Imported Intermediate Goods and Product Innovation: Evidence from India

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Abstract

In this study, we build a structural model of multi-product firms that illustrates how access to new foreign intermediate goods contributes to product innovation. We establish a stochastic dynamic model of firm evolution allowing firms to be heterogeneous in their efficiency levels. Through introducing importing decision to this dynamic framework, we show that the effects of importing intermediate goods are twofold: i) it increases the revenues per each product created and ii) through the knowledge spillovers obtained from importing, firms become more likely to introduce new varieties. Calibration of the model to Indian data shows that the model can successfully explain the dynamics of product evolution and other moments related to importing and product distribution. The comparison of autarky with trade equilibrium shows how liberalizing trade increases innovation performance and product growth. We also measure the direct impact of cost-reducing spillover from imported intermediates on R&D activity. When we do not allow for this spillover channel, R&D effort of importing firms comes down by 33%.

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1 Introduction

An important role of international trade is the exposure of firms to new goods. The purpose of this study is to build an analytical framework to illustrate the potential gains in an economy that international trade can generate as new products are introduced resulting from the access to new foreign intermediate goods. We illustrate this mechanism through a trade liberalization episode. We first show how aggregate innovation rates as well as individual firm's dynamics change after the liberalization period, and then we study counterfactual exercises by highlighting the channels through which imported intermediates affect general equilibrium outcomes.

The process through which international trade expands the set of traded goods has been the subject of recent studies. Kehoe and Ruhl (2013) find that trade liberalization episodes are characterized by a significant increase in the traded volume of goods not previously traded. They further document that such role of the extensive margin of trade seems proper channel of structural changes in the economy (e.g. trade liberalization episodes) and absent from higher frequency events like business cycles. (Arkolakis, 2010) derives a similar result within his model of heterogeneous firms and costly access to trade. In our framework, trade brings access to new intermediate goods previously unavailable to firms, provided they find it profitable to pay the fixed costs associated with international trade.

At the firm level, (Goldberg, Khandelwal, Pavcnik, and Topalova, 2010a) establish a robust relationship between access to new foreign intermediate inputs and the introduction of new products by domestic firms. Using data for India, they find that 31 percent of the expansion of product diversity of firms could be explained by declining input tariffs. Similar to their empirical specification, at the center of our theoretical framework is a set of heterogeneous multi-product firms that endogenously decide on investing in the development of new product varieties. In our model access to new intermediate inputs increase firms' revenues of existing products and they also improve firms' innovation capacity.

(Klette and Kortum, 2004) (KK henceforth), (Lentz and Mortensen, 2008), (Luttmer, 2011), and (Şeker, 2012) have studied firm size and industry dynamics where multi-product firms could innovate. (Bernard, Redding, and Schott, 2010, 2011) (BRS (2010,2011), henceforth) explained several attributes of multi-product firms at the detail of firm-product level. The findings of (Goldberg et al., 2010a) and their other accompanying work have highlighted the link between imported intermediates, and firms product scope. In this paper, we fill the missing link that has not been discussed in any of the above mentioned studies by explaining through a structural model the interaction between innovation and imported intermediates for multi-product firms. Imported intermediates boost innovative effort through two channels in our setup. The first one is the standard "love-of-variety" in production technology[1]. The second and more novel channel is the spillover effect of imported varieties on firm innovative activities.

The concept of innovation used throughout the paper is one of horizontal product innovation; that is, an innovation or discovery consists of the knowledge required to manufacture a new final good that does not displace existing ones. This type of framework is a natural complement to models of vertical innovation where the "creative destruction" process makes previous products obsolete. Horizontal innovation models have been extensively used in growth theory, where (Romer, 1990) and (Grossman and Helpman, 1993) represent seminal studies. Furthermore, it has been applied in trade theory to explain wage inequality, cross-country productivity differences among other topics (see (Gancia and Zilibotti, 2005)). Among disaggregated firm-level models, (Luttmer, 2011) and (Şeker, 2012) are relevant references. Both of these papers follow from KK (2004) which present a stylized model that explains the regularities in firm and industry evolution. In their model, an establishment is defined as a collection of products and each product evolves independently. Every product owned by an establishment can give rise to a new product as a result of a stochastic innovation process or can be lost to a competitor. This birth and death process of the products is the source of firm evolution. Through this model of innovation, they explain various stylized facts that relate R&D, productivity, and growth. The extension introduced by (Şeker, 2012) allows exogenous heterogeneity in firms' efficiency levels which provides a better fit to the data on explaining firm size distribution as well as correlations between size, age, exit and firm

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[1] (Kasahara and Lapham, 2013), (Halpern, Koren, and Szeidl, 2015), (Gopinath and Neiman, 2014) are among several others that have highlighted this channel.

growth[2]. Our basic framework, which draws extensively from (Şeker, 2012), is meant to illustrate the role of international trade in the introduction of new products. It is also relevant to mention that in contrast to frameworks of learning externalities, ours is one in which firms fully internalize the dynamic consequences of their innovation efforts.

In our framework international trade affects both the marginal benefit and the marginal cost of innovation efforts. As in (Şeker, 2012), each firm decides how much resources to invest in innovation, where a higher investment increases the probability of successfully introducing a new product and consequently enjoying a new stream of revenue. In the model, higher variety of intermediate goods benefits a given firm with larger returns to scale (productivity) both in its existing products as well as in any new product. Within the empirical literature, (Kasahara and Rodrigue, 2008) find a robust and significant increase in productivity among Chilean firms that import intermediate goods. (Halpern et al., 2015) find that among Hungarian firms, such gain comes mostly from increased variety of intermediates and to a lesser extent from quality improvements.

In our model, exposure to foreign intermediate goods reduces, ceteris paribus, the cost of innovation. As firms learn from the knowledge embodied in such goods, they could obtain a higher probability of success investing the same amount of resources. Empirical support for this assumption can be found in (Goldberg et al., 2010a). They find that: 1) firms that faced stronger input tariff reductions were found, ceteris paribus, more likely to introduce new products; 2) such firms were more likely to invest in R&D; and 3) the main channel of these effects was through new varieties of intermediate goods and not through the trade expansion of previously traded ones. In this study, we explain these empirical findings through a structural model.

A central element of our analysis is the heterogeneity among firms. Besides their exogenous component of productivity level, firms also differ in their portfolio of goods currently in production. A seminal work in the area of heterogeneous firms and trade is (Melitz, 2003). In his framework, more productive firms are self-selected into export markets as only they find it profitable to pay the fixed costs associated with exporting. In our study, we focus on the importing side of trade rather than exporting. However the

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[2] (Lentz and Mortensen, 2008) also present a model that also introduces heterogeneity in firms' innovation capacities. See (Şeker, 2012) for a comparison of both models.

mechanism is quite similar to the one presented in (Melitz, 2003). Firms face fixed costs of importing. Only the efficient ones can compensate these costs and import goods. Our analysis also highlights that trade liberalization can have a redistribution effect, where some firms could end worse-off (lower profits) after the trade liberalization episode even if they import intermediate goods. We further document a similar redistribution of innovation efforts toward most productive firms away from the least productive. The reallocation of the non-reproducible factor of production, labor, is central for this redistribution effect.

In the model, firms own multiple products and their evolution is roughly determined by the sum of the evolution of each of their products. In this respect, the model complements several existing models explaining product scope. BRS (2010, 2011) provide empirical evidence on how multi-product producers dominate total production in the U.S. economy[3]. Contribution of firms' product margin towards output growth significantly exceeds the contribution of entry and exit. They also construct a static model of multi-product firms and analyze their behavior during trade liberalization. They introduce two margins (intensive and extensive) of growth, and these margins are positively correlated with each other. However, their model lacks a dynamic framework of firm innovation behavior and evolution. The novelty of our study is its ability to analytically explain the evolution of firm's products through a structural model which we test through fitting it to the data on product dynamics and how this relates to importing decision.

We find that, aside from the productivity boost, knowledge spillovers from imported goods increases innovation effort of incumbent firms such that when we shut down the spillover channel innovation effort of importer firms decline by more than 33%. However, at the same time, knowledge spillovers from trade reduces innovation effort of entrants.

The remainder of the paper is organized as follows. Section 2 describes the model economy in two main parts: 1) selection of the mix of intermediate goods and labor, as well as firm's decision whether to import intermediate goods from abroad; 2) the selec-

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[3] Some other studies on multi-product firms are (Nocke and Yeaple, 2006), (Eckel and Neary, 2010), and (Baldwin and Gu, 2009).

tion of the optimal level of innovation. Section 3 describes the data source, section 4 presents the quantitative model and the calibration exercise, section **??** discusses counterfactual analyses, and section 7 discusses further topics. Section 8 concludes. We present detailed derivations of some of the key equilibrium conditions of the model in the appendix.

2 Model

Empirical evidence presented in (Goldberg, Khandelwal, Pavcnik, and Topalova, 2009; Goldberg et al., 2010a) show that imported intermediate goods lead to higher innovation rates of new products and hence faster growth. In this study, we present a structural model of firm evolution that can explain this evidence as well as other stylized facts on the dynamics of multi-product firms. Following KK (2004) and their extensions by (Lentz and Mortensen, 2008) and (Şeker, 2012) we introduce a model of firm growth that accounts for the heterogeneity in producers' efficiency levels as well as entry and exit decisions. In introducing the heterogeneity in efficiency levels, we follow (Melitz, 2003). There is a continuum of final good producers each of which produce a different variety. Unlike (Melitz, 2003), here firms produce multiple products. The dynamic nature of the model with heterogeneous firms allows us to capture the differences in the evolution of importers and non-importers and contribution of importing to new product creation and consequently the aggregate growth.

The model is presented in a general equilibrium framework. First, we discuss the demand side of the economy. Then, we introduce the static profit maximization problem of producers. We incorporate this static problem into a dynamic framework to discuss the growth patterns of firms distinguishing the differences between importers and non-importers.

2.1 Consumers

Consider an economy with $N+1$ identical countries. In a continuous time setup, each country consists of a representative consumer with an inter-temporal utility function given as

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \ln C_\tau d\tau,$$

where ρ is the discount rate and C_t is the aggregate consumption of the composite good at time t. Instantaneous utility obtained from consumption at time t is $\ln C_t$. The consumer is free to borrow or lend at the interest rate r_t. Aggregate expenditure at time t is $E_t = P_t C_t$ where P_t is the price of the composite good. The optimization problem of the consumer yields $\frac{\dot{E}}{E} = r_t - \rho$. We let total expenditure be the numeraire and set it to a constant for every period.

In each country there is a mass of available products each of which is indexed by j. The total mass of products is represented by J. Consumers have a taste for variety and consume $y_t(j)$ units of variety j. Goods are substitutes with elasticity of substitution $\sigma > 1$. The composite good is determined by the following constant elasticity of substitution aggregator.

$$C_t = \left(\int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Composite good producer makes zero profit. Solution of this problem yields

$$y_t(j) = C_t \left(\frac{p_t(j)}{P_t} \right)^{-\sigma}.$$

Price of the composite good P_t can be found as follows

$$\begin{aligned} P_t C_t &= \int p_t(j) \, y_t(j) \, dj \\ P_t &= \left(\int_{j \in J} p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \end{aligned} \tag{2}$$

2.2 Intermediate Goods Producers

In the intermediate goods sector, firms are perfectly competitive. All firms have identical linear technologies and have the same productivities which are set as one. Labor is the only input used in the production. There is a unit continuum of domestic intermediate goods produced within a country. There is also free entry in this sector. Each intermediate good producer is a price taker hence price of these products equals to marginal cost which is the wage rate w. Interaction among the intermediate goods producers (and between intermediate and final goods producers, as explained below) happens through the labor market as they compete for the same resource.

2.3 Final Goods Producers: Import Decision

In this section, we solve the producer's decision problem regarding its importing status. Producers are characterized by their efficiency level φ, the number of products in their portfolio n and by their importing status: m if they decide to import some of their intermediate goods or h if they decide not to import intermediate goods. There are two main decisions a given firm takes, 1) whether to import or not, and 2) the amount of resources to invest in innovation, having as its ultimate goal to maximize the present discounted value of profits, $rV_{\varphi}(n)$. We assume that firms decide whether to import when they introduce their first product and that such decision is permanent. This assumption is analogous to deciding to import in every period for every product variety produced.

