International Patenting with Heterogeneous Firms*

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Abstract

How do firms decide where and when to patent? This paper develops a heterogeneous firm model of trade with endogenous rival entry where innovating firms compete with rival firms on price. Patenting reduces the number of rival firms and provides higher expected profits and increased markups from reduced competition and greater appropriability. Countries with higher technology levels, more competition and better patent protection have a higher proportion of entrants who patent. Industries follow a U-shaped pattern of patenting depending on the variability of production and substitutability. Using bilateral international patent flows, the model is calibrated to obtain measures of country technology states and IP benefits. A look at the benefits of international patent treaties and its member countries highlights how nearly all countries benefit from participating in patent treaties and that patent treaties have reduced administrative fees for innovating firms by more than $7.2B in 2012. Further simulations suggest that technology gains and trade liberalization between 1996 and 2012 both contributed substantially to the rise of international patents.

Keywords: Patents, international trade, heterogeneous firms, endogenous markups, intellectual property rights, imperfect competition, patent agreements

JEL Codes: F12, F23, K11, L11, L60, O34

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1 Introduction

Most worldwide innovation is concentrated in a few countries, with the majority of global innovation increasingly taking place in the U.S., Japan, China, South Korea and Europe. Countries outside of this group rely on the international diffusion of new technologies for technological change and productivity growth (Eaton and Kortum (1996)). These technologies diffuse across international borders through multiple channels, with trade and foreign direct investment (FDI) both playing prominent roles (Eaton and Kortum 1996; Archaya and Keller 2009; Branstetter 2006). Much work has been done in assessing which national policies have most directly affected the dynamics of technology diffusion with trade policy and intellectual property rights (IPR) receiving considerable attention (Grossman and Helpman 1991; Ethier and Markusen 1996; Branstetter et al. 2006; Keller 2004; Archaya and Keller 2009; Falvey et al. 2006).

In finding ways to narrow the current technological divide and promote international technology diffusion, patents offer a promising solution. Nearly all patentable innovations undergo the patenting process (Dernis et al. (2001)). International patents are often a precondition for collaborative technology transfer. Only after a firm holds a patent right in a country is it likely to actively share proprietary technology either through joint ventures with an already established company, granting licenses for production or relocating production to that country. For example, a rigorous analysis of multinational firms and their affiliates concludes that strengthening patent protection in the affiliate country increases subsequent patent applications, R&D investment, and technology transfer (Branstetter et al. (2006)). While a developing country can choose to use a patented invention without the authorization of the inventor, many developing countries will struggle to successfully leverage the invention without the interest and cooperation of the patent holder. For such countries, the patenting decision can be an important precursor for the transfer of cutting-edge technology, which can subsequently spark the long-run diffusion of technology and knowledge.

Firms consider many different strategies when it comes to patenting, with two of the most important factors being cost and timing (Livne 2006; Schneiderman 2007). In a survey of U.S. manufacturing firms, Cohen et al. (2000) find that firms patent mainly to prevent imitation and counterfeiting, but also for reasons such as patent blocking, negotiating licensing agreements with other firms, preventing lawsuits and bullying their competitors to exit the market. More recent work by Graham and Sichelman (2008) find that start-ups patent for many of the same reasons listed above, but also for reasons such as making the firm appear more attractive for investment or acquisition. The cost of patenting can pile up very quickly, with filing fees, agent fees and translation fees bringing the total application cost to more than $10,000 per country application (Source: WIPO) in addition to the considerable legal fees appropriated each year to defend patents. In addition, there are also transaction costs, interaction costs with licensing professionals and knowledge costs of exposing ideas to potential imitators. On the other hand, the benefits of patenting give the firm additional market power, prevent counterfeiting and allows them to charge higher markups and seek higher profits over the life of the patent (Horstmann et al. 1985; Owen-Smith and Powell 2001).

While the firm has many incentives to patent, our understanding of the market conditions that attract patents is still vague. Much of the focus in the literature is focused on the impact of IPR, but other country and industry-specific conditions may also play an important role. For instance, a recent paper by Bilir (2014) finds that product life-cycle is a key determinant in multinational activity abroad, where firms with longer product life-cycles are more sensitive to intellectual property rights. This paper looks at how competition, production variability, substitutability and country technology-levels impact international patenting strategies. The paper accomplishes this by introducing a patenting decision component into a Ricardian heterogeneous firm model of trade (similar to Bernard et al. (2003)) with imperfect competition (similar to de Blas and Russ (2015)). In the model, innovating firms compete internationally with domestic rival firms on price (Bertrand competition). The number of rivals and their productivities depend on the innovating firm's own productivity so that more productive innovating firms face a greater number of more competitive rivals. This creates more incentive to patent. The model manages to maintain the producer-level facts regarding the behavior and composition of exporting firms and multi-nationals in the 'new' new trade theory, while allowing for cross-country and cross-industry differences to determine the flows of international patenting. These cross-country and cross-industry differences play a key role in deciding spatial patenting outcomes.

International patenting has important implications for development and is the topic of much debate regarding international technology transfer. The role of patent rights figured prominently in the original negotiations of the Trade-Related Aspects of Intellectual Property Rights Agreement (TRIPS) in the Uruguay Round of GATT in 1994. Throughout these negotiations developing countries expressed concern that stronger IPR would only benefit wealthy countries that had already developed strong innovation capacity. Developed countries therefore agreed to a provision to provide incentives for firms to transfer technology to developing countries and enable them to build a viable technological base (Article 66.2). Implicit in this provision is the hope that offering stronger patent protection to foreign innovators might speed up the process of technology transfer. Where firms choose to patent is therefore central in this debate.

Since the passing of the TRIPS agreement, the number of non-resident patent application filings has risen from 305,000 in 1994 to more than 800,000 in 2012 (Source: WIPO IP Stat), with the share of non-resident filings rising from 32% of total filings to 40% by 2006. In addition, the set of "core" countries that countries choose to apply to has also changed over this time period, with firms focusing less on Europe and countries outside of the periphery, and shifting their attention towards Asia, and specifically China, Russia and Japan. Figure 1 plots the probability of patenting in the top 12 jurisdictions based on the size of the patent family in 1990, 2000 and 2010. The United States remains firmly entrenched as the top destination for international patents, followed by China, Japan and the European Patent Office (EPO). The hierarchy of patent destinations has shifted dramatically, led by China's extraordinary rise as a key patenting destination.

[Figure 1 about here]

While trade-related factors are a key determinant of international patenting behavior, these trends cannot only be explained by existing trade models. One of the key contributions of this paper is to provide a specific framework to analyze the determinants of international patenting. Using a database of patent families compiled by the World Intellectual Property Organization (WIPO) and the EPO, the model backs out measures of country intellectual property (IP) benefit, which considers both the degree of protection (probability that the firm will receive monopolistic profits), reduction in competition and gain in markups for firms. This measure of IP benefit differs from the traditional measures of IPR found in Park (2008) since it considers additional factors such as the degree of competition within a country and market size. Hence, small countries with limited competition who have strong IPR measures, may have low measures of IP benefit.

The calibration of the model also provides annual measures for country technology states and industry variability. The combination of parameters allows for comparisons of patent flows based on these same parameters through policy experiments. The paper finds that the net impact of global technology gains since 1996 led to a predicted gain of 38% more patents in 2012. Similarly, the impact of a reduction in trade costs between 1996 and 2012 contributed to a nearly 43% increase in the number of global patents.

The contributions of this paper to the literature are several. The paper introduces a new framework to analyze firm patenting and trade decisions, with endogenous rival entry and endogenous markups. Empirically, this paper obtains measures of country IP benefit and technology levels using existing international patent flows. This former measure considers both the degree of protection, as well as the level of competition and market size by country. This paper also measures the effectiveness of patent agreements on patent filings, finding a significant increase in the number of patents exchanged worldwide and a reduction in administrative costs resulting from these agreements. The next section defines the model and outlines the process to calculate a numerical equilibrium. Section 3 describes the properties of the equilibrium using simulations and parameter estimates. Section 4 describes the empirical portion and constructs market-based measures of patent benefit. This is followed by the conclusion.

2 Model

The core elements of the model are taken from a heterogeneous firm trade model, with some key differences in the composition of rivals and ability of innovating firms to patent. We start by describing the innovating firm whose productivity is drawn from a boundless distribution. If/when the innovating firm competes in a market, the firm faces a number of rivals whose distribution is bounded by the innovating firm's productivity capabilities. The innovating firm competes against the low-cost rival and the markup is determined by Bertrand competition. The innovating firm can reduce the number of rivals the firm faces by paying an additional fixed cost and patenting. No closed-form equilibrium solution is possible and the model is solved analytically through numerical simulations, before parameterization.

2.1 Demand

Assume that that there are $i = 1, ..., I$ countries where each country has the ability to produce $k = 1, ..., K$ different goods or industries. Next, assume only one factor of production, labor L_i, which is perfectly mobile across industries but not countries and paid wage w_i. Each good k is comprised of an infinite number of varieties, which will be indexed by $\omega \in \Omega$.

In each country, preferences are given by a representative consumer with a two-tier utility function. The upper-tier utility function is Cobb-Douglas where the share of expenditure on varieties from industry k in country i are given by α_i^k where $0 \leq \alpha_i^k \leq 1$. The lower-tier utility function is CES with elasticity of substitution σ^k between varieties. Thus, in any country i, the total expenditure on variety ω of good k will be given by:

$$x_i^k(\omega) = \left(\frac{p_i^k(\omega)}{P_i^k}\right)^{1-\sigma^k} \alpha_i^k w_i L_i \tag{1}$$

where P_i^k is the CES price index[1]. Given these assumptions, the consumer price index in country i is given by $P_i = \prod_{k=1}^{K} \left(p_i^k\right)^{\alpha_i^k}$.

2.2 Production and Innovation

Labor is the only factor used in production and is assumed to be perfectly mobile across types and goods, but immobile across countries. Denote $z_i^k(\omega)$ to be the measure of productivity of variety ω in industry k. Next, assume that there are two types of firms in the world economy: i.) Innovating firms who pay a one-time fixed cost of innovation I_i^k that allows them to draw their productivity parameter z from an unbounded distribution and ii.) Imitating or rival firms who do not pay an entry fee but are bounded in their productivity draws by the innovating firms' productive capability.

Both types of firms draw their productivity $z_i^k(\omega)$ from the same distribution type. If we think of innovating firms as repeatedly drawing ideas from a set of existing ideas, then the best (i.e. most efficient) surviving idea takes on a Fréchet (inverted Weibull) distribution $F_i^k(z)$ with positive support (see Eaton and Kortum (2009), Chapter 4). The Fréchet distribution will be governed by two separate parameters: a country specific technology parameter T_i which will govern the mean of the distribution and an industry specific shape parameter $\theta^k > 1$ that determines the heterogeneity of efficiency levels. The distribution for the innovating firms is given by

$$F_i^k(z|T_i, \theta^k) = \Pr\left[z_i^k \leq z\right] = e^{-T_i z^{-\theta^k}} \tag{2}$$

[1]Given by $P_i^k = \left(\sum_{\omega\prime\in\Omega} p_i^k(\omega\prime)^{1-\sigma^k}\right)^{1/\left(1-\sigma^k\right)}$

A higher T_i implies higher technology and greater productivity on average, while a higher θ^k means lower variability in labor efficiencies so that producers are more homogeneous. In order to guarantee the existence of a well-defined CES price index P_j^k, assume that the elasticity of substitution $\sigma_i^k < 1 + \theta^k$.

Next, drop the superscript k and assume that the following holds for each industry type $k = 1, ..., K$. Once the innovating firm in country i draws this parameter z, the firm decides whether to pay a per-period fixed cost to enter the market and sell their good in market j, f_{ij}^*. The innovating firm can choose to either export, paying a per-period fixed cost of f_{ij}^X, along with iceberg trade costs d_{ij} and the home country wage of w_i[2]. Or the firm can relocate abroad, paying a higher per-period fixed cost f_{ij}^F and the destination country's wages as in Helpman et al. (2004). For the purposes of this model, it makes no difference which method the firm chooses to sell the good in destination j. The outcomes of the model will depend only on the cost function after the firm makes this choice, which is denoted as c_{ij}^I[3].

Given CES demand, the optimal price for the innovating firm will be to charge a CES or monopolistic markup. Without any rivals and with the exception of different productivity distributions, the equilibrium and properties of the equilibrium are similar to the results obtained in Helpman et al. (2004).

2.3 Production and Imitation

Assume that for each new variety ω in each market j, the innovating firm faces some number r_j of rivals or imitators who compete with the firm on price (Bertrand) as in Bernard et al.

[2]All trade costs are positive ($d_{ij} \geq 1$)and I assume that trade barriers obey the triangle inequality so that $d_{ij} \leq d_{in} d_{nj}$ for all i, j and n.

[3]Formally, c_{ij}^I is written as

$$c_{ij}^I = \begin{cases} \frac{w_j}{z_i^I} & z_i^I \geq \left[\frac{(\sigma)^\sigma}{(\sigma-1)^{\sigma-1}} \frac{f_{ij}^F - f_{ij}^X}{\alpha_j Y_j} \left(w_j^{1-\sigma} - (w_i d_{ij})^{1-\sigma} \right) \right]^{\frac{1}{\sigma-1}} P_j \\ \frac{w_i d_{ij}}{z_i^I} & \text{Otherwise} \end{cases}$$

Where Y_j is country-level income (equal to $w_j L_j$). Assuming $\left(\frac{w_j}{w_i}\right)^{\sigma-1} f_{ij}^F > d_{ij}^{\sigma-1} f_{ij}^X$. The firm will make the choice to either export or commit FDI using the CES/monopolistic markup.

(2003) and de Blas and Russ (2015). Unlike the previous models, however, the number of rivals and possible imitators in each country is endogenously determined by the productivity parameter of the innovating firm[4] The rival firms' marginal cost distributions will be bounded by the marginal costs of the innovating firm, so they are never more efficient at producing variety ω than the firm who invented it. Since the firms compete on price, this implies that the rival firms will never make positive profits unless the innovating firm decides to exit. However, each rival draws only one time and do not pay a fixed cost of entry or innovation cost and are able to enter and exit at any given time, thereby behaving as a 'credible threat' to the innovating firm.

