Cities, Productivity and Trade*

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PRELIMINARY AND INCOMPLETE – DO NOT CIRCULATE

Abstract

We document a novel stylized fact: Using data for several countries, we show that export activity is disproportionately concentrated in larger cities – even more so than overall economic activity. We account for this fact by marrying elements of international trade and economic geography. We build a model with agglomeration economies where firms with heterogeneous productivity sort across city sizes and select into exporting. The model allows us to study the geographic implications of trade policy, as well as the international trade effects of place-based policies. We show that (i) weaker restrictions on housing supply increase not only the aggregate productivity of the economy but also its export intensity, by allowing more firms to locate in larger cities and profit from agglomeration effects; (ii) conversely, falling trade costs increase the fraction of the population that inhabits larger cities, where exporting firms tend to locate; (iii) when trade costs fall, the well-known process of reallocating resources to more efficient producers is slowed down by congestion in larger cities. We structurally estimate the model using data for the universe of Chinese manufacturing firms and study the general equilibrium effects of trade liberalization and of place-based policies. We find that the effects of these policies are quantitatively different from those predicted by trade models that ignore economic geography, and by economic geography models that omit international trade.

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1 Introduction

Over the last decades, two mega-trends have shaped economies across the globe: urbanization and international trade.[1] The simultaneous surge of trade and city growth naturally raises the question to what extent the two are connected. Two strands of the literature have examined the underlying drivers: international trade and urban economics. While these fields have a common connection in Paul Krugman's (1979, 1991) seminal contributions, they have largely developed along parallel paths, without much interaction. Canonical trade models struggle to explain the geographic distribution of trade activity, and economic geography models typically overlook how urban characteristics affect international trade.[2] Yet, there is a natural overlap: On the one hand, a large body of evidence in economic geography has shown that overall economic activity is unevenly distributed across space,[3] and that firms are more productive – and thus more concentrated – in larger cities (Combes, Duranton, Gobillon, Puga, and Roux, 2012). On the other hand, the trade literature has established that among a distribution of heterogenous firms, the more productive ones tend to become exporters (Pavcnik, 2002; Melitz, 2003). Combining these two facts, one should expect a higher share of exporting firms in larger cities, and – vice versa – a positive impact of growing city size on trade openness. Neither of these connections has been previously examined.

In this paper we study the economic geography of exporting activity both empirically and theoretically. We first show – using data for China, the United States, and Brazil – that larger cities systematically export a higher fraction of their output than smaller cities, even after controlling for differences in cities' geographic characteristics. Over three-fourths of the association between export intensity and city size can be attributed to variation within industries. We show that the higher within-industry export intensity of large cities is driven by a higher export participation of firms in larger cities. This suggests that economic geography has important implications for exporting activity.

To explain the stylized facts described above we extend the systems of cities framework of Gaubert (2018) to a multi-country setting and augment it with a mechanism of selection into exporting in the spirit of Melitz (2003). We study a setup with an arbitrary number of symmetric countries, each subject to an identical distribution of potential entrants in each sector. Within countries, cities form endogenously on sites that are ex-ante identical and grow in population as firms choose to locate there and increase local labor demand. For firms, the main benefit of locating in cities is given by agglomeration externalities, such as thick labor markets or knowledge

[1] The average urbanization rate in the world grew from 43 to 55 percent between 1990 and 2010. During the same period, exports as a share of GDP have grown from 30 to 46 percent (https://data.worldbank.org/indicator).

[2] A notable exception are Melitz and Ottaviano (2008), who develop an analytically tractable model to explore the link between market size, trade, and competition within markets. However their simple model does not allow to analyze a central dimension of economic geography – the location of exporting firms and industries within economies.

[3] See reviews for the United States by Holmes and Stevens (2004), for European countries by Combes and Overman (2004), and for China and Japan by Fujita, Mori, Henderson, and Kanemoto (2004).

spillovers (Duranton and Puga, 2004). Firms are heterogeneous in productivity and produce in a variety of sectors. They sort across cities of different sizes within their country.[4] When choosing their location, firms trade off the gains in productivity generated by local externalities in large cities against the higher labor costs prevailing in these cities. Moreover, in line with Gaubert (2018) and Combes et al. (2012) we assume that more efficient firms benefit relatively more from these local externalities. This generates positive assortative matching: More efficient firms locate in larger cities, reinforcing their initial productivity advantage. Finally, as in Gaubert (2018), city developers operate within each country and compete to attract firms to their city. They act as a coordinating device in the economy, leading to a unique spatial equilibrium.

The model explains the disproportionate share of exporting firms in larger cities: More productive firms sort into larger cities. As a result, they augment their productivity advantage and are more likely to export. Consequently, within sectors, a higher fraction of firms in larger cities become exporters, which accounts for the positive association between export intensity and city size at the level of city $\times$ industry cells.[5] Moreover, if two cities have similar sectoral compositions, the model predicts that the larger one will have a larger aggregate export intensity, as it will have a higher export intensity in every sector. Similar to Melitz (2003), the model also predicts that – conditional on exporting – export *intensity* at the firm level is unrelated to productivity. This is consistent with our findings.

Our paper is related to several strands of the literature. First, we document a series of novel stylized facts regarding the economic geography of exporting activity ("exporter facts", as in Bernard, Jensen, Redding, and Schott, 2007, etc). To the traditional stylized facts about exporters (being larger and more productive) we add a new one: Exporters tend to locate disproportionately in large, successful and expensive cities that allow them to leverage agglomeration economies. This, in turn, leads to an economic geography of exporting within countries that is even more uneven than that of overall economic activity.

Second, by combining a tractable model of spatial equilibrium featuring heterogeneous firms with a Melitz (2003) style mechanism of selection into exporting we contribute both to the systems of cities literature, pioneered by Henderson (1974), and to the international trade literature. From the perspective of the former, our contribution is most closely related to Gaubert (2018), who first proposed the modelling strategy of urban systems that we employ.[6] However, her study focuses on

[4]Firms cannot choose the country they enter; they choose a city within a pre-determined country.

[5]An important strand of the trade literature predicts the opposite: A direct implication of the gravity model – and the underlying Armington assumption – is that larger cities (or countries) are *less* open (Anderson and van Wincoop, 2004).

[6]Some of our modeling assumptions are motivated by the empirical findings of Combes and Overman (2004), who show that the productivity advantage of firms in large cities is not driven by tougher competition (and hence stronger selection) in larger cities, but by agglomeration effects. They also find that the most efficient firms are disproportionately more productive in large cities, indicating potential complementarities between firm productivity and city size.

the sorting and agglomeration of heterogeneous firms in a single country setting, and it does not allow for the possibility of international trade and of selection into exporting. A related seminal contribution is Behrens, Duranton, and Robert-Nicoud (2014) who study the spatial sorting of entrepreneurs who produce non-tradable intermediates.[7] We study the case of producers of goods that are perfectly tradable within countries but subject to transportation frictions across countries.[8]

From the perspective of the trade literature our contribution is most closely related to the theoretical body of work that analyzes firms' decisions to enter into exporting (Melitz, 2003; Bernard et al., 2007). We embed a Melitz style mechanism into a model of firm location across space to show that the same firm level fundamentals that lead firms to select into exporting may also cause them to locate in large, productive but expensive cities. This allows us to account for the uneven economic geography of exporting and also to study the interplay of location decisions and exporting decisions. Our paper is also related to an older theoretical literature that analyzes the joint determination of international trade flows and within-country economic geography (Krugman and Livas Elizondo, 1996; Monfort and Nicolini, 2000; Paluzie, 2001; Behrens, Gaigne, Ottaviano, and Thisse, 2006a,b, 2007). As in some of these models, in our framework trade policy affects the configuration of economic geography; while spatial policy can affect trade flows. Moreover, as in these models, our framework also captures the fact that domestic policy decisions can have spillovers on other countries via trade channels.[9] However, unlike these earlier stylized models, our quantitative model can be taken to the data.

Finally, as in Gaubert (2018), Desmet and Rossi-Hansberg (2013) and Behrens et al. (2014), we also use structural estimation of a model of a system of cities to assess the welfare implications of the spatial equilibrium. In doing so, we contribute to the literature that measures agglomeration externalities, as reviewed in Rosenthal and Strange (2004).[10] Moreover, we also employ the model to run policy experiments in order to study the general equilibrium effects of place-based policies. We thus contribute to the strand of literature that quantifies productivity and output losses from policies that distort location decisions, such as restrictive housing policies (Hsieh and Moretti, 2019; Gaubert, 2018; Parkhomenko, 2018). Relative to this literature the main innovation of our paper is an assessment of the indirect effect of (policy induced) spatial distortions on productivity,

[7] Another closely related strand of the literature uses similar conceptual tools, borrowed from the assignment literature, to study how workers rather than firms sort across space (Eeckhout, Pinheiro, and Schmidheiny, 2014; Davis and Dingel, 2014, 2019)

[8] Our setup is, however, sufficiently tractable to be extended to feature trade costs also within countries.

[9] For example, spatial policies that limit agglomeration in a country reduce productivity and entry into exporting, thus hurting foreigners that benefit from fewer and more expensive varieties of consumption goods from that country.

[10] This literature provides some evidence that sorting across space matters for the understanding of the wage distribution. Some papers in this literature use detailed data on workers' characteristics or a fixed effect approach to control for worker heterogeneity and sorting in a reduced form analysis (Combes, Duranton, and Gobillon, 2008; Mion and Naticchioni, 2009; Matano and Naticchioni, 2012). By contrast, we follow Gaubert (2018) use a structural approach to explicitly account for the sorting of firms when measuring agglomeration economies.

output, and welfare via their effect on the gains from international trade.

2 Empirical Evidence

2.1 Data

Our main empirical analysis uses firm-level data from the 2004 Chinese Economic Census of Manufacturing. One important advantage of the Chinese data is that it details the geographical location of the firms at the at the county-level. This allows us to study the sorting of firms and exporters across cities. In addition, we use more aggregate information at the city-level from the United States (at the MSA level) and Brazil for 2012. We use these datasets to confirm the main patterns we derive in this section. To derive industry, and firm-level patterns, we use information for China, which is our main dataset. We begin discussing the Chinese data in detail, and then we turn to describing the main features of the US and Brazilian data.

2.1.1 China

Data for the Chinese Economic Census of Manufacturing is collected by the *National Bureau of Statistics*, and covers the universe of firms in China, irrespective of their size. It contains detailed information on plant characteristics, such as sales, spending on inputs and raw materials, employment, investment, and export value. In the data, the location of firms is defined in terms of the county where the firms' headquarters is based. We argue that this feature most likely plays a minor role in our results, because as Brandt, Van Biesebroeck, and Zhang (2014) shows, over 90 percent of firms in China are single-plant firms.

In our main analysis, we rely on information from the Census of manufacturing to compute measures of city, industry and firm-level export activity. Although we also have access to official exports information from the Chinese Customs Agency, we avoid using it for three reasons. First, customs exports only consider direct exports, while Census exports consider both direct and indirect exports through intermediaries. Second, data from customs provides no information for the location of the exporters, and the data cannot be matched in a straightforward way to the Census of manufacturing, leading to poor matching rates.[11] Finally, when computing export intensity with Customs information, many firms have unreliable export intensities – about 10% of the firms identified as exporters using customs data have export intensities above 100%. Nevertheless, as we show in the online appendix, both export measures are highly correlated across firms. Computing export intensities with customs exports lead to confirmation of the main patterns we derive using the information from the Census.

In our main analysis, we define cities in terms of metropolitan areas defined as contiguous

[11] Firms in the Census and Customs datasets does not share a common identifier. The only way to match both dataset is through fuzzy matching algorithm using firm names. These procedure yields poor matching rates: Only two-thirds of the export value in Customs can be matched to firms in the Census of manufacturing.

areas of lights in nighttime satellite images. We use the correspondence constructed by Dingel, Miscio, and Davis (2019) to map counties into metropolitan areas with a threshold for light intensity equal to 30. This value is in the middle of the set of threshold provided by Dingel et al. (2019). Importantly, our results are not dependent on the particular value chosen for threshold of light intensity.[12] We define city size in terms of urban population obtained from the Chinese Population Census of 2010.This information is available at the county-level, which we aggregate up to the level of metropolitan areas using the correspondences provided by Dingel et al. (2019).

The Census of Manufacturing contains information for approximately 1,272,000 firms with positive output in 2004. However, our main sample only considers firms located in cities with more than 100,000 inhabitants (1,178 metropolitan areas). We drop firms with zero or missing sales (66,887 observations, corresponding to 5.3% of the sample) or industry codes (20,882 observations, 1.7% of the sample), or with export intensity above 100% (6,261 observation, 0.5% of the sample). Our final sample consists of 1,169,258 firms, accounting for 95.7% of sales in cities with more than 100,000 inhabitants.

2.1.2 United States

We now turn to the description of the city-level data for the United States. In the case of the United States, we define cities in terms of Metropolitan Statistical Areas (MSA). MSAs are defined as one or more adjacent counties with at least one urban core area with a population of at least 50,000 inhabitants, with a high degree of social and economic integration with the core, as measured by commuting flows to work and school. As Dingel et al. (2019) show, MSAs are well approximated by cities defined in terms of contiguous areas of lights in nighttime satellite images, as we do in the case of China. Our analysis considers 312 U.S. metropolitan areas with a population over 100,000 inhabitants in 2012.

To develop our main analysis, we combine data from several sources. Data for exports at the MSA level are provided by the International Trade Administration of the U.S. Department of Commerce and include overall exports. We combine this with establishment-level information of sales and employment aggregated at the MSA level from the 2012 Economic Census. In our baseline analysis we use information for the manufacturing sector (NAICS 31-33), which is closer to our theoretical framework. Consequently, city-level export intensity is constructed as overall exports over manufacturing sales. Finally, we use MSA population from the population projections of the U.S. Census Bureau.

