Flexibility and Productivity Heterogeneity of Multiproduct Exporters

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Abstract

We document three stylized facts for Chinese exporters that challenge the traditional models of multiproduct firms in which productivity is the only determinant of firm's sales and scope. First, firm-destination specific shocks explain more than 50% of the variation in scope across firms and destinations. Second, there is a disconnect between sales and scope across firms within a destination: for any level of sales, there are several single product firms and few wide-scope firms. Third, the scope of exporters conditional on sales depends on measurable characteristics of firms, such as capital intensity and R&D expenditures. We rationalize the three stylized facts in a model featuring firm heterogeneity in productivity and in flexibility, namely the ability to introduce new varieties in a destination at low costs. Heterogeneity in firm's flexibility explains more than 20% of the total variation in scope across firms and destinations.

Keywords: Multiproduct firms, Flexible Manufacturing, Firm Heterogeneity, Firm-level data, China.

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1 Introduction

Standard models of multiproduct firms assume that one feature of the firm - productivity - drives both the ability to produce a good efficiently and the ability to produce many goods (Bernard et al., 2011; Mayer et al., 2014). More productive firms export a larger number of varieties, or scope, and enjoy larger sales. The positive relationship between productivity, sales, and scope is intuitive: a rise in firm's productivity increases the sales of each existing variety, and restores the profitability of discontinued varieties. This study provides new evidence suggesting that firms differ in their productivity and the ability to introduce new varieties. As multiproduct firms dominate trade flows (Bernard et al., 2007), modeling such a two-dimensional heterogeneity has major implications for the margins of trade.

Using Chinese firm-level data from the China Custom Dataset and the Annual Surveys of Industrial Production (Brandt et al., 2014), we document three new stylized facts for multiproduct exporters. First, firm-destination specific shocks explain more than 50% of the variation in scope across firms and destinations. Models of multiproduct firms predict that destination characteristics such as size (Bernard et al., 2007), per capita income (Macedoni, 2017), and competition (Mayer et al., 2014), along with firm's productivity, determine the scope of exporters. The evidence suggests that firm-destination specific shocks, as those modeled by Arkolakis et al. (2014) and Mayer et al. (2016), are quantitatively relevant in determining firm's scope.

Second, we document a disconnect between total sales of a firm, or scale, and its number of varieties, or scope, in a given destination. Standard models of multiproduct firms predict a positive relationship between sales and scope (Bernard et al., 2011), driven by productivity. However, there are several single product firms and wide-scope firms at any level of sales by Chinese firms. While any kind of firm-destination specific shocks to demand (Kee and Krishna, 2008) or entry costs (Arkolakis et al., 2014) could explain the variation of scope across firms and destinations, only shocks to the within-firm extensive margin can rationalize the disconnect between scale and scope. To generate the disconnect between sales and scope, a model needs firm-destination specific shocks that affect the choice of scope but that leave unchanged the sales per variety.

The observed disconnect is robust to alternative measures of firm's size, it emerges in all manufacturing industries, and it is independent of the level of differentiation of products. Moreover, we document a similar disconnect between scope and firm's productivity: there are single product firms and wide-scope firms for any level of productivity. In addition, we divide firms in quartiles by their productivity, and study the distribution of scope conditional on productivity. The distribution of the conditional scope approximates a Pareto distribution, as in each quartile the largest mass of firms produces only one product, and smaller fractions of firms export a wider scope.

Finally, in the spirit of Hallak and Sivadasan (2013)[1], we investigate the determinants of the shock to the within-firm extensive margin, studying the relationship between exporter's scope,

[1] Hallak and Sivadasan (2013) study the exporters premia conditional on sales

conditional on scale, and other firm level characteristics[2]. The conditional scope of Chinese exporters is affected by productivity, capital stock, capital intensity and R&D spending. The signs of the relationships vary across industries, suggesting that the determinants of the shock to the within-firm extensive margin differ significantly across industries. For instance, the conditional scope of textile firms declines with productivity: when two firms have the same sales, the lower the productivity, the higher the scope. However, such a relationship is absent or even positive in other industries.

We rationalize the three stilyzed facts in a general equilibrium, multi-country model of multiproduct firms based on Bernard et al. (2011) and Arkolakis et al. (2014). Consumers have Constant Elasticity of Substitution (CES) preferences over a continuum of varieties offered by a mass of monopolistically competitive multiproduct firms. Following Eckel and Neary (2010) and Mayer et al. (2014), we assume that firms have a core competence: one variety that has the lowest marginal cost of production. The marginal costs of additional varieties increase as the firm moves away from its core competence. As a result, the first variety of each firm has the highest revenue, which falls as the firm introduces varieties farther from the core.

Firms differ in their productivity and in the ability to introduce a new variety in a destination at low cost. In particular, expanding the scope in a destination requires the payment of a fixed cost, to adapt production and distribution processes to the new variety. The fixed cost per variety is subject to firm-destination specific shocks. The heterogeneity in the fixed cost can be interpreted as heterogeneity in the flexibility of firm's production processes. Eckel and Neary (2010) define flexible manufacturing as the ability of firms to introduce new varieties with minimal adaptations to production processes[3]. While in Eckel and Neary (2010) firms are fully flexible and the fixed cost per variety is absent, in our framework firm's flexibility is subject to shocks: the higher the fixed cost per variety, the lower the firm's flexibility.

The scope of a firm in a destination depends on the size of the destination, bilateral trade costs, and firm's productivity, which is common across the destinations reached by the firm. Moreover, the scope falls with the realization of the firm-destination specific fixed cost: the larger the realization of the shock, the less flexible the technology of the firm, and the smaller the scope. If the fixed cost is large enough, the firm decides not to be active in a destination. The fixed cost shock is the firm-destination specific variable that appears quantitatively relevant in the data. Moreover, the model explains the disconnect between scale and scope, and between productivity and scope. In fact, firms with the same level of productivity or sales export a different scope depending on the realization of the fixed cost shock, or flexibility. We assume the flexibility shock follows a Pareto distribution to match the distribution of scope conditional on productivity that we observe

[2]By conditional scope we refer to the scope conditional on firm's sales.

[3]The IO literature first dealt with flexible manufacturing (Eaton and Schmitt, 1994). A more general definition by Milgrom and Roberts (1990) states that flexible manufacturing allows for a quick response to market conditions. Hence, flexible manufacturing is related to the ability of introducing more varieties, of reducing delivery times (Tseng, 2004), and of changing production scale with minor adjustment costs (Gal-Or, 2002).

in the data. The larger the dispersion of the flexibility shock, the larger the dispersion of the scope distribution.

The main purpose of the model is to understand how heterogeneity in flexible manufacturing, combined with firm's productivity, shape the exporting decisions of firms and aggregate trade flows. To preserve tractability, we avoid modeling the determinants of firm's flexibility, and instead use a shorthand by allowing the distribution of flexibility draws to be correlated with firms' productivity. As we let the sign of the correlation unspecified, the model generalizes the results that arise from alternative frameworks of multiproduct firms (Arkolakis et al., 2014; Nocke and Yeaple, 2014) and of flexible manufacturing (He, 1992). Moreover, the model can shed some light on industry specific determinants of flexible manufacturing, which can range from cash flows to R&D expenditures (Parisi et al., 2006), and the relevance of these determinants is heterogeneous across industries (Klette and Kortum, 2004; Bertschek, 1995).

Consistent with the empirical evidence, our model predicts that the conditional scope of active firms depends on firm's observable characteristics - in our case, productivity. In fact, the flexibility shock received by active firms is related to firm's productivity through two channels. First, there is a selection effect, which is independent of the correlation between flexibility and productivity draws. Only more productive firms survive a low flexibility shock while less productive firms remain active if they receive a high flexibility shock. As a result, firms' selection generates a negative relationship between expected flexibility shock and firm's productivity. Therefore, the conditional scope of active firms tends to be negatively related to firms' productivity or other variables positively related to productivity, such as R&D (Klette and Kortum, 2004), which we observe in some industries.

However, in other industries, conditional scope and firm's productivity are uncorrelated or even positively related. To explain this relationship, our model offers a second channel, which originates from the correlation between the draws of flexibility and productivity. When flexibility and productivity are negatively related, the probability of a firm surviving in a destination declines with productivity. The more negatively related productivity and flexibility are, the lower the average fixed cost shock received by surviving high-productivity firms is. Hence, when the two draws are negatively correlated, the conditional scope of firms tends to be uncorrelated or positively related to firm's productivity: the second channel mitigates or even reverses the first channel. In contrast, when productivity and flexibility are positively related the effects of the first channel are magnified.

The model is consistent with several established regularities on multiproduct firms in international trade. On average, firms export only a fraction of the goods they sell domestically (Iacovone and Javorcik, 2010), and a reduction in trade costs reduces the scope of domestic firms (Bernard et al., 2011). Because of the core competence assumption, firms' sales are skewed towards the core, in line with (Arkolakis et al., 2014). Even when productivities are Pareto distributed, the model generates a distribution of sales which approximates a log normal distribution, similar to the empirical distribution of sales of Chinese firms. Finally, because of the fixed cost shock there is no hierarchy of destinations for exporters (Eaton et al., 2011), and the relationship between productivity and sales is far from perfect (Kee and Krishna, 2008).

The model predicts that the elasticity of firm's sales with respect to trade costs declines with the firm's scope. Suppose that a positive trade shock reduces the cost of exporting in a destination. Firms react by increasing their scope and the sales per variety. However, wide-scope firms, either because of high productivity or high flexibility, introduce new varieties that are far from the core, and, thus, have a lower impact over total firms' sales. Furthermore, firm's heterogeneity in flexibility modifies the aggregate responses of firms to trade shocks. In fact, the intensive margin of trade - the change in trade flows that arises from changes in firms' sales - depends on the distribution of firms' flexibility. The more disperse the distribution of flexibility, the larger the number of wide-scope firms, and the smaller the change in the intensive margin.

The heterogeneity of firm's flexibility affects aggregate trade flows, as both its dispersion, and its relationship with productivity, determine the trade elasticity[4]. When flexibility and productivity are positively correlated, productivity differences are magnified and, therefore, entry of new firms in a destination is dampened. Thus, the trade elasticity declines with the correlation between productivity and flexibility. The effect of the dispersion in firms' flexibility on the trade elasticity with respect to variable trade costs depends on the correlation between productivity and flexibility. If the correlation is positive, higher dispersion of flexibility increases the trade elasticity, and vice versa if the correlation is negative. In contrast, the trade elasticity with respect to fixed cost always increases with the dispersion of flexibility.

Using the scale and scope disconnect predicted by the model, we estimate the firm-destination specific fixed cost shock. The model, in fact, generates a simple equation that relates the scale and scope of a firm to their fixed cost shock. The estimation technique is appropriate, given that the scale and scope disconnect can only be rationalized with a firm-destination shock to the within-firm extensive margin. The fixed cost shock has an explanatory power that is comparable to firm's specific characteristics: heterogeneity in firms' flexibility explains approximately 20-30% of the variation in scope across firms and destinations.

The remainder of the paper is organized as follows. Section 2 surveys the related literature. Section 3 documents three new empirical regularities for Chinese firms. Section 4 presents our model of multiproduct firms, highlighting how the model rationalizes the empirical evidence, and discussing its implications for aggregate trade flows. In Section 5, we estimate the fixed cost shock and study its contribution to the scope decisions of firms. Section 6 concludes the paper.

[4]Following a long tradition (Chaney, 2008), we assume that productivity is Pareto distributed. Arkolakis et al. (2012) show that for a large class of models, including ours, the welfare gains from trade are proportional to the change in the domestic expenditure share, which is weighted by the inverse of the trade elasticity.

2 Related Literature

Since the seminal work of Melitz (2003), a large body of literature assumes that heterogeneity in a single attribute of the firm, usually productivity, drives the heterogeneity in firms' sales and exports. In "single attribute" models, as Hallak and Sivadasan (2013) label them, there is a one-to-one mapping of firm's productivity into its sales and number of destinations reached: the more productive a firm is, the larger its sales and number of destinations reached are (Melitz, 2003). Additionally, in the context of multiproduct firms, there is a positive relationship between productivity, sales and scope (Bernard et al., 2011)[5].

Recent evidence challenges the "single attribute" assumption. In fact, the correlation between sales and access to export markets is far from perfect, as there are small firms that export and large firms that only sell domestically (Kee and Krishna, 2008; Eaton et al., 2011). Moreover, while a standard model predicts a hierarchy in the destinations reached by firms, there are several firms that export to destinations that are difficult to reach but not to others that are easier to serve Eaton et al. (2011). To rationalize these facts, the literature proposes additional layers of heterogeneity across firms, which either affect the demand or the supply side.

On the demand side, Kee and Krishna (2008), Eaton et al. (2011), Demidova et al. (2012), Roberts et al. (2012), and Cherkashin et al. (2015) add exogenous firm-destination specific demand shifters[6]. Demand shifters cannot replicate the observed scale and scope disconnect: a positive demand shock increases the scope of a firm, by increasing the sales of each variety - even those with zero initial sales. Hence, when two firms, with different productivity and demand shocks, have the same sales, they also have the same scope.

On the other hand, a fixed cost shock only affects the extensive margin of a firm's sales, and, as a result, it can rationalize the scale and scope disconnect. Eaton et al. (2011), Armenter and Koren (2015), and Arkolakis et al. (2014) introduce firm-destination specific shocks to the fixed cost of exporting[7]. The fixed cost shock represents differences in the ability with which firms introduce new products in a destination at low costs. Such an assumption finds support in the evidence documented by Parisi et al. (2006): firms' investments in physical capacity and R&D differentially affects their productivity and scope.

In the context of multiproduct firms, Arkolakis et al. (2014) augment the Bernard et al. (2011) model with product-firm-destination specific demand shocks and with firm-destination specific shocks[8]. Such shocks are introduced in the spirit of Eaton et al. (2011) to better match the data.

[5]More productive firms sell more varieties and higher volumes for each variety. The one-to-one mapping of sales into scope is independent of the type of competition chosen, as it arises in models of monopolistic competition (Allanson and Montagna, 2005; Brambilla, 2009; Bernard et al., 2011; Mayer et al., 2014) and oligopoly and cannibalization (Feenstra and Ma, 2007; Eckel and Neary, 2010; Macedoni, 2017). A similar relationship arises in models where firms can choose between product and process innovation (Dhingra, 2013).

[6]Demand shifters can be interpreted as product quality (Hallak and Sivadasan, 2013; Fasil and Borota, 2013).

[7]Moreover, Harrigan and Reshef (2015) model heterogeneity in productivity and skill intensity while Lileeva and Trefler (2010) add heterogeneity in the productivity gains due to investment in innovation.

[8]Mayer et al. (2016) also model demand shocks in the context of multiproduct firms.

Our empirical work further supports the authors' assumption. In fact, the firm-destination specific shock to the fixed cost per variety generates the observed scale and scope disconnect: depending on the realization of the shock firms with the same level of total sales may export a different number of varieties.