Denote the rival productivity in country j as z_j^R. Each of the rivals have constant returns to scale and their marginal costs are given by $c_j^R(\omega) = \frac{w_j}{z_j^R(\omega)}$. The rivals face the same demand functions as their counterparts, so that the profit function is similar to the innovating firm's profit function, with the exception that each rival pays a per-period fixed cost f_j^R to enter into the market. Due to this per-period fixed cost, there exists a non-zero cutoff cost parameter $\tilde{c}_j^R$ that governs whether the rival has the low-cost necessary to compete and serve as a credible threat. This cutoff condition $\tilde{c}_j^R$ is determined by assuming monopolistic pricing and setting the profit equal to zero so that $\tilde{c}_j^R$ corresponds to the productivity threshold sufficient to cover the fixed per-period overhead costs.

$$\tilde{c}_j^R = \left[\frac{(\sigma)^\sigma}{(\sigma - 1)^{\sigma-1}} \frac{f_j^R}{\alpha_j Y_j} \right]^{\frac{1}{1-\sigma}} P_j \tag{3}$$

Rivals who draw a cost parameter $c \leq c_j^R$ are permanent entrants and remain as credible threats in the market for as long as the innovating firm competes. Rivals who draw $c > c_j^R$

[4]As de Blas and Russ (2015) note, Bernard et al. (2003) assumes that the number of rivals for any given product is a random variable determined by a Poisson distribution. This assumption allows the number of rivals to cancel out in the analysis (see Eaton and Kortum (2009), Chapter 4) as the primary focus is on the low-cost rival. On the other hand, de Blas and Russ (2015) assume that the number of rivals is determined solely by the free-entry condition, and is therefore unaffected by the productivity of the innovating firm. In this paper, the number of rivals is determined jointly by the zero-profit condition and innovating firm's productivity, providing a bound to both the number of rival firms and their productivities.

can never enter and are not deemed credible. Each rival draws their cost parameters c from a similar shape distribution as the innovating firm (Wiebull), but their support is truncated by the marginal costs of the innovating firm which will be denoted as c_{ij}^{I}[5]. The CDF of the rivals' cost function in country j is[6]

$$G_j^R(c|c_{ij}^I, T_j, \theta) = 1 - e^{-T_j w_j^{-\theta}\left(c^{\theta} - \left(c_{ij}^I\right)^{\theta}\right)} \tag{4}$$

A depiction of the rival and innovating firms' distributions is given below in Figure 2. In each chart, the leftmost c represents the cost parameter for the innovating firm and shows a left-truncation of the rivals' cost distribution. The rightmost c is the equivalent cost parameter for the cutoff condition for rival entry (given as the inverse of z_j^R, so that the area in between the two lines is the ex-ante probability of successful entry by the rivals in country j. The figure depicts three separate charts that are differentiated by the technology levels in the destination country. Both the state of technology and variability in productivity in each country-industry will determine the number of rivals and their productivities. In Figure 2a, the technology for the innovating firm's country is higher than the country of the rival firms. Figure 2a shows that the innovating firm from country i will not only face relatively fewer rivals, but also those rivals have lower average productivity than the innovating firm. In Figure 2c, the opposite occurs where the country of the rival firms has higher technology and the innovating firm not only faces more competition, but each competitor will have cost parameters that are closer to the innovating firm's cost parameter.

[5]Note that the definition of c_{ij}^I will vary by innovating firm type (exporting or FDI).

[6]Formula for a left-truncated Weibull distribution can be found on pages 134-135 in Rinne (2009). Note that the corresponding productivity CDF for the rivals is given by

$$F_j^R\left(z|z_i^I, T_j, \theta\right) = e^{-T_j\left(z^{-\theta} - \left(\frac{w_j}{w_i d_{ij}}\right)^{-\theta}\left(z_i^I\right)^{-\theta}\right)}$$

for exporting firms and

$$F_j^R\left(z|z_i^I, T_j, \theta\right) = e^{-T_j\left(z^{-\theta} - \left(z_i^I\right)^{-\theta}\right)}$$

for FDI firms, where z_i^I is the productivity parameter of the innovating firm.

[Figure 2 about here]

The area under the curve gives the ex-ante probability of successful entry by a rival and is determined by the following formula

$$\int_{c_{ij}^I}^{\tilde{c}_j^R} g_j^R \left(c|c_{ij}^I, T_j\theta\right) dc = 1 - e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_j^R\right)^\theta - \left(c_{ij}^I\right)^\theta\right]} \tag{5}$$

When c_{ij}^I decreases, the ex-ante probability of successful entry by the rivals increases, so that more productive innovators are more likely to face more rivals. The intuition behind is that more profitable firms will face a higher number of entrants than less profitable firms. This is supported in the literature by Luttmer (2007) who claims that there are stronger incentives for entry by imitators when the incumbent firm is larger and more profitable. This is also supported in Costinot et al. (2012) who model 'follower' firms in a similar manner where they do not exceed the productive capabilities of the innovating firm. Assuming that the number of potential (ex-ante) rivals in each country j is exogenously given by R_j[7], then each innovating firm can expect to compete against the following number of rivals

$$R_{ij}(\omega) = R_j \left[1 - e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_j^R\right)^\theta - \left(c_{ij}^I\right)^\theta\right]}\right] \tag{6}$$

Below are some preliminary properties of the rivals and their production capabilities[8].

Result 1: The number of rivals $R_{ij}(\omega)$ and their average efficiency increases as the state of technology T_j increases.

Result 2: The number of rivals $R_{ij}(\omega)$ increases with the cutoff condition of rival entry $\tilde{c}_j^R$.

Result 3: The number of rivals $R_{ij}(\omega)$ and their productivity increases with the productivity parameter of the innovating firm.

To summarize, more productive innovating firms not only face more rivals, but these rivals

[7]One could also make it proportional to market size Y_j or allow for rivals from outside countries who adhere to the same conditions with bounded distributions.

[8]Proofs can be found in the Appendix.

are also more productive on average. Innovating firms can reduce the number of rivals they face by increasing the cutoff condition for rival entry. Moving forward, consider the case where $c_{ij}^I < \tilde{c}_j^R$ so that at least one rival exists at all times.

2.4 The Distributions of Markups and Prices

The rivals in each country j ensure that the innovating firm does not charge a dubious markup. The innovating firm competes with the low-cost rival through price (Bertrand). Denote the marginal cost function of the low-cost rival in country j as c_{ij}^{R*}. The price will be determined as the minimum of the low cost rival's cost function and the CES/monopolistic price of the innovating firm. Denoting the price under the Bertrand competition scenario as p_{ij}^B and the price in the monopoly scenario as p_{ij}^M, prices are then

$$p_{ij}(\omega) = \min\left\{p_{ij}^B = c_{ij}^{R*}, p_{ij}^M = \overline{m}c_{ij}^I\right\} \tag{7}$$

With markup

$$m_{ij}(\omega) = \min\left\{m_{ij}^B = \frac{c_{ij}^{R*}}{c_{ij}^I}, \overline{m} = \frac{\sigma}{\sigma - 1}\right\}$$

Where $\overline{m}$ is the Dixit-Stiglitz CES markup. The price, markup and firm profits are all determined by the cost ratio of the low-cost rival and innovating firm. If the cost function of the low cost rival is greater than the monopolistic price, than the innovating firm will be able to charge a monopolistic price and obtain monopolistic profits. However, if the cost function of the low-cost rival is lower than the monopolistic price, then the innovating firm obtains profits generated by the Bertrand markup.

In order to determine when the low-cost rival's cost functions is greater than or less than the monopolistic prices, define the distribution of the cost ratio as . [9]

$$h\left(\frac{c_{ij}^{R*}}{c_{ij}^{I}}\right) = h(m_{ij}^{B}) = \begin{cases} \frac{R_{ij}T_iT_j\theta(w_iw_jd_{ij})^{\theta}\left(m_{ij}^{B}\right)^{\theta-1}}{\left[R_{ij}T_j(w_id_{ij})^{\theta}\left(\left(m_{ij}^{B}\right)^{\theta}-1\right)+T_iw_j^{\theta}\right]^2} & \text{for } 1 \leq m_{ij}^{B} \leq \overline{m} \\ \int_{\overline{m}}^{\infty} \frac{R_{ij}T_iT_j\theta(w_iw_jd_{ij})^{\theta}\left(m_{ij}^{B}\right)^{\theta-1}}{\left[R_{ij}T_j(w_id_{ij})^{\theta}\left(\left(m_{ij}^{B}\right)^{\theta}-1\right)+T_iw_j^{\theta}\right]^2} dm_{ij}^{B} & \text{for } m_{ij}^{B} = \overline{m} \\ 0 & \text{for } m_{ij}^{B} > \overline{m} \end{cases} \quad (8)$$

With a mass point at $\overline{m}$. The distribution of the markup is determined primarily by the number of rivals R_{ij}, which governs the distribution of the low-cost rival. In the symmetric case with no trade costs and one rival, $h(m_{ij}^{B}) = \theta\left(m_{ij}^{B}\right)^{-\theta-1}$ is identical to the Pareto density for markups obtained in Bernard et al. (2003). Figure 3 shows the distribution of $h(m_{ij}^{B})$ for varying levels of R_{ij}.

[Figure 3 about here]

Integrating $h(m_{ij})$ over the values from $\overline{m}$ to ∞ gives the probability that the innovating firm in country i charges the monopolistic markup in country j and achieves profit π_{ij}^{M}. This probability is denoted as ϕ_{ij}.

$$\phi_{ij} = \Pr\left[m_{ij}^{B} \geq \overline{m}\right] = \int_{\overline{m}}^{\infty} h(m_{ij}^{B})dm_{ij}^{B} = \frac{T_iw_j^{\theta}}{R_{ij}T_j\left(w_id_{ij}\right)^{\theta}\left(\overline{m}^{\theta}-1\right)+T_iw_j^{\theta}} \quad (9)$$

This value is identical to the probability that a supplier charges the unconstrained markup in de Blas and Russ (2015). Having defined when the markup is Bertrand and when the

[9] Combine the CDF of the low-cost rival with the cost distribution of the innovating firm. The formulation for the low-cost rival comes from Rinne (2009) on pages 224 and 237 which gives

$$G_{ij}^{R*}(c_{ij}^{R*}|c_{ij}^{I}, r_{ij}, T_j, \theta) = 1 - e^{-r_{ij}T_jw_j^{-\theta}\left[\left(c_{ij}^{R*}\right)^{\theta}-\left(c_{ij}^{I}\right)^{\theta}\right]}$$

The full derivation is found in Appendix section A.

markup will be CES, the innovating firm's profit equation is

$$\begin{aligned} \mathrm{E}\left[\pi_{ij}^{I}(\omega)\right] &= \phi_{ij}\pi_{ij}^{M} + (1-\phi_{ij})\,\mathrm{E}\left[\pi_{ij}^{B}\right] \\ &= \left(\frac{c_{ij}^{I}}{P_j}\right)^{1-\sigma} \alpha_j Y_j \left[\phi_{ij} V\left(\bar{m}\right) + (1-\phi_{ij})\, V\left(\bar{m}_{ij}^{B}\right)\right] - f_{ij}^{*} \end{aligned} \tag{10}$$

where $V(x) = x^{-\sigma}(x-1)$ and $\bar{m}_{ij}^{B} = \mathrm{E}\left[m_{ij}^{B} | m_{ij}^{B} \leq \overline{m}\right]$ is the expected value of the markup when it is less than the CES markup[10]. This leads to the next set of results.

Result 4: The probability the innovating firm charges the CES markup in country j is decreasing in contestability R_{ij}[11].

Result 5: The probability the innovating firm charges the CES markup in country j increases as the cutoff condition for rival entry decreases and decreases as the innovating firm becomes more productive (lower costs)[12].

Result 6: The innovating firm's expected profit $\mathrm{E}\left[\pi_{ij}^{I}(\omega)\right]$ is decreasing in contestability r_{ij}.

Result 7: The price of variety ω charged to consumers in country j is decreasing in contestability R_{ij}.

To sum up the results, the number of rivals negatively effects the innovating firm's expected profits, so that holding the innovating firm's productivity constant, they will want to reduce the number of rivals. Note that despite the increased contestability, innovating firms with higher productivities still receive larger profits due to CRS and capturing a larger market share. The next section looks at when the innovating firm decides to patent.

[10]The expected value of this is given by the formula $\int_{1}^{\overline{m}} \frac{m_{ij} h(m_{ij})}{1-H(\overline{m})} dm_{ij}$ which has no closed-form solution.

[11]This result is similar to the findings in de Blas and Russ (2015) who similarly show that lower markups occur with increased contestability.

[12]It may seem counterintuitive that more productive firms are less likely to be monopolists, but note that the expected markup for the innovating firm increases with their productivity so that they still generate more profits than low productivity firms.

2.5 The Decision to Patent

The innovating firm's operating profits depends on the number of rivals which is dependent on the firm's productivity draw and zero-profit condition. Patenting is a mechanism that allows the innovating firm to reduce the number of rivals they face in any given market j by increasing the zero-profit condition $\tilde{c_{ij}}^R$. When firms patent in country j, they pay a fixed cost f_j^P and in return, rival firms will have to pay an additional overhead cost of $f_{j,pat}^R$ where $f_{j,pat}^R \geq f_{j,not}^R$. One way to interpret this is that patenting causes the rival firms to pay additional legal/licensing fees. As patent protection gets stronger (for instance, if breadth increases), the cost for rivals to enter increases, leading to fewer rivals. This reduction in rivals has a doubly positive effect on firm profits as it not only increases the probability for the innovating firm to charge their optimal markup, but also increases the expected markup should the firm operate in Bertrand competition. The expected profits from patenting are:

$$\mathrm{E}\left[\pi_{ij,pat}^I(\omega)\right] = \phi_{ij,pat}\pi_{ij}^M + (1-\phi_{ij,pat})\,\mathrm{E}\left[\pi_{ij,pat}^B\right] - f_{ij}^* - f_j^P \tag{11}$$

The firm will patent when $\pi_{ij,pat}^I(\omega) - \pi_{ij,not}^I(\omega) \geq f_j^P$. Figure 4 plots the two expected profits for when the firm elects to patent and when they do not. Also included are the entry conditions for the rival firms where the Z^R is the corresponds to the marginal cost sufficient to cover the fixed per-period overhead costs so that $Z_{j,pat}^R > Z_{j,not}^R$. When productivity of the innovating firm z_i^I is greater than or equal to the corresponding patenting threshold z_{ij}^P, then the innovating firm will elect to patent.

