# IS SKILL DISPERSION A SOURCE OF PRODUCTIVITY AND EXPORTING IN DEVELOPING COUNTRIES?

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ABSTRACT. Recent literature claims that skill mix within firms, in contrast to average human capital, influences the entire economy. This paper provides theoretical and empirical evidence of the linkage from skill mix to output, inequality, productivity and exports. I develop a multisector model of firms who employ teams of workers in production. In this setting I derive two main results. First, I consider what impact changes in the skill distribution from migration, education or outsourcing have on output. I find an increase in industry specific workers boosts output, but in contrast to classical models, worker spillovers to other industries may attenuate output. Second, I consider the impact of price changes as caused by tariff reductions or subsidies. I show a rise in output prices raises the total wages of a worker team but changes relative wages within the team. This is because relative wages depend on the supply of team members to the industry. Inequality will increase if the supply of high skilled workers is tight. This possibility of a sector boom coincident with higher inequality provides a new explanation of inequality trends beyond skill biased technical change. Empirically, my model motivates a novel specification that characterizes industries as "intensive in skill diversity" or "intensive in skill similarity." Productivity differences explained by skill mix intensity are comparable to the magnitude of training and imported inputs combined. I also find skill mix differences explain intrasector export variation better than physical or human capital.

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## 1. Introduction

This paper revisits the theory of the firm to incorporate the role of worker teams in production. The mix of skills employed within a firm may range from one extreme of skill similarity to another extreme of skill diversity. By skill similarity I refer to production processes which benefit when worker teams are composed of similarly skilled workers. An example is a production line where jobs have been broken down into equally difficult tasks, as typified by the O-Ring production process of Kremer (1993). By skill diversity I refer to superstar production processes which depend on the most skilled member of a team. An example is hierarchical production structures where success is heavily tied to the quality of individuals at the top, an idea which goes back to Rosen's Economics of Superstars Rosen (1981). Such a fundamental distinction between methods of production has important implications for growth, labor markets with imperfect information, theories of the firm, wage inequality and trade.[1] This paper contributes new theoretical implications by considering the role of worker scarcity when firms form teams. In addition, this paper supports the theoretical literature by providing an empirical linkage from skill mix to productivity within and across industries.

Theoretically I build on Grossman and Maggi (2000) who consider worker teams employed in either skill similar or skill diverse production. Each firm hires from a population of skill levels to form worker teams and produce goods. I extend their model to multiple sectors and arbitrary skill distributions to exhibit new, rich forms of the Rybczynski and Stolper-Samuelson theorems. The Rybczynski Theorem under skill diversity predicts an increase in the mass of industry specific workers increases industry output. However, spillovers of workers into other sectors may attenuate this increase in output. The Stolper-Samuelson theorem under skill diversity shows that a rise in output prices raises the total wages of worker teams but changes relative wages within the team. Furthermore, an increase in industry output price may decrease the wages of some workers in the industry due to changes in the division of surplus. This possibility of growth concurrent with decreasing wages provides a new channel for inequality beyond skill biased technical change.

[1]In connection to growth, Das (2005) considers research and development as a submodular process while consumption goods are produced using a supermodular process. In labor, Jones (2009) builds on the idea of complementary production processes and intermediate goods linking sectors while Delacroix (2003) links unemployment and wage dispersion. See also Acemoglu (1999), Grossman (2004) and Moro and Norman (2005). Organizational theories of the firm include Hong and Page (2001) who theoretically explore role of diversity in production in a novel way, suggest a search for additional empirical evidence of richer organizational theories.

The primary empirical focus of this paper is establishing that skill mix, in contrast to the average level of skill, is an important determinant of productivity. There is a paucity of systematic evidence for the role of skill similarity and skill diversity at the firm level, although some evidence is suggestive.[2] The theoretical model motivates an empirical specification that characterizes industries as "intensive in skill diversity" or "intensive in skill similarity." I use the production structure of the model to arrive at estimable primitives of a skill diverse, multisector economy. The approach is to specify production in a neoclassical form where skill mix enters as labor augmenting technology. The labor augmenting technology may be "similar skill loving", "diverse skill loving" or neutral for each industry. The specification is estimated for a cross section of firms in over thirty developing countries using the Enterprise Surveys collected by the World Bank. In developing countries productivity differences due to skill mix should be pronounced due to both greater heterogeneity in educational attainment and labor abundant production.

I find that over two-thirds of developing country firms belong to sectors which are significantly characterized as either "similar skill loving" or "diverse skill loving." The estimates provide a ranking of intensity across industries from skill similar to skill diverse. This parallels the concept of factor intensity in classical trade theory. As the model predicts linkages from such intensities to inequality, the magnitudes of the estimates are particularly relevant in a developing country context.[3] Within industries, I rank firms by the level of productivity explained by skill mix. Using this ranking, I find the difference between the 75th and 25th percentiles are 9-13%, comparable to the effects of training and imported inputs combined.

The second empirical focus of this paper is testing the linkage from skill mix to comparative advantage at the firm level.[4] Recent theoretical work has shown that skill mix may predict patterns

[2]Andersson et al. (2009) find that firms with high potential payoffs for selecting the right products (a task assisted by worker talent) pay higher starting salaries and select superstars who have a history of success. Martins (2008) also uses matched employer-employee data to tackle competing theories of the relationship between wage dispersion on firm performance, finding a positive relationship which becomes negative once firm and worker fixed effects are considered.

[3]The relationship between trade and inequality has generated a vast literature. For surveys, see Kremer and Maskin (2003), Winters et al. (2004) and Goldberg and Pavcnik (2007).

[4]Again evidence is sparse or suggestive. Mamoon and Murshed (2008) find developing countries with a high level of schooling experience smaller increases in wage inequality following trade. This is consistent with the idea that high skill countries have a comparative advantage in complementary production relative to low skill countries which best utilize scarce high skill workers in diverse production.

of trade and expected wage shifts resulting from trade.[5] My approach to testing trade implications joins two strains of the trade literature. The first literature emphasizes the role of heterogeneous firm level productivity as a selection mechanism for exports in the presence of trade frictions (e.g. Melitz (2003)). The second literature explains productivity differences as outcomes of heterogeneous worker matches to jobs.[6] If productivity differences arise from a relatively high endowment of diverse or similar labor, the predicted pattern of trade depends on moments of the skill distribution beyond the mean. I use a non-linear function of skill mix to explain firm productivity and account for firm exports, revealing a new connection from skill similarity and skill diversity to trade. Productivity differences explained by skill mix predict intrasector exports better than physical or human capital.

The rest of this paper is organized in six sections. Section 2 lays out the model setting while Section 3 develops implications in the form of new Rybczynski and Stolper-Samuelson theorems. Section 4 puts forth the production specification and estimates the relative importance of skill mix within industries. Section 5 estimates the relationship between propensity to export and skill mix. Section 6 concludes.

## 2. A Skill Diverse Multi-Sector Economy

This section begins with the model setting where firms form production teams from a heterogeneous pool of workers. After defining a perfectly competitive equilibrium, I consider the allocation of workers within and across firms. Finally, I construct the wage schedule which supports the efficient equilibrium. General equilibrium implications are pursued in the following section.

2.1. **Model Setting.** Consider an economy populated with a mass $L$ of workers of varying skill levels $q$. Denote the distribution of skills within the population by $\Phi(q)$, and assume that $\Phi$ has a continuous pdf with full support on $[0, \infty)$ and finite mean.[7] Equilibrium wages received by a

[5]Ohnsorge and Trefler (2007) construct a model in which a high correlation between worker attributes amounts to a relative abundance of one of the factors, resulting in a pattern of trade based on a second moment of the skill distribution. In Bougheas and Riezman (2007) both first order stochastic dominance and mean preserving spreads which alter the skill distribution predict a pattern of trade.

[6]From a comparative advantage perspective, both Manasse and Turrini (2001) and Yeaple (2005) result in the selection of high skill inputs into export activities. Both papers are competing explanations for the stylized facts of a growing skill premium and productivity differences between exporters and non-exporters. In contrast, I investigate a different channel based on the *skill mix of worker teams* rather than the skill of individual workers.

[7]I show in the appendix that the value of production in the economy is bounded if and only if $\Phi$ has a finite mean so this assumption entails no loss of generality.

worker of skill level $q$ are written as $w(q)$. Workers supply labor inelastically, and choose employment at the highest wage available in one of $N+1$ sectors $S_i$, $i \in [0, N]$. This results in an optimal sorting problem of workers within firms and across sectors.

Each sector $i$ produces a single type of good, whereas goods within a sector $i$ are indistinguishable. A firm within sector $i$ has a production technology $F^i$ which produces goods by pairing workers with skill levels $q$ and $q'$, producing a quantity $F^i(q, q')$. Each production technology $F^i$ is symmetric, homogeneous of degree one and twice continuously differentiable. All firms maximize profits. Firms face perfectly competitive output prices $p_i$ and input prices $w(q)$. Thus each firm takes $p_i$ and $w(q)$ as set by the market and chooses to pair workers of skills $q, q'$ in order to maximize profits $\pi^i(q, q')$ where

$$\pi^i(q, q') \equiv p_i F^i(q, q') - w(q) - w(q') \tag{2.1}$$

Output prices can be considered fixed by world prices exogenous to the economy (after accounting for tariffs) or endogenously fixed by identical homothetic preferences. In both cases, output price ratios are fixed and can be converted to price levels by taking the sector $S_0$ good as numeraire. Since specifying either world prices and tariffs or homothetic preferences both effectively result in assuming levels for prices, I simply take $\{p_i\}$ as given to focus on production behavior and endogenous input prices.

I now formalize the idea that different industries might better use different mixes of skill within their workforce. This is done by adding structure to the production technologies, which will result in a ranking of the skill diversity content across sectors. $S_0$ is a complementary sector where the production technology $F^0$ supermodular in skill inputs. In this case, supermodularity of $F^0$ is equivalent to $F^0_{12}(q, q') \geq 0$ so worker skills are complementary. Grossman and Maggi (2000) show that the revenue maximizing skill pairing within such a supermodular sector is to pair workers of identical skills and this pairing also occurs in the equilibrium of this model. It is only necessary to consider a single supermodular sector $S_0$ as investigation shows one such complementary sector dominates the others and crowds them out in equilibrium.

For each sector $S_i$, with $i > 0$ the production technology $F^i(q, q')$ is submodular. Submodularity of $F^i$ is equivalent to $F^i_{12}(q, q') \leq 0$ so workers skills are substitutes for each other. This implies production becomes mostly dependent on the highest skill worker of the team. Holding total skills $q + q'$ constant, firm revenues in these sectors increase as the skill levels of employed workers diverge.[8] However, the allocation of labor across many sectors creates a conflict: which sectors will get the most diverse workers and how will they be paired? I show that the most important difference between sectors is their input intensities, akin to other models based on factor intensity.[9] Intuitively, the "most submodular" sector can best utilize a diverse workforce and should therefore be allocated the most diverse workers. This intuition bears out in equilibrium, provided the production technologies satisfy the following *diversity ranking* $\succeq$ between sectors.

**Definition** (Diversity Ranking). *$S_i$ is more diverse than $S_j$, written $S_i \succeq S_j$, if and only if $p_i F^i(1,x)/p_j F^j(1,x)$ is strictly increasing for $x \geq 1$.*

If for each pair of sectors $S_i$ and $S_j$ either $S_i \succeq S_j$ or $S_i \preceq S_j$ then $\succeq$ is a *complete diversity ranking*. Intuitively, $S_i \succeq S_j$ says that the relative output of $S_i$ over $S_j$ increases as more diverse workers are employed. In order to make this ranking concrete, consider the ranking in the context of CES production functions. In this case it is straightforward to show that the elasticity of substitution measures the capacity of a technology to efficiently use diverse labor. The diversity ranking induced is given in the following Lemma.

**Lemma 1** (CES Ranking). *Suppose each $F^i$ is CES, specifically $F^i(q,q') \equiv A_i(q^{\rho_i} + q'^{\rho_i})^{1/\rho_i}$. Then $S_i \succeq S_j$ if and only if $\rho_i > \rho_j$ so CES production technologies imply a complete diversity ranking.*

*Proof.* See Appendix. □

The supermodularity of $F^0$ guarantees that $S_i \succeq S_0$ for all $i > 0$. This makes sense as the productivity of a submodular technology $F^i$ increases in diversity, while the productivity of $F^0$ is maximized when identical workers are employed. It is also clear that $\succeq$ is transitive, i.e. $S_i \succeq S_j$ and $S_j \succeq S_k$ implies $S_i \succeq S_k$, so $\succeq$ provides a natural way to rank all of the sectors $S_i$. Consequently, I hereafter maintain Assumption 1.[10]

**Assumption 1.** *The diversity ranking $\succeq$ is complete and $S_N \succeq S_{N-1} \succeq \ldots \succeq S_1 \succeq S_0$.*

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[8]It is worth remarking that the often used Cobb-Douglas form for skill inputs cannot distinguish skill inputs as being supermodular or submodular since the cross partials are always positive, implying a supermodular form. For instance, Ohnsorge and Trefler (2007) model heterogeneous workers with "human capital" and "brawn" attributes which enter production in a Cobb-Douglas fashion, implying complementarity between the attributes across all sectors.

[9]Of course, if some sectors strictly dominate others, for example through a superior Hicks neutral technology or high price, such dominant sectors will soak up all the workers. Consequently the most interesting sectors are those which employ workers in equilibrium and therefore consideration is restricted to producing sectors.

[10]A more empirically motivated criterion which implies completeness (and is almost equivalent) is to define the "elasticity of diversity" for each sector $i$ by $\mathcal{E}_i \equiv \partial \ln F^i(1,x)/\partial x$. Then there is a complete ranking of sectors $S_N \succeq S_{N-1} \succeq \ldots \succeq S_1 \succeq S_0$ so long as $\mathcal{E}_N \geq \mathcal{E}_{N-1} \geq \ldots \geq \mathcal{E}_0$.

I consider competitive equilibria in which all firms maximize profits and economy wide revenue is maximized. To account for the sorting of workers, define two assignment maps $M(q) = q'$ and $\iota(q) = i$. $M(q)$ pairs a worker of skill $q$ to a worker of skill $q'$ while $\iota(q)$ assigns the paired workers $(q, M(q))$ to sector $i$. A competitive equilibrium therefore consists of assignments $M(q), \iota(q)$ and wages $w(q)$ which satisfy the following conditions:

**Definition.** An efficient, competitive equilibrium is a wage schedule $w(q)$ and assignments $M(q), \iota(q)$ which satisfy:

(1) (Profit Maximization) Each triple $(q, M(q), \iota(q))$ is consistent with profit maximization:

$$\pi^{\iota(q)}(q, M(q)) \geq \pi^i(q, q') \quad \forall i, q'$$

(2) (Perfect Competition) Each triple $(q, M(q), \iota(q))$ yields zero profits, $\pi^{\iota(q)}(q, M(q)) = 0$.

(3) (Efficiency) Economy wide revenue is maximized.

Conditions (1) and (2) together guarantee a "no arbitrage" condition for workers. They guarantee that breaking up an equilibrium pair of workers $(q, M(q))$ and reassigning them to different sectors or teams can only yield weakly negative profits. Under conditions (1) and (2) workers cannot be lured away from equilibrium through arbitrage. Condition (3) is a type of equilibrium selection constraint that restricts attention to first best equilibria.

I now provide implications for the allocation of workers to each other through teams $(q, M(q))$ and across sectors through $\iota(q)$. Shortly these allocations will be supported by wages so that the allocative results discussed here are the *equilibrium* skill allocation, fixing production and endogenous sorting in the economy.

