical integration to estimate $\tilde{S}_{v_j}$ and $\tilde{H}_{v_j}$. Compute the sample mean of r_j conditional on $v_j \geq z_j^*$ at a grid of z_j^* and apply smoothing via LOESS to estimate $\tilde{K}_{v_j}(z_j^*)$. Solve the cutoff values by jointly solving the zero profit cutoff condition and (20).

2. **Estimating f_x:** Given a value of f_d, choose the value of f_x to minimize the absolute value of the left-hand-side of (19). To do so, use (f_d, f_x) to impute $\{(v_d, v_x)\}$ and follow Step 1 to estimate the distributional functions and the cutoffs.

3. **Estimating f_d:** Choose f_d to minimize the gap between f_d and $\frac{z_d^*}{\sigma-1}$, where the cutoff z_d^* is computed by following Step 2 and Step 1.

4. **Estimating $\frac{f^E}{f_d \mathcal{S}_d}$:** With f_d and corresponding cutoffs estimated, estimate the adjusted entry cost from the free entry condition.

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