While Arkolakis et al. (2014) interpret the fixed cost per variety as a market access costs, our study argues that the fixed cost per variety is a measure of firms' flexibility in production (Eckel and Neary, 2010), and, therefore, it is related to firm's observable characteristics, such as capital intensity and R&D expenditures. In our model, we capture market access costs with a fixed entry cost that is independent of scope. Such a cost prevents the least efficient firms from entering in any market, which is possible in Arkolakis et al. (2014). Moreover, we refine the distributional assumptions of Arkolakis et al. (2014), guided by the new evidence. The authors assume that the fixed cost shock follows an *i.i.d.* log normal distribution. Since we observe that, conditional on productivity, a large mass of firms exports a single product, we assume that the fixed cost shock follows a Pareto distribution. Second, as the shock is related to observable firm's characteristics, we assume that the distribution of such shocks is firm-specific.

Another model of multiproduct firms that features two sources of heterogeneity is that of Nocke and Yeaple (2014). In their model, firms differ in terms of productivity and organizational capital. Firms allocate their capital endowment across their varieties. Firm's technology exhibits diseconomies of scope: more productive firms produce larger quantities of a variety given the same amount of capital. Hence, more productive firms prefer to produce fewer varieties more efficiently while firms that have larger endowments of capital produce a wide scope.

3 Stylized Facts for Multiproduct Exporters

In this Section, we document three stylized facts for Chinese exporters. First, the variation in scope across destinations and firms is largely explained by firm-destination specific shocks. Second, within a destination, there is a disconnect between scope and sales, and between scope and productivity. Third, observable firm's characteristics are correlated with the scope conditional on sales. We begin by describing the sources of data and examining the distribution of exporters' sales and scope.

3.1 Data

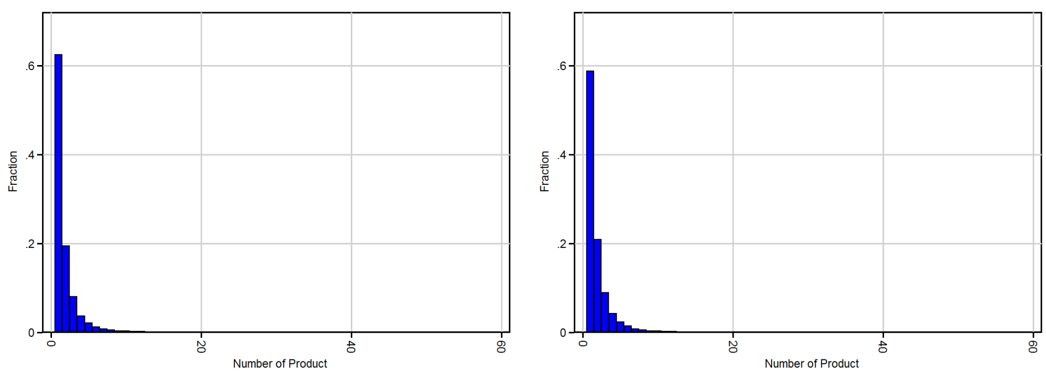
We rely on two sources of data. The first is the China Custom Dataset, which provides data on export values at the product-firm-destination level for all international transactions from China. A product is a Harmonized System (HS) eight-digit code. To understand which firms' characteristics influence the scope decisions of firms, we use the Annual Surveys of Industrial Production (ASIP) that is conducted by the National Bureau of Statistics of China (Brandt et al., 2014). The dataset covers manufacturing firms with more than five million RMB in annual sales ($\approx \$700k$). For each firm, the ASIP provides data on employment, output, and elements of accounting statements. We use the China Custom Dataset to document the first two stylized facts, and we combine it with the ASIP for the third empirical regularity.

We use names, location, zip code, and telephone number to match the firms in the two datasets. We match 30% to 40% of exporters involved in ordinary trade to the information provided in ASIP[9]. The China Custom Dataset ranges from 2000 to 2006 while the ASIP from 1998 to 2007. While the results of our paper hold for each year separately, for the sake of exposition we focus on 2006, which has the largest number of matched firms. Moreover, we refine our sample to the firms involved in ordinary trade only[10].

Chinese multiproduct firms dominate the country's exports: 77% of exporters sell at least two products in a destination, and they account for 94% of total export value. Such results are in line with the evidence documented for several other countries by Bernard et al. (2007), Mayer et al. (2014), Arkolakis et al. (2014), and Macedoni (2017). Our sample of matched firms exhibits a similar distribution: 78% the firms in the sample are multiproduct and they account for 96% of the sample's total exports.

Figure 1: Distribution of Product Scope in the U.S. (2006)

(a) All Sample (b) Matched Sample



As the main empirical results of the paper focus on the distribution of firms' scope, sales, and productivity within a destination, we focus on the US, the most popular destination for Chinese exports[11]. Figure 1 illustrates the distribution of the number of HS eight-digit goods per firm that Chinese exporters sell to the US. The first graph uses all exporters while the second focuses on the sample of firms with matched characteristics from ASIP. The two distributions are remarkably

[9]We follow the conventional method to match firms from the China Custom Data to the ASIP (Feenstra et al., 2014; Yu, 2015; Manova and Yu, 2016).

[10]According to Dai et al. (2016), firms involved in processing exports behave abnormally in China. Therefore, we focus on the firms that export in ordinary mode only (the code for trade mode is 18). In the robustness analysis, we additionally exclude the state-owned ordinary exporters.

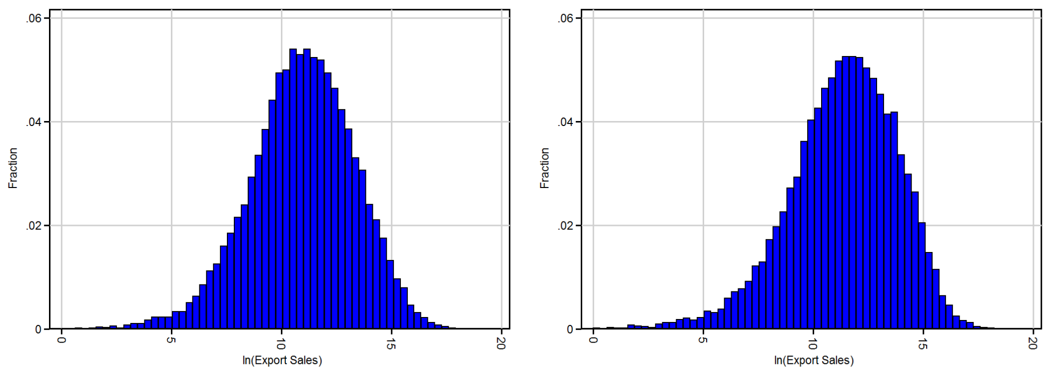
[11]The results are similar when we consider other popular destinations for Chinese exporters: South Korea, Germany, and the UK.

similar, as they both exhibit the largest mass for a scope of one. In particular, 36% of all Chinese exporters and 40% of our matched sample export a single HS eight-digit good to the US. However, matched firms have a wider scope, on average, than the firms in the entire sample.

While the scope distribution across firms resembles a Pareto distribution, sales appear to be lognormally distributed. Figure 2 shows the distribution of log sales within the US, for the entire sample of firms and the matched sample. Although the average sale is larger for the matched sample, the two distributions look virtually identical.

Figure 2: Distribution of ln(Export Sales) in the U.S. (2006)

(a) All Sample (b) Matched Sample



3.2 Exporter Scope Across Firms and Destinations

"Single attribute" models of multiproduct firms predict that the scope of an exporter f, from i to j, depends on three sets of variables. The first set involves firm's f productivity and country i's factors' costs. Second, firms choose their scope according to several characteristics of the destination j, such as aggregate size (Bernard et al., 2011), level of development (Macedoni, 2017), and intensity of competition (Mayer et al., 2014). Finally, the scope of an exporter is influenced by bilateral trade costs from i to j (Bernard et al., 2011). How much do these three types of variables explain the variation of scope across firms and destinations?

To answer this question, we regress the number of products that a firm f exports to a destination j on a firm fixed effect a_f and a destination fixed effect d_j:

$$\ln(\#\ \text{Products}_{fj}) = a_f + d_j + c_{fj} \tag{1}$$

where the error c_{fj} captures firm-destination specific shocks to the scope. The firm level fixed effect absorbs firm-specific characteristics that are common across the destinations while the destination fixed effect captures both destination's features and bilateral trade costs, since we have only one country of origin.

In table 8, we show the R^2 of the regression, as well as the contribution to the model fit of the firm and destination fixed effects. The simple model (1) explains 37% of the scope variation for all exporters and 44% for the sample of matched exporters. More than a half of the variation in the scope of Chinese exporters is explained by firm-destination specific shocks. Firm characteristics account for almost all the explanatory power of standard models, since destination features have a smaller impact[12].

Table 1: Decomposition of Product Scope Variation

Sample	Model Fit (R^2)	a_f	d_j
All Exporters	0.37	0.36	0.01
Matched Exporters	0.44	0.42	0.02

R^2 from (1), and from regressing the log of the scope on firm (a_f) and destination (b_j) fixed effects only.

The results are robust to analyzing the additional years available, and the R^2 becomes even smaller as we consider years prior to 2006. With a regression model similar to (1), Macedoni (2017) finds that firm and destination characteristics explain about 50-60% of the scope variation of multiproduct exporters from Mexico, Peru and other low to middle income countries.

3.3 Scale and Scope Disconnect

Let us now focus on the within-destination distribution of product scope, sales and productivity. "Single attribute" models of multiproduct firms predict a positive relationship between number of products exported by a firm f in j and its sales:

$$(\#\ \text{Products})_{fij} = G(b_{ij}, \text{Revenue}_{fij}) \tag{2}$$

where b_{ij}, depending on the model, captures destination characteristics and bilateral trade costs. Moreover, since firm's revenues are proportional to the firm's productivity, (2) can be written as:

$$(\#\ \text{Products})_{fij} = G^{'}(b'_{ij}, \text{Productivity}_f) \tag{3}$$

How do (2) and (3) perform in the data? Choosing the US as destination, we plot the scope of Chinese exporters against the natural logarithm of their sales in Figure 3.

There is a positive relationship between sales and scope, as the standard models would predict. The regression coefficient is positive and significant and exhibits a similar magnitude for both samples. However, Figure 3 highlights that such a relationship is far from perfect. At any level of sales, there are single product firms and multiproduct firms. Although only at medium-high level

[12]Destination characteristics mainly affect the within-firm R^2 of the scope regression. Estimating (1) by industry yields similar results

of sales it is possible to observe wide-scope multiproduct firms, the standard models would fail to explain why, at those level, there are narrow scope firms as well.

Figure 3: Product Scope and Sales of Exporters in the U.S. (2006)

(a) All Sample (b) Matched Sample

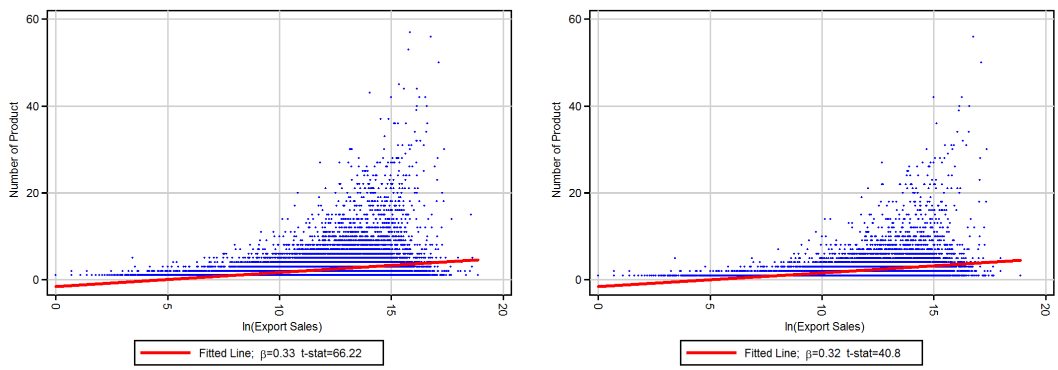
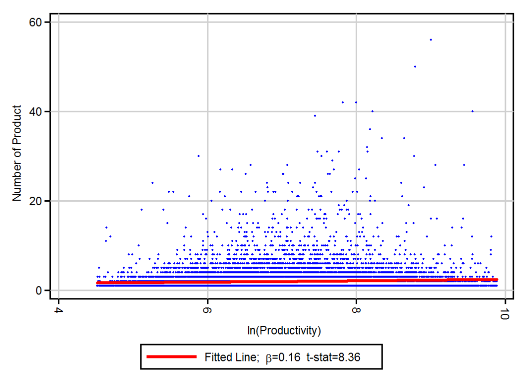


Figure 4: Product Scope and Productivity of Exporters in the U.S. (2006)

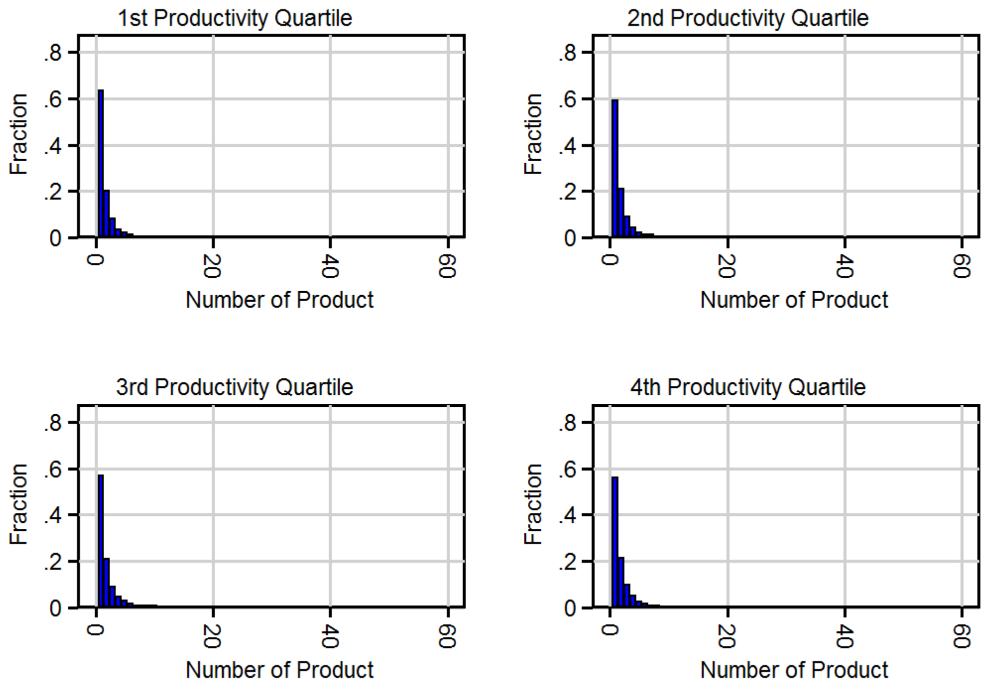


To evaluate the relationship between scope and firm's productivity, we focus on the sample of matched exporters, for which ASIP provides data on employment, value added, capital and intermediate inputs. We use the Levinsohn and Petrin (2003) method to estimate productivity, and exclude firms whose productivity values are $\leq 1\%$ or $\geq 99\%$[13]. The relationship between productivity and scope (Figure 4) is similar to that between sales and scope (Figure 3). In fact, there is a positive and statistically significant relationship between productivity of a firm and scope in a destination. At any level of productivity, there are single and multiple product firms. Moreover, wide-scope firms are spread across all ranges of productivity. "Single attribute" models of multiproduct firms can hardly be reconciled with this evidence.