[Figure 4 about here]

Analytically, this patenting cutoff condition will be defined as c_{ij}^P so that whenever $c \leq c_{ij}^P$ innovating firms will elect to patent.[13] Note that country j's patent protection will have no effect on the firm's decision to enter into a foreign market, thereby preserving the properties

[13]To solve for the patenting cutoff condition, set the patent profits equal to the non-patent profits.

uncovered in the 'new' new trade theory models. There are now four types of firms in every market: Non-patenting exporting and FDI firms where $c_{ij}^I > c_{ij}^P$ and patenting exporting and FDI firms where $c_{ij}^P \geq c_{ij}^I$.[14]

Unfortunately, no closed-form solution for an equilibrium exists, but it is possible to solve for a numerical equilibrium. One can normalize the wages by introducing an homogeneous good that is freely traded and then introduce a free-entry condition. It is then possible to solve for the cutoff productivities for each firm type in a system of equations. Given the following parameters: T, θ, σ, L, d, R and the various fixed costs, one can then solve for a numerical equilibrium.

3 Properties of the Model

This section runs several simulations with different parameter values to assess how country and industry characteristics impact the decision to patent. The first subsection looks at the closed economy simulation with emphasis on industry characteristics before turning to the open economy scenario.

$$c_{ij}^P = \left(\frac{f_j^P}{\alpha_j Y_j}\right)^{\frac{1}{1-\sigma}} P_j \left\{\phi_{ij,pat}\left[V\left(\overline{m}\right) - V\left(\bar{m}_{ij,pat}^B\right)\right] - \phi_{ij,not}\left[V\left(\overline{m}\right) - V\left(\bar{m}_{ij,not}^B\right)\right] + V\left(\bar{m}_{ij,pat}^B\right) - V\left(\bar{m}_{ij,not}^B\right)\right\}^{\frac{1}{\sigma-1}}$$

Next, since not all firms elect to patent, it must be the case that c_{ij}^P is less than the market entry condition c_{ij}^E. Given the entry condition

$$c_{ij}^E = \left(\frac{f_{ij}}{\alpha_j Y_j}\right)^{\frac{1}{\sigma-1}} P_j \left[\phi_{ij,not} V\left(\overline{m}\right) + \left(1 - \phi_{ij,not}\right) V\left(m_{ij,not}^B\right)\right]^{\frac{1}{\sigma-1}}$$

It must be the case that

$$f_j^P \geq f_{ij}^* \left[\frac{\phi_{ij,pat}\left(V\left(\overline{m}\right) - V\left(m_{ij,pat}^B\right)\right) + V\left(m_{ij,pat}^B\right)}{\phi_{ij,not}\left(V\left(\overline{m}\right) - V\left(m_{ij,not}^B\right)\right) + V\left(m_{ij,not}^B\right)}\right]$$

.

[14] Depending on the level of patent protection, there is no pre-determined ranking of productivities for each type of firm. It is simply the case that $z_{ij,not}^{EXP} < z_{ij,not}^{FDI}$ and $z_{ij,not}^{EXP} < z_{ij,pat}^{EXP}$ and $z_{ij,not}^{FDI} < z_{ij,pat}^{FDI}$.

3.1 Closed Economy with Different Industry Parameters

The simulation starts by normalizing wages and assuming the following values: Market size $L_i = 100$, Number of potential rivals $R_i = 20$, Fixed operating costs $f_i^* = 50$, Patenting costs $f_i^P = 10$ and Technology $T_i = 1$. Assume that patenting increases the fixed cost of rival entry by 25 times [15]. Figure 5 plots how the number of rivals and markups change as the productivity of the innovating firm z_i^I increases. As z_i^I gets larger, the number of rivals increases, while the markup declines until the dotted line, at which the innovating firm patents, clearing the market of rivals and allowing the innovating firm to charge the full monopolistic mark-up.

The industry variants of interest include the elasticity of substitution σ and variance in production capabilities, θ. The parameter values for θ are taken from Bernard et al. (2003), while maintaining the condition $\sigma \leq \theta + 1$. As each parameter value changes, the following is expected to occur: As σ begins to increases, the monopolistic mark-up for rival firms decreases, leading to fewer rivals faced by innovating firms and more entrants. With reduced competition, the proportion of entrants who patent will initially decline, while the levels may increase (depending on the number of entrants). For higher θ, there is less variability in firm productivities within that industry. While θ does not affect the number of rivals faced by the innovating firm, it will effect the rivals' productivities. As θ increases, the competition within each industry becomes more fierce, leading to higher entry conditions for the innovating firm and fewer entrants. Patent levels may increase, while the proportion of entrants who patent increase, depending on whether the monopolistic markup makes patenting worthwhile. In industries where competition is fiercest (high θ and low σ), only the most productive innovators will enter the market, and most of them will patent. The baseline results of the closed-economy simulation are in Table 1.

[Figure 5 about here]

[15] Note that the simulation values should not be interpreted in any meaningful way

[Table 1 about here]

The simulations suggest a U-shaped pattern on the *proportion* of entrants that patent, with monotonic changes in the *levels* of patents depending on the values of σ and θ. Reduced variability in productivity (high θ) leads to an increased share of entrants that patent, but only when it is interacted with a large number of rivals (low σ). Otherwise, increased θ consistently reduces the total number of patents. A plot on the propensity to patent based on given values of σ and θ is in Figure 6.

[Figure 6 about here]

To summarize, industry patterns of patenting are variable and depend on the import demand elasticity and production variability of the firms within each industry. As the elasticity increases, the model predicts higher levels of patents due to increased entry of innovators resulting from fewer competitors. On the other hand, as variability declines, the anticipated competition between the innovator and the rivals is more fierce, leading to fewer patents as a result of reduced entry. Proportionally, patenting follows a U-shaped pattern where firms in industries with very low elasticity interacted and high levels of θ are the most likely to patent. The next section looks at the open economy simulations.

3.2 Open Economy

In the open economy simulations, the same starting parameter values are used as before, keeping both $\sigma = 4$ and $\theta = 3.60$ constant. Trade costs of 10% are assumed in the baseline scenario ($d_{ij} = 1.1$). Note that because of the addition of trade costs, innovating firms require a higher z to break-even, leading to fewer overall entrants. Along with the trade costs, the market size, fixed costs and the technology parameter of the destination country will influence the decision to patent by the innovating firm from the origin country. The impact of changes to these simulated values is expected to be straightforward. As the population or market size increases, the number of rival entrants and innovating firm entrants will increase. Due to more entrants and more rivalry, innovating firms would be more likely to patent, resulting in both a higher proportion and higher level of patents. For the destination country technology parameter, under Result 1, the model predicts an increase in both the number and composition of the rival firms. This makes it more difficult for new innovating firms to enter the market, and more likely that the entrants will patent in order to reduce the absolute number of rivals. For trade costs, higher trade costs will impact the entry condition for innovating firms, meaning that while there are fewer entrants, each of the entrants will be more productive on average (and hence more likely to patent). The changes to the cost of patenting and the new overhead charge for rival firms should have no effect on the innovating firms' entry condition but will impact the proportion who patent. Higher patenting costs are expected to lead to fewer patents, while better protection leads to increased patenting. Table 2 displays the results.

[Table 2 about here]

The open country simulation shows that increasing market size leads to slightly fewer entrants but a greater proportion of patenting entrants resulting from increased competition. Increasing the technology levels of the destination country further reduces the number of entrants while leading to a higher proportion of patenting entrants (but reduced levels). Increasing the origin country's technology has the opposite effect and leads to higher entrants, but a reduction in the percentage of entrants that patents. By increasing trade costs, the number of entrants is reduced, so that entrants have on average higher productivity, leading to more patents. Finally, the costs and penalties associated with patenting have the expected effects. The impact of each of these country-specific factors is either monotonically increasing or decreasing on the percentage of firms who patent.

To conclude, the simulations tell us that countries that are technologically advanced, have good patent protection (in the form of low costs for innovating firms and higher overhead costs for rival firms) and lots of competition from rivals experience a higher share of patenting firms.

4 Data and Structural Estimation

In this section, the technology levels, industry productivity variation and measures of country IP benefits are calibrated using patent and trade data. These measures are then used to estimate the impact of changes to the global economy, such as increased technology levels, higher IP benefits, reduced trade costs and the proliferation of patent agreements.

4.1 Data

The key variable used to calibrate the model are bilateral patent flows found in PATSTAT (2016 version), the patent database compiled by WIPO and the EPO. PATSTAT contains patent data organized by the date of application, whether the patent was granted, technology classification of the patent (International Patent Classification (IPC)) and destination office of the patent. The origin of the patent is constructed from the country code of the inventor(s). [16] Patents that have not been granted are excluded from the tabulation and the earliest application date of the original patent is used as the primary application date for all patents. The technology classification code for each patent is given by IPC, an alphanumeric code with more than 60,000 subdivisions and describes the technology classification of the patent. Patents may have multiple IPC codes and are aggregated into 4-digit IPCs that are weighted equally by the number of times they are listed in the patent.

To translate the number of patents by technology into an industry, the IPC4-SITC (Rev. 2) crosswalk from Lybbert and Zolas (2014) is used. When backing out the industry parameters, the estimation uses the elasticity of substitution measures from Broda and Weinstein (2006),

[16]For simplicity, the modal country of inventors is assumed to be the country of origin. In cases where the modal country is equal (occurs in roughly 3.8% of cases), the country is selected randomly).

which are also expressed in SITC Rev. 2.

In addition to patents, the estimation uses country-level estimates for market size, wages, patent fees and trade costs. Trade costs, denoted as d_{ij}, are calculated by combining covariate estimates from a standard gravity equation of bilateral trade flows (at the country-level) similar to Fieler (2011).

$$d_{ij} = \exp\left(\delta_1 + \delta_2 DIST_{ij} + \delta_3 DIST_{ij}^2 + \delta_{border} + \delta_{lang} + \delta_{TA}\right)$$

The trade data used for this calculation come from UN Comtrade and are combined with distance and trade cost figures from CEPII (Eeckhout and Jovanovic (2002)). The estimates are performed for each cross-sectional year so that trade costs can vary over time. Market size and wages are assumed to be GDP and GDP per capita for each country.

The calibration is a two-step process where the country-level estimates (T_i and IP benefit) are obtained first and then used to estimate the industry-level parameters (θ). A summary of the data sources can be found in Table 3.

[Table 3 about here]

Step 1: Estimation of Country IP Benefits and Technology States

For the technology parameter to be estimated for a country, the country needs to have applied for and been granted at least 1 patent. For the IP benefit parameter to be estimated, a country needs to have received at least 1 patent (resident or non-resident). The IP benefit parameter is obtained from the patenting cut-off condition. For the first set of calibrations that focus on country measures, industry parameters do not vary and expenditure shares are equivalent to 1 ($\alpha_j = 1$). The patent cut-off condition written in terms of productivity parameter z_{ij}^P is

$$z_{ij}^{P}=\left(f_{j}^{P}\right)^{\frac{1}{\sigma-1}}\left(w_{i}d_{ij}\right)\underbrace{Y_{j}^{\frac{1}{\sigma-1}}P_{j}\left\{\phi_{ij,pat}\left[V\left(\overline{m}\right)-V\left(\bar{m}_{ij,pat}^{B}\right)\right]-\phi_{ij,not}\left[V\left(\overline{m}\right)-V\left(\bar{m}_{ij,not}^{B}\right)\right]+V\left(\bar{m}_{ij,pat}^{B}\right)-V\left(\bar{m}_{ij,not}^{B}\right)\right\}^{\frac{1}{1-\sigma}}}_{\gamma_{j}\in[0,(Y_{j}V(\bar{m}))^{\frac{1}{1-\sigma}}]} \tag{12}$$

Where γ_j is heretofore known as the *IP benefit* and measures the net benefit to the innovating firm from taking out a patent in country j.[17] The formal definition of the IP benefit measure is the difference in markup and monopolistic profits probability between patenting and non-patenting firms. This measure encapsulates the full competitive effects of patenting and considers both market size (Y_j) and the impact of patenting on reducing the number of rivals for each innovating firm. Hence, small countries with few rivals who have a high degree of IPR may have low IP benefits.

The range of values for γ_j can be pinned down by assuming that the minimum benefit from patenting is zero when the probability of obtaining monopolistic profits does not differ for patenting and non-patenting firms ($\phi_{ij,pat} = \phi_{ij,not}$) and when the markup is the same for patenting and non-patenting firms ($\bar{m}_{ij,pat}^{B} = \bar{m}_{ij,not}^{B}$). On the other hand, maximum benefit from patenting happens when perfect competition occurs in the non-patenting state ($\phi_{ij,not} = 0$ and $\bar{m}_{ij,not}^{B} = 1$) and monopolistic competition occurs in the patenting state ($\phi_{ij,pat} = 1$ and $\bar{m}_{ij,not}^{B} = \overline{m}$). This yields the range of $\gamma_j \in [0, (Y_j V(\bar{m}))^{\frac{1}{1-\sigma}}]$.

The main estimating equation combines the patenting cut-off condition with the Fréchet distribution, and takes double logs to get

$$\ln\left(-\ln\left(F\left(z_{ijt}^{P}\right)\right)\right)=\ln\left(T_{it}\right)-\theta\ln\left[\left(f_{j}^{P}\right)^{\frac{1}{\sigma-1}}\left(w_{it}d_{ijt}\right)\gamma_{jt}\right] \tag{13}$$

By combining patent data with country-level starting parameters, it is now possible to estimate T_{it} and γ_{jt}. Assume the starting parameters for θ and σ are taken again from Bernard et al. (2003) so that $\theta = 3.6$ and $\sigma = 3.79$. The patent application cost is $f_j^P = \$10,000$, α_j.

[17] This step also makes the simplifying assumption of assigning the IP benefit measure for country j regardless of the origin country. While it is possible to back out bilateral measures of country IP benefit, there is very little variation in this measure and assigning a single country value is more practical.

Trade costs d_{ij} are estimated using the technique used in Fieler (2011). The last remaining variable definition is LHS variable $F\left(z_{ijt}^{P}\right)$, which is defined as the probability an innovating firm from country i patents in country j at time t. This can be defined in a number of ways, which will primarily impact the scalar parameter T_{it} [18]. For now, it is assumed to be the number of bilateral patents divided by GDP (in $ million) for country i at time t. [19]

Note that the initial estimating equation does not vary by industry and the industry-specific variables, namely σ and θ have been aggregated and are unchanging. Each year t has $I \times J$ observations to estimate I number of technology parameters and J number of IP benefit parameters. These parameters are estimated using ordinary least squares. The values for T_{it} and γ_{jt} are then used to calibrate the industry-level productivity variation θ.

