Heterogeneous Firms, Quality and Trade*

Alexis Antoniades

Draft

Aug 11, 2008

Abstract

We present a model of quality choice in a world of heterogeneous firms and non-constant markups. In equilibrium, some firms exit the market, others produce output but choose no quality upgrades, and the most productive firms produce both output and quality upgrades. For these firms, the more productive they are, the more quality they choose.

In line with recent empirical findings, the model shows that prices can increase in productivity, that developing countries have an incentive to export more quality goods to developed countries than developing, and that competition in endogenous sunk costs industries does not have to lead to market fragmentation. The model also generates the Linder hypothesis from the supply side, and the Balassa-Samuelson effect. Finally, trade partner characteristics do affect the level of competition in the host country when quality choice is taken into consideration.

Keywords: Intra-industry trade, firm heterogeneity, qualilty choice, markups

*aa658@georgetown.edu. I would like to thank Don Davis, James Harrigan, Amit Khandelwal, Eric Verhoogen, David Weinstein, and participants at the International Trade Colloquium at Columbia University for helpful comments and suggestions.

Recently, the role of quality has received a lot of attention in international trade. Bernard et al. (2006) document that capital and skill abundant countries use their endowment advantage to produce vertically superior varieties that have higher prices. Similarly, Hallak (2006) shows that the quality of goods produced and consumed varies systematically with the income level. Baldwin and Harrigan (2007) find that export unit-prices increase with distance. The authors point out that leading trade models can only account for this price pattern if quality is introduced. Verhoogen (2008) shows that the rising wage inequality gap in developing countries can be explained in a model where firms export high quality goods to developed countries and hence, they demand high skill labor. Finally, Hummels and Klenow (2002) find that in rich countries, a large portion of the increase in exports occurs through quality upgrades.

Three main observations come from analyzing the studies above. First, quality is essential in trying to understand the empirical findings. Second, in order to account for these findings, several ad-hoc assumptions are made. For instance, it is assumed that consumers in rich countries have higher preferences for quality than consumers in poor countries (Murphy and Schleifer [1997], Choi et al[2006]). It is also assumed that developing countries export goods of higher quality to developed countries than developing (Verhoogen [2008]), or that the level of quality increases with productivity (Baldwin and Harrigan [2007]). Third, in isolation, none of the assumptions above can account for the entire set of stylized facts.

This paper presents a model of quality choice with heterogeneous firms and non-constant markups that matches the stylized facts mentioned, but also generates endogenously several of the assumptions discussed above. The model is based on Melitz and Ottaviano (2007) model of heterogeneous firms with linear demand functions. As in Melitz and Ottaviano (MO), firms pay a fixed cost to be able to draw a marginal cost (productivity) parameter from a distribution. Firms with high marginal costs (low productivity) exit. Firms with lower marginal costs stay in the market and produce heterogeneous goods. The MO framework is augmented to include quality. On the demand side, the consumers like quality and hence, a new utility function is presented to account for this fact. On the supply side, the firms not only choose whether to produce or not, but they also choose whether to undertake quality upgrades for their products. If all firms choose not to upgrade quality, then this model collapses to the MO framework. In this sense, our model nests that of MO. A firm has incentive to invest in quality upgrade since consumers like quality. However, quality comes at a cost. If a firm chooses to produce quality, it needs to pay an extra fixed cost (with respect to quantity) that increases with the level of quality upgrade it chooses to undertake. Any choice of quality upgrade does not affect the marginal cost of each unit produced, but it affects the markup a firm charges for that unit and therefore, its price. The choice of quality upgrade is continuous and non-negative.

Several interesting features come out of the model. First, at equilibrium there are three types of firms. Some firms draw high marginal costs and choose to exit the market. Others, produce output but choose not to undertake any quality upgrades. For these firms, their productivity is high enough to enable them to produce output, but not high enough to enable them to recover the fixed cost of innovation. And finally, there are firms that produce both output and quality upgrades. These are the most productive firms in the market.

Second, for the set of firms that choose quality upgrade, the higher their productivity is (lower marginal cost), the more quality upgrade they choose. As firms get more productive, it becomes easier for them to recover the fixed cost of innovation. More innovation raises quality and more quality raises the consumers' willingness to pay. Hence, the model can generate endogenously the first assumption made by Baldwin and Harrigan (2007) that quality increases in productivity[1]. This point is also made by Johnson (2007) in a similar paper that also endogenizes quality choice in a CES framework.

Third, if the elasticity of markups with respect to quality choice is high, prices increase in productivity even though marginal costs fall. In standard heterogeneous firms models (eg. Melitz [2003], Melitz and Ottaviano [2007]) the opposite is true. The more productive a firm is, the lower its marginal cost and the lower price it charges. This is because firms set prices as markups over marginal cost, and markups are assumed to be constant. Baldwin and Harrigan study how US export unit prices vary with export destination distance and find that unit prices increase in distance. They argue that their finding is consistent with models where prices increase in productivity. Here, a firm with high productivity will have lower marginal costs. However, it will also have an incentive to produce more quality and raise the markup of its product. If the increase in markup is sufficiently large to offset the drop in marginal cost, then prices can rise with productivity.

[1] The second assumption in their paper is that quality raises the marginal cost.

Fourth, the model predicts that the share of quality goods produced and consumed in developed/large countries will be higher than in developing/small countries. As the size of the country gets larger, or the country becomes more developed (ability to innovate increases), more firms undertake quality upgrades, and the level of quality upgrade undertaken by each firm increases. Intuitively, it is easier to recover the fixed cost of innovation when the market size is large or when the cost of innovation is low. Therefore, in rich and/or large countries, the model predicts a higher share of quality goods produced and consumed. This prediction is known as the Linder hypothesis. In past work, the explanation of the Linder hypothesis came from the demand side, by assuming that consumers in rich countries have higher preferences for quality. In contrast, here we offer a supply-side explanation of the Linder hypothesis.

Fifth, trade with developed countries results in more quality upgrade. Verhoogen (2007) documents that Mexican firms have an incentive to export more quality goods when they trade with the US. We show that trade with developed countries encourages firms to upgrade quality, both for the export market and the domestic. That is, not only do Mexican firms export more quality goods when they trade with the US, but they also upgrade the quality of the goods produced for the domestic market.