The average scope and average sales vary by industry but the disconnect between the scale and scope persists. Moreover, results are robust to dividing firms between those that produce homogeneous and differentiated goods according to the definition introduced by Rauch (1999).

[13] The appendix provides the detailed procedure for estimating firm's productivity.

Figure 5: Scope Distribution by Productivity Quartiles



Figures 4 and 3 hide the distribution of product scope conditional on productivity and sales. If the presence of single product firms at any level of sales and productivity is simply an outlier event, the case for adding an additional layer of firms' heterogeneity to the standard model would be weakened. To address such a concern, we divide firms in quartiles by productivity, and plot the distribution of product scope conditional on the firm belonging to a certain quartile of the productivity distribution.

Figure 5 shows that in each quartile, the majority of firms exports a single product to the US. The distribution of scope conditional on productivity resembles a Pareto distribution, and it is similar to that arising when we divide firms in quartiles by their sales. The result is in stark contrast to a standard model of multiproduct firms, which, even allowing for some noise in the data, would predict that the peak of the distribution would be shifting to higher scope as productivity increases.

3.4 What Causes the Disconnect?

In this Section, we investigate whether the deviations of the exporter's scope from the value predicted by export values are random or if they depend on other observable characteristics of firms. Such hypothesis is motivated by the literature, which suggests that the scope of firms may depend on firms' characteristics that are related to productivity. Using Italian data, Parisi et al. (2006) find that R&D expenditure and cash flows increase the likelihood of product innovation by firms. Timoshenko (2015) finds that product switching occurs more often in new exporters, suggesting that firm's learning could affect the scope decisions of firms[14].

We first run the following regression for Chinese exporters in the US:

$$\ln(\#\text{ Products}_{fUS}) = b_0 + b_1 \ln(\text{Revenue}_{fUS}) + \epsilon_f \tag{4}$$

and record the error term ϵ_f. Next, we run univariate regressions of ϵ_f on the following firm's level variables: value added per worker, capital intensity, total assets, R&D expenditures, and advertisement fees. Since the results are industry specific, we apply the procedure for each industry separately. Moreover, we focus on the sample of matched firms, as only for those we have information on the explanatory variables.

Table 2: Correlation Between Residual and Firm Characteristics

Firm Characteristics	Textile	Machinery
VA per Worker	-0.027*	0.001
Capital Intensity	-0.265***	0.033
Total Asset	0.019*	0.020***
R&D Fee	-0.006	0.016**
Advertisement Fee	0.004	0.015**

Coefficients from regressing ϵ_f from (4) on firm characteristics. *** p<0.01, ** p<0.05, * p<0.1.

For the sake of exposition, let us focus on the two largest export industries by value: textile and machinery (Table 2). The residuals from (4) are correlated to several firm level characteristics, although there is heterogeneity in the industry specific coefficients. Conditional on sales, the scope of textile exporters declines in value added per worker and capital intensity. This means that given two firms with the same level of sales, the firm with higher value added per worker, or capital intensity, exports fewer varieties. On the other hand, the conditional scope of machinery exporters increases in R&D and advertisement expenditures. Total asset increases firm's conditional scope for both textile and machinery.

There is considerable heterogeneity in the relationship between conditional scope and firm's characteristics across industries. In the appendix, we report the industry-level correlations. Total capital stock improves the conditional scope for chemicals, plastic, wood and footwear while it reduces the conditional scope of firms producing stone and glass, metals and transportation. A positive relationship between capital stock and conditional scope may suggest the presence of

[14] We do not find a role for firm's age in explaining the conditional scope, suggesting that firm's experience does not affect the fixed cost per variety. Such finding is not in contrast with Timoshenko (2015), where firm's experience is linked to firm specific demand shock.

economies of scope (Arkolakis et al., 2014): the larger the capital stock, the lower the fixed cost per product. A negative relationship, on the other hand, suggests the presence of diseconomies of scope (Nocke and Yeaple, 2014).

In addition to textile, higher capital intensity has a negative and significant effect on the conditional scope of firms producing food, footwear, and stone and glass. A similar relationship arises between conditional scope and value added per worker in the same industries. Finally, expenditures on R&D improve the conditional scope in the chemicals, wood and miscellaneous industries. The result is consistent with the evidence from Parisi et al. (2006) who document that R&D improves product innovation. Moreover, Klette and Kortum (2004) showed substantial heterogeneity in R&D across industries.

4 Model

To rationalize the three stylized facts, we build a general equilibrium model of monopolistically competitive multiproduct firms based on Bernard et al. (2011) and Arkolakis et al. (2014). The first part of this Section illustrates the general model and describes how trade shocks affect firm-level and aggregate exports. Second, we choose functional forms for the distribution of shocks to replicate, qualitatively, the distributions observed in the data. We then discuss how the model can explain the empirical evidence. The Section concludes with the description of the margins of adjustments to trade shocks and the corresponding elasticities.

4.1 Consumer's Problem

There are I countries: subscript i denotes an origin and j a destination. In each country j, L_j consumers, with per capita income y_j, enjoy the consumption of varieties of a differentiated good. Consumers in each country j have the same CES preferences:

$$U_j = \left[\sum_{i=1}^{I} \int_{\Omega_{ij}} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where $\sigma > 1$ is the elasticity of substitution across varieties, and Ω_{ij} is the set of varieties exported from i to j. As in Bernard et al. (2011) and Mayer et al. (2014), varieties are horizontally differentiated, and the elasticity of substitution within firms is the same as the elasticity of substitution across firms. Thus, we assume away cannibalization effects that would be generated by a nested preference structure, which only quantitatively affects the results of this model.

Let $x_{ij}(\omega) = L_j q_{ij}(\omega)$ be the aggregate demand for variety ω from i to j. Solving the consumer's problem and aggregating across consumers yield the following inverse demand function[15]:

$$p_{ij}(\omega) = A_j x_{ij}(\omega)^{-\frac{1}{\sigma}} \tag{6}$$

4.2 Firm's Problem

Each firm produces a continuum of varieties[16], with a constant returns to scale technology and labor is the only input and is paid a wage w_i. Each firm produces the first variety, or *core*, with the lowest labor requirement, and the marginal cost of new varieties increases with their distance from the core. Such an assumption generates a within-firm distribution of sales which is consistent with the data (Eckel and Neary, 2010; Mayer et al., 2014).

Introducing a new variety in a destination additionally requires a fixed cost, which represents the costs of adjusting production and distribution processes to the new variety (Bernard et al., 2011; Arkolakis et al., 2014). We interpret the fixed cost per variety as a measure of firm's flexibility in adjusting its production and delivery processes to new varieties (Eckel and Neary, 2010). Such a fixed cost is subject to shocks that are firm-destination specific, and are common to all goods produced by one firm. As a result, in addition to productivity differences, firms differ in terms of their flexibility. Heterogeneity in flexibility generates the observed scale and scope disconnect[17].

As in the Melitz (2003) model, in each country there is a pool of potential entrants. Upon entry, a firm pays a fixed cost f_E in domestic labor unit and discover the productivity $\frac{1}{c}$ of its core variety, where c is drawn from a distribution $G_i(c)$, with pdf $g_i(c)$ and support $[0, \bar{c}_i]$. Only a mass N_i of firms pays the fixed cost of entry. Free entry drives the expected profits to zero and, therefore, per capita income y_i equals the wage rate w_i.

To export to a destination j, a firm faces a two-stage problem similar to Demidova et al. (2012) and Cherkashin et al. (2015). In the first stage, the firm decides whether to pay a fixed cost F_{ij} and discovers its realization of the fixed cost per variety in j. Only a subset of firms pays F_{ij}. To introduce a new variety in a destination j, each firm pays a fixed cost $f_{ij}\beta$, where f_{ij} is the deterministic component while β is subject to firm-destination specific shocks. The firm draws β from a distribution $B_c(\beta)$, with pdf $b_c(\beta)$ and support $[0, \beta_{max,c}]$. The distribution of β could be firm specific, since we documented that the conditional scope is related to observable characteristics of the firm.

In the second stage, given c and β, each firm chooses quantity $x_{ij}(\omega, c, \beta)$ for each produced variety $\omega \in [0, \delta_{ij}(c, \beta)]$, and scope $\delta_{ij}(c, \beta)$. To produce a variety ω from i to j, the firm pays

[15] A_j is a demand shifter that equals $A_j = y_j L_j \left[\sum_{i=1}^{I} \int_{\Omega_{ij}} x_{ij}(\omega)^{\frac{\sigma-1}{\sigma}}\right]^{-1}$.

[16] The assumption of a continuum of varieties within the firm is made for tractability, as in Bernard et al. (2011) and Eckel and Neary (2010). For models where a continuum of firms produces a discrete number of varieties, see Mayer et al. (2014) and Arkolakis et al. (2014).

[17] Although firm-destination specific demand shocks may be empirically relevant (Kee and Krishna, 2008; Eaton et al., 2011; Arkolakis et al., 2014; Mayer et al., 2016), they do not generate the observed disconnect between scope and sales.

a marginal cost $\tau_{ij} w_i c h(\omega)$, where τ_{ij} is the iceberg trade cost, and $\tau_{ii} = 1$. To capture the core competence assumption, we assume that $h'(\omega)$ is increasing in ω. Moreover, we normalize $h(0) = 1$, so that $\frac{1}{c}$ is the productivity of the core variety of a firm with cost draw c[18].

4.2.1 Second Stage

To solve the firm's problem we proceed backwards, starting from the second stage. The constant returns to scale assumption allows us to analyze the profit maximizing choices of firms by destination. The profits of a firm from i to j are given by:

$$\Pi_{ij}(c,\beta) = \int_0^{\delta_{ij}(c,\beta)} [(p_{ij}(\omega) - \tau_{ij} w_i c h(\omega)) x_{ij}(\omega) - f_{ij}\beta] d\omega \tag{7}$$

with $p_{ij}(\omega)$ from (6). Profit maximization yields the standard constant markup pricing rule:

$$p_{ij}(\omega, c) = \frac{\sigma}{\sigma - 1} \tau_{ij} w_i c h(\omega) \tag{8}$$

It is convenient to re-write the profits of a variety ω as follows:

$$\pi_{ij}(\omega, c, \beta) = \frac{1}{\sigma - 1} \left[A_j \frac{\sigma - 1}{\sigma} \right]^{\sigma} (\tau_{ij} w_i c h(\omega))^{1-\sigma} - f_{ij}\beta \tag{9}$$

From (9), and given that $h^{'}(\omega) > 0$, there exists a value of the fixed cost shock $\beta^*_{ij}(c)$, such that a firm with cost draw c makes zero profits by selling its core variety $\omega = 0$. If a firm with productivity $\frac{1}{c}$ draws $\beta < \beta^*_{ij}(c)$, it sells a positive scope to consumers in j. Otherwise, if $\beta > \beta^*_{ij}(c)$ the firm is inactive in j. The fixed cost cutoff declines with the marginal cost of production and delivery:

$$\beta^*_{ij}(c) = \frac{1}{f_{ij}(\sigma - 1)} \left[A_j \frac{\sigma - 1}{\sigma} \right]^{\sigma} (\tau_{ij} w_i c)^{-(\sigma-1)} \tag{10}$$

Less productive firms need to be more flexible in order to be active in j. Moreover, the cutoff declines in the iceberg trade cost: for a given productivity, less flexible firms can sell a positive scope in closer destinations.

Let us rewrite per variety profits as a function of the fixed cost cutoff $\beta^*_{ij}(c)$ and β:

$$\pi_{ij}(\omega, c, \beta) = f_{ij}\beta \left[\frac{\beta^*_{ij}(c)}{\beta} h(\omega)^{1-\sigma} - 1 \right] \tag{11}$$

The firm introduces new varieties until the profits of the last variety $\delta_{ij}(c, \beta)$ drop to zero. The

[18] Assuming that the core variety has zero marginal cost, as Macedoni (2017), or that it has an infinite demand shock, as in Bernard et al. (2011) would require an additional fixed cost to determine firms' entry.

optimal scope of the firm is then implicitly defined by:

$$h(\delta_{ij}(c,\beta)) = \left(\frac{\beta^*_{ij}(c)}{\beta}\right)^{\frac{1}{\sigma-1}} \tag{12}$$

The left-hand side of (12) is increasing in the scope of the firm, by the core competence assumption. The right-hand side is increasing in firm's productivity ($\frac{1}{c}$). However, because of the shock β, a given scope can be attained by a high productivity firm with low flexibility, and by a low productivity firm with high flexibility.

Following the notation of Arkolakis et al. (2014), let $H(\delta_{ij}(c,\beta)) = \left[\int_0^{\delta_{ij}(c,\beta)} h(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ be a measure of the productivity of the firm across its varieties. $H(\delta_{ij}(c,\beta))$ is declining in scope: as the firm introduces new varieties far from the most productive core, its average productivity falls. The total sales of a firm are:

$$R_{ij}(c,\beta) = \sigma f_{ij} \beta^*_{ij}(c) H(\delta_{ij}(c,\beta))^{1-\sigma} \tag{13}$$

Using our definition of scope (12) we can rewrite (13) as:

$$\left(\frac{h(\delta_{ij}(c,\beta))}{H(\delta_{ij}(c,\beta))}\right)^{\sigma-1} = \frac{R_{ij}(c,\beta)}{\sigma f_{ij}\beta} \tag{14}$$

which represents the scope and scale disconnect illustrated in the previous Section. The left-hand side is increasing in the scope. Hence, there is a positive relationship between scope and scale which is, however, disconnected by the shock β. In standard models of multiproduct firms (Bernard et al., 2011; Mayer et al., 2014), β is constant across firms, and, therefore, these models predict a positive relationship between scope and sales. In our model, for a given level of sales the scope is determined by the realization of β. Since β is drawn from a firm-specific distribution, it may be related to firm's characteristics as the evidence suggests.

