Comparative Advantage and Within-Industry Firms Performance.*

Matthieu Crozet[†] and Federico Trionfetti[‡]

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Abstract

Guided by empirical evidence we consider firms heterogeneity in terms of factor intensity. We show that Heckscher-Ohlin comparative advantage and firm-level relative factor-intensity interact to jointly explain the observed differences in relative sales. Firms whose relative factor-intensity matches up with the comparative advantage of the country have lower relative marginal costs and larger relative sales than firms who do not. Our empirical analysis, conducted using data for a large panel of European firms, supports these predictions. Our findings also provide an original firm-level verification of the Heckscher-Ohlin model based on the effect of comparative advantage on firms relative sales.

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[†]Paris School of Economics (Paris I) and CEPII. crozet@univ-paris1.fr. Maison des Sciences Economiques, 106 boulevard de l'hopital, 75013 Paris, France.

[‡]GREQAM, Université de la Méditerranée, France. Federico.Trionfetti@univmed.fr

1 Introduction

There is a wealth of empirical evidence that factor intensities differ across firms even within the same industry. This observation contrasts with the assumption usually adopted in trade models where firms are either assumed to be identical or are assumed to differ by Hicks-neutral productivity differences. In either case the resulting factor intensities are identical across firms within any industry. But this is not in line with facts. In our data, for instance, only 33 percent of the total variance in firm-level capital/labor ratios is between country-industry groups, 66 percent is within the same country-industry group. Neglecting this source heterogeneity misses an important aspect of within industry differences among firms. Thus, guided by empirical observation we abandon the assumption of Hicks-neutral productivity differences. Specifically, we consider a Heckscher-Ohlin model where firms differ in factors relative marginal productivity (RMP). As a result firms have different factor intensities even within the same industry. This assumption offers a closer adherence to the observed heterogeneity across firms than the assumption of Hicks-neutral heterogeneity usually adopted in the literature.

The main result emerging from this assumption is that the comparative advantage matters for within-industry relative performance of firms. The key firm-level variables in our analysis are relative factor intensity and relative sales. The relative factor intensity of a firm is given by the ratio between the factor intensity of the firm and the factor intensity of the average firm in the same industry-country. The relative sales of a firm is the ratio between the sales of the firm and the sales of the average firm in the same industry-country. We show that comparative advantage and relative factor intensity jointly determine firm relative sales. To fix ideas, consider two firms in different industries and different countries and whose capital intensity is ten percent larger than their respective country-industry average. The firm in the capital intensive industry and capital abundant country will have a relative cost advantage over the firm in the other industry and country. The relative cost advantage is reflected in larger relative sales. If we consider two firms whose capital intensity is ten percent lower than their respective country-industry average the firm in the capital intensive industry and capital abundant country will have a relative cost disadvantage and lower relative sales. To sum up, firms whose relative factor intensity matches up with the comparative advantage of the country have lower relative cost and larger relative sales than firms whose relative factor intensity does not match up with the comparative advantage of the country.

This prediction does not obtain in models where the only source of heterogeneity is in Hicks-neutral productivity differences. In these models, two firms in different industries or countries and whose productivity is ten percent larger (or smaller) than their respective country-industry average have exactly the same relative sales. The reason is that when the factor intensity is identical across firms the comparative advantage benefits or hurts all firms in the same country-industry in the same way. When factors RMP differs across firms, instead, the comparative advantage benefits or hurts different firms differently even within the same country-industry. There is therefore an interaction between firms characteristics and comparative advantage in our model that is absent in models where firms have the same factor intensities. Our empirical investigation, conducted on a dataset which comprises over four hundred thousands firms in twenty-six European countries and eighty-seven industries, strongly supports the presence of this interaction. Both structural and non-structural estimations show that the relationship between relative sales and relative factor intensity is affected by comparative advantage in the way predicted by the model. For instance, the non-structural estimations show that two firms with capital intensity 10% above their respective country-industry average have different relative sales: the sales of the firm in the capital intensive industry and capital abundant country are 34 percent larger than the average firm in the same industry-country whereas the sales of the firm in the labor intensive industry and labor abundant country are only 27 percent larger that the average firm in the same industry-country.

