The Methodology for the Mid-Term Forecasting of the Financial Results of Firms

Belikov V.V., Gataullina R.I., Tregub I.V.

Abstract. The article is dedicated to the mid-term forecasting of the Company financial results. The recommended approach is based on simulation and stochastic modeling on two levels: item level modeling and asset level modeling. Also several scenarios are proposed to reflect different options of financial results change.

Key words. Mid-Term Forecasting, Simulation and Stochastic Modeling, Item Level Modeling, Asset Level Modeling, Monte-Carlo Method.

Introduction

The article presents practical approaches (or methodology) to the mid-term forecasting of Company financial results, as well as quantify exposure of calculated financial results to risk factors.

Risk factors, mentioned in the article, include commodities and raw materials prices, base interest rates, credit spreads, foreign exchange rates, as well as production, sales, transportation and storage volumes of commodities products. The methodology of mid-term forecasting is based on simulation and stochastic modeling of risk factors taking into account possible linear and nonlinear relationships, and also scenery calculation of commodity and cash flows. Modeling results are expected values and probabilistic distributions of mid-term commodity and cash flow forecasting with details on calendar months, which makes it possible to use this approach for financial risk analysis.

To construct a scenario for commodity and cash flow forecasting and financial results calculating two methods (different in the degree of input data detail for modeling) can be used:

- *Item level modeling* - modeling entire items values without their factorial analysis and components modeling [1]. This approach to modeling is based on consolidated financial forecast data;
- *Asset level modeling* – modeling of company operating, financing and investing activities at the level of production assets and financial instruments, including the commodity prices, exchange rates, interest rates and other indices modeling. The approach is used for modeling based on detailed operational and financial data [2].

In practice, it is necessary to combine these approaches to different items depending on detail level, availability of data, corporality of items, and work capacity to receive and process data.

1. General Approach to Mid-Term Forecasting

The mid-term cash flows and liquidity forecasting approach is based on the principles of simulation and stochastic modeling of financing, operating and investing activities for creating the stochastic and deterministic forecasts of Company financial results.

Application of simulation and stochastic (probabilistic) methods allows to obtain a large amount of analytical information. In the article the next scenarios of mid-term liquidity forecasting are used:

- *Expected scenario* - monthly forecast of financial results, the average value of the total company multicurrency residues (for which modeling is carried out) at the end of each calendar month period;
- *Conservative scenario* - monthly forecast of financial results corresponding to 95% percentile of the total company multicurrency residues (for which the simulation is carried out) at the end of each calendar month period;
- *Positive scenario* – monthly forecast of financial results corresponding to 5% of total company multicurrency residues (for which the simulation is carried out) at the end of each calendar month period.

2. Modeling algorithm

Modeling algorithm consists of the next steps:

- Specify the modeling parameters, download the necessary data;
- Perform cycle iterations for the Monte Carlo method. For each iteration perform the following steps:
  - Calculate all index values based on their relationship;
  - Calculate costs, profit and loss values, cash flows of financial instruments (if that approach to modeling is used);
  - Calculate commodity flows (if applicable), profit and loss values, cash flows of productive assets (if that approach to modeling is used).
  - Iterative calculation of Profit and Loss Tree items according to specified calculation algorithm. Cycle termination is reached when root element of the Tree is calculated;
- Calculate necessary statistics of financial results indices according to values calculated at every cycle iteration.

In case of deterministic forecasting one iteration of cycle should be performed. In case of stochastic forecasting more that 10000 iterations of Monte-Carlo method should be performed in order to gain almost 100% percentile quality for finance results values [3].