Sixth, more competition does not necessarily result in market fragmentation as most trade models predict. Sutton (1989, 1991) argues that in an endogenous sunk cost industry, more competition encourages the dominant firms to differentiate their products, either vertically or horizontally, by paying a higher sunk cost. As the products become more differentiated, the consumers' willingness to pay increases along with the firms' market share. Sutton's insight is reflected in this model. As the market size expands, competition increases. However, the propensity to upgrade quality also increases as it becomes easier to recover the fixed cost of innovation. For the least productive firms the former of the two effects is stronger resulting in firms loosing market share due to competition. For the most productive firms, the latter of the two effects is stronger. These firms choose to pay a higher sunk cost and differentiate their products more as Sutton suggests[2]. Enough product differentiation may actually increase their market share even if competition increases.

Seventh, the model generates the Balassa-Samuelson effect. Trade with rich countries encourages the most productive firms to increase quality, and forces the least productive to lower quality. The most productive

[2] The model is also consistent with two other insights of Sutton, namely that there is a lower bound to quality and the window of quality levels increases with trade liberalization (Sutton [2007]).

firms raise quality because they now have access to a larger market and thus, recovering the fixed cost of innovation is easier. However, firms that are not very productive have to cut back on quality and lower their markups because this extra competition reduces their market share. Since the least productive firms are the ones that produce the non-traded goods, the price of a basket of non-traded goods falls, while the price of a basket of traded goods rises, for these goods now have higher quality relative to the non-traded goods.

Eighth, the model shows that trade partner characteristics do affect domestic market competition and the cost threshold between the firms that produce and those that do not. Trade with large countries creates more competition than trade with small countries does, and trade with developed countries induces more quality upgrades than trade with developing.

In the benchmark version of the model, we assume that quality raises the endogenous fixed cost but not the marginal cost of production. Usually, in quality models, the higher the level of quality choice is, the higher the marginal cost, while markups remain constant. Given that the existing literature has provided very important insights on how quality affects prices and competition through altering marginal costs, we decide to focus our attention on the link between quality choice and markups. That is, we do not reject the fact that adding quality raises marginal costs. Rather, we try to augment our understanding of the dynamics of quality choice on trade through the impact it has on markups. Exploring this channel is precisely the objective of the paper. As discussed above, once we endogenize quality choice and markups, a lot of interesting dynamics and predictions come out of the model. In an (unpublished) appendix we solve the model by relaxing this assumption and show that the main predictions of the model are not altered.

In earlier versions of this work we worked with CES preferences[3]. In the surface, both models are similar. Both models predict that quality choice increases in productivity. However, there are four substantial differences. First, in a CES framework the level of quality choice does not affect markups, so we cannot exploit the trade-off between the cost of adding quality and the gain from raising markups. Second, if prices do increase in productivity, the CES framework can only attribute this to rising MC. However, in this paper, we show that prices can also rise due to an increases in markups. Third, the choice of quality in the CES framework does not depend on domestic (and foreign) country characteristics (such as size and

[3] Robert Johnson (2007) develops a CES version in more detail.

ability to innovate), whereas in the paper presented below we show that quality does depend on country characteristics. Therefore, under CES, the level of quality choice will not be larger in developed, rich and/or large countries, in contrast to evidence from recent work by Khandelwal (2008) and Schott (2004). As a result, the CES model cannot generate the Linder hypothesis. Also, the Verhoogen (2008) insight that trade with developed countries encourages firms to produce and export high quality goods cannot be explained in the CES model without assuming that consumer in rich countries have stronger preference for quality. Furthermore, with CES preferences more competition results in market fragmentation, in contrast to the Sutton insight. Fourth, the CES framework can only be solved for the symmetric case, whereas this model can be solved for the asymmetric case (with the symmetric case being a special case).

The paper proceeds as follows: Section 2 presents the closed economy version of the model. A reader familiar to MO will observe that our model looks almost identical to MO. However, underneath there is a much richer world. Consumers have different preferences and firms not only choose price and quantity, but they also choose quality. One important contribution of the model is that it describes this rich environment without giving up simplicity or tractability. Simplicity helps the reader obtain the intuition behind the model, and tractability allows one to extend this model is several meaningful directions that we discuss at the end. Section 3 presents a two-country version of the model that can be used to study how openness and trade policy affect equilibrium when quality choice is taken into consideration. Section 4 parameterizes a version of the model and obtains expressions for aggregate variables such as average cost, price, markups and quality. Section 5 analyzes the model when parameters change or the country trades. Section 6 concludes this work.

1 One-Country Model

1.1 Consumers

The preferences for a typical consumer are given by

$$U = q_o^c + \alpha \int_{i\in\Omega} q_i^c di + \alpha \int_{i\in\Omega} z_i di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 di - \frac{1}{2}\gamma \int_{i\in\Omega} (z_i)^2 di + \gamma \int_{i\in\Omega} (q_i^c z_i) di - \frac{1}{2}\eta \left\{ \int_{i\in\Omega} \left(q_i^c - \frac{1}{2} z_i \right) di \right\}^2 \quad (1)$$

where q_o^c and q_i^c represent the individual consumption of the numeraire good and each variety i. The quality upgrade for each variety is represented by z_i. If all firms choose no quality upgrade ($z_i \equiv 0$), the preference relation is identical to that in MO. The parameters α and η capture the degree of substitution between each variety and the numeraire, and the parameter γ captures the degree of differentiation between the varieties. All parameters are assumed to be positive. The inverse demand for each variety is given by

$$p_i = \alpha - \gamma q_i^c + \gamma z_i - \eta Q^c \tag{2}$$

where $Q^c = \int\limits_{i \in \Omega} \left(q_i^c - \frac{1}{2} z_i\right) di$. By inverting (2) we can obtain the demand for each variety consumed

$$q_i = L q_i^c = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_i + L z_i + \frac{\eta N L}{\gamma(\eta N + \gamma)} \bar{p} - \frac{1}{2} \frac{\eta N L}{\eta N + \gamma} \bar{z} \tag{3}$$

where N is the number of consumed varieties, L is the size of the country, $\bar{p} = (1/N) \int_{i \in \Omega^*} p_i di$, $\bar{z} = (1/N) \int_{i \in \Omega^*} z_i di$, and $\Omega^* \subset \Omega$ is the subset of varieties consumed. The specified preferences ensure that the demand for good i is linear in price and quality. As price increases, the demand falls. As quality increases, the demand increases as well. Aggregate quantities also affect the demand for good i. For a given price, consumers buy more of the good if average price is high since the price of the good relative to all the goods is not as high. Similarly, if the average quality of all goods in the economy is high, consumers buy fewer units of the good since its quality relative to all the other goods is low.