[12] A large body of research using information for China defines cities in terms of prefecture-level cities. A prefecture-level city is an integrated political and economic unit, but it often includes rural areas. We avoid defining cities in terms of prefectures, because administrative boundaries may fragment economically integrated areas into distinct cities or circumscribe places, including rural areas. Nevertheless, in the appendix we check the robustness of our findings when using prefecture cities.

2.1.3 Brazil

Finally, for the case of Brazil we consider microregions as the main unit of analysis. Microregions are defined by the Brazilian Institute of Geography and Statistics (IBGE) as urban agglomerations of economically integrated contiguous municipalities with similar geographic and productive characteristics.[13] Although microregions do not directly capture of commuting flows as U.S. Metropolitan Areas, they are constructed according to information on integration of local economies, which is closely related to the notion of local labor markets. Our sample includes 420 microregions with more than 100,000 inhabitants in 2012.

To construct export intensity, we use overall exports – available at the level of municipalities – from the COMEX Stat database (which is compiled by the Brazilian *Ministry of Industry, External Commerce and Services*), and complement it with municipal-level GDP from IBGE. We aggregate both exports and GDP at the level of microregions using the correspondence provided by the IBGE, and compute city-level export intensity as the ratio of overall exports over GDP (across all sectors). Finally, we use population projections from the 2010 population Census.

2.1.4 Summary Statistics

Before turning to our empirical results, we show descriptive statistics for the sample of cities considered in the analysis for China, the United States and Brazil. Table 1 shows statistics for the distribution of population and export intensity for the three samples. Average city size varies importantly across the three datasets. U.S. cities are larger on average (about 800 thousands inhabitants), followed by China (522 thousands) and Brazil (439 thousands). These reflect the fact that population in the U.S. is more concentrated in larger cities. Indeed, as Figure 4 shows, both China and Brazil have a relatively higher density of small cities than the United States.[14] While for the U.S. two-thirds of the cities in our sample have populations over 500 thousands people, in China and Brazil only 16 percent of the cities surpass the 500 thousand people threshold.

In terms of export intensity, the most noticeable difference between the three countries is the prevalence of zeros. In the U.S., all cities have exporting firms; in contrast, in China and Brazil about 10 percent of the cities record no export activity. We argue that the existence of cities with zero exports does not affect the quantitative implications of our results, because these cities represent a small fraction of output (2.5% and 1.5% of the production in China and Brazil, respectively). As with population, the distribution of export-intensity is positively skewed for the three countries in our sample, with the distribution of the U.S. dominating the distributions of China and Brazil.

[13] A number of researchers have used microregions as their main unit of analysis (see Kovak, 2013; Dix-Carneiro and Kovak, 2015, 2017, 2019; Costa, Garred, and Pessoa, 2016; Chauvin, Glaeser, Ma, and Tobio, 2017).

[14] This is consistent with evidence in Au and Henderson (2006), who shows that about half of prefecture-level cities in China are smaller than their optimal size. They argue that this is most likely due to the existence of strong migration restrictions.

Table 1: Descriptive Statistics for City Size and Export Intensity Across Datasets

	Population ('000s)			Export Intensity		
	(1)	(2)	(3)	(4)	(5)	(6)
	China	U.S.	Brazil	China	U.S.	Brazil
Observations	1,178	312	420	1,178	312	420
Mean	522.5	798.8	439.7	.0800	.1131	.0604
25th percentile	149.4	157.5	152.1	.0107	.0445	.0031
50th percentile	215.8	277.5	224.7	.0396	.0853	.0266
75th percentile	364.2	636.6	378.5	.1031	.1370	.0711
90th percentile	692.5	1,929.2	717.2	.2075	.2250	.1629
95th percentile	1,225.6	3,176.1	1,254.8	.3093	.3292	.2460
Cities without exports	—	—	—	116	0	48

Notes: The Table analyzes the relationship between city size and export intensity. Cities are defined in terms of Metropolitan Areas for China (as defined by Dingel et al., 2019, using lights at night with a threshold equal to 30 to define metropolitan areas) and the United States; and in terms of Microregions for the case of Brazil. For all countries, the analysis only considers cities with positive exports and population over 100,000 inhabitants. City-level export intensity is defined as manufacturing exports over manufacturing sales for China; overall exports over manufacturing sales for the United States, and as overall exports over GDP for the case of Brazil.

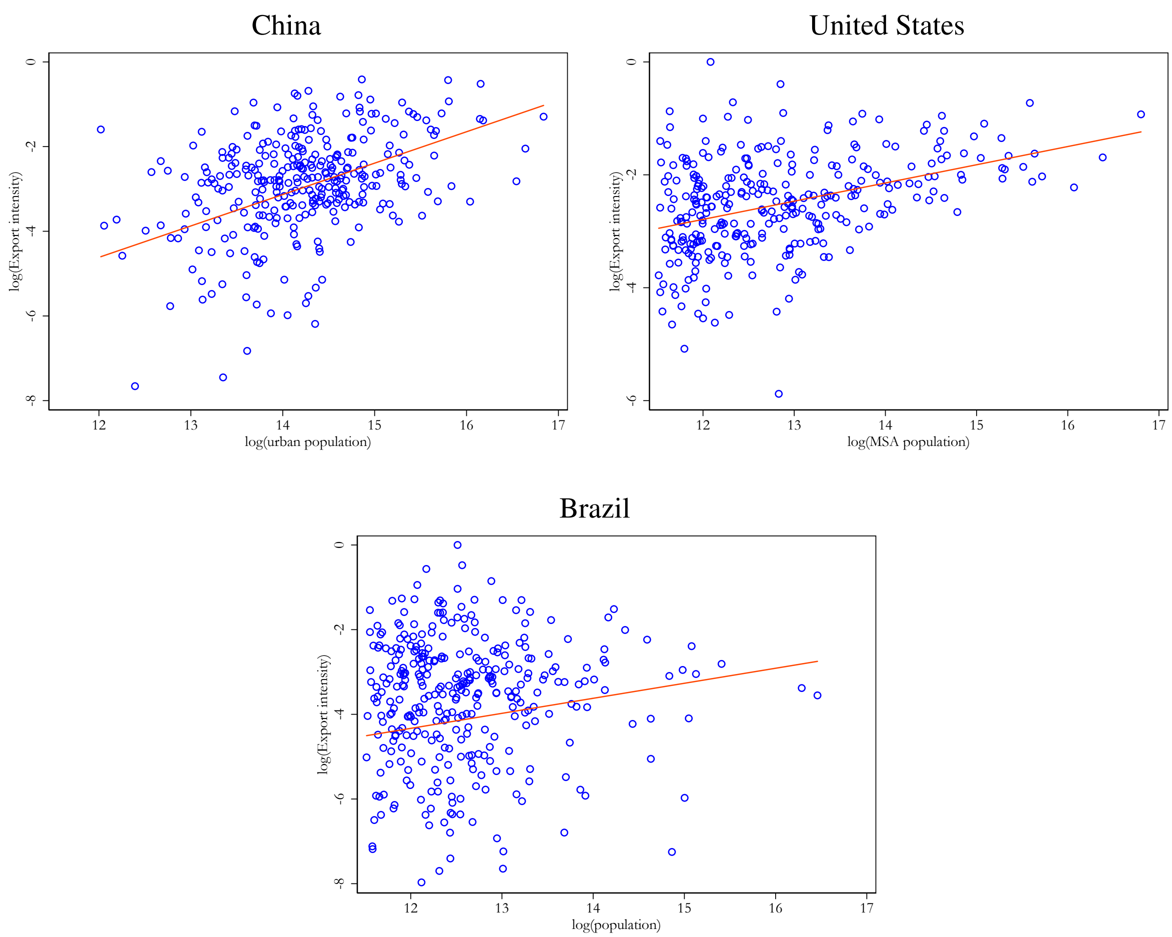
2.2 Stylized Facts

In this section we present our empirical results. We first present results about the distribution of export activity across cities. Next, we show to what extent the city-level results reflect differences in sectoral composition. Finally, we provide evidence on firm-level sorting into exporting and into cities.

2.2.1 Export activity and City Size

Figure 1 presents our main result – the relationship between export intensity and city size. For all three countries, we begin plotting the logarithm of export intensity against the logarithm of city size, measured in terms of cities' population, for cities with more than 100,000 people. The figure shows a remarkable positive relation between export intensity and the size of the cities. The relationship is more precisely estimated for China and the United States than for Brazil. Nevertheless, even for Brazil we estimate a significant positive relationship between export intensity and cities' population. Table 1 shows the point estimates for the elasticity between export intensity and city size. For all countries, the elasticity is highly significant, ranging from 0.32 for the United States and Brazil (column 2 and 3) to 0.45 for China (column 1). Importantly, it remains positive and highly significant when we include geographical controls for distance to the nearest port and a categorical variable for cities located in the coastline (columns 2, 4 and 6).[15] In all these cases, the elasticity varies between 0.30 and 0.41.

Figure 1: Export intensity and City size in China, United States and Brazil



Notes: The figure shows the relationship between city size and export intensity. Cities are defined in terms of metropolitan areas in the cases of China and the United States, and in terms of microregions for the case of Brazil. For all countries, the analysis only considers cities with positive exports and population over 100,000 inhabitants. City-level export intensity is defined as manufacturing exports over manufacturing sales for China; overall exports over manufacturing sales for the United States, and as overall exports over GDP for the case of Brazil.

We implement several tests to check the robustness of our findings, focused on our baseline dataset, China. First, an important body of literature uses prefecture-level Chinese cities as their main unit of analysis (e.g. Au and Henderson, 2006). We show in the appendix that our main findings are qualitatively unchanged; the elasticity we estimate is actually larger in this case (0.73 for

[15]The distance to the nearest port variable is computed as the shortest straight distance from the center of the city to the nearest port.

Table 2: Export intensity and City size in China, United States and Brazil

	China		United States		Brazil	
	(1)	(2)	(3)	(4)	(5)	(6)
log City Size	.447***	.308***	.323***	.305***	.322***	.410***
	(.0476)	(.0434)	(.0351)	(.0366)	(.1188)	(.1442)
Geographical Controls	No	Yes	No	Yes	No	Yes
R^2	.046	.171	.158	.166	.013	.021
Observations	1,062	1,062	312	312	372	372

Notes: The Table analyzes the relationship between city size and export intensity. Cities are defined in terms of Metropolitan Areas for China and the United States; and in terms of Microregions for the case of Brazil. For all countries, the analysis only considers cities with positive exports and population over 100,000 inhabitants. City-level export intensity is defined as manufacturing exports over manufacturing sales for China; overall exports over manufacturing sales for the United States, and as overall exports over GDP for the case of Brazil. Geographical controls include a dummy variable for cities located in coastal areas, and the log of the linear distance between the city center and the nearest port. Robust standard errors in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

the unconditional correlation and 0.38 once geographical controls are included). Second, a distinctive element of China is the existence of Special Economic Zones (SEZ) and Coastal Development Areas (CDA), which are intended to promote exports and overall economic activity in particular areas. We show that our main results are not affected by the inclusion of categorical variables for SEC and CDA cities. Finally, we show that our results are unchanged when we control by the average prevalence of processing trade. In sum, the strong correlation between cities export intensity and city size establishes our first stylized fact:

Stylized Fact 1. *Export intensity increases with city size*

2.2.2 Within- and between-industries variation

To what extent does the positive correlation between export intensity and city size reflect within-industry variation? To address this question, we replicate the analysis of the previous section at the industry-city level. Importantly, we can only perform this analysis for our main dataset, China, because for Brazil and the United States we only have access to aggregate city-level information. For each industry j (at the 4-digit ISIC level), we run versions of the equation:

$$y_{jc} = \alpha_j + \beta_j \log(Population)_c + \gamma X_c + \varepsilon_{jc} \tag{1}$$

where y denote different outcomes for export activity defined at the city-industry level, and X_j corresponds to the set of geographical controls we use in Table 2. We run this equation industry-by-industry, and also pooling industries while allowing for industry fixed-effects. Note that in this last case, the coefficient β_j is restricted to be homogeneous across industries. In all regressions, we only include information for industries located in cities with positive exports and population above 100,000 people (but allow industries to have zero exports in any given city).

Table 3 shows the results when we pool industries and cities. In columns 1-3 we explore the overall variation in export intensity, within and across industries. We avoid applying logarithms to the ratio of exports to output as in the previous sub-section, because the issue of zeros in exports at the industry-city level becomes endemic. For comparability with Table 2, we show the estimated export intensity semi-elasticity using aggregate city-level information (column 1). The estimated coefficient is positive and significant at the 1% level, as in Table 2. Then, in columns 2-3 we show results defining export intensity at the city-industry level. As it can be seen, the estimated coefficient for city size is largely unaffected by the inclusion of industry fixed effects: It varies from .0139 (no industry FE) to .133 (with industry FE). These values are also in the ballpark of the coefficient we estimate with city-level information in column 1. The stability of the coefficient on log city size in columns 1-3 suggests that the positive correlation between export intensity and city size reflect to an important extent, variation occurring within industries.

Table 3: Export Activity and City Size: Pooled Industry-City Level Regressions

	——Export Intensity ——			——$\mathbb{I}$(Exports>0) ——		
	(1)	(2)	(3)	(4)		
log City Size	.00825***	.0139***	.0133***	.0554***	.112***	.118***
	(.00309)	(.000693)	(.000651)	(.00832)	(.00167)	(.00163)
Geographical Controls	yes	yes	yes	yes	yes	yes
Industry FE	no	no	yes	no	no	yes
R^2	.241	.049	.158	.094	.134	.213
Observations	1,178	64,139	64,139	1,178	64,139	64,139

Notes: The Table shows the results of estimating 1 at the city-industry level. Regressions 1-3 uses export intensity as dependent variable. Columns 4-6 uses dependent variable a categorical variable that takes the value one for positive exports. Regressions in columns 1 and 4 are run at the city level, for comparability with results in Table 3. Cities are defined in terms of Metropolitan Areas. The analysis only considers cities with population over 100,000 inhabitants. Geographical controls include a dummy variable for cities located in coastal areas, and the log of the linear distance between the city center and the nearest port. Robust standard errors in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

In columns 4-6 we explore whether is more likely to observe positive industry-level exports in larger cities. For this, we define a categorical variable that takes the value one for industries with strictly positive exports, and use it as the dependent variable in (1). Columns 4-6 of Table 3 show the results. In column 4 we show results for aggregate city-level export intensity. The estimated coefficient suggest that doubling the city size increases the likelihood of positive export in 5.5 percentage points. Then, in columns 5-6 we show results using data aggregated at the city-industry level, with and without fixed effects, respectively. In both cases, the coefficients are positive and statistically significant at the 1% level. As in the case of overall export intensity, the coefficients on the log city size is remarkably stable when we include industry fixed-effects. The point estimates suggest that doubling city size increases the probability of positive export in 11.2-11.8 percentage points.