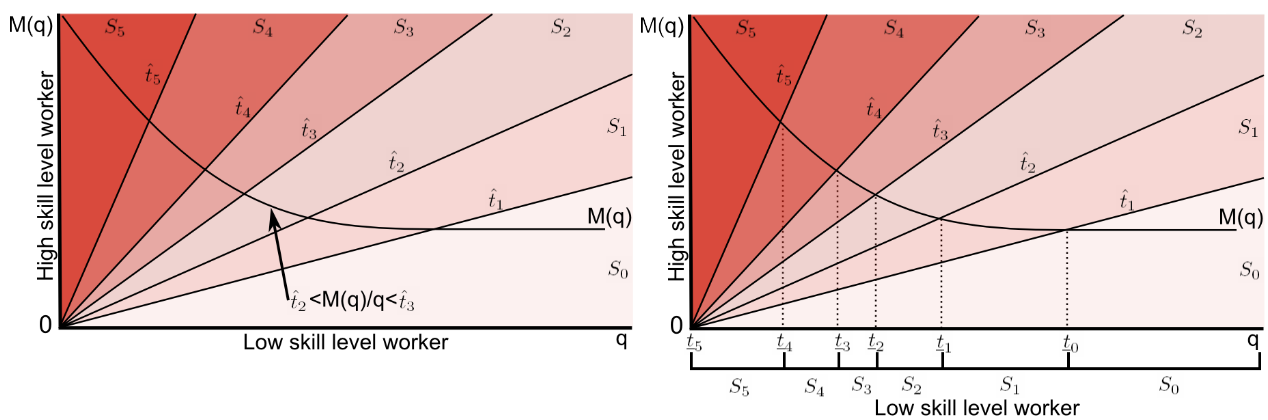
2.2. **Allocation of Skill Diversity by Sector.** I now derive the allocation of workers to teams and sectors. This allocation hinges heavily on the concept of skill dispersion. While there are potentially many ways to measure skill dispersion within a firm, this paper uses the following idea. For an equilibrium matching function $M(q)$, suppose firm $i$ employs workers with skill levels $(q_i, M(q_i))$ and firm $j$ employs the team $(q_j, M(q_j))$, where by convention $q_i \leq M(q_i)$ and $q_j \leq M(q_j)$. Then a natural definition is that firm $i$ employs more diverse labor than firm $j$ if $M(q_i)/q_i \geq M(q_j)/q_j$. If this holds for every firm $i$ in sector $S_i$ and every firm $j$ in sector $S_j$, then sector $S_i$ employs more diverse labor than sector $S_j$. Assumption 1 in fact guarantees that entire sectors can be ranked by the diversity of their labor force using the ranking $\succeq$.

**Proposition 1.** *Under Assumption 1, for any allocation in which firms maximize profits, there are skill ratios $\{\hat{t}_i\}$, increasing in $i$, where if $(q, M(q))$ are assigned to $S_i$ then $M(q)/q \in [\hat{t}_i, \hat{t}_{i+1}]$.*

*Proof.* See Appendix. □

Figure 2.1(a) illustrates Proposition 1 for a hypothetical $M(q)$. For a team $(q, M(q))$, the x-axis tracks the skill level of the low skill worker $q$ and the y-axis plots the skill level of his partner $M(q)$. As the Figure shows, teams where the skill ratio $M(q)/q$ is the highest (close to $q = 0$) must work in the most diverse sector, $S_5$. If the skill ratio within a team decreases slightly, profit maximizing firms will employ the team in the next most diverse sector, $S_4$ and so on. In conclusion, skill intensity ratios play a similar role as factor intensities in the multi-sector Heckscher-Ohlin model, except that in Heckscher-Ohlin there is a unique intensity in each sector whereas here there is a range of skill ratios.

FIGURE 2.1. Skill Ratio Cutoffs by Sector




(A) Skill Ratio Cutoffs by Sector (B) Sector Assignments fixed by $M(q)$

So far, Proposition 1 has pinned down $\iota(q)$ (the equilibrium sector of a worker of skill $q$), given $M(q)$. This follows because when $M(q)$ is known, $M(q)/q$ is known and consequently the skill ratio cutoffs $\{\hat{t}_i\}$ imply the sectoral assignment (2.2).

$$\iota(q) = \begin{cases} N, & M(q)/q \in [\hat{t}_N, \infty) \\ \vdots & \vdots \\ 1, & M(q)/q \in [\hat{t}_1, \hat{t}_2) \\ 0, & M(q)/q \in [1, \hat{t}_1) \end{cases} \tag{2.2}$$

I now discuss how $M(q)$ is determined in order to fix the equilibrium allocation $(q, M(q), \iota(q))$. Getting from the assignment (2.2) to the matching function $M(q)$ proceeds in three steps:

(1) Aggregate the sectors $\{S_i\}_{i\geq 1}$ into a single submodular sector using Assumption 1.

(2) Show in the $S_0$ sector workers pair assortatively, i.e. $M(q) = q$. In the sectors $\{S_i\}_{i\geq 1}$ workers pair according to maximal cross matching.

(3) Show efficiency requires the skill set of workers assigned to the $S_0$ sector is of the form $[\underline{t}, \bar{t}]$. With (2), this fixes $(q, M(q))$ for the sectors $\{S_i\}_{i\geq 1}$. Proposition 1 then assigns workers among the $N$ sectors since $M(q)$ is known.

As these steps are somewhat involved, they are left to the appendix. Here I focus on the resulting form of the matching function $M(q)$. Within the $S_0$ sector, workers are paired with identical skills. As the diversity loving sectors $\{S_i\}_{i>0}$ best utilize pairs of low and high skill workers, it is sensible that the skill range of workers employed in $S_0$ is the range $(\underline{t}_0, \bar{t}_0)$ with

$$\underline{t}_0 \leq \text{median skill level of population} \leq \bar{t}_0$$

So the "middle of the road" workers are employed in $S_0$. I now detail an intuitive argument for fixing the cutoffs $(\underline{t}_0, \bar{t}_0)$. The revenue generated per unit of skill in the $S_0$ sector must be at least as high as in any other sector. This results in skill cutoffs $(\underline{t}_0, \bar{t}_0)$ as fixed in Equation (2.3):

$$\text{(2.3)} \qquad \underbrace{p_0 F^0(1,1)/2}_{\text{Revenue per unit of skill in } S_0} = \underbrace{p_1 F^1(\underline{t}_0, \bar{t}_0)/(\underline{t}_0 + \bar{t}_0)}_{\text{Revenue per unit of skill in least submodular sector}}$$

The remaining workers are employed in pairs $(q, m^S(q))$, where $m^S(q)$ is the maximal cross matching of workers unassigned to $S_0$. The function $m^S(q)$ pairs the highest and lowest skilled workers first, the second highest skilled with the second lowest skilled, and so forth. Formally, letting $\Phi^S(t) \equiv \int_0^t \mathbf{1}_{[0,\underline{t}]\cup[\bar{t},\infty)} d\Phi$ be the distribution of workers employed in the sectors $\{S_i\}_{i>0}$, the maximal cross matching $m^S(q)$ must satisfy $\Phi^S(q) + \Phi^S(m^S(q)) = 1$. Therefore $m^S(q)$ can be written $m^S(q) = (\Phi^S)^{-1}(1 - \Phi^S(q))$. The final allocation is therefore given by Equation (2.4), where the skill cutoffs $\underline{t}_i, \bar{t}_i$ are fixed by $(\underline{t}_0, \bar{t}_0)$ and Equation (2.2).

$$\text{(2.4)} \qquad M(q) \equiv \begin{cases} m^S(q), & q \leq \underline{t}_0 \\ q, & q \in (\underline{t}_0, \bar{t}_0) \\ m^S(q), & q \geq \bar{t}_0 \end{cases} \qquad \iota(q) = \begin{cases} N, & q \in (0, \underline{t}_{N-1}] \\ \vdots & \vdots \\ 0, & q \in (\underline{t}_0, \bar{t}_0) \\ \vdots & \vdots \\ N, & q \in [\bar{t}_N, \infty) \end{cases}$$

Although the geometry of Equation (2.4) is simple, as illustrated in Figure 2.1(b). Once the worker match function $M(q)$ is fixed by Equation (2.4), Proposition 1 then pins down $\iota(q)$ as in Figure 2.1(a). This induces the skill cutoff ranges for each sector $S_i$ of low skill workers $q \in (\underline{t}_i, \underline{t}_{i-1}]$ paired with high skill workers $M(q) \in [M(\underline{t}_{i-1}), M(\underline{t}_i))$. These allocations are next supported by wages.

2.3. **Wages Under Multisector Diversity.** Up until now, I have only derived necessary conditions for a competitive equilibrium, which fix a unique assignment of workers to firms. I now construct wages across sectors which support this assignment so that the allocation of workers described above is a competitive equilibrium. The assumption of multiple sectors will yield differing skill premiums across sectors for both low and high skill workers. This highlights a principle of worker symbiosis and parasitism: the division of surplus between high and low skill workers depends on the joint skills of the workers, as well as aggregate labor market conditions.

For brevity I only consider wages $w(q)$ which are continuous and differentiable in the interior of each sector. Since there is a continuum of firms, competitive behavior must drive profits to zero. In the complementary $S_0$ sector where $q \in (\underline{t}_0, \bar{t}_0)$, workers are paired with identical skills so the zero profit condition implies that

$$w(q) + w(q) = p_0 F^0(q, q) = p_0 F^0(1, 1) \cdot q$$

So for $q \in (\underline{t}_0, \bar{t}_0)$, $w(q) = w_0 q$ where $w_0 \equiv (p_0/2) \cdot F^0(1, 1)$. However, wages in the sectors $\{S_i\}_{i \geq 1}$ are more interesting. This is because a firm in $\{S_i\}_{i \geq 1}$ must lure both high and low skill workers away from $S_0$ by offering wages above $w_0 q$, and I term the additional wages required a *diversity premium*. Notice that the diversity premium is different from a skill premium: both low and high skill workers receive a diversity premium in addition to $w_0 q$ which increases in skill.

Next I construct wages recursively starting with sector $S_1$ and proceeding to $S_2, S_3$, and so on. Workers in $S_1$ have skills $q \in (\underline{t}_1, \underline{t}_0] \cup [M(\underline{t}_0), M(\underline{t}_1))$ and once wages are found for $q \in (\underline{t}_1, \underline{t}_0]$, the zero profit condition fixes wages for $q \in [M(\underline{t}_0), M(\underline{t}_1))$. First, a worker with skill $\underline{t}_0$ employed at the cusp between $S_0$ and $S_1$ must be employable in either sector, so $w(\underline{t}_0) = w_0 \underline{t}_0$ since $w_0$ is the wage rate paid per unit of skill in the $S_0$ sector. Second, for any wages $w$ the first order necessary conditions for profit maximization by firms must hold. The profit maximization conditions are

Equations (2.5) for $i \geq 1$.[11]

$$p_i F_1^i(q, M(q)) = w'(q) \quad \text{and} \quad p_i F_2^i(q, M(q)) = w'(M(q)) \tag{2.5}$$

Thus, for $q \in (\underline{t}_1, \underline{t}_0]$, equilibrium requires that $w'(q) = p_1 F_1^1(q, M(q))$ and integrating from $\underline{t}_0$ down to $q < \underline{t}_0$ gives

$$w(q) = w(\underline{t}_0) - \int_q^{\underline{t}_0} p_1 F_1^1(s, M(s)) ds, \qquad q \in (\underline{t}_1, \underline{t}_0]$$

Proceeding inductively, the recursive relationship for wages across sectors at skill levels below the median are given by

$$w(q) = \begin{cases} w_0 q, & q \in (\underline{t}_0, \bar{t}^0) \\ w(\underline{t}_0) - \int_q^{\underline{t}_0} p_1 F_1^1(s, M(s)) ds, & q \in (\underline{t}_1, \underline{t}_0] \\ w(\underline{t}_1) - \int_q^{\underline{t}_1} p_2 F_1^2(s, M(s)) ds, & q \in (\underline{t}_2, \underline{t}_1] \\ \vdots & \vdots \\ w(\underline{t}_{N-1}) - \int_q^{\underline{t}_{N-1}} p_N F_1^N(s, M(s)) ds, & q \in (0, \underline{t}_{N-1}] \end{cases} \tag{2.6}$$

Equation (2.6) highlights the fact that the wage an individual receives depends not only on his raw skill but also the wages of those in less diverse sectors. Consider a worker who is the lower skill member of a team in sector $S_i$, $i \geq 0$. His wage is a sector specific wage $w(\underline{t}_{i-1})$ plus an integral related to the curvature of $F^i$. Since $w(\underline{t}_{i-1})$ is tied to wages in sector $S_{i-1}$, wages in sector $S_i$ are recursively "pegged" to wages in $S_{i-1}$ and so on up to $S_0$.

Although "pegged", wages given by Equation (2.6) have the usual properties. For instance, they are strictly increasing in skill. Wages are also bounded below by the $S_0$ shadow wages of $w_0 q$ since the sectors $\{S_i\}_{i\geq 1}$ must lure away workers from $S_0$. Wages are also convex in skill for the following economic reason: wages are fixed by the profit maximization Equations (2.5). Through duality wages are also fixed by cost minimization in skill choice, implying some variety of convexity. The specific property is that $w'(q)$ is (essentially) increasing.[12] These properties of the wage structure are summarized in Proposition 2.

**Proposition 2.** *The wage schedule $w(q)$ has the following properties:*

[11]Other approaches to obtaining wages for heterogeneous workers in the absence of firm competition include bargaining between workers (e.g. Delacroix, 2003) and the $N$-worker coalitional model of (Sherstyuk, 1998).

[12]At the cusp levels of skill $\{\underline{t}_i\}$ and $\{M(\underline{t}_i)\}$, $w'(q)$ is not defined but essentially increases discontinuously as the cusp from lower to higher skill is crossed. Technically, the one sided derivatives of $w(q)$ are defined at the cusps $\{\underline{t}_i\}$ and $\{M(\underline{t}_i)\}$ and the left hand limit is less than the right hand limit.

(1) $w(q)$ *is strictly increasing and convex.*
(2) $w(q)$ *is bounded below by the* $S_0$ *shadow wages* $w_0 q$.
(3) $w'(q)$ *is increasing where defined, and elsewhere* $\lim_{q \to x_-} w'(q) \leq \lim_{q \to x_+} w'(q)$.

*Proof.* See Appendix. □

The wage schedule of Equation (2.6) supports a competitive equilibrium. In order to show this, it is sufficient to guarantee that if a firm deviates from a skill pairing $(q, M(q))$, then the firm obtains non-positive profits. This can be done by exploiting the convexity of the wage structure as given by Proposition 2. The basic argument relies on checking the first order conditions of any firm who deviates from equilibrium. Leaving details to the appendix, the fact that wages support the proposed equilibrium is stated as Proposition 3

**Proposition 3.** *Wages given by Equation (2.6) support an efficient competitive equilibrium.*

*Proof.* See Appendix. □

This section has detailed the allocation of workers to both teams and sectors in any efficient, competitive equilibrium. The next section proceeds to consider the effects of price and endowment changes in general equilibrium, deriving variations of the Rybczynski and Stolper-Samuelson theorems.

## 3. General Equilibrium Implications

This section extends canonical results of the trade literature, the Rybczynski and Stolper-Samuelson theorems. The Rybczynski theorem predicts how production changes in response to a change in endowments. In this model setting, the most interesting changes in the skill distribution are those which in some way change the diversity of skills in the workforce. Here I provide a definition of such "changes in diversity" and show that increases in availability of a sector's inputs increase the mass of workers employed in the sector. Associated with the Rybczynski theorem is the "magnification effect", which means that an increase in sector endowments results in a more than proportional increase in output. In this section, I show changes in the skill distribution cause spillovers of workers across sectors. Such spillovers may amplify or eliminate magnification effects, conditional on economy wide endowments.