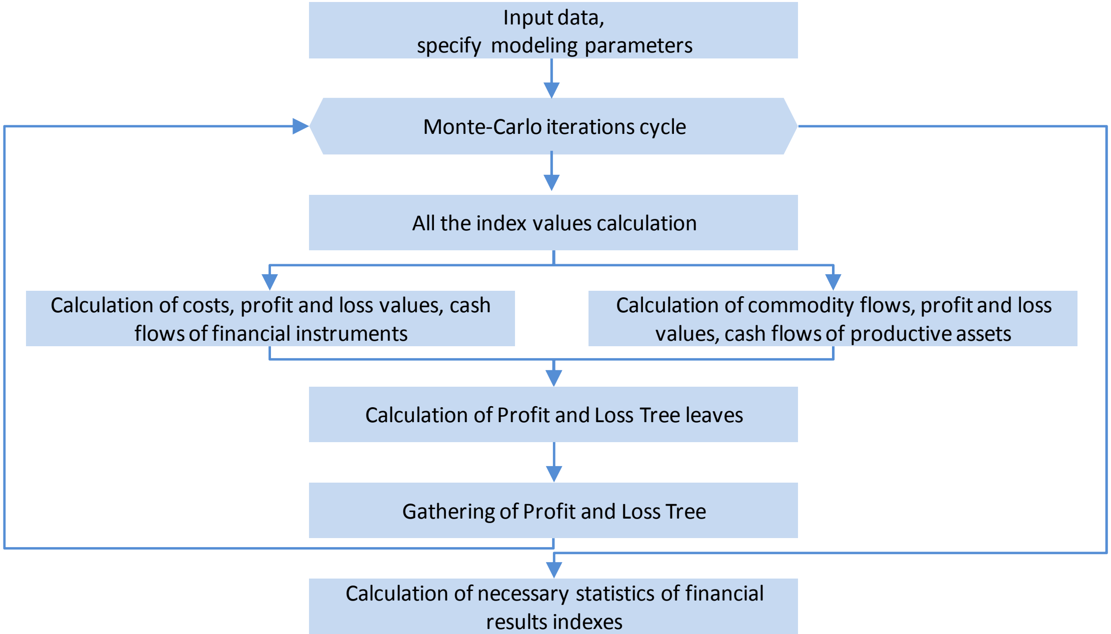

Figure 1. The general scheme of modeling algorithm

3. Indices modeling

Indices modeling is necessary for forecasting financial (foreign exchange rates, interest rates, commodities data, etc.) and production data (production volumes, sales volumes, etc.). By means of indices it’s possible to Profit and Loss Tree without further detail.

The main stages of index values calculation are:

- Indices and their relationships creation;
- Calculation of correlated values of a random perturbations of “main” indices defined by the stochastic method, including:
  - Calculation of singular perturbations;
  - Calculation of correlated singular perturbations using Cholesky decomposition;
  - Calculation of values using set of stochastic differential equations (SDE) and correlated singular perturbations;
- Calculation of values of “derivative” indices, based on the values of indices, calculated at the previous stage.

3.1. Indices

Indices are the approximation of the risk factors and items-leaves of Profit and Loss Tree is given in sufficient detail for modeling.

Indices can be specified in the following ways:

- *Deterministic method* - index value forecast is given by the deterministic vector-column of future values for each month in the period of mid-term forecasting;
- *Stochastic method* - index is specified using SDE;

- *Formula method* - index is given by the formula, where indices specified by deterministic and stochastic methods are used as variables.

Deterministic method

While using this method, the indices are set by the vectors of the deterministic values with the dimension $1 \times n$, where n is the mid-term forecasting horizon, expressed in calendar months. Most often deterministic method is used for forecasting the least volatile cash flows, in particular, salary costs, revenues and expenses on financial activities, etc.

Stochastic method

When using this method, the indices are set using a particular SDE (for each index), its parameters, and vector of deterministic initial values. Specific forecasted index values for all months of the period of mid-term prediction is calculated using a Monte Carlo-based SDE formula, parameters of SDE, and initial values.

For collaborative modeling of indices, determined using the stochastic method, one should consider their linear relationship – using correlation matrix M of indices (dimension $m \times m$).

Formula method

When using this method, indices are set by a formula, where the indices defined using deterministic, stochastic and formula ways are used as variables.

3.2.Monte-Carlo method

Monte-Carlo method provides collaborative generation of correlated random perturbations of indices defined using the stochastic method.

Based on the symmetric positively oriented correlation matrix M, defined by stochastic method, one should perform the Cholesky decomposition: $M = U^T U$, where U is an upper triangular matrix of size $m \times m$, where m is the number of indices determined using stochastic method.