1.2 Firms

As in Melitz and MO, labor is the only factor of production. Firms pay a fixed fee, f_E, required for entry and then draw a productivity parameter that determines their marginal cost, c. The distribution of c is $G(c)$

with support on $[0, c_m]$. Firms with high marginal cost (low productivity) exit the market. The remaining firms maximize their profits by taking the number of firms N, the average price $\bar{p}$, and the average level of quality upgrade $\bar{z}$ as given. Here, a firm not only chooses optimal price, but it also chooses whether to enhance its product with quality and if so, how much quality upgrade to undertake. The cost function of surviving firm i is given by

$$TC_i = c_i q_i + f_z I_z + \theta (z_i)^2 \tag{4}$$

where f_z is the fixed cost of quality upgrade, I_z is a dummy that takes the value of 1 if the firm undertakes quality upgrade and 0 otherwise, and θ shifts the marginal cost of quality. There are two parts to the cost structure. The first part (1^{st} term) is the cost of producing a variety and is identical to MO. The second part (2^{nd} and 3^{rd} terms) is a novelty of this paper and represents the cost of undertaking quality upgrades. If the firm undertakes no quality, the last two terms in the cost structure will be 0. However, if the firm chooses to undertake quality upgrade, then the firm has to do two things. First, it needs to pay a fixed cost f_z and then it has to pay a variable cost $\theta (z_i)^2$ with respect to the level of quality choice it chooses to undertake (but fixed with respect to the level of output). Intuitively, to add quality, one needs to invest in R&D. This implies that a research facility is needed (costs f_z) and a certain number of scientists will be hired. (cost $\theta (z_i)^2$). A firm that wants more quality upgrade will need to hire a larger number of scientists, or it will require more hours of labor from the same scientists than a firm that wants to undertake small quality upgrades. Alternatively, if a firm wants to convince the consumers that its product is of high quality, then this firm has to pay an ad agency to come up with an advertisement. This is the fixed cost f_z. Next, the firm decides how many times to show the ad. The more aggressive the advertising campaign is, the higher the cost to the firm is $(\theta (z_i)^2)$ and the more convinced the consumers are that the product is of high quality.

The firm's problem has three stages. In the first stage, the firm sets price as markup over marginal cost for a given level of quality upgrade. Let c_D be the marginal cost threshold between the firms that produce and the firms that exit. The firm with marginal cost c_D earns zero profits and its demand $q(c_D)$ is driven to 0. Following MO, we can express all performance measures as functions of c, c_D, and z.

$$p(c,z) = \frac{1}{2}\left[c_D + c\right] + \frac{\gamma}{2}z \tag{5a}$$

$$q(c,z) = \frac{L}{2\gamma}\left[c_D - c\right] + \frac{L}{2}z \tag{5b}$$

$$r(c,z) = \frac{L}{4\gamma}\left[(c_D)^2 - c^2\right] + \frac{L}{2}zc_D + \frac{L\gamma}{4}z^2 \tag{5c}$$

$$\pi(c,z) = \frac{L}{4\gamma}\left[c_D - c\right]^2 + \frac{L}{2}z\left[c_D - c\right] + \frac{L\gamma}{4}z^2 - f_z I_z - \theta z^2 \tag{5d}$$

The first proposition in the paper relates the amount of quality upgrade a firm chooses given its marginal cost and the threshold cost c_D for the economy.

Proposition 1 *The amount of quality upgrade a firm chooses is increasing in the productivity of the firm, in the ability to innovate (low θ), in the degree of product differentiation (γ)and in the size of the economy (L).*

Proof. In the second stage, the firm chooses the level of quality that maximizes the profit function above. Taking the derivative of $\pi(c,z)$ with respect to z yields the optimal level of quality upgrade:

$$z^* = \lambda\left[c_D - c\right] \tag{5f}$$

where $\lambda = L/(4\theta - L\gamma)$. ■

Since we are interested in situations where quality upgrade is positive, we assume that $\theta > L\gamma/4$. The parameter $-\lambda$ is country (or industry) specific and is the slope of quality with respect to cost. The higher the cost of a firm is, the lower the level of quality it chooses. The further away a firm is from the cost threshold c_D, the more quality upgrade it chooses.

The performance measures can now be re-written as functions of c, and c_D.

$$p(c, z^*) = \frac{1}{2}[c_D + c] + \frac{\gamma\lambda}{2}[c_D - c] I_z \tag{7a}$$

$$q(c, z^*) = \frac{L}{2\gamma}[c_D - c] + \frac{L\gamma\lambda}{2}[c_D - c] I_z \tag{7b}$$

$$r(c, z^*) = \frac{L}{4\gamma}\left[(c_D)^2 - c^2\right] + \frac{L\lambda}{2}[c_D - c] c_D I_z + \frac{L\gamma\lambda^2}{4}[c_D - c]^2 I_z \tag{7c}$$

$$\pi(c, z^*) = \frac{L}{4\gamma}(1 + \gamma\lambda.I_z)[c_D - c]^2 - f_z I_z \tag{7d}$$

For the firms that choose not to produce quality upgrades, $I_z = 0$, and the variables above are identical to those in MO. One important difference from MO is that prices can now increase with productivity, even if marginal costs fall. In MO, as the marginal cost drops, the markup $\mu = p - c = \frac{1}{2}[c_D - c]$ increases, but the overall price falls. However, in this setup, the markup for a quality-producing firm increases even more ($\mu_z = \frac{1}{2}(1 + \gamma\lambda)[c_D - c]$). The increase in markup can offset the drop in marginal cost, and the price can increase with productivity. This is the next proposition.

Proposition 2 *Prices can increase with productivity in the presence of quality upgrade iff $L > 2\theta/\gamma$.*

Proof. Just take the derivative of price with respect to c using the equation above to see this. ■

The proposition above is the Baldwin and Harrigan (2007) observation: prices can increase with productivity. The authors attribute this to the fact that more productive firms choose more quality that pushes marginal costs up. We show here that even if you maintain the assumption that marginal costs decrease as productivity increases, you can still get prices to increase through markups. On one hand, as productivity increases, marginal costs fall. On the other hand, quality also increases, pushing markups up. If the country is large enough, firms choose more quality (see Proposition 1), the increase in markups dominates the decrease in marginal cost, and as a result prices increase.