To check the robustness of the pooled results, we run specification (1) industry-by-industry. Table 4 summarizes the results for the 118 four-digit ISIC industries.[16] Column 1 shows results using export intensity, and column 2 when using a categorical variable for the probability of positive exports. As it can be seen in the first row of Table 4, in both cases the average semi-elasticity is close to the average effect estimated in Table 3. More importantly, the bottom part of Table 4 shows a positive coefficient for practically all industries at different confidence levels. In the case of export intensity, in two-third of the industries export intensity increases with city size at least at the 10% level (column 1). For the case the case of the probability of positive industry-level exports, in over 97% of industries the probability of positive export increases with city size at the 10% level. All this evidence is reassuring for our results in Table 3, and suggests that the patterns we found before are also observed within industries. This leads us to our second stylized fact:

Stylized Fact 2. *Within industries, export intensity increases with city size*

Stylized fact 2 can be interpreted as a refinement to Stylized fact 1. It suggests that the positive elasticity between export intensity and city size we document in section 2.2.1 for China, the United States and Brazil, actually reflect industries becoming more export-oriented in larger cities.

Next, we turn into determining whether the correlation between export intensity and city size can also be accounted for by differences in sectoral composition – i.e., larger cities being more intensive in more export-oriented industries. To answer this question we construct an imputed measure of city-level export intensity. This measure is constructed in the following way. First, we compute for each sector its national-level export intensity. We then impute city-level export intensity by interacting (national-level) industry export intensities with each city's industrial composition. Thus, this counterfactual measure of city-level export intensity solely reflects variation in the sectoral composition of each industry across cities.

Table 5 compares the elasticity between city size and actual export intensity at the city-level (column 1) with the imputed measure where only sectoral composition varies across cities (column 2). As it can be seen in column 2, the counterfactual export intensity is significantly related to city size, suggesting that more export-intensive industries represent a higher share of economic activity

[16] We exclude 5 industries with activity in less than 100 cities out of the total of 1,178 cities in our sample.

Table 4: Export Activity and City Size: Pooled Industry-City Level Regressions

	(1)	(2)
Dependent Variable	Export intensity	Export Probability
Mean	0.0148	0.1152
5th percentile	0.0003	0.0450
10th percentile	0.0014	0.0500
25th percentile	0.0048	0.0772
Median	0.0112	0.1181
75th percentile	0.0213	0.1469
90th percentile	0.0323	0.1797
95th percentile	0.0392	0.1917
% [t-stat>0.000]	95.8	100.0
% [t-stat>1.645]	63.6	97.5
% [t-stat>1.960]	54.2	95.8
% [t-stat>2.326]	45.8	93.2

Notes: The Table shows the results of estimating 1 industry-by-industry. Column 1 uses export intensity as dependent variable, while column 2 a categorical variable that takes the value one for positive exports as dependent variable. The analysis only considers cities with positive exports and population over 100,000 inhabitants. Geographical controls include a dummy variable for cities located in coastal areas, and the log of the linear distance between the city center and the nearest port. Robust standard errors in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

in larger cities. However, the coefficient turns non-significant once we add the set of geographical controls, most likely reflecting the fact that more export oriented industries does not only benefit from larger city size, but also from locating in cites with good access to ports.

Note that the imputed export-intensity measure can also be used to assess the robustness of the correlation between city-level export intensity and city size (stylized facts 1 and 2). For this, we run a horse-race between our agglomeration variable and the imputed city level export intensity measure presented above. The result of this exercise is shown in columns 3 (no controls) and 6 (geographical controls) of Table 5. As it can be seen, the conditional correlation between city-level export intensity and city size goes down by one-fourth (no controls) and one-tenth (with geographical controls) once the imputed city-size measure is considered. Nevertheless, the agglomeration elasticity remains highly significant suggesting that variation at the city-industry level is important for accounting for the relationship between agglomeration and export intensity at the overall city level.

2.2.3 Firm Sorting

To improve our understanding of the drivers of the association between aggregate export intensity and city size, we study exporting behavior at the firm-level. In exploring this relationship, we face

Table 5: Sectoral composition and City-level Export Intensity

Specification:	No Controls			Geographical Controls		
	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var.:	$\log(X/Y)$	$\log(\widehat{X/Y})$	$\log(X/Y)$	$\log(X/Y)$	$\log(\widehat{X/Y})$	$\log(X/Y)$
log(City Emp.)	.447***	.066***	.326**	.308***	.0207	.275***
	(.0476)	(.0175)	(.0415)	(.0437)	(.0155)	(.0403)
$\log(\widehat{X/Y})$	—	—	1.841***	—	—	1.587***
			(.0918)			(.0970)
Controls	—	—	—	✓	✓	✓
Observations	1,062	1,062	1,062	1,062	1,062	1,062
R^2	.046	0.013	.307	.171	.179	.332

Notes: The Table studies the extent to which sectoral composition account could account for the positive relation between export intensity and city size. (X/Y) are city-level export intensity (total exports over sales), and $(\widehat{X/Y})$ is a counterfactual measure of city-level export intensity that holds fix city-industry export intensity at the national average for each industry (i.e, across all cities with positive production in each industry). The sample includes all Chinese metropolitan areas with positive exports and population over 100,000 people. Geographical controls include a dummy variable for cities located in coastal areas, and the log of the linear distance between the city center and the nearest port. Robust standard errors in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

an important challenge related to the composition of our data. Small firms (less than 25 employees) dominate the Chinese Census of Manufacturing: They account for over 50 percent of firms in all city sizes, and within small firms about half of them have less than 10 employees. At the same time, small firms account only for 6 percent of the aggregate production value, and less than 2% of them are exporters. This is in stark contrast with export activity among medium- and large -sized firms, where over 20 percent of the firms are exporters.[17] The dominance of small firms in our sample will most likely tend to dilute the coefficient between export activity and city size in a simple unweigthed regression, because they are distributed more or less homogeneously across all city sizes and have a low unconditional export probability. In the following, we address this issue reporting results with firms weighted by their share of city-level sales.[18] In this way, we aim to identify the coefficient on city size by the set of firms where the forces of agglomeration could most likely induce selection into exporting. Nevertheless, in light of the limitations imposed by our data, our results should be interpreted as an exploratory analysis.[19]

[17]This is consistent with a large literature shows that larger, more productive firms, sort into exporting (e.g. Melitz, 2003). See Bernard et al. (2007) for evidence for the United States. As a reference, almost 5 percent of plants with less than 25 employees in Chile are exporters.

[18]We could also weight observations by firms' sales. This alternative, however, would implicitly give a higher weight to larger cities.

[19]A second limitation of our analysis is related to the endogeneity of firm's location. To alleviate this concern, we control for the logarithm of total factor productivity: We expect more productive firms to have a higher probability of

Table 6 shows the main firm-level results. Columns 1–2 use firm-level export intensity as dependent variable, while columns 3–4 use a categorical export variable as dependent variable. In all regressions, we include geographical controls and industry fixed-effects (at the 2-digit level), and cluster standard errors at the city-level.

We begin discussing results for overall export intensity. Column 1 shows a positive correlation for the coefficient on city size. This suggests that export intensity tend to be higher for firms that locate in larger cities: Doubling the city size leads to an export intensity 0.8 percentage points higher. Then, column 2 includes firm-level productivity as a control.[20] This allows us to assess the role played by firm level productivity in mediating the relationship between city size and export intensity. Our results indicate that firm productivity indeed plays a role, as the TFP control is positive and highly significant, and reduce in about one-third the magnitude of the city scale measure. Indeed, once we control for TFP, the coefficient on city size is only significant at the 10 percent level.

Table 6: Export Activity and City Size: Firm-Level Regressions

	—Export Intensity —		—$\mathbb{I}$(Exports>0) —	
	(1)	(2)	(3)	(4)
log City Size	.00737*** (.00265)	.00463* (.00271)	.0664*** (.00620)	.0537*** (.00591)
log TFP	—	.0195*** (.00205)	—	.0900*** (.00541)
Geographical Controls	yes	yes	yes	yes
Industry FE	yes	yes	yes	yes
R^2	.153	.159	.107	.144
Observations	1,035,046	1,035,046	1,035,046	1,035,046

Notes: The Table shows the results of estimating 1 at the firm-level. Regressions 1-2 uses export intensity as dependent variable. Columns 3-4 uses dependent variable a categorical variable that takes the value one for positive exports. All regressions are weighted by the sales share of each firm in city-level sales. Cities are defined in terms of Metropolitan Areas. The analysis only considers cities with population over 100,000 inhabitants. Geographical controls include a dummy variable for cities located in coastal areas, and the log of the linear distance between the city center and the nearest port. Standard errors are clustered at the city level. Key: *** significant at 1%; ** 5%; * 10%.

locating in larger cities In order to accurately pinning-down the relationship between export activity and city size, we would need a source of exogenous variations.

[20]For computing firm-level total factor productivity, we estimate a value added production function for each 2-digit industry using the proxy-function method proposed by Ackerberg, Caves, and Frazer (2015). Given that this methodology is dynamic, we only use information for the subset of firms in the Census of Manufacturing available in the Annual Survey of Manufacturing from 2004 to 2007. We proxy for unobserved using materials' expenditure, and correct for non-random exit.

Results in columns 1–2 of Table 6 suggests that the intensive export margin is relatively weak in explaining the positive correlation between city-level export activity and city size. Next, we explore if the extensive margin of exporting could account for the higher export intensity of large cities. In particular, we replicate the regressions in columns 1–2 using as the dependent variable an export dummy for firms with positive exports. Columns 3–4 of Table 6 show the results. In both columns, city size is positive and statistically significant at the 1 percent level. This suggests that exporting is more likely for firms located in larger cities. We stress that this results can only be interpreted as a correlation, because as our model in next section suggests, firm location is endogenous.[21] Next, in column 4 we include log productivity as a control. As in the case of export intensity (column 2), the coefficient on firm-level productivity is positive and significant, suggesting that more productive firms are more likely to be exporters. At the same time, the coefficient on city size stays positive and significant, experiencing a modest drop (less than 20 percent) in its magnitude once we control for firm productivity. Taken together, these results suggest that the higher within-industry export intensity of large cities is driven by a higher export participation of firms in large cities. We summarize this in the following stylized fact:

Stylized Fact 3. *Within-industries, there is a positive – although statistically small– relationship between export intensity and city size. On the other hand, the extensive margin of firms export participation is relatively strong: Export participation is more likely to occur in larger cities.*

These suggestive findings provide partial justification for our theoretical framework, that emphasizes the role of firm level productivity, which is typically high in large cities due to sorting and agglomeration mechanisms, in explaining the correlation between city size and aggregate (city-level) export intensity. We turn to the presentation of our model in the next section.

3 Model

In this section we present a model of sorting and agglomeration of firms across cities, together with selection into exporting, that can account for the stylized facts documented in the previous section. The model combines a multi-country version of the firm location model of Gaubert (2018) with a standard mechanism of selection into exporting as in Melitz (2003).

3.1 A Multi-Country Gaubert (2018) Model

In what follows we describe the setup of the Gaubert (2018) model, which we extend to a multi-country setting to make it amenable to our analysis of the economic geography of exporting. Consider a world economy featuring C symmetric countries. Each country has an endowment N of workers and contains a continuum of potential city sites that are ex-ante identical. Each site has

[21] Conversely, the coefficient could also be reflecting the fact that exporting leads to efficiency gains (see Garcia-Marin and Voigtländer, 2019, for establishment-level evidence for Chile, Colombia and Mexico).

a given stock of land normalized to one. Cities with different population levels L may emerge endogenously on these sites. Crucially, workers are assumed to be perfectly mobile across cities within countries, but immobile internationally.

Within these countries production takes place in cities in an arbitrary number of sectors, denoted by S. In each country and sector, production is undertaken by heterogeneous firms which produce differentiated varieties in cities making use of local labor. Land scarcity in cities gives rise to congestion but cities are also the locus of non-market interactions that generate positive agglomeration economies. Moreover, these agglomeration effects are assumed to be heterogeneous across firms, with more efficient firms benefiting disproportionately from local agglomeration forces. Like workers, firms are also assumed to be mobile within countries but immobile internationally.

Economic geography is primarily driven by the location choices of firms. When choosing which city to locate in, firms trade off the strength of local productivity externalities, the local level of input prices, and the generosity of any local subsidies. Firms can ship their goods costlessly within their home country but need to pay trade costs when shipping internationally. Moreover, all locations within each country have symmetric access to foreign markets. Heterogeneous firms face different incentives which leads them to make different choices regarding location and export status.

Following Gaubert (2018), we posit that, within countries, each potential city site is administered by a city developer who represents local landowners and competes against other sites to attract firms. These developers play a coordination role in the creation of cities, leading to a unique equilibrium. In what follows we fix a country and describe the rest of the model's setup from the perspective of one "home" country. Given that all other countries are symmetric, the setup would look identical from other countries' perspectives.

With the setup described above, city size is sufficient to characterize all the key economic forces at play at the local level. In particular, the distance between two cities plays no role in the model because goods produced in the economy are freely traded within the country, all cities have by assumption equal access to foreign markets, and housing (the only other good in the economy) is non-tradable. Consequently, in what follows we index all relevant city-level parameters by city size L. We now proceed to describe in greater detail the optimization problems faced by the key agents in the model, namely by workers, housebuilders, firms, and city developers.