Our specification of the production function is similar to (Kasahara and Lapham, 2013) which extends (Melitz, 2003) by incorporating importing decision to the firm's optimization problem. The difference in our model is that here firms produce multiple products. The final goods sector is combined of a continuum of monopolistically competitive firms producing horizontally differentiated goods. The production of each variety requires employment of labor and intermediate goods which might be either domestically produced or imported. Producers of the final goods are distinguished from each other by their efficiency levels, indexed by $\varphi > 0$ which is randomly drawn from a continuous cumulative distribution $F(\varphi)$. As in (Melitz, 2003) higher efficiency level means producing a symmetric variety at a lower marginal cost. We assume that efficiency levels exogenously grow at rate g for all firms. Solution of the monopolistic competition model yields revenue $r(\varphi)$ and profit $\pi(\varphi)$ from each product j as follows[4]

$$\begin{aligned} r(\varphi) &= p(\varphi)\,y(\varphi) = E\left(\frac{p(\varphi)}{P}\right)^{1-\sigma} \text{ for } \forall\, j \in J \\ \pi(\varphi) &= \frac{r(\varphi)}{\sigma}. \end{aligned} \tag{3}$$

Each product is produced by a single producer, hence we can write the equilibrium levels of revenue and profit from each product as a function of firm's efficiency level φ.

Firms employ labor and intermediate goods in the production of final goods. We follow the macro-growth and trade literature and assume that there is increasing returns to variety in intermediate goods. All intermediate goods enter symmetrically in the production function, implying that none of them is intrinsically better or worse than any other. Similar approach has been used in (Ethier, 1982) and (Romer, 1987) among many others[5]. This specification is consistent with the empirical findings in (Amiti and Konings, 2007), (Halpern et al., 2015), and (Kasahara and Rodrigue, 2008) which associate the use of foreign intermediate goods with higher productivities.

Importing firms incur a fixed cost f_m and iceberg transport cost $\tau > 1$. Let $I \in \{0, 1\}$ refer to the import status of the firm. Production function for final goods is given as

$$y(\varphi, I) = \varphi l^{\alpha}\left[\int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}}\,dj + I\int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}}\,dj'\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}, \tag{4}$$

where l shows the labor, $q_d(j)$ shows the domestic and $q_f(j)$ shows the imported intermediate goods employed in production and I is equal to one if the firm imports. The output elasticity of labor in production is represented by $0 < \alpha < 1$ and the elasticity of substitution between intermediate inputs is $\gamma > 1$. N is the number of partner countries that are traded with.

The solution of the final good producer's static optimization problem is given in the appendix. In the solution we get $q_d(j) = q_d$, $q_f(j) = q_f$ for $\forall\, j$ and $q_f = \tau^{-\gamma} q_d$. This finding

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[4] Since in this section we only solve for firm's static optimization problem, we exclude the time subscript t from the variables for brevity.

[5] See (Kasahara and Lapham, 2013) for a discussion of the benefits of using this specification.

simplifies the production function to

$$y\left(\varphi, I\right)=\varphi\left(1+N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}}l^{\alpha}\left[q_{d}+IN\tau q_{f}\right]^{1-\alpha}.$$

Here $\varphi\left(1+N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}}$ could be interpreted as a total factor productivity term. Price of a final good which only uses domestic intermediate goods is $p^{h}\left(\varphi\right)$ for a $\varphi-$type producer. This price is found as

$$p^{h}\left(\varphi\right)=\frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varphi}.$$

If imported intermediates are used in production of the good, then the price is $p^{m}\left(\varphi\right)=p^{h}\left(\varphi\right)/(1+N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}$. Since $(1+N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}>1$, $p^{m}\left(\varphi\right)<p^{h}\left(\varphi\right)$ for $\forall\ \varphi$. Once the prices are determined, it is straightforward to find revenues gained by importing and non-importing firms. Define $r^{h}\left(\varphi\right)$ as the per-product revenue generated by a $\varphi-$type firm that does not import which is calculated as

$$r^{h}\left(\varphi\right)=\left(\frac{p\left(\varphi\right)}{P}\right)^{1-\sigma}E=\left(\frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varphi P}\right)^{1-\sigma}E. \tag{5}$$

Similarly, per-product revenue of an importing firm $r^{m}\left(\varphi\right)$ is

$$r^{m}\left(\varphi\right)=Z_{m}r^{h}\left(\varphi\right), \tag{6}$$

where $Z_{m}=(1+N\tau_{m}^{1-\gamma})^{\frac{(1-\alpha)(\sigma-1)}{\gamma-1}}>1$. From equations 5 and 6 we see that revenue is proportional to the firm's efficiency level. Defining $\pi_{i}^{h}\left(\varphi\right)$ and $\pi_{i}^{m}\left(\varphi\right)$ as the profit level of a non-importing and an importing firm respectively (*the latter net of the fixed cost of importing* $wf_{m}N$) we are now ready to describe the importing decision problem.

As mentioned earlier, beside its effects on productivity, importing intermediates also affects firms' innovation technology so that the decision to import is also of a dynamic nature. Firms basically contrast the fixed costs necessary to import intermediates, with not only the extra productivity today but also with the benefit of increasing their prospects for introducing new products in the future. That is, both today's and future profits would be affected by the import decision (equations 7 and 8). To illustrate this point, consider the import decision of a firm with one product. In the following equations $V_{\varphi}(\mathbf{1})$ represents the value of a firm with one product and efficiency level φ (when characterizing the solution of the model we will also denote such value as $v(\varphi)$ whenever there is no risk of confusion).

$$rV_{\varphi}(\mathbf{1}) = \max\left[rV_{\varphi}^{m}(\mathbf{1}), rV_{\varphi}^{h}(\mathbf{1})\right] \tag{7}$$

$$rV_{\varphi}^{j}(\mathbf{1}) = \max_{\lambda \geq 0} \left\{ \begin{array}{c} \pi^{j}(\varphi) - wc^{j}(\lambda) + \\ \lambda\left[V_{\varphi}(\mathbf{2}) - V_{\varphi}(\mathbf{1})\right] \\ +\mu\left[-V_{\varphi}(\mathbf{1})\right] \end{array} \right\}, j = m, h \tag{8}$$

In the following sections, we apply numerical methods to compute the cutoff efficiency level for importing φ_m^* so that firms for which $\varphi \geq \varphi_m^*$ will find it profitable to import.

2.4 Final Goods Producers: Innovation Decision

The dynamic optimization problem of the firm follows from (Şeker, 2012) which is an extension of KK (2004). The monopolistic competition in the final goods sector results in each product being produced by a single firm. Firms can produce multiple varieties. The number of products n determines the portfolio of the producer. This portfolio increases by innovating new product varieties and it decreases by destruction of the existing products [6]. This process determines evolution of the firm.

2.4.1 Innovation technology

The innovation production function determines the Poisson rate of arrival of the next product, based on the firm's investment in research and development (R&D) R, which is measured in labor units, and knowledge capital. The key distinction between importers and non-importers is the interaction of knowledge capital n with the imported intermediate goods.

The innovation production function is strictly increasing and strictly concave in R. It is strictly increasing in the knowledge capital and homogeneous of degree one in R

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[6] An alternative way of how innovations materialize in the economy is quality improvements. In KK (2004) and (Lentz and Mortensen, 2016) innovations arrive in this way.

and knowledge capital. For non-importing firms knowledge capital is measured by the total knowledge accumulated through past innovations. We use the number of products innovated n to represent this capital stock. For the importing firms, knowledge can be accumulated through their own past innovations as well as through knowledge embodied in the imported intermediate goods, $(1+N)^{\zeta} n$.

(Grossman and Helpman, 1993) discuss several ways in which international knowledge spillover is possible. They argue that residents of a country may find occasions to learn technical information from meeting with foreign counterparts that contributes to their stock of general knowledge. Also the use of differentiated intermediate goods that are not available in the domestic market can increase the insights that local producers gain from inspecting and using these goods. (Grossman and Helpman, 1993) use the cumulative volume of trade between countries as the international spillover for innovation. We use a function of the total flow of intermediate goods that are imported. We measure the magnitude of the spillover by $\zeta > 0$.

(Goldberg et al., 2010a) provide empirical evidence that motivates the inclusion of imported intermediate goods in the innovation function. They show that larger availability of foreign intermediate products have increased the innovation capacity of firms in the Indian manufacturing sector. They find that reduction of input tariffs led to imports of new varieties in the economy and this has led to an expansion of within-firm product scope by almost 8 percent.

As in KK (2004), innovations arrive at a Poisson rate represented as $I = \lambda n$, where λ would be denoted the innovation intensity. The innovation function for an importing firm can be formalized as

$$\begin{aligned}
\lambda n &= R^{\theta}\left((1+N)^{\zeta} n\right)^{1-\theta} \text{ where } 0<\theta<1 \text{ and } \zeta>0 \\
\lambda n &= R^{\theta} n^{1-\theta} (1+N)^{\zeta(1-\theta)} \\
R &= \left(\frac{\lambda}{(1+N)^{\zeta(1-\theta)}}\right)^{\frac{1}{\theta}} n
\end{aligned}$$

If we define $1 + c_1 = \frac{1}{\theta} > 1$ then $\frac{1-\theta}{\theta} = c_1 > 0$. We can rewrite R&D investment as

$$R = \frac{\lambda^{1+c_1}}{(1+N)^{\zeta c_1}} n = c(\lambda) n$$

This equation captures that higher use of imported intermediate goods reduces the cost of R&D and therefore increases the probability of innovating new varieties for a given level of R&D spending. In the calibration we parametrize this equation for importers as $wc^m(\lambda)\, n = wc_0 \frac{\lambda^{1+c_1}}{(1+N)^{\zeta c_1}} n$ for $c_0,\ c_1 > 0$ where c_0 is a scale factor. The R&D cost for the non-importing firm is same as the cost for importing firm without the spillover term $wc^h(\lambda)\, n = wc_0 \lambda^{1+c_1} n$.

2.4.2 Bellman equation

Each producer also faces a Poisson hazard rate μ of losing any of its products capturing that products in practice get obsolete. A similar interpretation is presented in (Luttmer, 2011). He assumes that producer of a differentiated commodity needs a blueprint to produce and these blueprints depreciate in a one-hoss-shay fashion. While each firm takes the value of μ as given, its equilibrium value, would be determined by the aggregate creation in the economy which is endogenously determined in the model. Firms exit if all of their products are destroyed. There is no re-entering once exit occurs.

Assuming a constant interest rate, r, the Bellman equation for a producer of efficiency, φ, and number of products, n, is formulated as follows[7]

$$rV_\varphi^j(\mathbf{n}) = \max_{\lambda \geq 0} \left\{ \begin{array}{c} \pi^j(\varphi)\, n - wnc^j(\lambda) + \\ n\lambda \left[V_\varphi^j(\mathbf{n}+\mathbf{1}) - V_\varphi^j(\mathbf{n}) \right] \\ +\mu n \left[\left(V_\varphi^j(\mathbf{n}-\mathbf{1}) - V_\varphi^j(\mathbf{n}) \right) \right] \end{array} \right\}, for\ j = m, h \tag{9}$$

This equation shows that current value of firm is equal to the sum of three terms. The first term on the right hand side shows the current profit net of R&D costs. The other two terms show the net future value of the firm. The second one is the gain in value caused by the innovation of a new variety and the last one is the expected loss associated with

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[7] Constancy of interest rate is obtained from the consumer's optimization problem.

a loss of a randomly chosen product[8].

Since the value function is linear in the number of products, it is possible to obtain an analytical solution to optimization problem. Similar to (Lentz and Mortensen, 2008), we conjecture that the value function is

$$V_{\varphi}^{j}(\mathbf{n}) = \left(\frac{\pi^{j}\left(\varphi\right)}{r+\mu} + \Theta^{j}\left(\varphi\right)\right) n \tag{10}$$

where $\Theta\left(\varphi\right)$ is the type conditional continuation value of innovation. Then we incorporate this conjecture into the Bellman equation which simplifies to

$$\begin{aligned} \left(r+\mu\right)\Theta^{j}\left(\varphi\right) &= \max_{\lambda\geq 0}\left\{\lambda\left(\frac{\pi^{j}\left(\varphi\right)}{r+\mu} + \Theta^{j}\left(\varphi\right)\right) - wc^{j}\left(\lambda\right)\right\} \\ \Theta^{j}\left(\varphi\right) &= \max_{\lambda\,\geq\,0}\left\{\frac{\frac{\lambda\pi^{j}(\varphi)}{r+\mu} - wc^{j}\left(\lambda\right)}{r+\mu-\lambda}\right\}. \end{aligned} \tag{11}$$

In order to have the conjectured value function solve this problem, all variables have to be stationary. In the appendix we show how the economy grows on a balanced growth path. Solving the simplified Bellman equation, we get

$$\frac{\pi^{j}\left(\varphi\right)}{r+\mu} + \Theta^{j}\left(\varphi\right) = wc^{j\prime}(\lambda). \tag{12}$$

We define $v^{j}\left(\varphi\right) = \frac{\pi^{j}(\varphi)}{r+\mu} + \Theta^{j}\left(\varphi\right)$ as the expected value of a single product for a $\varphi-$type producer. It is the sum of discounted stream of the profits and the innovation option value. Using the value of $\Theta^{j}\left(\varphi\right)$ from equation 11 and implementing it into equation 12 we get

$$wc^{j\prime}\left(\lambda\right) = \frac{\pi^{j}\left(\varphi\right) - wc^{j}\left(\lambda\right)}{r+\mu-\lambda}. \tag{13}$$