I then present two variations on the Stolper-Samuelson theorem. The first shows an increase in output price increases total factor returns in a sector, namely total wages per worker pair. This increase in wages is independent of endowments, in the spirit of the Stolper-Samuelson theorem (e.g. the review in Lloyd, 2000). Unlike the Stolper-Samuelson theorem, an increase in output price also reallocates workers to the more profitable sector.[13] In contrast, the second variation of the Stolper-Samuelson theorem shows that surplus sharing within worker teams generally changes in response to price changes. In contrast to Grossman and Maggi (2000), the wages of either member of a worker team may decrease once a sector booms, providing a new channel from growth to inequality.

3.1. **Rybczynski Under Skill Diversity.** Characterizing precisely what diversity means for a given distribution of skills $\Phi$ is a difficult task. This is because without additional structure on $\Phi$ it is hard to predict equilibrium changes, or come to terms about the meaning of a particular change. I settle on a "results based" definition which allows for a Rybczynski type prediction, although there could easily be others. To simplify the exposition I first consider only two sectors $S_0$ (which pairs identical workers) and $S_1$ (which pairs diverse workers). Having seen the mechanics of the two sector case, the multisector Rybczynski theorem is presented.

Define a change in a skill endowment $\Phi$ to a new skill endowment $\tilde{\Phi}$ a *skill shock* and normalize the mass of labor to one for both distributions. I define three types of skill shifts from $\Phi$ to $\tilde{\Phi}$: Low Skill Shocks (LSS), High Skill Shocks (HSS) and Diverse Skill Shocks (DSS).

**Definition** (Skill Shocks)**.** Let $\Phi$ and $\tilde{\Phi}$ be two skill distributions and $(\underline{t}_0, \bar{t}_0)$ the range of worker skills employed in $S_0$ under $\Phi$.

(1) (LSS) $\Phi(\underline{t}_0) \leq \tilde{\Phi}(\underline{t}_0)$ and $\tilde{\Phi}(M(\underline{t}_0)) = \Phi(M(\underline{t}_0))$.
(2) (HSS) $\Phi(\underline{t}_0) = \tilde{\Phi}(\underline{t}_0)$ and $\tilde{\Phi}(M(\underline{t}_0)) \leq \Phi(M(\underline{t}_0))$.
(3) (DSS) $\Phi(\underline{t}_0) \leq \tilde{\Phi}(\underline{t}_0)$ and $\tilde{\Phi}(M(\underline{t}_0)) \leq \Phi(M(\underline{t}_0))$.

In the case of a Low Skill Shock, the pool of workers in the low skill range of $S_1$, $q \in (0, \underline{t}_0]$, expands at the expense of $S_0$. An High Skill Shock is similar. In both cases, the addition of workers to $S_1$ will increase the mass of workers employed in $S_1$ in the post-Shock equilibrium. A Diverse Skill Shock is the simultaneous combination of a Low Skill Shock and High Skill Shock. A special case of a DSS is a "median preserving spread." The fact that all of these skill shocks increase the mass of workers in $S_1$ is recorded as Lemma 2.

**Lemma 2.** *Let $\Phi$ and $\tilde{\Phi}$ be two skill distributions and $\underline{t}_0, \tilde{\underline{t}}_0$ be the respective skill cutoffs in equilibrium. Consider a Skill Shock from $\Phi$ to $\tilde{\Phi}$. Then:*

(1) *Following a DSS the mass of workers in $S_1$ increases.*
(2) *Following a HSS, the mass of $S_1$ workers increases and $\tilde{\underline{t}}_0 \geq \underline{t}_0$ and $M(\tilde{\underline{t}}_0) \geq M(\underline{t}_0)$.*

[13]Upward sloping supply and demand for labor in each sector is an important feature missing from classical trade theory, as discussed by Ohnsorge and Trefler (2007). Details are relegated to the appendix.

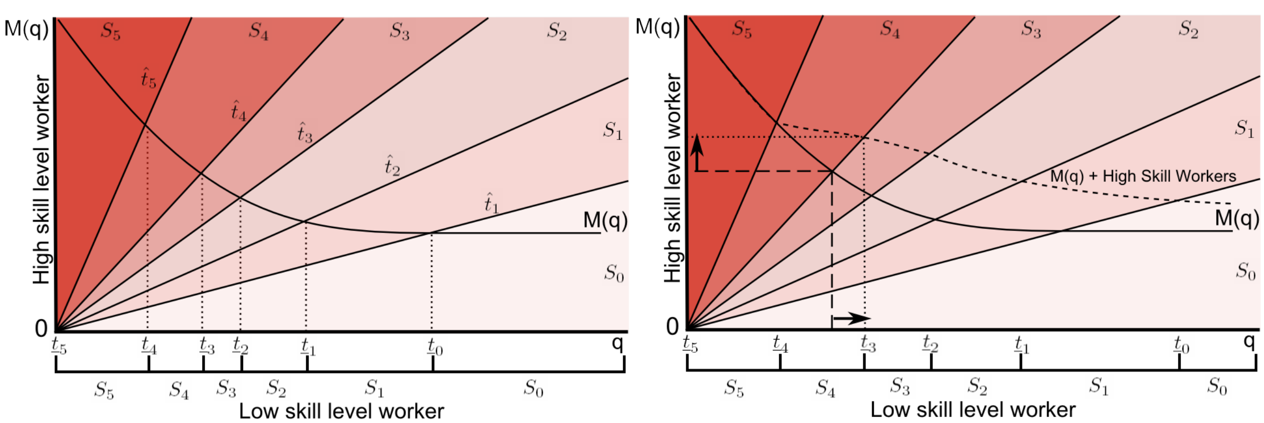
(3) *Following a LSS, the mass of $S_1$ workers increases and $\tilde{\underline{t}}_0 \leq \underline{t}_0$ and $M(\tilde{\underline{t}}_0) \leq M(\underline{t}_0)$.*

*Proof.* See Appendix. □

The Lemma also shows that changes in the skill distribution also change the skill range $(\underline{t}_0, \bar{t}_0)$ of workers in $S_0$ and similarly $S_1$. Thus re-matching of workers creates spillovers of workers across sectors. This effect can more be interpreted in an economy with more than two sectors with the aid of Figures 3.1(a-b). Figure 3.1(a) displays the allocation of workers across sectors before a skill shock, while Figure 3.1(b) displays the reallocation after the endowment of high skill workers increases. Figure 3.1(b) shows that adding high skill workers to the economy shifts up the matching function $M(q)$. This is because low skill workers can be matched with new, higher skill team members, increasing the ratio of skills per team.

Once $M(q)$ shifts up, it determines new skill ranges for each sector. This can be seen comparing the two Figures, as emphasized by arrows in Figure 3.1(b). As shown, the $S_4$ sector expands and the range of low skill workers $[\underline{t}_4, \underline{t}_3]$ employed in $S_4$ increases by shifting $\underline{t}_3$. To restore skill diversity in the sector, $M(\underline{t}_3)$ must rise so all workers in $S_4$ are more skilled on average. Thus while new low skill workers are admitted to $S_4$, their inclusion "raises the bar" for $S_4$ workers and some high skill $S_4$ workers will spill over into $S_3$.

FIGURE 3.1. Structural Changes Across Sectors




(A) Sector Assignments fixed by $M(q)$

(B) Re-matching from a High Skill Shock

By keeping track of how workers are employed as in Figures 3.1(a-b), I arrive at a parallel version of the original Rybczynski Theorem. Consider an economy with an initial skill endowment $\Phi$ and a mass of workers $L$, to which is added a distribution of workers $\Psi$ with mass $P$. This added endowment of workers might be migrants, newly trained workers, or the result of outsourcing.

To mirror the concept of a Diverse Skill Shock, further assume $\Psi$ is only composed of workers currently working as low or high skill workers in one sector $S_i$. Formally, $\Psi$ has support on some set $[\underline{t}_i, \underline{t}_{i-1}] \cup [M(\underline{t}_{i-1}), M(\underline{t}_i)]$. Once $\Psi$ is pooled with the initial endowment $\Phi$, the supply of workers to the sector $S_i$ has increased. The new equilibrium allocation will result in both more workers employed in $S_i$, but more precisely, the total output in the sector will increase. This result is summarized as Proposition 4.

**Proposition 4.** *[Rybcszynski under Diversity] If the endowment of skills specific to a sector increases then output of the sector increases.*

*Proof.* See Appendix. □

A stronger form of the Rybczynski theorem is the "magnification effect," meaning that an increase in sector specific endowments results in a more than proportional increase in output. As seen above, the addition of workers to the economy causes spillovers of workers across sectors. Such spillovers preclude any unconditional magnification effect because added endowments can contain a high percentage of workers who will spillover into other sectors. For instance, suppose the skill distribution is $\Phi$ and a distribution of workers $\Psi$ migrate to join the $\Phi$ workers. To be specific, say the new $\Psi$ workers have low skills in the range $[\underline{t}_i, \underline{t}_{i-1}]$, those skills used by the $S_i$ sector. A Heckscher-Ohlin setting would predict a boom in the $S_i$ sector through magnification. However, in this model, the outcome depends on the supply of high skill workers available to work with the new migrants in $S_i$. If the host country's distribution $\Phi$ is thin on workers in the range $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$, then new migrants will force spillovers into other sectors successively; first the $S_{i-1}$ sector then $S_{i-2}$, etc. Conversely, if the host country is abundant in the $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$ skill range, then magnification effects could be large since the migrants might be very efficient additions to the sector.

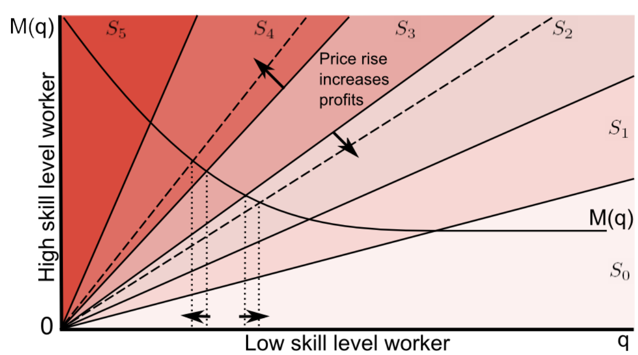
These results are in sharp contrast to model settings which ignore the team aspect of production. In particular, migration and education policies designed to support particular industries or growth should consider the relative scarcity of differently skilled workers. Policies targeting groups of fairly homogeneous skills (refugees, the uneducated, specialist workers) should keep in mind the availability of their potential co-workers as this may influence their final industry of employment. In addition, the distribution of skills impacts relative wages within teams which I now turn to.

### 3.2. Stolper-Samuelson under Skill Diversity.

The Stolper-Samuelson Theorem, along with modifications and extensions, has a long history (e.g. Neary, 2004). I now develop two Theorems in the spirit of Stolper-Samuelson with surprising implications. The first is in line with the canonical result that a rise in the world price $p_i$ of a good increases returns to the intensive factor of a sector. In this case I consider the aggregate wages of the two workers $(q, M(q))$ paired in a sector relative to all other sectors. The second version I consider concerns the wage distribution between low and high skill workers. A rise in a sector's good price will raise aggregate wages in the sector, but will also change relative wages within worker teams due to the endogenous reallocation of workers across sectors. I show that the indirect reallocation effect of worker sorting may dominate the direct "Stolper-Samuelson" effect of a rise in good price. In particular, the welfare implications of Grossman and Maggi (2000) depend on the assumption of a symmetric skill distribution and are overturned once empirically motivated distributions are considered.

For the first Stolper-Samuelson result, consider a rise in output prices $p_i$ in a diverse sector $S_i$ with $i \geq 1$. In contrast to a Heckscher-Ohlin setting in which all outputs would remain constant, in this model supply is upward sloping. Increases in $p_i$ make production in $S_i$ more profitable, thereby expanding the range of worker teams employable in the sector. This effect is depicted in Figure 3.2(a). As more worker teams become profitable to employ the sector expands, stealing away worker teams from adjacent sectors.[14]

FIGURE 3.2. Stolper Samuelson Effects




(A) Effect of Increased Output Price

Now consider the effect of output prices on wages. If the output price $p_i$ rises to $\tilde{p}_i$, then for any team $(q, M(q))$ employed in $S_i$, revenues $p_i F^i(q, M(q))$ rise. Perfect competition implies that firm revenue equals $w(q) + w(M(q))$ and therefore total wages increase with output price. At the same

[14]Note there is only one instance in which a change in output prices can affect the composition a worker team. This is when the team is either created by pulling $(q, M(q))$ from the $S_0$ where workers are paired with identical skills, or a team is destroyed by moving from $S_1$ into $S_0$.

time, "new" teams $(q, M(q))$ who join $S_i$ after the price increase also receive $w(q) + w(M(q))$ equal to firm revenue. As revenue is maximized in equilibrium, it must be that the "new" team receives total wages at least as high as the revenue generated in their "old" employment. Finally, revenues remain unchanged outside of $S_i$, so total wages paid by any firm outside of $S_i$ do not change. Therefore relative wages $w(q) + w(M(q))$ for incumbent and new employees in $S_i$ increase. These effects of a rise in output price are recorded as Proposition 5.

**Proposition 5.** *If output price $p_i$ increases and a worker team is employed in $S_i$ then total wages for the team increase relative to every other sector. In addition, output and employment in $S_i$ expand.*

In contrast to models where inputs are mobile but homogeneous, Proposition 5 shows that output price effects on total factor returns are quarantined to the affected sector. What is surprising is that despite this quarantine on *total* wages for a team, individual wages far from the affected sector change. This occurs because worker reallocation changes the division of total wages within a team. Thus there is a second Stolper-Samuelson effect on input prices which I now consider.

3.3. **Stolper-Samuelson within Sectors.** Although total factor returns increase following an increase in good price, it is not necessarily true that wages for all workers in a team rise. In fact, the wages of a high or low skill worker employed in a "booming" sector may fall. To see why, consider a low skill worker with $q \in (\underline{t}_i, \underline{t}_{i-1}]$ in sector $S_i$, $i \geq 1$, and denote his wages by $w_i(q)$. From the wage structure derived in Equation (2.6), $w_i(q)$ is given by

$$w_i(q) = w(\underline{t}_{i-1}) - \int_q^{\underline{t}_{i-1}} p_i F_1^i(s, M(s))ds$$

Thus $w_i(q)$ is implicitly a function of both output price $p_i$ and wages $w(\underline{t}_{i-1})$.

Increases in sector output price $p_i$ will have two effects on wages: direct effects through $p_i$ within the sector and indirect effects through $\underline{t}_{i-1}$ by drawing new workers into the sector. As seen in Figure 3.2(a), the indirect effect will expand the skill range $(\underline{t}_i, \underline{t}_{i-1}] \cup [M(\underline{t}_{i-1}), M(\underline{t}_i))$ of workers employed in $S_i$. The two effects are decomposed in Equation (3.1).

$$dw_i(q)/dp_i = \underbrace{\partial w_i/\partial p_i}_{\text{Direct Effect on Wages}} + \underbrace{\partial w_i/\partial \underline{t}_{i-1} \cdot d\underline{t}_{i-1}/dp_i}_{\text{Indirect Effect of New Workers}}$$

$$(3.1) \qquad = \underbrace{-\int_q^{\underline{t}_{i-1}} F_1^i(s, M(s))ds}_{(-)} + \underbrace{\left[p_{i-1}F_1^{i-1}(\underline{t}_{i-1}, M(\underline{t}_{i-1})) - p_i F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))\right]}_{(+)} \cdot \underbrace{d\underline{t}_{i-1}/dp_i}_{(+)}$$

The decomposition of Equation (3.1) is striking because the direct effect of an increase in output price is to decrease wages for low skill workers.[15] Paradoxically, when revenues rise, low skill wages fall in the absence of entry by new workers. These lost wages "trickle up" to each low skill worker's team member. From the perspective of the firm, the importance of high skill workers increases with revenues since they are the essential ingredient to a diversity loving technology. For high skill workers, the direct effect of an increase in output prices is to capture all new revenues, and also some wages formerly paid to low skill workers.

