Firm-Level Comparative Advantage*

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Abstract

Drawing on empirical evidence we consider firms' heterogeneity in terms of factor intensity. We show that Heckscher-Ohlin comparative advantage begets a comparative advantage at firm-level. Firms whose relative factor-intensity matches with the comparative advantage of the country have lower relative marginal costs and higher relative sales. Our empirical analysis, conducted using data for a large panel of European firms, supports these predictions. These predictions and results provide a novel approach to the verification of the Heckscher-Ohlin theory and new evidence on its validity.

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1 Introduction

There is a wealth of empirical evidence that factor intensity differs across firms even within the same country-industry groups. In our data, for instance, only 31 percent of the total variance in firm-level capital/labor ratios is between country-industry groups, 69 percent is within the same country-industry groups. Empirical literature in international trade has documented that differences in factor intensities matter for firms' performances (e.g. Bernard, Jensen, Redding and Schott 2007). These observations contrasts with the assumption usually adopted in trade models whereby firms are either assumed to be identical or are assumed to differ by Hicks-neutral productivity differences. In either case, the resulting factor intensities are identical across firms within any industry. Following this empirical observation, we consider a Heckscher-Ohlin model in which firms differ in factors' relative marginal productivity (RMP). As a result, firms have different factor intensities even within the same country-industry group. The main result emerging from this model is that countries' comparative advantage begets a comparative advantage at firm level. The three key firm-level variables in the model are relative capital/labor ratio (κ), relative marginal costs, and relative sales; all three relative to the country-industry average. A firm is capital- (labor)-intensive with respect to the country-industry average if $\kappa > 1$ ($\kappa < 1$). A firm has a comparative advantage over another firm if it has lower relative marginal cost. Our main theoretical result may be stated as follows:

Consider any two firms with same κ but in different countries and industries. The firm that is intensive in the factor intensively used in its industry and of which its country is relatively well-endowed has a comparative advantage over the other firm. Because of the comparative advantage, the firm will also have higher relative sales.

The statement above is the natural generalization of the Heckscher-Ohlin theorem to an environment with heterogenous firms. In the Heckscher-Ohlin theorem, the comparative advantage of a country is generated by the matching of country and industry characteristics. In the statement above, the comparative advantage of a firm is generated by the matching of its characteristics (κ) with those of the industry and country. In the Heckscher-Ohlin theorem, the comparative advantage of countries gives rise to differences in the relative size of industry output (international specialization). In the statement above, the comparative advantage of firms gives rise to differences in the relative size of firm output (relative sales).

As an example, consider two firms in different industries and different countries but with an identical κ. Assume, for instance, $\kappa > 1$. Then, the firm in the capital intensive industry and capital abundant country will have a comparative advantage (lower relative marginal cost) over the firm in another industry and country. The firm's comparative advantage will show up in larger relative sales. If, instead, we consider two firms whose capital intensity is lower than their respective country-industry average ($\kappa < 1$) then the firm in the capital intensive industry and capital abundant country will have a comparative disadvantage and lower relative sales. This prediction does not obtain in models where heterogeneity is in Hicks-neutral productivity differences. In these models, two firms with same relative productivity but in different countries and industries have identical relative marginal cost and identical relative sales.

Our empirical investigation, conducted on a dataset which comprises over 400,000 firms in 28 European countries and 84 industries, strongly supports the theoretical result. Both structural and non-structural estimates show that the relationship between relative sales and relative factor intensity is affected by comparative advantage in the way predicted by the model. For instance, the non-structural estimates show that two firms with capital intensity ten percent above their respective country-industry average ($\kappa = 1.1$) have different relative sales: the sales of the firm in the capital intensive industry and capital abundant country are 4.3 percent larger than the average firm in the same country-industry whereas the sales of the firm in the labor intensive industry and the labor abundant country are only 1.5 percent larger that the average firm in the same country-industry.

We are not alone to assume heterogeneity in factor intensity. Costinot and Vogel (2010) and Burstein and Vogel (2010) are notable examples. Their models differ from ours in terms of the market structure, technology, and preferences. There are also important differences in the mechanisms driving the results; theirs rest on skilled biased heterogeneity, ours does not. Lastly, the focuses are very different; they study the effect of trade liberalization on wage inequality, we study instead how countries comparative advantage begets comparative advantage at firm level. Yet, in their works like in ours, heterogeneity in factor intensity harnesses to a better extent the potentials of heterogenous firms models in the understanding of international trade issues.

Our paper contributes to the literature in two ways. First, it provides a novel approach to the verification of the Heckscher-Ohlin (HO) model. Seminal contributions, e.g., Leamer (1980), Trefler (1993, 1995), Davis and Weinstein (2001), Romalis (2004) have provided solid evidence on the empirical merits of the factors proportion theory. In their works, comparative advantage is revealed by the effect it has on aggregate variables (the factor content of trade or industry specialization). We proposes a different approach. In our model, comparative advantage is revealed by the effect it has on firm-level variables (firm-level comparative advantage). Comparative advantage is the mechanism driving the HO theorem but it remains behind the scenes in homogenous-firms models, as well as in Hicks-neutral heterogeneity models, because it does not give rise to a comparative advantage at firm level. Being able to observe the firm-level comparative advantage generated by the comparative advantage of countries brings to light the fundamental mechanism driving the HO theorem. Approaches based on aggregate variables are, of course, unsuited to bring this mechanism to light.

Our second contribution is to show that countries' comparative advantage matters for explaining relative performances within industries. Seminal contributions by Bernard, Eaton, Jensen and Kortum (2003) and Melitz (2003) as well as many subsequent important developments use a one factor model and take out countries' comparative advantage.[1] They are therefore unsuited to study the effect of country' comparative advantage on firms performances. Bernard, Redding and Schott (2007) are the first to introduce firm heterogeneity in a HO model, but they consider only Hicks-neutral productivity differences. With Hicks-neutral productivity differences firms relative sales are insensitive to comparative advantage. Thus, the one-factor assumption or Hicks-neutral heterogeneity make that within industry relative performances are independent from comparative advantage and depend only on exogenously given differences in productivity. As a result, one may be left with the impression that there is a dichotomy between within-industry effects and across-industry effects, the former being driven by firm-level differences and the latter by countries' comparative advantage. Our work, instead, highlights precisely how within-industry effects are determined jointly by firm-level characteristics (κ) and countries' comparative advantage.

We conclude this section by mentioning that the four core theorems of

[1]See, e.g., Yeaple (2005), Chaney (2008), Melitz and Ottaviano (2008), Arkolakis, Costinot and Andrés Rodríguez (2011), Bustos (2011), Eaton, Kortum and Kramarz (2011), Crozet, Head and Mayer (2011). See also Manasse and Turrini (2001), Egger and Kreickemeier (2009), Helpman, Itskhoki and Redding (2010), Davis and Harrigan (2011), Amiti and Davis (2011) for particular focus on the distribution effects of trade integration.

international trade remain valid in our model. But the degree of international specialization, the intensity of the Stolper-Samuelson and Rybczynski magnification effects, and the size of the factor price equalization set are all affected. We briefly present these results in Appendix Section 9.2.8.

The remainder of the paper is as follows. Section 2 outlines the model, Section 3 describes the theoretical results, Section 4 derives the estimable equation, Section 5 presents the data, Sections 6 and 7 present the empirical results for the structural and non-structural estimates respectively, and Section 8 concludes. The Appendix contains the detailed description of the model, the proofs of propositions, additional technical matters, a brief discussion on the four core theorems, and some numerical simulations.

2 Heterogeneity in Factor Intensity

In this section we outline the model focusing on its key elements, namely, heterogeneity in factor intensity. A detailed description of the model is given in Appendix Section 9.1. The world economy is assumed to have two countries indexed by $c = H, F$; it produces two differentiated goods indexed by $i = Y, Z$, by using two primary factors indexed by $j = K, L$. Each country is endowed with a share $\nu_j^c > 0$ of world's endowments, $\overline{K}$ and $\overline{L}$. Production requires fixed and variable inputs in each period. The variable input technology takes the CES form $q_i = \phi \left(\lambda_i \left(\alpha L \right)^{\frac{\sigma-1}{\sigma}} + (1 - \lambda_i) \left(\beta K \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$ where q_i is output, L and K are factors inputs, $\sigma > 1$ measures gross substitutability between factors, $\lambda_i \in (0, 1)$ is a constant technology parameter, and ϕ, α, and β are random variables.[2]

To fix ideas, throughout the paper it is assumed that H is *K-abundant,* i.e., $\nu_K^H > \nu_L^H$, and that Y is *K-intensive*, i.e., $\lambda_Y < \lambda_Z$.

The marginal cost for a firm in industry i of country c, mc_i^c, is

$$mc_i^c = \frac{1}{\phi} \left[(\lambda_i)^\sigma \left(\frac{w^c}{\alpha} \right)^{1-\sigma} + (1 - \lambda_i)^\sigma \left(\frac{r^c}{\beta} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} . \tag{1}$$

[2]When factors are gross complement ($\sigma < 1$) our main result, namely, that countries comparative advantage begets comparative advantage at firm level, remains valid. See Appendix Section 9.2.4 for further discussion.

where r^c and w^c denote, respectively, the price of K and L in country c. Models that focus on Hicks-neutral heterogeneity assume α and β constant, and identical across firms, and let ϕ vary across firms. We instead focus on the heterogeneity in α and β which, regardless of variations in ϕ, influences factors' RMP and, thereby, factor intensity. For the time being, therefore, we keep ϕ constant and equal to 1 and let α and β vary across firms. There is no difficulty in letting ϕ vary as well as α and β. But this would unnecessarily burden the exposition with a more intricate notation. We shall reintroduce ϕ in Sections 4, as well as in the empirical part of the paper. The relative marginal productivity of K is $\frac{1-\lambda_i}{\lambda_i}\left(\frac{L}{K}\right)^{\frac{1}{\sigma}}\left(\frac{\beta}{\alpha}\right)^{\frac{\sigma-1}{\sigma}}$ and the *K-intensity* in production, θ_i^c, is

$$\theta_i^c = (\omega^c)^\sigma \Lambda_i^\sigma \left(\frac{\beta}{\alpha}\right)^{\sigma-1}, \tag{2}$$

where $\omega^c \equiv w^c/r^c$ and $\Lambda_i \equiv (1-\lambda_i)/\lambda_i$.