Proposition 3 *At equilibrium there will be two sectors in the economy: one sector contains firms that choose no quality upgrades and the other sector contains firms that choose quality upgrades. The firms in the latter sector are the most productive ones.*

Proof. In the third stage of the production process, the firm calculates the level of profit is can achieve without undertaking quality upgrade $\pi(c, 0)$, and compares it with the level of profit it can achieve with quality upgrade $\pi(c, z^*)$. If the former is greater, the firm will choose no quality upgrade, but if the latter is greater, the firm will undertake quality upgrade. It is easy to show that for $c < c_d - \left(\frac{4f_z}{\lambda L}\right)^{1/2}$ the firm will undertake quality upgrades. From the firms that choose to stay in the market and produce, only the most productive will choose to undertake quality upgrades. Denote c_z the lower marginal cost threshold that separates the quality undertaking firms with these that do not undertake quality upgrades.

$$c_z = c_D - \left(\frac{4f_z}{\lambda L}\right)^{1/2} \tag{7h}$$

The range of firms that do not undertake quality upgrades $c_d - c_z$ increases in the fixed cost of innovation f_z, in the convexity of the variable cost of innovation θ, and it decreases is country size L. ■

Propositions 1 and 3 can be illustrated graphically. Figure 1 plots the level of quality upgrades firms choose (y-axis) given their cost (x-axis). There are three important observations in the figure. First, firms with cost higher than c_D exit. Second, the firms that stay in the market are separated in two. One sector (in the middle) is the set of firms that chooses not to upgrade quality. These are firms productive enough to produce output, but their productivity is not high enough to enable them to recover the fixed cost needed to undertake quality upgrades. The other sector in the economy (far left) is the set of firms that both produce output, and also choose quality upgrades. These are firms productive enough to produce output, but also they are productive enough to recover the fixed cost needed for quality upgrades. Third, quality (for the quality producing firms) increases in productivity. This is shown by the solid, negatively slope line that measures how much quality a firm chooses ($z(c)$). The slope of the $z(c)$ line is λ.

1.3 Free-Entry Equilibrium

In equilibrium, the expected profit of a firm is 0. Therefore

$$f_E = \int_0^{c_d} \pi(c, z) dG(c) = \frac{L}{4\gamma} \int_0^{c_D} [c_D - c]^2 \, dG(c) + \frac{L}{4} \lambda \int_0^{c_z} [c_D - c]^2 \, dG(c) - \int_0^{c_z} f_z dG(c) \tag{9}$$

The condition above determines the cutoff cost c_D. The number of surviving firms can be found from (2). Set $q_i = 0$, then

$$c_D = \frac{1}{\eta N + \gamma}(\alpha\gamma + \eta N\bar{p} - \frac{1}{2}\eta N\gamma\bar{z}) \tag{10}$$

It can be shown that

$$N = \frac{2\gamma}{\eta}\frac{(a - c_D)}{(c_D - \bar{c})} \tag{11}$$

Readers familiar with the MO work will notice that our expression for the number of firms is identical to that in MO. This is not a mistake. The dynamics of quality choice on prices, competition, markups, etc are summarized by the term c_D. It is in the sense that we argue simplicity is not lost even though a much richer framework is introduced.

2 Two-Country Model

2.1 Consumers

We now extend the closed economy model to a two-country setting. There is a home (H) and a foreign (F) country. Each country is endowed with L^H and L^F workers (consumers). For simplicity, assume that consumers have identical preferences across the two countries and there is no labor mobility. As in the closed-economy setting, the demand for good i in country l ($l = \{H, F\}$) is given by

$$q_i^l = L^l q_i^c = \frac{\alpha L^l}{\eta N^l + \gamma} - \frac{L^l}{\gamma}p_i^l + L^l z_i + \frac{\eta N^l L^l}{\gamma(\eta N^l + \gamma)}\bar{p}^l - \frac{1}{2}\frac{\eta N^l L^l}{\eta N^l + \gamma}\bar{z}^l \tag{12}$$

where p_i^l and q_i^l is the price of good i and quantity demanded in country l, respectively. Average price and quality in country l are given by $\bar{p}^l$ and $\bar{z}^l$. There are N^l firms selling in country l. These are both domestic firms and foreign exporters. A firm does not discriminate in quality between domestic and foreign sales, but it does discriminate in price. This is why quality does not have a country superscript by price does. Later in the paper, we relax this assumption and allow firms to set differentiated levels of quality for the domestic and the foreign markets.

2.2 Firms:

As in the closed economy setting, a firm chooses whether to produce or not, and whether to install quality upgrades. The firm now has the option to export. There is cost to exporting, so not all firms choose to export. A firm that exports sets two different prices, one for the domestic market and one for the foreign market. The level of quality it chooses, however, is fixed. There is no difference in the quality of a product the firm exports to the one it sells domestically. The delivery cost of a unit with cost c to country l is $\tau^l c$[4]. Let $p^l(c)$ and $q^l(c)$ be the domestic level of the profit maximizing price and quantity, respectively. The operating profit (ignores fixed costs and cost to quality upgrade) from domestic and foreign sales is given by

$$\pi_D^l(c,z) = \left[p_D^l(c,z) - c\right] q_D^l(c,z) \tag{13a}$$

$$\pi_X^l(c,z) = \left[p_X^l(c,z) - \tau^h c\right] q_X^l(c,z) \tag{13b}$$

The profit maximizing prices and quantities must satisfy

$$q_D^l(c,z) = \frac{L^l}{\gamma}\left[p_D^l(c,z) - c\right] \tag{14a}$$

$$q_X^l(c,z) = \frac{L^h}{\gamma}\left[p_x^l(c,z) - \tau^h c\right] \tag{14b}$$

The production cutoffs are defined as

$$c_D^l = \sup\left\{\pi_D^l(c) > 0\right\} = p^l \tag{15a}$$

$$c_X^l = \sup\left\{\pi_X^l(c) > 0\right\} = \frac{p^h}{\tau^h} \tag{15b}$$

[4]In the appendix I present the solution of the model when a per-unit cost of export is impossed. That is, the cost of delivering a unit with cost c to country l is now $\tau^l + c$ instead of $\tau^l c$. Prof James Harrigan suggested this exercise.