4.2.2 First Stage

Given β and c a firm chooses prices and scope according to (8) and (12). Its profits are given by:

$$\Pi_{ij}(c,\beta) = f_{ij}\beta \left[\left(\frac{h(\delta_{ij}(c,\beta))}{H(\delta_{ij}(c,\beta))}\right)^{\sigma-1} - \delta_{ij}(c,\beta)\right] \tag{15}$$

which are declining in c, provided that $\frac{d}{d\delta}\left(\frac{h(\delta)}{H(\delta)}\right)^{\sigma-1} > 1$. Firm's expected profits in j equal:

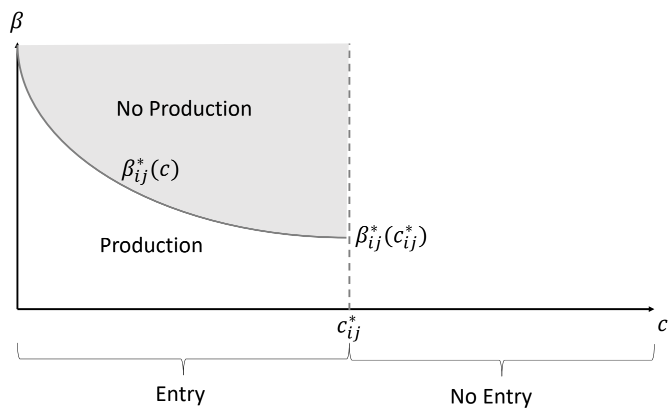
$$E[\Pi_{ij}(c)] = \int_0^{\beta^*_{ij}(c)} \Pi_{ij}(c,\beta) b_c(\beta) d\beta \tag{16}$$

A firm with cost draw c enters a destination j as long as its expected profits over the possible realizations of the fixed cost shock exceed the fixed cost of entry F_{ij}[19]. Hence, there exists a marginal firm with cost draw c^*_{ij} such that:

$$E[\Pi_{ij}(c^*_{ij})] = F_{ij} \tag{17}$$

Figure 6 summarizes the entry and production decisions of firms. A firm with cost draw $c > c^*_{ij}$ does not pay the fixed cost F_{ij} and, thus, decides not to enter the destination. On the other hand, a firm with $c < c^*_{ij}$ pays the fixed cost F_{ij} and discovers its draw of β. If $\beta > \beta^*_{ij}(c)$ the firm does not produce any variety. If β equals the cutoff, the firm is indifferent between producing the core variety and not producing, thus, having a scope of zero. For β below the threshold, the firm has a positive scope and sales. For a given c, firms with low β have a higher scope $\delta_{ij}(c, \beta)$.

Figure 6: Entry, Production and Scope



4.2.3 Firms' Entry

Firms pay the fixed cost f_E if their expected profits across all destinations exceed the fixed cost of entry. In particular, ex-ante expected profits across all destinations are given by:

$$\pi_i = \sum_{j=1}^{I} G_i(c^*_{ij}) \int_0^{c^*_{ij}} E[\Pi_{ij}(c)]\mu_i(c)dc \tag{18}$$

where $E[\Pi_{ij}(c)]$ is defined in (16), and $\mu_c(c)$ is the distribution of cost draws conditional on c being below the threshold c^*_{ij}. In particular $\mu_i(c) = \frac{g_i(c)}{G_i(c^*_{ij})}$ if $c < c^*_{ij}$ and zero otherwise. Free entry implies that firms' expected profits equal the fixed cost of entry:

$$\pi_i = w_i f_E \tag{19}$$

[19] The expected profits are declining in c if $\int_0^{\beta^*_{ij}(c)} \left[\frac{\partial \Pi_{ij}(c,\beta)}{\partial c} b_c(\beta) + \Pi_{ij}(c,\beta)\frac{\partial b_c(\beta)}{\partial c}\right] d\beta < 0$.

4.3 Equilibrium

Following Cherkashin et al. (2015), we compute the mass of firms N_{ij} that sell to a destination j by integrating the probabilities over the white area (labeled "Production") of figure (6):

$$N_{ij} = N_i \int_0^{c_{ij}^*} \int_0^{\beta_{ij}^*(c)} dB_c(\beta) dG_i(c) \tag{20}$$

The total revenues from i to j are:

$$T_{ij} = N_i \int_0^{c_{ij}^*} \int_0^{\beta_{ij}^*(c)} R_{ij}(c, \beta) b_c(\beta) g_i(c) d\beta dc \tag{21}$$

where firm's revenues are defined in (13). Goods market clearing implies that:

$$\sum_{i=1}^{I} T_{ij} = w_j L_j \tag{22}$$

In equilibrium, firms choose scope and prices according to (12) and (8), free entry drives expected profits equal to the fixed cost of entry (19), goods markets clear (22) and trade is balanced $T_{ij} = T_{ji}$.

4.4 Parametrization of the Model

To derive the model's predictions, we consider the following functional form assumptions for the distributions of productivity, flexibility and within-firm marginal costs. Following a long literature started by (Chaney, 2008), the marginal cost c is drawn from a Pareto distribution with CDF $G_i(c) = \left(\frac{c}{\bar{c}_i}\right)^{\kappa}$, where $c \in [0, \bar{c}_i]$, κ is the shape parameter, common across all countries, and $\bar{c}_i$ is an origin specific location parameter.

The distribution of the fixed cost shock β follows a Pareto distribution with CDF $B_c(\beta) = \left(\frac{\beta}{\beta_m c^{\alpha}}\right)^{\gamma}$, where $\beta \in [0, \beta_m c^{\alpha}]$, γ is the shape parameter and $\beta_m c^{\alpha}$ is a firm specific location parameter. The choice of such distribution makes the model tractable and generates a distribution of the scope of firms, conditional on their productivity, which is consistent with the data. If $\alpha > 0$, the fixed cost shock is positively correlated with the cost draw, and, thus, productivity and flexibility are positively correlated. If $\alpha < 0$, productivity and flexibility are negatively correlated while they are uncorrelated if $\alpha = 0$.

The firm specific location parameter is a shorthand that captures alternative ways with which firm's characteristics affect firm's flexibility. The evidence, in fact, suggests that several firm-level variables, from capital stock to R&D expenditures, influence the conditional scope. Assuming an exogenous firm-specific location parameter for the distribution of the fixed cost generalizes alternative endogenous mechanisms that explain firm's heterogeneity in the ability to introduce new varieties. Parisi et al. (2006) document that R&D expenditures increase the probability of product innovation, and the survey of the literature by Klette and Kortum (2004) finds a positive relationship between R&D and productivity. These findings could be matched with a positive correlation ($\alpha > 0$) between productivity and flexibility. Our model can be thought of as a reduced-form expression of He (1992), in which firms can choose between an output-specific or a flexible technology[20]. Moreover, the model of economies of scope of Arkolakis et al. (2014) can be represented by $\alpha > 0$ while the presence of diseconomies of scope of Nocke and Yeaple (2014), due to a fixed physical capacity, is rationalized by $\alpha < 0$.

Finally, our marginal cost per variety ω is given by:

$$h(\omega) = \exp(\theta\omega) \tag{23}$$

which has the desired properties of $h(0) = 1$ and $h'(\omega) > 0$. The implication of such functional form is that firms with wider scope respond less to trade shocks than narrow scope firms[21].

4.5 Model and Stylized Facts

This Section shows how the parametrized model explains the three stylized facts we document. We assume that the fixed cost to enter a destination F_{ij} is a constant and expressed in destination labor units: $F_{ij} = Fw_j$ but we let f_{ij} unspecified. We leave the derivation of the model solution to the appendix.

4.5.1 Exporter Scope across Firms and Destinations

Let $\tilde{\theta} = \theta(\sigma - 1)$ and $\tilde{\gamma} = (\gamma + 1)(\sigma - 1)$. The scope (12) of the firm with cost draw c and fixed cost shock β becomes:

$$\delta_{ij}(c, \beta) = \frac{1}{\tilde{\theta}} \left[\ln \beta^*_{ij}(c) - \ln \beta\right] \tag{24}$$

The scope increases with productivity and declines with the fixed cost β. By solving the fixed cost cutoff $\beta^*_{ij}(c)$ as a function of the model parameters, we can write the scope of exporters, conditional on the firm being active, as:

$$\delta_{ij}(c, \beta) = \bar{\delta} + \frac{\alpha\gamma + \tilde{\gamma}}{\tilde{\theta}(\gamma + 1)} \ln \bar{c}_i - \frac{1}{\tilde{\theta}(\gamma + 1)} \ln \left(\frac{f_{ij}}{w_j}\right) + \frac{\alpha\gamma + \tilde{\gamma}}{\kappa\tilde{\theta}(\gamma + 1)} \ln \left(\frac{\lambda_{ij} L_j}{L_i}\right) - \frac{1}{\theta} \ln c - \frac{1}{\tilde{\theta}} \ln \beta \tag{25}$$

[20] If the cost for the flexible technology is constant, more productive firms are endogenously more flexible too. In contrast, if choosing the flexible technology reduces the efficiency of production, the opposite case would occur.

[21] Similar results can be obtained with $h(\omega) = (1 + \omega)^{\theta}$, used by Arkolakis et al. (2014), which matches the data less precisely.

where $\bar{\delta}$ is a positive constant that depends on the parameters of the model, and λ_{ij} is the trade share of country i's exports over total expenditures in j. The scope of an exporter is decomposed into several determinants. As already mentioned, the scope declines the firm specific marginal cost draw c, and the firm-destination specific fixed cost draw β.

Both the origin size L_i and the shift parameter of the distribution of productivity $\bar{c}_i$ affect the scope of exporters. In line with the evidence of Macedoni (2017), exporter's scope increases with the size of the destination L_j and with its per capita income w_j[22]. Since the fixed cost of entry is expressed in foreign labor units, as w_j increases, only firms with high enough productivity - and, thus, larger scope - are active in the destination. Firms enjoy higher total and per product sales in larger destinations and, thus, can more easily cover the fixed costs per product and of entry in larger destinations. Finally, the scope depends on two bilateral variables: f_{ij}, and the export trade share λ_{ij}. The easier it is to reach a destination j, the larger the scope of the firm.

The model is consistent with the first stylized fact. The scope of exporters from an origin depends on firm specific and destination specific characteristics that would be captured by firm and destination fixed effects. Moreover, the presence of the fixed cost shock β introduces the firm-destination specific component that is suggested in the data.

4.5.2 Scale and Scope Disconnect

Consider the disconnect between scope of a firm in destination j, defined in (24), and productivity $\frac{1}{c}$. Figure 7 illustrates the disconnect between productivity and scope arising in a numerical example for different values of γ and α. The appendix illustrates the algorithm and main parameters of the numerical example. The model can replicate the disconnect observed in the data: for any level of productivity, there are several narrow and wide-scope firms. Moreover, the relationship between scope and productivity tends to be positive.

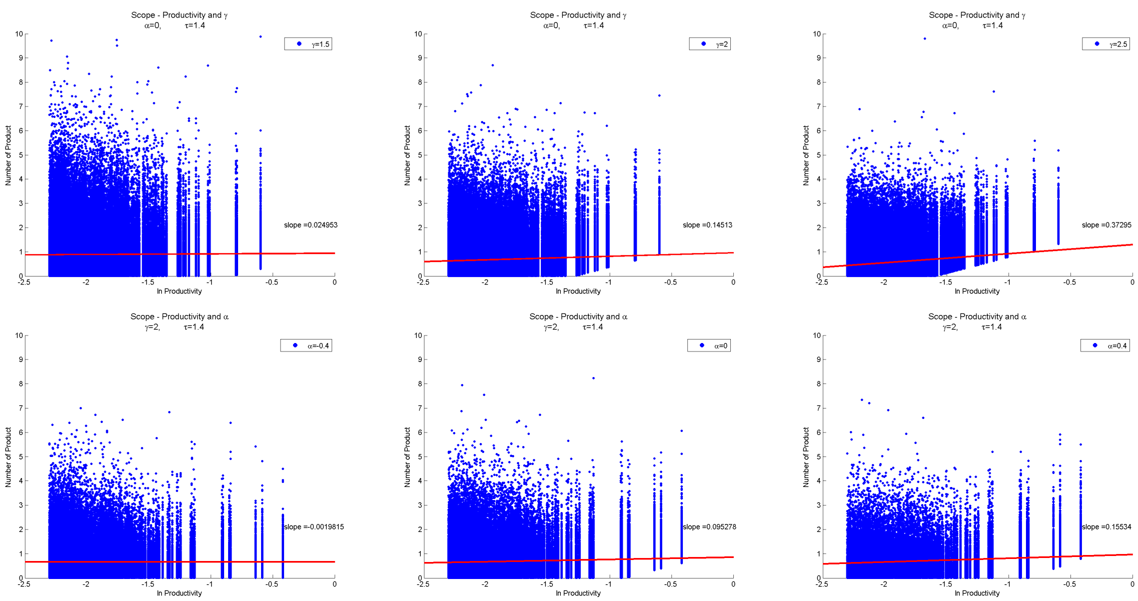
Larger values of γ increase the slope of the line of best fit between scope and productivity. The greater the value of γ, the lower the dispersion in the distribution of the fixed cost shock β, which becomes more concentrated towards higher values of β. As γ increases, favorable fixed cost shocks become rarer, and, thus, there are fewer wide-scope firms. Moreover, as γ increases, fewer low-productivity firms enter or are active in a destination. Therefore, with higher γ there are relatively more high-productivity firms in a destination, which increases the correlation between productivity and scope.

Recall that α controls the correlation between productivity and flexibility draws. For negative values of α, more productive firms are more likely to receive high fixed cost shock draws: as a result, the relationship between productivity and scope may become negative. For positive values of α more productive firms are also more likely to receive low values of β: the correlation between

[22]The mechanism is different than Macedoni (2017), in which non-homothetic preferences drive the positive relationship between per capita income and scope. If f_{ij} is expressed in foreign labor units, such a positive relationship, however, disappears

scope and productivity improves.

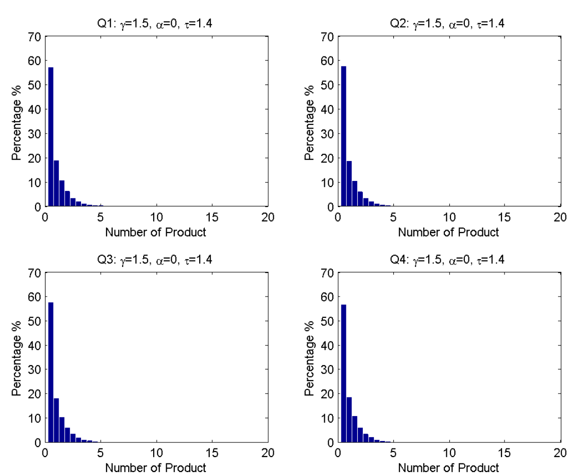
Figure 7: Disconnect between Productivity and Scope



Results from numerical example of a two symmetric country model. Details in appendix.

The distribution of the scope, conditional on the realization of c, follows a distribution, which is approximately similar to the distribution observed in the data. In our numerical example, we divide firms in quartiles by productivity, and generate the distribution of scope conditional on $\frac{1}{c}$ which is similar to the empirical distribution of Chinese firms' conditional scope (Figure 8). The larger the γ, the larger the mass of firms producing a narrow scope. The larger the α, the larger the average productivity in a destination, and the smaller the skewness of the simulated distribution.

Figure 8: Distribution of Scope Conditional on Productivity



Results from numerical example of a two symmetric country model. Details in appendix.