This direct "trickle up" effect is counteracted by the indirect effect from new workers entering the sector. This indirect effect can be understood in terms of the relative supply of low and high skill workers to the sector. If low skill workers are abundant, the indirect effect will be small since even a small increase in wages will draw many low skill workers to the sector. Potential low skill entrants have skills in the range $(\underline{t}_{i-1}, \underline{t}_{i-1} - \epsilon)$ for small $\epsilon$ and the mass of such entrants is approximately $\Phi'(\underline{t}_{i-1})$. Therefore large values of $\Phi'(\underline{t}_{i-1})$ attenuate the indirect effect. In contrast, if high skill entrants are abundant, as indicated by large values of $\Phi'(M(\underline{t}_{i-1}))$, then the indirect effect will be strengthened. These relationships of relative scarcity are seen by rewriting the Indirect Effect of Equation (3.1) as

$$\text{Indirect Effect of New Workers} = \frac{F^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})) \cdot M(\underline{t}_{i-1})/\underline{t}_{i-1}}{M(\underline{t}_{i-1})/\underline{t}_{i-1} + \underbrace{\Phi'(\underline{t}_{i-1})/\Phi'(M(\underline{t}_{i-1}))}_{\text{Relative Scarcity of Skills}}} \tag{3.2}$$

Finally, if output price $p_i$ rises, the workers who enter $S_i$ have left the adjacent sectors $S_{i+1}$ and $S_{i-1}$. This creates an opposite indirect effect on wages in adjacent sectors. Thus if prices $p_i$ increase, low skill wages in $S_{i+1}$ and $S_{i-1}$ decrease as given by Equation (3.2), while high skill wages increase. These direct and indirect effects on wages emphasize the joint role of prices, technology and labor supply in predicting wages in diversity loving sectors. This second version of the Stolper-Samuelson theorem is stated in Proposition 6.

**Proposition 6.** *Suppose output prices $p_i$ rise in a sector $S_i$ with $i \geq 1$. The effects on wages can be decomposed into a direct price effect and an indirect entry effect where:*

(1) *The direct effect increases high skill wages proportionally more than revenues and decreases low skill wages.*
(2) *The indirect effect increases low skill wages and decreases high skill wages. This effect is magnified when there are few low skill entrants or many high skill entrants.*

[15] Equation (3.1) holds for workers in $S_1$ by interpreting $p_0 F_1^0(\underline{t}_0, M(\underline{t}_0)) = p_0 F_1^0(\underline{t}_0, \underline{t}_0)$.

In contrast to the rich wage dynamics just discussed, the $S_0$ sector operates identically to a "piece-rate" sector with constant wages $w_0 = p_0 F^0(1,1)/2$ per unit of skill. Here the effects of a rise in output price are more conventional. The direct effect of an increase in $p_0$ is simply $\partial w_0 q / \partial p_0 = w_0 q / p_0$, so wages in $S_0$ increase. The range of skills employed $[\underline{t}_0, \bar{t}_0]$ have no influence on wages in the $S_0$ sector, so the indirect effect of a price rise is zero. Therefore wages in $S_0$ rise exactly in proportion to the increase in $p_0$, and gains are equally captured by all workers.

I now compare the Stolper-Samuelson theorem in this model to Grossman and Maggi's model, who consider symmetric skill distributions. In the context of Stolper-Samuelson, symmetric skill distributions cause the indirect effect to dominate the direct effect. This occurs because under symmetry the relative scarcity component of the indirect effect is neutralized. To see this note that if $\Phi$ is symmetric, then $\Phi'(q) = \Phi'(M(q))$ for all $q$ so the indirect effect reduces to

$$\text{Indirect Effect under Symmetry} = \frac{F^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})) \cdot M(\underline{t}_{i-1})/\underline{t}_{i-1}}{M(\underline{t}_{i-1})/\underline{t}_{i-1} + \underbrace{1}_{\text{Relative Scarcity of Skills}}}$$

In contrast, under general skill distributions the scarcity term $\Phi'(\underline{t}_{i-1})/\Phi'(M(\underline{t}_{i-1}))$ can range from zero (where the indirect effect dominates) to infinity (where the direct effect dominates). In this light, perhaps it is less surprising that the restriction of the scarcity term to unity causes the indirect effect to dominate. Leaving algebraic details to the appendix, the implications of symmetric skills are recorded as Proposition 7.

**Proposition 7.** *If the skill distribution is symmetric then the indirect Stolper-Samuelson effect dominates the direct effect. An increase in output price raises low skill wages.*

*Proof.* See Appendix. □

Contrasting Propositions 6 and 7 shows that considering general asymmetric skill distributions can have surprising implications.[16] Introducing general skill distributions into worker sorting models is important to capture empirically motivated skill distributions, such as those from wage regressions. Also of empirical interest is the relationship between worker scarcity and predicted wage changes. This relationship depends on both the immediate sector and adjacent sectors which employ similarly skilled workers. In particular, this model shows that heterogeneous worker teams may generate unintended consequences with regard to inequality. The Stolper-Samuelson

[16] Analytical wage effects across two sectors can be worked out for the Log-Normal and Pareto distributions.

results show that subsidization of an industry will cause the sector to expand, but low skill wages may fall as high skill workers in the sector are disproportionately rewarded.

Having laid out the model and described several testable implications, I now step back and look for structural evidence of the production side. An essential assumption of this setting is the presence of sectors which benefit from skill diversity in varying degrees. Does productivity vary by team composition holding average human capital constant? If so, does skill diversity affect productivity in different ways across sectors? I address these questions in the next section.

## 4. Estimation: Production and Skill Mix

This section tests for evidence of production technologies which benefit from skill diversity or skill similarity. The model above (Lemma 1) shows that the CES form allows a clear interpretation of productivity differences arising from skill complementarity or skill dependence.[17] In this section I first develop a simple, but novel, specification to estimate productivity differences explained by skill mix, then incorporate controls for differences in foreign and domestic markups. I then discuss the econometric approach for productivity estimation and describe the data used. The section concludes with the production estimation results which show that most sectors vary in productivity with skill diversity.

4.1. **Productivity Specification.** I begin with a general form $Y_i = G(K_i, L_i, \psi_i)$ which relates value added output $Y_i$ for a firm $i$ to capital $K_i$, labor $L_i$ and a firm specific skill mix measure $\psi_i$. While the model above considers worker teams with two types of workers, more generally firms employ several different types of workers. Interpreting the skill mix measure $\psi_i$ more broadly as the distribution of workers in the firm, this paper uses data which differentiates between four different educational levels of workers. Consequently $\psi_i$ is defined as a $k-$dimensional distribution of skill levels, $\psi_i = \begin{pmatrix} \psi_{1,i} & \dots & \psi_{k,i} \end{pmatrix}$ measured as a percentage of total employment by the firm. Although $\psi_i$ denotes the distribution of worker skills rather than skill levels of discrete workers, a diversity and similarity interpretation of skill mix holds as developed shortly.

[17] The closest work I am aware of is Iranzo et al. (2008) who examine skill dispersion and firm productivity using Italian employer-employee panel match data. They find that productivity is associated with a higher overall dispersion of skills and evidence of complementarity between production and non-production workers.

Modeling the effect of $\psi_i$ as a labor augmenting factor $\phi(\psi_i)$, I rewrite the production function using a neo-classical production function $F$

$$Y_i = G(K_i, L_i, \psi_i) = F(K_i, \phi(\psi_i) \cdot L_i) \tag{4.1}$$

Letting $F$ be the Cobb-Douglas form $F(K, L) = AK^\alpha L^\beta$ and allowing both $F$ and $\phi$ to be specific to each sector $S$, denoted $F_S$ and $\phi_S$, value added output $Y_i$ for a firm $i$ in sector $S$ is given by Equation (4.2).

$$Y_i = F_S(K_i, \phi_S(\psi_i) \cdot L_i) = A_S K_i^{\alpha_S} L_i^{\beta_S} \phi_S(\psi_i)^{\beta_S} \tag{4.2}$$

In Equation (4.2), the contributions of capital and labor are assumed to be sector specific, with both sector specific and firm idiosyncratic productivity terms $A_S$ and $\phi_S(\psi_i)^{\beta_S}$.

I now connect the productivity term $\phi_S(\psi_i)^{\beta_S}$ of Equation (4.2) to the composition of skills employed in the firm via supermodularity and submodularity in skill inputs. This is done by assuming $\phi_S$ be the CES form, with a sector specific substitution parameter $\rho_S$. Specifically, I assume $\phi_S$ takes the form

$$\phi_S(\psi_i) = \left( \frac{1}{k} \psi_{1,i}^{\rho_S} + \frac{1}{k} \psi_{2,i}^{\rho_S} + \ldots + \frac{1}{k} \psi_{k,i}^{\rho_S} \right)^{1/\rho_S} \tag{4.3}$$

The CES specification (4.3) parallels that of the model above, but with a twist that $\psi_i$ is the distribution of workers across skill groups instead of the skill levels of individual workers. To help fix ideas about the meaning of $\rho_S$, consider the limiting cases as $\rho_S \to \infty$ and $\rho_S \to -\infty$. As is well known, these limiting cases of the CES are

$$\lim_{\rho_S \to \infty} \phi_S(\psi_i) = \max\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\} \quad \text{Similarity Loving} \tag{4.4}$$

$$\lim_{\rho_S \to -\infty} \phi_S(\psi_i) = \min\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\} \quad \text{Diversity Loving} \tag{4.5}$$

Therefore a firm in a sector with $\rho_S > 1$ will approximately choose a $\psi_i$ which maximizes $\max\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\}$ subject to prevailing wage rates. Such a choice of $\psi_i$ is typified by vectors of the form $\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$, etc. which is to say a mix of workers with similar skill levels. Thus firms in a sector with $\rho_S > 1$ benefit from skill similarity and correspond to sector $S_0$ of the model.

Conversely, in a sector with $\rho_S < 1$, firms will pick $\psi_i$ to roughly maximize $\min\{\psi_{1,i}, \psi_{2,i}, \ldots, \psi_{k,i}\}$. This implies a mix of workers with diverse skills, and a representative choice of $\psi_i$ would be

$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$. Therefore sectors with $\rho_S < 1$ correspond to the skill diverse sectors $S_i$ for $i \geq 1$ of the model. Finally, at $\rho_S = 1$, $\phi_S(\psi_i)$ collapses to $\sum \psi_i = 1$, implying that differences in skill mix have no influence on productivity. Thus $\phi_S$ nests the null hypothesis that skill mix is irrelevant for productivity at $\rho_S = 1$. To summarize, for sectors where $\rho_S < 1$, skill diversity increases productivity. For $\rho_S > 1$, skill diversity decreases productivity.

Combining the specification (4.2) with the basic production Equation (4.2) and adding an idiosyncratic productivity term $\epsilon_i$ for estimation yields

$$Y_i = F_S(K_i, \phi_S(\psi_i) \cdot L_i)\epsilon_i = \underbrace{A_S K_i^{\alpha_S} L_i^{\beta_S}}_{\text{Cobb-Douglas}} \cdot \underbrace{\left(\sum \psi_{e,i}^{\rho_S}\right)^{\beta_S/\rho_S}}_{\text{Explained Productivity}} \cdot \epsilon_i \tag{4.6}$$

The specification (4.6) allows identification of "diversity loving" and "similarity loving" technologies through estimation of $\rho_S$. The relative magnitudes of $\rho_S$ rank sectors as $S_0, \ldots, S_N$. In addition, the specification allows for testing against the null hypothesis $\rho_S = 1$, which implies $\left(\sum \psi_{e,i}^{\rho_S}\right)^{1/\rho_S} = 1$. Equation (4.6) shows that the null hypothesis $\rho_S = 1$ corresponds to a standard neo-classical form with no labor augmentation from skill mix. This paper uses the term $\phi_S(\psi_i)^{\beta_S} = \left(\sum \psi_{e,i}^{\rho_S}\right)^{\beta_S/\rho_S}$ to explain intrasector productivity and later, export propensity.

4.2. **Econometric considerations and Data Description.** In the data, only the value of sales generated by a firm are observed instead of the direct quantities of different goods produced. Fernandes and Pakes (2008) have also used a similar data set and emphasize that the data allows for estimation of the "sales generating function" rather than the production function. For brevity I stick to the label "production function." Since sales are observed, it is important to control for the effect of producing for both the domestic and foreign market. For this reason, as further developed in the appendix, I include sector level controls ($M_S$) for markups on firm level exports ($X_i$).[18] Since the firms considered produce in developing countries, generally one should expect $M_S \geq 0$. This is because of the high value exported to the economic North suggesting $M_S^X \geq M_S^D$ (see OECD (2006)).

After a transformation of Equation (4.6) by logs, letting lower case letters represent log-values, $\tilde{\epsilon}_i \equiv \ln \epsilon_i$ and incorporating the control for the effect of exports on sales, $M_S X_i$, the specification

[18]The controls used here are of an accounting nature, although more involved derivations are possible. Depending on the focus and data available, various methods can be used. For instance, Melitz (2000) is primarily concerned with washing out biases in measuring firm productivity for differentiated product firms. In a similar vein, De Loecker (2009) estimates productivity gains from liberalization using an explicit demand system which implies constant markups, the latter being an assumption I impose below.

derived is

$$y_i = M_S X_i + \frac{\beta_S}{\rho_S} \ln \sum_{e=1}^{4} \psi_{e,i}^{\rho_S} + a_S + \alpha_S k_i + \beta_S l_i + \tilde{\epsilon}_i \tag{4.7}$$

Here $a_S$ is a sector level effect while $k_i$ and $l_i$ are measured by the value of capital and labor inputs. The skill diverse or skill similar term $\left(\sum \psi_{e,i}^{\rho_S}\right)^{\beta_S/\rho_S}$ is specialized to reflect the data which has the percentage of workers within four educational bins by firm. Equation (4.7) is non-linear in the CES coefficients $\rho_S$ which leads to some choices about the estimation method.[19] Equation (4.7) is estimated via non-linear least squares using feasible generalized least squares to control for heteroskedasticity across all country-sector pairs. Issues of identification for this form are discussed in the Appendix.

The most closely related paper in approach is Iranzo et al. (2006) who estimate a linearized CES specification for Italian firms which allows techniques to control for firm level fixed effects and other endogeneity issues (for a brief overview of such techniques see Arnold (2005)). In comparison to my approach, Iranzo et al. have a larger sample across time and so suffer from fewer endogeneity issues, although questions regarding theoretical implications of their linearized CES system remain. In this respect, the scarcity of cross-country firm level panels is a limitation to controlling for endogeneity issues in this paper.[20]

While endogeneity may be a problem for estimates of $\alpha_S$ and $\beta_S$, it is less important for the main parameters of interest, namely $\rho_S$. Even if unexpected shocks in productivity $\epsilon_i$ influence labor choices $L_i$ and capital is fixed before the shock, the term $\left(\sum \psi_{e,i}^{\rho_S}\right)^{\beta_S/\rho_S}$ is Hicks neutral and should not be affected by $\epsilon_i$ if the firm faces competitive input markets. For example, productivity shocks to the firm might alter the number of man hours employed, but are less likely to alter the optimal composition of the workforce. This argument is additionally supported by specification tests of the model below.