The paper brings the literature forward in three ways. First, it shows that factor intensity is an important source of heterogeneity across firms. The seminal contributions by Bernard, Eaton, Jensen and Kortum (2003) and Melitz (2003) and many subsequent important developments have used models where there is only one factor of production thus ruling out heterogeneity in factor intensity altogether.[1] Bernard, Redding and Schott (2003)

[1] See, e.g., Chaney (2008), Melitz and Ottaviano (2008), Arkolakis, Costinot and Andrés Rodríguez (2010), Bustos (2010), Eaton, Kortum and Kramarz (2010) for new models development. See, e.g., Egger and Kreickemeier (2009), Costinot and Vogel (2010), Helpman, Itskhoki and Redding (2010), Davis and Harrigan (2010), Amiti and Davis (2008) for particular focus on the distribution effects of trade opening. In some models of the latter group, for instance in Costinot and Vogel (2010), there is more than one factor in the sense that heterogenous labor is applied to a continuum of tasks (goods) but all goods are produced by identical firms (with identical factor intensities, therefore).

are the first to introduce firms heterogeneity in a Heckscher-Ohlin model but they consider only Hicks-neutral productivity differences and, therefore, within-industry factor intensities are identical across firms. Burstein and Vogel (2009), like us, assume heterogeneity in factor intensity across firms. Their model differs from ours in many respects and their objective is very different from ours but, like ours, their study shows that taking account of heterogeneity in factor intensity allows for a deeper exploration of the consequences of firms heterogeneity on international trade issues.[2]

The second contribution of our work is to show that comparative advantage matters for within-industry relative performance. If Hicks-neutral productivity differences were the only source of heterogeneity firms relative sales would just reflect the exogenous distribution of productivity and this even in the presence of comparative advantage. If, instead, heterogeneity is in factor intensity firms relative sales are endogenously determined by the interaction between firms factor intensity and comparative advantage. This interaction makes that differences in factors intensity across firms show up as magnified or dampened on relative sales. One of the interesting aspects of the heterogenous firms literature is that it allows investigating within-industry effects of international trade. Yet, Hicks-neutral heterogeneity (or the one-factor assumption) makes that within industry effects are independent from comparative advantage and depend only on exogenously given differences in productivity. As a result, one may be left with the impression that there is a dichotomy between within-industry effects and across-industry effects, the former driven by heterogeneity and the latter driven by comparative advantage. Our work, instead, highlights precisely how within-industry effects are determined jointly by firms characteristics and comparative advantage.

This focus leads to our third contribution which consists in a novel ap-

[2]Burstein and Vogel (2009) assume that each country produces one non-traded final good by use of a continuum of intermediated goods (sectors) traded at iceberg costs. Within each sector there is a continuum of sub-sectors. There is perfect competition in all goods and heterogeneity results from draws of factor intensities whose distribution is exponential as in Eaton and Kortum (2002). They assume a skill-bias in technology. Their Objective is to study the effect of trade opening on the skill premium. They identify a between effect (essentially driven by the Stolper-Samuelson effect) and a within effect (driven by within-sector reallocation of factors towards the most skill-intensive firms). Further, they show that the between effect is attenuated by an increase in the dispersion of technology. The skill premium is not the focus of our analysis but our model too gives rise to the between effect and to its attenuation when the technology becomes more dispersed. The within effect obtains from our model after a simple modification.

proach to the verification of the Heckscher-Ohlin model. Seminal contributions, e.g., Leamer (1980), Trefler (1993, 1995), Davis and Weinstein (2001), Romalis (2004) have provided solid evidence on the empirical merits of the factors proportion theory. In their works, comparative advantage is revealed by its effect on the factor content of trade flows or the specialization pattern (aggregate variables). Our work proposes a different approach. In our model, comparative advantage is revealed by the effect it has on relative sales individual firms in different industries and countries (a firm-level variable). Absent comparative advantage, factors relative marginal productivity differences would not result in differences in relative sales across industries and countries. Therefore, evidence of a relationship between firms relative sales and country-industry characteristics is evidence of the existence of a comparative cost advantage of the Heckscher-Ohlin type. Being able to capture the effect of comparative advantage on relative sales (relative marginal cost) is particularly interesting because such effect is, after all, at the heart of the international specialization mechanism in the Heckscher-Ohlin model. Specialization is driven by reallocation of firms across industries due to the comparative cost advantage but this mechanism remains behind the scenes in homogenous firms model as well as in Hicks-neutral heterogeneity models. Looking at the firm-level effect of comparative advantage therefore brings to light the fundamental mechanism of international specialization. Approaches based on aggregate variables are, of course, unable to bring this mechanism to light.