To begin, let us consider the simple case in which $\alpha = 1$ and firms draw β from a probability distribution $g(\beta)$ with support in $(0,\infty)$ and with a cumulative distribution $G(\beta)$. Let β_i^{*c} be the smallest value of β, such that profits are non-negative. Firms with a productivity draw larger or equal to β_i^{*c} engage in production while the other firms exit the market.[3] Therefore, the average β over producing firms, denoted $\widetilde{\beta}_i^c$, is

$$\widetilde{\beta}_i^c = \left[\frac{1}{1-G(\beta_i^{*c})}\int_{\beta_i^{*c}}^{\infty}\beta^{\sigma-1}g(\beta)\,d\beta\right]^{\frac{1}{\sigma-1}}, \tag{3}$$

which allows writing the average factor intensity, $\overline{\theta}_i^c$, as[4]

$$\overline{\theta}_i^c = (\omega^c)^\sigma \Lambda_i^\sigma \left(\widetilde{\beta}_i^c\right)^{\sigma-1}. \tag{4}$$

Lastly, the average marginal cost, denoted $\widetilde{mc}_i^c$, is

$$\widetilde{mc}_i^c = \left[\frac{1}{1-G(\beta_i^{*c})}\int_{\beta_i^{*c}}^{\infty}\left[mc_i^c(\beta)\right]^{1-\sigma}g(\beta)\,d\beta\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

[3] It is worth mentioning that the propositions in this paper hold even if we had assumed that all firms survived in the market. See Appendix Section 9.2.5

[4] Firms with a very high β in industry Z may have a higher *K-intensity* than firms with a low β in industry Y. Yet, $\Lambda_Y > (<)\,\Lambda_Z$ is a sufficient condition for the average *K-intensity* to be larger (smaller) in industry Y than in Z (see Appendix Section 9.2.6).

Turning to the fixed input technology, whether it is homogenous or heterogenous across firms gives qualitatively the same results. For clarity of exposition, it is necessary to retain one of the alternatives throughout the paper. We retain the former since it allows focusing on heterogeneity in the production process (which is the heart of the matter). Specifically, we assume that the fixed-input takes the form of output that must be produced but cannot be sold.[5] Thus, the fixed production cost is $F_i \widetilde{mc}_i^c$ where F_i is a positive constant. Analogously to fixed production cost, the fixed exporting cost is $F_{ix} \widetilde{mc}_i^c$ where F_{ix} is a positive constant. As is well known, the presence of fixed exporting costs generates endogenously a partitioning of firms by export status. Analogously for β_i^{*c}, let β_{ix}^{*c} be the least value of β such that foreign profits are non-negative. Firms with productivity draw $\beta < \beta_i^*$ will exit immediately, firms with productivity β such that $\beta_i^* \leq \beta < \beta_{ix}^*$ will produce for the domestic market only, and firms with a productivity draw $\beta \geq \beta_{ix}^*$ will produce for the domestic and the foreign market.

We conclude this section with three remarks. First, we do not impose any restriction on the relationship between *K-intensity* and productivity. This means that we allow for both types of normalization: $\alpha = 1$, $\beta \in (0, \infty)$ or $\beta = 1$, $\alpha \in (0, \infty)$. We see from expression (1) and (2) that with the first normalization, a higher *K-intensity* corresponds to a lower marginal cost, whereas with the second normalization a higher *K-intensity* corresponds to a higher marginal cost. We allow for both normalizations. Indeed, analogous definitions for $\widetilde{\alpha}_i^c$ and $\widetilde{mc}_i^c$ may be given if we assume $\beta = 1$ and let α be distributed according to $g(\alpha)$ with support in $(0, \infty)$. We detail these definitions in Appendix Section 9.2.7. To keep the notation simple we continue to present the model choosing $\alpha = 1$ and $\beta \in (0, \infty)$, but we shall present the results in Section 3 for both normalizations. The second remark concerns possible factor bias. We can see from expression (4) that, with respect to models where firms have identical factor intensities, there is a factor bias in this model, whenever $\widetilde{\beta}_i^c \neq 1$. The bias is endogenous since it depends on the cut-off values β_i^{*c}. It may go in either direction - to a *K-bias* or a *L-bias* - and the direction may differ in different industries or countries. None of our results depend on the direction or on the existence of such bias.

Turning to demand, the representative consumer has Dixit-Stiglitz prefer-

[5] The assumption of homogenous fixed inputs is adopted also in Melitz (2003), Yeaple (2005), Bernard, Redding and Schott (2007) and many others. In our model, as in theirs, results are robust to assuming heterogeneity in fixed costs. Fixed inputs in the form of output is also widely adopted in the literature.

ences represented by a Cobb-Douglas index, with shares $\gamma_i \in (0,1)$, $\gamma_Y + \gamma_Z = 1$, and defined over CES aggregates whose elasticity of substitution between varieties is $\varsigma > 1$. Given this structure, the sales of any firm relative to any other firm in the same industry and country - with both being either exporters or non-exporters - depend solely on the ratio of marginal costs. That is, for any two firms with draws β' and β'' such that $\beta_{ix}^* > (\beta', \beta'') \geq \beta_i^*$ or $(\beta', \beta'') > \beta_{ix}^*$ we have

$$\frac{s_i^c(\beta')}{s_i^c(\beta'')} = \left[\frac{mc_i^c(\beta')}{mc_i^c(\beta'')}\right]^{1-\varsigma}, \tag{6}$$

where s stands for sales.

3 Comparative Advantage

For clarity of exposition we present our results under the assumption that $F_{ix} = 0$. While this makes the reading smoother it also highlights that our results do not depend on the partitioning of firms by export status.[6] We shall reintroduce fixed exporting costs in the Appendix Section 9.3, however, where numerical simulations confirm the analytical results obtained in this section. With no fixed exporting costs all existing firms export and Equation (6) holds for any existing firm, i.e., for any $(\beta', \beta'') \geq \beta_i^*$.

Our objective is to show that countries' comparative advantage generates a comparative advantage at firm level. We begin by defining three variables. These are firms' relative factor intensity, κ, relative marginal cost, μ_i^c, and relative sales, RS_i^c (all three relative to the country-industry average):

$$\kappa = \frac{\theta_i^c}{\overline{\theta}_i^c}, \qquad \mu_i^c \equiv \frac{mc_i^c}{\widetilde{mc}_i^c}, \qquad RS_i^c \equiv \frac{s_i^c}{\overline{s}_i^c} \tag{7}$$

Relative factor intensity, κ, is a parameter that serves the purpose of comparing firms across industries and countries. We shall compare firms belonging to different countries and industries that have identical κ, for any $\kappa > 0$. Naturally, μ_i^c and RS_i^c are endogenous and depend, among other variables, on κ as well.

[6] The partitioning of firms by export status plays a crucial role in many papers in the literature, e.g., Manasse and Turrini (2001), Yeaple (2005), Costinot and Vogel (2010), Helpman, Itskhoki and Redding (2010), Amiti and Davis (2008).

Definition 1 *A firm is K-intensive if $\kappa > 1$. A firm is L-intensive if $\kappa < 1$.*

Definition 2 *A firm has a comparative cost advantage over another firm iff it has lower relative marginal cost.*

Our main theoretical result may be stated as follows:

Theorem 1 *A firm in industry i and country c has a comparative cost advantage over another firm with same κ but in a different country and industry, if it is intensive in the factor intensively used in i and of which country c is relatively well endowed. In our notation, and recalling that H is K-abundant Y is K-intensive industry, this means:*

$$\text{For any } \beta' \text{ such that } \theta_i^c(\beta') = \kappa\overline{\theta}_i^c, \text{ we have } \mu_Y^H \lesseqqgtr \mu_Z^F \text{ as } \kappa \gtreqqless 1. \tag{8}$$

Proof. *See Appendix Section 9.2.2* ■

This means that a *K-intensive* firm ($\kappa > 1$) has a comparative cost advantage over another firm with same κ, if it is in the *K-intensive* industry of the *K-abundant* country, and if the other firm is in the *L-intensive* industry of the *L-abundant* country. Likewise, *mutatis mutandi*, for two firms with $\kappa < 1$.

The HO theorem relates the comparative advantage to industries' factor intensity and to countries' relative factor abundance. Theorem 1 relates firms comparative advantage to countries comparative advantage. Thanks to heterogeneity in factor intensity we can *observe* HO comparative advantage *in action*, as it generates the comparative advantage of firms. Observing this effect in the data provides a novel empirical verification of the HO theory which goes to the very heart of its functioning mechanism.

An individual firm's comparative advantage is reflected in its relative sales via Equation (6). We then establish formally the relationship between factor intensity and relative sales which we shall verify empirically.

Let $a \in A = \left\{a_Y^H, a_Z^F\right\}$ be an index of comparative advantage where $a_i^c = (\Lambda_i)^\sigma (\omega^c)^{\sigma-1} \left(\widetilde{\beta}_i^c\right)^{\sigma-1}$.[7] Let $\kappa \in B \subseteq \mathbb{R}^+$. Then:

[7] In Appendix Section 9.2.1 we show that selection into entry results in $\widetilde{\beta}_Y^H > \widetilde{\beta}_Z^F$. If, instead, we had assumed that all firms survived in the market then the cut off values β_i^{*c} and the averages $\widetilde{\beta}_i^{*c}$ would be the same for all c and i. In either case $a_Y^H > a_Z^F$ since H is *K-abundant* and Y is *K-intensive.*

Proposition 1 *The function RS: $A \times B \to \mathbb{R}^{+}$ is strictly log-supermodular in (a, κ). Further $RS(a, 1) = 1$ $\forall a$. This, implies that for any β' such that $\theta_i^c(\beta') = \kappa\overline{\theta}_i^c$, $RS_Y^H \gtreqless RS_Z^F$ as $\kappa \gtreqless 1$.*

Proof. *See Appendix Section 9.2.3 .* ■

Proposition 1 says that for any two firms with same κ, the firm in the industry and country of its comparative advantage has larger relative sales. Intuition for this result is served by analyzing the two underlying mechanisms giving rise to it.