KK (2004) show that the optimal value of λ is an increasing function of profit level. Since the profit level monotonically increases in φ, this result shows that firms with high efficiencies are more likely to innovate. Plugging the value of $c^{j}\left(\lambda\right) = \frac{c_0\lambda^{1+c_1}}{(1+N)^{\zeta c_1}}$ for

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[8] Existence of a solution for a similar dynamic optimization problem under heterogeneous firms is proven in (Lentz and Mortensen, 2005).

an importing firm into this equation we get,

$$\begin{aligned} \frac{wc_0 \left(1+c_1\right) \lambda^{c_1}}{\left(1+N\right)^{\zeta c_1}} &= \frac{\pi^m \left(\varphi\right) - w \frac{c_0 \lambda^{1+c_1}}{(1+N)^{\zeta c_1}}}{r+\mu-\lambda} \\ wc_0 \left(1+c_1\right) \lambda^{c_1} &= \frac{\left(1+N\right)^{\zeta c_1} \pi^m \left(\varphi\right) - wc_0 \lambda^{1+c_1}}{r+\mu-\lambda} \text{ for } \varphi > \varphi_m^*. \end{aligned} \tag{14}$$

Similarly for the non-importing firm the solution is

$$wc_0 \left(1+c_1\right) \lambda^{c_1} = \frac{\pi^h \left(\varphi\right) - wc_0 \lambda^{1+c_1}}{r+\mu-\lambda} \text{ for } \varphi \leq \varphi_m^*.$$

A key difference between the solution for importing and non-importing firms is the spillover term which is captured by $\left(1+N\right)^{\zeta c_1} > 1$. It enters into the equation as a positive multiplier of the profit level. Hence importing gives extra advantage to firms to innovate. To guarantee the existence of a stationary size distribution in equilibrium the condition $\mu > \lambda\left(\varphi\right)$ for all efficiency levels φ must hold. If the innovation rate of a firm exceeds or becomes equal to the aggregate destruction rate, size and age of some firms diverge to infinity which precludes having a stationary size distribution[9]. This condition can be written as an upper bound on the efficiency type distribution which is presented in the appendix.

2.5 Entrant's Problem

From a constant potential pool of entrants, successful ones enter the economy as a result of an innovation of a new variety. Firms discover their efficiency types immediately after they enter. Entrants face the same innovation cost function as incumbent firms and they innovate at rate λ_e. Entry rate is determined by the free entry condition given as

$$\underbrace{wc'\left(\lambda_e\right)}_{\text{marginal cost of innovation}} = \underbrace{\int v\left(\varphi\right) \phi\left(\varphi\right) d\varphi}_{\text{net gain from innovation}}. \tag{15}$$

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[9] The condition needed to guarantee $\mu > \lambda\left(\varphi\right)$ for $\forall\ \varphi$ is $wc'\left(\mu\right) > \frac{\pi - wc(\mu)}{r}$ which follows from equation 13.

Here $\phi(\cdot)$ is type distribution for entrants and $v(\varphi)$ is the value of a single product for a $\varphi-$type producer. Entry rate η is the product of mass of potential entrants M_e and innovation rate of entrants λ_e, $\eta = M_e\lambda_e$. From incumbent firm's optimization problem we had $wc'(\lambda(\varphi)) = v(\varphi)$. Incorporating this result in equation 15 we can solve for the innovation rate of entrants as a function of innovation rates of incumbent firms

$$\begin{aligned} wc_0(1+c_1)\lambda_e^{c_1} &= \int wc'(\lambda(\varphi))\phi(\varphi)\,d\varphi \\ \lambda_e &= \left[\int \lambda(\varphi)^{c_1}\phi(\varphi)\,d\varphi\right]^{1/c_1} \\ &= \left[\int_0^{\varphi_m^*} \lambda(\varphi)^{c_1}\phi(\varphi)\,d\varphi + \int_{\varphi_m^*}^{\infty} \frac{\lambda(\varphi)^{c_1}}{(1+N)^{\zeta c_1}}\phi(\varphi)\,d\varphi\right]^{1/c_1}. \end{aligned}$$

This result shows that keeping all else constant, higher knowledge spillover from trade decrease the innovation rate of entrants, hence reduces the entry rate. This is because in an environment with higher knowledge spillover a firm can attain the same gain from an innovation with lower R&D investment.

2.6 General Equilibrium

Firms evolve as a result of a birth and death process of products. We define $M_n(\varphi)$ as total mass of φ-type firms with n products. $M(\varphi) = \sum_{n=1}^{\infty} M_n(\varphi)$ is total mass of φ-type firms. Then $\delta(\varphi) = M(\varphi)/M$ is defined as the steady state type distribution where M is the total mass of firms. Mass of products produced by $\varphi-$type firms is represented as $\Lambda(\varphi) = \sum_{n=1}^{\infty} nM_n(\varphi)$. In steady state, the rate of product destruction μ should be equal to the sum of the product creation rates of entrants and incumbent firms

$$\mu = \eta + \int \lambda(\varphi)\Lambda(\varphi)\delta(\varphi)\,d\varphi. \tag{16}$$

In equilibrium, total mass of products produced by $\varphi-$type firms is found as[10]

$$\Lambda(\varphi) = \frac{\eta\phi(\varphi)}{\mu - \lambda(\varphi)}. \tag{17}$$

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[10] For completeness of the model, derivation of this equation is reproduced in the appendix.

In the appendix, we also derive the entry type distribution $\phi(\cdot)$ as a function of the steady state type distribution $\delta(\cdot)$ .The final equilibrium condition is labor market clearing condition.

2.6.1 Labor Market Equilibrium

There is a fixed measure of workers in the economy which is denoted as $L.$ Labor is allocated across four activities for every φ-type incumbent firm: final good production l_f, intermediate good production l_i, R&D investment $c(\lambda)$, and fixed cost of importing Nf_m. There is also a part of the labor force allocated to research and development for potential entrants $c(\lambda_e)$. The labor market clearing condition is stated as

$$L = \int_0^\infty (l_f(\varphi) + l_i(\varphi) + c(\lambda(\varphi)) + I(\varphi) Nf_m) \Lambda(\varphi) \delta(\varphi) d\varphi + M_e c(\lambda_e) \tag{18}$$

where $I(\varphi) = 1$ for $\varphi > \varphi_m^*$. In our setup $l_i(\varphi)$ is increasing in firm efficiency φ regardless of the import decision of the firm. Labor employed in R&D activity $c(\lambda(\varphi))$ is also monotonically increasing in φ[11].

Given these equilibrium conditions, a stationary equilibrium for this economy consists of aggregate destruction rate μ, wage rate w, and interest rate r such that for given values of (μ, w, r) , $i)$ any $\varphi-$type incumbent producer chooses the optimal innovation rate $\lambda(\varphi)$, decides on whether to import or not, and solves equation 13 to maximize its value, $ii)$ potential entrants choose λ_e in solving equation 15 and break even in expectation, $iii)$ representative consumer maximizes utility subject to budget constraint, $iv)$ labor market clears as in equation 18, and $v)$ equations 16 and 17 hold. The equilibrium level of aggregate price index decreases with rate g as it is derived in the appendix.

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[11] In Lentz and Mortensen (2008), since producing any intermediate input requires labor and capital in fixed proportions more profitable firms not necessarily employ relatively more labor. In their setup, labor demand for production of a product is decreasing in product profitability whereas labor demand for innovation increases in product profitability and dominant one of the two forces determines the direction of net impact of firm profitability on labor demand.

3 Data

The data used for the analysis is obtained from the Prowess database which is collected by Center for Monitoring the Indian Economy (CMIE). The data is constructed as a panel of relatively large firms spanning the period from 1989 to 1997. These firms account for 60% to 70% of the all economic activity in India. We restrict our analysis to manufacturing firms. Prowess records detailed product-level information at firm level and it enables us to track firm's product mix over time. This information is available for 2927 firms which correspond to 85% of the manufacturing firms and 90% of total output in Prowess. The data on product mix of firms allows us to test the model's ability to explain product evolution of firms.

Definition of a product is based on the CMIE's internal product classification which is based on the Harmonized System and National Industry Classification. This data on product-mix of firms has been used in several studies by (Goldberg et al., 2009; Goldberg, Khandelwal, Pavcnik, and Topalova, 2010b; Goldberg et al., 2010a). (Goldberg et al., 2010b) in an unpublished appendix explain data cleaning process in obtaining the product level data. We follow their methodology in getting this information. They define 1886 products linked to 108 four-digit NIC industries. In the data multi-product firms account for almost half of the firms and they account for 80% of the output.

4 Quantitative Model and Calibration

The model is calibrated to match the data from Indian manufacturing sector for 1989-1997 time period. The novel contribution of the model is explaining the product evolution of firms and how this evolution is related to importing intermediate goods. To test the model's ability to explain the data we choose eight data moments as presented in Table 1. The moments that relate to product distribution help identify efficiency-type distribution and innovation cost parameters. First two moments are the mean and standard deviation of product distribution[12]. Since the model is capable of explaining firm

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[12] In the data maximum number of products obtained by any firm is 35. In the calibration we use this value as the maximum number of products that any firm can produce.

evolution, we include three moments that relate to firm dynamics: mean and standard deviation of product growth distribution for surviving firms and percentage of firms that do not change their product scope in a year. Data show that 90% of firms are inactive in the sense that they do not add or subtract products in a given year[13]. In order to highlight the model's capacity in explaining the significance of multi-product firms in economic activity, we include the contribution of their sales to aggregate output (82%). All these data moments are obtained from the Prowess database and represent average values for 1989-1997[14].

Table 1. Data Moments

Definition	*Value*
Product distribution moments	
Average number of products	1.97
Standard deviation of number of products	1.68
Firm & Product dynamics moments	
Average of product growth (conditional on survival)	1.9%
Standard deviation of product growth (conditional on survival)	0.17
Fraction of firms with no net annual change in products	90%
Contribution of multi-product firms to total sales	82%
Imported intermediate moments	
Fraction of firms importing intermediates	19%
Import intensity (Cost of foreign inputs/Sales)	19%

Two moments that relate to the decision of importing identify fixed cost of importing as well as the elasticity of substitution between intermediate inputs. The Prowess database does not include any information about importing. Hence these moments are obtained from Annual Survey of Industries for 2001-2002 period. In India during this period roughly 19% of the firms participate in import activity. We also compute average import intensity of the importing firms as 19%. For each firm, import intensity is

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[13] Using five year averages of firm activity instead of one year, (Goldberg et al., 2010b) show that the share of inactive firms are 72%.

[14] India went through a balance of payments crisis at the end of 1991. We exclude this year in computation of the moments.

computed as the share of total foreign input costs to total sales.

We assume that exogenous efficiency type distribution is lognormal $(\varphi\text{~}LN(\mu_\varphi,\sigma_\varphi))$ whereμ_φ and σ_φ are the mean and standard deviation of this distribution[15]. The parameter vector that is chosen for identification is $\Delta_1 = \{c_0, c_1, \zeta, \mu_\varphi, \sigma_\varphi, f_m, \alpha, \gamma\}$. Innovation cost has three parameters: scaling factor c_0, the convexity of the cost c_1, and the spillover from imports parameter ζ. There are two parameters from the production function which are the labor share of production α and the elasticity of substitution between intermediate products γ. We estimate γ analytically from the model to match average import intensity of firms. Cost of imported intermediates for a product is $w\tau Nq_f$. Dividing this value by the revenue obtained from a product which is derived in equation 6, we get import intensity $Int = \frac{\sigma-1}{\sigma}(1-\alpha)\tau^{1-\gamma}\frac{N}{(1+N\tau^{1-\gamma})}$. This equation allows us to get the value of γ once the other parameters are determined. The final parameter to calibrate is the fixed cost of importing f_m.

We combine several methods to determine the rest of the parameters $\Delta_2 = \{g, N, r, \sigma, \tau, M_e\}$. In the model there are two components of growth: growth of efficiency levels which is exogenously set and evolution of products which is endogenously determined. The data allow us to compute the contribution of each part to the aggregate growth. Average growth rate of sales per product (the intensive margin) over the sample period is 8.9%. This value corresponds to the value of g. Mass of potential entrants M_e is set to unity. As in (Lentz and Mortensen, 2008) real interest rate is set as 5%. Elasticity of substitution between final goods σ, is set as 2.8 which is the median value of elasticities calculated by (Broda and Weinstein, 2006). Tariff rate on intermediate inputs is computed as 24% which yields the iceberg cost as 1.24 following (Topalova and Khandelwal, 2011) who compute input tariffs by running industry-level tariffs through India's input-output table for 1993-1994 period.