Step 2: Estimation of Industry Productivity Variation

Once the estimates of technology T_{it} and IP benefit γ_{jt} are obtained, they are both used to pin down the industry-variability measure θ_{kt}. To do this, the patent data is further disaggregated to the origin-destination-SITC-year level. The same estimating equation (14) is used, but σ is allowed to vary by industry. The values for the elasticity of substitution σ are taken from Broda and Weinstein (2006). Expenditure shares for each country j (α_{jkt}) are assumed to be the share of total imports in product k to country j at time t

The terms of equation (13) are rearranged so that the bilateral patent percentage stays on the LHS, but now the technology measures derived in the previous section are subtracted and the observable variable on the RHS includes the destination country's IP benefit measure (along with the starting value assumptions for patent costs, sigma and trade costs). The unknown variable, θ can then be measured for each k so that

[18] Because the scalar T_{it} has no real world basis, it is allowed to take on any value that is comparable across countries over time.

[19] Alternative scalars have used R&D expenditures as a share of GDP and combination of R&D and patents with very similar results

$$\ln\left(-\ln\left(F\left(z_{ijkt}^{P}\right)\right)\right)-\ln\left(\hat{T}_{it}\right)=\theta_k\ln\left[\left(\frac{f_{jt}^{P}}{\alpha_{jkt}}\right)^{\frac{1}{\sigma_k-1}}\left(w_{it}d_{ijt}\right)\hat{\gamma}_{jt}\right] \quad (14)$$

For each industry k, there are approximately $I \times J$ observations at time t. This parameter is estimated using ordinary least squares. This completes the calibration. The next section describes the results from the calibration and simulates changes to the some of the key parameters.

5 Results

The full parameter estimates for technology states, IP benefit and industry productivity variability for each year, country and industry are available online. The estimated parameters are used to predict bilateral patent flows and are compared with the actual flows in the data. A summary of the results are provided in Table 4. The calibration is mostly successful. The model is able to estimate total patent flows within 10% of actual patent flows by country across all years. A plot of actual and predicted patent flows is provided in Figure 7. A perfect fit would be clustered around the 45-degree line. The lowess smoothing line mostly runs parallel to this, albeit shifted slightly to the right as the model tends to underestimate domestic patents and overestimate foreign patent flows. Taken at face value, this might also be suggestive that foreign patent flows should be higher than what is seen in the data. A discussion of each of the parameters is given below.

[Table 4 about here]

[Figure 7 about here]

5.1 Technology States

Country technology levels take an extreme value form in Figure 8, with the majority of countries possessing low technology levels. The technology parameter is driven primarily by the number of originating patents and is consistent with the high concentration of patents originating from relatively few countries. The technology levels are also consistent over time, slowly rising in the late 1990s, declining somewhat in the early 2000s and then rising again after 2006. The average growth rate between 1994 and 2012 across all countries with non-zero patent flows in 2012 is 4.0% annually. Among the countries with the highest rates of technology growth include China (18.2%), Turkey (10.4%) and the Czech Republic (9.3%), while the countries with the slowest technology growth are South Africa (-22.5%), Algeria (-16.6%) and Thailand (-1.6%). Unsurprisingly, country technology levels are highly correlated with R&D expenditures as a share of GDP as found in Figure 9.

[Figure 8 about here]

[Figure 9 about here]

5.2 IP Benefit

Figure 10 plots both the distribution and the level changes over time of the IP benefit γ parameter. The IP benefit parameter values are driven by foreign flows of patents and follow a more normalized distribution with significantly less variation than the technology parameters estimated. The values stay constant until 2006 before slowly rising. Certain countries have made investments toward improving the IP benefits over time, while other countries have seen the benefit of IP fall over this time period. Countries who have seen the largest annual increase in benefit of IP are Ireland (3.9%), Lithuania (3.6%) and Denmark (3.4%). Meanwhile, countries who have seen the lowest annual growth in IP benefit are

Azerbaijan (-4.8%), Singapore (-2.8%) and China (-0.9%). The inclusion of China as one of the slower growing countries in terms of IP benefit may be surprising. However, despite the growth of foreign patents to China over this time period, this growth appears to have not kept pace with the economic growth.

[Figure 10 about here]

Despite the importance of market size in terms of calibrating IP benefits, IP protection also figures prominently in the calculation. Figure 11 plots the IP benefit with traditional measures of IPR, as found in Park (2008). While the IP benefit is positively correlated with figures found in Park (2008), there are a number of notable differences. Namely, countries who are also part of patent treaties tend to have lower IP benefits than might otherwise be predicted. This is primarily due to firms electing to apply for a patent within the patent treaty, as opposed to the individual country. Hence, the individual country benefit may not be so high. When the patent treaty member countries are removed as the right hand side shows, the relationship between IP benefit and IPR is more robust.

[Figure 11 about here]

5.3 Industry Variability

Finally, Figure 12 plots the density of θ across all SITC Rev. 2 industries. The industry variability parameter has a mean of 4.00, which is not too far off from the aggregate measure of 3.60. This parameter has declined by approximately 0.8% each year over this time period, hinting at more concentrated competition on average across all SITC Rev. 2 industry codes.

[Figure 12 about here]

5.4 Patent Agreements

As hinted in the previous section, the IP benefits for countries in patent treaties tend to be lower than what one would predict given a country's IPR and market size. One of the benefits of the model is that it is possible to measure the IP benefit of existing patent treaties and compare them with IP benefit of the countries that make up the patent treaty. A patent treaty's effectiveness can be determined by the number of patents that are applied through the agreement versus the number of patents that are applied for in the individual countries that make up each patent agreement. Across the entire sample, the average difference in IP benefit between the patent treaty and the countries that make up the patent treaty is 0.41, an almost 1 s.d. increase in the average country's IP benefit.
There are 5 separated international patent treaties listed in the patent data. These include the European Patent Organization (EPO), Eurasian Patent Organization (EAO), African Regional Intellectual Property Organization (ARIPO), the African Intellectual Property Organization (AIPO) and Gulf Cooperation Council (GCC). Note that AIPO did not have enough data points of the member countries to make a proper comparison. In nearly all cases, one would expect that patent agreements lead to increased patenting resulting from scale effects and lower administrative fees. In cases where the benefit of the individual country would be higher, this may be due to enforcement issues within the agreement.

[Figure 13 about here]

Figure 13 compares the IP benefit of each of the patent agreements with the IP benefits in each of the member countries. Member countries who receive no patents outside of the patent treaty are not included in the plots. These countries are the largest beneficiaries of the patent agreements as they would otherwise have had zero access to patents. The plots show some interesting trends over time. In nearly all of the treaties, the benefit of belonging to the treaty declines over the time period. Starting with the EPO treaty (which has the highest IP benefit), all of the countries are better off from participating with member countries of Great Britain (GB), Germany (DE) and France (FR) benefiting the least from the agreement. Over time, the role of the EPO looks to be slowly diminishing as individual country IP benefits increase and converge.

For countries in the Eurasian Patent Organization (EAO), roughly half of the countries have higher individual benefits than the ones implied by the treaty, with Russia (RU) and Kazakhstan (KZ) never benefiting directly from the treaty. On the other hand, smaller countries like Moldova (MD) and Azerbaijan (AZ) consistently benefit from being part of the EAO treaty. For the African Regional Intellectual Property Organization (ARIPO) treaty, we find the benefit split between the two member countries that also received patents as individual countries (Kenya (KE) and the Sudan (SD)), with the differences in IP benefit being very slight. There is also a benefit for all of the member countries in the Gulf Cooperation Council (GCC).

The differences in IP benefit across patent treaties are driven by two factors: levels of IPR (both relative and absolute) and market size (both relative and absolute). Table 5 describes a regression looking at how the net benefit of a patent treaty is impacted by the interaction of these two factors. The results support the idea that member countries who have larger market size or higher measures of IP rights tend to benefit less from patent treaties. On the other hand, there does appear to be a positive effect for countries having both a large market size and high IP rights. This may perhaps be indicative that patent treaties benefit disproportionately more from having large countries with substantial IP protection as member countries.

[Table 5 about here]

These results are the first empirical study that looks at the benefits of patent treaties. While it would seem to be obvious that patenting through a treaty would be advantageous for the innovating firm since it reduces the fixed costs of patenting, many firms still opt to patent through the individual country only. This may be due to a variety of factors relating to the patent treaty, such as differences in enforcement, stricter/looser guidelines, greater exposure of ideas, redundancy, additional transaction costs over applying to a single country or other reasons. The next section looks at the changes to worldwide patent flows resulting from existing policy measures enacted over the time period.

5.5 Policy Experiments

The last empirical exercise looks at how changes to the parameter values impact the flow of domestic and international patents. The comparison will look at the predicted patent flows in 2012 resulting from the following policy experiments: i.) Having no patent treaties in place (but countries have similar IP benefits as patent agreements), ii.) All countries having the maximum technology state in 2012, iii.) All countries having the minimum technology state in 2012, iv.) Constant technology states (1996 levels), v.) Cost of patenting declines by 50%, vi.) Cost of patenting increases by 100%, vii.) Trade costs decline by 25% from 2012 levels, viii.) Trade costs increase by 25% from 2012 levels, ix.) Constant trade costs (1996 levels). The first policy exercise (i.) will provide a dollar value measure of the benefits of patent treaties for firms.

Table 6 estimates the cumulative worldwide patent flow resulting from each of these experiments. Based on the starting values, Table 6 shows that without patent agreements (but keeping the IP benefits of the patent agreement in place for member countries), foreign patent flows would increase by more than 90%. A back-of-the-envelope calculation can measure the economic benefits of patent agreements by comparing the differences in administrative fees that patenting firms would face if no patent agreements were in place. For EPO member countries, they received approximately 77,000 actual patents in 2012 according to PATSTAT data. If the EPO was not in place, firms applying for a patent in all of the member countries of the EPO where the IP benefit was high enough to justify the expenditure would have to submit and apply for approximately 802,000 patents across all of the countries, a difference of 725,000 patents. If the average application cost of these patents is $10,000, then firms experience a cost savings of approximately $7.25B in administrative fees from the EPO patent agreement just in 2012.[20] This is quite substantial and highlights the cost benefit of patent treaties for innovating firms.

[Table 6 about here]

The other policy experiments tend to move in the direction predicted by the model and the table can give a sense of how innovation levels would increase/decrease from changes to technology levels, patent costs and trade costs. According to the model, if technology states had remained at 1996 levels, innovation in 2012 (as measured by domestic patents) would be 38% lower than the levels predicted. For patent costs, a reduction in the administrative fees of 50% would lead to a 41% increase in patent output, while a doubling of patent costs would lead to a reduction of around 24%.

Finally, trade costs also figure prominently in reducing or increasing patent flows. A 25% reduction in trade costs are predicted to increase foreign patent flows by 44%. If trade costs were to increase by 25%, corresponding foreign patent flows would decline by 28%. Finally, if trade costs had remained the same as in 1996 in 2012, then foreign patent flows would have declined by more than 75%.

To quickly summarize the empirical findings from the calibration exercise, nearly all countries benefit from being a part of a patent treaty, with smaller markets benefiting more than larger markets. The EPO patent treaty by itself, reduced patenting administrative costs by approximately $7.25B in 2012. Policy experiments highlighted the role of technology investments in increasing innovation rates, as well as how reducing administrative fees and liberalizing trade can promote technology transfer. Had countries kept their 1996 technology levels, patenting would be 38% lower than the levels predicted in 2012.

[20] This does not include the additional cost savings from having to continuously monitor and protect these patents

6 Conclusion

The goal of this paper is to better understand the conditions for multinational firms in deciding where to seek international patent protection. These decisions are shown to have critical implications for future investment, technology diffusion and economic growth, especially for developing countries who linger outside of the patent core. This paper proposes a new type of patenting decision model that borrows elements from the Ricardian heterogeneous firm trade literature and can explain significant portions of spatial patenting patterns. The model explains why countries with higher levels of technology, better patent protection and more competition are able to solicit a greater number of patents. Using a generalized version of the patenting cutoff condition, the model was able to calibrate patent benefit measures for more than 80 countries of various size and income between 1994 and 2012. These patent benefit measures take into account the actual patent flows to each country and differ with alternative IPR indices by accounting for competition. In addition, the calibrated results can also demonstrate how countries benefit from participating in patent treaties, with the average country experiencing a nearly 1 s.d. improvement.

Although the model is described in full detail, several properties of the model remain unknown. One of the more interesting aspects that has yet to be explored are the welfare effects that arise from strengthening IPRs and increasing technology transfer through patents. While increased patenting activity would lead to higher expected profits of innovating firms and greater diffusion, it is not clear what the negative effects are for consumers who may gain new varieties, but must now pay higher markups. It may be the case that the gain in welfare from the availability of new varieties outweighs the welfare loss from higher prices, which is the argument put forth by developed countries in the TRIPS agreement. Analyzing this question will help in addressing whether Article 66.2 of the TRIPS agreement has had a positive or negative impact on developing countries who were forced to make improvements to their IPRs.

Other possible extensions to the model include allowing foreign entry of rivals and incorporating an innovation component. Under the current framework, all of the potential rivals are local. Given the assumptions on the productivity constraints of rivals, it makes little sense to include foreign rivals since they would have to pay for the additional trade costs, making it unlikely that they would ever become the low-cost rival. On the other hand, by making the number of potential rivals in each country proportional to market size, it leaves open the possibility of foreign entry. Including foreign rivals would add robustness to the model since in many cases, multinational firms use patents as a deterrence and blocking device for outside competitors trying to gain access to a particular market. Second, although the model includes innovating firms, there is no decision variable for innovation. It is certainly possible to include this component, since the profits for innovating firms are well-defined and it would be interesting to see how rivals, patent protection and country variables interact in influencing this decision.

Modeling international patent flows is an important step in understanding the process of technology diffusion and the transfer of knowledge abroad. Many developing countries have improved upon their intellectual property protection and this has shown to be increasingly effective, but it is not clear whether firms will continue to respond positively to these changes. There also appears to be a trade-off between intellectual property rights and developing industrial capacity (Falvey et al. (2006)). Nevertheless, the model provides a testable framework for international patenting decisions and may lead to better policy in developing more effective patent regimes.