1. Factor-intensity and industry technology. The first mechanism relates firm-level factor intensity (κ) to the technology of the industry (Λ_i). Consider two firms in the same country but in different industries and with identical $\kappa > 1$. Although these firms have same κ, the relative marginal cost is lower and relative sales are higher for the firm in the *K-intensive* industry, because the factor whose relative marginal productivity is higher for both firms with respect to the industry average (K) is used more intensively in the *K-intensive* industry. Consider now two firms whose *K-intensity* is lower than their respective industry average ($\kappa < 1$). The relative marginal cost is higher and relative sales lower for the firm in the *K-intensive* industry, because the factor whose relative marginal productivity is lower for both firms with respect to the industry average (K) is intensively used in this industry. Formally:

Let $\Lambda \in \underline{\Lambda} = \{\Lambda_Y, \Lambda_Z\}$, the function RS: $\underline{\Lambda} \times B \to \mathbb{R}^{+}$ is strictly log-supermodular in (Λ, κ). Further, $RS(\Lambda, 1) = 1$ $\forall \Lambda$. This implies that for any β' such that $\theta_i^c(\beta') = \kappa\overline{\theta}_i^c$, $RS_Y^c \gtreqless RS_Z^c$ as $\kappa \gtreqless 1$, $\forall c$.

2. Factor intensity and factors abundance. The second mechanism relates firm-level factor intensity (κ) to countries' relative factors endowments via relative factors price (ω). Consider two firms in the same industry but in different countries and with identical $\kappa > 1$. Relative marginal costs are lower and relative sales are higher for the firm in the *K-abundant* country, because the factor that both firms use intensively with respect to the industry average (K) is relatively cheaper in the *K-abundant* country. Consider now two firms in the same industry but in different countries and whose *K-intensity* is instead lower than their respective industry average ($\kappa < 1$). The relative marginal cost is higher and sales lower for the firm in the *K-abundant* country,

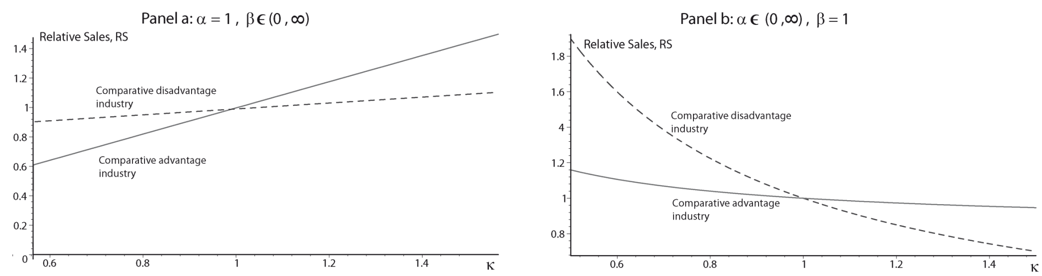

Figure 1: Graphical representation of Proposition 1

because the factor that both firms save with respect to the industry average (K) is relatively cheaper in the *K-abundant* country. Formally:

Let $\omega \in \Omega = \left\{\omega^H, \omega^F\right\}$, *the function* RS: $\Omega \times B \rightarrow \mathbb{R}^+$ *is strictly log-supermodular in* (ω, κ) *and* $RS\,(\omega, 1) = 1\ \forall\omega$. *This implies that for any* β' *such that* $\theta_i^c\,(\beta') = \kappa\overline{\theta}_i^c$, $RS_i^H \gtreqless RS_i^F$ *as* $\kappa \gtreqless 1$, $\forall i$ *and for any* $\tau \in (0,1)$.

Figure 1 offers a graphical representation of the relationship between relative sales (RS_i^c) and relative *K-intensity* (κ) stated in Proposition 1. Panel a) shows it when κ has a positive impact on relative sales, i.e., $\alpha = 1$, $\beta \in (0, \infty)$. Panel b) shows it when κ has a negative impact on relative sales, i.e., $\alpha \in (0, \infty)$, $\beta = 1$.[8] In either case, the ranking of relative sales is as stated in Proposition 1, i.e., $RS_Y^H\,(\kappa) \gtreqless RS_Z^F\,(\kappa)$ *as* $\kappa \gtreqless 1$. As should be clear by now, relative factor intensity may have a positive or a negative impact on relative marginal cost and, thereby, on relative sales; but in either case a firm has larger relative sales if it is in the country and industry of its (the firm's) comparative advantage.

From the discussion above, it is clear that Hicks-neutral heterogeneity has no impact on relative sales. This is easily shown by replacing $\alpha = \beta = 1$ throughout the model and by letting ϕ be a random variable. Then:

Proposition 2 *When heterogeneity is Hicks-neutral, relative sales do not depend on country-industry characteristics.*

[8] Under the normalization $\alpha \in (0, \infty)$ and $\beta = 1$, Proposition 1 is restated, except that α' replaces β'. Likewise for the two underlaying mechanisms.

Proof. *Compute average marginal cost from expressions (1) and use Equation (6) to obtain that for any* ϕ'*, such that* $\phi = \varphi \widetilde{\phi}_i^c$*. We have:*

$$RS_i^c = \varphi^{\varsigma - 1}; \qquad c = H, F; \quad i = Y, Z; \quad \forall \varphi > 0; \quad \forall \tau \in [0, 1] \,. \tag{9}$$

which proves the Proposition. ■

4 Empirical Implementation

We now assemble the results obtained in the previous section in a single estimable equation. A firm with draws $\phi' = \varphi \widetilde{\phi}_i^c$ and β' such that $\theta_i^c(\beta') = \kappa \overline{\theta}_i^c$ will have log of relative sales given by:[9]

$$\ln RS_i^c = (\varsigma - 1) \ln \varphi + \frac{1 - \varsigma}{1 - \sigma} \ln \left[\frac{1 + \kappa a_i^c}{1 + a_i^c} \right], \tag{10}$$

where we recall that $a_i^c = (\Lambda_i)^\sigma (\omega^c)^{\sigma - 1} \left(\widetilde{\beta}_i^c \right)^{\sigma - 1}$. Equation (10) summarizes what we have learnt so far. Both the Hicks-neutral productivity difference (φ) and relative factor intensity (κ) influence relative sales. But, while the effect of the former is independent of country and industry characteristics (Proposition 2) the effect of the latter depends on country and industry characteristics, condensed here in a_i^c (Proposition 1). Specifically, the log-supermodularity of relative sales stated in Propositions 1 is supported empirically, if the estimated elements of A are such that $\hat{a}_Y^H > \hat{a}_Z^F$.

Equation (10) is log-linear in the first term but not in the second. It can be estimated with non-linear least squares, but parameters a_i^c and σ cannot be identified independently. In order to obtain an estimate of a_i^c, we must set an arbitrary value for σ. We therefore also propose an estimation based on a second order Taylor expansion around $\kappa = 1$, of the second term in (10). We obtain a linear equation, which is more convenient to estimate and allows a_i^c, σ and ς to be estimated simultaneously:

$$\begin{aligned} \ln RS_i^c \;\; &= \;\; (\varsigma - 1) \ln \varphi + \frac{1 - \varsigma}{1 - \sigma} \frac{a_i^c}{1 + a_i^c} (\kappa - 1) \\ &\quad - \frac{1}{2} \frac{(1 - \varsigma)}{(1 - \sigma)} \left(\frac{a_i^c}{1 + a_i^c} \right)^2 (\kappa - 1)^2 + \varepsilon_i^c, \end{aligned} \tag{11}$$

[9] Use (2), (4), (6) and (9) to obtain (10).

where ε_i^c is the remainder of the Taylor expansion times $(1-\varsigma)/(1-\sigma)$, which can be decomposed into a country-industry dyadic fixed effect and a structural error term.

5 The Data

Our empirical examination combines two sources of data: firm-level balance sheets and country-level capital and labor endowments. Firm-level data are provided by Bureau Van Dijk's Amadeus database.[10] Amadeus compiles balance sheet information for a very large number of companies located in 41 European countries. Its coverage is increasing progressively. To get the most comprehensive database, we use the two most recent years available at the time of writing, 2006 and 2007. When companies are present in the database in both years, we simply retain the mean value of the information for 2006 and 2007. We take the information needed from Amadeus to estimate the influence of firms' capital intensity on their sales. We proxy the capital intensity by the ratio of tangible fixed assets to total employment and sales by the turnover of the firm, without distinction between exports and domestic sales. Firms in Amadeus are classified according to their primary activity. Each company is assigned to a single 3-digit NACE-Rev2 code. We restrict our empirical analysis to manufacturing sectors (including agrifood), i.e. to firms with a primary activity code between 101 and 329.[11] All firms with only one employee or less and firms with a capital/labor ratio 200 times below or 200 times above the corresponding country-industry median value are dropped from the sample. Moreover, we drop all country-industry pairs which contain too few observation to perform robust regressions. We fix an arbitrary limit and retain country-industry pairs with more than 20 firms.

Capital abundance for each country (K^c/L^c) is derived from several sources. We use ILO and United Nations data for workforce figures. Capital stocks are estimated by the perpetual inventory method, using investment data from the World Bank and national sources.[12] Industry-level capital intensity is computed directly with our data. For each country and industry, we compute the average firm-level capital-labor ratio, weighted by firms' sales.

[10] http://www.bvdep.com/en/AMADEUS.html

[11] We also exclude manufacturers of coke and refined petroleum products.

[12] We are indebted to Jean Fouré for giving us these country-level data. See Bénassy-Quéré *et al.* (2010) for a description of the source data and the methodology.

Then, *K-intensity* for industry i, (K_i/L_i), is the industry-level average of these values across all countries, weighted by countries' output of good i.

The final database is a panel of 412,386 firms in 84 industries and 28 European countries.[13] The country-industry panel is unbalanced because all countries do not have more than 20 firms in all the 84 industries. We have data for 1,471 country-industry pairs, for a total of 2,352 possible combinations. The average number of firms per country-industry pair is 280, but the population within each group varies greatly. The median country-industry pair has only 91 firms, and the largest group contains 8,858 observations.[14]

For many firms and countries only limited information is available. Considering these data limitations we can only provide quite rough estimates of firms' total factor productivity (TFP), which only require data on total turnover, tangible assets and employment levels. However, to be consistent with our theoretical framework, we estimate firms' TFP assuming a CES production function with constant returns to scale. We further assume identical technologies across countries. We estimate the second-order Taylor series expansion of the CES function proposed by Kmenta (1967),[15] for each industry separately, and controlling for country fixed effects. While our estimates of TFP are not very sophisticated, they are highly correlated to alternative TFP estimates based on Cobb-Douglas assumptions and do explain a very substantial part of the observed firm-level variance in total sales.[16]

Table 1 shows a variance decomposition analysis for firm-level total factor productivity and capital intensity in our sample. The first column gives the total variance of each variable while the four last columns report the shares of variance (R^2) in the log of total factor productivity, and in the log capital intensity that are explained respectively by different sets of fixed effects. Column 2 introduces industry level (NACE 3) fixed effects. Column 3 reports the explanatory power of country level fixed effects. We use the two sets of fixed effects together in Column 4 and country-industry pairs fixed effect in Column 5. It appears first that firms are much more heterogeneous in terms of capital intensity than in terms of productivity. More importantly for the

[13] Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Czech republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Romania, Russian Federation, Serbia, Slovakia, Slovenia, Spain, Sweden, Ukraine and United Kingdom.