Next to other parameters of the model, τ, ζ, and N also affect the gains from importing and their individual identification is also achieved. Number of trading partner countries N is set to one as a normalization. The difference in innovation effort, $c(\lambda)$ of

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[15] There is no particular reason why log normal distribution is used here. The same distribution is also used in Şeker (2012). In the following section, we verify robustness of the model results to allowing the efficiency types to come from a Pareto distribution.

an importer and non-importer firm reflects the total impact of imported intermediates on firm productivity and also the spillover effect on R&D costs. In our moments we target fraction of firms importing intermediates and the difference in revenue per product between an importer and non-importer firm helps identify the impact of imported inputs on unit production costs, namely, the productivity. After controlling for this impact of imported inputs on efficiency the remaining difference in R&D efforts between importer and non-importer firms identifies the spillover impact of importing, ζ, on the R&D effort.

Solution of the model requires lengthy fixed point iterations for each choice of model parameter candidates. To avoid these iterations and in the spirit of (Lentz and Mortensen, 2008) we set the wage level at 200 and set total labor supply L that would satisfy the labor market clearing condition in equation 18. In the simulation results, the average ratio of wage bill to sales is about 10 percent, slightly higher than the 7 percent found for India by (Topalova and Khandelwal, 2011)[16].

To solve the model, we simulate a panel of 10,000 firms for two periods to obtain the moment values. Then we seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. Since the model is highly nonlinear, we use down-hill simplex method (amoeba) for optimization. The steps to computationally solve the equilibrium and calibrate the model are described in the appendix.

4.1 Estimation Results

The parameter estimates are presented in Table 2. The estimated parameter values are broadly consistent with those obtained from several recent studies. (Kasahara and Lapham, 2013) use the same production function for final goods in estimating trade premia of importing and exporting firms using data from Chilean manufacturing sector. Their estimates of the fixed importing cost parameter f_m varies between 0.3 and 0.55 across three industries (wearing apparel, plastic products, and structural metal).

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[16] In the calibration exercise we do not match any data moment like total sales, productivity, or total compensation that relies on the scale of the economy. Thus the wage value used in estimation is not critical for the values of simulated moments chosen for calibration. Aggregate expenditure is normalized to 10,000.

We estimate relatively lower importing costs for all Indian industries which may also be highly influenced by the position of Indian firms in global value chains. The share of labor in their production function is around 0.26 while we estimate it to be 0.39 for Indian manufacturing firms. Our estimate of elasticity of substitution between intermediate goods differs from their estimates of 4.5-5. The convexity of the innovation cost c_1 is comparable to the estimate obtained in Lentz and Mortensen (2008) which fit their model to Danish firms. They find the value of this parameter as 3.73[17]. Similarly, (Şeker, 2012) estimate this parameter for five Chilean manufacturing industries with values varying between 3.8 and 5.7.

Table 2. Parameter Values

	Calibrated Parameters	
Parameter	*Description*	*Value*
$c_0, c_{1,} \zeta$	Innovation cost parameters	1490.7^a, 3.27^a, 1.07^a
$\mu_\varphi, \sigma_\varphi$	Efficiency distribution	3.03^a, 2.87^a
f_m	Fixed cost of importing	0.08^a
α	Labor share in production	0.39^a
γ	Elasticity of substitution between intermediate goods	1.26^a
	Other Parameters	
r	Real interest rate	0.05
g	Growth rate of efficiency levels	0.089
τ	Iceberg costs of importing	1.24
N	Number of trading partner countries	1
M_e	Potential pool of entrants	1
σ	Elasticity of substitution between varieties	2.8

Notes: ^{a} means $p < 0.01$. While calculating the standard errors for parameters we follow the path described in (Gourieroux and Monfort, 2002).

The calibration results are presented in Table 3. The model performs reasonably well in matching the data. It over-estimates the contribution of multi-product firms to total sales. Percentage of importers is also matched quite well. Import intensity is exactly

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[17] The parameter c_0 is a scaling factor and our estimate is likely to differ from (Lentz and Mortensen, 2008).

matched as it was derived analytically from the model. In addition to these targeted moments, the model performs well in capturing several other data moments. Two of these un-targeted moments, variation of output with respect to extensive margin and between share of annual aggregate growth of continuing firms, are presented at the bottom of the table. (Goldberg et al., 2010b) show that approximately 8.5% to 11.5 % of the variation in output across firms can be attributed to the variation in extensive margin which is defined as the number of products firms produce. Number of products at steady state has a logarithmic distribution which is derived in KK (2004). Using the formulas for the mean and variance of this distribution, we can decompose the total variation in total sales into its components of intensive and extensive margins. Total variation in size is determined as

$$Var_{Total} = \underbrace{\int Var\left[r\left(\varphi\right)n|\varphi\right]\delta\left(\varphi\right)d\varphi}_{Var_{Int.\ Mar.}} + \underbrace{\int \left(E\left[r\left(\varphi\right)n|\varphi\right] - \bar{E}\right)^{2}\delta\left(\varphi\right)d\varphi}_{Var_{Ext.\ Mar.}}. \tag{19}$$

where total sales is $\bar{E} = \int E\left[r\left(\varphi\right)n|\varphi\right]\delta\left(\varphi\right)d\varphi$ where $\delta\left(\varphi\right)$ is defined as the efficiency type distribution of firms at steady state, $r\left(\varphi\right)$ is revenue per product derived in equation 3 and $E\left[\cdot\right]$ is the expectation operator. Using the estimated parameter values, total variation decomposition shows that almost 12% of the variation in output is attributed to the variation in the extensive margin. Another moment is decomposition of aggregate growth of output for continuing firms. Following (Goldberg et al., 2010b) we decompose aggregate annual change in output of continuing firms into the changes in product mix (between component), and changes in output of existing products (within component). The contribution of between component to aggregate growth is 15% in the data where it is 11.8% in the model. In this decomposition aggregate growth is equal to g, and we try to understand which of the two margins[18] contribute more to the aggreagete output growth of continuing firms. For the continuing products, that are neither newly

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[18] Although (Goldberg et al., 2010b) label the two margins as extensive and intensive, for the purposes of growth decomposition of continuing firms, we label those two margins ”between” and ”within”, respectively, not to cause a confusion with the previous variance decomposition. Following their definition, let y_{ijt} denote the output of product i produced by firm j at time t. C the set of products that a firm produces in both periods t and $t-1$ (within component) and E the set of products that the firms produce in only t or $t-1$ (between component). Then changes in a firms's total output between t and $t-1$ can be decomposed as follows: $\sum_j \Delta y_{jt} = \sum_j \sum_{i\in E} \Delta y_{ijt} + \sum_j \sum_{i\in C} \Delta y_{ijt}$.

innovated nor discarded, of continuing firms, the growth rate of output, the within component, is $\Delta y_{ijt} = g$. Close accordance with the data in these moments relies on a novel feature of the model. The model incorporates two forces that generate persistent differences in firm performance. In his review of models on firm evolution, (Sutton, 1997) lists these forces as: (i) intrinsic efficiency differences that are determined before entering the economy (ii) differences that are generated through idiosyncratic innovations that accumulate through the life of the establishment. Both forces have drawn great attention in the literature[19]. Our model incorporates both types of forces which allows it to produce rich firm dynamics.

Table 3. Model Fit to the Data

Targeted Moments	*Data*	*Model*
Average number of products	1.97	1.13
Standard deviation of number of products	1.68	1.19
Average of product growth (conditional on survival)	1.9%	1.2%
Standard deviation of product growth (conditional on survival)	0.17	0.18
Fraction of firms with no net change in products	90%	94%
Contribution of multi-product firms to total sales	82%	92%
Fraction of firms importing intermediates	19%	17%
Import intensity (Cost of foreign inputs/Sales)	19%	19%
Other Moments		
Variation of output wrt. extensive margin	8.5-11.5%	12%
Between share of annual growth of continuing firms	15%	11.8%

We also look at the model's ability to explain product distribution of multi-product firms. The model slightly underestimates the product distribution in mean and standard deviation of the product distribution when all firms are included. Restricting the comparison to multi-product firms we get the results presented in Table 4. The product distribution that emerges from the simulation has a longer right tail than the one observed in the data. Although mean, median, and some of the percentiles are quite

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[19] For a review and comparison of both types of models see (Klette and Raknerud, 2003) and (Şeker, 2012).

comparable, the 99th percentile is much higher in the model.

Table 4. Product Distribution of Multi-Product Firms

		Percentiles				
	Mean	25th	50th	75th	90th	99th
Data	3.1	2	2	3.2	5	10
Model	3.9	2	2	3	7	28

Notes: Single-product firms are exclude in this table.

4.2 Autarky versus Trade

(Goldberg et al., 2010b,a) show the significant contribution of trade liberalization to innovation performance of Indian firms. In order to show the success of these policy reforms in the model, we take an extreme case and compare autarky with free trade equilibrium. In order to make this comparison, we let the wage rate be determined by the labor market clearing condition keeping labor supply fixed. We randomly draw 10,000 efficiency values and use the same set of firms in both autarky and trade.

First, we compare the profit levels of importing firms in autarky and trade. We define $\pi^m(\varphi)$, P^m, w^m as profit, aggregate price, and equilibrium wage in trade equilibrium. Replacing superscripts to a refers to the same variables in autarky. Profit levels $\pi^m(\varphi)$ and $\pi^a(\varphi)$ are calculated as

$$\begin{aligned}
\pi^m(\varphi) &= \frac{E}{\sigma}\left[\frac{\sigma-1}{\sigma}\frac{P^m}{w^m}\alpha^\alpha(1-\alpha)^{1-\alpha}\left(1+N\tau_m^{1-\gamma}\right)^{\frac{(1-\alpha)}{\gamma-1}}\varphi\right]^{\sigma-1} - w^m N f_m \\
\pi^a(\varphi) &= \frac{E}{\sigma}\left[\frac{\sigma-1}{\sigma}\frac{P^a}{w^a}\alpha^\alpha(1-\alpha)^{1-\alpha}\varphi\right]^{\sigma-1}.
\end{aligned}$$

For an importing firm, the comparison of these profit levels yield

$$\begin{aligned}\pi^{a}(\varphi) &< \pi^{m}(\varphi)-w^{m}f_{m},\ iff\\ w^{m}f_{m} &< \frac{E}{\sigma}\left[\frac{\sigma-1}{\sigma}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\varphi\right]^{\sigma-1}\left[\left\{\left(1+N\tau_{m}^{1-\gamma}\right)^{\frac{(1-\alpha)}{\gamma-1}}\frac{P^{m}}{w^{m}}\right\}^{\sigma-1}-\left\{\frac{P^{a}}{w^{a}}\right\}^{\sigma-1}\right].\end{aligned}$$

Recall that the cutoff efficiency level for the decision of importing φ_{m}^{*} was determined through equation 7. As long as $D=\left[\left\{\left(1+N\tau_{m}^{1-\gamma}\right)^{\frac{(1-\alpha)}{\gamma-1}}\frac{P^{m}}{w^{m}}\right\}^{\sigma-1}-\left\{\frac{P^{a}}{w^{a}}\right\}^{\sigma-1}\right]>0$ in the equation above, there is an efficiency level $\hat{\varphi}>\varphi_{m}^{*}$ such that all firms with efficiency levels greater than $\hat{\varphi}$ are going to gain higher profits in trade. Unlike the setup in (Melitz, 2003) it is not analytically possible to prove that $D>0$. Hence we simulate the model to compare the autarky and trade equilibrium.

Using the parameter values obtained from the simulation exercise, we compute average profit level $\pi(\varphi)$ per product and innovation rates $\lambda(\varphi)$ for each efficiency level φ. Figure 1 shows the relationship between efficiency levels and per-product profit levels. Cutoff efficiency level φ_{m}^{*} is 97.7. The graph shows that not all importers gain from trade in profit due to higher costs of production. This finding is in accordance with (Melitz, 2003). He shows that among exporting firms only the most efficient ones gain from trade ($\varphi\geq\hat{\varphi}$). In our exercise this threshold value of efficiency levels is $\hat{\varphi}\cong 227$. Trade induces reallocation of resources toward the more efficient firms through competing for labor.