We assumed here that the quality cost threshold is to the left of the export cost threshold. Combining the cutoffs conditions for the two countries, it is easy to show that $c_X^h = c_D^l/\tau^l$. The optimal price and quantity for the domestic and the foreign market can now be expressed as functions of the cutoff cost thresholds.

$$p_D^l(c,z) = \frac{1}{2}(c_D^l + c) + \frac{\gamma}{2}z \tag{16a}$$

$$p_X^l(c,z) = \frac{\tau^h}{2}(c_X^l + c) + \frac{\gamma}{2}z \tag{16b}$$

$$q_D^l(c,z) = \frac{L^l}{2\gamma}(c_D^l - c) + \frac{L^l}{2}z \tag{16c}$$

$$q_X^l(c,z) = \frac{L^h}{2\gamma}\tau^h(c_X^l - c) + \frac{L^h}{2}z \tag{16d}$$

Given prices and quantities, operating profit of the firm in each market is given by

$$\pi_D^l(c,z) = \frac{L^l}{4\gamma}(c_D^l - c)^2 + \frac{L^l}{2}z(c_D^l - c) + \frac{\gamma L^l}{4}z^2 \tag{17a}$$

$$\pi_x^l(c,z) = \frac{L^h}{4\gamma}(\tau^h)^2(c_X^l - c)^2 + \frac{L^h}{2}\tau^h z(c_X^l - c) + \frac{\gamma L^h}{4}z^2 \tag{17b}$$

Adding up the two operating profits and including the cost to quality upgrade yields the total profits of the firm with cost c.

$$\pi^l(c,z) = \pi_D^l(c,z) + \pi_X^l(c,z) - f_z.I_z - \theta z^2 \tag{18}$$

where I_z is the dummy that take the value of 1 if the firm chooses to install quality upgrades and 0 if it does not. The level of quality upgrade z^* that maximizes the profit above is given by

$$z^*(c) = \lambda_D(c_D^l - c) + \lambda_X(c_X^l - c) \tag{19}$$

where $\lambda_D = L^l/(4\theta - L\gamma)$, $\lambda_X = \tau^h L^h/(4\theta - L\gamma)$, and $L = L^l + L^h$.

By substituting the optimal value of z into (18) we obtain

$$\pi^l(c) = \frac{L^l}{4\gamma}(1+\gamma\lambda_D.I_z)(c_D^l - c)^2 + \frac{L^h}{4\gamma}(\tau^h)^2(1+\frac{\gamma\lambda_x}{\tau^h}.I_z)(c_X^l - c)^2 + \frac{1}{2}L^l\lambda_X(c_D^l - c) - f_z.I_z \tag{20}$$

The total profit is the sum of the profits from domestic and foreign sales plus. With quality upgrade, a firm can shift the two components of the profit up since $\gamma\lambda_D$ and $\frac{\gamma\lambda_x}{\tau^h}$ are both positive. However, the firm has to pay the fixed cost to quality upgrade. There is, however, an extra positive term (3^{rd}) that shifts profits up. This term represents gains from trade that are only realized in the presence of quality upgrade.

2.3 Free-Entry Condition

In equilibrium, the expected profit of a firm is 0. Therefore

$$f_E = \int_0^{c_d^l} \pi_D^l(c)dG(c) + \int_0^{c_x^l} \pi_X^l(c)dG(c) \tag{21}$$

3 Parameterization

A way to better illustrate the properties of the model is to parameterize the cost distribution. Above we studied how firms respond to the environment in which they operate by choosing price and quality. Now we study how their choices affect aggregate variables in the economy. Parameterizing the cost distribution yields simple expressions for the cost threshold, aggregate prices, quantities, markups, and productivity.

For simplicity, we ignore the fixed cost f_z here ($f_z = 0$) and work in a world where all firms choose quality. By not having two sectors in the economy, the model becomes very similar to the MO. This enables for a more direct measure on the marginal gains from adding quality in their model. Also, we assume that firms choose separate levels of quality upgrade for the domestic and foreign markets. This makes the algebra of aggregation simpler without compromising the predictions of the model.

As in MO, suppose the distribution of cost draws is given by

$$G(c) = \left(\frac{c}{c_M}\right)^k, c \in [0, c_M] \tag{22}$$

3.1 Closed Economy

Given that cost draws come from the pareto distribution above, the cost threshold in the closed economy is

$$c_D = \left[\frac{\gamma\phi}{(1+\gamma\lambda)L}\right]^{\frac{1}{k+2}} \tag{23}$$

where $\phi = 2c_m^k(k+1))(k+2)f_E$. Remember that the parameter λ is the slope of the quality line in Figure 1 and it increases with the size of the economy and the ability to innovate $(1/\theta)$. An increase in the size of the economy or the ability to innovate generates more competition and pushes the cost threshold downwards.

The average cost in the economy is

$$\bar{c} = \frac{kc_D}{k+1} \tag{24}$$

As competition increases in this economy, the cost threshold falls and average productivity increases ($\bar{c}$ falls). There are a couple of interesting points worth making here. First, notice that the ability to innovate affects average productivity. Most static models on firms' behavior do not explicitly explore the link between innovation and productivity. Here, the link is a strong one. In countries with low cost of innovation, firms choose to undertake more quality upgrade. This generates extra competition that pushes the cost threshold down and average productivity up. Therefore, not only do we get that the size of the country matters in determining key parameters, but we also show that the ability to innovate matters. For example, if Germany and Ethiopia have the same population, they can differ in average productivity if Germany has higher ability to innovate. Any policy that changes the ability of/cost to innovation automatically affects productivity, as well as all the other aggregates measures in the economy. This insight is one of the main contributions of this work.

This brings up the second point. Differences in productivity across countries can be explained by differences in the ability to innovate. Usually, we think of developed countries as countries whose firms have higher productivity on average than firms in developing countries. But what is the driving force behind these differences in productivity? One explanation may be that the ability to innovate across countries differs. Consider two countries, one with a lower cost of innovation, θ, than the other but identical productivity distributions. In the country with the lowest cost of innovation, quality upgrade is higher and competition is tougher. More competition lowers the cost threshold, and pushes average productivity up. Even though these two countries start with the same productivity distribution, variations in the ability to innovate result in differences in productivity. Therefore, part of the observed variation in average productivity across countries can be attributed to variations in the ability to innovate. For the remaining of the paper, we will refer to a country with low θ (high ability to innovate) as a developed country and country with high θ (low ability to innovate) as a developing country.

The average quality in the economy is

$$\bar{z} = \lambda \left(\frac{c_D}{k+1} \right) \tag{25}$$

An increase in L or a decrease in θ can have an ambiguous effect of average quality. This is because such changes reduce the cost threshold c_D but increase λ.

Given average productivity $\bar{c}$ and the cost threshold c_D, we can use (11) to solve for the number of firms.

$$N = \frac{2(k+1)\gamma}{n} \frac{(a - c_D)}{c_D} \tag{26}$$

The expression is identical to the expression one gets when no quality choice is allowed (see MO). The main difference, however, is that in the economy where quality choice is allowed, the cost threshold will always be lower (since the denominator in (23) is now multiplied by $1+\gamma\lambda$) and the number of varieties will be higher.