The matrix of random perturbations D with the dimension $n \times m$ is filled by pairwise independent random numbers with distribution $N(0;1)$. The matrix D stores perturbations for each index, defined by the stochastic method, in each month of the period mid-term forecasting.

The matrix of correlated random perturbations dW with the dimension $n \times m$ is calculated by multiplication the next matrices: the matrix of random perturbations D, matrix U, and scalar value dt (forecast step size):

$$dW = D \times U \times dt$$

At final step, the elements of matrix dW substitute for variables dW_t^i by index i at month t in SDE formulas.

3.3. Simulation and stochastic indices modeling

For indices modeling one can use the following set of stochastic differential equations (SDEs), commonly applied in financial mathematics. The purpose of the SDE use is to obtain a vector of stochastic index modeling values for future periods based on the set of SDE parameters.

Model selection is carried out on the basis of common practices and expert opinions. The key criteria are the following:

- Compliance of historical probability distribution of monthly simulated index values with theoretical distribution of the SDE, including distribution tails and corresponding statistical tests;
- Stability of SDE parameters, calculated based on different historical periods.

Below SDU, featured for the purposes of this methodology, and recommendations for their practical application in mid-term forecasting.

List of SDEs for indices modeling is following [4]:

- Brownian motion $dS_t = Gdt + \Sigma\, dW_t$
- Geometric Brownian Motion $dS_t = \mu S_t dt + \sigma S_t dW_t$
- Merton Jump Diffision Model $dS_t = \mu S_t dt + \sigma S_t dW_t + S_t dJ_t$, $dJ_t = Y * N_t$
- Mean Reverting Jump Diffision Model $dS_t = \theta(M - S_t)dt + \sigma dW_t + dJ_t$, $dJ_t = Y * N_t$
- Vasicek Model $dS_t = \theta(M - S_t)dt + \sigma dW_t$
- Cox-Ingersoll-Ross Model $dS_t = \theta(M - S_t)dt + \sigma\sqrt{S_t} dW_t$
- Logarithm Mean Revering Model Diffusion Model

$$d \operatorname{Ln} S_t = \theta(\operatorname{Ln} M - \operatorname{Ln} S_t)dt + \sigma_t dW_t$$

- Logarithm Mean Revering Jump Diffusion Model

$$d \operatorname{Ln} S_t = \theta(\operatorname{Ln} \mathrm{M} - \operatorname{Ln} S_t)dt + \sigma_t dW_t + dJ_t,\ dJ_t = Y * N_t$$

4. Item level modeling

Item level modeling is related to “top down” principle, due to the complexity of the quantitative estimates of exposure of some assets to risk factors. The advantage of this approach is its relative simplicity and considerably lower requirements to the input data detail. Item level modeling allows to estimate cumulative exposure of cash flow to risk factors. It is recommended to use this approach to model relatively insignificant items, but also significant items, information on which is difficult or impossible to get .

The main steps of the calculation of the Tree item values are:

- Gathering of the Tree based on calculated indices values within Monte-Carlo method iterations, which includes:
    - Calculation of the Tree leaves based on index values;

- Calculation of the Tree (non-leaves) vertices based on values of underlying items;
- Keeping the values of Tree root and other significant vertices;

- Calculation of probability indicators for root (and other significant) Tree items, based on calculated values of these items on all iterations of Monte Carlo method.

4.1. Items

Items of Profit and Loss Tree form a multi-level Tree (in the mathematical aspect), consisting of vertices and leaves. Tree branches can have any length independently from each other.