Preferences Workers live in a city of their choice within their home country, and consume a bundle of goods and housing while being paid the applicable local wage $w(L)$. Crucially, as described in detail below, the wage earned by workers depends on the size of the city chosen as a residence. Workers' preferences are characterized by the utility function:

$$U = \left(\frac{c}{\eta}\right)^{\eta} \left(\frac{h}{1-\eta}\right)^{1-\eta} \tag{2}$$

where h denotes housing and c is a Cobb-Douglas composite of tradable goods across the S sectors of the economy

$$c = \prod_{j=1}^{j=S} c_j^{\xi_j} \text{ with } \sum_{j=1}^{j=S} \xi_j = 1 \tag{3}$$

Moreover, within each sector $j \in \{1, \ldots, S\}$ consumers choose varieties according to the CES aggregator:

$$c_j = \left[\int c_j(i)^{\frac{\sigma_j - 1}{\sigma_j}} di\right]^{\frac{\sigma_j}{\sigma_j - 1}} \tag{4}$$

Housebuilding In each city, housing is built by atomistic local landowners by combining land with local labor according to the technology:

$$h^S = \gamma^b \left(\frac{l}{1-b}\right)^{1-b} \tag{5}$$

where h^S denotes housing supply, γ denotes land, and b denotes share of the cost of producing housing attributable to land. Both land and housing markets are assumed to be perfectly competitive at the local level, and landlords take the local wage level $w(L)$ as given.

Production Within each country and sector, firms produce differentiated tradable varieties using labor. Firms differ exogenously in their "raw"efficiency z. For a firm of efficiency z in sector j and city of size L the production technology is given by

$$y_j(z, L) = \psi(z, L, s_j)l \tag{6}$$

where l denotes labor inputs and $\psi(z, L, s_j)$ is a firm-specific productivity shifter.The productivity of a firm $\psi(z, L, s_j)$ increases with its own 'raw'efficiency z and with local agglomeration externalities that depend on city size L. The productivity function is also indexed by a sector-specific parameter s_j, with sectors that benefit from stronger agglomeration economies for each city size being assigned higher values of this parameter. Moreover, the key assumption of the Gaubert (2018) model, which we also adopt, is that the productivity of a firm $\psi(z, L, s_j)$ exhibits a strong complementarity between local externalities and the 'raw'efficiency of the firm. More precisely, we assume that $\psi(z, L, s)$ is twice differentiable, log-supermodular in city size L, firm raw efficiency z and sectoral characteristic s and strictly log-supermodular in (z, L)[22]. That is,

$$\frac{\partial^2 \log \psi(z, L, s)}{\partial L \partial z} > 0 \quad ; \qquad \frac{\partial^2 \log \psi(z, L, s)}{\partial L \partial s} \geq 0 \quad ; \qquad \frac{\partial^2 \log \psi(z, L, s)}{\partial z \partial s} \geq 0$$

Following Gaubert (2018), we also assume that agglomeration externalities have decreasing elasticity to city size. Given that the congestion forces increase with city size with a constant elasticity, this guarantees that the firm's problem is well defined and concave for all firms, absent any local subsidies. Intuitively, we require that the positive effects of agglomeration externalities are not too strong compared to the congestion forces to preclude a degenerate outcome with complete agglomeration of all firms in the largest city of each country.

Firms engage in monopolistic competition and aim to maximize profits by their choice of location and pricing. In doing so, they take the sectoral price index (which by symmetry is the same across countries) as given. Moreover, there is an infinite supply of potential entrants in each country and sector. Firms pay a sunk cost f_j in terms of the final good in order to enter. They then draw a raw productivity level z from a distribution given by $F_j(.)$. Once firms discover their raw efficiency they choose the size of the city where they want to produce and whether they want to export to other countries.

City developers Within countries, each potential city site is administered by a city developer. Each city developer i announces a city size L and competes with other city developers to attract firms to their city by subsidizing firms'operational profits (understood to mean total revenues minus variable costs of production or profits gross of any fixed production costs). Thus city developers also announce the level of subsidies to local firms'operational profits in sector j, that may vary with firm type z, $T^i_j(L, z)$. Developers are funded by fully taxing the profits made by landlords on the housing market. City developers are therefore the residual claimants on local land value and their objective is to maximize land rents net of the cost the of policies they put in place to maximize local land value[23]. There is perfect competition and free entry among city developers, which drives their profits to zero in equilibrium.

[22]This set of assumptions is denoted as Assumption A in Gaubert (2018).

[23]As is standard in the literature (e.g. Henderson 1974), the role of these developers is to solve a coordination failure: atomistic agents such as firms, workers or landowners alone cannot create new cities. This results in multiple equilibria in which cities of suboptimal size persist due to the failure of atomistic to coordinate on creating new cities. City developers are, in contrast, large players at the city level and act as a coordinating device that allows a unique equilibrium to emerge in terms of city-size distribution.

3.2 International Trade Costs and Selection Into Exporting

To complete the link between the multi-country version of the Gaubert (2018) model presented in the previous section and the analysis of the economic geography of exporting that we aim to undertake, we now specify the international trade frictions faced by firms that aim to ship their goods internationally. To export to other countries firms need to pay a sector-specific fixed export cost f_j^e in terms of the final good for each foreign country it wants to export to, and their exports are also subject to iceberg transportation costs τ. Importantly, these costs are symmetric for all locations within the source country (i.e. a firm locating in any city in the source country will face the same international trade costs) and across all destination countries (the same trade costs apply to all country pairs). This setup yields a standard mechanism of selection into exporting, with firms above a certain sector specific threshold of "realized"productivity $\underline{\psi_j}$ selecting to export, while firms below that threshold remain domestic. Moreover, given the symmetry of the problem, firms will either find it optimal to be purely domestic or to export to all countries (if it is profitable for a firm to export to one country, it is profitable to export to all countries).

With the above setup in place we now proceed to describe the key spatial equilibrium conditions, those characterizing workers and firms.

Spatial Equilibrium: Workers and Firms We begin our discussion of the key spatial equilibrium conditions with an analysis of workers. Denoting by P the aggregate price index for the composite tradable good in the home country, and by $c(L)$ and $h(L)$ the consumption of the tradable composite good and housing, respectively, for a worker residing in a city of size L, we can write the budget constraint for such a worker as:

$$Pc(L) + p_H(L)h(L) = w(L).$$

Since goods are freely tradable within countries, all cities have symmetric access to foreign markets, and countries are symmetric, the price indices denoted by P are the same across all cities in all countries. Moreover, given the housebuilding technology given in equation (5) and the housing market clearing condition, the quantity of housing consumed in equilibrium by each worker in a city of size L is given by:

$$h(L) = (1-\eta)^{1-b}L^{-b} \tag{7}$$

Intuitively, housing consumption is lower in more populous cities because cities are land constrained. This yields a congestion force that counterbalances the positive production externalities that occur in cities and thus precludes the complete agglomeration of each country's economy into only one city.

The free mobility of workers and the symmetry of countries guarantees that in equilibrium worker utility must be equalized across all inhabited locations in all countries. We denote this common level of utility $\bar{U}$. As a result, wages must increase with city size to compensate workers for the higher cost of housing in these locations:

$$w(L) = \bar{w}\left((1-\eta)\, L\right)^{b\frac{1-\eta}{\eta}} \tag{8}$$

where following Gaubert (2018) $\bar{w} = \bar{U}^{\frac{1}{\eta}} P$ denotes a country-wide constant that is determined in general equilibrium. However, this constant will be the same in all countries due to symmetry.

We can now proceed to characterize the spatial equilibrium condition for firms, whose location choices are the main driver of economic geography in the model. Firms choose city size based on three factors. First, the price of the labor varies by city size. Second, firm productivity increases with city size, as a result of stronger agglomeration externalities. Third, the firms stand to benefit from subsidies to operational profits (profits gross of any fixed exporting costs paid) offered by local city developers. The firm's problem can thus be solved recursively. For a given city size, the problem of the firm is to hire labor and set prices to maximize profits, taking as given the size of the city (and hence the size of the externality term), input prices, and subsidies. Then, firms choose location to maximize this optimized profit. When maximizing profits, firms treat local productivity as exogenous, so that the agglomeration economies take the form of external economies of scale.

Consider a firm of efficiency z producing in sector j and in a city of size L. Denoting by P_j the price index in sector j (which again by symmetry will be the same in all countries) and given CES preferences, firms face demand curves of the type:

$$c_j(i) = \left(\frac{p_j(i)}{P_j}\right)^{-\sigma_j} c_j \tag{9}$$

which can be rewritten:

$$c_j(i) = p_j(i)^{-\sigma_j} P_j^{\sigma_j - 1} E_j \tag{10}$$

Where E_j represents total expenditure in sector j in the (home) country (by symmetry this will be the same in all countries). Given monopolistic competition, firms set constant mark-ups over marginal costs yielding profits before subsidies on the domestic market:

$$\pi_j^D(z, L) = \frac{1}{\sigma_j^{\sigma_j}} (\sigma_j - 1)^{\sigma_j - 1} \left[\frac{\psi(z, L, s_j)}{w(L)}\right]^{\sigma_j - 1} E_j P_j^{\sigma_j - 1} \tag{11}$$

Moreover, for each foreign country c', a firm may make profits from exporting given by the expression

$$\pi_j^{Exp\ c'}(z, L) = \begin{cases} \frac{\pi_j^D(z,L)}{\tau^{\sigma_j - 1}} - P f_j^e & \text{if } \frac{\pi_j^D(z,L)}{\tau^{\sigma_j - 1}} >= P f_j^e \\ 0 & \text{if } \frac{\pi_j^D(z,L)}{\tau^{\sigma_j - 1}} < P f_j^e \end{cases} \tag{12}$$

Given that in equilibrium each firm will either export to no foreign countries or to all foreign countries, a firm's total profits from exporting will be given by

$$\pi_j^{Exp}(z,L) = \begin{cases} \frac{(C-1)\pi_j^D(z,L)}{\tau^{\sigma_j-1}} - (C-1)Pf_j^e & \text{if } \frac{\pi_j^D(z,L)}{\tau^{\sigma_j-1}} >= Pf_j^e \\ 0 & \text{if } \frac{\pi_j^D(z,L)}{\tau^{\sigma_j-1}} < Pf_j^e \end{cases} \tag{13}$$

It is straightforward to show that domestic profits given by (11) are increasing in z when holding L constant. As a result, for each sector and city size there may exist a $z_j^*(L)$ such that a firm remains domestic if $z < z_j^*(L)$ and exports to all countries if $z \geq z_j^*(L)$[24]. As a result we can write a firm's operational profits as

$$\pi_j^o(z,L) = \begin{cases} \pi_j^D(z,L) & \text{if } z < z_j^*(L) \\ \left[1 + \frac{(C-1)}{\tau^{\sigma_j-1}}\right]\pi_j^D(z,L) & \text{if } z \geq z_j^*(L) \end{cases} \tag{14}$$

While a firm's total profits before subsidies are given by

$$\pi_j^T(z,L) = \begin{cases} \pi_j^D(z,L) & \text{if } z < z_j^*(L) \\ \left[1 + \frac{(C-1)}{\tau^{\sigma_j-1}}\right]\pi_j^D(z,L) - (C-1)Pf_j^e & \text{if } z \geq z_j^*(L) \end{cases} \tag{15}$$

Finally, firms receive subsidies to operational profits (profits gross of any fixed costs of exporting paid) from the city developers, which yields total profits after subsidies

$$\pi_{Sub,j}^T(z,L) = \begin{cases} (1 + T_j(z,L))\,\pi_j^D(z,L) & \text{if } z < z_j^*(L) \\ (1 + T_j(z,L))\left[1 + \frac{(C-1)}{\tau^{\sigma_j-1}}\right]\pi_j^D(z,L) - (C-1)Pf_j^e & \text{if } z \geq z_j^*(L) \end{cases} \tag{16}$$

The problem of the firm thus is to choose the city size L to maximize (16).

3.3 Equilibrium Existence, Uniqueness and Stability

With the setup outlined in the previous two sections, we can define a spatial equilibrium of the world economy as follows:

Definition 1. *An equilibrium is, for each country, a set of cities $\mathcal{L}$ characterized by a city-size distribution $f_L(.)$, a wage schedule $w(L)$, a housing-price schedule $p_H(L)$ and for each sector $j = 1, ..., S$ a location function $L_j(z)$, an employment function $l_j(z)$, a production function $y_j(z)$, a price index P_j and a mass of firms M_j such that:*

[24] This $z_j^*(L)$ satisfies the condition $\pi_j^D(z_j^*(L), L) = Pf_j^e$. If for a certain sector j and city size L such a $z_j^*(L)$ does not exist, it means that in that sector and at that size level we either have that firms of all productivities would be domestic, or firms irrespective of productivity would be exporters. In this case the relevant expressions for profits would prevail.

1. *workers maximize utility given $w(L)$,$p_H(L)$ and P_j,*

2. *utility is equalized across all inhabited cities,*

3. *firms maximize profits given $w(L)$ and P_j, and choose whether to participate in export markets,*

4. *landowners maximize profits given $w(L)$ and $p_H(L)$*

5. *city developers choose $T_j(L, z)$ to maximize profits given $w(L)$ and the firm problem,*

6. *labor, goods and housing markets clear; in particular, the labor market clears in each city,*

7. *firms and city developers earn zero profits.*

Building on the work of Gaubert (2018) it is possible to show that the there exists an unique equilibrium of the model (proofs are relegated to Appendix A). Moreover this equilibrium is stable.[25] Intuitively, our assumptions guarantee that, within each sector and country, for each firm type there exists a unique optimal city size that maximizes profits. Moreover, due to the assumed complementarity between intrinsic productivity z and city size, the optimal city size is increasing in the firm's intrinsic productivity. The presence of competitive city developers ensures that, within countries, the optimal city size of each firm type and sector, is provided in equilibrium. As a result, the assignment of firms to city sizes can be uniquely pinned down in equilibrium for all countries and sectors, which in turn uniquely pins down the realized productivity of all firms. This in turn allows us to recover the values of general equilibrium quantities: total expenditure for each country, the mass of firms by sector in each country, the sectoral price indices in each country and sector, the export productivity threshold in each country and sector. Finally, the mass (or "number") of cities of each type endogenously adjusts in equilibrium such that labor markets clear.