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Tables

	$\theta = 3.60$		
	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$
P(Entry ∥ Innovator)	0.022%	2.278%	8.678%
P(Patent ∥ Entry)	11.857%	7.940%	11.121%
P(Patent ∥ Innovator)	0.0026%	0.1809%	0.9651%
	$\theta = 8.28$		
	$\sigma = 3$	$\sigma = 5$	$\sigma = 8$
P(Entry ∥ Innovator)	0.001%	0.789%	7.396%
P(Patent ∥ Entry)	7.135%	4.283%	10.655%
P(Patent ∥ Innovator)	0.0001%	0.0342%	0.7880%
	$\theta = 12.86$		
	$\sigma = 3$	$\sigma = 5$	$\sigma = 8$
P(Entry ∥ Innovator)	0.000%	0.035%	1.322%
P(Patent ∥ Entry)	23.639%	1.404%	4.928%
P(Patent ∥ Innovator)	0.0000%	0.0005%	0.0651%

Notes: Simulations assume a market size of $L_i = 100$, # of potential rivals $R_i = 20$, fixed operating costs $f_i^* = 50$, patenting costs $f_i^P = 10$ and technology level $T_i = 1$.

Table 1: Closed Economy Simulation Results

Parameters	Baseline	$\uparrow L_j$	$\uparrow T_j$	$\uparrow T_i$	$\uparrow d_{ij}$	$\uparrow f^R_{j,pat}$	$\uparrow f^P_j$
L_j	100	200	200	200	200	200	200
T_j	1	1	2	2	2	2	2
T_i	1	1	1	3	3	3	3
d_{ij}	0.1	0.1	0.1	0.1	0.2	0.2	0.2
$f^R_{j,pat}$	25	25	25	25	25	35	35
f^P_j	10	10	10	10	10	10	15
P(Entry ‖ Innovator)	4.988%	4.953%	2.437%	5.947%	4.675%	4.675%	4.675%
P(Patent ‖ Entry)	14.415%	36.625%	45.478%	26.851%	32.328%	39.015%	20.446%
P(Patent ‖ Innovator)	0.719%	1.814%	1.108%	1.597%	1.511%	1.824%	0.956%

Notes: Simulations assumes # of potential rivals $R_i = 20$, fixed operating costs $f^*_i = 50$, elasticity of substitution $\sigma = 4$, and production variability $\theta = 3.60$.

Table 2: Open Economy Simulation Results

Parameterization Values, 1996-2012	Source
GDP (USD)	World Bank
GDP per Capita (Wages, USD)	World Bank
Worldwide Foreign and Domestic Patents	PATSTAT
Patenting Cost (USD)	10000
θ	3.60
σ (Country Calibration)	3.79
σ (Country-Industry Calibration)	Broda & Weinstein (2006)
Trade Flows (USD)	UN Comtrade
Gravity Variables	CEPII

Table 3: Data Sources for Calibration

	Mean	S.D.	Median	Minimum	Maximum
Predicted Patents/Patents (by Country)	107.1%	-	-	-	-
Domestic Patents/Patents (by Country)	90.4%	-	-	-	-
Foreign Patents/Patents (by Country)	135.8%	-	-	-	-
Technology Parameter	0.66	1.7	0.06	8.24E-08	18.11
IP Benefit	1.17	0.48	1.12	0.24	3.17
Industry Variability	4.00	3.04	2.91	0.86	22.13

Notes: Model was estimated using bilateral patent data combined with estimated trade costs, wages and market sizes from Table 3. The model was estimated using ordinary least squares (OLS). The patent ratios are the total predicted patents divided by the actual patents.

Table 4: Calibration Summary Statistics

	(1) Net Benefit from Treaty	(2) Net Benefit from Treaty
ln GDP	-0.261***	
	(0.0204)	
IP Rights	-0.502**	
	(0.153)	
ln GDP × IP Rights	0.0323***	
	(0.00600)	
Relative GDP		-0.288***
		(0.0362)
Relative IP Rights		-0.224**
		(0.0716)
Relative GDP × Relative IP Rights		0.170***
		(0.0325)
Year FE	Yes	Yes
Observations	1729	1729
R-squared	0.232	0.349

Notes: Robust standard errors in parentheses. Standard OLS Regression measuring the yearly difference between the treaty IP benefit and member country IP benefit across countries. *, **, and *** denote significance at 5, 1 and 0.1% confidence level.

Table 5: Determinants of Net Patent Treaty IP Benefits, 1996-2012

	Total (2012)	Domestic (2012)	Foreign (2012)	Foreign Share
Starting Value (000s)	843.9	361.3	482.5	57.2%
	% Change in Total	% Change in Domestic	% Change in Foreign	Change in Foreign Share
Patent Agreement IP Benefits	90.1%	9.1%	150.7%	18.2%
Maximum Technology Level	73.9%	57.7%	86%	4%
Minimum Technology Level	-65.8%	-90.8%	-47.1%	31.4%
1996 Technology Level	-38.5%	-56.2%	-25.2%	12.4%
Patent Costs Decrease by 50%	41%	37.8%	43.4%	1%
Patent Costs Increase by 100%	-23.7%	-22.6%	-24.6%	-0.6%
Trade Costs Decrease by 25%	25.4%	-	44.5%	8.7%
Trade Costs Increase by 25%	-16.3%	-	-28.5%	-8.3%
Trade Costs Revert to 1996 Levels	-43.4%	-	-75.8%	-32.8%

Notes: Each policy experiment started with the baseline 2012 predicted patent levels. Each experiment changed one of the parameters for the predicted values. Relative changes are relative to baseline values.

Table 6: Policy Experiment Impacts on Patent Flows

Figures

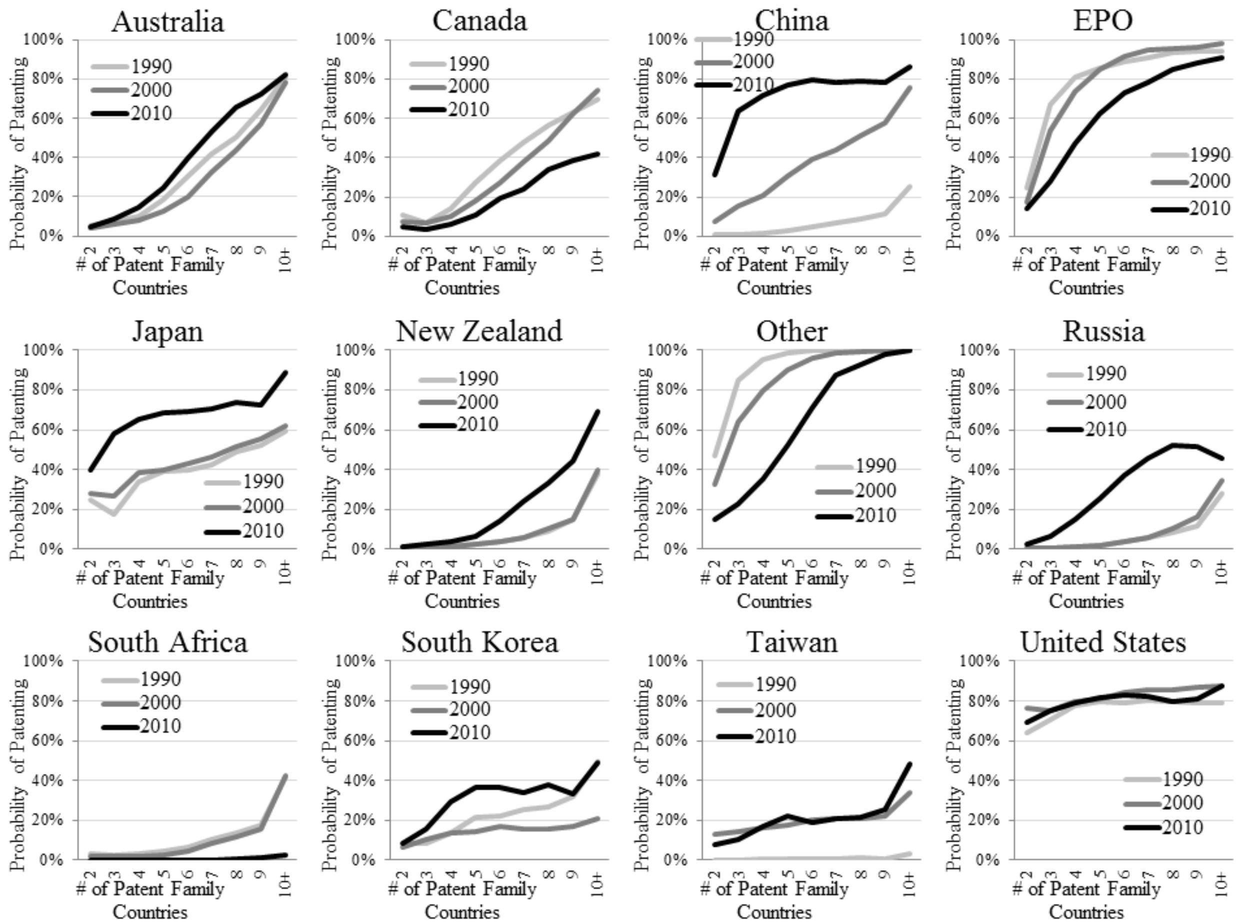
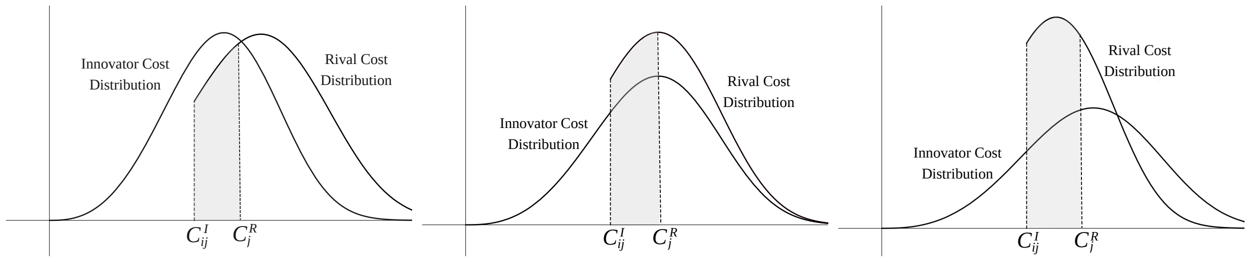


Figure 1: Probability of Patenting in Given Jurisdiction based on Size of Patent Family, 1990-2010



a. $T_i > T_j$ b. $T_i = T_j$ c. $T_i < T_j$

Figure 2: PDF of Cost Distribution $G(c)$ of Innovating (T_i) and Rival (T_j) Firms