As far as I am aware, this paper is unique in the breadth of developing country firms examined ($\approx$ 6700 firms across 36 countries).[21] The main data set consists of firm level data generally called

[19]Estimation of CES production technologies goes back to Kmenta (1967) who surmounts computational issues using a second order approximation to the production function. This became a popular technique, e.g. Klump et al. (2007). However, simulation work indicates that Kmenta's approximation suffers from efficiency problems, resulting in "unacceptable standard errors" (Hansen and Knowles, 1998; Tsang and Persky, 1975; White, 1980).

[20]Although there is a large literature on production function estimation, the techniques beginning with Olley and Pakes (1996) have been developed using panel data and assuming Cobb-Douglas forms, continuing on through Levinsohn and Petrin (2003) and more recently Ackerberg et al. (2006).

[21]For discussion of issues and findings regarding manufacturing firms in developing countries see Tybout (2000).

the Enterprise Surveys, conducted by the World Bank. To the best of my knowledge, this is the largest set of cross country data with firm level distributions of employed skill. Firms in the surveys were randomly sampled, in some cases with stratification. The country/year pairs in the survey span from 2002-2005 as tallied in Table 7 of the Appendix. The break down of firms and sales by sector are presented in Table 1. Monetary values have been converted to 2004 US Dollars (CPI adjusted) based on the 2008 International Financial Statistics published by the IMF.[22]

TABLE 1. Observations and Sales Percentages in Sample by Sector

| | **Agro-industry** | **Autos/ Components** | **Beverages** | **Chemicals/ Pharma** | **Electronics** | **Food** | **Garments** |
|---|---|---|---|---|---|---|---|
| Observation % | 1.884 | 1.884 | 5.129 | 6.191 | 1.600 | 12.876 | 17.497 |
| Sales % | 6.968 | 0.994 | 1.132 | 8.062 | 0.852 | 39.202 | 4.947 |
| Value Added % | 8.102 | 1.161 | 1.280 | 7.804 | 0.839 | 33.721 | 6.675 |
| | **Leather** | **Metals/ Machinery** | **Non-metal/ Plastics** | **Other manufact.** | **Paper** | **Textiles** | **Wood/ Furniture** |
| Observation % | 4.187 | 17.482 | 6.909 | 3.215 | 2.348 | 8.479 | 10.319 |
| Sales % | 0.508 | 14.879 | 3.994 | 2.104 | 1.004 | 3.049 | 12.307 |
| Value Added % | 0.509 | 14.964 | 5.069 | 1.868 | 1.171 | 3.551 | 13.285 |

The factor endowments of firms vary considerably across sectors, as shown in Table 2. Capital-labor ratios, measured as the dollar value of capital per dollar of wage, range from labor intensive (Garments and Leather) industries up to capital intensive (Agroindustry). Skill, as measured by mean years of education, range from low skill intensive in Garments and Leather to high skill intensive in Chemicals, Electronics and (surprisingly) Paper production. Skill dispersion, as measured by the Gini of education, ranges from skill similar in Agroindustry to skill diverse in Paper production. Finally, Table 2 shows that the typical developing country firm is a marginal exporter at best, underscoring the well known fact that most exports accrue to a disproportionately small share of firms.

I now briefly discuss variable construction. Value added sales were constructed as total sales reported, less raw material costs and energy costs, with the caveat that energy costs are only known to be non-zero for about 70% of observations. Sector-year and country controls were included to pick up a variety of effects. Five controls, indicators for product line and technology upgrading, ISO certification, internal worker training and firm size (>50 permanent workers) are used to capture productivity differences associated with these characteristics. ISO certification in particular

[22]For some countries, this publication simultaneously provides market and official exchange rates. The market rate was preferred when available.

TABLE 2. Endowment Intensities and Exports by Sector

| | Agro-industry | Autos/ Components | Beverages | Chemicals/ Pharma | Electronics | Food | Garments |
|---|---|---|---|---|---|---|---|
| Median K/L | 7.985 | 4.305 | 6.667 | 7.559 | 4.231 | 7.400 | 2.881 |
| Median Yrs Educ | 8.317 | 9.185 | 9.850 | 10.135 | 10.250 | 8.975 | 8.175 |
| Median Educ Gini | 0.415 | 0.441 | 0.485 | 0.497 | 0.518 | 0.504 | 0.503 |
| Median Export % | 0.050 | 0.010 | 0.000 | 0.000 | 0.020 | 0.000 | 0.000 |
| | **Leather** | **Metals/ Machinery** | **Non-metal/ Plastics** | **Other manufact.** | **Paper** | **Textiles** | **Wood/ Furniture** |
| Median K/L | 2.822 | 4.869 | 6.210 | 4.216 | 6.004 | 5.146 | 3.701 |
| Median Yrs Educ | 7.872 | 9.700 | 9.150 | 10.050 | 10.705 | 8.714 | 8.478 |
| Median Educ Gini | 0.502 | 0.492 | 0.487 | 0.517 | 0.565 | 0.503 | 0.499 |
| Median Export % | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.000 |

has been found to be associated with increases in exports, quality upgrading and higher productivity firms; for evidence and a theoretical mechanism see Verhoogen (2008). Three continuous controls are also used. First, the imported fraction of inputs control for quality and technology differences of imported processes and inputs. Second, the fraction of foreign ownership helps control for imported management, organization and general know-how. Third, foreign versus domestic markup differences are controlled for using the fraction of exports, as discussed above.

4.3. **Production Estimates.** The production estimates are provided in Table 3. Parameter estimates have been segmented into three groups: Controls, Production and Markups, and the CES skill parameter by sector. The association of value added sales with ISO certification and training have the expected positive and significant signs. ISO certification explains an additional 6.6% of increased productivity while training accounts for 2.5%. I also find a productivity increase of roughly 4.8% if a firm has upgraded their product line in the last three years. The effect of new production technologies is surprisingly insignificant. Also as expected, the percentage of inputs which are imported has a significant positive sign but is modest: roughly a 14% increase in imported inputs is associated with a 1% increase in value added sales. The controls for firm size and foreign ownership are also positive.

The estimates for production and markups show highly significant capital and labor production parameters which have a sum slightly less than one for each sector, except Leather which has a sum slightly greater than one. Wald tests of the restriction $\alpha_S + \beta_S = 1$ fail to reject constant returns in eight of the sectors.[23] This suggests constant or slightly decreasing returns to scale in

[23] The constant returns sectors are Beverages, Chemicals/Pharma, Food, Garments, Metals/Machinery, Other manufacturing, Textiles and Wood/Furniture.

capital and labor in almost all sectors. The controls for the effects of markups ($\hat{M}_S$) are generally insignificant, with the notable exceptions of Garments, Leather, Paper and Textiles where they are positive as expected, and Non-Metals/Plastics which has a negative sign. This surprising finding for Non-Metals/Plastics would be consistent if this industry produces a large quantity of intermediate inputs for strong domestic markets, but this is purely conjecture.

The final group of estimates characterizes sectors as "diverse skill loving" ($\rho_S < 1$) or "similar skill loving" ($\rho_S > 1$). Here tests of significance that $\rho_S \neq 1$ are often highly significant and are jointly significant at the 1% level. The $\rho_S$ estimates actually have even more coverage than Table 3 suggests: skill mix explains productivity differences in 11 of 14 sectors which comprise over 90% of firms and 90% of sales in the sample. This high proportion of sales is largely driven by Food, which comprises the lions share of both sales and value added sales.

TABLE 3. Non-linear FGLS production function estimates

| Dependent Variable: | Value Added Sales | | | | | |
|---|---|---|---|---|---|---|
| **Controls** | **Estimate** | **SE** | *p*-value | **Other Controls** | | |
| Upgraded Products | 0.0476*** | (0.0132) | 0.0001 | Sector-Year Dummies | | |
| New Production Tech | 0.0059 | (0.0127) | 0.3207 | Country Dummies | | |
| ISO Certification | 0.0656*** | (0.0165) | 0.0000 | | | |
| Worker Training | 0.0254** | (0.0122) | 0.0188 | **Summary Statistics** | | |
| % Imported Inputs | 0.0697*** | (0.0175) | 0.0000 | Obs: | 6687 | |
| Large Firm | 0.0547*** | (0.016) | 0.0000 | Pseudo $R^2$: | 0.8951 | |
| Foreign Own | 0.0938*** | (0.0211) | 0.0000 | | | |
| **Production and Markups** | **Capital ($\alpha_S$)** | **Labor ($\beta_S$)** | **Markup ($M_S$)** | **CES Parameter ($\rho_S$) Estimate** | **SE** | *p*-value |
| Agroindustry | 0.3810*** | 0.5690*** | −0.1622 | 1.2710 | (0.5944) | 0.3242 |
| Autos & components | 0.2461*** | 0.7354*** | 0.3060 | 0.6799** | (0.1401) | 0.0112 |
| Beverages | 0.8068*** | 0.1486*** | −0.0596 | 0.6332*** | (0.1383) | 0.0040 |
| Chemicals/Pharma | 0.4491*** | 0.5126*** | 0.1260 | 0.6243*** | (0.0772) | 0.0000 |
| Electronics | 0.3011*** | 0.6748*** | −0.0256 | 0.7120** | (0.1407) | 0.0203 |
| Food | 0.4478*** | 0.5230*** | −0.0345 | 1.5072*** | (0.2178) | 0.0099 |
| Garments | 0.3650*** | 0.5966*** | 0.0894** | 1.1321* | (0.0961) | 0.0846 |
| Leather | 0.2606*** | 0.7687*** | 0.1502* | 0.9856 | (0.1228) | 0.4532 |
| Metals and machinery | 0.5228*** | 0.4533*** | −0.0073 | 0.8501** | (0.0759) | 0.0241 |
| Non-metal/Plastics | 0.3896*** | 0.5923*** | −0.2473*** | 0.6394*** | (0.0567) | 0.0000 |
| Other manufacturing | 0.2912*** | 0.6447*** | −0.0349 | 0.6716*** | (0.0833) | 0.0000 |
| Paper | 0.5768*** | 0.4195*** | 0.1486*** | 1.3376 | (0.2836) | 0.1169 |
| Textiles | 0.3263*** | 0.6339*** | 0.1018** | 0.8896* | (0.0799) | 0.0836 |
| Wood and furniture | 0.2921*** | 0.6764*** | 0.0700 | 0.7663*** | (0.0593) | 0.0000 |

*/**/*** denote .1/.05/.01 Significance levels

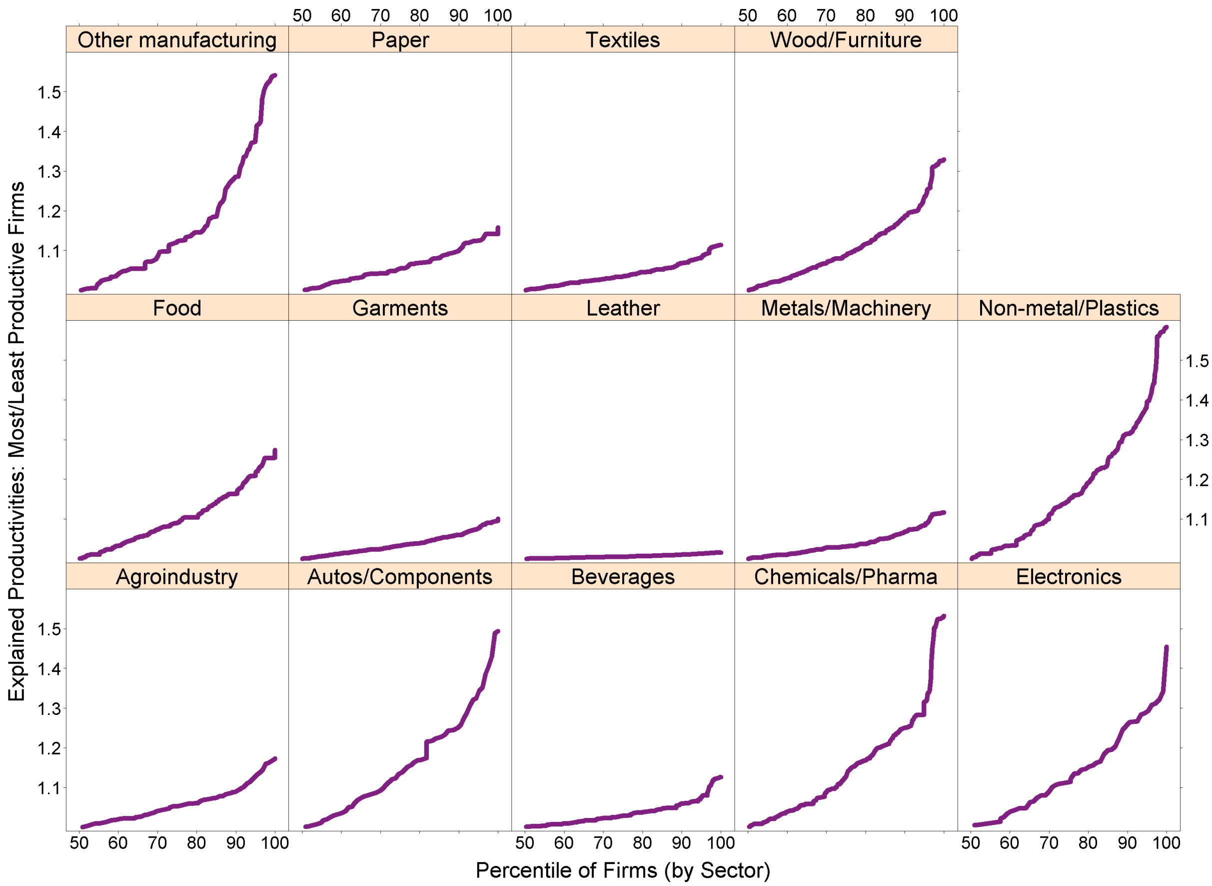
The significance of each $\rho_S$ and whether each $\rho_S$ is greater or less than 1 is fairly robust across controls. Similar estimates are obtained using optimal GMM. As an additional robustness check, the estimates of Table 3 are repeated using the translog form for $F(K, L)$ in Table 8 of the Appendix.. For the translog, ten of fourteen sectors are found to have $\rho_S$ significantly different from one.

Finally, if skill mix can help explain productivity through a labor augmenting effect, are the labor coefficients biased when skill mix is not accounted for? I approach this question by comparing the labor augmented specification (4.2) with the restricted specification that $\phi(\psi_i) \equiv 1$, which ignores the labor augmenting role of skill mix. The labor coefficients for both models are extremely close, and applying a Hausman specification test fails to reject the null hypothesis that labor coefficients are affected by skill mix. This suggests that skill mix explains productivity rather than unaccounted for elements of a firm's wage bill. The evidence also suggests the presence of endogeneity bias in production coefficients is unlikely to affect $\rho_S$.

4.4. **Interpretation of Explained Productivity.** I now examine how the labor augmenting productivity term $\phi_S(\psi_i)^{\beta_S}$ explains within sector productivity differences. The terms $\phi_S(\psi_i)^{\beta_S}$ can explain within sector variation, but are not directly comparable in levels across sectors because $\phi_S(\psi_i)^{\beta_S}$ is decreasing in $\rho_S$. This implies the average level value of labor augmenting productivity in a sector is indistinguishable from the sector fixed effect, so only variation within $\phi_S(\psi_i)^{\beta_S}$ is identified. While $\rho_S$ can pick up the submodularity and supermodularity of skill mix within a sector, this is inherently a sector specific measure.