[14] Spain-Manufacture of structural metal products.

[15] $\ln(q_i) = \ln(\phi) + \lambda_i \ln(L) + (1 - \lambda_i)\ln(K) - \frac{1}{2}\frac{\sigma-1}{\sigma}\lambda_i(1-\lambda_i)(\ln(L) - \ln(K))^2$.

[16] The log of TFP explains more than 45% of the variance in the log of sales.

premise of the paper, the different sets of fixed effects explain systematically a larger share of variance in TFP than in capital-intensity. The first R^2 reported in Column 5 establishes that 57 percent of the total TFP variance results from common country and industry characteristics. In other words, 43 percent of firm-level heterogeneity in terms of TFP is within countries and industries. This is quite a lot, but is still relatively low compared to capital intensity's variance. The R^2 reported in Column 5 for this variable is slightly higher than 0.31, which means that about 69 percent of the observed firm-level heterogeneity is within country-industry groups. This finding clearly confirms that the assumption of homogeneous factor intensity within industries, largely adopted in the literature, contrasts with actual observations.

Table 1: Variance decomposition of firm's TFP and Capital intensitiy: explanatory power (R^2) of different set of fixed effects

	Total Variance	Fixed Effects: Industry	Country	Country and Industry	Country-Industry pairs
ln(TFP)	1.4215	0.0909	0.5023	0.5530	0.5701
ln(K/L)	2.5920	0.0754	0.2162	0.2963	0.3132
Nb. obs	412386	412386	412386	412386	412386

6 Structural Estimates

This section presents the structural estimates of our model, based on Equation (10) and its Taylor expansion (11). In both cases, the dependant variable ($\ln RS_i^c$) is the log of firms' total sales (s_i^c) relative to the corresponding country-industry average ($\bar{s_i^c}$) in country c and industry i. The right-hand side variables are the total factor productivity and capital intensity of this firm, relative to the country-industry averages. The parameters to be estimated are ς, σ and a_i^c. The model imposes $a_i^c > 0$, $\sigma > 0$ and $\varsigma > 1$. More importantly, Proposition 1 states that a_i^c should be higher for *K-abundant* countries and *K-intensive* industries than for *L-abundant* countries and *L-intensive* industries.

We start with the Taylor expansion. Table (2) shows the estimates resulting from Equation (11). Since the Taylor expansion approximates better the true function the closer the independent variable is to the expansion point (i.e. a relative *K-intensity* close to 1) we use a restricted sample of firms. Within each country-industry pair, we retain firms with a relative *K-intensity* between the 5th and the 95th percentiles. Column (1) reports the results obtained pooling all the industries and countries.[17] Then, we restrict the sample to country-industry pairs that exhibit the prerequisite for comparative advantage. Column (2) retains countries whose *K-abundance* is above the median and industries whose *K-intensity* is above the median. We shall refer to this sample as the *KK-group*. Similarly, Column (3) retains countries with lower-than-median *K-abundance* and industries with lower-than-median *K-intensity* industries. We shall refer to this sample as the *LL-group*.

The empirical results strongly support proposition 1. All the estimated coefficients are very significant and have the expected sign. Moreover, the structural parameters we can infer from these estimates are in line with theoretical requirements. Our estimates for ς appear to be very robust across the different samples of countries and industries. They are always strictly larger than one as expected. They vary between 2.123 and 2.201. These values of ς are relatively small according to some of the estimates proposed by the existing literature. For instance, Anderson and Van Wincoop (2004), surveying several empirical trade analyses, consider that a reasonable range for the elasticity of substitution between varieties in a CES utility function is between 5 and 10. But our results are very close to Broda and Weinstein (2006) who report a median value for this parameter of 2.2, when they conduct their estimates using a 3-digit product classification. Our result is also in line with Imbs and Méjean (2009) who find a value ranging from 2.5 to 3 when they force the elasticities to be equal across sectors, as we do. More importantly, the ς reported in Table (2) are not significantly different from each other. This finding corroborates Proposition 2 which states that the marginal influence of Hicks-neutral heterogeneity on relative sales should not be related to the characteristics of countries and industries determining comparative advantages.

The parameter σ ranges between 1.763 and 2.22. These values imply a quite strong substitutability between labor and capital. They are very

[17] All regressions pooling different countries and industries are weighted by the importance of each country-industry groups in terms of total production.

high compared to the elasticities usually reported in the literature[18], but consistent with the evidence that relative *K-intensity* has on relative sales and productivity.

Turning to the heart of the matter, we observe a higher value of the parameter a_i^c for the *KK-group* (0.921) than for the *LL-group* (0.721). The parameter we obtain using all the data lies logically between these two values. This result corroborates our Proposition 1.

Table 2: Impact of relative TFP and *K-intensity* on relative sales: structural estimates of Taylor Expansion (Eq. 11)

Dependant variable: ln firms' relative sales ($\ln(s_i^c/\overline{s}_i^c)$)			
Countries	All	$K-abundant$	$L-abundant$
Industries	All	$K-intensive$	$L-intensive$
	(1)	(2)	(3)
$(\varsigma-1)$	1.123^a	1.201^a	1.129^a
	(0.031)	(0.039)	(0.048)
$\frac{1-\varsigma}{1-\sigma}\frac{a_i^c}{1+a_i^c}$	0.572^a	0.755^a	0.388^a
	(0.032)	(0.048)	(0.034)
$-\frac{1}{2}\frac{1-\varsigma}{1-\sigma}\left(\frac{a_i^c}{1+a_i^c}\right)^2$	-0.132^a	-0.181^a	-0.081^a
	(0.026)	(0.041)	(0.014)
R^2	0.315	0.329	0.343
Observations	372829	1112015	58045
ς	2.123	2.201	2.129
σ	1.909	1.763	2.22
a_i^c	0.862	0.921	0.721

Notes: Equation (11). Linear regressions with country-industry fixed effects. Regressions are weighted by the total production within country-industry groups. Firms with a $K-intensity$ beyond their respective country-industry 5th and 95th percentiles are excluded from the sample. Robust standard errors adjusted for country-industry clusters in parentheses. Significance level: ^{a} $p < 0.01$.

We now move to estimating Equation (10). In order to estimate the parameter a_i^c in Equation (10), we must set an arbitrary value for σ. We use

[18]Most of the literature on the elasticity of substitution between labor and capital concludes in favor of a complementarity between the two factors, with a σ ranging between 0.4 and 0.6 (Chirinko, 2008).

the estimated values obtained with the Taylor expansion (11) and reported in Table 2. Results are shown in Table 3. The top panel of Table 3 imposes the same value of σ for all samples of countries and industries. In the bottom panel, we use the value of σ which has been estimated for each corresponding sample: 1.91 for the whole sample, 1.76 for the *KK-group* and 2.22 for the *LL-group*. We also introduce an intercept to give the model some flexibility. The intercept is not in the structural equation but may be introduced nevertheless to alleviate the consequence of a possible systematic measurement error in any variable.

Once again, all estimates of ς are very stable and they are not statistically different from each other. In contrast, the estimated values of a_i^c vary greatly and can be ranked strictly. The smallest parameter is obtained with the *LL-group* and the largest with the *KK-group*. These structural estimates undoubtedly reveal that comparative advantages magnify the consequences of firm-level heterogeneity in *K-intensity* (as predicted by Proposition 1) while it has no influence on the relationship between firms' relative TFP and firms' relative sales (as predicted by Proposition 2).

7 Non-Structural Estimates

Equations (10) and (11) impose strict structural constraints on the key parameters of the model. In this section we abandon structural estimations and focus on verifying empirically the validity of the relationships stated in Propositions 1 and 2 and of the two constitutive mechanisms of Proposition 1. This is important since it provides an empirical assessment not only of our model but, potentially, of an entire class of models exhibiting heterogeneity in factor intensity.

The two mechanisms giving rise to proposition 1 are tested with a two-steps procedure. The first step consists in estimating the following non-structural form of Equation (10) with our firm-level data:

$$\ln\left(\frac{s_i^c}{\bar{s}_i^c}\right) = \digamma + \psi \ln\left(\frac{\theta_i^c(\beta')}{\bar{\theta}_i^c}\right) + \eta \ln\left(\frac{\phi'}{\widetilde{\phi}_i^c}\right) + \epsilon_i^c, \tag{12}$$

where $\digamma$ is an intercept and ϵ_i^c is an error term. The first term on the right hand side is the relative *K-intensity* of the firm, and the second term is its relative TFP. This specification is much more flexible and comprehensive than

Table 3: Impact of relative TFP and *K-intensity* on relative sales: structural estimates of Eq. (10)

Dependant variable: ln firms' relative sales ($\ln(s_i^c/\overline{s}_i^c)$)			
Countries	All	*K-abundant*	*L-abundant*
Industries	All	*K-intensive*	*L-intensive*
		$\sigma = 1.91$	
	(1)	(2)	(3)
ς	2.046^a	1.982^a	2.146^a
	(0.029)	(0.0413)	(0.043)
a_i^c	0.675^a	2.469^a	0.014^a
	(0.079)	(0.488)	(0.007)
Intercept	-1.029^a	-1.002^a	-0.997^a
	(0.034)	(0.051)	(0.025)
R^2	0.286	0.302	0.339
Observations	412386	123965	64102
σ	1.91	1.76	2.22
	(4)	(5)	(6)
ς	2.046^a	2.004^a	2.144^a
	(0.029)	(0.042)	(0.043)
a_i^c	0.675^a	1.236^a	0.032^a
	(0.079)	(0.205)	(0.012)
Intercept	-1.029^a	-1.084^a	-0.997^a
	(0.034)	(0.053)	(0.025)
R^2	0.286	0.291	0.339
Observations	412386	123965	64102

Notes: Equation (10). Non-linear least squared. Starting values: $a_i^c = 1$ and $\varsigma = 3$. Regressions are weighted by the total production within country-industry groups. Robust standard errors adjusted for country-industry clusters in parentheses. Significance level: ^{a} $p < 0.01$.

the structural equation and should provide more robust results. We estimate this equation separately for each of the 1,471 countries-pairs and collect the corresponding estimated coefficients on firms' relative *K-intensity*. The second step consists in testing whether these estimated coefficients, $\widehat{\psi}_i^c$, now specific to each country c and industry i, vary with the industry-level *K-intensity* and the country-level *K-abundance*. According to Proposition 1, we expect $\widehat{\psi}_i^c$ to be significantly larger in the *KK-group* than in the *LL-group*. According to Proposition 2, the determinants of comparative advantages should not influence the coefficient associated with the relative TFP, $\widehat{\eta}$.