The average (unweighted) price in the economy is given by

$$\bar{p} = (1+\gamma\lambda)\frac{2k+1}{2k+2} \tag{27}$$

In a world with no quality choice, average price is constant (only the second term exists). When quality choice is allowed, developing and/or large countries will have higher average price. Intuitively, for large and/or developed countries, there are more firms. These firms choose high levels of quality upgrade resulting in higher markups that push prices up. Notice that since we assumed that in this version of the model marginal costs do not increase with quality, any increase in price comes only from increases in markups. That does not imply however, that all prices go up. In fact, any parameter change that induces more competition and lowers the cost threshold, causes some firms to increase quality and markups and others to decrease quality and markups. We discuss this in the next section.

Average markups are

$$\bar{\mu} = \frac{1}{2}(1+\gamma\lambda)\frac{1}{k+1}c_D \tag{28}$$

An increase in the size of the country does not necessarily imply that average markups fall. An increase in L reduces c_D, but at the same time it increases λ. By substituting (23) into the expression above and taking the derivative of $\bar{\mu}$ with respect to L we can easily obtain conditions under which markups increase or decrease in L.

Finally, average profits are given by

$$\bar{\pi} = f_E\left(\frac{c_M}{c_D}\right)^k \tag{29}$$

3.2 Open Economy

We now turn our attention to the open economy and examine the implications of the model when cost draws come from a pareto distribution. As mentioned earlier, we deviate slightly from the model presented in Section 2 and assume that firms choose two quality levels, one for the domestic market and one for the foreign. Given this assumption, the cost to innovation is now $\theta(z_l + z_h)^2$, where z_l and z_h are the domestic and foreign levels of quality upgrade, respectively. The benefit (to us) is that the algebra of aggregation becomes simpler. The cost (to the firms) is that they will incur higher costs of innovation since they will need to "reproduce" every level of quality upgrade for each market. By having to reproduce the quality, they no longer receive the extra gains from trade captured by the third term of (20). This assumption understates the gains from trade, but does not alter the predictions and dynamics of the model in any significant way. Again, we assume that the fixed cost of innovation f_Z is zero so all firms produce quality upgrades.

First, we present the optimal level of quality choice for each market. The domestic level of quality upgrade is given by $z^l_D = \lambda^l_D(c^l_D - c)$ and the foreign by $z^l_X = \lambda^l_X(c^l_x - c)$, where $\lambda^l_D = L^l/(4\theta^l - L^l\gamma)$ and $\lambda^l_x = \tau^h L^h/(4\theta^l - L^h\gamma)$. Using the assumption of the pareto distribution for the costs along with the free-entry condition stated in equation (21) we obtain the following relation:

$$L^l(1 + \gamma\lambda^l_D)(c^l_D)^{k+2} + L^h(\tau_h)^2(1 + \frac{\gamma\lambda^l_x}{\tau^h})(c^l_X)^{k+2} = \gamma\phi \tag{30}$$

By solving the free-entry condition of the foreign country and using the fact that $c^l_X = c^l_D/\tau^l$, we can derive the cost threshold

$$c^l_D = \left\{ \frac{\gamma\phi(1 - \rho^h\sigma^h)}{L^l(1 + \gamma\lambda^l_D)[1 - (\rho^l\sigma^l)(\rho^h\sigma^h)]} \right\}^{\frac{1}{k+2}} \tag{31}$$

where $\rho = 1/\tau$, $\sigma^\mu = [1 + \gamma\lambda^\mu_D(\theta^\nu)]/[1 + \gamma\lambda^\mu_D(\theta^\mu)]$, $\mu = \{l, h\}$, $\nu = \{h, l\}$, and $\lambda^\mu_D(\theta^\nu) = L^\mu/(4\theta^\nu - L^\mu\gamma)$. The sigmas are shift parameters that measure how similar or dissimilar the trading partners are. A few insights stem from the expression above. First, the cost threshold does depend on the parameters of the other country, namely size and ability to innovate, once we allow for quality choice. This eases any concerns that trading partner characteristics do not affect the cost threshold in linear demand functions. Second, free trade does not blow up the solution. Setting ρ^h and ρ^l to 1 causes problems only if the countries are identical in size and/or productivity ($\sigma^\mu = 1$). Third, the degree of heterogeneity between the trading partners affects the sigmas, and therefore, affects the responsiveness of the cost threshold to trade liberalization. That is, the effect that a 10% reduction in tariffs has on c_D does depend on characteristics of the trading partners, namely how similar or dissimilar they are in size and ability to innovate. We elaborate on this later.

4 Analysis

Based on the parameterized version of the quality choice model, we proceed to show what happens to such economies when key parameters change. First, we consider an increase in the size of the economy. Then, we consider an increase in the ability to innovate. In both cases, we focus on the closed economy version of the model. Looking at size and the ability to innovate helps us compare and contrast large versus small economies, as well as developed versus developing. Next, we see what happens when an economy opens up to trade[5].

4.1 Size and Ability to Innovate

For the sake of building intuition, suppose that at time t_0 the economy is small (L) or the cost of innovation (θ) is low and at time t_1 its size increases or the cost of innovation decreases. An increase in L or a decrease in θ induces more competition. This extra competition lowers the cost threshold. Some firms now decide to exit the market since they cannot compete. Given that the cost threshold in now lower, product variety and average productivity increase. From the firms that stay in the market, some are encouraged to increase quality choice (and markups) and others to lower quality. Since the slope of the quality line is λ, as L increases or θ falls, the cost threshold moves inwards and the slope increases. This is shown in Figure 2. The black line represents quality choice at time t_0 for the small or developing country. The grey line represents

[5]We do not analyze unilateral trade liberalizations in this version. The reason is that by assuming that export quality has to be reproduced, we get similar results to the MO framework (although differences between the trading partners will have some impact). What makes a difference is when we move to a more realistic assumption where we assume that export firms only have to pay the incremental cost of innovation if they choose to export goods of higher quality. In such case, unilateral trade liberalizations do not necessarily imply less competition for the host country. This analysis is coming soon.

quality choice at time t_1 for the large or developed country. The firms to the right of the intersection of the two lines represent firms that lower their quality choice, markups and prices as a result of the extra competition. Firms to the right are firms that increase their quality choice and markups. Prices need not increase for these firms. Based on (16a), a drop in c_D puts downward pressure on prices. But there is also an upwards pressure on prices since the quality choice is now higher. Whether the total effect on prices will be positive or negative depends on Proposition 2. In contrast to MO, the extra competition does not necessarily imply that prices and markups will fall. For some firms this will be true, but for others it will not be. Therefore, average prices and markups also need not fall. Notice also that the range of available qualities is higher for rich and developed countries. This can be seen from the graph by observing that the z-intercept is higher for the gray line than it is for the black line.