The equilibrium is unique in terms of distribution of outcomes within countries, such as firm-size distribution, city-size distribution and matching functions between firms and city sizes within countries. It is not unique in terms of which site is occupied by a city of a given size, as all sites are identical ex ante.

4 Equilibrium Properties: Matching the Stylized Facts

In this section we highlight the main characteristics of the equilibrium, with a focus on describing how the model matches the stylized facts we've documented above. To set the stage for presenting our main results, it is helpful to note that as in Gaubert (2018), the equilibrium is characterized by

[25] The equilibrium is said to be stable if no deviation of any small mass of individuals or firms from a given city to another city or empty site enhances their utility. This definition of stability is commonly used in the literature (see Behrens et al. (2014) for example).

strict ranking of firms in terms of productivity, profits and revenues vis a vis city size. We restate this result, already present in Gaubert (2018), more formally in the lemma below

Lemma 1. *In equilibrium, within each country and sector, (average) firm revenues, profits and productivity increase with city size in the following sense. For any* $L_H, L_L \in \mathcal{L}$ *such that* $L_H > L_L$*, take* z_H *such that* $L_j^*(z_H) = L_H$ *and* $L_j^*(z_L) = L_L$*. Then* $r^*(z_H) > r^*(z_L), \pi^*(z_H) > \pi^*(z_L), \psi^*(z_H) > \psi^*(z_L)$*.*

These strong predictions are a direct consequence of the perfect sorting of firms, which naturally yields a ranking of firm productivity with respect to city size. In turn this productivity ranking is reflected in an identical ranking in terms of firm profits and firm size by revenues (as the mapping from firm productivity to revenues and profits is a monotonic bijection in equilibrium). Notably, Lemma 1 is silent on the association between employment and city size. This is because the relationship between (average) firm employment and city size is ambiguous: firm employment can be either positively or negatively associated with city size due to the effect of wages. More precisely within a sector, it is straightforward to see that $l^*(z) \propto r^*(z)/w(L^*(z))$, where both firm revenues and wages increase with city size. Firms may thus have lower employment in larger cities, even though they are more productive and profitable.

We now proceed to describe the properties of the equilibrium concerning the distribution of exporting activity across space. These properties speak directly to the stylized facts we have documented and are described in the following proposition:

Proposition 1. *In equilibrium, within each country and sector, (average) firm exports and export intensity (i.e. exports/sales) weakly increase with city size in the following sense. For any* $L_H, L_L \in \mathcal{L}$ *such that* $L_H > L_L$*, take* z_H *such that* $L_j^*(z_H) = L_H$ *and* $L_j^*(z_L) = L_L$*. Then* $Exp^*(z_H) \geq Exp^*(z_L), Expint^*(z_H) \geq Expint^*(z_L)$*.*

Corollary 1. *Across city*sector cells, export intensity weakly increases with city size.*

Corollary 2. *If two cities have similar sectoral compositions, the larger one will feature weakly larger overall export intensity.*

As intrinsically more productive firms sort into bigger cities, they become even more productive as they benefit from agglomeration economies. This in turn means that firms in larger cities are more likely to jump over the “Melitz barrier”and engage in exporting. This produces a positive correlation between export intensity within sectors and city size. One feature of the model is important to note at this stage: within sectors, larger cities only export strictly more than smaller cities in the case of a pair of cities that are “on the opposite sides of the sector specific exporting threshold z_j^*. Above and below the exporting threshold export intensities for a given sector are constant with city size - export intensity is zero for all cities hosting firms with intrinsic productivity $z < z_j^*$ and given by $\frac{C-1}{\tau^{\sigma_j - 1}}$ for all cities hosting firms with intrinsic productivity $z > z_j^*$. This result is an artefact of the perfect sorting predicted by the model, with each city having a degenerate firm productivity (and hence firm size) distribution within sectors. An extension of the model to allow for imperfect sorting would predict a smooth, monotonically increasing relationship between export intensity and city size.[26].

Aggregating the exporting result from the firm level to city*sector cell level (Corollary 1) is trivial, given that in the model each city size bin only hosts a single type of firm, so city*sector cells preserve all the properties of the unique firm size that they host. For the proof of corollary 2 note that export intensity at the city level is given by:

$$Expint_c = \frac{Exports_c}{Output_c} = \frac{\sum_{j=1}^{S} Exports_{cj}}{Output_c} = \frac{\sum_{j=1}^{S} Expint_{cj} Output_{cj}}{Output_c} \tag{17}$$

Which can be rewritten as

$$Expint_c = \sum_{j=1}^{S} Expint_{cj} \frac{Output_{cj}}{Output_c} \tag{18}$$

In the last equation, if the sectoral shares $\frac{Output_{cj}}{Output_c}$ (i.e. the sectoral composition) are identical for two cities of different sizes, then the relative export intensity of the two cities will be driven by the within sector export intensity terms (i.e. the $Expint_{cj}$ terms), which the main proposition has shown to be weakly higher in larger cities.

It is important to note that the results outlined in Proposition 1 do not depend on our assumptions regarding the presence of city developers. While the presence of city developers ensures the uniqueness of equilibria, the properties outlined in Proposition 1 would apply to any equilibrium (in other words, in the absence of city developers the model will have multiple equilibria, but all equilibria will satisfy the properties outlined in Proposition 1).

Finally, as in Gaubert (2018), the model is able to account for Zipf's law for cities, which posits that the city size distribution follows a power law (more precisely a Pareto distribution with exponent -1). This feature of the model is captured in the next proposition:

Proposition 2. *If the firm size distribution in domestic revenues within countries follows Zipf's law, a sufficient condition for the upper tail of the city size distribution to follow Zipf's law is that domestic revenues increase with constant elasticity with respect to city size in equilibrium.*

[26] Indeed, in the quantitative section of the paper, we present a stochastic extension of the model that allows for imperfect sorting of firms across cities of different sizes. In this extended model the results on exporting are stronger. If we allow firm productivity to be given by a deterministic component given by $\psi(.,.,.)$ and stochastic multiplicative shock distributed independently of city size, then we obtain the result that average export intensity strictly increase with city size, at least beyond a certain city size threshold.

5 Welfare Analysis

The competitive equilibrium derived in section 3.3 can be shown to be inefficient, as firms tend to locate in cities that are too small. The intuition for this result is as follows. The social marginal benefit of choosing a larger city is higher than the private benefit perceived by firms through their profit function. There are two related benefits of choosing a larger city that are not fully internalized by firms: (1) first, choosing a larger city increases the productivity of the economy which lowers the entry cost of firms into a sector (Pf_j); (2) second, the same productivity effect of choosing larger cities lowers the entry cost into exporting (Pf_j^e). The latter is a new effect that appears in our open-economy, multi-country model and was absent in existing work. Fostering entry and entry into exporting increases welfare, by the love of variety effect. Firms ignore the effect of their choice of city size on the cost of entry and the cost of entry into exporting, and therefore choose cities that are too small compared to the social optimum. This general equilibrium cross-city and cross-country effect is not internalized by firms nor by city developers who, despite being large local players, are still atomistic at the national and international levels.

The model is also amenable to the welfare analysis of interesting policy experiments such as place based policies or trade liberalizations. In particular, it can be shown that existing urban economics models may understate the benefits of a multilateral policy aimed at increasing housing supply elasticities as they ignore the indirect effects operating via increased gains from trade. Similarly, it can be shown that existing trade models that ignore economic geography considerations may overstate the gains from trade liberalization as they overestimate the extent of factor reallocation to more productive firms brought about by trade liberalization. Intuitively, more productive firms tend to locate in larger cities and face higher wages, and thus will tend to expand less in response to trade liberalization than in standard trade models where wage differences across space are omitted from the analysis.

6 Trade and Economic Geography Implications

One of the key features of the the model is that allows us to study the joint determination of international trade and economic geography. In this section we briefly outline some of the comparative static properties of the model and highlight how the model allows us to study the impact of geographic policy (i.e. housing supply restrictions) on international trade (and exporting activity in particularly) and, conversely, the impact of trade policy on within country economic geography.

We begin with exploring the implications of the model concerning the impact of geographic policies, such as housing supply restrictions on international trade.

Proposition 3. *Weakening housing supply restrictions (i.e. lowering b) increases the export intensity of the economy in all sectors.*

We begin by deriving an expression for aggregate exports and aggregate export intensity in a sector j:

$$Exp_{agg,j} = \sigma_j k_{1j} E_j P_j^{\sigma_j - 1} M_j \frac{C-1}{\tau^{\sigma_j-1}} \int_{z_j^*}^{z_{j,max}} \left[\frac{\psi(z, L_j^*(z), s_j)}{w(L_j^*(z))} \right]^{\sigma_j - 1} \tag{19}$$

$$E_j = \sigma_j k_{1j} E_j P_j^{\sigma_j - 1} M_j S_j(z_j^*) \tag{20}$$

Dividing (19) by (20) yields an expression for (national) export intensity in sector j:

$$Expint_j = \frac{Exp_{agg,j}}{E_j} = \frac{\frac{C-1}{\tau^{\sigma_j-1}} \int_{z_j^*}^{z_{j,max}} \left[\frac{\psi(z, L_j^*(z), s_j)}{w(L_j^*(z))} \right]^{\sigma_j - 1}}{S_j(z_j^*)} \tag{21}$$

Differentiating the last equation with respect to b yields

$$\left(\frac{\partial Expint_j}{\partial b} \right) = \underbrace{\left(\frac{\partial Expint_j}{\partial z_j^*} \right)}_{<0} \underbrace{\left(\frac{\partial z_j^*}{\partial b} \right)}_{>0} < 0 \tag{22}$$

where the sign of the first bracket on the RHS can be shown easily by direct differentiation, whereas the sign of the second can be established by applying the implicit function theorem to equation (A.16).

Intuitively a lowering of housing supply restrictions increases export intensity by lowering the intrinsic export productivity thresholds in all sectors, thus causing a higher fraction of firms to export. As housing supply elasticity is increased the wage gradient in city size becomes flatter and all firms locate in larger cities and become more productive. As a result the, within sectors the firm profit distribution shifts to the right, which means that keeping the fixed cost of exporting and price levels constant, more firms jump over the Melitz barrier associated with exporting. However, the price level does not remain constant: As productivity increases, all firms cut their prices thus lowering the price index in all sectors and hence the aggregate price index. Moreover, increased profits trigger both more entry and more entry into exporting which both cause a reduction of the overall price index. As a result the fixed cost of exporting declines, further increasing the fraction of firms that export and further increasing export intensity.

Finally, the model makes predictions about the economic geography implications of reduced international trade frictions. These implications are complex so we provide a characterization of a special case below.

Proposition 4. *Consider a very small city as a city containing no exporters in any sector, both before and after a change in transportation costs, and a very large city a city in which firms in all sectors are exporters. Then a reduction in transportation costs τ (assumed as in the main model* *to be identical across sectors) has the following implications*

1. *The relative mass (or "number") of very large cities relative to very small cities increases. This happens in the following sense. Take two city sizes $L_S < L_B$ with L_S sufficiently small that no firms in any sector export either before or after the change in international transport costs. Then we have that $\partial\tau\left(\frac{f_L(L_B)}{f_L(L_S)}\right) > 0$.*

2. *City sizes associated with firms in any sector that experience changes in export status experience a discrete jump in their densities.*

It can easily be seen from equation (A.8) that reducing transportation costs has no impact on the matching function between firms and cities in any sector, and hence no impact on realized firm productivities. Under the assumption of the presence of city developers this in turn implies that the support of the city size distribution does not change when trade costs change.

What reducing transportation costs does is increase the size of exporters relative to non-exporters, and also it reduces the exporting productivity threshold in all sectors. In response, the mass of cities accommodating the workers of these firms needs to change for labor markets to clear. The fact that exporters tend to increase in size relative to non-exporters will tend to increase the size of the relatively larger cities that house exporting firms. However, the more stark prediction of the model is that the mass of mid-sized cities that accommodate firms around the export productivity threshold will jump as this threshold is reduced by a fall in trade costs. This is because the model predicts a discontinuity in the city size distribution around exporting thresholds (as exporters are discretely larger than non-exporters - check the proof of the existence of equilibrium in Appendix A) and this point of discontinuity shifts in response to changes in trade costs.

Note however that under the more realistic assumption of heterogeneous trade costs and heterogeneous trade liberalization across sectors, the spatial implications of trade liberalization are highly complex, as we need to keep track of which sectors are affected by liberalization and where these sectors tend to locate. While a homogeneous trade liberalization across sectors will tend to shift population from small to middle sized and large cities as per our result above, if we had a limited liberalization affecting a small number of sectors this could shift population to any city size category depending on whether the affected sector is located in large, mid-sized or small cities.

7 Quantitative Analysis

In this section we take the model to the data. We first present the main features of the estimation procedure. Next, we show how the model fits salient patterns in the Chinese data. Finally, we provide quantitative results for the effect of (i) spatial policies and (ii) trade liberalization on productivity.

7.1 Structural Estimation

Functional Forms

The first step to estimate the model is to specify the productivity process. In the model, firms sort perfectly into cities according to their raw efficiency z. This produces the stark prediction that small cities have no productive firms. Yet, in the data, small cities feature both productive and unproductive firms. To accommodate this fact, we introduce a disturbance term in ex-post productivity that varies across firms and cities. This reflects the fact that firms may be more productive in certain locations, for example because they have better knowledge of the local culture and can organize production in a more efficient way. The resulting productivity process features two sources of randomness: raw productivity (z), and a idiosyncratic productivity shock ($\varepsilon_{i,L}$) that varies across firms and cities. In this way, we allow firms to sort imperfectly into cities of different sizes.