The first step gives extremely robust results. Table 4 reports the estimates of this non-structural equation obtained on the pooled dataset.

Table 4 confirms the results obtained from the structural specification (see Tables 2 and 3). Column (1) omits the total factor productivity term. The positive and very significant coefficient confirms that firms with higher relative *K-intensity* have significantly higher relative sales. A firm with a *K-intensity* 10% above the country-industry mean would have relative sales that are 3% higher than the average firm. Column (2) introduces firms' relative TFP. Not surprisingly, productivity has a great influence on firms' performances. The coefficient on TFP is highly significant and very large in magnitude. The introduction of this variable also improves greatly the global fit of the regression, raising the R^2 by a factor of 5.5. More importantly, controlling for TFP does not effect the coefficient on relative capital abundance, suggesting that these two variables can be reasonably considered as only weakly correlated. Indeed, regressing relative *K-intensity* on relative TFP with a full set of country-industry fixed effects fails to reveal a significant relationship between the two variables.[19] Column (3) verifies more directly the log-supermodularity of the *RS* function interacting the relative *K-intensity* variable with more precise elements of the set of comparative advantages. These elements are dummies denoting - for each country-industry pair - whether the product of its *K-abundance* and *K-intensity* is lower than the 20th percentile, between the 20th and the 40th, between the 40th and 60th, between the 60th and the 80th, or higher than the 80th percentile. All the coefficients of the interacted variables are significantly positive and show an almost perfect ranking. Relative *K-intensity* is systematically associated with relatively better performances and, more importantly, this relationship

[19] The correlation between the two variables is only 0.0095, and the regression coefficient is $1.04e-08$ with a standard error of 0.003.

Table 4: Impact of relative TFP and K-intensity on relative sales: non-structural log-linear model

	(1)	(2)	(3)	(4)	(5)
	lrelSale	lrelSale	lrelSale	lrelSale	lrelSale
Countries	All	All	All	*K-abundant*	*L-abundant*
Industries	All	All	All	*K-intensive*	*L-intensive*
	(1)	(2)	(3)	(4)	(5)
Ln Rel. TFP		1.1189^a	1.120^a	1.184^a	1.145^a
		(0.028)	(0.028)	(0.031)	(0.049)
Ln Rel. K-intensity	0.297^a	0.315^a		0.426^a	0.151^a
	(0.017)	(0.017)		(0.020)	(0.022)
Ln Rel. K-intensity 1			0.147^a		
			(0.020)		
Ln Rel. K-intensity 2			0.252^a		
			(0.011)		
Ln Rel. K-intensity 3			0.226^a		
			(0.014)		
Ln Rel. K-intensity 4			0.309^a		
			(0.024)		
Ln Rel. K-intensity 5			0.491^a		
			(0.025)		
Constant	-1.250^a	-0.878^a	-0.874^a	-0.944^a	-0.845^a
	(0.016)	(0.022)	(0.017)	(0.024)	(0.018)
Observations	412386	412386	412386	123965	64102
R^2	0.062	0.342	0.349	0.368	0.359

Notes: Country-Industry fixed effects for all columns. Regressions are weighted by the total production within country-industry groups. Robust standard errors adjusted for country-industry clusters in parentheses. Within R^2 are reported. Significance levels: ^{a} $p < 0.01$

becomes stronger with higher indices of comparative advantage. Finally, Columns (4) and (5) replicate the tests shown in Table 3. Column (4) reports the results obtained on the sample restricted to the *KK-group*, while Column (5) shows the coefficient obtained when considering the *LL-group*. The estimated coefficient on relative *K-intensity* is significantly larger for the *KK-group* than for the *LL-group* and the TFP coefficients in Columns (4) and (5) are not significantly different from each other. Both findings are consistent with the theoretical results. The differences between coefficients on relative *K-intensity* reported in Columns (4) and (5) are not only statistically significant, but also important in magnitude. The slope of the relationship between relative *K-intensity* and relative sales is more than 3 times larger in *KK-group* than in the *LL-group*: a *K-intensity* 10% above the country-industry average results in a relative sales 1.5% larger in the *LL-group*, but more than 4.3% in the *KK-group*.

When estimating Equation (12) separately for each of the 1,471 country-industry pairs, we obtain quite robust results. Only 6 country-industry pairs (0.4 percent) show an unexpected, significantly negative coefficient $\widehat{\psi}_i^c$. In 422 cases (28.7 percent), the coefficient is not significantly different from zero (60 are negative, and 362 are positive). Finally, we obtain strictly positive coefficients for a huge majority of country-industry pairs (1,043 cases, representing 71 percent of the sample).[20]

Figure (2) illustrates the relationships between $\widehat{\psi}_i^c$ coefficients and the determinants of comparative advantages. Panel (a) plots the mean values of $\widehat{\psi}_i^c$ for each industry i, with the corresponding mean standard deviations, against the industry's capital intensity. Panel (b) relates the country means of $\widehat{\psi}_i^c$ and its standard deviations to countries' capital abundance. While it is barely significant in Panel (b), the two graphs exhibit the positive slope predicted by our model. This is confirmed by the regression results shown in Table (5).

In the top half of Table 5, we regress the estimated slope of the relationship between firms' relative *K-intensity* and firms' relative sales, $\widehat{\psi}_i^c$, on industry-level capital intensity and country fixed effects. The positive coefficient reported in Column (1) explicitly validates Mechanism 1 of Proposition 1. It says that, in a given country, the payoff, in terms of relative sales, of having a higher relative capital-labor ratio is bigger in *K-intensive* indus-

[20]The coefficients $\widehat{\psi_i^c}$ range between -0.16 and 1.7, with a mean of 0.34 and a median of 0.31.

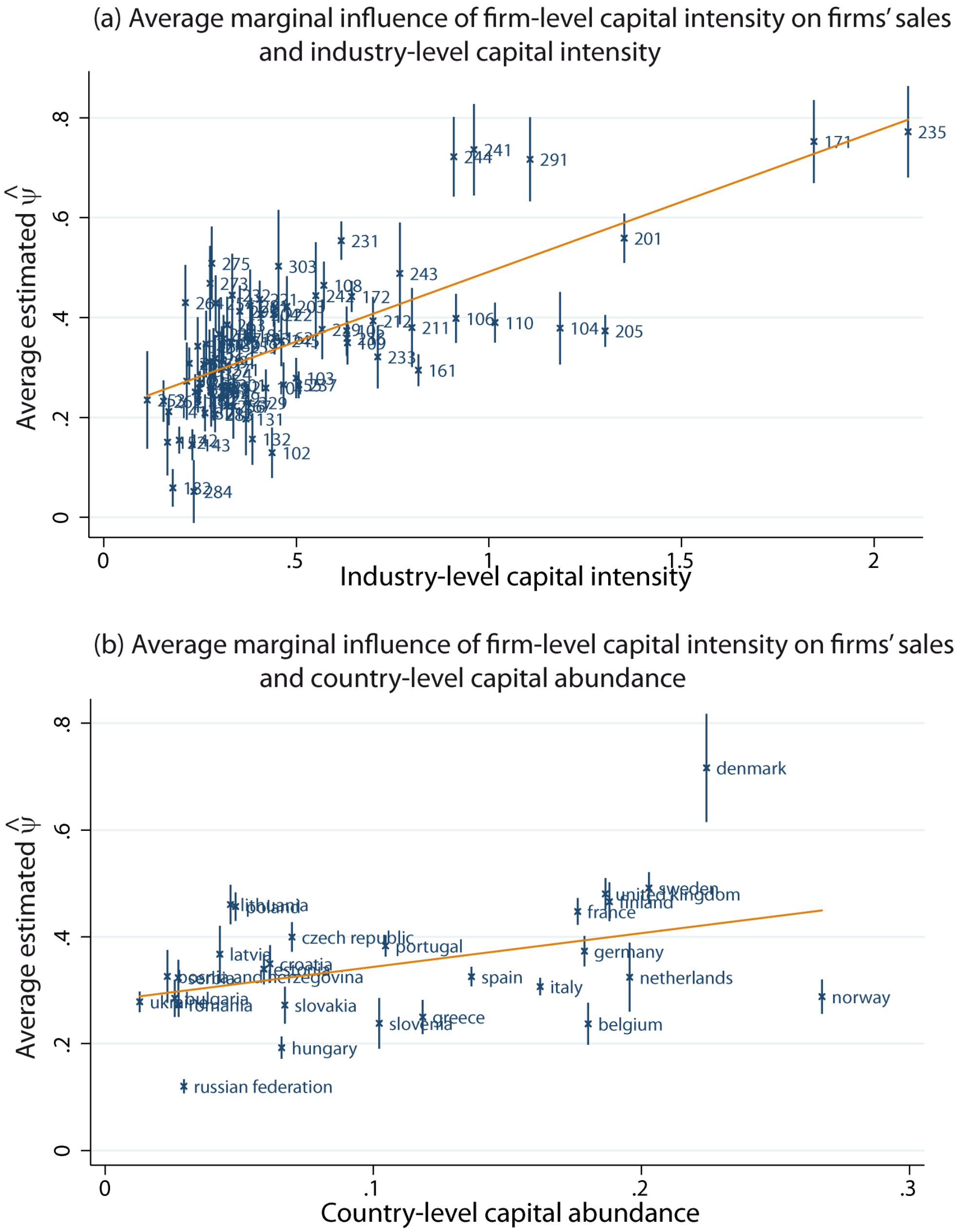

Figure 2: Average $\widehat{\psi}_i^c$, industry's $K-intensity$ and country's $K-abundance$

tries than in relatively *L-intensive* industries. The regression reported in the first column only considers the estimated coefficients $\widehat{\psi}_i^c$ without controlling for their significance level or economic relevance. Regressions reported in Columns (2), (3) and (4) make use of information we have on the precision of each estimate. In Columns (2), we retain only significantly positive coefficients $\widehat{\psi}_i^c$. Saxonhouse (1976) points out that regressions using estimated parameters as dependant variables are likely to be affected by heteroschedasticity. He suggests weighting the observations in order to give more importance to more significant estimates. In Column (3), the weight we give is the inverse of the standard error reported for each $\widehat{\psi}_i^c$. A second possible weight we can use to control for the significance of the estimates is the degree of freedom in the first step regressions. Regressions in Column (4) are performed giving a weight equal to the square root of the number of firms within each country-industry group minus 3. All these robustness checks confirm the result shown in Column (1).