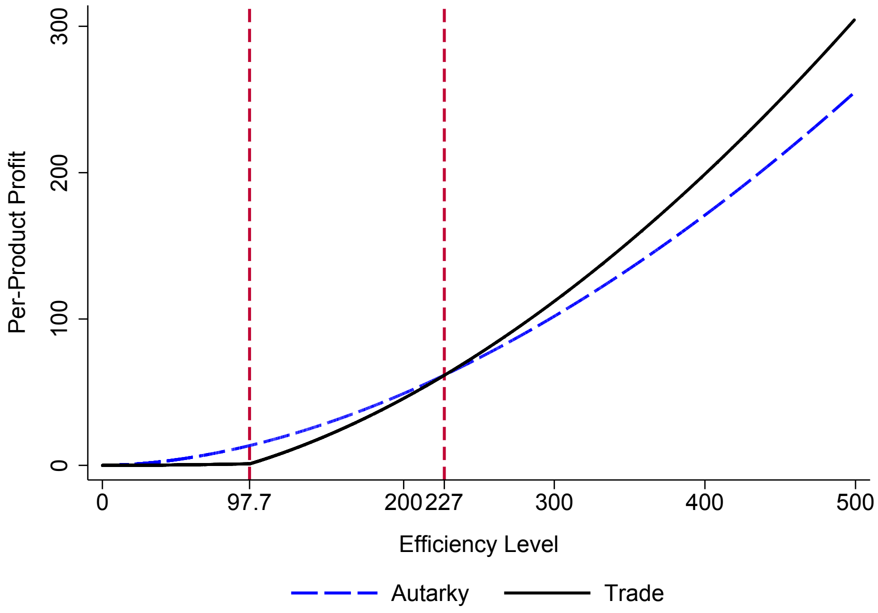

Figure 1. Profit levels (π) versus efficiency levels(φ)

Next in Figure 2, we look at how innovation rates are affected by trade. In trade equilibrium importing firms innovate at higher rates than they would in autarky. Importing increases firms' profits and the knowledge spillover from using new input varieties reduce the cost of innovation. When both factors are combined we observe a faster innovation rate of importers in trade equilibrium. This finding is in accordance with the empirical evidence presented in (Goldberg et al., 2010a). They show that as a result of the tariff liberalization in 1990's, India experienced a surge in imported inputs mostly in inputs that were not available before the liberalization. Higher usage of new input varieties led to introduction of new products in the domestic market. Another result that can be obtained from the graph is that although some of importers with efficiency levels φ such that $\varphi_m^* < \varphi < \hat{\varphi}$ observe drops in their per-product profits $\pi^m(\varphi)$ relative to autarky, they may still gain higher aggregate profits $\pi^m(\varphi)\, n$ in trade equilibrium because they are likely to innovate more products when trade is allowed. In the open economy setup 18% of firms import and 10% incur higher profits while the remaining ones attain lower profits. However, averages profits are higher by 22% for importer firms compared to the autarky setup.

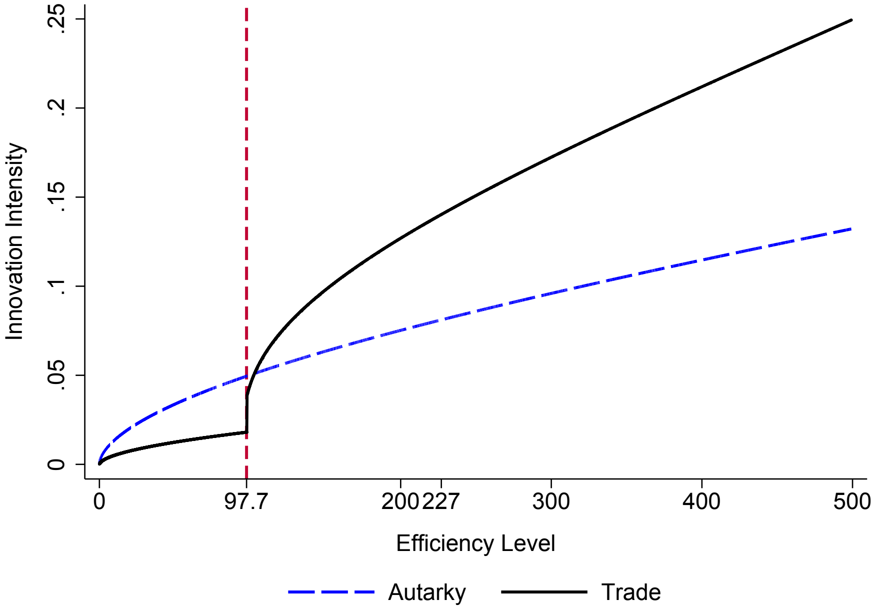

Figure 2. Innovation rates (λ) versus efficiency levels (φ)

Table 5 present a comparison of autarky and trade equilibriums. In trade equilibrium, the spillover from the use of imported intermediate goods and the increase in profit levels push up average innovation rate of incumbent firms. This leads to an increase in product portfolio of multi-product firms from 2.95 to 3.89. It also increases the average and aggregate innovation rates in the economy.

After the trade liberalization, aggregate price level is lower. Those firms which find importing intermediate varieties optimal are able to reduce marginal production cost of their products, and decrease their price. Also, the new entrants are on average more efficient than the entrants of autarky regime due to entry costs being higher. These more efficient entrants also charge lower prices than the entrants of the autarky regime contributing to attaining a lower aggregate price index[20]. Since aggregate price level decreases, wage rate increases and hence innovation gets costlier in trade, entry of new firms decreases in trade.

In Table 6 we see allocation of aggregate labor across different activities in both of the

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[20] Although the firms that do not import after liberalization, and face higher wages will increase their prices, the impact of these higher prices is suppressed by the lower prices charged by the importer ones and the new aggregate price index is lower than the one in the autarky environment.

Table 5. Autarky and Trade Comparison

Moments	(1) *Autarky*	(2) *Trade*
Ave # of products (multi-product firms)	2.95	3.89
Ave innovation rate	3.3%	3.5%
Aggregate innovation rate (by incumbents)	19.2%	34.8%
Entry Rate	5.9%	4.6%
Average product growth rate	1.15%	1.18%
Aggregate price level	0.21	0.07
Equilibrium wage	150.5	200

Notes: This table exhibits some moments from equilibriums of autarky, trade and trade with knowledge spillover from imports eliminated.

equilibriums. When trade is liberalized, more of the aggregate labor is allocated to final goods production l_f. Since imported intermediate goods are also employed in final goods production, less labor is allocated to domestic intermediate goods production. Besides, It is observed that aggregate labor dedicated to R&D activities slightly goes down when the economy is liberalized. Although this may look a bit unexpected, it is mainly because of the less productive firms making more R&D investments in the autarky setup and their reduced R&D effort in the liberalized environment. This phenomenon is also highlighted by (Lentz and Mortensen, 2016) and will be discussed more thoroughly in the following counterfactual exercises (section 6.1).

Table 6. Allocation of Labor across Different Activities: Autarky vs. Trade (prct.)

	l_f	$l_{R\&D}$	l_i	l_{f_m}	Total
Autarky	0.372	0.048	0.579	-	1
Trade	0.395	0.047	0.556	0.002	1

Notes: This table shows how aggregate labor supply is allocated across different types of productive activities in autarky, and trade environments. Each cell in a row shows the share total labor allocated to the activity represented by the column.

4.3 Firm Size, Growth Rate and Hazard Rate of Exit

Trade liberalization reduces marginal production costs for importer firms and hence make them decrease their prices and increase their output per product. With the spillover effect of imported intermediates, importer firms also increase their innovation effort. As a results, in equilibrium they innovate more products. These two forces allow more efficient firms to import and become even larger firms. At the same time, due to the higher efficiency cutoff for entry, the new entrants are on average larger than the new entrants of the autarky regime.

The increasing returns from imported intermediate inputs increases the dispersion of firm growth rates, as well. Due to the total spillover effect of imported inputs on innovative effort, importer firms exert more R&D effort when trade is allowed in the model. Hence, dispersion of growth rate increases as long as some firms do not import while others do.

As to the exit hazards, the exit hazard of a one-product firm is directly linked to the common rate of creative-destruction μ which is presented in equation 16. Since M_e is normalized to 1, entry rate η is equal to the innovation rate of entrants λ_e which is an increasing function of φ_m^* because of the spillover from trade. Compared to the autarky setup, innovation rate of entrants is lower when trade is allowed which will reduce μ (this impact can also be observed in Figure 2). At the same time, the second component of μ in equation 16, which is the aggregate arrivals of innovations to incumbent firms is higher. This is due to the availability of more imported intermediate varieties in the domestic market which makes incumbent producers more innovative. Consequently, when the economy is opened to trade the creative-destruction rate μ will be determined by two forces; first is the entry rate which is relatively lower, and the second is innovation rate of incumbents which is relatively higher in the open economy (See Table 5). In our simulations when trade is liberalized aggregate innovation rate by incumbents go up from 19.2% to 34.8% and entry rate goes down from 5.9% to 4.6%. Drop in innovation rate of entrants is more than compensated by the increase in innovation of incumbents and the overall creative-destruction rate μ increases from 0.25 to 0.39.

5 Counterfactuals

5.1 Lowering Fixed Costs of Importing

5.2 Increasing Number of Trading Partners

5.3 Government policies that lower innovation costs/ Subsidies to Innovation

6 Robustness Exercises

To get a firmer grasp of the mechanisms at work and sensitivity of the results to different aspects of the model, in this section we conduct a series of counterfactual analyses and robustness checks. First, we analyze the spillover effect of imported intermediates on innovation costs. To understand the impact of this channel on the economic activity, we shut it down by setting $\zeta = 0$. Second, we study sensitivity of the equilibrium outcomes to demand elasticity specification of the model. Finally, we change our assumption of Lognormal distribution of efficiencies to Pareto distribution, and check robustness of the results.

Tables 7-9 exhibit comparison of some important equilibrium outcomes across different robustness tests. We realize that in some cases although aggregate labor allocated to the same activity is similar across different specifications, allocation of the aggregate labor across firms may differ substantially (e.g., exercise with σ). That's why, we also report the average labor dedicated to different economic activities across firms. Panel B of Table 8 compares average labor employed at a firm for different activities.

Table 7. Eqilibrium Outcomes in Different Specifications

Equilibrium Aggregates	*Benchm.*	ζ=0	σ=2.5
Wage (w)	1	0.995	0.994
Agg. Price (P)	1	0.922	1.058
Agg. Entry (η)	1	1.153	1.217
Av. Lambda	1	0.615	1.515
Av. Profit	1	0.868	1.135
Agg. Innov.	1	0.504	1.150
Agg. Cons. (C)	1	1.085	0.945
Efficiency cutoff for importing	1	1.077	0.526
% of importers	1	0.948	1.679
Av. import intensity (Cost of foreign inputs/Sales)	1	1.000	0.933
Variation of output wrt. extensive margin	1	1,071	0.983
Between share of annual aggregate growth	1	0.693	1.507
Overall destruction rate (μ)	1	0.579	1.158

Notes: This table shows how some important equilibrium outcomes vary across equilibriums with different specifications. Benchmark results are produced using the parameters in Table 2. Each column presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a row is normalized with the benchmark value of that row. All simulations are performed under general equilibrium setup.

Table 8. Labor Reallocation in Counterfactuals

	Panel A. Comparison of Totals					*Panel B. Comparison of Means*				
	l_f	$l_{R\&D}$	l_i	l_e	l_{f_m}	l_f	$l_{R\&D}$	l_i	l_e	l_{f_m}
Benchmark	1	1	1	1	1	1	1	1	1	1
No Spillover	1.005	0.928	1.005	1.839	0.883	0.877	1.006	0.877	1.839	0.984
Low Elasticity	0.939	1.847	0.939	2.318	1.009	0.654	1.812	0.654	2.318	1.135

Notes: Benchmark results are produced using the parameters in Table 2. Each row presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a column is normalized with the benchmark value of that column. In all of the computed equilibriums aggregate labor supply is the same, and labor markets clear. Panel A of the table shows how much total labor allocated for different activities deviates from the benchmark specification. Panel B compares the mean values of labor employed at a firm for different types of activities across different equilibriums.

Table 9. Decomposition of Labor Across Activities in Counterfactuals

	l_f	$l_{R\&D}$	l_i	l_e	l_{f_m}	$Total$
Benchmark	0.387	0.066	0.544	0.000	0.002	1
No Spillover ($\zeta = 0$)	0.389	0.062	0.547	0.000	0.002	1
Low Elasticity ($\sigma = 2.5$)	0.363	0.123	0.511	0.000	0.002	1

Notes: Benchmark results are produced using the parameters in Table 2. Each row presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a row shows the share total labor allocated to the activity represented by the column.

6.1 Shutting Down the Spillover Effect

When we shut down the spillover effect of imported inputs, average innovation effort λ and aggregate innovation rate of the economy declines by 39% and 50%, respectively. These declines show how important the spillover channel is on domestic innovative activity. (Goldberg et al., 2010a) finds out a significant variety channel of imported inputs on firm product scope. However, they do not thoroughly lay out the mechanism between imported varieties and innovation. Our findings highlight that spillover effects of imported inputs on innovation are quite substantial.

Share of growing firms declines by 32 percent from 2.8% to 1.9% while fraction of multi-product firms stayed with multiple products between two periods increases by 4 percent from 37.6% to 54.2%[21]. This is mainly because of the overall destruction rate which comes down to 0.23 from 0.39. Eliminating the spillover effect reduces the aggregate price level and increases consumption by 8%. Again, this welfare effect results from the inherent inefficiency in the decentralized economy of the KK (2004) model[22]. Low efficiency firms are pushed out of business and resources are allocated towards more efficient firms and hence consumption is higher. (Lentz and Mortensen, 2016) point out that planner's solution discourages innovation by low efficiency innovators as well as entry. Persistent firm types, monopolist innovators, creative destruction of new innovations contribute to this inefficiency of the decentralized equilibrium.