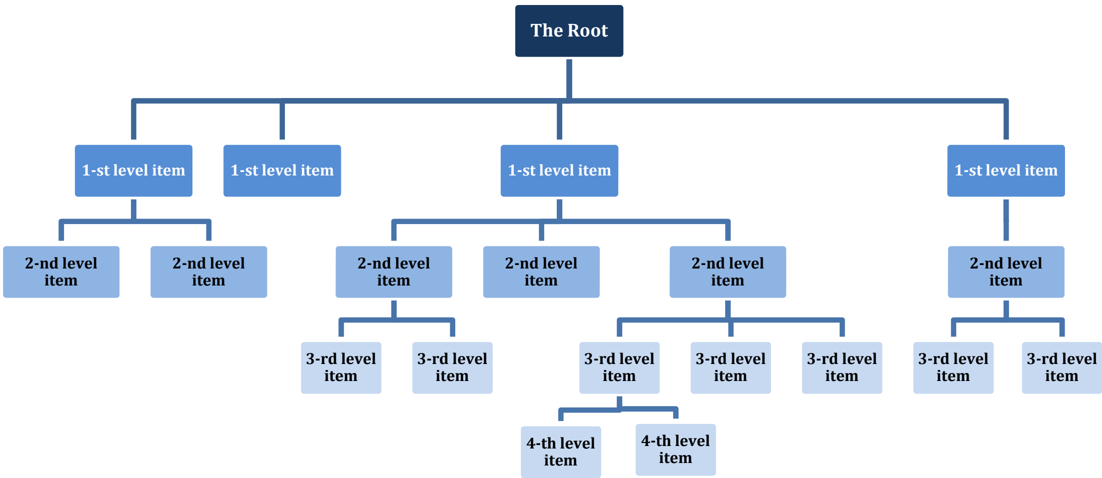

Figure 2. Example of structure of multi-level Tree

4.2. Gathering of the Tree of items

Gathering of the Tree of items is the calculation of the values for all Tree items for all months of mid-term forecasting period t. The end of the gathering process is the calculation of the Tree root.

The process of Tree gathering depends on method of indices determination.

4.2.1. All indices are specified by deterministic method

Indices are defined by vectors of size $n \times 1$: $< index_1, \ldots, index_m >$, where m is the number of indices specified through deterministic values.

Items (that don't have sub-items) are calculated according to the next formulas:

$$item_1 = f_1\left(index_{1,t-p}, \ldots, index_{1,t}, \ldots, index_{m,t-q}, \ldots, index_{m,t}\right) = \begin{pmatrix} item_{11} \\ item_{12} \\ \ldots \\ item_{1,n} \end{pmatrix}, \ldots,$$

$$item_k = f_k\left(index_{1,t-p}, \dots, index_{1,t}, \dots, index_{m,t-q}, \dots, index_{m,t}\right) = \begin{pmatrix} item_{k,1} \\ item_{k,2} \\ \dots \\ item_{k,n} \end{pmatrix}$$

- $item_{i,j}$ is a forecasted value of item i in month j;
- $index_{s,t-l}$ is an index s value in month $t-l, l \in \overline{0,t}$;
- k is a number of modeling items which don't have sub-items;
- n is horizon of mid-term forecasting;
- m is the number of indices, specified through deterministic values.

The calculation starts with the leaves of the Tree and ends with the root of the Tree. In case of necessity of transition from deterministic method to stochastic method (for items calculation), the transformation of the vector into the matrix (size $n \times r$) is performed according to the formula:

$$item_p = \begin{pmatrix} item_{p,1} \\ item_{p,2} \\ \dots \\ item_{p,n} \end{pmatrix} \rightarrow \begin{pmatrix} item_{p,1} & \dots & item_{p,1} \\ \dots & \dots & \dots \\ item_{p,n} & \dots & item_{p,n} \end{pmatrix}$$

4.2.2. There are indices, which specified by stochastic method

The indices are specified by matrixes of size $n \times r$: $index_1, \dots, index_m$, where m is the number of indices specified through stochastic method.

Items (that don't have sub-items) are calculated according to the next formulas:

$$item_1 = f_1(index_1, \dots, index_m), \dots, item_k = f_k(index_1, \dots, index_m)$$

- k is a number of modeling items which don't have sub-items;
- m is the number of indices, specified through deterministic values.

The calculation starts with the leaves of the Tree and ends with the root of the Tree.

4.2.3. Calculation of probabilistic indicators

According to the algorithm above, one can calculate the Root of the Tree (net income in the reporting currency) for each scenario of Monte-Carlo method. In the result, there is the matrix, each column of which is the result of some scenario modeling:

$$\left(\begin{pmatrix} NI_{11} \\ NI_{12} \\ \dots \\ NI_{n,1} \end{pmatrix} \begin{pmatrix} NI_{12} \\ NI_{22} \\ \dots \\ NI_{n,2} \end{pmatrix} \dots \begin{pmatrix} NI_{1,r} \\ NI_{2,r} \\ \dots \\ NI_{n,r} \end{pmatrix} \right)$$

- NI_{ij} is forecasted Net Income value in month i according to the scenario j;
- n is horizon of mid-term forecasting;
- r is the number of scenarios of Monte-Carlo method.