Of practical importance is the magnitude of productivity differences explained by skill mix. For example, consider the inter-quartile range: is the productivity difference between the most productive 25% and least productive 25% of firms (as accounted for by skill mix) the same magnitude as other productivity controls? To answer this question for each sector, I first introduce the shorthand $P_i^S \equiv \phi_S(\psi_i)^{\beta_S}$. We can examine the ratio $P^S_{(75)}/P^S_{(25)}$, where $P^S_{(x)}$ denotes the $x^{th}$ percentile of explained productivity. This forms a measure of productivity differences. If $P^S_{(75)}/P^S_{(25)}$ is equal to say, 1.17 then any firm $H$ picked from the top 25% of explained productivity and any firm $L$ picked from the bottom 25% must have a ratio of productivity $P^S_H/P^S_L$ of at least 1.17. This translates into at least a 17% productivity difference between the firms $H$ and $L$. Accordingly, the productivity ratios $P^S_{(x)}/P^S_{(1-x)}$ for $x \geq$50% are graphed in Figure 4.1.

FIGURE 4.1. Productivity Ratios and Mean Productivity Ratios by Percentile




In interpreting Figure 4.1, consider the expected patterns of productivity ratios. First, sectors with $\rho_S$ terms close to unity, especially if they are insignificant, imply a production techonology which is skill mix neutral. Thus if $\rho_S \approx 1$ then the productivity ratios accounted for by skill mix should be close to 1. These are the Agroindustry, Garments, Leather, Metals/Machinery, Paper and Textiles sectors. It is also appropriate to include the Beverages sector which has a significant $\rho_S$ but stands out as being exceedingly capital intensive so labor augmentation does not amount to large differences. Second, with regard to significant sectors (excluding Beverages), differences in productivity of at least 5-9% at the inter-quartile range would imply that skill mix is as important as any of the controls considered individually. In fact, Table 4 shows inter-quartile productivity differences of 9-13% under the inter-quartile measure $P^S_{(75)}/P^S_{(25)}$ for significant sectors. Thus productivity differences explained by skill mix are comparable to the magnitude of training and imported inputs combined. These results are emphasized in Table 4. By comparison, the inter-quartile measure in the capital intensive Beverage sector is roughly comparable to the effect of training. Of course, all of these differences become more pronounced if we consider the 90%/10% firm measure in Table 4. These results support Iranzo et al. (2006) who find that firms in the last productivity decile have dispersion almost 35% higher than those in the first decile.

TABLE 4. Productivity Ratios Explained by Skill Mix

| **Sector** | **Skill Mix Estimate** | **Diverse or Similar Skill Intensive** | **75%/25% Explained Productivity Ratio** | **90%/10% Explained Productivity Ratio** |
|---|---|---|---|---|
| Agroindustry | 1.2710 | – | 1.0538 | 1.0934 |
| Autos/Components | 0.6799** | Diverse | 1.1384 | 1.2542 |
| Beverages | 0.6332*** | Diverse | 1.0289 | 1.0593 |
| Chemicals/Pharma | 0.6243*** | Diverse | 1.1299 | 1.2502 |
| Electronics | 0.7120** | Diverse | 1.1226 | 1.2611 |
| Food | 1.5072*** | Similar | 1.0911 | 1.1720 |
| Garments | 1.1321* | Similar | 1.0330 | 1.0610 |
| Leather | 0.9856 | – | 1.0049 | 1.0100 |
| Metals/Machinery | 0.8501** | Diverse | 1.0306 | 1.0679 |
| Non-metal/Plastics | 0.6394*** | Diverse | 1.1492 | 1.3143 |
| Other manufacturing | 0.6716*** | Diverse | 1.1236 | 1.2858 |
| Paper | 1.3376 | – | 1.0551 | 1.1054 |
| Textiles | 0.8896* | Diverse | 1.0354 | 1.0682 |
| Wood/Furniture | 0.7663*** | Diverse | 1.0887 | 1.1874 |

*/**/*** denote .1/.05/.01 Significance levels that $\rho_S \neq 1$.

Table 4 shows that the majority of sectors best utilize diverse skills. However, since the largest sector by sales is Food which best utilizes similarly skilled workers, it would be inaccurate to conclude that the bulk of developing country sectors are "diversity intensive." Rather, the amount of diversity intensive production depends on the stage of development or export orientation into sectors beyond Food and basic manufactures. In the long run, it is likely countries transition into manufacturing which allows for specialized jobs that encourage employment of a diverse workforce. If the theory, which implies a convex structure of wages in diverse sectors, is correct then the expansion of the manufacturing sector caused by growth and trade implies a widening wage gap. This suggests further work looking at the link between inequality and the growth of diverse sectors as enumerated in Table 4.

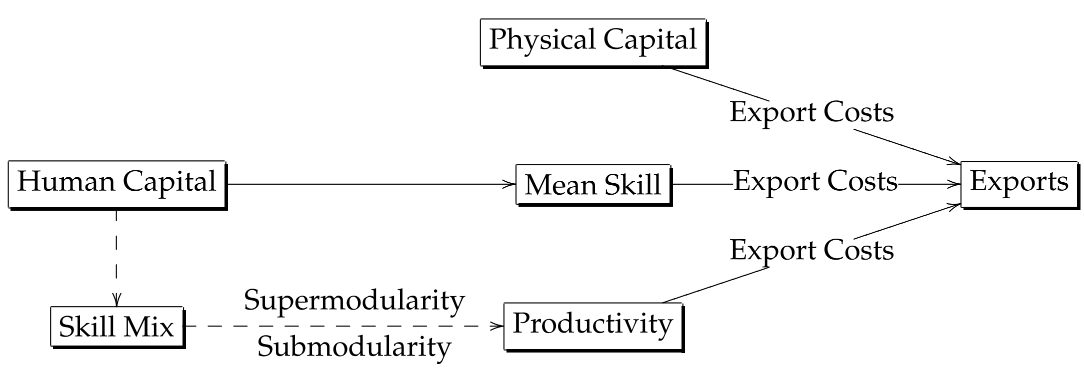
This section has provided structural evidence at the firm level for the model of this paper as well as the literature on the role of skill mix in production. The fact that sectors have been ranked by $\rho_S$ opens the door to testing other implications of the model, in particular the new versions of the Rybczynski and Stolper-Samuelson Theorems. Taking a step in this direction, I look for evidence of comparative advantage in the next section by testing if differences in productivity explained by skill mix result in greater exports for firms.

## 5. Estimation: Exports and Skill Mix

The last section showed that a skill diverse workforce is positively related to productivity in some sectors and vice versa for others. Is there evidence of increased exports for firms which capture such productivity gains? This section tests whether skill dispersion is a basis for exports. I quickly detail a two-tailed Tobit specification explaining exports through skill mix, controlling for overall skill and capital intensity. I then estimate the specification and assess the relative magnitude of skill dispersion as a determinant of exports.

5.1. **Productivity-Export Linkage.** While there are many possible relationships between skill mix and propensity to export, I frame the question by considering the selection mechanism caused by export costs. As discussed by Roberts and Tybout (1997), export costs are one of the most important determinants of exports in developing countries. In the presence of trade frictions, only the most productive firms export.[24] Therefore higher productivity should assist in amortizing trade frictions, as should higher physical and human capital intensity. These relationships are depicted with solid lines in Figure 5.1. Productivity differences arising from skill mix will increase exports when the skill endowment available is comparatively advantageous. These relationships are depicted with dashed lines in Figure 5.1.

FIGURE 5.1. Effects of Endowments and Diversity on Firm Level Exporting




[24] For instance Bernard and Jensen (1999) find that US exporters are "larger, more productive, more capital-intensive, more technology-intensive, and pay higher wages" (Pg. 2)

I now use explained productivities $\phi_S(\psi_i)^{\beta_S}$ to capture the effect of productivity differences through skill diversity. Since the percentage of exports is between zero and one, any linear model examining exports is necessarily truncated so the specification is a two-tailed tobit:

$$\text{Export \%of Sales}_i = \text{Sector Effects} + \text{Country Effects} + \alpha \cdot K_i/L_i + \beta \cdot \text{Skill}_i + \gamma \cdot \text{Productivity(Skill Mix)}_i + \delta \cdot \text{Unexplained Productivity}_i \tag{5.1}$$

This specification tests whether productivity explained by skill mix predicts exports after controlling for average skill and capital intensity. In order to operationalize Equation (5.1) using the explained productivities $\phi_S(\psi_i)^{\beta_S}$, I address the fact that $\phi_S(\psi_i)^{\beta_S}$ only identifies within sector variation as mentioned above. Accordingly, define the Z-score for each $\phi_S(\psi_i)^{\beta_S}$ within a sector by $Z_i^S \equiv (\phi_S(\psi_i)^{\beta_S} - \mu^S)/\sigma^S$, where $\mu^S$ and $\sigma^S$ are the estimated mean and standard deviation of $\phi_S(\psi_i)^{\beta_S}$. A one unit increase in $Z_i^S$ is precisely an increase of one standard deviation in productivity. Since $Z_i^S$ has an approximately normal distribution, the inter-quartile difference between the 75th and 25th percentiles, $Z_{(75)}^S - Z_{(25)}^S$, is approximately 1.35. Finally, $Z_i^S$ increases in productivity so positive estimates of $\gamma$ imply a positive relationship between exports and productivity explained by skill mix.[25]

TABLE 5. Skill Determinants of Export Sales (two sided Tobit)

| | **Firm Export %** | | | **Firm Export %** | | |
|---|---|---|---|---|---|---|
| Trade Variables | **Estimate** | **SE** | *p*-value | **Estimate** | **SE** | *p*-value |
| K/L | 0.191∗∗ | (0.093) | 0.039 | 0.124 | (0.091) | 0.173 |
| Mean Skill | 0.358 | (0.592) | 0.545 | −0.940 | (0.585) | 0.108 |
| Skill Mix Z-Score | 4.723∗∗∗ | (1.072) | 0.000 | 3.801∗∗∗ | (1.054) | 3e-040 |
| Unexplained Prod Z-Score | 1.884∗ | (1.047) | 0.072 | 2.314∗∗ | (1.018) | 0.023 |
| Trade Dummies | **Estimate** | **SE** | *p*-value | **Estimate** | **SE** | *p*-value |
| ISO Certification | – | – | – | 29.693∗∗∗ | (2.709) | 0.000 |
| Worker Training | – | – | – | 17.931∗∗∗ | (2.403) | 0.000 |
| Upgraded Product Line | – | – | – | 12.557∗∗∗ | (2.603) | 0.000 |
| New Production Tech | – | – | – | 6.401∗∗∗ | (2.317) | 0.006 |
| Controls | | | | | | |
| Sector Effects: | | Yes | | | Yes | |
| Country Effects: | | Yes | | | Yes | |

*/**/*** denote .1/.05/.01 significance levels

[25] This two stage estimation process is suboptimal in the sense that standard errors of the second stage are not derived from joint estimation. This issue could be addressed in the future using the strategy of Newey (1984).

5.2. **Export Propensity and Productivity From Diversity.** The estimates of Equation (5.1) are reported in Table 5, with and without controls such as ISO certification which have been linked to exports.

To assess the magnitude of skill mix effects, I appeal to the "rule of thumb" inter-quartile difference $Z^S_{(75)} - Z^S_{(25)} \approx 1.35$, suggesting the difference in exports explained by inter-quartile skill mix differences is 6-7%. In contrast, the remaining unexplained productivity from the production estimates do not explain exports as well. This supports the claim that skill mix is an especially important determinant of trade. Furthermore, skill mix is robust in predicting exports under the inclusion of controls, unlike physical and human capital. In order to compare the predictive power of skill mix to physical and human capital, I provide the inter-quartile firm differences by sector in Table 6. Considering the significant effect of physical capital on exporting, I find an implied export propensity of .7-2.8%, very small compared to the magnitude explained by skill mix. Similarly, mean skill shows inter-quartile differences of 1.7-3.5%. Therefore inter-quartile differences of exporting due to skill mix are more than those from physical and human capital combined. I conclude that skill diversity is a relatively strong determinant of exports through its effects on firm productivity.

TABLE 6. Inter-quartile Endowment Differences by Sector

| **Sector** | **Interquartile K/L** | **Interquartile Mean Skill** | **Interquartile Skill Mix** | **Sector** | **Interquartile K/L** | **Interquartile Mean Skill** | **Interquartile Skill Mix** |
|---|---|---|---|---|---|---|---|
| Agroindustry | 14.233 | 3.775 | 1.333 | Leather | 4.9195 | 2.850 | 1.309 |
| Autos/Components | 7.012 | 2.516 | 1.521 | Metals/Machinery | 5.9239 | 2.255 | 1.241 |
| Beverages | 8.198 | 2.105 | 1.237 | Non-metal/Plastics | 9.5572 | 2.530 | 1.412 |
| Chemicals/Pharma | 10.612 | 3.005 | 1.407 | Other manufacturing | 7.2056 | 1.800 | 1.295 |
| Electronics | 6.376 | 2.300 | 1.455 | Paper | 8.6811 | 1.983 | 1.429 |
| Food | 10.563 | 2.788 | 1.433 | Textiles | 7.8246 | 3.153 | 1.414 |
| Garments | 3.616 | 2.858 | 1.434 | Wood/Furniture | 5.3937 | 2.330 | 1.369 |

## 6. Conclusion and Directions for Further Work

This paper has developed a model in which the technical capacity to use diverse workers varies by sector. The model serves the joint role of extending the theoretical literature and motivating a structural form for estimation. Theoretically, this paper generates new predictions about how worker teams are employed and what team members are paid. I provide a new version of the

Rybczynski Theorem: a one percent increase in workers with skills specific to a sector increases output, but potentially by less than one percent due to worker spillovers. I also provide new versions of the Stolper-Samuelson theorem. An increase in sector output price will cause a sector to boom, but superstar effects and entrant workers change the pay structure of the sector. The new pay structure will increase team wages but may decrease individual wages.

These results suggest growth and trade can exacerbate inequality. In particular, this paper places emphasis on the role of worker teams formed from asymmetric skill distributions. Since low and high workers cannot efficiently produce without the other, growth depends on the available supply of both groups. Wages also hinge on the relative scarcity of low and high skill workers. Therefore this paper establishes ideas about the structure of production which have fundamental implications for inequality, among other themes. This suggests further work looking at the link between inequality and changes in the size of "skill diversity loving" sectors through growth and trade. Further work might also consider a political economy framework considering protection as pro/anti-tariff coalitions can form which cut along both sector and skill lines.

Empirically, I characterize manufacturing sectors by skill mix intensity (diversity or similarity) helps explain productivity. I find that greater than two thirds of firms in a large cross country sample belong to sectors where skill mix is an important determinant of productivity. Inter-quartile productivity differences explained by skill mix are comparable to the magnitude of training and imported inputs combined, and the magnitude in four sectors is comparable to training, imported inputs and ISO combined. Furthermore, the majority of sectors best utilize diverse skills, and theory suggests this explains higher wage inequality through "superstar" wage effects.

Having established a linkage from skill mix to productivity, this paper also evaluates the effect of productivity differences on exports. I find that differences in skill mix explain intrasector export variation better than the physical and human capital combined. I conclude that skill diversity is a relatively good determinant of exports, through its effects on firm productivity. This result clearly has implications for the human capital *content* of traded goods. Put together, the results of this paper show that a more detailed view of human capital, beyond that of a simple average, yields insights into both productivity and export patterns.

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## Appendix A. Supplemental Empirical Details

A.1. **Firm distribution by Sector and Country.** The distribution of firms by sector and country is presented in Table 7. In order to facilitate heteroskedasticity by sector-country pairs, sector-country pairs containing exactly one observation were dropped from the sample, making the minimum number of observations two for each pair.