Since the wage rate does not change much, labor allocated to final good production, intermediate goods production remains at similar levels with the benchmark setup. However, labor allocated to R&D declines by 7.2%. As aggregate entry is higher, labor allocated to entry almost doubles going up by 83.9%. R&D labor also shuffles across importer and non-importer firms. Panels A and B of Table 8 also highlights that although aggregate labor allocated to final good production slightly changes, average firm size declines 12.3%.

6.2 Robustness to Changing the Elasticity Parameter

Demand elasticity is an important determinant of per-product profits, expected returns to new product innovations and consequently innovation effort that is exerted by firms. In this section, we analyze the impact of a change in substitutability of final goods for consumers on firm innovation behavior. As explained in the previous sections, we calibrated demand elasticity based on existing studies, and set it equal to 2.8. To under-

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[21] Further equilibrium outcome comparisons can be found in Appendix Table A1.

[22] (Lentz and Mortensen, 2016) find that the welfare loss of the decentralized economy in (Lentz and Mortensen, 2008) is equivalent to 21% tax on social planner consumption path.

stand the role of this elasticity we reduce σ to 2.5[23].

When the demand elasticity is lowered to 2.5, aggregate price, and average profits are higher by 5.8%, and 13.5%, respectively. Less elastic demand increases profits to per product created by a firm. Consequently, average innovative effort λ increases by 51% while aggregate innovations increase by 15%. In the labor market, aggregate labor allocated to final and intermediate goods production declines by almost 6%. While, average number of employees per final good producer firm is lower by 35% which means that there are more final good producer firms in equilibrium. Total labor allocated to R&D is higher by 85% and the increase in mean R&D labor of incumbent firms is by 81%. Incumbent firms increase their innovation effort mainly because the final goods are less substitutable to consumers now, and average profitability of a new good is higher. Since returns to a new good are higher more firms prefer to be an importer which is also evidenced by 13.5% higher fix import costs in Panel B of Table 9.

(Lentz and Mortensen, 2008) also highlight that the simulation results are very responsive to selection of the elasticity parameter σ. The novel spillover channel of our paper for imported intermediates adds to the benefits from importing and hence amplifies the impact of a change in demand elasticity on firm innovation and production. When demand elasticity changes, although the directions in analyzed relationships do not change, magnitudes change.

6.3 Pareto Distribution of Efficiencies

Pareto distribution has proven effectiveness in studying several dimensions of firm behavior and size. Although we have preferred another commonly used distribution which is Lognormal distribution, we have estimated our model where we have allowed firm efficiencies to come from a Pareto distribution. We estimated the optimal shape and scale parameters of the Pareto distribution at 22.58 and 0.53, respectively. We present parameters that are estimated under the Pareto assumption in Table 10.

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[23] When we increase the elasticity to 3.1 the same mechanisms described in this section work in the reverse direction. Hence we only present results where σ is lowered.

Table 10. Parameter Values

Parameter	Description	Value
	Calibrated Parameters	
Parameter	Description	Value
$c_0, c_{1,} \zeta$	Innovation cost parameters	1507, 3.40, 1.10
$k_\varphi, \alpha_\varphi$	Efficiency distribution	22.58, 0.53
f_m	Fixed cost of importing	0.08
α	Labor share in production	0.37
γ	Elasticity of substitution between intermediate goods	1.54

Notes: This table shows the parameter estimates where we draw firm efficiencies from a Pareto distribution.

Comparison of results that stem from the Lognormal and Pareto assumptions are exhibited in Table 11. In general, not much difference in the estimated moments is observed between the two specifications. Estimates for average number of products, standard deviation of number of products, average product growth, fraction of firms with no net change in products, and contribution of multi-product firms to total sales improved with lower distances between the target and estimated moments. On the other hand, distance slightly grew up for standard deviation of product growth, fraction of firms importing intermediates and average import intensity (cost of foreign inputs/sales).

Table 11. Model Fit to the Data

Targeted Moments	*Data*	*Model (Lognormal)*	*Model-Pareto (Pareto)*
Average number of products	1.97	1.13	1.26
Standard deviation of number of products	1.68	1.19	1.83
Average of product growth (cond. on survival)	1.9%	1.17%	2.5%
Standard deviation of product growth (cond. on surv.)	0.17	0.18	0.27
Fraction of firms with no net change in products	90%	94.4%	89%
Contribution of multi-product firms to total sales	82%	92%	91%
Fraction of firms importing intermediates	19%	17%	23%
Import intensity (Cost of foreign inputs/Sales)	18.6%	18.9%	19.7%
Other Moments			
Variation of output wrt. extensive margin	8.5-11.5%	12%	11.3%
Between share of annual growth of continuing firms	15%	11.8%	23%

Notes: This table presents the targeted moements derived from the data in the Data column. Model column presents the same moments that the model produced using the parameter values presented in Table 2, and Model-Pareto column presents the moments that the model produces when the efficiency type distribution is specified as a Pareto distribution instead of the Lognormal specification of the benchmark setup.

Advantages and drawbacks of both Pareto and Lognormal distributions in fitting firm size distribution has been widely discussed in the literature[24]. Since we target neither firm nor product size distributions in this study, we are indifferent between using either distribution. We target several moments related number of products of a firm and KK (2004) has already showed that in this model number of products has a logarithmic distribution. (Şeker, 2012) successfully explains firm size distribution and dynamics using the same exogenous efficiency type distribution (Lognormal), and the model of

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[24] Lognormal distribution fits well on the left tail of firm size distribution whereas it diverges from the actual distribution on the right tail, and right tail is explained better with Pareto distribution. (Luttmer, 2007) points this out and provides an explanation through entry and imitation costs. (Levy, 2009) underlines a similar phenomenon for city size distribution. (Di Giovanni, Levchenko, and Ranciere, 2011) highlight the role of trade on firm size distribution and state that because of trade, existing power law component estimates of the literature are systematically lower than the true values. (Arkolakis, 2016) states that random entry (or exit) and a process that exhibits size independence are sufficient to generate a cross-sectional firm size distribution with Pareto right tails.

this paper draws on (Şeker, 2012). Since we do not target to match any moment on firm size distribution, the moments matched by the model do not differ significantly across the different distributional assumptions.

7 Further Discussions

In this study, we focused our attention to the interaction between imported intermediate goods and product innovation. However, our model has implications on firm productivity and dynamics. It can also be extended to study different questions such as economic geography and heterogeneous products of firms. In this section, we discuss implications of our model on firm productivity and its measurement, firm dynamics, and how the model can be extended to an environment with asymmetric countries. We also discuss the link between our study and the literature on multi-product firms.

7.1 Firm Productivity across Counterfactuals

Measured productivity changes between the benchmark trade environment and the counterfactual environments. Different types of firms contribute to these changes to varying extents, and in this subsection, we shed some more light onto these underlying productivity dynamics of the study.

Our stationary equilibrium is characterized by a steady mass of products and a steady efficiency distribution over those products where we use productivity as a grand result of exogenously growing intrinsic firm efficiency, import status choice, and distribution of domestic and imported intermediates. If one product is destroyed, it is by the nature of stationarity replaced with a new product. In steady state, since the labor share of each firm in total employment is constant, between firm and cross components of a (Baily, Hulten, and Campbell, 1992) (BHC, henceforth) growth decomposition is zero[25].

Worker reallocation allows growth in the model. Every time when a new product is invented labor resources are shifted to the inventor firm and the invented product from

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[25] BHC growth decomposition literature identifies the terms ”within”, ”between”, ”cross ”entry” and ”exit” as growth in the productivity due to productivity improvements by incumbents, productivity growth from reallocation of labor from less to more productive firms, contribution of correlation between input shares and productivity growth, firm entry and exit, respectively.

firms with products that recently become obsolete. However, this reallocation does not illustrate a reallocation of labor resources across producers of different efficiency. On the other hand, across counterfactual exercises labor resources switch across producers of different efficiency levels, and in this section we measure the extent of this reallocation using different methodologies. Although we keep firms' intrinsic efficiencies the same across different counterfactual environments, the changes in environments result in changes in production and trade behaviors of firms. Table 12 exhibits number of firms, and how they contribute to changes in average measured aggregate productivity between the benchmark and counterfactual environments from the perspectives different measurement strategies[26].

In the benchmark environment of trade equilibrium 18% of firms import intermediate goods, and in the case where we shut down the spillover channel, the cutoff efficiency for importing goes up and the number of firms that import goes down. Hence, average efficiency of the firms that import in both environments, labeled as the Yes-Yes group, goes up.

82% of firms import in neither the benchmark nor the setup where we shut down the spillover effect, and 11.5% of them start importing only in the case where we reduce the demand elasticity. We also report two more ways to understand how reallocation of resources across firms affect measured productivity. Following the definition in (Petrin and Levinsohn, 2012), change in allocative efficiency allows changes in final demand and markups to influence the measured productivity. Numbers reported for changes in allocative efficiency are higher than changes in productivity measured a la (Baily et al., 1992) which is mainly change in revenue weighted efficiency.

When the spillover channel is closed or the demand elasticity is lowered again almost all change in allocative efficiency takes place among importer firms and the efficiency loss adds up to around 87%. When we compare these numbers with their counterparts from a calculation à la Baily Hulten Campbell, we get more modest numbers. Measured productivity change is around 5% and -12.2% in no spillover and low elasticity environments, respectively. Again, all gain and loss occur through the importer firms. BHC

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[26] We basically compare simple average firm efficiency with the measurement methods proposed in (Baily et al., 1992) and (Petrin and Levinsohn, 2012).

methodology measures a positive productivity change for the firms that import in both setups with and without the spillover effect (Yes-Yes group). With this 5% measured productivity change BHC is mainly capturing the selection effect for the most productive firms. 111 firms, that imports when the spillover channel is active, stop importing when the spillover is shut down. Since these are the lowest efficiency importing firms, when they stop importing the BHC productivity change of importing firms goes up.

To sum, findings of this section show us that it is the importer firms that make up the biggest margin where productivity gains(losses) occur across different environments. Also, the differentiation in both size and direction of measured productivity gains across different measurement methodologies confirm the findings of (Petrin and Levinsohn, 2012).

Table 12. Firm Productivity across Counterfactuals

		$\zeta = 0$	$\sigma = 2.5$
Yes-Yes	Av. Efficiency	398.4	380.1
	Ch. in Allocative Efficiency	-0.869	-0.873
	Ch. in BHC Productivity	0.049	-0.122
	No. of Firms	1,684	1,795
Yes-No	Av. Efficiency	101.9	-
	Ch. in Allocative Efficiency	0.000	-
	Ch. in BHC Productivity	0.000	-
	No. of Firms	111	0
No-Yes	Av. Efficiency	-	71.09
	Ch. in Allocative Efficiency	-	0.002
	Ch. in BHC Productivity	-	0.000
	No. of Firms	0	1,153
No-No	Av. Efficiency	23.22	15.39
	Ch. in Allocative Efficiency	0.000	0.000
	Ch. in BHC Productivity	0.000	0.000
	No. of Firms	8,205	7,052

Notes: This table shows how measured efficiency (productivity) levels differ across different counterfactuals in comparison to the benchmark setup. Each column compares one counterfactual with the benchmark results. Yes-Yes, Yes-No, No-Yes, and No-No stands for the firms that import in both benchmark and counterfacual, import in only the benchmark, import in only counterfacual, import in neither benchmark nor counterfactual environments, respectively. Average efficiency reports the simple average efficiency (φ) for firms under different setups. Change in Allocative efficiency (ΔAE) reports the relative change in allocative efficiency between the benchmark environment and the counterfactual environments following the definition introduced in (Petrin and Levinsohn, 2012). $\Delta AE_i = (\Delta Revenue_i - \Delta Cost_i)/\sum_i (Revenue_{i,Benchmark} - Cost_{i,Benchmark})$ where i represents the four different firm groups. Change in BHC productivity$_i = \Delta(RevenueShare_i * \varphi_i)/\sum_i (RevenueShare_{i,Benchmark} * \varphi_{i,Benchmark})$.

7.2 Multi-Product Firms and Firm Size Dispersion

This paper contributes also to the literature on multi-product firms, and in this subsection we discuss the link of our study to this literature and also discuss how dispersion of number of products is effected by changes in the structural parameters of the model.

Similar to ours, a key implication of the model in BRS (2010) is that a firm's product range increases in its efficiency. In their setup, a firm incurs fixed production cost for each product while in our specification innovation effort is endogenously determined, and is not attached to any specific product. BRS (2010) highlight the importance of shocks that are idiosyncratic to firm-product pairs in explaining the product switching behavior observed for the US firms. In our framework, we do not allow heterogeneity across products of a firm. Hence, the quantitative analysis cannot target some moments related to asymmetries across products within a firm such as the share of each product in firm output (Table 10 of BRS (2010)), and abstracts from some of the findings of BRS (2010, 2011). On the other hand, by making this assumption we are able to develop a general equilibrium model that allows the study of innovation behavior in multi-product firms.