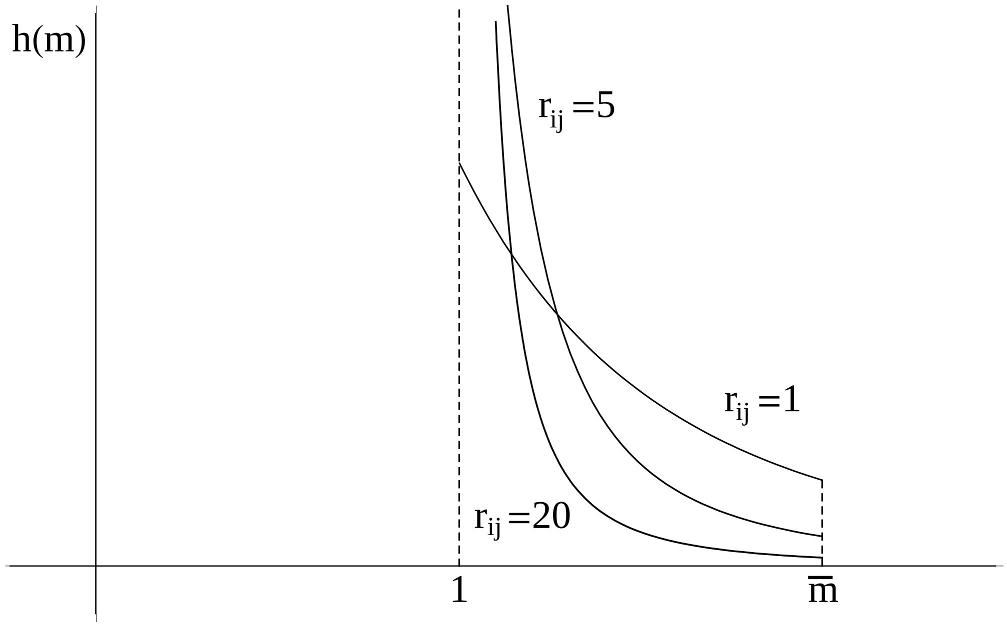


Figure 3: Density of the Markup

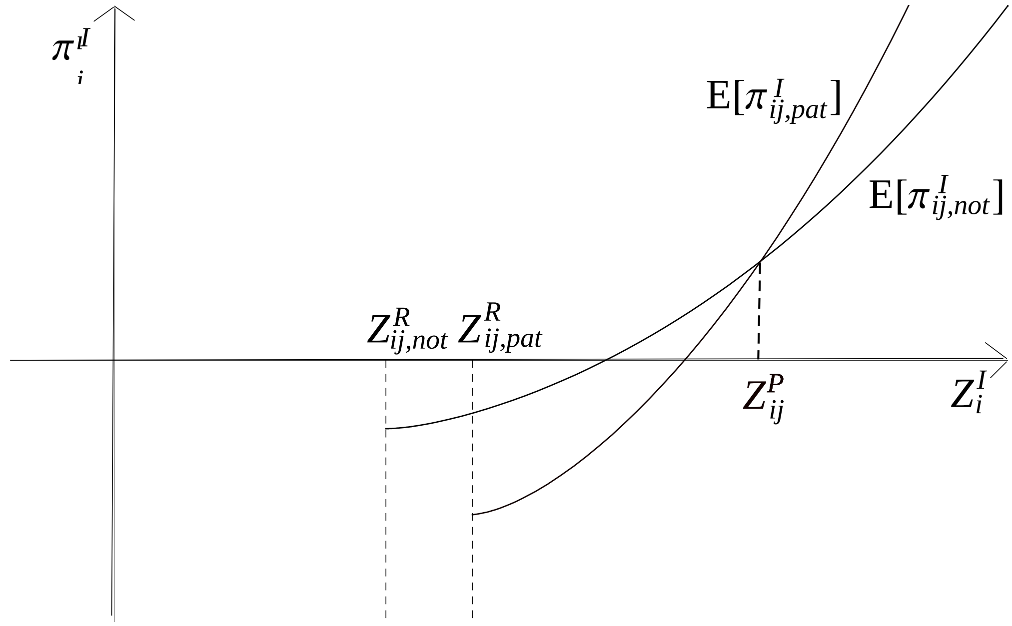


Figure 4: Innovating Firm's Expected Profit

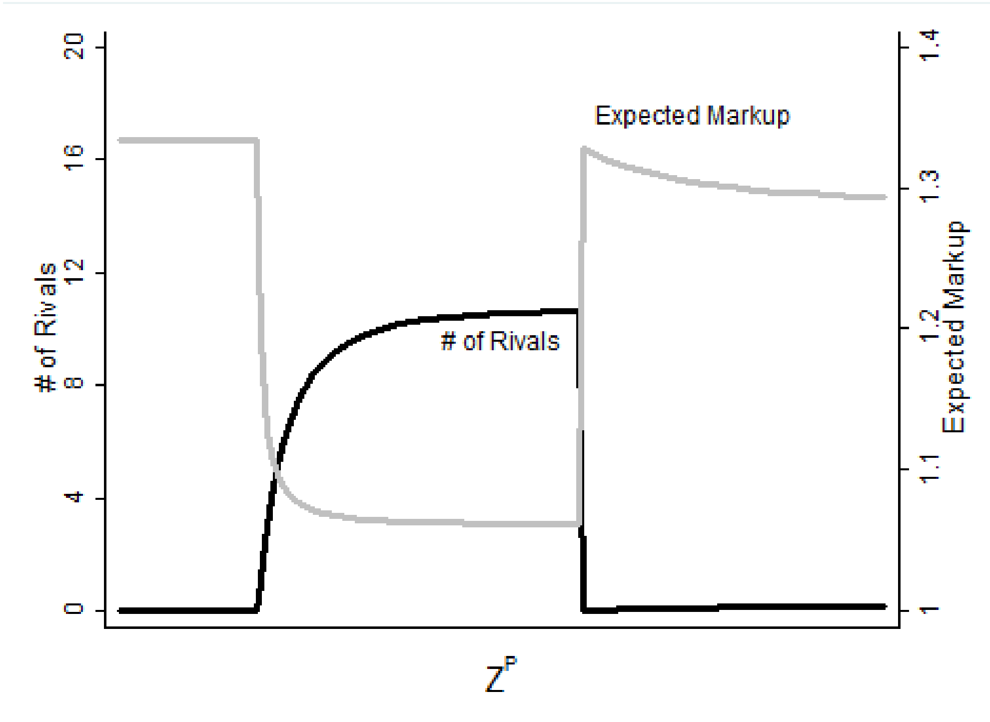


Figure 5: Number of Rivals and Expected Markup by Z, for $\theta = 3.60$ and $\sigma = 4$

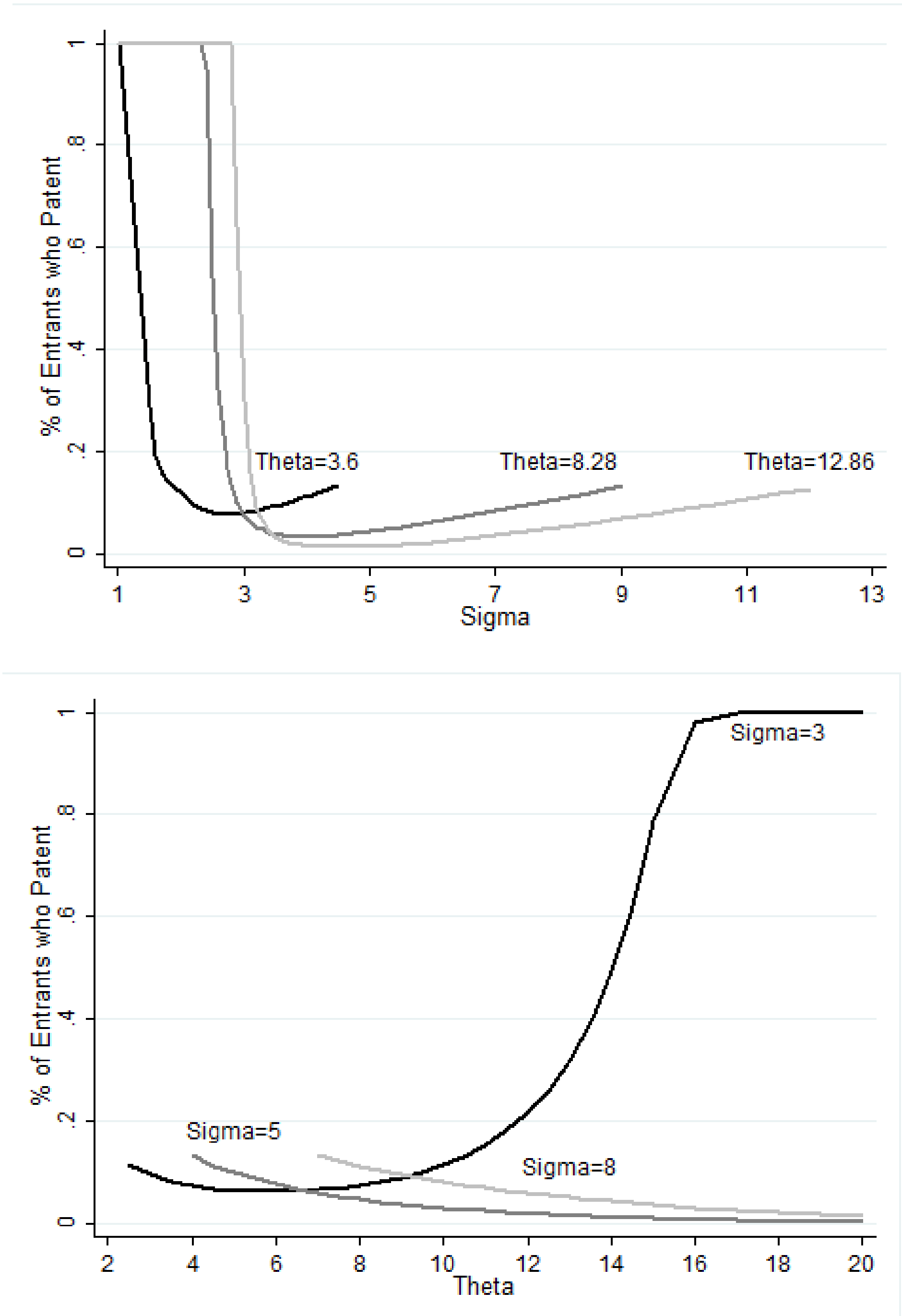


Figure 6: Propensity to Patent for given values of σ and θ

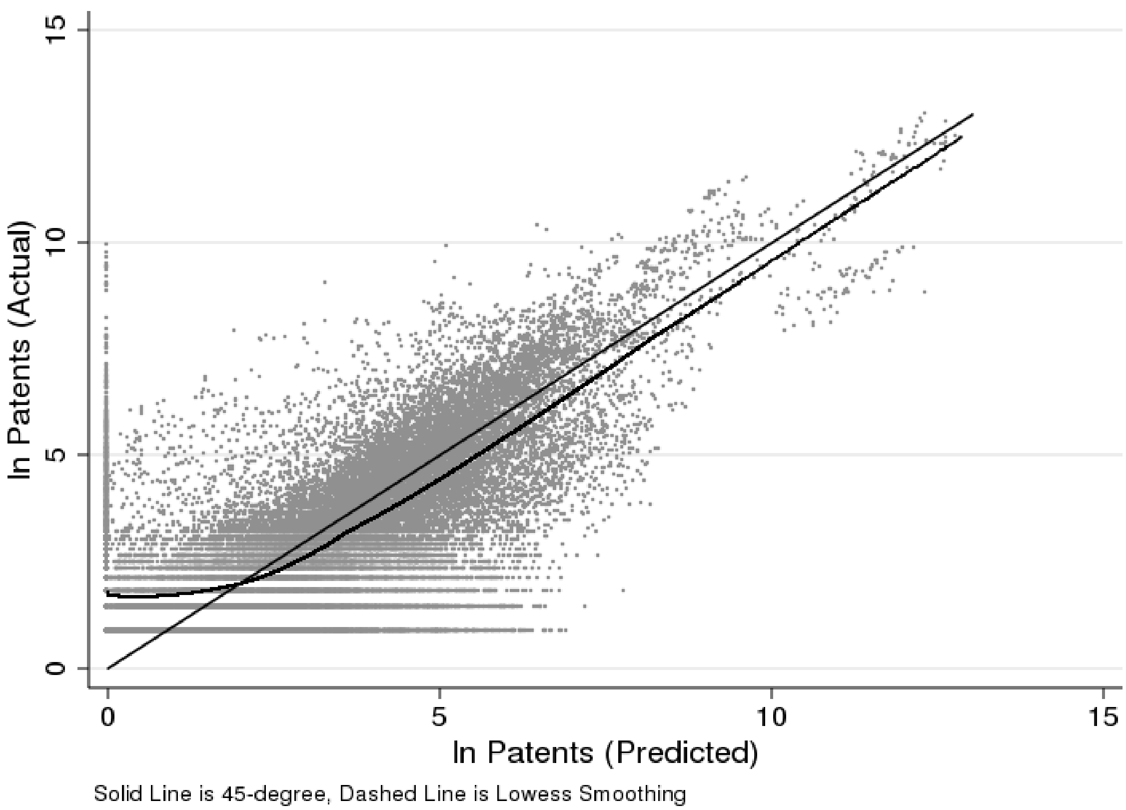


Figure 7: Lowess Smoothing of Predicted Patents and Actual Patents

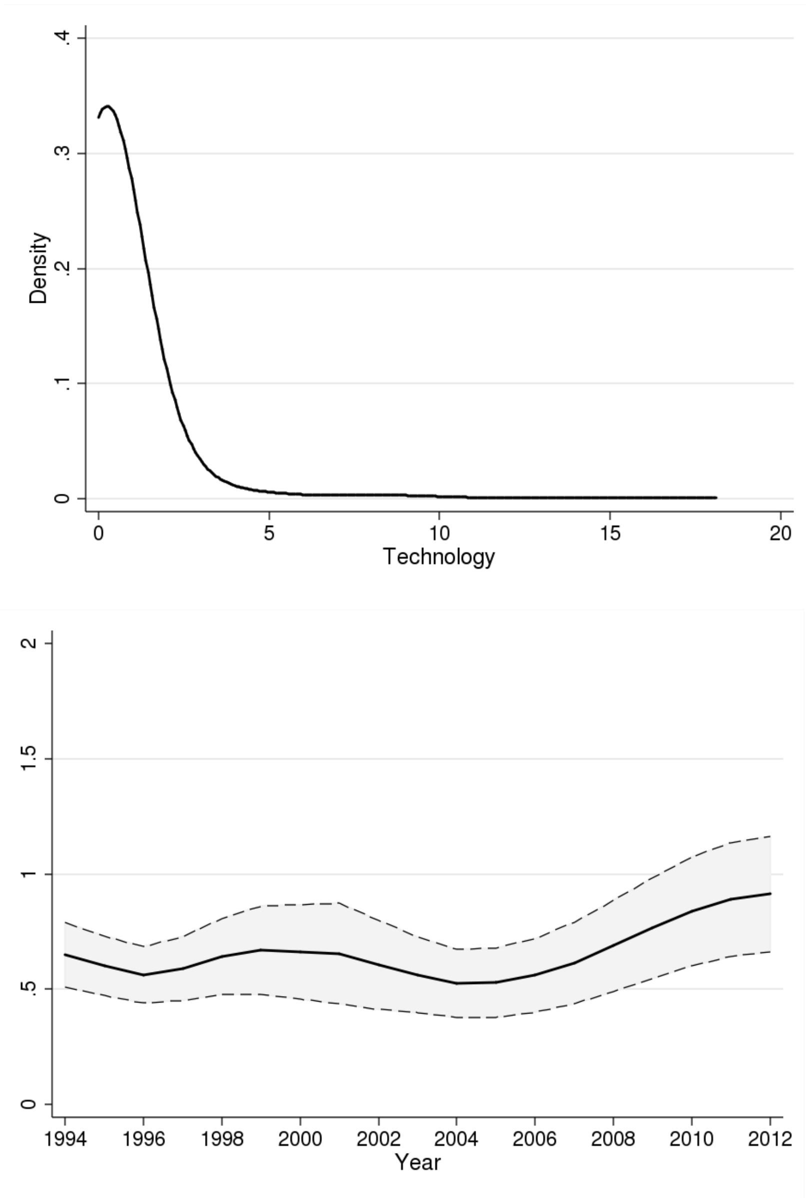


Figure 8: Density and time changes of tech parameter estimates

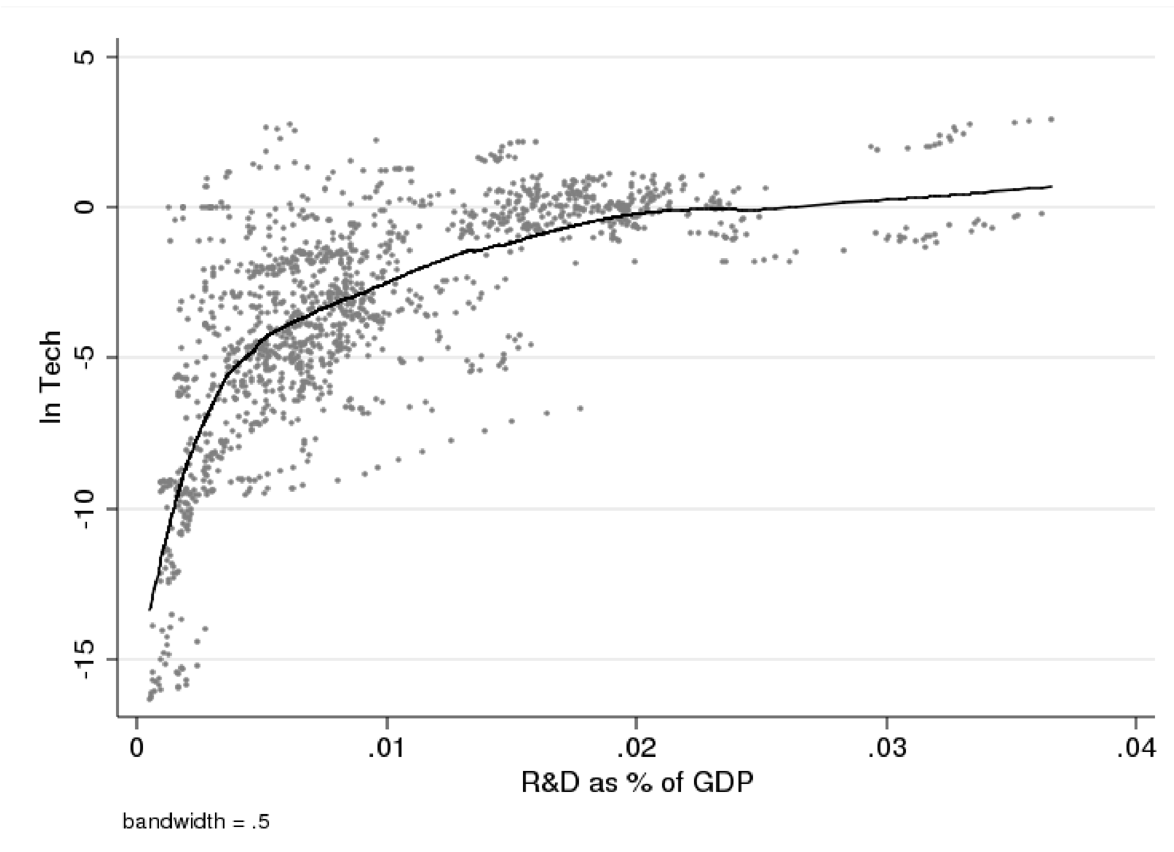


Figure 9: Tech parameter and R&D as Proportion of GDP (Lowess Smoothing)