TABLE 7. Distribution of Firms by Country and Sector

| Country | Year | Agroindustry | Autos/Components | Beverages | Chemicals/Pharma | Electronics | Food | Garments | Leather | Metals/Machinery | Non-metal/Plastics | Other Manufacturing | Paper | Textiles | Wood/Furniture | Country Totals |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Albania | 2005 | 0 | 0 | 8 | 3 | 0 | 5 | 5 | 0 | 4 | 2 | 0 | 0 | 0 | 4 | 31 |
| Armenia | 2005 | 0 | 0 | 70 | 3 | 0 | 13 | 9 | 0 | 23 | 6 | 0 | 4 | 3 | 6 | 137 |
| Belarus | 2005 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 3 | 0 | 0 | 3 | 18 |
| Benin | 2004 | 13 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 8 | 0 | 0 | 0 | 0 | 25 | 51 |
| Brazil | 2003 | 0 | 118 | 0 | 68 | 57 | 103 | 405 | 154 | 157 | 0 | 0 | 0 | 96 | 275 | 1433 |
| Bulgaria | 2005 | 0 | 0 | 6 | 2 | 0 | 2 | 3 | 0 | 7 | 2 | 0 | 0 | 5 | 3 | 30 |
| Cambodia | 2003 | 3 | 0 | 0 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 |
| Chile | 2004 | 39 | 0 | 16 | 85 | 0 | 134 | 0 | 0 | 119 | 0 | 0 | 40 | 0 | 71 | 504 |
| CostaRica | 2005 | 0 | 0 | 4 | 0 | 2 | 16 | 8 | 2 | 19 | 40 | 19 | 2 | 10 | 12 | 134 |
| Croatia | 2005 | 0 | 0 | 3 | 2 | 0 | 0 | 0 | 0 | 6 | 4 | 0 | 2 | 0 | 2 | 19 |
| Egypt | 2004 | 0 | 8 | 0 | 48 | 0 | 95 | 80 | 19 | 107 | 119 | 25 | 0 | 85 | 31 | 617 |
| ElSalvador | 2003 | 0 | 0 | 0 | 3 | 0 | 5 | 3 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 16 |
| Estonia | 2005 | 0 | 0 | 3 | 0 | 0 | 0 | 4 | 0 | 4 | 2 | 0 | 3 | 2 | 4 | 22 |
| FYROM | 2005 | 0 | 0 | 5 | 0 | 0 | 2 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 2 | 14 |
| Georgia | 2005 | 0 | 0 | 8 | 0 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 2 | 0 | 14 |
| Guyana | 2004 | 0 | 0 | 0 | 3 | 0 | 51 | 6 | 0 | 0 | 0 | 0 | 0 | 5 | 28 | 93 |
| Hungary | 2005 | 0 | 0 | 6 | 0 | 0 | 14 | 23 | 2 | 110 | 7 | 0 | 7 | 5 | 3 | 177 |
| Kazakhstan | 2005 | 0 | 0 | 85 | 0 | 0 | 3 | 16 | 0 | 46 | 4 | 0 | 4 | 0 | 0 | 158 |
| Kyrgyzstan | 2003 | 0 | 0 | 16 | 3 | 0 | 14 | 12 | 0 | 4 | 6 | 0 | 0 | 7 | 2 | 64 |
| Latvia | 2005 | 0 | 0 | 2 | 0 | 0 | 0 | 4 | 0 | 3 | 0 | 0 | 2 | 0 | 4 | 15 |
| Lithuania | 2004 | 0 | 0 | 0 | 0 | 0 | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 33 | 9 | 64 |
| Madagascar | 2005 | 0 | 0 | 0 | 5 | 0 | 10 | 11 | 2 | 4 | 4 | 12 | 2 | 4 | 10 | 64 |
| Mauritius | 2005 | 0 | 0 | 3 | 5 | 0 | 12 | 0 | 0 | 4 | 0 | 3 | 10 | 14 | 4 | 55 |
| Moldova | 2005 | 0 | 0 | 39 | 0 | 0 | 5 | 16 | 0 | 16 | 2 | 3 | 2 | 0 | 5 | 88 |
| Morocco | 2004 | 0 | 0 | 0 | 56 | 30 | 60 | 319 | 76 | 17 | 70 | 0 | 3 | 151 | 3 | 785 |
| Oman | 2003 | 0 | 0 | 0 | 3 | 0 | 5 | 0 | 0 | 9 | 12 | 4 | 2 | 0 | 2 | 37 |
| Poland | 2005 | 0 | 0 | 8 | 0 | 0 | 32 | 46 | 0 | 78 | 3 | 0 | 4 | 4 | 3 | 178 |
| Romania | 2005 | 0 | 0 | 36 | 9 | 0 | 29 | 65 | 2 | 62 | 4 | 0 | 5 | 5 | 6 | 223 |
| Serb&Mont | 2005 | 0 | 0 | 6 | 0 | 0 | 2 | 0 | 0 | 5 | 0 | 0 | 2 | 2 | 0 | 17 |
| SouthAfrica | 2003 | 0 | 0 | 0 | 35 | 7 | 44 | 20 | 7 | 100 | 38 | 72 | 11 | 17 | 69 | 420 |
| Tajikistan | 2005 | 0 | 0 | 4 | 0 | 0 | 4 | 4 | 0 | 5 | 6 | 0 | 6 | 6 | 0 | 35 |
| Turkey | 2005 | 0 | 0 | 0 | 32 | 6 | 84 | 48 | 4 | 107 | 38 | 3 | 2 | 56 | 6 | 386 |
| Ukraine | 2005 | 0 | 0 | 13 | 3 | 0 | 3 | 9 | 4 | 13 | 4 | 11 | 6 | 4 | 9 | 79 |
| Uzbekistan | 2003 | 0 | 0 | 0 | 0 | 0 | 8 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 17 |
| Vietnam | 2005 | 0 | 0 | 2 | 23 | 5 | 82 | 34 | 8 | 95 | 83 | 60 | 31 | 34 | 85 | 542 |
| Zambia | 2002 | 71 | 0 | 0 | 18 | 0 | 0 | 0 | 0 | 19 | 0 | 0 | 7 | 17 | 4 | 136 |
| **Sector Totals** | | 126 | 126 | 343 | 414 | 107 | 861 | 1170 | 280 | 1169 | 462 | 215 | 157 | 567 | 690 | 6687 |

A.2. **Controlling for the sales effects of exporting.** Ideally, the data set would include production information that differentiates the use of inputs for domestic and foreign use. Of course, many inputs are used in both domestic and foreign production activities, so the true counterfactuals of exporting are not directly observable. I therefore impose structure on the domestic and foreign decomposition of revenues to control for market segmentation.

Suppose firm $i$ produces a quality adjusted quantity $q_i$, of which a fraction $X_i$ is sold to foreign markets at price $p_i^X$, and the remainder is sold domestically at price $p_i^D$. Then value added sales $Y_i$ are given by Equation (A.1)

$$Y_i = X_i \cdot [p_i^X - c_i^X]q_i + (1 - X_i) \cdot [p_i^D - c_i^D]q_i \tag{A.1}$$

where $c_i^X$ and $c_i^D$ are foreign and domestic material input costs. Defining the foreign ($M_i^X$) and domestic ($M_i^D$) markups in the usual way by $M_i^X \equiv (p_i^X - c_i^X)/c_i^X$ and $M_i^D \equiv (p_i^D - c_i^D)/c_i^D$, rewrite Equation (A.1) as

$$Y_i = \underbrace{(p_i^D - c_i^D)q_i}_{\substack{\text{Value added sales at} \\ \text{domestic prices and input costs}}} \cdot \underbrace{1 + \frac{M_i^X(c_i^X/c_i^D) - M_i^D}{M_i^D} X_i}_{\substack{\text{Export weight to account for} \\ \text{foreign markups and input costs}}} \tag{A.2}$$

Equation (A.2) shows value added sales decomposed into domestic value added sales times an export weight that depends on markups and input costs. Introducing the shorthand

$$M_i \equiv \left[ M_i^X(c_i^X/c_i^D) - M_i^D \right] / M_i^D$$

and taking logs, Equation (A.2) becomes

$$\ln Y_i \approx \ln(p_i^D - c_i^D)q_i + M_i X_i$$

where the approximation is the commonly used fact that $\ln(1 + x) \approx x$ for small values of $x$. This approximation halves the number of non-linear parameters to recover. Although the individual elements of $M_i X_i$ are not identified, $X_i$ is observed. Assuming the term $M_i$ does not vary across some grouping, (as would be implied by constant markups within the grouping and $c_i^X/c_i^D$ fixed), $M_i$ can be used to control for the sales effects of exporting. In what follows, I assume $M_i \equiv M_S$ for all firms $i$ in sector $S$ in order to focus on within-sector differences.[26] Since the firms considered

[26] Constant markups for both exporting and domestic production are, for example, consistent with Melitz (2003), but no longer hold once that model is modified to allow for scale effects as in Melitz and Ottaviano (2008). For a concise summary and comparison of these models see Dhingra and Morrow (2008).

produce in developing countries, generally one should expect $M_S \geq 0$. This is because of the high value exported to the economic North, suggesting $M_S^X \geq M_S^D$ (see OECD (2006)) and because of potentially higher costs for "export quality" goods ($c_S^X \geq c_S^D$).

A.3. **Translog production estimates.** After adding the labor augmenting effect of skill mix, the translog specification implies

$$\ln F(K, \phi(\psi_i) \cdot L) \equiv \alpha_S \ln K_i + \beta_S \ln(\phi(\psi_i) L_i) + \sum_{i,j} \gamma_{ij} \ln K_i \ln\left(\phi(\psi_i) L_j\right) \tag{A.3}$$

Estimates are reported in Table 8.

TABLE 8. Translog production estimates

| Dependent Var: | Value Added Sales | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Controls** | **Estimate** | **SE** | *p*-value | | | | **Other Controls** | |
| Upgraded Products | 0.0357*** | (0.0124) | 0.0019 | | | | Sector-Year & | |
| New Prod Tech | 0.0169* | (0.012) | 0.0797 | | | | Country Dummies | |
| ISO Certification | 0.0379*** | (0.0163) | 0.0100 | | | | | |
| Worker Training | -0.0104 | (0.0122) | 0.1973 | | | | **Summary Stats** | |
| % Imported Inputs | 0.0856*** | (0.0159) | 0.0000 | | | | Obs: | 6687 |
| Foreign Own | 0.0633*** | (0.0194) | 5e-040 | | | | $R^2$: | 0.899 |
| **Production and Markups** | **Capital ($\alpha_S$)** | **Labor ($\beta_S$)** | **Markup ($M_S$)** | **Capital$^2$** | **Labor$^2$** | **Capital· Labor** | **CES Param ($\rho_S$) Estimate** | *p*-value |
| Agroindustry | 0.3562* | 0.2956 | -.1255 | .0025 | .0351 | -.0031 | 1.5585 | 0.2664 |
| Autos & comp | -0.2343 | 1.4013*** | .6052 | .0730*** | .0153 | -.1163*** | 0.7720* | 0.0906 |
| Beverages | 0.2204* | 0.7253*** | -.0427 | .0743*** | .0580*** | -.1345*** | 0.6019*** | 0.0000 |
| Chem/Pharma | 0.0185 | 1.0526*** | -.0419 | .0780*** | .0871*** | -.1702*** | 0.7168*** | 0.0030 |
| Electronics | 0.0249 | 0.6666*** | .0241 | .0292 | .0269 | -.0326 | 0.6947** | 0.0205 |
| Food | 0.1557** | 0.9155*** | .0755 | .0586*** | .0557*** | -.1244*** | 1.5216** | 0.0115 |
| Garments | 0.0924* | 1.0049*** | .0834** | .0835*** | .1085*** | -.1963*** | 1.0753 | 0.1760 |
| Leather | 0.0276 | 1.1396*** | .1685** | .0473** | .0293 | -.0929** | 0.8835 | 0.1166 |
| Metals/Machinery | -0.0103 | 0.9904*** | -.0233 | .0922*** | .0912*** | -.1811*** | 0.8871* | 0.0747 |
| Non-metal/Plastics | -0.3054*** | 1.0338*** | -.2751*** | .0709*** | -.0319* | -.0465* | 0.6364*** | 0.0000 |
| Other manufact. | 0.3653** | 0.6903*** | -.0796 | .0070 | .0049 | -.0197 | 0.7089*** | 0.0086 |
| Paper | -0.7113*** | 1.7950*** | .2883*** | .1995*** | .2129*** | -.4195*** | 0.7707*** | 0.0058 |
| Textiles | 0.1809** | 1.0182*** | .0829* | .0417*** | .0291** | -.0914*** | 1.0058 | 0.4711 |
| Wood and furniture | 0.0474 | 1.0394*** | .0705 | .0646*** | .0712*** | -.1426*** | 0.8126*** | 0.0024 |

*/**/*** denote .1/.05/.01 Significance levels

This specification coincides with that of the main text, with the exception that firm size is not included as a control since the specification (A.3) already captures much variation in possible scale effects. Note that the estimates are somewhat inefficient as the share equations associated with this specification were not used in estimation, see Kim (1992).

## APPENDIX B. PROOFS

This section of the appendix includes most proofs from the main text. The construction of wages as those which support the efficient competitive equilibrium is considerably involved and available from the author upon request.

**Lemma.** *If $\Phi$ has finite expectation then the value of all production in the economy is bounded.*

*Proof.* For the result it is necessary to show that for all admissible allocations $\mathcal{A} \equiv (q, M(q), \iota(q))$ of workers to firms that

$$\sup_{\mathcal{A}} \frac{1}{2} \int p_{\iota(q)} F^{\iota(q)}(q, M(q)) d\Phi < \infty \tag{B.1}$$

Since the integrand in equation (B.1) is always positive, for the result it is sufficient to show it holds for $N+1$ different economies with technologies $F^i$ and the entire distribution of skill to allocate. Formally, it is sufficient to show for the allocations $\mathcal{A}_i$, $i = 0, \ldots, N$ which employ all workers in each sector $i$ that

$$\sup_{\mathcal{A}_i} \int p_i F^i(q, M(q)) d\Phi < \infty \quad \forall i = 0, \ldots, N \tag{B.2}$$

Since each $F^i$ is increasing in each argument and is homogeneous of degree one, $p_i F^i(q, M(q)) \leq \max\{q, M(q)\} \cdot p_i F^i(1,1) \leq (q + M(q)) \cdot p_i F^i(1,1)$. Thus for equation (B.2) to hold, since the terms $p_i F^i(1,1)$ are irrelevant for boundedness, it is enough that

$$\sup_{\mathcal{A}_i} \int (q + M(q)) d\Phi < \infty \quad \forall i = 0, \ldots, N \tag{B.3}$$

Examining equation (B.3), $\int q d\Phi$ is the mean of $\Phi$ which is assumed finite so showing $\sup_M \int M(q) d\Phi < \infty$ will prove the result. I claim in fact that $\int M(q) d\Phi = \int q d\Phi$ for any admissible $M$. This follows from the allocative nature of $M$ which requires $\int_{\{q:\, M(q) \in [a,b]\}} 1 d\Phi = \Phi(b) - \Phi(a)$ so in particular for any integers $j$ and $K$,

$$\left| \int_{\frac{j}{K}}^{\frac{j+1}{K}} q d\Phi - \int_{\{q:\, M(q) \in [\frac{j}{K}, \frac{j+1}{K}]\}} M(q) d\Phi \right| \leq [\frac{j+1}{K} - \frac{j}{K}] \left| \int_{\frac{j}{K}}^{\frac{j+1}{K}} 1 d\Phi - \int_{\{q:\, M(q) \in [\frac{j}{K}, \frac{j+1}{K}]\}} 1 d\Phi \right|$$
$$\leq \frac{1}{K} [\Phi(\frac{j+1}{K}) - \Phi(\frac{j}{K})]$$

Summing both sides over $j$ implies

$$\left| \int_0^\infty q d\Phi - \int_{\{q:\, M(q) < \infty\}} M(q) d\Phi \right| \leq \frac{1}{K} \lim_{j \to \infty} \Phi(\frac{j+1}{K}) = \frac{1}{K}$$

and letting $K \to \infty$ shows $\int M(q) d\Phi = \int q d\Phi$ which is finite, proving the result. □