BRS (2010) also find out that the intensive margin of output per product is dominant in determining firm size. In fact, (Şeker, 2012) successfully explains firm size distribution in a parsimonious way by only introducing an exogenous efficiency type distribution. Since the model of this paper builds on (Şeker, 2012), it is also able to explain firm size distribution without any need to product level shocks.

A concordance between the vector of the taste shocks idiosyncratic to firm-product pairs in BRS (2010) setup and the knowledge capital in KK (2004) and our setup can be built. If we treat groupings in the knowledge capital n in the KK (2004) framework as distinct products, then each of these groups can be treated as the firm-product taste shocks of the BRS (2010).

An extension of product heterogeneity within a firm would make the model closer to the data at the expense of making the model less tractable. In our setup we emphasize innovation and the role of imported intermediate varieties in the innovation effort of firms. Still, achieving a better comprehension of the interrelation between firm innovation behavior and heterogeneity of products within a firm would be useful, and it is a part of future research agenda.

When we evaluate the implications of our model regarding firm size dispersion, we see that sources of size dispersion in steady state are the type conditional innovation intensity $\lambda(\varphi)$, the rate of destruction μ and entry rate η. They have the pivotal roles in explaining the size dispersion, and determine the distribution of number of products conditional on firm type φ.

To understand the impact of economic environment on firm size dispersion, in this section we compare standard deviation as well as right tail of the number of products distribution for the firms in different environments. As seen in Table 13, in the benchmark setup the standard deviation is 15.34. The deviation goes down to 12.7 and 14.4 when the spillover channel is shut down and the demand elasticity is lowered, respectively. When the spillover channel is shut down both average innovation intensity λ and overall destruction rate μ decreases and entry rate η increases (see Table 7) and consequently number of products fall down especially for the most efficient firms. When the demand becomes less elastic, λ, μ and η increase altogether. Increasing entry increase the number of single product firms, increasing innovation intensity raises the number of multi-product firms while increasing overall destruction rate suppresses both of these effects, and the resulting effect on dispersion is the net effect of all these changes.

Table 13. Firm Size Dispersion

	Std. Dev.	Mean	Median	$\%95$	$\%99$
Benchmark	15.35	2.45	1	2	19
No Spillover ($\zeta = 0$)	12.71	2.02	1	2	6
Low Elasticity ($\sigma = 2.5$)	14.4	2.29	1	2	10

Notes: This table shows the statistics related to firm size distribution for different equilibrium specifications. Column Std. Dev. reports the standard deviation of number of products across firms. Columns $\%95$ and $\%99$ report number of products for the firms which are at the 95th and 99th percentiles of the size distributions, respectively.

To sum, the novel spillover channel of the paper implies an increase in firm size dispersion upon trade liberalization, and size of the response evidently depends on the demand elasticity in the economy.

8 Conclusion

In this study we develop a general equilibrium model of multi-product firms to explain the relationship between importing, innovation, and firm growth. Following the structural model of KK (2004) and their extension in (Şeker, 2012), we present a stochastic dynamic model of firm and industry evolution. In the model we introduce heterogeneity in firms' efficiency levels. The novel feature of our model is that unlike many of the recent trade models that follow (Melitz, 2003), our model has a dynamic feature. Firms invest in R&D which results in introduction of new varieties to the economy. Each period these products face a probability of being destructed. The birth and death process of products yield the stochastic growth process of firms. Incorporating the importing decision in this setup allows us to relate trade with innovation and growth.

Firm's efficiency is the main driver of its evolution. Only the most efficient firms can participate in import markets as they can compensate the sunk costs of trade. These firms are also innovative more. With the learning they obtain through knowledge spillover from the use of foreign intermediates, their innovation rates increase even further relative to non-importing firms. With the additional benefits of importing on their revenues and innovation rates, these firms grow faster and exit less often. One other novelty of the study for understanding the impact of trade on innovation rates which is the knowledge spillover next to the standard productivity boost coming from the "love-of-variety" attribute of the production function.

We test the model's ability to explain product distribution, dynamics of firm evolution, and the relationship between importing and innovation through a calibration exercise. We fit the model to Indian panel of firms for 1989-1997 time period. The model explains the targeted moments in the data relatively well. It also produces reasonable estimates of some un-targeted moments such as the variation in output with respect to extensive margin, between share of annual growth, and product distribution of multi-product firms.

We also present a comparison of autarky and trade equilibrium. This exercise shows that as in Melitz (2003), in trade resources are reallocated to more efficient firms and this leads to an increase in average size and number of products produced by firms. We measure the direct cost-reducing spillover of imported intermediates on R&D activity. We find that when we do not allow for this spillover, R&D effort of importing firms comes down by 33%. Moreover, average product creation rate increases which leads to faster growth. Higher innovation rates of importing firms cause a reduction in the entry rate. Our counterfactual exercises also prove that our model is robust to underlying efficiency distribution and elasticity assumptions.

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A

A.1 Solving Firm's Static Problem

In a symmetric equilibrium, the static profit maximization problem of a final good producer can be formalized as follows

$$\begin{aligned} & Max\ PC^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} - w\int_0^1 q_d(j)\,dj - w\tau\int_0^N q_f(j)\,dj - wl \\ & st \\ y &= \varphi l^{\alpha}\left[\int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}}\,dj + I\int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}}\,dj\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}. \end{aligned}$$

The first order conditions of this maximization problem with respect to $q_d(j)$, $q_f(j)$ for $\forall$ j and l in respective order are given as follows:

$$\begin{aligned} w &= \\ &PC^{\frac{1}{\sigma}}\frac{\sigma-1}{\sigma}y^{\frac{\sigma-1}{\sigma}-1}\varphi l^{\alpha}\frac{(1-\alpha)\gamma}{\gamma-1}\left[\int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}}dj+I\int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}}dj\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1}\frac{\gamma-1}{\gamma}q_d(j)^{\frac{\gamma-1}{\gamma}-1} \end{aligned} \tag{A1}$$

$$\begin{aligned} w\tau &= \\ &PC^{\frac{1}{\sigma}}\frac{\sigma-1}{\sigma}y^{\frac{\sigma-1}{\sigma}-1}\varphi l^{\alpha}\frac{(1-\alpha)\gamma}{\gamma-1}\left[\int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}}dj+I\int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}}dj\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1}\frac{\gamma-1}{\gamma}q_f(j)^{\frac{\gamma-1}{\gamma}-1} \end{aligned} \tag{A2}$$

$$w = C^{\frac{1}{\sigma}}\frac{\sigma-1}{\sigma}y^{\frac{\sigma-1}{\sigma}-1}\alpha\varphi l^{\alpha-1}\left[\int_0^1 q_d(j)^{\frac{\gamma-1}{\gamma}}dj+I\int_0^N q_f(j)^{\frac{\gamma-1}{\gamma}}dj\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}. \tag{A3}$$

From equation A1, we get $q_d(j) = q_d$ for all j, and from equation A2 we get $q_f(j) = q_f$ for $\forall$ j. Then, taking ratios of equations A1 and A2, for all firms that import intermediate products we get

$$\begin{aligned} \frac{1}{\tau} &= \left(\frac{q_d}{q_f}\right)^{-\frac{1}{\gamma}} \qquad \text{(A4)} \\ q_f &= \tau^{-\gamma}q_d \text{ for } \forall j. \end{aligned}$$

Total output of a final good is found as

$$\begin{aligned} y(\varphi, I) &= \varphi l^{\alpha}\left[q_d^{\frac{\gamma-1}{\gamma}}+IN\left(\tau^{-\gamma}q_d\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \\ &= \varphi l^{\alpha}\left[\left(1+IN\tau^{1-\gamma}\right)q_d^{\frac{\gamma-1}{\gamma}}\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \\ &= \varphi l^{\alpha}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}}q_d^{1-\alpha}. \qquad \text{(A5)} \end{aligned}$$

This equation could further be simplified. Taking the ratios of equations A1 and A3 and using the results that q_d and q_f are constant, we get

$$\frac{l^{\alpha}\frac{(1-\alpha)\gamma}{\gamma-1}\left[q_d^{\frac{\gamma-1}{\gamma}}+INq_f^{\frac{\gamma-1}{\gamma}}\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}-1}\frac{\gamma-1}{\gamma}q_d^{\frac{\gamma-1}{\gamma}-1}}{\alpha\varphi l^{\alpha-1}\left[q_d^{\frac{\gamma-1}{\gamma}}+INq_f^{\frac{\gamma-1}{\gamma}}\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}}=1.$$

Simplifying this ratio and using the finding $q_f = \tau^{-\gamma} q_d$, we get

$$\begin{align}
\frac{l\left(1-\alpha\right) q_d^{-\frac{1}{\gamma}}}{\alpha\left[q_d^{\frac{\gamma-1}{\gamma}} + INq_f^{\frac{\gamma-1}{\gamma}}\right]} &= 1 \notag\\
\frac{l\left(1-\alpha\right) q_d^{-\frac{1}{\gamma}}}{\alpha\left[(1+IN\tau^{1-\gamma}) q_d^{\frac{\gamma-1}{\gamma}}\right]} &= 1 \notag\\
(1+IN\tau^{1-\gamma}) q_d^{\frac{\gamma-1}{\gamma}} q_d^{\frac{1}{\gamma}} &= l\left(\frac{1-\alpha}{\alpha}\right) \notag\\
(1+IN\tau^{1-\gamma}) q_d \left(\frac{\alpha}{1-\alpha}\right) &= l. \tag{A6}
\end{align}$$

Now combining the result from equation A3, the finding that $p\left(j\right) = \left(\frac{y}{C}\right)^{-\frac{1}{\sigma}} P$, and the result in equation A6, we get an equation that gives the relationship between price and wage rate

$$\begin{align}
w &= \frac{\sigma-1}{\sigma} p\alpha\varphi l^{\alpha-1}\left[\int_0^1 q_d\left(j\right)^{\frac{\gamma-1}{\gamma}} dj + I\int_0^N q_f\left(j\right)^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} \notag\\
w &= \frac{\sigma-1}{\sigma} p\alpha\varphi l^{\alpha-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha} \notag\\
p &= \frac{\sigma}{\sigma-1}\frac{w}{\alpha\varphi l^{\alpha-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha}} \notag\\
&= \frac{\sigma}{\sigma-1}\frac{w}{\alpha\varphi\left((1+IN\tau^{1-\gamma})q_d\left(\frac{\alpha}{1-\alpha}\right)\right)^{\alpha-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha}} \notag\\
&= \frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varphi(1+IN\tau^{1-\gamma})^{\alpha-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}}} \notag\\
p &= \frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varphi(1+IN\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}}. \tag{A7}
\end{align}$$

The result obtained from equation A7 shows that the final good price for a good that only uses domestic intermediate goods is $p^h\left(\varphi\right) = \frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\varphi}$. If imported intermediate goods are used in production, then the price is $p^m\left(\varphi\right) = p^h\left(\varphi\right)/(1+N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}$.