Let us now consider the disconnect between scope and sales. The revenues of a firm with cost draw c and fixed cost draw β are given by:

$$R_{ij}(c, \beta) = \frac{\sigma f_{ij}}{\tilde{\theta}} \left[\beta^*_{ij}(c) - \beta \right] \tag{26}$$

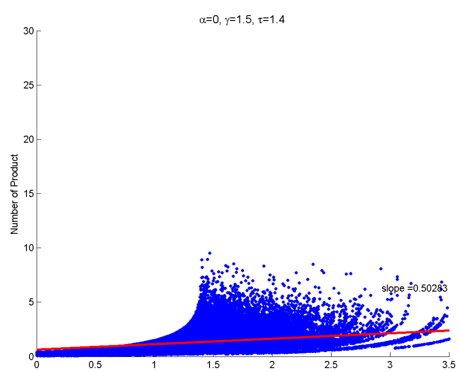
where we restrict the parameter space so that revenues are positive: $\tilde{\theta} < 1$. Both scope (24) and total sales increase with firms productivity and decline with the realization of the fixed cost β. However, since the effects of c and β on sales and scope are different, there are combinations of (c, β) that generate the same sales but different scope and vice versa.

In particular, we can rewrite (14) as:

$$\exp(\tilde{\theta}\delta_{ij}(c, \beta)) - 1 = \frac{\tilde{\theta} R_{ij}(c, \beta)}{\sigma f_{ij} \beta} \tag{27}$$

where the left-hand side is increasing in the scope. For a given level of revenue R, larger values of β imply a smaller scope. Similarly, two firms with the same scope δ, have different revenues depending on the realization of the fixed cost shock β. Figure 9 shows the simulated relationship between scope and sales, which is similar to the relationship uncovered in the data. Narrow scope firms are present at any level of sales, and wide-scope firms tend to be more frequent for medium to high levels of sales.

Figure 9: Sales and Scope Disconnect



Results from numerical example of a two symmetric country model. Details in appendix.

4.5.3 What Causes the Disconnect?

In the empirical analysis, we found that the scope conditional on sales is related to firm's characteristics. In the model, the conditional scope of active firms depends on the realization of the fixed cost shock (27). To gather some intuition, let us consider the expected realization of the fixed cost shock β for active firms, conditional on a firm drawing c. In other words, let us compute the expected value of β conditional on a firm being active (i.e. $\beta \leq \beta_{ij^*(c)}$), and weight it by the probability of a firm with cost c being active:

$$E[\beta|c] = \frac{\gamma}{\gamma+1} \frac{\beta^*_{ij}(c^*_{ij})^{\gamma+1} c^{*\sigma-1}_{ij}}{\beta^\gamma_m} \, c^{-\alpha\gamma-\tilde{\gamma}} \tag{28}$$

The expected value of β depends on firm's productivity $\frac{1}{c}$ with an elasticity of $(\alpha\gamma)+\tilde{\gamma}$. The reason why the expected realization of the fixed cost shock varies with firm's productivity is twofold. First, there is a selection effect ($\tilde{\gamma}$), which is independent of the correlation between β and c. Since only firms with high productivity are able to remain active despite high fixed cost shocks, we observe that, on average, more productive firms experience higher shocks.

The correlation between fixed cost draw and marginal cost draw, represented by the parameter α, drives the second mechanism. With positive values of α, more productive firms are likely more flexible. Hence, they are more likely to be active, thus, increasing the positive relationship between average shock and productivity. On the other hand, with a negative α, more productive firms are more likely to draw high fixed cost shocks. As more productive firms are then more likely to be inactive, the relationship between $E[\beta|c]$ and productivity flattens.

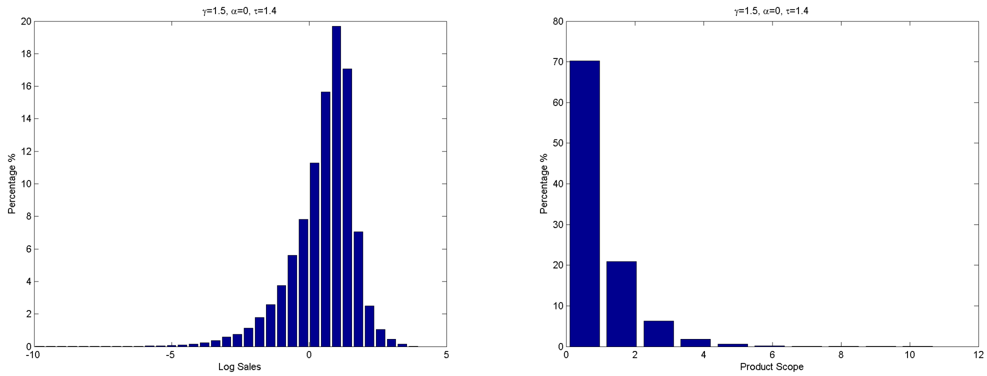
If productivity and flexibility draws are uncorrelated or positively related, the model predicts a negative relationship between conditional scope and productivity, which we observe for Chinese firms in textile, food, footwear and metals. On the other hand, if productivity and flexibility are negative related, the model predicts that conditional scope is independent or negatively related to productivity, as in Chinese machinery, chemicals, vegetable products and transportation.

4.5.4 Scope and Sales Distributions

Using Chinese data, we document a distribution for firms' scope and sales that approximate a Pareto and a log normal distribution. Figure 10 shows that the distributions of scope and sales that arise in our numerical example is consistent with the evidence.

In "single attribute" models, the distribution of sales follows the distribution of productivities (Mrázová et al., 2016). Hence, if productivity is Pareto distributed, so are the sales. In our model, the two underlining Pareto distributions of productivity and flexibility generate a distribution of sales that resembles a log normal distribution. To understand the result, consider the firms at the bottom of the productivity distribution. As the Pareto distribution has a large mass at the bottom, there is a large mass of less productive firms with low sales. The shock to the fixed cost per variety causes some of these firms to not produce, and others to produce more varieties than a "single attribute" model would predict. Both channels reduce the mass of firms at low sales and shift it towards higher levels of revenues.

Figure 10: Distribution of Scope and Sales



Results from numerical example of a two symmetric country model. Details in appendix.

4.6 Margins of Trade

How does the fixed cost shock affect aggregate trade flows? To answer this question, let us derive the gravity equation, obtained by aggregating the revenues of all firms from i to j, and dividing by the total expenditures in j:

$$\lambda_{ij} = \frac{L_i \bar{c}_i^{\kappa} (\tau_{ij} w_i)^{-\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}} f_{ij}^{-\frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}}}}{\sum_{v=1}^{I} L_v \bar{c}_v^{\kappa} (\tau_{vj} w_v)^{-\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}} f_{vj}^{-\frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}}}} \tag{29}$$

The trade elasticity is a constant that depends on the shape parameter κ of the distribution of productivities, the shape parameter γ of the distribution of fixed cost shocks, the parameter α that controls the correlation between the two distributions, and the elasticity of substitution σ:

$$-\frac{d\ln\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right)}{d\ln\tau_{ij}} = \underbrace{\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}} - \tilde{\gamma}}_{\text{Extensive}} + \underbrace{\tilde{\gamma}}_{\text{Intensive}} = \frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}$$

$$-\frac{d\ln\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right)}{d\ln f_{ij}} = \underbrace{\frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}} - \gamma}_{\text{Extensive}} + \underbrace{\gamma}_{\text{Intensive}} = \frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}}$$

First, firm's sales - the intensive margin - are more responsive to trade shocks for larger values of γ. Larger values of γ imply that the draws of β are more concentrated towards the right tail of the distribution. Larger realizations of β reduce the scope of firms, and firms with smaller scope are more reactive to changes in trade costs. As it is common with models where productivity is Pareto distributed (Chaney, 2008), the contribution to aggregate trade flows from the intensive margin is, however, offset by the change in the extensive margin. Thus the trade elasticity only reflects the changes in the extensive margin.

Let us consider the trade elasticity with respect to variable trade costs τ_{ij}. If $\alpha = 0$, the elasticity becomes κ, and is identical to that arising from "single attribute" models of single or multiproduct firms (Chaney, 2008; Bernard et al., 2011). The reason for this result lies in the fact that aggregate trade flows are not affected by a change in the number of active firms, because firms with $\beta = \beta^*_{ij}(c)$ have zero revenues. Moreover, since the intensive margin is offset by an equal change in the extensive margin, the only adjustment mechanism that affects total export is the change in the number of entrants (change in c^*_{ij}).

If $\alpha > 0$, productivity and flexibility are positively related and the trade elasticity is smaller than κ. For higher levels of α, less productive firms expect lower profits, and, therefore, their entry in a destination is dampened. Another way to understand the effects of α is to think that a positive α magnifies the productivity differences across firms, which could be achieved in a standard model with a smaller κ. On the contrary, when α is negative, the trade elasticity is larger than the standard one.

The effect of γ depends on whether α is positive or negative. If $\alpha > 0$, the trade elasticity with respect to variable trade costs declines with γ while it increases with γ for $\alpha < 0$. When flexibility and productivity are positively related, entry is further dampened by higher values of γ that reduce expected profits. In contrast, when flexibility and productivity are negatively related, less productive firms are likely more flexible and they benefit from less disperse fixed cost shock (higher γ).

On the other hand, for $\alpha = 0$ the elasticity with respect to fixed trade cost depends on γ as well. In particular, it is increasing in γ and approaches the standard case (Bernard et al., 2011) when $\gamma \to \infty$. Hence, the elasticity is smaller than "single attribute" model: heterogeneity in flexibility reduces the responsiveness of trade to changes in f_{ij}. This arises because a fixed cost shock only affects the expected profits of firms that are active, conditional on drawing a β below the cost cutoff.

5 Estimation of Firms' Flexibility

In this Section, we estimate the firm-destination specific fixed cost shock β using Chinese data. We follow a two-step procedure. First, we estimate $\tilde{\theta}$, which captures how fast revenues fall as the firm introduces varieties far from the core, either because of an increase in marginal costs θ or because of the high substitutability between varieties σ. Given $\tilde{\theta}$, we use the scale and scope disconnect to obtain the fixed cost shock β. As the scale-scope disconnect can only be explained by a shock to the within-firm extensive margin, it appears reasonable to use such statistic from the data to estimate β.

We estimate $\tilde{\theta}$ using the within-firm distribution of sales. Revenues from i = China to a destination j of a variety ω by firm f with cost c_f and fixed cost shock β_{fij} are given by:

$$r_{ij}(\omega, c_f, \beta_{fij}) = A_j^{\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (\tau_{ij} w_i)^{1-\sigma} c_f^{1-\sigma} \exp(-\tilde{\theta}\omega) \tag{30}$$

Taking logs of (30) yields:

$$\ln r_{ij}(\omega, c_f, \beta_{fij}) = \underbrace{\ln \left[\left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \right]}_{\text{Constant}} + \underbrace{\ln \left[A_j^{\sigma} (\tau_{ij} w_i)^{1-\sigma} \right]}_{\text{Destination Fixed Effects}} + \underbrace{(1 - \sigma) \ln c_f}_{\text{Firm Fixed Effects}} - \tilde{\theta}\omega \tag{31}$$

We can rewrite (31) in the following reduce form to estimate $\tilde{\theta}$:

$$\ln r_{ij}(\omega, c_f, \beta_{fij}) = b_0 + b_1 g_{fj}(\omega) + \eta_j + u_f + \epsilon \tag{32}$$

where $g_{fj}(\omega) = 1, 2, 3, ...,$ denotes the sales rank of variety ω within firm f in destination j, $b_1 = -\tilde{\theta}$, and ϵ stands for the measurement error. Arkolakis et al. (2014) show that there is a certain degree of heterogeneity in the sales-rank elasticity within firms depending on the firm's scope. Following Arkolakis et al. (2014), we estimate (32), restricting the sample to firm-destination pairs with scope $\delta_{fd} = 6$. We also try with $\delta_{fij} = 4, 8, 10, ...,$ finding similar results.

Table 3 reports the point estimate $\tilde{\theta}$ using equation (32), under different specification and using different samples. Panel a uses the whole sample of Chinese exporters and Panel b uses the matched sample. In each panel, we report the results from restricting the sample to firms $\delta_{fij} = 6$ and $\delta_{fij} = 8$, finding similar results.

Rearranging our scale-scope disconnect equation (27), and taking logs, we obtain:

$$\ln \Xi_{ij}(c, \beta) = \ln \sigma + \ln f_{ij} + \ln \beta_{fij} \tag{33}$$

where $\Xi_{ij}(c, \beta) \equiv \tilde{\theta} R_{ij}(c, \beta) / \Big(\exp\left(\tilde{\theta} \delta_{ij}(c, \beta) \right) - 1 \Big)$. Given the $\tilde{\theta}$ estimated in the previous step, we recover the fixed cost shocks $\ln \beta$ from the residual term of (33), after controlling for the destination fixed effects. The above estimation method of $\ln \beta_{fij}$ would suffers from measurement error bias, since we cannot separate the white noise from the true values of $\ln \beta_{fij}$.

Given the estimated $\ln \beta_{fij}$, we include it as additional variable into the scope regression (1) to study the extent whereby the fixed cost explains the scope dispersions observed in data. Table 4 reports the performance of the model. The fixed cost shock improves significantly the performance of the model: the R^2 rises to 64% for the whole sample and 62% for the matched sample. Besides the sizable magnitude of the additional model fit brought by fixed cost shocks, we believe that the improvement of model fit by $\ln \beta$ is still underestimated, due to the existence of measurement error problem.

Table 3: Estimation of Model Parameter: $\tilde{\theta}$

Panel a: All Chinese Exporters				
	$\delta_{fij} = 6$		$\delta_{fij} = 8$	
Var. \ Specification	(1)	(2)	(3)	(4)
Estimated $-\tilde{\theta}$	-0.804***	-0.789***	-0.625***	-0.614***
	(0.012)	(0.012)	(0.009)	(0.009)
Observations	44,460	44,348	25,992	25,885
R-squared	0.815	0.823	0.847	0.855
Destination Country FE	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES
HS 4-Digit FE	NO	YES	NO	YES
Panel b: Matched Chinese Exporters				
	$\delta_{fij} = 6$		$\delta_{fij} = 8$	
Var. \ Specification	(5)	(6)	(7)	(8)
Estimated $-\tilde{\theta}$	-0.879***	-0.856***	-0.679***	-0.656***
	(0.016)	(0.015)	(0.012)	(0.011)
Observations	16,128	16,006	8,936	8,825
R-squared	0.805	0.817	0.843	0.858
Destination Country	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES
HS 4-Digit FE	NO	YES	NO	YES

Robust standard errors are clustered at firm-destination country level and reported in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 4: Contribution of β to the Model Fit (R^2)

Sample	Exclusion of $\ln\beta$	Inclusion of $\ln\beta$
All Exporters	0.37	0.64
Matched Exporters	0.44	0.62

Exclusion of $\ln\beta$: R^2 from regressing the log of the scope on firm (a_f) and destination (b_j) fixed effects. Inclusion of $\ln\beta$; R^2 including estimated $\ln\beta$, in addition to a_f and b_j. Robust standard errors in parentheses, clustered at firm-destination level; *** p<0.01, ** p<0.05, * p<0.1.