We specify the same functional form for ex-post productivity ψ (including agglomeration economies related to the firm's optimal city choice) as Gaubert (2018):

$$\log(\psi_j(z_i, L, s_j)) \quad = \quad a_j \log L + log(z_i) \left[1 + \log \frac{L}{L_0}\right]^{s_j} + \varepsilon_{i,L} \tag{23}$$

where L_0 denotes the size of the smallest city, and $\{a_j, s_j\}$ are sectoral parameters. Equation (23) shows that ex-post (log) productivity ψ is composed by three terms. The first term ($a_j \log L$) represents the classical agglomeration mechanism: Firms are more efficient when they locate in larger cities. The second term represents the log-modularity between firms' raw efficiency z and (normalized) city size (L/L_0). According to this, firms' raw productivity z and city size L are complementary: Initially more productive (high z) firms benefit relatively more from locating in larger cities (provided that s is greater than zero). Finally, the last term $\varepsilon_{i,L}$ is an idiosyncratic term that varies across firms and cities. Importantly, this term is distributed independently of firm's raw productivity z. Thus, regardless of the level of raw productivity z, firms can still find optimal to locate in smaller cities.

We assume that raw productivity z follows a log-normal distribution with mean zero and variance σ_Z. We restrict the process for $\log z$ to be non-negative to ensure that ex-post productivity ψ increases with city size. Consequently, the distribution for $\log z$ is truncated at zero. Regarding the idiosyncratic term $\varepsilon_{i,L}$, we assume that it is distributed type-I extreme value. We restrict the parameters so that the mean of the process is equal to zero. With this restriction, the distribution is determined solely by the scale parameter β_ε.[27]

[27] The location parameter λ can be recovered explicitly as a function of the scale parameter β_ε. In particular, the restriction $\mathbb{E}(\varepsilon) = 0$ implies that $\lambda = -\gamma\beta_\varepsilon$, where γ is the Euler-Mascheroni constant.

Estimation Procedure

To estimate the model, we use the data from the Chinese Census of Manufacturing (see Section 2, for details). To match the relative size of China in the world economy, we consider a world with 5 symmetric countries. The estimation is carried out sector-by-sector for each 2-digit manufacturing ISIC industry.[28]

The estimation strategy proceeds in two steps and follows Gaubert (2018). We first calibrate all parameters that can be directly linked to the data $\{\sigma_j, \xi_j, b(1-\eta)/\eta, \tau_j\}$. The elasticity of substitution σ_j is set to match the average 2-digit markup, computed at the the establishment-level using the procedure outlined by De Loecker and Warzynski (2012). The Cobb-Douglas sectoral share ξ_j is computed as the share of each sector's value added within the manufacturing sector. The composite parameter $b(1-\eta)/\eta$ corresponds in the model to the elasticity of wages to city size. This elasticity is equal to the difference between the elasticity of average value-added to city size minus the elasticity of average employment to city size.[29] Thus, we run regressions for the logarithm of average city-level value-added, and the logarithm of average city-level employment as dependent variables, against the logarithm of urban population of the city size, and then subtract the coefficient on log city size from the former regression to the corresponding coefficient on the latter regression. Finally, the iceberg variable trade cost τ_j is set to match the average export intensity within exporting firms.[30]

In the second stage, we estimate the remaining parameters $\{a_j, s_j, \sigma_Z, \beta_\varepsilon, f_j^e\}$ through simulated method of moments (SMM). This method compares the objective moments in the data to the moments derived from a simulated economy, for candidate values of the parameters to be estimated. The vector of estimated parameters $\hat{\theta}_{SMM}$ are such that they minimize the weighted distance between the moments in the data ($\mathbf{m}_j$) and the simulated economy ($\hat{\mathbf{m}}_j(\theta_j)$):

$$\hat{\theta}_{j,SMM} = \arg\min_{\theta_j} (\hat{\mathbf{m}}_j(\theta_j) - \mathbf{m}_j)^T W_j (\hat{\mathbf{m}}_j(\theta_j) - \mathbf{m}_j) \tag{24}$$

In equation (24), the matrix W_T weights the vector of moments. We set this matrix to be equal to the inverse of the variance-covariance matrix of the moments. To compute this matrix, we follow Eaton, Kortum, and Kramarz (2011) and compute bootstrapped standard errors of the moments resampling (with replacement) 5,000 artificial economies with the same number of firms as in the Chinese Census of Manufacturing in 2004.

[28]We consider a total of 19 industries. We exclude manufactures of Tobacco products, and we merge (i) manufactures of coke, refined petroleum products and nuclear fuel with manufactures of chemicals and chemical products, and (ii) office, accounting and computing machinery with manufactures of electrical machinery.

[29]To see why this is the the case, note that $w(L)l_j(z,L) = (\sigma_j - 1)/\sigma_j r_j(z,L)$, where r represents firm revenues.

[30]In the model, the average export intensity conditional on exporting is equal to $\left(1 + \frac{\tau^{\sigma_j - 1}}{(C-1)}\right)^{-1}$.

Choice of Moments and Identification

We now discuss the moments we choose to target in the SMM estimation. Table 7 summarizes these moments, together with the parameter each moment aims to identify. The first set of moments relate to $\{a_j\}$. This parameter summarizes classical agglomeration forces: as a_j gets larger, productivity and revenues increase with city size. Accordingly, we define the target moment as the share of value added produced by firms located in cities of different sizes. We construct the moment in the following way. For each sector, we sort cities in terms of population, and define city groups in terms of quartiles of cumulative population (e.g., the first group contains all smallest cities in the economy, until that their population add up to 25 percent of the overall population). Then, we compute four moments as the share of value-added produced by firms located in each of the population quartiles. Thus, the first set of moments match by how much the share of sectoral value added increases with the size of the cities.

Table 7: Target Moments

Parameter	**Moment**
I. Calibrated Parameters	
σ_j	Average sectoral markup (De Loecker & Warzinsky, 2012)
ξ_j	Sectoral value added share
$\frac{b(1-\eta)}{\eta}$	Elasticity of wages to city size
τ_j	Average export intensity across exporting firms
II. Estimated Parameters	
a_j	Share of value added across city sizes
s_j	Average value added across city size (upper quartiles)
$\nu_{j,Z}$	Establishments' size distribution (pctiles 25th, 50th, 75, 90th)
$\nu_{j,R}$	Average value added across city size (lower quartiles)
f_j^e	Export probability

Notes: The Table summarizes the moments targeted by the parameter of the model when taking the model to the data. With the exception of the composite parameter $b(1-\eta)/\eta$, all parameters are computed at level of 2-digit ISIC sectors (revision 3). The quantitative analysis considers a mixed strategy, calibrating parameters that can be directly mapped to particular moments of the data (upper panel), and estimating the remaining parameters (bottom panel) through simulated method of moments.

The second set of moments relates to $\{s_j\}$, which determines the strength of the complementarity between raw productivity z and city size. To identify this parameter, we seek to match the average value added of firms in relatively large cities. Intuitively, for a given productivity z, the higher is the value of s_j, the stronger is the increase of firm productivity and revenues in city size. Formally, we divide cities in four quartiles by size, and then compute the average value-added of the firms locating in each quartile. We emphasize the two upper quartiles of the city size distribution: differences in s_j will affect relatively more the slope of average value added in relatively large cities, while it will tend to have a more modest impact in relatively smaller cities.

The third set of moments relate to the scale parameter of the idiosyncratic productivity shock (β_ε). This parameter varies by firm and city size, and it accounts for relatively productive firms locating in small cities. To identify this parameter, we target the average value added of firms in small cities. Through the lens of the model, if average value added in small cities is high, it must be because some highly productive firms are choosing to locate in these cities. Thus, as β_ε increases, we expect the average value added of firms locating in small cities to increase. Formally, this moment is defined analogously to the second set of moments, but with an emphasis on the first two quartiles of the city size distribution.

The fourth set of moments relate to the variance of the truncated log-normal distribution of raw productivity, σ_z. To identify this parameter, we target the firm size distribution. More specifically, we compute the 20, 50, 75 and 90th percentiles of normalized sales across all cities. Finally, to identify the fixed export cost we target the fraction of of firms that are exporters in the data. Intuitively, a higher fixed export cost affects the extensive margin of exporting. As this cost increases, fewer firms will be sufficiently profitable to pay the fixed export cost and participate in export markets.

7.2 Export Intensity and City Size

After presenting the main features of the estimation procedure, we turn to the main quantitative results. We begin by studying whether the baseline model can fit the fact that in the data export intensity increases with city size. This pattern is not directly targeted by our estimation strategy. Thus, our results in this section can be used to evaluate the mechanisms highlighted by the model – firm sorting and agglomeration, plus selection into exporting.

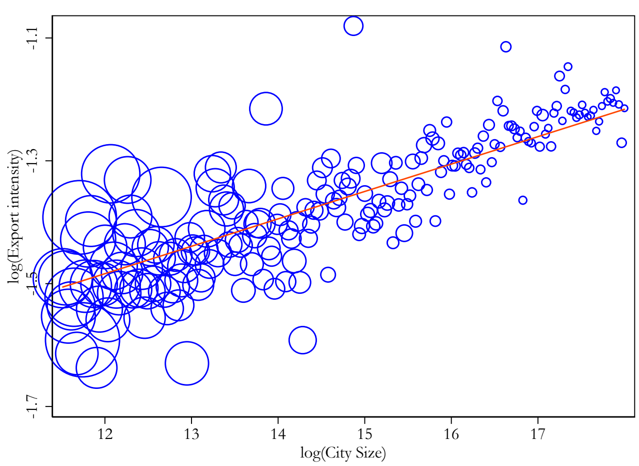
We simulate an economy with 200 equally-spaced city size bins. The support of the city size distribution in the simulated economy resembles the Chinese data described in section 2. Note that although the grid of city sizes is fixed, the effective city size distribution is determined endogenously in the model as a result of sorting and agglomeration forces. For each sector, we draw 20,000 realizations of raw productivity z and 20,000$\times$200 realization of idiosyncratic productivity shocks (one for each potential city size). Then, we solve the firms' problem and determine: (i) optimal city size and (ii) export participation.[31] Conditional on these choices, we solve the general equilibrium problem, taking the effective number of firms in each sector as the equilibrium mass of firms $\{M_j\}$ of the economy. This leads us to the equilibrium values for sectoral prices $\{P_j\}$,

[31] In the model, these decisions are independent from each other. Firms' location choice weights the strength of agglomeration economies over ex-post productivity ψ against congestion forces leading to more expensive labor costs. Thus, once firms choose their optimal city, the export decision affects the level of revenues and employment demand.

aggregate revenues $\{R\}$ and the export productivy threshold $\{\psi^*(L)\}$.[32] Once we obtain these values, we compute revenues and export value, and construct city-level export intensity as the share of aggregate exports to revenues, both defined at the level of city sizes.

Figure 2 shows the main result. It plots (log) export intensity against (log) city size. In the figure, the size of each bubble represents the number of cities in each city size bin, and the solid line represent the regression line that best fits the data, weighted by the number of cities in each bin. The model produces a remarkable positive relationship between city size and export intensity: In the model – as in the data – bigger cities are more export intensive. The regression coefficient is very precisely estimated at a value 0.06. This is about a 20 percent of the value in the data. Thus, while the model reproduces the data qualitatively, it cannot explain a substantial fraction of the the variation in the data.

Figure 2: Export Intensity and City Size in the Baseline Model



Notes: The figure shows the relationship between city size and export intensity predicted by the model. It simulates an economy with 200 city size bins, and 20,000 firms in each sector 2-digit sector. In the simulated economy, we define a log-linear grid over 200 equally spaced city-size bins. The support of the grid of city sizes in the simulated economy resembles the distribution of city sizes in the data. The size of each bubble denotes the number of cities in each city size. The actual number of cities of each size are determined endogenously within the model as a consequence of firms sorting into cities.

One explanation for the poor fit of the main stylized fact relates to how the selection-into-exporting mechanism operates in the model. Conditional on productivity, the probability that a

[32] Unlike the theoretical model, in the empirical model the export productivity thresholds varies with city size. This is directly related to the fact that in the theoretical model firms sort perfectly into city sizes. As a consequence, there is only one city size featuring both domestic firms and exporters. This city defines the only relevant export productivity threshold. In contrast, in the model with imperfect sorting all cities may feature exporters. Since labor costs vary across cities, exporting requires a higher productivity threshold in larger cities.

firm exports in the model decreases with city size. Firms in larger cities have to pay higher labor costs, which ultimately reduces the probability that they can generate enough profits to pay the fixed export costs.[33] In contrast, export activity in the data increases with city size, even after controlling for firm-productivity (see columns 2 and 4 in Table 6). Then, unless we introduce an additional force, the model's ability to perfectly fit this dimension of the data is limited.[34]

7.3 Counterfactual Analysis

We now move towards analyzing the general equilibrium effect of trade and spatial policies. Our goal is to illustrate how economic geography and international trade interact in the model. In particular, we seek to determine first, how international trade affect the effectiveness of spatial policies, and second, how economic geography affect the effectiveness trade policies.

International Trade and the Effect of Spatial Policies

In this section, we replicate the reduction in land-use restrictions studied by Gaubert (2018), and compare the response in the open and closed economy cases. We operationalize this policy as a (multilateral) reduction in the parameter b, which measures the intensity of land use in the housing production function.[35] Changing this parameter affects both housing supply and the cost of labor across cities. In particular, a reduction in the value of b increases the housing supply elasticity, and flattens the wage schedule across city sizes.

In the model, a less restrictive spatial policy lead to a higher level of aggregate productivity. As b decreases, firms have incentives to move (in average) to larger cities. Ultimately, this relocation process generates improvements in aggregate total factor productivity, due to within-firm efficiency gains, and gains from reallocation of resources. On the one side, firms that move to larger cities benefit of larger agglomeration economies, leading to within-firm efficiency gains. On the other side, these firms become larger, and hire relatively more workers. This produces a reallocation of resources within the economy, which reinforces the within-firm effect and leads to additional gains in efficiency.