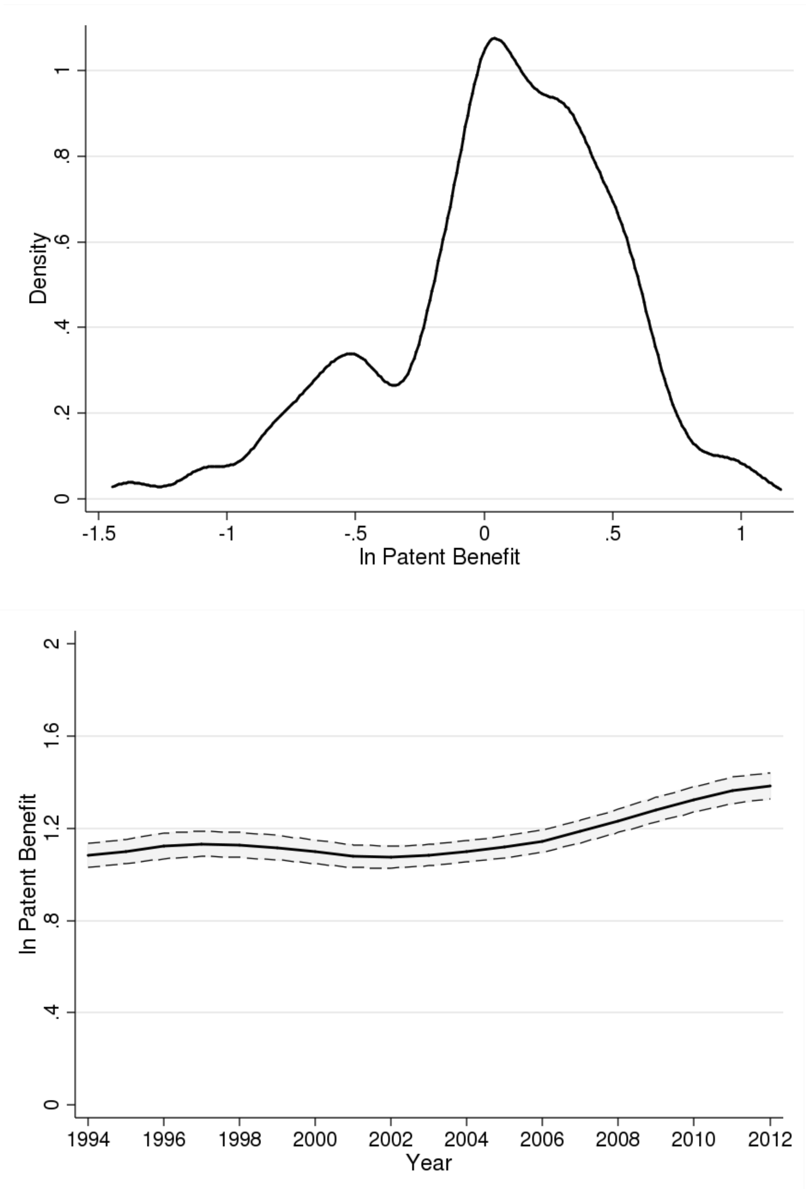
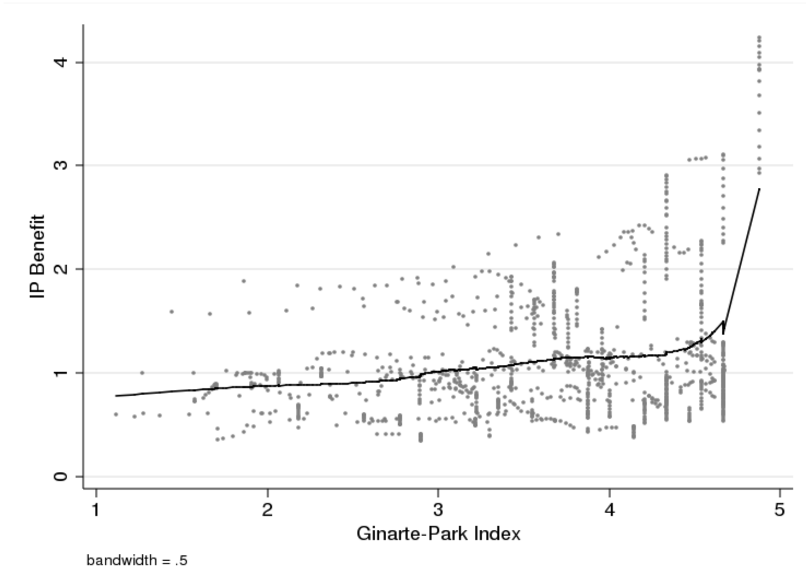
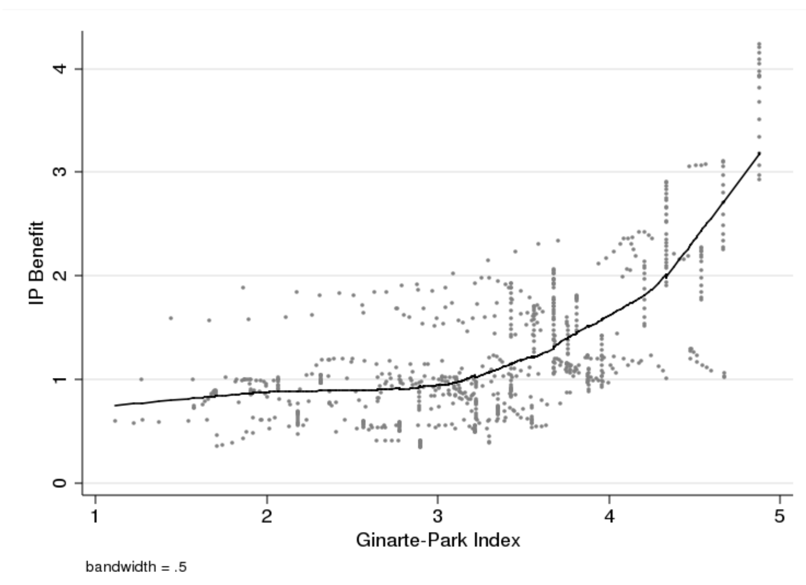


Figure 10: Density and time changes of IP benefit parameter estimates



a. All Countries, 1996-2012



b. No EPO Countries, 1996-2012

Figure 11: IP Benefit and Park & Ginarte IPR Comparison (Lowess Smoothing)

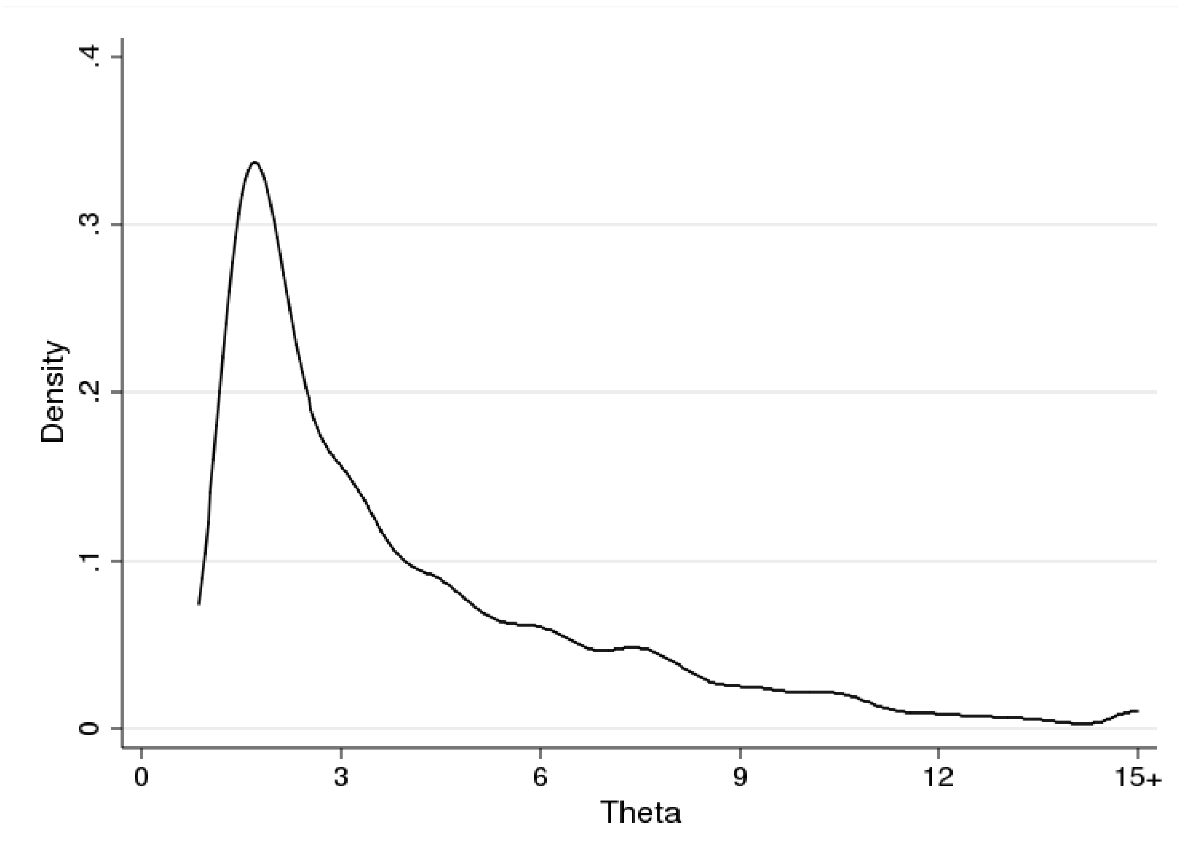
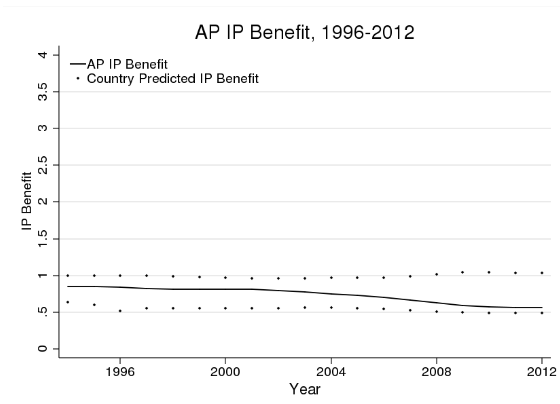
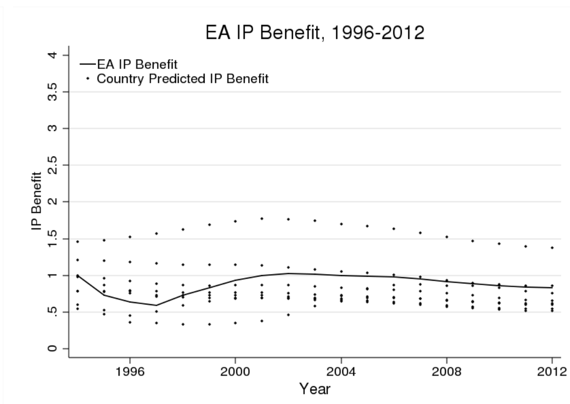


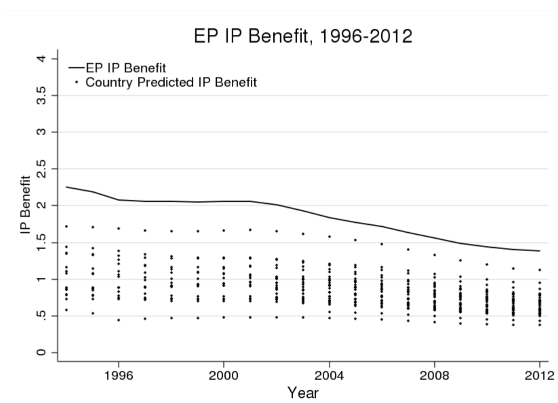
Figure 12: Density Plot of θ



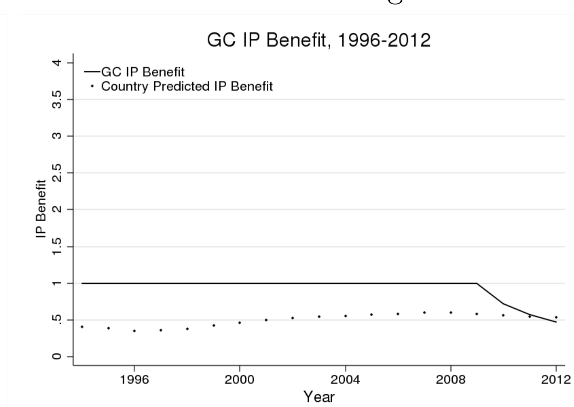
a. ARIPO



b. Eurasian Patent Organization



c. European Patent Organization



d. Gulf Cooperation Council

Figure 13: IP Benefit and International Patent Agreements, 1996-2012

Appendix A. Proofs

Result 1: The number of rivals $r_{ij}(\omega)$ and their average efficiency increases as the state of technology T_j increases.
Proof: First, I show that the number of rivals increases as with T_j:

$$\frac{\partial r_{ij}}{\partial T_j} = w_j^{-\theta} Y_j^R \left[\left(\tilde{c}_{ij}^R\right)^{\theta^k} - \left(c_{ij}^I\right)^{\theta} \right] e^{-T_j w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right]} > 0$$