6 Conclusions

We have argued that "single attribute" models, in which productivity is the only source of firm's heterogeneity, fail to explain several new stylized facts for multiproduct exporters. We build a model in which firms differ in productivity and flexibility - the ability with which they introduce new varieties at a low costs. Firm's flexibility explains more than 20% of the total variation of scope across firms and destinations.

The empirical evidence shows that firm's flexibility is related to firm's observable characteristics, such as capital intensity or R&D expenditures. However, the effects of these characteristics on the scope vary by industries. For instance, given two firms with the same level of sales in the textile industry, the more capital intensive firm has lower scope while in machinery industry higher R&D yields a wider scope conditional on sales.

Our model predicts that the larger the dispersion in flexibility, the larger the scope dispersion at any level of productivity, and the smaller the change in the intensive margin of trade. Moreover, since more productive firms can be active in a market despite low flexibility, the expected scope conditional on sales tends to decline with firm's productivity. However, if firm's flexibility and productivity are negatively related, the relationship between conditional scope and productivity can become flat or positive. Finally, heterogeneity in firm's flexibility affects the trade elasticity: the more correlated flexibility and productivity are, the larger is the trade elasticity.

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7 Appendix

7.1 Data Appendix

7.1.1 Estimation of Productivity

We use Levinsohn and Petrin (2003) method to estimate firm productivity. This method is applied to our dataset of Chinese firms in ASIP as follows. Let y_{it} denotes the value added of firm i in year t, and the production function is

$$\begin{aligned} y_t =& \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \epsilon_{it} \\ =& \beta_l l_{it} + \phi(k_{it}, m_{it}) + \epsilon_{it} \end{aligned}$$

where $\phi(k_{it}, m_{it}) \equiv \beta_0 + \beta_k k_{it} + \omega(k_{it}, m_{it})$. We substitute $\phi(k_{it}, m_{it})$ in equation of y_{it} with a third-order polynomial approximation in k_{it} and m_{it}, after which we are able to obtain the consistent estimation of the value-added equation using OLS as

$$y_{it} = \delta_0 + \beta l_{it} + \sum_{s=0}^{3} \sum_{j=0}^{3-s} \delta_{sj} k_{it}^s m_{it}^j + \epsilon_{it}$$

This provides the first stage of estimation, from which we obtain $\hat{\beta}_l$ and an estimate of $\hat{\phi}_{it}$.

Next, we estimate $\hat{\beta}_k$, which begins by computing the estimated value of $\hat{\phi}_{it}$:

$$\hat{\phi}_{it} = \hat{y}_{it} - \hat{\beta}_l l_{it} = \hat{\delta}_0 + \sum_{s=0}^{3} \sum_{j=0}^{3-s} \hat{\delta}_{sj} k_{it}^s m_{it}^j - \hat{\beta}_l l_{it}$$

For any candidate value of β_k^*, we compute the prediction for ω_{it} as $\hat{\omega}_{it} - \hat{\phi}_{it} - \beta_k^* k_{it}$. We assume a consistent (nonparametric) approximation to $E(\widehat{\omega_{it}|\omega_{it-1}})$ as

$$\hat{\omega}_{it} = \gamma_0 + \gamma_1 \omega_{t-1} + \gamma_2 \omega_{t-1}^2 + \gamma_3 \omega_{t-1}^3 + e_{it}$$

Given $\hat{\beta}$, β_k^* and $E(\widehat{\omega_{it}|\omega_{it-1}})$, we rewrite the sample residuals as

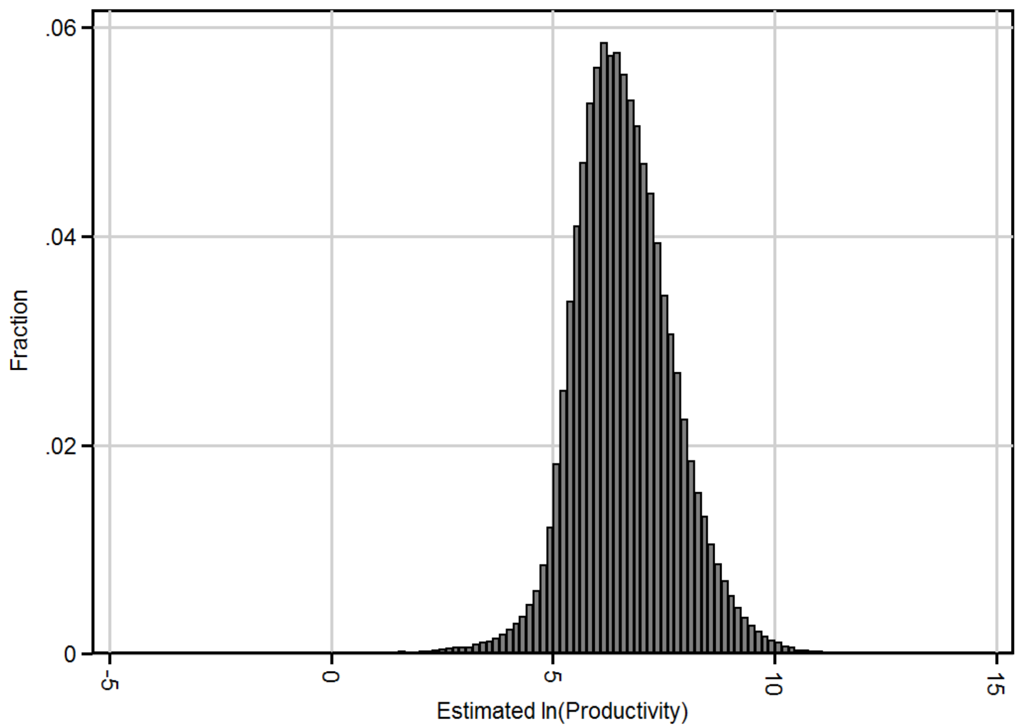
$$\widehat{\epsilon_{it} + e_{it}} = y_{it} - \hat{\beta}_l l_{it} - \beta_k^* k_{it} - E(\widehat{\omega_{it}|\omega_{it-1}})$$

Finally, we estimate $\hat{\beta}_k$ as the solution to:

$$\min_{\beta_k^*} \sum_i \sum_t \left(y_{it} - \hat{\beta}_l l_{it} - \beta_k^* k_{it} - E(\widehat{\omega_{it}|\omega_{it-1}}) \right)^2$$

In practice, we use data of ASIP for 2005 and 2006. We measure value added as outcome variable y_{it}, employment as the freely chosen variable (l_{it}), total fixed asset as capital (k_{it}), and intermediate input and the sales cost[23] as the proxy variables.

Figure 11: Distribution of Estimated Productivity (ASIP 2006)



Distribution of estimated productivity for 2006, using ASIP of year 2005 and 2006.

[23]The sales cost for a firm is the cost of merchandise in its beginning inventory plus the net cost of merchandise purchased minus the cost of merchandise in its ending inventory. We try different specifications using different variables to measure the m_{it}, and the estimated productivity are highly positively correlated.

7.1.2 Data

Table 5: Summary Statistics: Cross-section 2006 (All Sample)

Total Exporters				
Product Scope	Number of Firms		Export Value	
	N	% of total	Value	% of total
1	34,879	23.02	26,624.71	6.40
2	21,728	14.34	23,346.14	5.61
3	14,860	9.81	21,976.69	5.28
4	10,870	7.17	18,912.32	4.54
5	8,496	5.61	15,538.85	3.73
$6-10$	22,380	14.77	53,065.52	12.75
$11-30$	21,391	14.12	79,534.55	19.10
$31-50$	5,290	3.49	31,152.95	7.48
$51-70$	2,982	1.97	17,115.56	4.11
> 70	8,658	5.71	129,058.06	31.00
Total	151,534	100	416,325.3	100
Matched Sample				
Product Scope	Number of Firms		Export Value	
	N	% of total	Value	% of total
1	10,892	21.51	13,197.05	8.74
2	7,946	15.69	12,402.61	8.21
3	5,936	11.72	11,797.43	7.81
4	4,462	8.81	10,838.26	7.18
5	3,528	6.97	8,911.25	5.90
$6-10$	9,186	18.14	29,835.19	19.75
$11-30$	7,406	14.62	37,467.43	24.81
$31-50$	934	1.84	10,839.69	7.18
$51-70$	219	0.43	4,691.00	3.11
> 70	139	0.27	11,051.66	7.32
Total	50,648	100.00	151,031.57	100.00

Products are defined at HS 8-digit level. Export value is in million of U.S. dollars. We refine data to the firms involved in ordinary trade.

Table 6: Match Statistics Between ASIP and China Custom Data

Year	Matched Firm Number	% of Total Number	% of Total Export
2000	16,596	35.32%	20.91%
2001	19,597	37.44%	24.41%
2002	23,112	37.51%	27.17%
2003	27,333	35.49%	26.35%
2004	41,955	41.52%	37.95%
2005	44,212	35.62%	35.61%
2006	50,648	33.42%	36.28%

Notes: Statistics on the matched firms is compared to the total sample of firms involved in ordinary trade in China Custom Dataset. We also filter the matched firms to the ones that are one-to-one mapping between ASIP and Custom Data.

Table 7: Classification of Industries

Industry Description	Range of HS 2-Digit Code
Animal & Animal Products	01-05
Vegetable Products	06-15
Foodstuffs	16-24
Mineral Products	25-27
Chemicals & Allied Industries	28-38
Plastics & Rubbers	39-40
Raw Hides, Skins, Leather & Furs	41-43
Wood & Wood Products	44-49
Textile	50-63
Footwear & Headgear	64-67
Stone & Glass	68-71
Metals	72-83
Machinery & Electrical	84-85
Transportation	86-89
Miscellaneous	90-97

We drop HS 27: Mineral Fuels etc.

7.1.3 Exporter Scope across Firms and Destinations

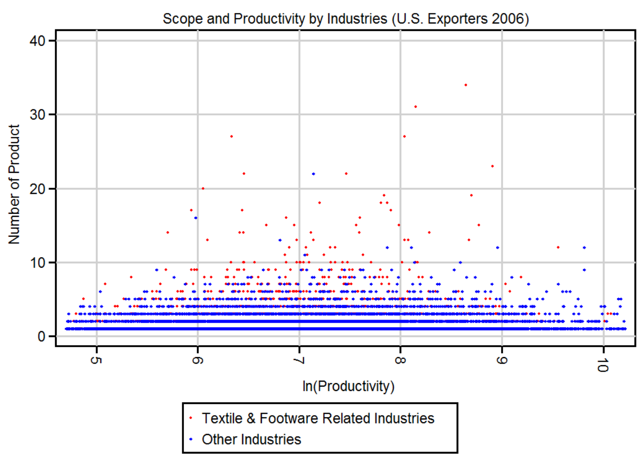
Table 8: Decomposition of Product Scope

Sample Year	Model Fit (R^2)	d_j	a_f
2000	0.30	0.01	0.29
2001	0.31	0.01	0.30
2002	0.32	0.01	0.31
2003	0.34	0.01	0.32
2004	0.35	0.01	0.34
2005	0.36	0.01	0.35
2006	0.37	0.01	0.36

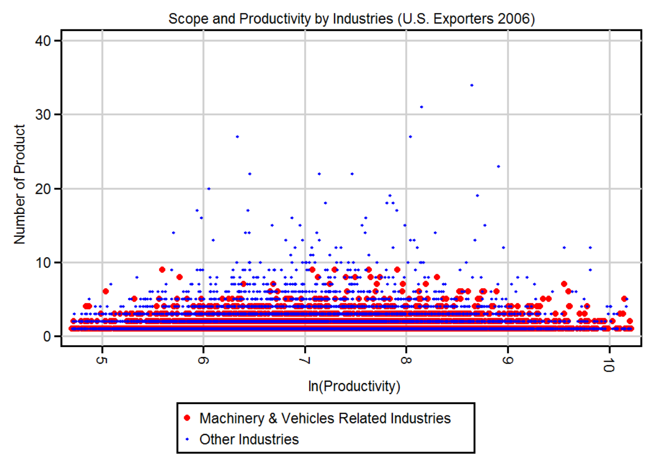
All Chinese Exporters. R^2 from regression (1).

7.1.4 Scale and Scope Disconnect

Figure 12: Scope and Productivity by Industries (Matched U.S. Exporters)



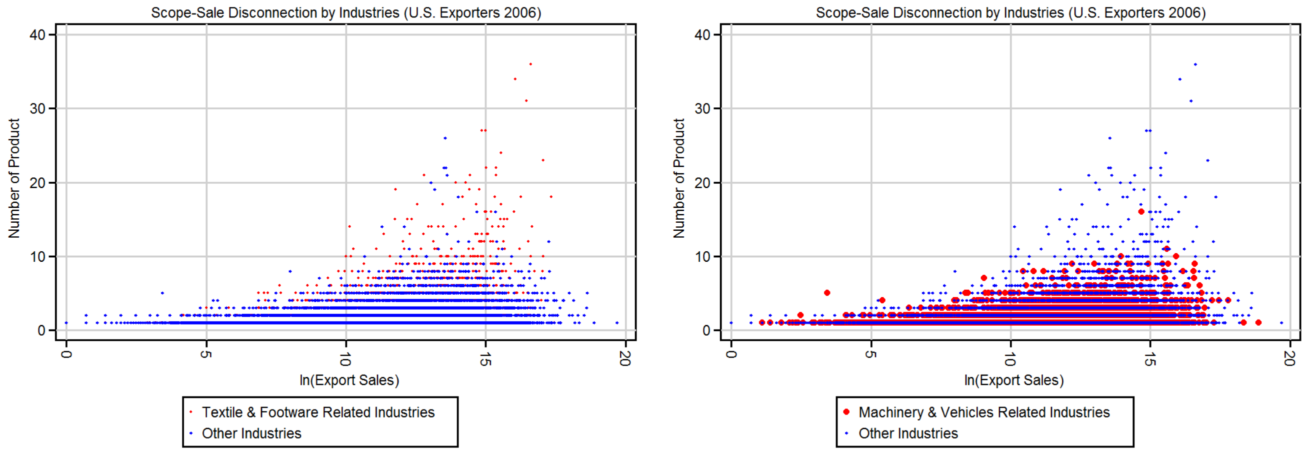
(a) Textile Related Industries



(b) Machinery Related industries

Matched Chinese exporters to the U.S. in 2006. We refine firms involved in ordinary exporting and drop firms active in multiple industries. We exclude the firms with outlier productivity ($\leq 1\%$ or $\geq 99\%$).

Figure 13: Scale and Scope Disconnect by Industries (U.S. Exporters)



(a) Textile Related Industries (b) Machinery Related industries

All Chinese exporters to the U.S. in 2006. We refine firms involved in ordinary exporting and drop firms active in multiple industries.