The idea that more competition encourages some firms to upgrade quality is not new. In a series of papers, Sutton shows that firms in an endogenous sunk cost industry can gain market share as competition increases. In his words (1991, p. 47): "If it is possible to enhance consumers' willingness-to-pay for a given product to some minimal degree by way of a proportionate increase in fixed cost (with either no increase or only a small increase in unit variable costs), then the industry will not converge to a fragmented structure, however large the market becomes." Sutton's insight is reflected in Figure 2. As the size gets bigger and competition increases, the slope of the quality line goes up. As the slope increases, the least productive firms lower quality and experience a drop in their market share, but the most productive exploit the increase in market size by adding more quality. More quality increases the consumers' willingness to pay and hence, the firms' market share. In the absence of quality choice however, more competition will result to market fragmentation as firms loose market share.

4.2 Trade

Accounting for quality choice really pays off when we consider trade. As in the case of no quality choice, opening up to trade induces more competition that raises average productivity and the number of varieties. But now, characteristics of the trading partner, such a size and ability to innovate, matter. As we discuss next, trading with a large country creates more competition than trading with a small country, and trading with a developed country induces more quality upgrade than trading with a developing one. Adding more structure the model enables us to consider a two-dimensional heterogeneity across countries: size and ability to innovate.

First we discuss differences between trading with a large versus a small partner. For simplicity, assume that both trading partners have the same ability to innovate ($\sigma^l \sigma^h = 1$). The higher the size of the trading partner is, the lower the production cost threshold c_D and export cost threshold c_X are, and the higher the slope of the export quality choice λ_X^l is. This is illustrated in Figure 3. The black line represents the quality choice of domestic firms exporting to a small country, and the grey line the quality choice of domestic firms exporting to a large country. Since the domestic firms that export choose two separate levels of quality upgrades, one for domestic sales and one for foreign, we focus and plot their choice of export quality (dashed line). Clearly, for the large trading partner, the cost threshold is lower since competition is higher. Also, the export threshold is lower. When the size of the export partner increases, the slope of export quality increases, forcing the least productive export firms to reduce export quality and encourages the most productive to raise export quality. Although the slope of domestic quality choice does not change, the line shifts down as a result of a drop in the cost threshold. Therefore, domestic quality upgrade choice is reduced more when trade occurs with a large country. Notice that trade does generate the Balassa-Samuelson effect. Opening up to trade forces the least productive firms (those that do not export) to lower price and quality due to increase competition. It also encourages the most productive firms (those that export) to upgrade quality, and raise markups and prices. The larger the perceived market is with of trade (either because the partner is a rich and/or a large country), the bigger the quality upgrade of exports will be and the bigger the drop in price of non-exports. As a result, prices of traded to non-traded increase. This is the Balassa-Samuelson effect.

Next we discuss the impact of trade when the ability to innovate differs, but size does not. Opening to trade induces competition. The competition is higher when trade occurs with a developing country than when it occurs with a developed country[6]. Trading with a developed country creates more profitable export opportunities so competition is not as tough and the cost threshold does not have to fall as much. The

[6] This is because $\sigma^l \sigma^h = 1$ and and as θ decreases, σ^h decreases and c_D increases.

slope of both the export and domestic quality upgrade choice do not change as the θ of the trading partner varies. However, when trade occurs with a developed country, the cost threshold drops by less than if trade occurred with a developing country, and consequently, the quality upgrade line (both domestic and foreign) shifts up. This is the Verhoogen (2008) insight. As Mexico trades more with the US, more firms in Mexico have incentives to export high quality goods. Verhoogen's insight goes through in this model, albeit in a stronger version. Not only Mexican firms have an incentive to increase export quality, but they also have an incentive to increase domestic quality as well. Figure 4 illustrates the differences between trading with a developed and a developing country discussed above.

5 Conclusion:

Quality choice by firms is important. It explains why prices increase in productivity (Baldwin and Harrigan [2007], Johnson [2007]), why the wage gap between low and high skill labor increases in developing countries when they trade with developed (Verhoogen [2008]), and why rich countries trade more with each other than gravity models predict (Hummels and Klenow [2002]).

This paper presents a model of heterogeneous firms, quality and trade. In the model, firms not only choose price, but they also choose the optimal level of quality they desire. To upgrade quality a fixed cost must be paid. The level of this fixed cost is endogenous and depends on the level of quality upgrade a firm chooses to undertake. Markups are non-constant. A firm can increase the consumers' willingness to pay by adding more quality, but it has to pay a higher cost of innovation in order to obtain the extra quality. The model is solved for both a closed and an open economy and the equilibrium is analyzed.

The model can match the insights of Verhoogen, Baldwin and Harrigan, Johnson, and Hummels and Klenow discussed above. It also generate the Linder hypothesis. That is, there is a higher share of quality goods produced and consumed in rich countries. The explanation comes from the supply side. It is easier to recover the fixed cost of innovation in rich rather than poor countries. Furthermore, the model generates the Balassa-Samuelson effect. The richer the trading partners are, the higher the ratio between the price of traded to non-traded is. Trade among rich countries encourages quality upgrade but is also creates competition. The most productive firms (those that export) raise quality and price, and the least productive (those that do not trade) lower quality and price. The model is also consistent with the Sutton (89, 91) insight that in endogenous sunk costs industries, competition does not necessarily imply market fragmentation. When firms are allowed to upgrade quality, we observe that productive firms can respond to the pressure of competition by paying a higher fixed cost of innovation, differentiating their products more, and increasing the consumers willingness to pay and their market share. Finally, when the quality choice of firms is taken into account, we see that trading partner's characteristics affect the level of competition in the home country, as well as the cost threshold between the firms that produce and the ones that do not. And since the cost threshold changes, all other parameters in the economy such as product variety, average productivity, price and markups change. This would not be true if quality choice is not accounted for (Melitz and Ottaviano [2007]) or if quality is accounted for but markups are kept constant (Johnson [2007]).