Empirical tests of Mechanism 2 of Proposition 1 are shown in the bottom half of Table 5. Here, the second step consists in regressing $\widehat{\psi}_i^c$ on countries' *K-abundance* and industry fixed effects. While much smaller than those reported in the top panel, the positive coefficient on *K-abundance* in Column (5) supports Mechanism 2 of Proposition 1. In a given industry, differences in relative firm-level capital intensity generate greater heterogeneity in relative sales in capital-abundant countries. Very similar results are provided by considering only significantly positive $\widehat{\psi}_i^c$ or weighting the observations.

8 Conclusion.

In this paper we have shown that the comparative advantage of countries begets a comparative advantage at firm level. Two firms with identical relative factor intensities have different relative sales if they belong to different industries or countries. The firm in the country and industry of its comparative advantage has larger relative sales. This result is due to two distinct mechanisms: the interaction between relative factor intensity and the industry technology and the interaction between relative factor intensity and factors endowment. These results do not require any assumption about the direction of the technology bias (if any), or about the relationship between productivity and factor intensity (the normalization choice).

We have verified empirically the predictions of the model using firm-level

Table 5: Verification of Mechanisms 1 and 2 of Proposition 1

Dependant Variable: $\widehat{\psi}_i^c$				
	Test of Mechanism 1			
	(1)	(2)	(3)	(4)
Industry K-intensity	0.276^a	0.239^a	0.246^a	0.256^a
	(0.024)	(0.023)	(0.015)	(0.016)
Observations	1471	1043	1471	1471
R^2	0.172	0.199	0.389	0.378
Fixed effects		Country		
	Test of Mechanism 2			
	(5)	(6)	(7)	(8)
Country K-abundance	0.678^a	0.616^a	0.697^a	0.740^a
	(0.092)	(0.084)	(0.064)	(0.066)
Observations	1471	1043	1471	1471
R^2	0.056	0.058	0.361	0.359
Fixed effects			Industry	

Notes: Robust standard errors in parentheses. Significance levels: ^{b} $p < 0.05$, ^{a} $p < 0.01$. Within R^2 are reported. Regressions in Columns (2) and (6) only retain significantly positive values of $\widehat{\psi}_i^c$. Regressions in Columns (3) and (7) are performed with weight = 1/s.e.($\widehat{\psi}_i^c$). Regressions in Columns (4) and (8) are performed with weight = degree of freedom in the first step regression.

data for a large number of countries and industries. The data contains information on capital intensities and total sales for a panel of 412,386 European firms in 84 industries and 28 countries. The econometric analysis compares the influence of firms' capital intensity on their sales across country-industry pairs, characterized by different comparative advantages. We find that comparative advantage interacts with firms' relative factor intensities, in explaining the observed heterogeneity in relative sales. This result is robust to different empirical specifications. The structural estimates corroborate our theoretical conclusions and support our modeling choices. The non-structural estimates dissect the impact of comparative advantage into its two constitutive mechanisms. Within a given country, the premium in terms of firms' relative sales of having a higher ratio of capital per worker increases sharply with average capital intensity at the industry-level. Whereas the evidence is less striking, we also confirm that the premium is larger in capital abundant countries, within a typical industry.

Our study sheds light on factor intensity as an importance source of heterogeneity across firms and contributes to the literature in two ways. First, it provides the first firm-level verification of the HO model by verifying how countries' comparative advantage begets a comparative advantage for firms. Second, it shows that comparative advantage matters for relative sales within-industry, thus breaking the dichotomy between country-sector adjustments and within-sector adjustments.

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9 Appendix

Here, we describe the model and provide our analytical results as well as numerical solutions. Analytical results are derived for the model without fixed exporting cost, whereas we resort to numerical solutions for the model with fixed exporting cost.

9.1 The Model

The nature of heterogeneity was described in Section 2. We describe the rest of the model in more detail here.

Demand. Preferences are as described in Section 2. The dual price index, associated with each of the CES aggregates (P_i^c), is also a CES, defined over the prices of all varieties of the same industry. With Dixit-Stiglitz preferences, the demand function emanating from domestic residents, $s_{id}^H(\beta)$, and from foreign residents, $s_{ix}^H(\beta)$, for the output of a firm in industry i of country H with draw β (where s stands for sales, d for domestic, and x for foreign) is:

$$s_{id}^H(\beta) = \left(\frac{p_{id}^H}{P_i^H}\right)^{1-\varsigma} \gamma_i I^H; \qquad s_{ix}^H(\beta) = \left(\frac{p_{ix}^H}{P_i^F}\right)^{1-\varsigma} \gamma_i I^F \tag{13}$$

Demand depends negatively on the price faced by consumers (respectively p_{id}^H and p_{ix}^H) and positively on the price index (P_i^c) and national income $\left(I^c = w^c \nu_L^c \overline{L} + r^c \nu_K^c \overline{K},\right)$. Analogous demand functions obtain for the output of a firm with draw β in industry i of country F.

Production. Except in the free trade situation, firms wanting to export face fixed and variable exporting cost. Variable costs are paid in terms of the good transported: for one unit of good shipped, only a fraction $\tau_i \in [0,1]$ arrives at its destination. Analogously, for fixed production cost: the fixed exporting cost is $F_{ix}\widetilde{mc}_i^c$, where F_{ix} is a positive constant.

With monopolistic competition and under the large-group assumption, the profit-maximizing prices for the domestic and the foreign market are:

$$p_{id}^c(\beta) = \frac{\varsigma}{\varsigma-1} mc_i^c(\beta); \qquad p_{ix}^c(\beta) = \frac{\varsigma}{\varsigma-1}\frac{1}{\tau_i} mc_i^c(\beta) \tag{14}$$

Firms pay a fixed entry cost $F_{ie}^c \widetilde{mc}_i^c$ after which they draw β from $g(\beta)$. Profits in the domestic and foreign market are, respectively, $\pi_{id}^c(\beta) = s_{id}^c(\beta)/\varsigma - F_i \widetilde{mc}_i^c$ and $\pi_{ix}^c(\beta) = s_{ix}^c(\beta)/\varsigma - F_{ix}\widetilde{mc}_i^c$. The total profit of a firm with draw β, $\pi_i^c(\beta)$, is $\pi_{id}^c(\beta)$ if it does not export, and $\pi_{id}^c(\beta) + \pi_{ix}^c(\beta)$ if it exports. Analogously to β_i^{*c}, let β_{ix}^{*c} be the least value of β such that foreign profits are non-negative. Using the expressions for domestic and foreign profits β_i^{*c} and β_{ix}^{*c} are such that

$$s_{id}^c\left(\beta_i^{*c}\right) = \varsigma F_i \widetilde{mc}_i^c, \qquad s_{ix}^c\left(\beta_{ix}^{*c}\right) = \varsigma F_{ix}\widetilde{mc}_i^c. \tag{15}$$

Firms with productivity draw $\beta < \beta_i^*$ will exit immediately, firms with productivity β such that $\beta_i^* \leq \beta < \beta_{ix}^*$ will produce for the domestic market only, and firms with productivity draw $\beta \geq \beta_{ix}^*$ will produce for the domestic and the foreign market.

Aggregation. Average prices, sales, and profits can be expressed as functions of average marginal productivity. In addition to the average marginal productivity in the industry denoted $\widetilde{mc}_i^c$ and defined in expression (5), we make use of the average marginal productivity of exporting firms, $\widetilde{mc}_{ix}^c$, computed as in expression (5) except that β_{ix}^{*c} replaces β_i^{*c} as a lower limit of integration. Given the profit-maximizing prices (14), the average price and the average export price are, respectively:

$$\widetilde{p}_{id}^c = \frac{\varsigma}{\varsigma - 1}\widetilde{mc}_i^c; \qquad \widetilde{p}_{ix}^c = \frac{1}{\tau_i}\frac{\varsigma}{\varsigma - 1}\widetilde{mc}_{ix}^c \tag{16}$$

and the price indices are:

$$P_i^H = \left[M_i^H \left(\widetilde{p}_{id}^H\right)^{1-\varsigma} + \chi_i^F M_i^F \left(\widetilde{p}_{ix}^H\right)^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}} \tag{17}$$

$$P_i^F = \left[M_i^F \left(\widetilde{p}_{id}^F\right)^{1-\varsigma} + \chi_i^H M_i^H \left(\widetilde{p}_{ix}^F\right)^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}} \tag{18}$$

where M_i^c is the mass of firms and $\chi_i^c \equiv \frac{1-G(\beta_{ix}^{*c})}{1-G(\beta_i^{*c})}$ is the *ex-ante* probability of exporting, conditional to successful entry. Using equations (6) and (15) we can compute the average value of total firm's output, $\overline{s}_i^c$,

$$\overline{s}_i^c = \left[\frac{\widetilde{mc}_i^c}{mc_i^{*c}}\right]^{1-\varsigma} \varsigma F_i \widetilde{mc}_i^c + \chi_i^c \left[\frac{\widetilde{mc}_{ix}^c}{mc_{ix}^{*c}}\right]^{1-\varsigma} \varsigma F_{ix}\widetilde{mc}_i^c \tag{19}$$

where the first addendum is average domestic sales, $\overline{s}_{id}^c$ and the second is average foreign sales, $\overline{s}_{ix}^c$. Average profit is

$$\overline{\pi}_i^c = \left[\frac{\overline{s}_{id}^c}{\varsigma} - F_i \widetilde{mc}_i^c\right] + \chi_i^c \left[\frac{\overline{s}_{ix}^c}{\varsigma} - F_{ix} \widetilde{mc}_i^c\right]. \tag{20}$$

Equilibrium. In addition to profit-maximizing prices and to the zero profit conditions discussed above, there are five additional sets of equilibrium conditions. First, stationarity of the equilibrium requires the mass of potential entrants, M_{ei}^c, to be such that at any instant the mass of successful entrants, $[1 - G(\beta_i^*)] M_{ei}^c$ equals the mass of incumbent firms who die, δM_i^c:

$$[1 - G(\beta_i^*)] M_{ei}^c = \delta M_i^c, \qquad c = H, F \text{ and } i = Y, Z. \tag{21}$$

where δ is the instant probability of death. Second, the presence of an infinity of potential entrants arbitrages away any possible divergence between the expected value of entry and entry cost. Therefore, the free entry condition, is:

$$[1 - G(\beta_i^{*c})] \overline{\pi}_i^c / \delta = F_{ei} \widetilde{mc}_i^c; \quad i = Y, Z; \; c = H, F. \tag{22}$$