7.1.5 What Causes the Disconnect?

Table 9: Correlation Between Residual and Firm Characteristics (Matched U.S. Sample)

	VA per Worker	Capital	Capital Intensity	Asset	R&D.	Ads.
Animal & Animal Products	-0.027	0.021	-0.072	-0.001	0.009	-0.043
Vegetable Products	0.038	0.037	0.209	0.047	0.075	-0.042
Foodstuffs	-0.051**	-0.009	-0.291**	0.005	0.015	-0.015
Mineral Products	-0.015	0.002	0.013	0.007	-0.011	-0.011*
Chemicals & Allied Industries	0.040***	0.028***	-0.042	0.043***	0.036***	-0.001
Plastics & Rubbers	0.009	0.030***	-0.019	0.039***	0.004	0.009
Raw Hides, Skins, Leather & Furs	-0.036	0.016	-0.545	0.012	-0.047	0.023
Wood & Wood Products	-0.027	0.032***	0.003	0.042***	0.075***	0.001
Textile	-0.027*	0.009	-0.265***	0.019*	-0.006	0.004
Footwear & Headgear	-0.063***	0.053***	-0.901**	0.088***	0.029	0.014
Stone & Glass	-0.022	-0.027***	-0.187***	-0.031***	-0.029	0.031
Metals	-0.052***	-0.025***	-0.030	-0.034***	-0.003	-0.009
Machinery & Electrical	0.001	0.010**	0.033	0.020***	0.016**	0.015**
Transportation	0.010	-0.021*	-0.090	-0.004	0.002	0.008
Miscellaneous	-0.026*	0.034***	-0.015	0.050***	0.024*	0.023**

*** p<0.01, ** p<0.05, * p<0.1.

7.2 Theory Appendix

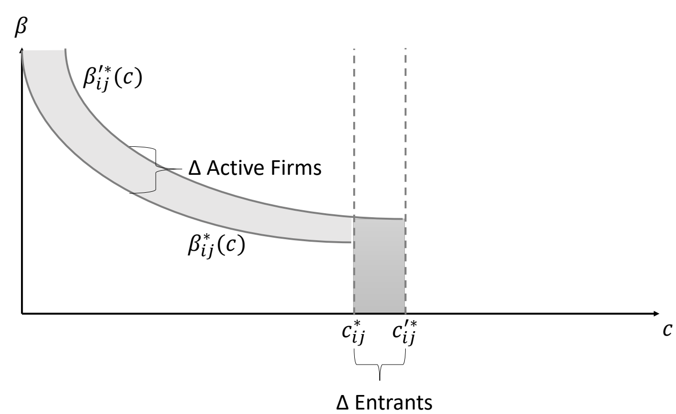
7.2.1 Effects of a Reduction in Trade Costs

To better understand the model, let us discuss the effects of a trade shock on trade flows. In particular, let us consider the effects of a change in either variable trade costs τ_{ij} or fixed cost per variety f_{ij} on total exports from i to j . Taking the total derivative of (21), keeping the mass of entrants constant, yields the following decomposition of the change in total exports:

$$
\begin{aligned}
dT_{ij} &= N_i \left[\int_0^{\beta^*_{ij}(c^*_{ij})} R_{ij}(c^*_{ij}, \beta) b_c(\beta) g_i(c^*_{ij}) d\beta \right] dc^*_{ij} + && \text{Change in Entrants} \\
&+ N_i \left[\int_0^{c^*_{ij}} R_{ij}(c, \beta^*_{ij}(c)) b_c(\beta^*_{ij}(c)) g_i(c) dc \right] d\beta^*_{ij}(c) + && \text{Change in Active Firms} \\
&+ N_i \int_0^{c^*_{ij}} \int_0^{\beta^*_{ij}(c)} dR_{ij}(c, \beta) b_c(\beta) g_i(c) d\beta dc && \text{Intensive Margin}
\end{aligned}
$$

Let us discuss each component with the help of Figure 14. First, a change in trade costs modifies the cost cutoff that pins down the mass of entrants in each destination j. Thus, a change in c^*_{ij} affects total exports by the level of sales of firms at the cost cutoff. Secondly, a change in trade costs affects the set of active firms, by shifting the $\beta^*_{ij}(c)$ curve. However, the contribution of this margin for a small variation in trade costs is zero. In fact, firms with β equal to the cutoff have zero revenues. As a result, a small change in the cutoff $\beta^*_{ij}(c)$ does not affect aggregate trade flows.

Figure 14: Trade Margins



The last term of the decomposition is the intensive margin: the change in the sales of each firm. This margin can be further decomposed into the change in the firm's sales from existing products - the within-firm intensive margin - and the change in the firm's number of products - the within-firm extensive margin. Let $\epsilon_H(\delta) = -\frac{d\ln H(\delta)}{\delta} > 0$ and $\epsilon_h(\delta) = \frac{d\ln h(\delta)}{\delta} > 0$ be the elasticity of the firm's average productivity, and marginal costs with respect to the scope. The within-firm trade margins can be written as:

$$-\frac{d\ln R_{ij}(c,\beta)}{d\ln f_{ij}} = \underbrace{0}_{\text{Intensive}} + \underbrace{\frac{\epsilon_H(\delta(c,\beta))}{\epsilon_h(\delta(c,\beta))}}_{\text{Extensive}}$$

$$-\frac{d\ln R_{ij}(c,\beta)}{d\ln \tau_{ij}} = \underbrace{\sigma-1}_{\text{Intensive}} + \underbrace{(\sigma-1)\frac{\epsilon_H(\delta(c,\beta))}{\epsilon_h(\delta(c,\beta))}}_{\text{Extensive}}$$

A reduction in the fixed cost per variety and destination f_{ij} only affects the extensive margin of trade while a variation in τ_{ij} both affect the intensive and the extensive margin. $\frac{\epsilon_H(\delta(c,\beta))}{\epsilon_h(\delta(c,\beta))}$ is positive but whether it increases or decreases with firm's scope depends on the shape of $h(\omega)$.

7.2.2 Model's Derivation

This section shows how to derive the predictions of the model applying the following exponential function for the marginal cost of variety ω:

$$h(\omega) = \exp(\theta\omega) \qquad \omega \in [0, \delta_{ij}(c,\beta)] \tag{34}$$

Let $\tilde{\theta} = \theta(\sigma-1)$. The optimal scope chosen by a firm is given by:

$$h(\delta_{ij}(c,\beta))^{\sigma-1} = \left(\frac{\beta^*_{ij}(c)}{\beta}\right)$$

$$\exp(\tilde{\theta}\delta_{ij}(c,\beta)) = \left(\frac{\beta^*_{ij}(c)}{\beta}\right) \tag{35}$$

$$\delta_{ij}(c,\beta) = \frac{1}{\tilde{\theta}}\left[\ln\beta^*_{ij}(c) - \ln\beta\right] \tag{36}$$

The revenues of a variety ω of firm with cost draw c and fixed cost draw β is given by:

$$r(\omega,c,\beta) = \sigma f_{ij}\beta^*_{ij}(c)\exp(-\tilde{\theta}\omega) \tag{37}$$

The firm's cost aggregator $H(\delta_{ij}(c,\beta))$ is given by:

$$H(\delta_{ij}(c,\beta)) = \left[\int_0^{\delta_{ij}(c,\beta)} h(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}} = \left[\frac{1}{\tilde{\theta}} - \frac{\exp(-\tilde{\theta}\delta_{ij}(c,\beta))}{\tilde{\theta}}\right]^{\frac{1}{1-\sigma}} \tag{38}$$

Thus, our scale and scope disconnect equation becomes:

$$\left(\frac{h(\delta_{ij}(c,\beta))}{H(\delta_{ij}(c,\beta))}\right)^{\sigma-1} = \frac{R_{ij}(c,\beta)}{\sigma f_{ij}\beta} \tag{39}$$

$$\exp(\tilde{\theta}\delta_{ij}(c,\beta))\left[\frac{1}{\tilde{\theta}} - \frac{\exp(-\tilde{\theta}\delta_{ij}(c,\beta))}{\tilde{\theta}}\right] = \frac{R_{ij}(c,\beta)}{\sigma f_{ij}\beta}$$

$$\exp(\tilde{\theta}\delta_{ij}(c,\beta)) - 1 = \frac{\tilde{\theta}R_{ij}(c,\beta)}{\sigma f_{ij}\beta}$$

$$\frac{R_{ij}(c,\beta)}{\exp(\tilde{\theta}\delta_{ij}(c,\beta)) - 1} = \frac{\sigma f_{ij}\beta}{\tilde{\theta}} \tag{40}$$

Using (36), our firm-level cost aggregate can be expressed as:

$$H(\delta_{ij}(c,\beta))\left[\frac{1}{\tilde{\theta}} - \frac{\beta}{\beta_{ij}^*(c)\tilde{\theta}}\right]^{\frac{1}{1-\sigma}} \tag{41}$$

Using (41), firm's aggregate revenues in destination j are given by:

$$R_{ij}(c,\beta) = \sigma f_{ij}\beta_{ij}^*(c)H(\delta_{ij}(c,\beta))^{1-\sigma} = \frac{\sigma f_{ij}}{\tilde{\theta}}\left[\beta_{ij}^*(c) - \beta\right] \tag{42}$$

Conditional on c, the expected revenues over the possible realizations of β are given by:

$$E[R_{ij}(c)] = \left(\frac{\beta_{ij}^*(c)}{\beta_m c^\alpha}\right)^\gamma \int_0^{\beta_{ij}^*(c)} R_{ij}(c,\beta)\frac{\gamma\beta^{\gamma-1}}{(\beta_{ij}^*(c))^\gamma} = \frac{\sigma f_{ij}}{\tilde{\theta}(\gamma+1)}\frac{(\beta_{ij}^*(c))^{\gamma+1}}{(\beta_m c^\alpha)^\gamma} \tag{43}$$

Let us now turn to profits. Using (39), (42) and (36), profits of a firm with cost draw c and fixed cost shock β are:

$$\begin{aligned}
\Pi_{ij}(c,\beta) &= f_{ij}\beta\left[\left(\frac{h(\delta_{ij}(c,\beta))}{H(\delta_{ij}(c,\beta))}\right)^{\sigma-1} - \delta_{ij}(c,\beta)\right] = f_{ij}\beta\left[\frac{R_{ij}(c,\beta)}{\sigma f_{ij}\beta} - \delta_{ij}(c,\beta)\right] = \\
&= f_{ij}\beta\left[\frac{1}{\beta\tilde{\theta}}\left[\beta_{ij}^*(c) - \beta\right] - \delta_{ij}(c,\beta)\right] = f_{ij}\beta\left[\frac{1}{\beta\tilde{\theta}}\left[\beta_{ij}^*(c) - \beta\right] - \frac{1}{\tilde{\theta}}\left[\ln\beta_{ij}^*(c) - \ln\beta\right]\right] = \\
&= \frac{f_{ij}}{\tilde{\theta}}\left[\beta_{ij}^*(c) - \beta - \beta\ln\beta_{ij}^*(c) + \beta\ln\beta\right]
\end{aligned}$$

Expected profits over the possible realizations of β are given by:

$$E[\Pi_{ij}(c)] = \frac{f_{ij}}{\tilde{\theta}(\gamma+1)^2}\frac{(\beta_{ij}^*(c))^{\gamma+1}}{(\beta_m c^\alpha)^\gamma} \tag{44}$$

Let $\tilde{\gamma} = (\sigma-1)(\gamma+1)$. Setting the expected profits (44) equal to the fixed cost of entry to a destination F_{ij}, and using the definition of $\beta^*_{ij}(c)$ yield the cost cutoff c^*_{ij}:

$$\frac{f_{ij}}{\tilde{\theta}(\gamma+1)^2}\frac{(\beta^*_{ij}(c^*_{ij}))^{\gamma+1}}{(\beta_m(c^*_{ij})^\alpha)^\gamma} = F_{ij} \tag{45}$$

$$\left[\frac{1}{f_{ij}(\sigma-1)}\left[A_j\frac{\sigma-1}{\sigma}\right]^\sigma(\tau_{ij}w_ic^*_{ij})^{-(\sigma-1)}\right]^{\gamma+1}\frac{f_{ij}}{\tilde{\theta}(\gamma+1)^2}\frac{1}{(\beta_m(c^*_{ij})^\alpha)^\gamma} = F_{ij}$$

$$\left[\frac{1}{\sigma-1}\left[A_j\frac{\sigma-1}{\sigma}\right]^\sigma\right]^{\gamma+1}\frac{(\tau_{ij}w_ic^*_{ij})^{-\tilde{\gamma}}f_{ij}^{-\gamma}F_{ij}^{-1}}{\tilde{\theta}(\gamma+1)^2}\frac{1}{(\beta_m(c^*_{ij})^\alpha)^\gamma} = 1$$

$$\left[\frac{1}{\sigma-1}\left[A_j\frac{\sigma-1}{\sigma}\right]^\sigma\right]^{\gamma+1}\frac{(\tau_{ij}w_i)^{-\tilde{\gamma}}f_{ij}^{-\gamma}F_{ij}^{-1}}{\beta_m^\gamma\tilde{\theta}(\gamma+1)^2} = (c^*_{ij})^{\alpha\gamma+\tilde{\gamma}}$$

Thus the cost cutoff c^*_{ij} equals:

$$c^*_{ij} = (\tau_{ij}w_i)^{-\frac{\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}}f_{ij}^{-\frac{\gamma}{\alpha\gamma+\tilde{\gamma}}}F_{ij}^{-\frac{1}{\alpha\gamma+\tilde{\gamma}}}\left[\left[\frac{1}{\sigma-1}\left[A_j\frac{\sigma-1}{\sigma}\right]^\sigma\right]^{\gamma+1}\frac{1}{\tilde{\theta}(\gamma+1)^2\beta_m^\gamma}\right]^{\frac{1}{\alpha\gamma+\tilde{\gamma}}} \tag{46}$$

The higher the variable or fixed costs of exporting, the lower the cutoff: a firm must have a low draw of c to decide to pay F_{ij} to reach a destination with high trade costs. Conveniently, c^*_{ij} can be written as a function of the domestic cost cutoff c^*_{jj}:

$$c^*_{ij} = c^*_{jj}\left(\frac{\tau_{ij}w_i}{\tau_{jj}w_j}\right)^{-\frac{\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}}\left(\frac{f_{ij}}{f_{jj}}\right)^{-\frac{\gamma}{\alpha\gamma+\tilde{\gamma}}}\left(\frac{F_{ij}}{F_{jj}}\right)^{-\frac{1}{\alpha\gamma+\tilde{\gamma}}} \tag{47}$$

Moreover, taking the ratio between $\beta^*_{ij}(c)$ and $\beta^*_{ij}(c^*_{ij})$ yields:

$$\beta^*_{ij}(c) = \beta^*_{ij}(c^*_{ij})\left(\frac{c}{c^*_{ij}}\right)^{-(\sigma-1)} \tag{48}$$

By (45),

$$\beta^*_{ij}(c^*_{ij}) = \left[\frac{\beta_m^\gamma F_{ij}\tilde{\theta}(\gamma+1)^2}{f_{ij}}\right]^{\frac{1}{\gamma+1}}(c^*_{ij})^{\frac{\alpha\gamma}{\gamma+1}} \tag{49}$$