Next, I show that the productivity of the rivals increases with technology T_j. I do this by showing that for any given cost parameter $c_{ij}^I \leq c'$, the probability that c is less than or equal to c' is increasing with T_j:

$$\frac{\partial \Pr\left[c \leq c^{'} | c_{ij}^I\right]}{\partial T_j} = w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right] e^{-T_j^k w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right]} > 0$$

■

Result 2: The number of rivals $r_{ij}(\omega)$ increases with the cutoff condition of rival entry $\tilde{c}_{ij}^R$.
Proof:

$$\frac{\partial r_i j}{\partial \tilde{c}_{ij}^R} = Y_j^R T_j w_j^{-\theta} \theta \left(\tilde{c}_{ij}^R\right)^{\theta - 1} e^{-T_j w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right]} > 0$$

■

Result 3: The number of rivals $r_{ij}(\omega)$ and their productivity increases with the productivity parameter of the innovating firm.
Proof: I start by first showing that the number of rivals increases as the cost parameter for the innovating firm declines:

$$\frac{\partial r_{ij}}{\partial c_{ij}^I} = -Y_j^R T_j w_j^{-\theta} \theta \left(c_{ij}^I\right)^{\theta - 1} e^{-T_j w_j^{-\theta} \left[\left(\tilde{c}_{ij}^R\right)^{\theta} - \left(c_{ij}^I\right)^{\theta} \right]} < 0$$

So that as c_{ij}^I declines (i.e. the innovating firm is more productive), the number of rivals increases. the Next, I show that for any given productivity $c_{ij}^I \leq c'$, the probability that c is less than or equal to c' is decreasing with c_{ij}^I, meaning that as c_{ij}^I decreases, it is more likely for c to be less than c':

$$\frac{\partial \Pr\left[z \geq z^{'}|z^{I}\right]}{\partial z^{I}} = -T_j w_j^{-\theta}\theta\left(c_{ij}^{I}\right)^{\theta-1} e^{-T_j w_j^{-\theta}\left[\left(\tilde{c}_{ij}^{R}\right)^{\theta}-\left(c_{ij}^{I}\right)^{\theta}\right]} < 0$$

So that the distribution of rival costs when the innovating firm has very low c_{ij}^{I}, first-order stochastically dominates the distribution with higher c_{ij}^{I} implying that the expected value of the rivals' productivity is increasing in z_{ij}^{I}. ■

Result 4: The probability the innovating firm charges the CES markup in country j is decreasing in contestability r_{ij}.

Proof:

$$\frac{\partial \phi_{ij}}{\partial r_{ij}} = \frac{-T_i T_j^{\theta}\left(w_i d_{ij} w_j\right)^{\theta}\left(\overline{m}^{\theta}-1\right)}{\left[r_{ij}T_j\left(w_i d_{ij}\right)^{\theta}\left(\overline{m}^{\theta}-1\right)+T_i w_j^{\theta}\right]^2} < 0$$

■

Result 5: The probability the innovating firm charges the CES markup in country j increases as the cutoff condition for rival entry decreases and decreases as the innovating firm becomes more productive (lower costs).

Proof:

$$\frac{\partial \phi_{ij}}{\partial \tilde{c}_{ij}^{R}} = \underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)}\underbrace{\frac{\partial r_{ij}}{\partial \tilde{c}_{ij}^{R}}}_{(-)} > 0 \qquad \text{and} \qquad \frac{\partial \phi_{ij}}{\partial c_{ij}^{I}} = \underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)}\underbrace{\frac{\partial r_{ij}}{\partial c_{ij}^{E}}}_{(+)} < 0$$

■

Result 6: The innovating firm's expected profit $\mathrm{E}\left[\pi_{ij}^{I}(\omega)\right]$ is decreasing in contestability r_{ij}.

Proof:

$$\frac{\partial \pi_{ij}^{I}}{\partial r_{ij}} = \left(\frac{c_{ij}^{I}}{P_j}\right)^{1-\sigma}\alpha_j w_j Lj\left[\underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} + \underbrace{\frac{\partial\left(1-\phi_{ij}\right)}{\partial r_{ij}}}_{(+)}\underbrace{\frac{\partial\left(\left(\bar{m}_{ij}^{B}\right)^{1-\sigma}\left(1-\left(\bar{m}_{ij}^{B}\right)^{-1}\right)\right)}{\partial r_{ij}}}_{??}\right]$$

So that the sign is going to depend on how the expected markup under Bertrand competition $\bar{m}_{ij}^{B}$ changes with r_{ij}. I show that for any given markup $1 \leq m' \leq \overline{m}$, the probability that m is great than or equal to m' decreases as r_{ij} increases.

$$\frac{\partial \Pr\left[m \geq m'\right]}{\partial r_{ij}} = \frac{-T_i T_j^{\theta}\left(w_i d_{ij} w_j\right)^{\theta}\left(\left(m'\right)^{\theta}-1\right)}{\left[r_{ij} T_j \left(w_i d_{ij}\right)^{\theta}\left(\left(m'\right)^{\theta}-1\right)+T_i w_j^{\theta}\right]^2} < 0$$

This implies that markup m_{ij}^{B} with a small amount of rivals r_{ij} first-order stochastically dominates m_{ij} with a high number of rivals r_{ij}, so that $\frac{\partial\left(\left(\bar{m}_{ij}^{B}\right)^{1-\sigma}\left(1-\left(\bar{m}_{ij}^{B}\right)^{-1}\right)\right)}{\partial r_{ij}} < 0$, which means that $\frac{\partial \pi_{ij}^{I}}{\partial r_{ij}} < 0$, thus completing the proof. ■

Result 7: The price of variety ω charged to consumers in country j is decreasing in contestability r_{ij}

From the price definition (Equation 9), I first compute the moment $1-\sigma$ for the expected marginal costs:

$$\mathrm{E}\left[\left(c_{ij}^{I}\right)^{1-\sigma}\right] = \int_{0}^{\infty}\left(c_{ij}^{I}\right)^{1-\sigma} g_{ij}\left(c_{ij}^{I}\right) dc_{ij}^{I} = \left(T_i \left(w_i d_{ij}\right)^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)$$

And[21]

$$\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right] = \int_{c_{ij}^{I}}^{\infty}\left(c_{ij}^{R*}\right)^{1-\sigma} g_{ij}^{R*}\left(c_{ij}^{R*}\right) dc_{ij}^{R*} = e^{r_{ij}\frac{T_j}{T_i}\left(\frac{w_i d_{ij}}{w_j}\right)^{\theta}}\left(r_{ij} T_j w_j^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}, r_{ij}\frac{T_j}{T_i}\left(\frac{w_i d_{ij}}{w_j}\right)^{\theta}\right)$$

From this, the proof is relatively straightforward.

[21]Note that after integrating $\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right]$, I have

$$\mathrm{E}\left[\left(c_{ij}^{R*}\right)^{1-\sigma}\right] = \int_{c_{ij}^{I}}^{\infty}\left(c_{ij}^{R*}\right)^{1-\sigma} g_{ij}^{R*}\left(c_{ij}^{R*}\right) dc_{ij}^{R*} = e^{r_{ij} T_j w_j^{-\theta}\mathrm{E}\left[\left(c_{ij}^{I}\right)^{\theta}\right]}\left(r_{ij} T_j w_j^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \Gamma\left(\frac{1+\theta-\sigma}{\theta}, r_{ij} T_j w_j^{-\theta}\mathrm{E}\left[\left(c_{ij}^{I}\right)^{\theta}\right]\right)$$

Next, I substitute the expected value $\mathrm{E}\left[\left(c_{ij}^{I}\right)^{\theta}\right]$ which is

$$\mathrm{E}\left[\left(c_{ij}^{I}\right)^{\theta}\right] = \int_{0}^{\infty}\left(c_{ij}^{I}\right)^{\theta} g_{ij}\left(c_{ij}^{I}\right) dc_{ij}^{I} = \frac{\left(w_i d_{ij}\right)^{\theta}}{T_i}$$

to complete the formula.

Proof:

$$\frac{\partial p_{ij}}{\partial r_{ij}} = \left(\underbrace{\frac{\partial \phi_{ij}}{\partial r_{ij}}}_{(-)} \overline{m} + \underbrace{\underbrace{\frac{\partial \left(1-\phi_{ij}\right)}{\partial r_{ij}}}_{(+)} \underbrace{\frac{\partial \bar{m}_{ij}^{B}}{\partial r_{ij}}}_{(-)}}_{(-)} \right) c_{ij}^{I} < 0$$

■

Derivation of Markup Distribution To derive the distribution of the markup m_{ij} it is necessary to look at the distribution of the ratio c_{ij}^{R*}/c_{ij}^{I}. To calculate this, I use the methodology in Nadarajah (2010) who use the following Lemma from Prudnikov et al. (1986)

Lemma 1 (Equation (2.3.1.13), Prudnikov et al. (1986), Vol. 1) For $\gamma > 1$, $a > 0$ and $s > 0$

$$\int_{0}^{\infty} x^{\gamma-1} e^{\left(-sx-ax^{k}\right)} dx = I(\gamma, a, k, s)$$

Where

$$I(\gamma,a,k,s) = \begin{cases} \sum_{j=0}^{q-1} \frac{(-a)^{n}}{j! s^{\gamma+kn}} \Gamma\left(\gamma+kj\right) {}_{p+1}F_{q}\left(1, \Delta\left(\rho, \gamma+kj\right); \Delta\left(q, 1+j\right); (-1)^{q} z\right) & 0 < k < 1 \\ \sum_{h=0}^{p-1} \frac{(-s)^{h}}{kh! a^{(\gamma+h)/k}} \Gamma\left(\frac{\gamma+h}{k}\right) {}_{q+1}F_{p}\left(1, \Delta\left(q, \frac{\gamma+h}{k}\right); \Delta\left(p, 1+h\right); \frac{(-1)^{p}}{z}\right) & k > 1 \\ \frac{\Gamma(\gamma)}{(a+s)^{\gamma}} & k = 1 \end{cases}$$

Where $k = p/q$ and $z = (p^{p}a^{q})/(s^{p}q^{q})$ and $\Delta(v,a) = (a/v, (a+1)/v, ..., (a+v-1)/v)$.

Given the following distributions for c_{ij}^{I} and c_{ij}^{R*}

$$G_{ij}^{I}(c_{ij}^{I}) = 1 - e^{-T_i (w_i d_{ij})^{-\theta} \left(c_{ij}^{I}\right)^{\theta}} \qquad \text{and} \qquad G_{ij}^{R*}(c_{ij}^{R*}) = 1 - e^{-r_j T_j w_j^{-\theta} \left(\left(c_{ij}^{R*}\right)^{\theta} - \left(c_{ij}^{I}\right)^{\theta}\right)}$$

The CDF of $m_{ij} = c_{ij}^{R*}/c_{ij}^{I}$ is:

$$
\begin{aligned}
H(m_{ij}) &= \int_0^\infty G_{ij}^{R*}(c_{ij}^I m_{ij}) g_{ij}^I(c_{ij}^I) dc_{ij}^I \\
&= \int_0^\infty \left[1 - e^{-r_j T_j w_j^{-\theta}\left(\left(c_{ij}^I m_{ij}\right)^\theta - \left(c_{ij}^I\right)^\theta\right)}\right] \theta T_i \left(w_i d_{ij}\right)^{-\theta} \left(c_{ij}^I\right)^{\theta-1} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} \left[1 - e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta}\right] e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= \int_0^\infty g^I(c_{ij}^I) dc_{ij}^I - \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I \\
&= 1 - \theta T_i \left(w_i d_{ij}\right)^{-\theta} \int_0^\infty \left(c_{ij}^I\right)^{\theta-1} e^{-r_j T_j w_j^{-\theta}\left(m_{ij}^\theta - 1\right)\left(c_{ij}^I\right)^\theta} e^{-T_i(w_i d_{ij})^{-\theta}\left(c_{ij}^I\right)^\theta} dc_{ij}^I
\end{aligned}
$$

I make the following substitution

$$
\begin{aligned}
x &= r_j T_j w_j^{-\theta} \left(m_{ij}^\theta - 1\right) \left(c_{ij}^I\right)^\theta \\
a &= \frac{T_i \left(w_i d_{ij}\right)^{-\theta}}{r_j T_j w_j^{-\theta} \left(m_{ij}^\theta - 1\right)}
\end{aligned}
$$

I can now rewrite my equation above as

$$
H(m_{ij}) = 1 - a \int_0^\infty e^{-x} e^{-ax} dx
$$

I can now apply Lemma 1 where $\gamma = 1$, $k = 1$ and $s = 1$ so that the CDF for markup m_{ij} is

$$
H(m_{ij}) = 1 - aI(1, a, 1, 1) = 1 - \frac{a\Gamma(1)}{a+1} = 1 - \frac{a}{a+1} = \frac{1}{a+1} = 1 - \frac{T_i w_j^\theta}{r_j T_j \left(w_i d_{ij}\right)^\theta \left(m_{ij}^\theta - 1\right) + T_i w_j^\theta}
$$

With PDF

$$
h(m_{ij}) = \frac{r_j T_i T_j \theta \left(w_i w_j d_{ij}\right)^\theta m_{ij}^{\theta-1}}{\left[r_j T_j \left(w_i d_{ij}\right)^\theta \left(m_{ij}^\theta - 1\right) + T_i w_j^\theta\right]^2}
$$