Relative to the closed economy case, we expect the reduction in land use restrictions to generate a larger effect on aggregate productivity when the economy is open. Most productive firms have a greater weight in the open economy case, because they can export and increase their revenues.

[33] Note that in the statistical model, this holds in expected values, because the conditional idiosyncratic productivity shocks $\varepsilon_{i,L}$ are distributed independently of firms' raw productivity z. As a consequence, two firms with the same z may draw very different $\varepsilon_{i,L}$ in large and small cities, leading them to have higher or lower export probability. However, because $\varepsilon_{i,L}$ has mean zero, it will still be true in expecation that – conditional on z – export participation decreases with city size.

[34] One easy way to improve the fit of the model to the data would be to allow the fixed export cost to fall with city size, perhaps reflecting the existence of better productive amenities – such as infrastructure – in larger cities.

[35] More generally, policies in the open economy case may lead to cross-country spillovers when they are not applied symmetrically in all countries. While this may lead to interesting quantitative results, for now we focus on the the case of multilateral policies to emphasize the different responses of the economies in the open and closed economy cases.

This amplifies the impact of the within-firm gains from the closed economy case. In addition, as we discuss in section 6, the model predicts that weakening housing supply restrictions increases the fraction of firms that are exporters. This leads to additional gains – relative to a closed economy – in the form of reallocation of resources from domestic firms to new exporters.

An important feature of the model is that the matching function between firms and cities does not depend on the degree of openness to trade of the economy. Indeed, optimal city choice only depends on the strength of agglomeration economies compared to congestion costs. These two forces are identical in open and closed economy, because city-sizes are predetermined: If firms want to relocate in larger cities, then city developers create new cities to accommodate the demand for housing. Thus, thus within-firm efficiency gains due to agglomeration will be identical for open and closed economies: Differences in aggregate productivity will only emerge due to a greater reallocation of resources in the open economy case.

The property of predetermined city sizes, although convenient analytically for solving the equilibrium of the model, is somehow unrealistic. At least in the short-run, cities grow when the they face increased housing demand. This, in turn, reinforces the within-firm gains and amplifies the overall productivity gains. Thus, when analyzing the general equilibrium effect of policies, we report results a less restrictive interpretation of the model where we allow cities to grow (but the number of cities of each size is fixed).[36]

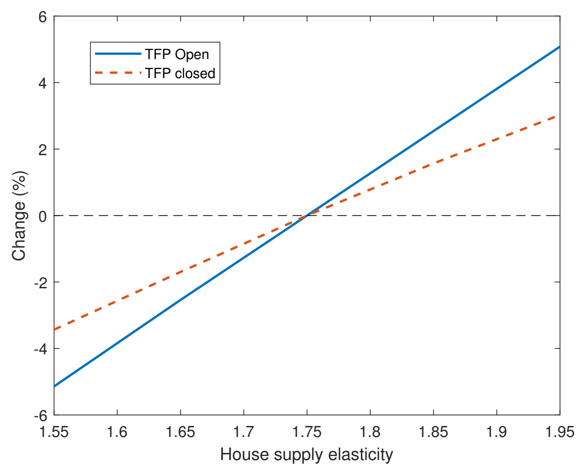
We proceed in three steps to analyze the effect of changes in b. First, we calibrate the land intensity parameter b. As in Gaubert (2018), we set this parameter to match the median housing supply elasticity across US cities (see Saiz, 2010). Second, we simulate the baseline economy as in section 7.2. Finally, we simulate the various counterfactual economies, where we change the value of b. This involves recomputing: (i) firms' optimal location, (ii) export decision, and (iii) general equilibrium objects. In particular, we vary b so that the housing supply elasticity varies between the 25th and the 75th percentile of the housing supply elasticity across U.S. cities (as defined by Saiz, 2010). Finally, we compute aggregate TFP for all economies. For the closed economy, we proceed in a similar way, but we set the variable trade cost equal to a large number, while we keep the rest of parameters fixed at their open economy values.

Figure 3 plots aggregate TFP against various levels of the housing supply elasticity. In order to simplify comparisons, we compute productivity relative to the level in the baseline economy. Accordingly, when the housing supply elasticity takes the value of the baseline economy (1.75), the value for normalized aggregate TFP is zero. In each panel, we plot the productivity trajectories for the closed (dashed line) and open (solid line) economy cases. Both cases show relatively large

[36]Operationally, the counterfactual exercise involves solving a fixed-point problem: A reduction in b leads firms to move to larger cities. This increases the size of these cities, and their attractiveness in terms of agglomeration economies. This leads to subsequent waves of firms moving to larger cities. This process continues up to the point that congestion costs counterbalance the benefits from agglomeration.

changes in aggregate productivity. Taking the economy from the first to the fourth quartile of the housing supply distribution increases aggregate productivity in approximately 10 percent relative to the baseline in the closed economy case. When we compute the same statistic for the open economy, the productivity gains scale up to almost 15 percent. Thus, open economy considerations increases the estimated effectiveness of spatial substantially. In our particular exercise, the effectiveness increases in about 50 percent.[37]

Figure 3: Aggregate Productivity Effect of a Reduction in Land Use Restrictions



Notes: The Figure shows the aggregate of aggregate productivity of reducing land-use restrictions. The horizontal shows the housing supply elasticity of the economy, while the vertical axis shows the aggregate TFP response relative to baseline economy. In the model, a less restrictive land use policy is mapped to an increase in the housing supply elasticity. The dashed line shows the closed economy response of aggregate TFP, while the solid line shows the open economy.

Once we show the effect of a reduction in the housing supply elasticity, we study the sources of productivity gains. It can be shown that log-deviations of aggregate TFP relative to the baseline value can be decomposed in three terms: (i) unweigthed average productivity, which reflects within-firm productivity gains; (ii) a covariance term between firms' productivity and their relative size, that reflect the extent of resource reallocation in the economy; and (iii) the change in the equilibrium mass of firms. [TO BE COMPLETED]

[37]Our estimates are significantly larger than the values estimated by Gaubert (2018) for a closed economy version of the model estimated for France. We note that our estimates are not directly comparable to hers: Gaubert (2018) solves the strict interpretation of the model, with predetermined city sizes. This dampens significantly the productivity response of the economy, as it misses agglomeration gains due to changes in the size of the cities.

Economic Geography and the Effect of Trade Liberalization [TBC]

In this section, we compare the productivity gains from trade liberalization predicted by a model without geography (e.g., Melitz, 2003) and our baseline model. As we discuss above, we expect that the model without geography will overestimate the productivity gains from trade liberalization. In our model, more productive firms tend to locate in larger cities and face higher wages, and thus will tend to expand less in response to trade liberalization than in standard trade models where wage differences across space are omitted from the analysis.

8 Conclusion

In this paper we study the interplay between agglomeration economies and international trade. Using information from three major trading nations – China, the United States and Brazil – we have documented a novel fact regarding the economic geography of exporting: Exporting is more unevenly distributed than overall economic activity, and in particular, it is disproportionately concentrated in larger cities. While about one-fourth of the association between export intensity and city size can be attributed to differences in sectoral composition across cities (i.e., export-intense sectors tend to locate in larger cities), this relationship remains highly significant (both statistically an economically) also within sectors.

Building on these findings, we show that a simple extension of models of sorting and agglomeration across space (a la Gaubert, 2018) to a multinational setting, combined with a mechanism of selection into exporting in the spirit of Melitz (2003), can account for the pattern that we documented in the data. The intuition of the model is straightforward: Due to both selection and agglomeration, larger cities feature more productive firms that are more likely to select into exporting. As a result, large and productive cities feature elevated aggregate export intensities in all sectors.

The model can be structurally estimated using firm level data to recover the shape of agglomeration externalities and the magnitude of fixed exporting costs. This allows us to disentangle the roles played by agglomeration forces on the one hand, and firm sorting on the other, in explaining the differences in productivity and export intensity between cities of different sizes.

Finally, in future work we aim to use the model to undertake policy analyses. The model naturally lends itself to assess the implications of both trade policies and (domestic) spatial policies on national-level economic indicators of interest (such as productivity and welfare). We aim to focus on the effect of spatial policies. In particular, we plan to analyze the general equilibrium impact of a prevalent class of spatial policies that influence the location choices of firms: regulations that limit city growth, such as zoning and building height regulations. We plan to assess how open economy considerations (i.e., the option to export/import) affect estimates of the aggregate productivity and welfare costs of urban development policies. This trade dimension has thus far been ignored in the analysis of spatial policies. Our counterfactual policy analysis offers a complementary approach to research that assesses the impact of specific place-based policies. The empirical literature has traditionally focused on estimating the local effects of these policies. A notable exception is Kline and Moretti (2014), who develop a methodology to estimate their aggregate effects. They estimate that, following a local productivity boost, additional positive local effects due to the endogenous creation of agglomeration externalities are offset by losses in other parts of the country. Our approach is to explicitly model the reaction of mobile firms to financial incentives. Preliminary results suggest a negative aggregate effect of policies that encourage firms to locate in smaller cities, such as regulations hampering urban growth.

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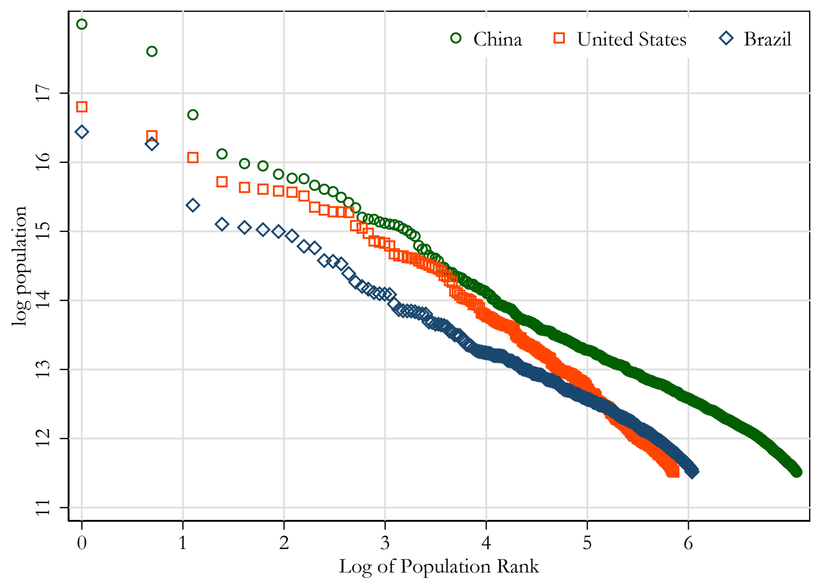
FIGURES

Figure 4: City Size Distribution in China, United States and Brazil



Notes: The figure shows the distribution of city size – measured in terms of population. It plots the logarithm of each city's ranking against its corresponding population (in logarithms). Cities in China and the United States are defined in terms of metropolitan areas, and as microregions in the case of Brazil (see section 2.1 for details). For all countries, the analysis only considers cities with population over 100,000 people.

Online Appendix

Cities, Productivity and Trade

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A Proof of Equilibrium Existence

The equilibrium is constructed in four steps. First, we solve for the equilibrium subsidy offered by city developers. Second, we show that it pins down how ïňĄrms match with city sizes within countries, as well as the set of city sizes generated in each country in equilibrium by city developers. Third, general equilibrium quantities are determined in each country by market clearing conditions and free entry conditions in the traded goods sectors, once we know the equilibrium matching functions from step 2. Finally, the city-size distribution within countries is determined by these quantities, using labor-market clearing conditions. In each step, the relevant functions and quantities are uniquely determined; hence, the equilibrium is unique.

Step 1: Equilibrium Subsidies

Lemma 1. *To determine the equilibrium subsidies, we first outline the optimization problem of city developers. City developers revenue comes from fully taxing the profits of local landlords. In turn, housing market clearing in each city implies that total landlord profits in each city are given by:*

$$\pi_H(L) = b(1-\eta)w(L)L \tag{A.1}$$

Thus, a city developer i developing a city of size L faces the following problem:

$$\max \Pi_L = b(1-\eta)w(L)L - \sum_{j=1}^{S} \int_z T_j^i(z,L)\pi_j^o(z,L)1_j(z,l,i)M_j dF_j(z) \tag{A.2}$$

such that

$$1_j(z,L,i) = 1 \textit{ if } L = \arg\max \pi_{Sub,j}^T(z,L) \textit{ and firm } z \textit{ chooses city } i$$
$$1_j(z,l,i) = 0 \textit{ otherwise}$$

In this expression, M_j denotes the mass of firms in sector j in each country (which will be the same in all countries by symmetry), $F_j(.)$ is the distribution of raw efficiencies in sector j in each country and $\pi_j^T(z,L)$ is the total profit before subsidy of a firm of efficiency z in sector j, as defined in equation (15) (which again will be the same in all countries by symmetry).

With the above set-up in place, it can be shown that, in equilibrium, city developers offer and firms *take-up operational profit subsidies according to the schedule:*

$$T_j(z, L) = T_j^* = \frac{\beta(1-\eta)(\sigma_j - 1)}{1-(1-b)(1-\eta)} \tag{A.3}$$

Proof: Competition among city developers means that the revenues raised from attracting any firm and sector category to the city exactly balance out the subsidies offered to that firm and sector combination:

$$\beta(1-\eta)w(L)L_j(z, L) = T_j(z, L)\pi_j^o(z, L) \tag{A.4}$$

where $L_j(z, L)$ represents the total number of workers drawn to the city when attracting a firm of productivity z in sector j in the productive conditions of a city of size L; while the LHS of (A.4) represents the total expenditures on housing of the workers drawn to the city as a result of the firm choosing to locate in the city. This expenditures represent the (marginal) revenue of the city developer from attracting a firm of productivity z in sector j in the productive conditions of a city of size L. If the above equation were not to hold the city developer would either be making losses or positive profits. In the latter case, another developer would have an incentive to start a city of size L and offer subsidies slightly lower than $T_j(z, L)$, draw all firms of productivity z in sector j and make a profit. This can't be the case in equilibrium.