Next we derive the equilibrium values of q_d and l. From the solution of the profit maximization problem of the final good producer, replacing the equilibrium value of labor l from equation A6 into equation A5 we get

$$\begin{align}
y\left(\varphi, I\right) &= \varphi\left((1+IN\tau^{1-\gamma})q_d\left(\frac{\alpha}{1-\alpha}\right)\right)^{\alpha}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\gamma}{\gamma-1}} q_d^{1-\alpha} \notag\\
y\left(\varphi, I\right) &= \varphi\left(1+IN\tau^{1-\gamma}\right)^{\frac{\gamma-\alpha}{\gamma-1}}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} q_d. \tag{A8}
\end{align}$$

Recall that from composite good producer's maximization problem we had

$$y\left(\varphi\right)=p\left(\varphi\right)^{-\sigma}\frac{E}{P^{1-\sigma}}. \tag{A9}$$

Combining equations A8 and A9, we get

$$\begin{aligned} p\left(\varphi\right)^{-\sigma}\frac{E}{P^{1-\sigma}} &= \varphi\left(1+IN\tau^{1-\gamma}\right)^{\frac{\gamma-\alpha}{\gamma-1}}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}q_d \\ q_d &= \frac{p\left(\varphi\right)^{-\sigma}\frac{E}{P^{1-\sigma}}}{\varphi\left(1+IN\tau^{1-\gamma}\right)^{\frac{\gamma-\alpha}{\gamma-1}}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}. \end{aligned}$$

We can get rid of the price in this equation by plugging in the value of $p\left(\varphi\right)$ from equation A7

$$\begin{aligned} q_d &= \frac{E}{P^{1-\sigma}}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\left[\varphi\left(1+IN\tau^{1-\gamma}\right)^{\frac{\gamma-\alpha}{\gamma-1}}\right]^{-1}\left[\frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varphi(1+IN\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}}\right]^{-\sigma} \\ &= \frac{E}{P^{1-\sigma}}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\varphi^{\sigma-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)\sigma-\gamma+\alpha}{\gamma-1}}\left(\frac{\sigma-1}{\sigma}\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{w}\right)^{\sigma}. \end{aligned}$$

Finally labor value can be found by incorporating q_d into equation A6

$$l=\frac{E}{P^{1-\sigma}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\varphi^{\sigma-1}\left(1+IN\tau^{1-\gamma}\right)^{\frac{(1-\alpha)(\sigma-1)}{\gamma-1}}\left(\frac{\sigma-1}{\sigma}\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{w}\right)^{\sigma}.$$

A.2 Balanced Growth in the Economy

We assume that efficiency levels grow at rate g (i.e. $\dot{\varphi}/\varphi=g$). Since there is no population growth, L is constant. Then from equation 4 we get $\dot{y}\left(\varphi,I\right)/y\left(\varphi,I\right)=g$. Equation 1 gives

$$\begin{aligned} \dot{C} &= \frac{\sigma}{\sigma-1}\left(\int_{j\in J}y_t\left(j\right)^{\frac{\sigma-1}{\sigma}}dj\right)^{\frac{\sigma}{\sigma-1}-1}\left(\int_{j\in J}\frac{\sigma-1}{\sigma}y_t\left(j\right)^{\frac{\sigma-1}{\sigma}-1}y\left(j\right)\frac{\dot{y}\left(j\right)}{y\left(j\right)}dj\right) \\ \dot{C} &= \frac{\sigma}{\sigma-1}\frac{\sigma-1}{\sigma}Cg \\ \frac{\dot{C}}{C} &= g. \end{aligned}$$

Since we normalize aggregate expenditure to a constant in the model $E = P_t C_t$ implies that P and $p\left(j\right)$ for $\forall\ j$ decreases at rate g.

$$\begin{aligned}
P_t &= \left(\int_{j\in\ J} p_t(j)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}} \\
\dot{P} &= \frac{1}{1-\sigma}\left(\int_{j\in\ J} p_t(j)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}-1}\left(\int \left(1-\sigma\right) p\left(j\right)^{-\sigma}\frac{\dot{p}\left(j\right)}{p\left(j\right)} p\left(j\right) dj\right) \\
\frac{\dot{P}}{P} &= -g = \frac{\dot{p}\left(j\right)}{p\left(j\right)} \text{ for } \forall\ j.
\end{aligned}$$

From the final good producer's optimization problem we found that

$$p\left(\varphi\right) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}.$$

Since $p\left(\varphi\right)$ decreases at rate g and φ grows at rate g, wage is constant. Then profit per product $\pi\left(\varphi\right) = \left(\frac{p(\varphi)}{P}\right)^{1-\sigma}\frac{E}{\sigma}$ is constant. This allows us to get the stationary solution in the Bellman equation. Note that although wage is constant, real wage which is w/P_t and real output $\frac{r(\varphi)}{P}$ grow at rate g.

B Steady State Size Distribution of Firms

Klette and Kortum (2004) show that in steady state mass of firms of any type converges to

$$M_n\left(\varphi\right) = \frac{\eta\phi\left(\varphi\right)}{\mu n}\left(\frac{\lambda\left(\varphi\right)}{\mu}\right)^{n-1}. \tag{A10}$$

Using this equation, we can derive total mass of firms of type φ as

$$\begin{aligned}
M\left(\varphi\right) &= \sum_{n=1}^{\infty} M_n\left(\varphi\right) = \frac{\eta\phi\left(\varphi\right)}{\mu}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\lambda\left(\varphi\right)}{\mu}\right)^{n-1} \\
&= \frac{\eta\phi\left(\varphi\right)}{\lambda\left(\varphi\right)}\ln\left(\frac{\mu}{\mu-\lambda\left(\varphi\right)}\right). \qquad \text{(A11)}
\end{aligned}$$

Here, convergence to a stationary size distribution requires $\mu > \lambda\left(\varphi\right)$ for all φ. Taking the ratio of equations A10 and A11 gives the steady state size distribution for the number of products

$$\frac{M_n\left(\varphi\right)}{M\left(\varphi\right)} = \frac{\frac{1}{n}\left(\frac{\lambda(\varphi)}{\mu}\right)^n}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}.$$

This is probability distribution for the logarithmic distribution with parameter value of $\lambda(\varphi)/\mu$. Finally total mass of products produced by $\varphi-$type firms is found as

$$\Lambda(\varphi) = \sum_{n=1}^{\infty} n M_n(\varphi) = \sum_{n=1}^{\infty} \frac{\eta\phi(\varphi)}{\mu}\left(\frac{\lambda(\varphi)}{\mu}\right)^{n-1} = \frac{\eta\phi(\varphi)}{\mu-\lambda(\varphi)}. \tag{A12}$$

C Deriving Entry Type Distribution in Steady State

From equation A11 we get

$$\begin{aligned}\eta\phi(\varphi) &= \frac{M(\varphi)\lambda(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)} \\ &= \frac{M\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}.\end{aligned}$$

Taking the integrals of both sides and using the fact that $\int \phi(\varphi)\,d\varphi = 1,$ we get

$$\begin{aligned}\eta &= \int \eta\phi(\varphi)\,d\varphi = M\int \frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}d\varphi \\ \phi(\varphi) &= \frac{\frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}}{\int \frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}d\varphi}. \qquad \text{(A13)}\end{aligned}$$

C.1 Deriving Aggregate Price Index

We defined $\Lambda(\varphi)$ as total mass of products produced by $\varphi-$type firms. Plugging the value of $p(\varphi)$ from equation A7 into aggregate price index from equation 2, we get

$$\begin{aligned}P^{1-\sigma} &= \int_0^{\varphi_m^*} p^h(\varphi)^{1-\sigma}\Lambda(\varphi)\,d\varphi + \int_{\varphi_m^*}^{\infty} p^m(\varphi)^{1-\sigma}\Lambda(\varphi)\,d\varphi \\ P &= \left(\frac{\sigma}{\sigma-1}\frac{w}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)\left(\int_0^{\varphi_m^*}\varphi^{\sigma-1}\Lambda(\varphi)\,d\varphi + \int_{\varphi_m^*}^{\infty}\left(\varphi(1+N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}\right)^{\sigma-1}\Lambda(\varphi)\,d\varphi\right)^{-\frac{1}{\sigma-1}}.\end{aligned}$$

In the simulation exercise, aggregate price index can be computed by replacing $\Lambda(\varphi)$ by its value in equation A12 and $\phi(\varphi)$ by its value in equation A13 which yields to

$$\begin{aligned}P^{1-\sigma} &= \int_0^{\infty} p(\varphi)^{1-\sigma}\Lambda(\varphi)\,d\varphi \\ &= \int_0^{\infty} p(\varphi)^{1-\sigma}\eta\frac{\frac{\lambda(\varphi)\delta(\varphi)}{(\mu-\lambda(\varphi))\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}}{\int\frac{\lambda(\varphi)\delta(\varphi)}{\ln\left(\frac{\mu}{\mu-\lambda(\varphi)}\right)}d\varphi}d\varphi.\end{aligned}$$

C.2 Maximum Attainable Level of Efficiency

Let's define $\pi^{\max}$ as the maximum attainable level of profit that would satisfy $\lambda(\varphi) < \mu$ for all φ.

$$\begin{aligned}
\frac{\pi^{\max} - \frac{wc_0\mu^{1+c_1}}{(1+N)^{\zeta c_1}}}{r + \mu - \mu} &= \frac{wc_0 (1 + c_1)\, \mu^{c_1}}{(1 + N)^{\zeta c_1}} \\
\pi^{\max} &= \frac{wc_0 \mu^{c_1}}{(1 + N)^{\zeta c_1}} \left(r\,(1 + c_1) + \mu \right).
\end{aligned}$$

The efficiency level that would correspond to this profit level can be found as follows

$$\begin{aligned}
\bar{\pi} &= \pi^{\max} - f_m \\
&= \frac{E}{\sigma} \left[\frac{\sigma - 1}{\sigma} \frac{P\alpha^{\alpha} (1 - \alpha)^{1-a}\, \varphi \left(1 + N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}}}{w} \right]^{\sigma-1} - f_m \\
\varphi^{\max} &= \left[(\bar{\pi} + f_m) \frac{\sigma}{E} \right]^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma - 1} \frac{w}{P\alpha^{\alpha} (1 - \alpha)^{1-a} \left(1 + N\tau^{1-\gamma}\right)^{\frac{1-\alpha}{\gamma-1}}}.
\end{aligned}$$

C.3 Algorithm Steps for the Model Solution

For a given set of model parameters $\Delta = \{\ \Delta_1, \Delta_2\}$ we simulate a panel of 10,000 firms which are identified by their unique efficiency levels. Using this panel, we compute the simulated moments and seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. The steps to compute the equilibrium are described as follows:

1. The parameter vector is initialized and the vertices of the simplex are determined.
2. For each parameter vector, equilibrium level of aggregate price index P and aggregate destruction rate μ are computed.
3. For each of the 10,000 efficiency level draws, Bellman equation is solved and innovation rates are found.
4. Using these values, moment are computed.
5. The value of the criterion function is checked and using the amoeba routine, the simplex of parameter vectors is updated.
6. The system is iterated until either the value of the criterion function or the parameter vector converges.

D Some Additional Equilibrium Outcomes

Table A1. Some Eqilibrium Outcomes in Different Specifications

	Benchm.	ζ=0	σ=2.5
Panel A: Firm Dynamics			
% of firms stayed (all)	1	1.017	0.971
% of firms stayed (single)	1	1.005	0.987
% of firms stayed (multi)	1	1.442	0.941
% of firms shrinks	1	0.714	1.505
% of firms grows	1	0.704	1.479
Panel B: Product Dynamics			
Average number of products	1	0.989	1.003
Std. dev. of number of products	1	0.913	0.884
Average of product growth (cond. on survival)	1	0.668	1.604
Std. dev. of product growth (cond. on survival)	1	0.800	1.294
Contribution of multi-product firms to total sales	1	1.013	0.943

Notes: This table shows how some important equilibrium outcomes vary across equilibriums with different specifications. Benchmark results are produced using the parameters in Table 2. Each column presents results of an equilibrium where only one parameter is changed keeping the rest of the parameters at their benchmark values. Each cell in a row is normalized with the benchmark value of that row. All simulations are performed under general equilibrium setup.

D.1 Asymmetric Countries

To further examine the attributes of the model we may allow countries to differ in their prices of intermediate goods. In the model, we have assumed that in each country one unit of intermediate input is produced with one unit of labor and its price is the local wage rate. If we define $wedge = w_f/w_h$ and $\hat{\tau} = wedge * \tau$, changing the asymmetry in prices of intermediate goods across countries is, in fact, isomorphic to changing the iceberg cost. This asymmetry highlights that lower the price of imported inputs higher the return from importing these inputs[27].

(Bloom, Draca, and Van Reenen, 2016) also studies the impact of imports on domestic innovation from a South-North perspective and finds that a higher threat of Chinese imports increase within-firm innovation (patenting and R&D) in selected European Union

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[27] In our specification, lower wage in a country implies lower average productivity of intermediate good producers in that country, and it may imply that imports from lower average productivity countries are far more beneficial for the importer country. However, such an interpretation may be misleading. Because we are forcing one unit of intermediate input to be produced with one unit of labor in each country regardless of the average productivity level of the labor in that country.

countries. Trapped factors of production keep firms in the industry and firms innovate new products to escape competition from the low wage country products.

The positive impact of imported intermediates that we are capturing through the spillover channel may be driven by the trapped factors laid out in (Bloom, Romer, Terry, and Reenen, 2013; Bloom, Romer, Terry, and Van Reenen, 2015). Since we do not allow a friction that hinders reallocating factors of production across firms we do not observe such a trapped factor phenomenon.

Although our model and (Bloom et al., 2015) have similar market size effects from being more integrated to the world market which boost profitability of domestic firms, we are depicting a different market mechanism than theirs where they focus on the direct competition from low-wage countries which interacts with the trapped factors of firms.

Using Canada-US Free Trade Agreement as a natural experiment for analyzing the relationship between trade costs and product scope, BRS (2011) find that in response to trade costs reductions U.S. firm concentrate production in their most successful products. Their revenues go up while the number of products they produce decline. In our setup liberalization brings similar increase in revenues. What we label as product scope of a firm n also increases. We have already discussed that the variable n can also be interpreted as the knowledge stock of the firm as KK does. In response to liberalization, through more research effort, knowledge stock of some firms expands. Nevertheless, our model is silent about heterogeneity across products of a firm.