**Lemma** (CES Ranking)**.** *Suppose each $F^i$ is CES, specifically $F^i(q, q') \equiv A_i (q^{\rho_i} + q'^{\rho_i})^{1/\rho_i}$. Then $S_i \succeq S_j$ if and only if $\rho_i > \rho_j$ so CES production technologies imply a complete diversity ranking.*

*Proof.* It is clear that a necessary condition for $S_i \succeq S_j$ is $\rho_i \neq \rho_j$. Some algebra along with the differentiability and homogeneity of each $F^i$ shows that another necessary condition is that $F_2^i(1,z)/F_2^j(1,z) \geq F^i(1,z)/F^j(1,z)$ for all $z > 1$ or rather

$$\frac{A_i z^{\rho_i - 1} (1 + z^{\rho_i})^{(1/\rho_i) - 1}}{A_j z^{\rho_j - 1} (1 + z^{\rho_j})^{(1/\rho_j) - 1}} \geq \frac{A_i (1 + z^{\rho_i})^{1/\rho_i}}{A_j (1 + z^{\rho_j})^{1/\rho_j}}$$

which holds iff $z^{\rho_i}/(1+z^{\rho_i}) \geq z^{\rho_j}/(1+z^{\rho_j})$. Since $x/(1+x)$ is strictly increasing for $x > 0$, the inequality holds iff $z^{\rho_i} \geq z^{\rho_j}$. Since $\rho_i \neq \rho_j$ equality cannot hold and since $z > 1$ this holds iff $\rho_i > \rho_j$. Working backwards, $\rho_i > \rho_j$ is sufficient for $S_i \succeq S_j$, giving the result. □

**Proposition.** *Under Assumption 1, for any allocation in which firms maximize profits there are skill ratios $\{\hat{t}_i\}$, increasing in i, where if $(q, M(q))$ are assigned to $S_i$ then $M(q)/q \in [\hat{t}_i, \hat{t}_{i+1}]$.*

*Proof.* Suppose $(q, M(q))$ are assigned to $S_i$ so profit maximization by firms requires that

$$p_i F^i(q, M(q)) \geq p_j F^j(q, M(q)) \quad \forall j \neq i \tag{B.4}$$

Considering sectors $j$ for $j > i$ and $j < i$ separately this implies that

$$\frac{p_j F^j(1, M(q)/q)}{p_i F^i(1, M(q)/q)} \leq 1 \quad \forall j > i \quad \text{and} \quad \frac{p_i F^i(1, M(q)/q)}{p_j F^j(1, M(q)/q)} \geq 1 \quad \forall j < i \tag{B.5}$$

For $j > i$, $S_j \succeq S_i$ says exactly that $\frac{p_j F^j(1,x)}{p_i F^i(1,x)}$ is increasing so Equation (B.5) shows there is an upper bound for the skill ratio $M(q)/q$. Call this upper bound for the skill ratio $\hat{t}_{i+1}$. Similarly for $j < i$ it holds that $S_j \preceq S_i$ so there is a lower bound for $M(q)/q$, say $\hat{t}_i$. Proceeding in this fashion for each $S_i$, if $(q, M(q))$ are assigned to $S_i$ then $M(q)/q \in [\hat{t}_i, \hat{t}_{i+1}]$ for some constants $\{\hat{t}_i\}$ with $\hat{t}_i$ increasing in $i$. Notice also that the $\{\hat{t}_i\}$ are fixed by the production technologies and prices, independent of the assignment $M(q)$. Specifically they are fixed by the implicit equations $p_i F^i(1, \hat{t}_{i+1}) = p_{i+1} F^{i+1}(1, \hat{t}_{i+1})$. □

**Proposition.** *The wage schedule $w(q)$ has the following properties:*

(1) *$w(q)$ is strictly increasing and convex.*
(2) *$w(q)$ is bounded below by the $S_0$ shadow wages $w_0 q$.*
(3) *$w'(q)$ is increasing where defined, and elsewhere $\lim_{q \to x_-} w'(q) \leq \lim_{q \to x_+} w'(q)$.*

*Proof.* **First Claim:** That $w(q)$ is strictly increasing is obvious from the form. Convexity follows from the third claim combined with Theorem 24.2 of (Rockafellar, 1970) by pasting across sectors.

**Second Claim:** The result clearly holds for workers in $S_0$ so consider workers in the sectors $\{S_i\}_{i \geq 1}$. First note that for $i \geq 1$, $F^i_{12} \leq 0$ and given that $F^i$ is homogeneous of degree one implies $F^i_{11} \geq 0$. Second, since at least some firms produce in the $S_0$ sector, efficiency implies that for all $i$, $p_i F^i(1,1) \leq p_0 F^0(1,1) = 2w_0$. Using these facts, for any worker of skill $q$ which is below the median of $\Phi$ it holds that

$$F^i_1(q, M(q)) = F^i_1(q/M(q), 1) \leq F^i_1(1,1) = F^i(1,1)/2 \leq w_0/p_i \tag{B.6}$$

We conclude that $p_i F^i_1(q, M(q)) \leq w_0$. This implies for any $q$ below the median employed in $\{S_i\}_{i \geq 1}$ wages are bounded below by

$$w(q) = w(\underline{t}_0) - \int_q^{\underline{t}_{i-1}} p_i F^i_1(s, M(s)) ds \geq w(\underline{t}_{i-1}) - \int_q^{\underline{t}_{i-1}} w_0 ds = w(\underline{t}_{i-1}) - w_0[\underline{t}_{i-1} - q] \tag{B.7}$$

Consequently, if $q$ is employed in $S_1$, using the fact that $w(\underline{t}_0) = w_0 \underline{t}_0$, Equation (B.7) implies $w(q) \geq w_0 q$. Proceeding inductively for $q$ below the median, suppose $w(q) \geq w_0 q$ for sector $S_{i-1}$ so for $S_i$, Equation (B.6) implies

$$w(q) \geq w(\underline{t}_{i-1}) - w_0[\underline{t}_{i-1} - q] \geq w_0 \underline{t}_{i-1} - w_0[\underline{t}_{i-1} - q] = w_0 q \tag{B.8}$$

Therefore by induction, Equation (B.8) implies $w(q) \geq w_0 q$ for all $q$ below the median of $\Phi$. A similar method applies to $q$ above the median of $\Phi$ by exploiting the analogous inequality of Equation (B.6) using $F^i_2$.

**Third Claim:** For $q$ not at a cusp $\{\underline{t}_i\}$ or $\{M(\underline{t}_i)\}$ this follows from $F^i_{11}, F^i_{22} \geq 0$ for $q$ in $\{S_i\}_{i\geq 1}$ and from $w'(q) = w_0$ for $q$ in $S_0$. Now suppose $q = \underline{t}_i$ for some $i$. For $\delta > 0$,

$$w'(\underline{t}_i - \delta) = p_{i+1}F^{i+1}_1(\underline{t}_i - \delta, M(\underline{t}_i - \delta)) \quad \text{and} \quad w'(\underline{t}_i + \delta) = p_i F^i_1(\underline{t}_i + \delta, M(\underline{t}_i + \delta)) \tag{B.9}$$

Therefore it is sufficient to show that

$$\lim_{\delta \to 0_-} p_{i+1}F^{i+1}_1(\underline{t}_i - \delta, M(\underline{t}_i - \delta)) \leq \lim_{\delta \to 0_+} p_i F^i_1(\underline{t}_i + \delta, M(\underline{t}_i + \delta))$$

and since each $F^i_1$ is continuous this inequality holds if $p_{i+1}F^{i+1}_1(\underline{t}_i, M(\underline{t}_i)) \leq p_i F^i_1(\underline{t}_i, M(\underline{t}_i))$. Since $S_{i+1} \succeq S_i$, by definition $\frac{\partial}{\partial x} \frac{p_{i+1}F^{i+1}(1,x)}{p_i F^i(1,x)} \geq 0$. This implies through homogeneity that

$$\frac{p_{i+1}F^{i+1}_2(y,z)}{p_{i+1}F^{i+1}(y,z)} \geq \frac{p_i F^i_2(y,z)}{p_i F^i(y,z)} \quad \forall z/y \geq \hat{t}_{i+1} \tag{B.10}$$

Also by definition, $M(\underline{t}_i)/\underline{t}_i = \hat{t}_{i+1}$ and $p_{i+1}F^{i+1}(\underline{t}_i, M(\underline{t}_i)) = p_i F^i(\underline{t}_i, M(\underline{t}_i))$ so letting $y = \underline{t}_i$ and $z = M(\underline{t}_i)$ in Equation (B.10) the denominators cancel and it holds that $p_{i+1}F^{i+1}_2(\underline{t}_i, M(\underline{t}_i)) \geq p_i F^i_2(\underline{t}_i, M(\underline{t}_i))$. Homogeneity then implies $p_{i+1}F^{i+1}_1(\underline{t}_i, M(\underline{t}_i)) \leq p_i F^i_1(\underline{t}_i, M(\underline{t}_i))$ as desired. The cusps $\{M(\underline{t}_i)\}$ can be shown similarly. □

**Lemma.** *Let $\Phi$ and $\tilde{\Phi}$ be two skill distributions and $\underline{t}_0, \tilde{\underline{t}}_0$ be the respective skill cutoffs in equilibrium. Consider a Skill Shock from $\Phi$ to $\tilde{\Phi}$. Then:*

(1) *Following a DSS the mass of workers in $S_1$ increases.*
(2) *Following a HSS, $\tilde{\underline{t}}_0 \geq \underline{t}_0$ and $M(\tilde{\underline{t}}_0) \geq M(\underline{t}_0)$ (and $S_1$ workers increase).*
(3) *Following a LSS, $\tilde{\underline{t}}_0 \leq \underline{t}_0$ and $M(\tilde{\underline{t}}_0) \leq M(\underline{t}_0)$ (and $S_1$ workers increase).*

*Proof.* I first show Claim 2. Let $\tilde{M}$ denote the equilibrium skill pairings under $\tilde{\Phi}$. Under a HSS, by assumption $\tilde{\Phi}(\underline{t}_0) = \Phi(\underline{t}_0)$ and $\tilde{\Phi}(M(\underline{t}_0)) \leq \Phi(M(\underline{t}_0))$ so that $\tilde{\Phi}(\underline{t}_0) + \tilde{\Phi}(M(\underline{t}_0)) \leq 1$. In equilibrium necessarily $M(\underline{t}_0)/\underline{t}_0 = \tilde{M}(\tilde{\underline{t}}_0)/\tilde{\underline{t}}_0 = \hat{t}_1$ since the $\{\hat{t}_i\}$ are fixed by prices and production technologies that are independent of the skill distribution. Therefore $\tilde{\Phi}(\underline{t}_0) + \tilde{\Phi}(\hat{t}_1\underline{t}_0) = \tilde{\Phi}(\underline{t}_0) + \tilde{\Phi}(M(\underline{t}_0)) \leq 1$ which implies

$$\tilde{\Phi}(\underline{t}_0) + \tilde{\Phi}(\hat{t}_1\underline{t}_0) \leq 1 = \tilde{\Phi}(\tilde{\underline{t}}_0) + \tilde{\Phi}(\hat{t}_1\tilde{\underline{t}}_0)$$

and since $\tilde{\Phi}$ is increasing, conclude $\tilde{\underline{t}}_0 \geq \underline{t}_0$. Therefore $\tilde{\Phi}(\tilde{\underline{t}}_0) \geq \tilde{\Phi}(\underline{t}_0) = \Phi(\underline{t}_0)$ so the mass of workers employed in $S_1$ under $\tilde{\Phi}$ is larger than under $\Phi$ as claimed. Claim 3 then follows by a symmetric argument. Claim 1 follows by decomposing any DSS into a HSS for changes above the median and a LSS for changes below the median and applying Claims 2 and 3 in succession. □

**Proposition** (Rybcszynski under Diversity)**.** *If the endowment of skills specific to a sector increases then output of the sector increases.*

*Proof.* (Sketch) This is clear for the $S_0$ sector. For other sectors, it can be shown that the mass of workers employed increases in a similar fashion as the skill shock Lemma, but with careful accounting of set of workers. Now fix a sector $S_i$ for $i \geq 1$, let $\Phi$ be given and first assume $\Psi$ has support on $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$. Normalize $\Phi$ and $\Psi$ to have mass one, forcing the normalized combined distribution to be $\tilde{\Phi} \equiv \frac{L}{L+P}\Phi + \frac{P}{L+P}\Psi$ and note $\tilde{\Phi}(q) = \frac{L}{L+P}\Phi(q)$ for $q \in [\underline{t}_i, \underline{t}_{i-1}]$. Let $\{\tilde{\underline{t}}_i\}$ be the new skill cutoffs between sectors and it is clear as in the text that $\tilde{\underline{t}}_i = \underline{t}_i$ and $\tilde{\underline{t}}_{i-1} \geq \underline{t}_{i-1}$. Also let $\tilde{M}$ denote the new matching function under $\tilde{\Phi}$ and the support of $\Psi$ implies $\tilde{M}(q) \geq M(q)$ for $q \in [\underline{t}_i, \underline{t}_{i-1}]$. Using these facts we have

$$\int_{\tilde{\underline{t}}_i}^{\tilde{\underline{t}}_{i-1}} F^i(q, \tilde{M}(q))d\tilde{\Phi} \geq \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, \tilde{M}(q))d\tilde{\Phi} \geq \int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\tilde{\Phi} = \frac{L}{L+P}\int_{\underline{t}_i}^{\underline{t}_{i-1}} F^i(q, M(q))d\Phi$$

The un-normalized value of the last line coincides with sector output before the skill shift. Thus for $\Psi$ with support on $[M(\underline{t}_{i-1}), M(\underline{t}_i)]$ the result follows, and for $\Psi$ with support on $[\underline{t}_i, \underline{t}_{i-1}]$ the result follows by symmetry. Combining the two cases gives the result. □

**Proposition.** *If the skill distribution is symmetric then the indirect Stolper-Samuelson effect dominates the direct effect.*

*Proof.* The indirect effect dominates when for any $i \geq 1$ and $q \in (\underline{t}_i, \underline{t}_{i-1}]$:

$$\int_q^{\underline{t}_{i-1}} F_1^i(s, M(s))ds \leq \frac{F^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})) \cdot M(\underline{t}_{i-1})/\underline{t}_{i-1}}{M(\underline{t}_{i-1})/\underline{t}_{i-1} + 1} \tag{B.11}$$

Noting that for any $s$, $F_1^i(s, M(s)) = F_1^i(s/M(s), 1)$ and $F_{11}^i \geq 0$, it follows $F_1^i(s, M(s)) \leq F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))$. Therefore

$$\int_q^{\underline{t}_{i-1}} F_1^i(s, M(s))ds \leq \int_q^{\underline{t}_{i-1}} F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))ds \leq F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))\underline{t}_{i-1} \tag{B.12}$$

Using the fact that $F_2^i(x, y) = F_1^i(y, x)$ it is also clear that

$$\begin{aligned} F^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})) &= F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))\underline{t}_{i-1} + F_2^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))M(\underline{t}_{i-1}) \\ &\geq F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})[\underline{t}_{i-1} + M(\underline{t}_{i-1})] \end{aligned} \tag{B.13}$$

Applying the inequalities of Equation (B.12) to the LHS of Equation (B.11) and Equation (B.13) to the RHS of Equation (B.11) it is sufficient for the result that

$$F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1}))\underline{t}_{i-1} \leq F_1^i(\underline{t}_{i-1}, M(\underline{t}_{i-1})[\underline{t}_{i-1} + M(\underline{t}_{i-1})] \frac{M(\underline{t}_{i-1})}{M(\underline{t}_{i-1}) + \underline{t}_{i-1}}$$

which clearly holds. □