Starting from (A.4) we can obtain the subsidy schedule in (A.3) by noting that the total number of employees drawn to a city when a firm of productivity z in sector j chooses the city is equal to the workforce of the firm plus a multiplier effect brought by the workers purchase of locally produced housing. Thus the number of workers drawn to the city by a firm is related to the firm's equilibrium employment via the equation:

$$l_j(z, L) = \left[1-(1-b)(1-\eta)\right] L_j(z, L) \tag{A.5}$$

Moreover, to obtain the subsidy schedule established by the above Lemma, we need to note that firm level can be written as:

$$l_j(z, L) = \begin{cases} \frac{(\sigma_j-1)\pi_j^D(z,L)}{w(L)} & \text{if } z < z_j^*(L) \\ \frac{(\sigma_j-1)\pi_j^D(z,L)}{w(L)}\left(1+\frac{C-1}{\tau^{\sigma_j-1}}\right) & \text{if } z \geq z_j^*(L) \end{cases}$$

which can be equivalently written:

$$l_j(z, L) = \frac{(\sigma_j - 1)\pi_j^o(z, L)}{w(L)} \tag{A.6}$$

Finally substituting (A.5) and (A.6) in (A.4) yields the subsidy schedule established in (A.3).

Moreover, substituting the optimal subsidies in (A.3) and the expressions for domestic profits from (11) into (16) yields the following expressions for total profits after subsidies:

$$\pi^T_{Sub,j}(z,L) = \begin{cases} k_{1j}\left(1+T_j^*\right)\left[\frac{\psi(z,L,s_j)}{w(L)}\right]^{\sigma_j-1} E_j P_j^{\sigma_j-1} & \text{if } z < z_j^*(L) \\ k_{1j}\left(1+T_j^*\right)\left[\frac{\psi(z,L,s_j)}{w(L)}\right]^{\sigma_j-1} E_j P_j^{\sigma_j-1}\left(1+\frac{C-1}{\tau^{\sigma_j-1}}\right) - (C-1)Pf_j^e & \text{if } z \geq z_j^*(L) \end{cases} \tag{A.7}$$

where the parameter k_{1j} is given by

$$k_{1j} = \frac{1}{\sigma_j^{\sigma_j}}\left(\sigma_j - 1\right)^{\sigma_j-1}$$

Step 2: Equilibrium City Sizes and the Matching Function

Within each country, the city developers'problem determines the equilibrium city sizes generated in equilibrium. Cities are opened up when there is an incentive for city developers to do so, i.e. when there exists a set of firms and workers that would be better off choosing this city size. Workers are indifferent between all locations within their countries, but firms are not, since their proïňĄts vary with city size. Given the equilibrium subsidies offered by city developers, the proïňĄt function of a firm with raw productivity z in sector j is given by (A.7).

Note that the firm's problem can be reduced to maximizing the expression in the square brackets in (A.7) which yields the first order condition:

$$\frac{\psi_2(z,L,s_j)L}{\psi(z,L,s_j)} = b\frac{1-\eta}{\eta} \tag{A.8}$$

with $w(L)$ being the wage schedule established by equation (8). There is a unique profit-maximizing city size for a firm of type z in sector j, under the regularity conditions we have assumed. Define the optimal city size as follows:

$$L_j^*(z) = \arg\max_{L\geq 0} \pi^T_{Sub,j}(z,L)$$

Assume that, for some firm type z and sector j, no city of size $L_j^*(z)$ exists. There is then a profitable deviation for a city developer on an unoccupied site to open up this city. It will attract the corresponding firms and workers, and city developers will make a positive profit by subsidizing firms at a rate marginally smaller than $T_j(z,L)$. The number of such cities adjusts so that each city has the right size in equilibrium. This leads to the following lemma, letting $\mathcal{L}$ denote the set of city sizes in equilibrium:

Lemma 2. *The set of city sizes for each country in equilibrium, $\mathcal{L}$, is the set of optimal city sizes for firms.*

Given this set of city sizes, the optimal choice of each firm in every country is fully determined. Define the matching function

$$L_j^*(z) = \arg\max_{L \in \mathcal{L}} \pi_{Sub,j}^T(z, L)$$

It is readily seen that the profit function of the firm (equation (A.7)) inherits the strict log-supermodularity of the productivity function in z and L. Therefore, the following lemma holds.

Lemma 3. *The matching function $L_j^*(z)$ is increasing in z.*

This result comes from a classic theorem in monotone comparative statics (Topkis (1998)). The benefit to being in larger cities is greater for more productive firms and only they are willing in equilibrium to pay the higher wages there. Furthermore, within each country the matching function is fully determined by the firm maximization problem, conditional on the set of city sizes $\mathcal{L}$. As seen from equation (A.7), this optimal choice does not depend on general equilibrium quantities that enter the proïňĄt function proportionally for all city sizes. Finally, under the regularity assumptions made on ψ as well as on the distribution of z, $F_j(.)$, the optimal set of city sizes for firms in a given sector and country is an interval (possibly unbounded). The sectoral matching function is invertible over this support. For a given sector, we use the notation $z_j(L)$ to denote the inverse of $L_j^*(z)$. It is increasing in L. The set of city sizes $\mathcal{L}$ available in equilibrium in each country is the union of the sector-by-sector intervals.

Moreover, given the bijection between productivity levels and city sizes within each sector and country in equilibrium, there will be a productivity threshold z_j^* associated with export participation (i.e. for all $z \geq z_j^*$ firms are involved in exporting to all countries, while for $z < z_j^*$ firms are purely domestic). Due to symmetry, these sectoral exporting thresholds z_j^* are the same in all countries.

Step 3: General Equilibrium Quantities

The equilibrium has been constructed up to the determination of the following general equilibrium values. The reference level of wages $\bar{w}$ defined in equation (8) is taken as the numeraire. The remaining unknowns are the aggregate revenues in the traded goods sector in each country E (identical in all countries), the mass of firms M_j in each sector and country (M_j's are the same in all countries due to symmetry), the sectoral price indexes P_j (identical in all countries), and the sectoral exporting thresholds z_j^*.

At the level of each country and sector, aggregate operational profits (ignoring fixed costs of exporting but including subsidies) are given by:

$$\pi^o_{Agg,j} = \int_{z_{min}}^{z_j^*} k_{1j}(1+T_j^*)\left[\frac{\psi(z,L_j^*(z),s_j)}{w(L_j^*(z))}\right]^{\sigma_j-1} E_j P_j^{\sigma_j-1} M_j dF_j(z)$$
$$+ \int_{z_j^*}^{z_{max}} k_{1j}(1+T_j^*)\left[\frac{\psi(z,L_j^*(z),s_j)}{w(L_j^*(z))}\right]^{\sigma_j-1} E_j P_j^{\sigma_j-1}\left(1+\frac{C-1}{\tau^{\sigma_j-1}}\right) M_j dF_j(z) \quad \text{(A.9)}$$

which can be rewritten

$$\pi^o_{Agg,j} = k_{1j}(1+T_j^*)E_j P_j^{\sigma_j-1} M_j \underbrace{\left[\int_{z_{min}}^{z_j^*}\left[\frac{\psi(z,L_j^*(z),s_j)}{w(L_j^*(z))}\right]^{\sigma_j-1} dF_j(z) + \int_{z_j^*}^{z_{max}}\left[\frac{\psi(z,L_j^*(z),s_j)}{w(L_j^*(z))}\right]^{\sigma_j-1}\left(1+\frac{C-1}{\tau^{\sigma_j-1}}\right) dF_j(z)\right]}_{S_j(z_j^*)} \quad \text{(A.10)}$$

Having an expression for aggregate operational profits at the sectoral level, it is straightforward to obtain an expression for aggregate revenues at the sectoral level:

$$R^o_{Agg,j} = E_j = \sigma_j k_{1j} E_j P_j^{\sigma_j-1} M_j S_j(z_j^*) \quad \text{(A.11)}$$

We can also derive an expression for the aggregate labor force used in sector j in every country

$$l_{Agg,j} = \int_z \frac{(\sigma_j-1)\pi^o_{Agg,j}}{(1+T_j^*)w(L_j^*(z))} M_j dF_j(z) \quad \text{(A.12)}$$

which can be expanded as

$$l_{Agg,j} = k_{1j}(\sigma_j-1)E_j P_j^{\sigma_j-1} M_j \underbrace{\left[\int_{z_{min}}^{z_j^*}\frac{\psi(z,L_j^*(z),s_j)^{\sigma_j-1}}{w(L_j^*(z))^{\sigma_j}} dF_j(z) + \left(1+\frac{C-1}{\tau^{\sigma_j-1}}\right)\int_{z_j^*}^{z_{max}}\frac{\psi(z,L_j^*(z),s_j)^{\sigma_j-1}}{w(L_j^*(z))^{\sigma_j}} dF_j(z)\right]}_{Emp_j(z_j^*)} \quad \text{(A.13)}$$

Finally, employing the expression for aggregate operational profits derived above we can find an expression for aggregate total profits

$$\pi^T_{Agg,j} = \pi^o_{Agg,j} - M_j \int_{z_j^*}^{z_{max}} (C-1)Pf_j^e dF_j(z)$$
$$\pi^T_{Agg,j} = k_{1j}(1+T_j^*)E_j P_j^{\sigma_j-1} M_j S_j(z_j^*) - M_j\left[1-F_j(z_j^*)\right](C-1)Pf_j^e \quad \text{(A.14)}$$

By the free entry condition we then have that:

$$E(\pi^T) = k_{1j}(1+T_j^*)E_j P_j^{\sigma_j-1} S_j(z_j^*) - \left[1-F_j(z_j^*)\right](C-1)Pf_j^e = Pf_j \quad \text{(A.15)}$$

All in all, within each country, equilibrium is defined by the system of equations (A.16) to (A.19):

$$k_{1j}(1+T_j^*)\left[\frac{\psi(z, L_j^*(z_j^*), s_j)}{((1-\eta)L_j^*(z_j^*))}\right]^{\sigma_j-1} E_j P_j^{\sigma_j-1}\frac{1}{\tau^{\sigma_j-1}} = f_j^e P \;\; \forall j \in 1,\ldots,S \tag{A.16}$$

$$P\left\{f_j + (C-1)\left[1-F_j(z_j^*)\right] f_j^e\right\} = k_{1j}(1+T_j^*)E_j P_j^{\sigma_j-1} S_j(z_j^*) \;\; \forall j \in 1,\ldots,S \tag{A.17}$$

$$1 = \sigma_j k_{1j} P_j^{\sigma_j-1} M_j S_j(z_j^*) \;\; \forall j \in 1,\ldots,S \tag{A.18}$$

$$N = \sum_{j=1}^{S} k_{1j}(\sigma_j-1)E_j P_j^{\sigma_j-1} M_j Emp_j(z_j^*) + N(1-b)(1-\eta) \tag{A.19}$$

where (A.16) comes from the definition of z_j^*, (A.17) comes from a re-writing of the zero profit condition in (A.15), (A.18) is a rewriting of (A.11), while (A.19) is a national labor market clearing condition.

This system of $3S+1$ equations characterizes the general equilibrium in each country. Given the symmetry of the countries, the same general equilibrium quantities apply for each country (in particular trade balance ensures that expenditures on all tradables are the same in each country). Inverting this system of $3S+1$ equations gives the $3S+1$ unknowns (for each country, but these quantities have the same values for all countries): P_j the price index for sector j in each country, M_j, the mass of firms that enters in sector j in each country, and z_j^*, the exporting threshold for each country, for all $j \in \{1, ..., S\}$. It also gives E, the aggregate revenues in the traded goods sector in each country by performing the substitution $E_j = \xi_j E$ in the equations above.

Step 4: Equilibrium City-Size Distribution

Within each country, the city developers'problem and the firms'problem jointly characterize (1) the set of city sizes that necessarily exist in equilibrium and (2) the matching function between firm type and city size. Given these, the city-size distribution is pinned down by the national labor market clearing conditions. The population living in a city of size smaller than any L in each country must equal the number of workers employed by firms in that country that have chosen to locate in these same cities, plus the workers hired to build housing. Thus, $\forall L > L_{min}$

$$\int_{L_{min}}^{L} u f_L(u)du = \sum_{j=1}^{S} M_j \int_{z_j(L_{min})}^{z_j(L)} l_j(z, L_j^*(z))dF_j(z) + (1-b)(1-\eta)\int_{L_{min}}^{L} u f_L(u)du \tag{A.20}$$

where $L_{min} = \inf \mathcal{L}$ is the smallest city size.

Differentiating this with respect to L and dividing by L on both sides gives the city size density

($f_L(L)$ is not normalized to sum to 1):

$$f_L(L) = k_2 \frac{\sum_{j=1}^{S} M_j 1_j(L) l_j(z_j(L), L) f_j(z_j(L)) \frac{\partial z_j(L)}{\partial L}}{L} \tag{A.21}$$

where $k_2 = \frac{1}{1-(1-b)(1-\eta)}$ is a constant and $1_j(L) = 1$ if sector j has firms in cities of size L and 0 otherwise. The equilibrium distribution of city sizes $f_L(.)$ is uniquely determined by equation (A.21), hence the following lemma:

Lemma 4. $f_L(.)$ *is the unique equilibrium of this economy in terms of the distribution of city sizes within countries. Note that this distribution is the same for all countries (due to symmetry).*

Note that for each city size, the share of employment in each sector can be computed using the same method, now sector by sector (and country by country). For a given city size, the average sectoral composition over all cities of a given size L is determined by the model. On the other hand, the model is mute on the sectoral composition of any individual city of size L, which is irrelevant for aggregate outcomes. This comes from the fact that agglomeration externalities depend on the overall size of the city, and not on its sectoral composition.

This step completes the full characterization of the unique equilibrium of the economy.