Then, it's necessary to create a probability distribution for indicators values of the financial result for the calculation of the relevant risk indicators «at risk»

5. Asset level modeling

Mid-term forecasting approach, based on modeling on the level of production and financial assets, is different from the approach, based on modeling on the item levels, as it requires input data in better detail.

The approach is based on commodity flows arising between the company and third parties (external commodity markets). According to pricing formulas for commodity trading (with fixed and variable expenses for mining), preparation for transport, transportation, processing and sales, the cash flow forecasts are formed with the respect for operating items [5]. Similarly, based on financial instruments parameters and their number, and also based on information about CAPEX and financial investments, cash flows forecasts on financial and investment items are formed.

Gathering of the Tree is produced with the methods described above. From a technical point of view, this approach is a particular case of the formula method for appropriate indices and items definition.

6. Results of mid-term forecasting

The result of the forecasting should financial results (Net income) of the company. According to scenarios, several forecasting values for each month should be calculated for the financial result (Net income):

- Average (expected) value of Net Income:

$$average_{NetIncome}(t) = \frac{\sum_{i=1}^{n} NetIncome_i(t)}{n}$$

  - $average_{NetIncome}(t)$ is the average value of Net Income in month t,
  - $NetIncome_i(t)$ is the value of Net Income according to scenario i of Monte-Carlo method, $i \in \overline{1,n}$, in month t,
  - n is the number of scenarios of Monte-Carlo method.

- Forecasted Net Income by Conservative scenario:

$$percentile_{95\%,NetIncome}(t) = percentile_{95\%}(NetIncome_1(t), \dots, NetIncome_n(t))$$

  - $percentile_{95\%,NetIncome}(t)$ is the forecasted value of Net Income in month t by Conservative scenario,
  - $NetIncome_i(t)$ is the value of Net Income according to scenario i of Monte-Carlo method, $i \in \overline{1,n}$, in month t,
  - n is the number of scenarios of Monte-Carlo method.

- Forecasted Net Income by Positive scenario:

$$percentile_{5\%,NetIncome}(t) = percentile_{5\%}(NetIncome_1(t), \dots, NetIncome_n(t))$$

- $percentile_{5\%,NetIncome}(t)$ is the forecasted value of Net Income in month t by Positive scenario,
- $NetIncome_i(t)$ is the value of Net Income according to scenario i of Monte-Carlo method, $i \in \overline{1,n}$, in month t,
- n is the number of scenarios of Monte-Carlo method.

For effective use of the methodology, on regular basis one needs to collect necessary for the mid-term forecasting structured historical forecasting and actual data on inflows and outflows, cash balances, volume of production, transport and sales, market data.

References

1. Jeremy C.Stein, Stephen E.Usher, Daniel LaGattuta, Jeff Youngen. A comparables approach to measuring CashFlow-at-Risk for non-financial firms. – Journal of Applied Corporate Finance, Volume 13, Number 4, Winter 2001.
2. Risk Metrics Group. Corporate Metrics. The Benchmark for Corporate Risk Management. Technical Document. – First edition (April, 1999).
3. Damiano Brigo, Antonio Dalessandro, MatthiasNeugebauer, Fares Triki. A Stochastic Processes Toolkit for Risk Management. – Fitch Solutions, Quantitative Research, November 2007.
4. Трегуб И.В. Особенности инвестирования инновационных проектов // Экономика. Налоги. Право. -2013. - №3. - С. 28-33.
5. Трегуб И.В., Трегуб А.В., Шкляев Л.О. Сравнительный анализ моделей оценки кредитного риска эмитента корпоративных облигаций на российском долговом рынке // Лесной вестник, № 3 (95), с. 215-221.
6. Трегуб И.В., Кузнецов В.Д. Математическое моделирование динамики экономических показателей (на примере выручки IT компании) // Современные проблемы науки и образования. – 2013. – № 6; URL:http://www.science-education.ru/113-10977