The left-hand-side is the expected profit stream until death multiplied by the probability of successful entry, and the right-hand-side is the entry cost. Third, replacing (16) into (13) gives average demands as functions of average prices, $s_{id}^c(\widetilde{p}_{id}^c)$ and $s_{ix}^c(\widetilde{p}_{ix}^c)$, which allows writing the goods markets equilibrium as

$$\overline{s}_i^c = s_{id}^c(\widetilde{p}_{id}^c) + \chi_i^c s_{ix}^c(\widetilde{p}_{ix}^c); \quad i = Y, Z; \; c = H, F. \tag{23}$$

Fourth, the optimal relationship between foreign and domestic sales is

$$\frac{mc_i^H(\beta_{ix}^*)}{mc_i^H(\beta_i^*)} = \left[\tau_i^{\varsigma-1}\left(\frac{P_i^F}{P_i^H}\right)^{\varsigma-1} \frac{I^F}{I^H}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\varsigma-1}}; \quad i = Y, Z. \tag{24}$$

$$\frac{mc_i^F(\beta_{ix}^*)}{mc_i^F(\beta_i^*)} = \left[\tau_i^{\varsigma-1}\left(\frac{P_i^H}{P_i^F}\right)^{\varsigma-1} \frac{I^H}{I^F}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\varsigma-1}}; \quad i = Y, Z. \tag{25}$$

Fifth, equilibrium in factor markets requires that factor demand inclusive of all fixed factors inputs, denoted L_i^c and K_i^c, be equal to factor supply

$$L_Y^c + L_Z^c = \nu_L^c \overline{L}; \qquad c = H, F. \tag{26}$$

$$K_Y^c + K_Z^c = \nu_k^c \overline{K}; \qquad c = H, F. \tag{27}$$

After replacing equations (16), (17)-(20) into (22)-(27) the model counts 15 independent equilibrium conditions that together with one normalization determine 16 endogenous variables. The equilibrium conditions are the four free-entry conditions (22), any three out of the four goods market equilibrium conditions (23), the four relationships between foreign and domestic sales (24)-(25), and the four factor market equilibriums (26)-(27). The endogenous are the four zero-profit productivity cut-off $\{\beta_i^{*c}\}$, the four zero exporting profit productivity cutoffs $\{\beta_{ix}^{*c}\}$, the four factor prices $\{w^c, r^c\}$ and the four masses $\{M_i^c\}$. The equilibrium value of all other endogenous variables can be computed from these.

9.2 Analytical Results

In this section $F_{ix} = 0$, which implies $\chi_i^c = 1$. Further, to isolate the effect of comparative advantage we eliminate any cross-industry differences in fixed cost and trade cost: i.e., $F_i = F$, $F_{ei} = F_e$, and $\tau_i = \tau$ for $i = Y, Z$. We begin by proving the ranking the cut off values and then use these results to prove Theorem 1, Proposition 1 and its two underlying mechanisms.

9.2.1 Ranking of cut-off values.

Replacing expressions (19) and (20) into Equation (22), we obtain a single equation which combines free entry and zero cut-off profit conditions, henceforth the FE-ZCP condition:

$$\Upsilon_i^c\left(\beta_i^{*c}, \Lambda_i, \omega^c\right) \equiv \int_{\beta_i^{*c}}^{\infty} \left\{ \left[\frac{mc_i^c\left(\beta\right)}{mc_i^c\left(\beta_i^{*c}\right)} \right]^{1-\varsigma} - 1 \right\} g\left(\beta\right) d\beta = \delta \frac{F_e}{F}. \tag{28}$$

Defining the right-hand side of the equation as $\Upsilon_i^c\left(\beta_i^{*c}, \Lambda_i, \omega^c\right)$ will save notation later. By simple calculus, we obtain the signs of the three partial derivatives of the right-hand-side

$$\frac{\partial \Upsilon_i^c}{d\beta_i^{*c}} < 0, \forall \sigma > 0; \ \frac{\partial \Upsilon_i^c}{d\Lambda_i} \gtrless 0 \text{ as } \sigma \gtrless 1; \ \frac{\partial \Upsilon_i^c}{d\omega^c} > 0, \forall \sigma > 0. \tag{29}$$

We can now establish two lemmas.

Lemma 1 *Within a country, the K-intensive industry has the highest (lowest) zero-profit productivity cut-off if $\sigma > 1$ ($\sigma < 1$). In our notation:*

$$\beta_Y^{*c} \gtrless \beta_Z^{*c} \quad \textit{as } \sigma \gtrless 1, \qquad \forall \tau \in [0, 1]\ . \tag{30}$$

Proof. Applying the implicit function theorem to Equation (28) gives:

$$\frac{d\beta_i^{*c}}{d\lambda_i} = -\left(\frac{\partial \Upsilon_i^c}{\partial \beta_i^*}\right) / \left(\frac{\partial \Upsilon_i^c}{\partial \Lambda_i}\right) \gtreqless 0, \text{ as } \sigma \gtreqless 1. \tag{31}$$

which proves the lemma. ■

Lemma 2 *Except under free trade, the K-abundant country has a higher zero-profit productivity cut-off in both industries. Furthermore, each cut-off value of the K-abundant country is larger with costly trade than under free trade, whereas each cut-off value of the L-abundant country is smaller under free trade than with autarky. In our notation:*

$$\left(\beta_i^{*H}\right)_{Costly\ Trade} \geqslant \left(\beta_i^{*}\right)_{Free\ Trade} \geqslant \left(\beta_i^{*F}\right)_{Costly\ Trade} \quad \forall i, \text{ and } \forall \sigma > 0 \tag{32}$$

with equality holding only in free trade.

Proof. Applying the implicit function theorem to Equation (28) gives:

$$\frac{d\beta_i^{*c}}{d\omega_i} = -\left(\frac{\partial \Upsilon_i^c}{\partial \beta_i^*}\right) / \left(\frac{\partial \Upsilon_i^c}{\partial \omega^c}\right) > 0. \tag{33}$$

Recalling that the *K-abundant* country has the highest relative price of L (i.e., $\omega^H > \omega^F$) proves the lemma.[21] ■

9.2.2 Proof of Theorem 1

In using (6), it is straightforward that proving Theorem 1 is equivalent to proving Proposition 1 since $\varsigma > 1$. We therefore move to the proof of Proposition 1.

9.2.3 Proof of Proposition 1

The function RS is strictly log-supermodular in (a_i^c, κ) iff

$$RS_Y^H(\kappa'')/RS_Y^H(\kappa') > RS_Z^F(\kappa'')/RS_Z^F(\kappa') \text{ for any } \kappa'' > \kappa'. \tag{34}$$

[21] Here we should demonstrate that for any positive level of trade cost the relative price of a factor is higher in the country where that factor is relatively scarce. This may be demonstrated analytically with a few pages of maths, but since it is a rather intuitive and standard result we omit the proof and resort to numerical simulations reported in supplementary online material.

Using (6), condition (34) becomes

$$\left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma}\left(\frac{\omega^H}{\omega^F}\right)^{\sigma-1}\left(\frac{\widetilde{\beta}_Y^H}{\widetilde{\beta}_Z^F}\right)^{\sigma-1}>1 \tag{35}$$

which is satisfied since $\widetilde{\beta}_Y^H > \widetilde{\beta}_Z^F$ from Lemma 1, $\frac{\omega^H}{\omega^F}>1$ (H is K-*abundant*), and $\Lambda_Y > \Lambda_Z$ (Y is $K-intensive$). Further, using the fact that $RS_i^c(1)=1$ for any c and any i proving that for any $\theta_i^c(\beta')=\kappa\overline{\theta}_i^c$ we have $RS_Y^H \gtreqless RS_Z^F$ as $\kappa \gtreqless 1$ is straightforward.

Mechanism 1: Log-supermodularity in (Λ,κ)**.** Using Equation (6), RS is strictly log-supermodular in (Λ,κ) iff

$$\left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma}\left(\frac{\widetilde{\beta}_Y^c}{\widetilde{\beta}_Z^c}\right)^{\sigma-1}>1 \tag{36}$$

which is satisfied since $\widetilde{\beta}_Y^C > \widetilde{\beta}_Z^C$ from Lemma 1 and $\Lambda_Y > \Lambda_Z$ since Y is K-*intensive*. Further, using the fact that $RS_i^c(1)=1$ for any c and any i proving that for any β' such that $\theta_i^c(\beta')=\kappa\overline{\theta}_i^c$ we have $RS_Y^c \gtreqless RS_Z^c$ as $\kappa \gtreqless 1$ is straightforward.

Mechanism 2: Log-supermodularity in (Ω,κ)**.** Using Equation (6), RS is strictly log-supermodular in (Ω,κ) iff

$$\left(\frac{\omega^H}{\omega^F}\right)^{\sigma-1}\left(\frac{\widetilde{\beta}_i^H}{\widetilde{\beta}_i^F}\right)^{\sigma-1}>1 \tag{37}$$

which is satisfied since $\widetilde{\beta}_i^H > \widetilde{\beta}_i^F$ from Lemma 2 and in costly trade $\omega^H > \omega^F$. Further, using the fact that $RS_i^c(1)=1$ for any c and any i proving that for any β' such that $\theta_i^c(\beta')=\kappa\overline{\theta}_i^c$, we have $RS_i^H \gtreqless RS_i^F$ as $\kappa \gtreqless 1$ is straightforward.

9.2.4 The substitutability and complementarity of factors

In the theoretical part of the paper we assumed that factors are substitutes ($\sigma>1$). Empirical estimates of σ confirmed that this assumption is tenable.

However, assuming that factors are substitutes is sufficient but not necessary for the results. Indeed, as it is apparent by using (29) in the proof of Mechanism 2, the latter remains valid regardless of whether $\sigma \gtrless 1$. Instead, using (29) in the proof of Mechanism 1 shows that the latter *may* not be satisfied if $\sigma < 1$. So, at worst, if factors are complements ($\sigma < 1$), Mechanism 1 is lost but Mechanism 2 remains valid. Since Proposition 1 results from the sum of the effects obtained through the two mechanisms, complementarity does not necessarily invalidate the proposition. Proposition 1 is reversed only if three conditions are met: (a) $\sigma < 1$, (b) the inequalities in Mechanism 1 are reversed, and (c) the effect of Mechanism 1 prevails on the effect of Mechanism 2. If these three conditions are met, then RS is strictly sub-modular in (a, κ). Then the comparative advantage of countries begets a comparative advantage at firm-level though in the opposite direction with respect to that stated in Proposition 1. Empirical investigation coherently with the model's parameters indicates the direction of the relationship between firms' and countries' comparative advantage.