Using (48) and (49) in (44) and (43), the expected profits and revenues can be writte as a function of c and c^*_{ij}:

$$E[\Pi_{ij}(c)] = \frac{f_{ij}}{\tilde{\theta}(\gamma+1)^2}\frac{(\beta^*_{ij}(c^*_{ij}))^{\gamma+1}}{(\beta_mc^\alpha)^\gamma}\left(\frac{c}{c^*_{ij}}\right)^{-\tilde{\gamma}} - F_{ij} = F_{ij}\left[\left(\frac{c}{c^*_{ij}}\right)^{-\tilde{\gamma}-\alpha\gamma}-1\right] \tag{50}$$

$$E[R_{ij}(c)] = \sigma(\gamma+1)F_{ij}\left(\frac{c}{c^*_{ij}}\right)^{-\tilde{\gamma}-\alpha\gamma} \tag{51}$$

Conditional on entry in a destination j, average profits and revenues are given by:

$$\bar{\pi}_{ij} = \int_0^{c_{ij}^*} E[\Pi_{ij}(c)]\kappa \frac{c^{\kappa-1}}{(c_{ij}^*)^\kappa} dc = \frac{F_{ij}(\alpha\gamma + \tilde{\gamma})}{\kappa - \alpha\gamma - \tilde{\gamma}} \tag{52}$$

$$\bar{R}_{ij} = \frac{F_{ij}\kappa\sigma(\gamma+1)}{\kappa - \alpha\gamma - \tilde{\gamma}} \tag{53}$$

Expected profits are then given by:

$$\pi_i^e = \sum_j \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa \bar{\pi}_{ij} = \frac{(\alpha\gamma + \tilde{\gamma})}{\kappa - \alpha\gamma - \tilde{\gamma}} \sum_j F_{ij} \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa$$

Setting the expected profits equal to the fixed cost of entry $w_i f_E$ yields:

$$\frac{(\alpha\gamma + \tilde{\gamma})}{\kappa - \alpha\gamma - \tilde{\gamma}} \sum_j F_{ij} \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa = w_i f_E \tag{54}$$

Total revenues from i to j are given by:

$$T_{ij} = N_i \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa \bar{R}_{ij} = N_i \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa \frac{F_{ij}\kappa\sigma(\gamma+1)}{\kappa - \alpha\gamma - \tilde{\gamma}} \tag{55}$$

Thus, market clearing and trade balance imply that:

$$\sum_j T_{ij} = w_i L_i$$

$$N_i \frac{\kappa\sigma(\gamma+1)}{\kappa - \alpha\gamma + \tilde{\gamma}} \sum_j F_{ij} \left(\frac{c_{ij}^*}{\bar{c}_i}\right)^\kappa = w_i L_i \tag{56}$$

Dividing (56) by (54) yields the mass of entrants N_i:

$$N_i = \frac{L_i(\alpha\gamma + \tilde{\gamma})}{f_E \kappa\sigma(\gamma+1)} \tag{57}$$

Revenues from i to j are then given by:

$$T_{ij} = \frac{\alpha\gamma + \tilde{\gamma}}{f_E(\kappa - \alpha\gamma - \tilde{\gamma})} L_i F_{ij} \bar{c}_i^{-\kappa} c_{ij}^{*\kappa} \tag{58}$$

Using (47), the trade share becomes:

$$\lambda_{ij} = \frac{T_{ij}}{\sum_v T_{vj}} = \frac{L_i \bar{c}_i^{-\kappa} (\tau_{ij} w_i)^{-\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}} f_{ij}^{-\frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}}} F_{ij}^{1-\frac{\kappa}{\alpha\gamma+\tilde{\gamma}}}}{\sum_v L_v \bar{c}_v^{-\kappa} (\tau_{vj} w_v)^{-\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}} f_{vj}^{-\frac{\kappa\gamma}{\alpha\gamma+\tilde{\gamma}}} F_{vj}^{1-\frac{\kappa}{\alpha\gamma+\tilde{\gamma}}}} \tag{59}$$

By market clearing, $T_{ij} = \lambda_{ij} w_j L_j$. Thus, using (58), the cost cutoff c^*_{ij} is given by:

$$c^*_{ij} = \bar{c}_i \left(\frac{\lambda_{ij} L_j w_j}{L_i F_{ij}} \right)^{\frac{1}{\kappa}} \left[\frac{f_E(\kappa - \alpha\gamma - \tilde{\gamma})}{\alpha\gamma - \tilde{\gamma}} \right]^{\frac{1}{\kappa}} \tag{60}$$

Hence, using (48), (49) and (60), the fixed cost cutoff for a firm with cost draw c is given by:

$$\begin{aligned}
\beta^*_{ij}(c) = \beta^*_{ij}(c^*_{ij}) \left(\frac{c}{c^*_{ij}} \right)^{-(\sigma-1)} &= \left[\frac{\beta_m^\gamma F_{ij} \tilde{\theta} (\gamma+1)^2}{f_{ij}} \right]^{\frac{1}{\gamma+1}} (c^*_{ij})^{\frac{\alpha\gamma}{\gamma+1}} \left(\frac{c}{c^*_{ij}} \right)^{-(\sigma-1)} = \\
&= \left[\frac{\beta_m^\gamma F_{ij} \tilde{\theta} (\gamma+1)^2}{f_{ij}} \right]^{\frac{1}{\gamma+1}} (c^*_{ij})^{\frac{\alpha\gamma+\tilde{\gamma}}{\gamma+1}} c^{-(\sigma-1)} = \\
&= \left[\frac{\beta_m^\gamma F_{ij} \tilde{\theta} (\gamma+1)^2}{f_{ij}} \right]^{\frac{1}{\gamma+1}} \left[\bar{c}_i \left(\frac{\lambda_{ij} L_j w_j}{L_i F_{ij}} \right)^{\frac{1}{\kappa}} \left[\frac{f_E(\kappa - \alpha\gamma - \tilde{\gamma})}{\alpha\gamma - \tilde{\gamma}} \right]^{\frac{1}{\kappa}} \right]^{\frac{\alpha\gamma+\tilde{\gamma}}{\gamma+1}} c^{-(\sigma-1)} = \\
&= \underbrace{\left[\beta_m^\gamma \tilde{\theta} (\gamma+1)^2 \right]^{\frac{1}{\gamma+1}} \left[\frac{f_E(\kappa - \alpha\gamma - \tilde{\gamma})}{\alpha\gamma - \tilde{\gamma}} \right]^{\frac{\alpha\gamma+\tilde{\gamma}}{\kappa(\gamma+1)}}}_{=\bar{B}} \left(\frac{F_{ij}}{f_{ij}} \right)^{\frac{1}{\gamma+1}} \bar{c}_i^{\frac{\alpha\gamma+\tilde{\gamma}}{\gamma+1}} \left(\frac{\lambda_{ij} L_j w_j}{L_i F_{ij}} \right)^{\frac{\alpha\gamma+\tilde{\gamma}}{\kappa(\gamma+1)}} c^{-(\sigma-1)} \\
&= \bar{B} \left(\frac{F_{ij}}{f_{ij}} \right)^{\frac{1}{\gamma+1}} \bar{c}_i^{\frac{\alpha\gamma+\tilde{\gamma}}{\gamma+1}} \left(\frac{\lambda_{ij} L_j w_j}{L_i F_{ij}} \right)^{\frac{\alpha\gamma+\tilde{\gamma}}{\kappa(\gamma+1)}} c^{-(\sigma-1)}
\end{aligned} \tag{61}$$

Using (61) into (36) yields:

$$\begin{aligned}
\delta_{ij}(c, \beta) &= \frac{1}{\tilde{\theta}} \left[\ln \beta^*_{ij}(c) - \ln \beta \right] = \\
&= \frac{\ln \bar{B}}{\tilde{\theta}} + \frac{\alpha\gamma + \tilde{\gamma}}{\tilde{\theta}(\gamma+1)} \ln \bar{c}_i + \frac{1}{\tilde{\theta}(\gamma+1)} \left[\ln \left(\frac{F_{ij}}{f_{ij}} \right) + \frac{\alpha\gamma + \tilde{\gamma}}{\kappa} \ln \left(\frac{\lambda_{ij} L_j w_j}{L_i F_{ij}} \right) \right] - \frac{1}{\theta} \ln c - \frac{1}{\tilde{\theta}} \ln \beta
\end{aligned} \tag{62}$$

Assuming that $F_{ij} = w_j F$, we obtain the expression outlined in the paper:

$$\delta_{ij}(c, \beta) = \bar{\delta} + \frac{\alpha\gamma + \tilde{\gamma}}{\tilde{\theta}(\gamma+1)} \ln \bar{c}_i - \frac{1}{\tilde{\theta}(\gamma+1)} \ln \left(\frac{f_{ij}}{w_j} \right) + \frac{\alpha\gamma + \tilde{\gamma}}{\kappa \tilde{\theta}(\gamma+1)} \ln \left(\frac{\lambda_{ij} L_j}{L_i} \right) - \frac{1}{\theta} \ln c - \frac{1}{\tilde{\theta}} \ln \beta$$

where $\bar{\delta} = \frac{\ln \bar{B}}{\tilde{\theta}} + \frac{\kappa - \alpha\gamma - \tilde{\gamma}}{\kappa \tilde{\theta}(\gamma+1)} \ln F$.

7.2.3 Intensive Margin

Let us examine the within-firm intensive and extensive margins of trade:

$$
\begin{aligned}
-\frac{d \ln R_{ij}(c,\beta)}{d \ln f_{ij}} &= \underbrace{0}_{\text{Intensive}} + \underbrace{\frac{1}{\exp(\tilde{\theta}\delta_{ij}(c,\beta)) - 1}}_{\text{Extensive}} \\
-\frac{d \ln R_{ij}(c,\beta)}{d \ln \tau_{ij}} &= \underbrace{\sigma - 1}_{\text{Intensive}} + \underbrace{\frac{\sigma - 1}{\exp(\tilde{\theta}\delta_{ij}(c,\beta)) - 1}}_{\text{Extensive}}
\end{aligned}
$$

The intensive margin is constant, as in the models of single product firms (Chaney, 2008). However, the extensive margin depends on the current scope of the firm: the larger the scope, the smaller the elasticity. The result arises because the scope of firms with wide-scope responds less to trade shock. Moreover, the contribution to total sales of additional varieties is smaller the farther away they are from the core. Hence, the wider the current scope the smaller their contribution to the firm's sales.

7.3 Numerical Example

To keep our numerical example simple, let us consider a world made of two symmetric economies. Without loss of generality we can normalize the wage rate to one in both economies. We assume that the deterministic component of the fixed cost per product and destination $f_{ij} = f$. The iceberg trade cost of exporting is denoted by τ, the location parameter of the distribution of productivities $\bar{c}_i = \bar{c}$, and country size $L_i = L$. Let c_D^* and c_X^* denote the cost cutoff for domestic production and for exports. Moreover, let $\beta_D^*(c)$ and $\beta_X^*(c)$ be the fixed cost cutoff for the domestic and export market of a firm with cost draw c. Let us now write the relationships between these cutoffs in order to simplify what follows:

$$c_X^* = c_D^* \tau^{-\frac{\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}} \tag{63}$$

Moreover, for each destination $i = D, X$ the fixed cost cutoff of a firm with cost draw c can be expressed as a function of the fixed cost and marginal cutoff of the marginal firm c_i^*:

$$\beta_i^*(c) = \beta_i^*(c_i^*) \left(\frac{c}{c_i^*} \right)^{1-\sigma} \tag{64}$$

and the domestic and export fixed cost cutoff are related by:

$$\beta_X^*(c) = \beta_D^*(c) \tau^{-(\sigma-1)} \tag{65}$$

From the zero expected profit condition ($\pi = f_E$), using (63) we obtain an expression for the domestic cost cutoff that only depends on the fundamental parameters of the model:

$$c_D^* = \left[\frac{\bar{c}^\kappa f_e (\kappa - \alpha\gamma - \tilde{\gamma})}{F(\alpha\gamma + \tilde{\gamma})} \right]^{\frac{1}{\kappa}} \left[\frac{\tau^{\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}}}{1 + \tau^{\frac{\kappa\tilde{\gamma}}{\alpha\gamma+\tilde{\gamma}}}} \right]^{\frac{1}{\kappa}} \tag{66}$$

From the zero expected profits in the domestic economy ($E[\Pi_D(c)] = F$), we can find an expression for the domestic fixed cost cutoff of the cutoff firm that only depends on the fundamental parameters of the model:

$$\beta_D^*(c_D^*) = \left[\frac{F \beta_m^\gamma \tilde{\theta} (\gamma + 1)^2}{f} \right]^{\frac{1}{\gamma+1}} (c_D^*)^{\frac{\alpha\gamma}{\gamma+1}} \tag{67}$$

Using (64), the revenues and scope of a firm with fixed cost draw β, and marginal cost c in a destination $i = D, X$ are given by:

$$R_i(c, \beta) = \frac{\sigma f_{ij}}{\tilde{\theta}} \left[\beta_i^*(c) - \beta \right]$$
$$\delta_i(c, \beta) = \frac{1}{\tilde{\theta}} \left[\ln \beta_i^*(c) - \ln \beta \right]$$

7.3.1 Simulation

In this Section we outline the simulation procedure. First, we choose the parameters for the economy. Second, we simulate the scale and scope of a large number of firms. Our purpose is to study how our model can replicate the disconnect between scale and scope, and between productivity and scope. Moreover, we want to study how the disconnect depends on the shape parameters of the distribution of flexibility (γ) and on the correlation between flexibility and productivity (α).

Table 10: Parameters

Parameter	Description	Value
τ	Iceberg trade cost	1.4
f_E	Fixed cost of entry	1
F	Fixed cost per destination	0.01
f	Fixed cost per product-destination	1
$\bar{c}$	Shift par. of productivity distr.	10
κ	Shape par. of productivity distr.	4
β_m	Shift par. of fixed cost distr.	10
γ	Shape par. of fixed cost distr.	[1.5, 2, 2.5]
α	Correlation productivity fixed cost	[-0.4, 0, 0.4]
θ	Elas. of m.cost with distance from core	0.75
σ	Elasticity of substitution	2

We have only one constraint:

$$\kappa - \alpha\gamma - \tilde{\gamma} > 0$$

Given the parameters, we can compute the domestic cost cutoff c_D^* using (66), the domestic fixed cost cutoff using (64), and the corresponding export cutoffs using (63) and (65). We simulate $10,000$ draws u from a uniform distribution from 0 to 1. The corresponding cost draws are $c = \left(\frac{u}{u_{max}}\right)^{\frac{1}{\kappa}} \bar{c}$. Given our cost cutoffs we divide firms in non-active, selling only domestically, and exporting. For each c, we draw simulate 100 uniform random draws u_b. The corresponding fixed cost shock is $\beta(c) = u_b^{\frac{1}{\gamma}} \beta_m c^{\alpha}$. We can then compute the scope of the firm with cost draws c and β and its revenues, and use these variables to study the scale and scope disconnect predicted by the model.