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Figure 1: Graphical Representation of Quality Choice for a Given Cost Level

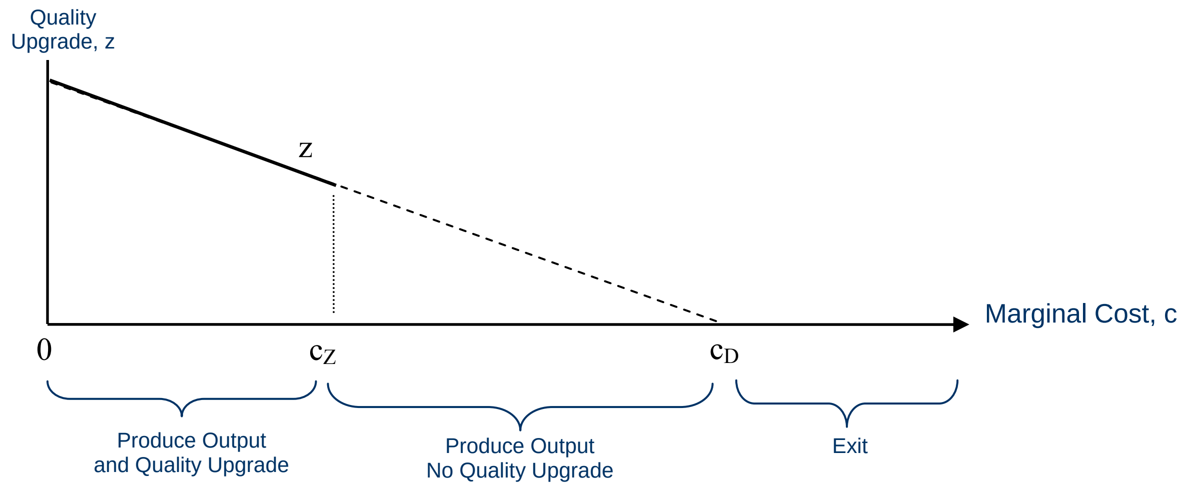

Figure 2: Quality Choices in a Closed Economy as L Increases or θ Falls.

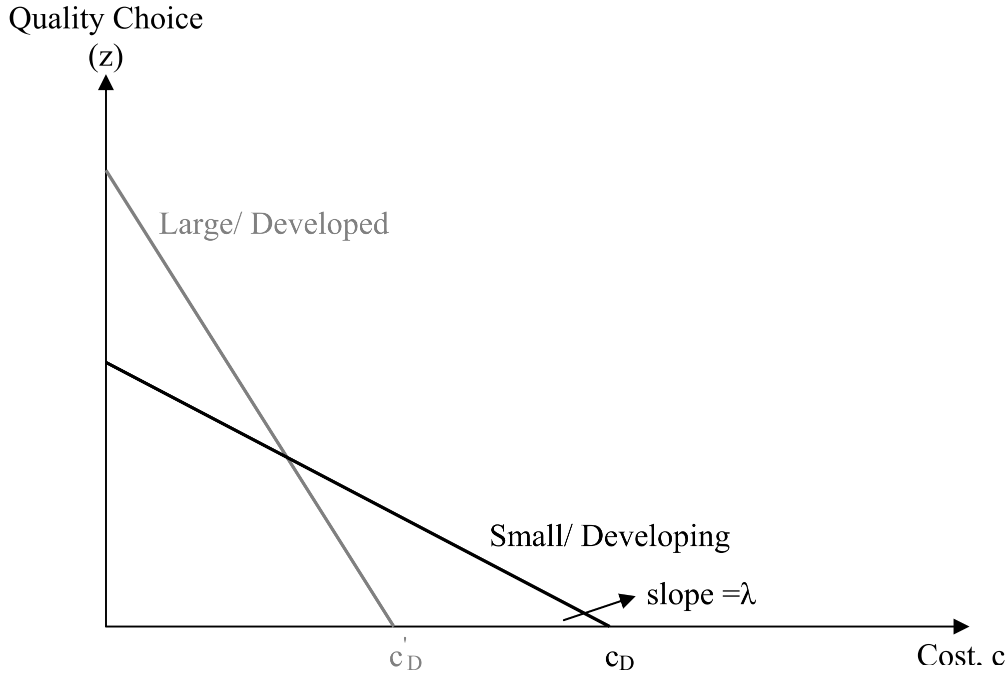

Figure 3: Quality Choice as the Size of the Trading Partner Increases

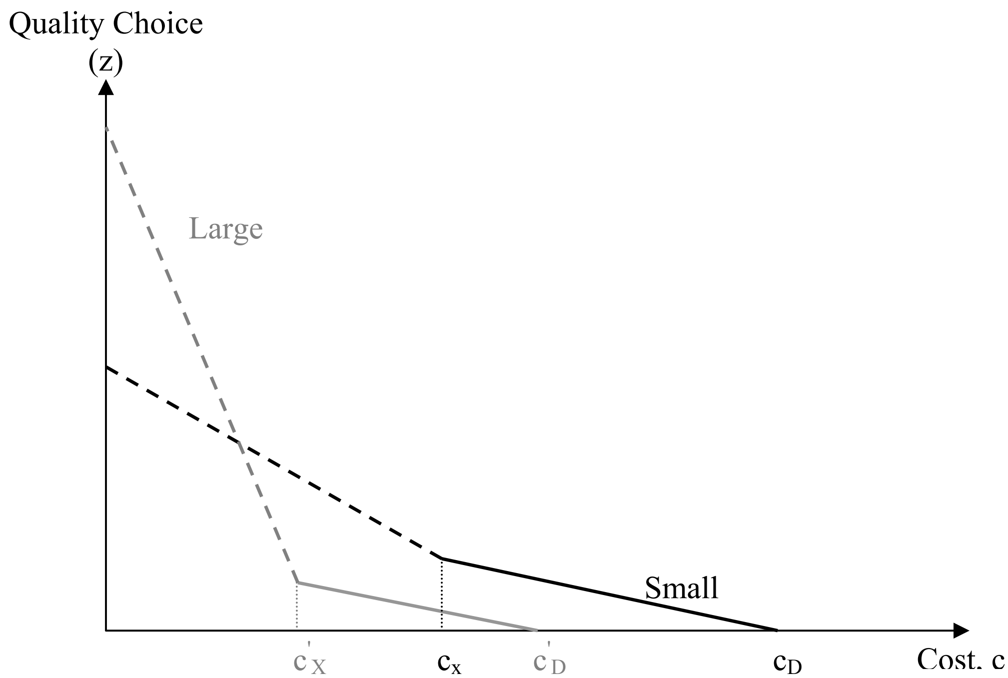

Figure 4: Quality Choice as the Ability to Innovate of the Trading Partner Increases