9.2.5 Entry

The two mechanisms giving rise to Proposition 1 are active, even if all firms can survive in the market, despite being heterogenous. In such case the cut off values β_i^{*c} and the averages $\widetilde{\beta}_i^{*c}$ would be the same for all c and i. Replacing an identical value of $\widetilde{\beta}_i^{*c}$ in (35)-(37) for all c and i (whatever this value is) shows that Proposition 1 and its two constitutive mechanisms remain valid. Allowing for entry, as we do in the model, makes Proposition 1 hold *a fortiori* as it is apparent by observing the ranking of cut-off values β_i^{*c} obtained in Lemma 1 and 2 and the resulting ranking of $\widetilde{\beta}_i^{c}$.

9.2.6 No average factor intensity reversal

From Equation (4) we have:

$$\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma} \left(\frac{\widetilde{\beta}_Y^c}{\widetilde{\beta}_Z^c}\right)^{\sigma-1} > 1. \tag{38}$$

With Hicks-neutral heterogeneity we would have $\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma} > 1$. With heterogeneity in factors' RMP the no-factor-intensity-reversal holds *a fortiori*, since $\widetilde{\beta}_Y^C > \widetilde{\beta}_Z^C$ from Lemma 1.

9.2.7 Robustness to normalization

Let $\beta = 1$ and $\alpha \in (0, \infty)$. The FE-ZCP (28) becomes:

$$\int_{\alpha_i^*}^{\infty} \left\{ \left[\frac{mc_i^c(\alpha)}{mc_i^c(\alpha_i^{*c})} \right]^{1-\sigma} - 1 \right\} g(\alpha)\, d\alpha = \delta \frac{F_e}{F}. \tag{39}$$

Applying to Equation (39) analogous differentiations to those for Equation (28) shows that:

$$\alpha_Z^{c*} > \alpha_Y^{c*} \quad , \quad \forall \tau \in [0,1], \quad \text{and} \quad \alpha_i^{H*} < \alpha_i^{F*} \ , \quad \forall \tau \in (0,1). \tag{40}$$

Let $\widetilde{\alpha}_i^c = \left[\frac{1}{1-G(\alpha_i^{c*})} \int_{\alpha_i^{c*}}^{\infty} \alpha^{1-\sigma} g(\alpha)\, d\alpha \right]^{\frac{1}{1-\sigma}}$. Proposition 1 requires that for any α_i' such that $\theta_i^c(\alpha_i') = \kappa \overline{\theta}_i^c(\widetilde{\alpha}_i)$ we have:

$$RS_Y^H \gtreqless RS_Z^F \text{ as } \kappa \gtreqless 1. \tag{41}$$

Substituting (6) into (41) shows that Proposition 1 remains valid since $\alpha_Z^{c*} > \alpha_Y^{c*}$, $\frac{\omega^H}{\omega^F} > 1$, and $\Lambda_Y > \Lambda_Z$. Robustness of the two underlying mechanisms obtains through the analogous procedure.

9.2.8 The Four Core Theorems

The Stolper-Samuelson, Rybczynski, Factor Price Equalization, and Heckscher-Ohlin theorems remain valid when heterogeneity exists in factors' RMP. But, compared to a model where heterogeneity is Hicks-neutral, their intensity is affected. Writing the closed economy (or integrated equilibrium) system in the canonical Jones (1965) form and applying "Jones Algebra" we obtain the following results.

The Stolper-Samuelson and Rybczynski magnification effects are attenuated (amplified) by heterogeneity in RMP if the average factor intensity is *K-biased* (*L-biased*).[22]

The FPE set is expanded by heterogeneity in RMP. This can be seen in inequality (38), which shows that the diversification cone is expanded by

[22]If heterogeneity is Hicks-neutral, the average factor intensity is $(\omega^c)^\sigma (\Lambda_i)^\sigma \quad \forall i, c$. If heterogeneity exists in factors' RMP, the average factor intensity is as given in expression (4) and exhibits a bias even if the technology is neutral; i.e., if $\int_0^\infty (\beta)^{\sigma-1} g(\beta)\, d\beta = \alpha^{\sigma-1}$. In such case, and if all firms could survive in the market, the average factor intensity would be exactly $(\omega^c)^\sigma (\lambda_i)^\sigma \quad \forall i, c$. Yet, because of selection in entry, a factor

heterogeneity in factor intensities.[23] The expansion of the FPE does not depend on the normalization choice nor on the direction of the factor bias. Changing the normalization we have $a_Z^* > a_Y^*$ and $\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma}\left(\frac{\widetilde{\alpha}_Z}{\widetilde{\alpha}_Y}\right)^{\sigma-1} > 1$.

The Heckscher-Ohlin specialization occurring when moving from autarky to free trade is attenuated. The attenuation is asymmetric: it is stronger (weaker) for the *L-abundant* (*K-abundant*) country when the average factor intensity is *K-biased*, and vice-versa when the average factor intensity is *L-biased*.

9.2.9 Robustness of Proposition 2 when $F_X > 0$

Proposition 2 remains valid when $F_x > 0$. This is proven by observing that by use of expressions (6) we obtain:

$$\frac{s_{i\zeta}^c\left(\phi'\right)}{s_{i\zeta}^c\left(\widetilde{\phi}_i^c\right)} = \varphi^{\sigma-1} \qquad \zeta = d, x; \quad c = H, F; \quad i = Y, Z. \tag{42}$$

9.3 Numerical simulations

In this section $F_x > 0$. Since firms may differ by export status, we need to make sure that we compare relative sales across firms for any κ, such that the associated pair of firms - in different countries and/or different sectors - belongs either to the set of exporting firms or to the set of non-exporting firms. Otherwise, relative sales may differ simply because one firm is an exporter and the other is not. When it comes to graphical representation, however, there is a considerable loss of esthetics because the lines representing relative sales are truncated for values of κ to which an importing and a non-importing firm correspond. We therefore take an easier route which rests on the fact that all firms sell at least some goods domestically. Therefore,

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bias emerges in equilibrium (a *K-bias* in this case) since $\left(\widetilde{\beta}_i^c\right)^{\sigma-1} > \alpha^{\sigma-1}$ even though $\int_0^\infty (\beta)^{\sigma-1} g(\beta)\, d\beta = \alpha^{\sigma-1}$. Naturally, one could use a $g(\beta)$ such that the average factor intensity would be *L-biased*. The direction of the bias determines in which way heterogeneity in RMP influences the intensity of the Stolper-Samuelson and Rybczynski magnification effects. As already mentioned, the existence and direction of the bias is irrelevant for the validity of Proposition 1.

[23] In a two-by-two setting, the size of the FPE set increases with the size of the diversification cone.

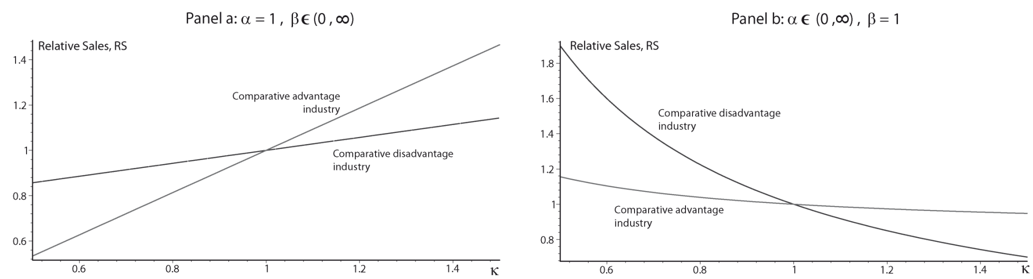

Figure 3: Graphical representation of Proposition 1 when $F_x > 0$

Proposition 1 applied to domestic sales holds for all firms, exporters and non-exporters, i.e., for any $\kappa > 0$. This is what we show below.

There are 15 parameter values to be assigned in order to solve the model numerically and, of course, a large number of possible combinations. The only requirement on parameters is that no firm can be an exporting firm without also selling in the domestic market. This is hardly restrictive given the large number of parameters. The only guidance to the choice of parameters concerns σ and ς. In accordance with empirical estimates of the substitution elasticity and with our own results, we assign to σ and ς the value of 2. Concerning size, preferences, and factors proportions, we have chosen to assume symmetry: goods are equally liked ($\gamma_Y = \gamma_Z = 1/2$) and countries have symmetric differences in endowments, with H being the K-*abundant* country ($\nu_K^H = \nu_L^F$, and $\nu_L^H = \nu_K^F$, with $\nu_K^H = 0.55$ and $\nu_L^H = 0.45$). Good Y is K-*intensive* and we have chosen symmetry in technology, $\lambda_Y = (1 - \lambda_Z) = 0.4$. World endowments are $\overline{K} = 2200$ and $\overline{L} = 2200$. Variable trade cost τ takes values that range from 0 to 1 with an interval of 0.2, that is: $\tau = \{0,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1\}$.[24] There is no empirical guidance as to the value of the three types of fixed costs. As a representative example of many simulations, we show the results for $F = 0.6$, $F_x = 0.4$, and $F_e = 0.2$. Lastly, we assume $g(\beta)$ to be Pareto with a lower bound β_M and a shape parameter b, to which we assign the value of 4. The value of β_M is irrelevant, but we may choose it in such a way that the *ex-ante* average factor intensity is the same as if there were no heterogeneity, i.e., $\int_{\beta_M}^{\infty} (\beta)^{\sigma-1} g(\beta)\, d\beta = \alpha^{\sigma-1}$

[24] In passing, we mention that $\tau = 0$ corresponds to autarky, while $\tau = 1$ does not correspond to free trade since there are fixed exporting cost $F_x > 0$.

which gives endogenously the value of β_M. In this way, if all firms were able to survive in the market, the average *K-intensity* would be the same as if there were no heterogeneity, that is, equal to $(\omega^c)^\sigma (\Lambda_i)^\sigma$. Naturally, nothing hinges on this particular parametrization. A final check is that all the zero profit productivity cut-off resulting from the simulations must be at least as large of β_M. Figure 3 report a representative example from many simulations which show that Proposition 1 remains valid.