We conclude this section by mentioning that the four core theorems of international trade remain valid in our model but the degree of international specialization, the intensity of the Stolper-Samuelson and Rybczynski magnification effects, and the size of the FPE set are affected. We briefly discuss these effects in the Appendix.

The remainder of the paper is as follows. Section 2 describes the model, Section 3 and 4 describe the theoretical results, Section 5 derives the estimable equation, Section 6 presents the data, Sections 7 and 8 show the empirical results for the structural and non-structural estimations respectively, and Section 9 concludes. The Appendix contains the proof of the propositions, a brief discussion on the four core theorems, and some numerical simulations.

2 The Model.

The model combines a two-by-two Heckscher-Ohlin model with monopolistic competition and heterogenous firms. The world economy is composed by two countries, indexed by $c = \{H, F\}$, produces two differentiated goods, indexed by $i = \{Y, Z\}$ by use of two primary factors, indexed by $j = \{K, L\}$. Each country is endowed with a share ν_j^c of world's endowments, $\overline{K}$ and $\overline{L}$.

2.1 Technology

Production requires fixed and variable inputs each period. To make the model more specific we assume that the variable input technology (inputs used for production) takes the following C.E.S. functional form:

$$q_i = \phi \left(\lambda_i \left(\alpha L\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\lambda_i\right)\left(\beta K\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where q_i is output, L and K are factors inputs, $\lambda_i \in (0,1)$ is a constant technology parameter, and ϕ, α, and β are random variables. The $K-$ *intensity* in the production process for a firm in industry i of country c, θ_i^c, is:

$$\theta_i^c = \left(\frac{w^c}{r^c}\right)^{\sigma} \left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma} \left(\frac{\beta}{\alpha}\right)^{\sigma-1}, \tag{2}$$

where r^c and w^c denote, respectively, the price of K and L in country c. An increase in β/α makes the firm more $K-$ *intensive*. An increase in the relative price of L, $\frac{w^C}{r^C}$, makes every firm in every industry more $K-$ *intensive*. The industry with the lowest λ_i has the $K-$*intensive* technology.[3] Models that focus on Hicks-netural heterogeneity assume α and β constant and identical across firms and let ϕ be vary across firms. We, instead, focus on heterogeneity in α and β which, regardless of variations in ϕ, influence factors RMP and, thereby, the factor intensity. Although our focus is on

[3]Given non-neutral heterogeneity it is necessary to distinguish between firm's factor intensity and average factor intensity of the industry. Indeed, firms with a vary high β/α in the industry whose technology is $L-$*intensive* (high λ_i) may have a higher $K-$*intensity* than firms with low β/α in the $K-$ *intensive* industry. Yet, it will be shown in the Appendix that the industry whose technology is intensive in a factor is also the industry which, on average, is intensive in that factor. Thus, no average factor intensity reversal holds in this model.

factor intensities we let ϕ in the model for two reasons. First, in the empirical implementation we control for total factor productivity (TFP) differences independent from the TFP-effect of changes in α and β. Second, in Section 4 we shall compare the effect on relative sales derived from heterogeneity in RMP with the effect derived from Hicks-neutral heterogeneity. For the time being, however, we keep ϕ constant (and equal to 1) and let α and β vary across firms. There is no difficulty in letting ϕ vary as well as α and β but this would unnecessarily burden the exposition with double integrals and a more intricate notation.

The marginal cost for a firm in industry i of country c, mc_i^c, is:

$$mc_i^c = \left[(\lambda_i)^\sigma \left(\frac{w^c}{\alpha} \right)^{1-\sigma} + (1-\lambda_i)^\sigma \left(\frac{r^c}{\beta} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

The relative marginal productivity of K is $\frac{1-\lambda_i}{\lambda_i} \left(\frac{L}{K}\right)^{\frac{1}{\sigma}} \left(\frac{\beta}{\alpha}\right)^{\frac{\sigma-1}{\sigma}}$. An increase in β/α increases the relative marginal productivity of K within a firm but it is clear that the relative marginal productivity of K does not necessarily reflect total productivity. Indeed, the firm with higher β/α could have very low absolute values of α and β and, therefore, have lower total productivity. In order to adhere to stylized facts we assume that total productivity is increasing with capital intensity. We therefore normalize $\alpha = 1$ for every firm in every industry and let β vary across firms. The chosen normalization reflects stylized facts about productivity often reported in the literature[4] and is also confirmed by our data as we shall see below. It is important to keep in mind, however, that the results in this paper do not depend on the normalization choice. Alternatively, we could normalize $\beta = 1$ and let α vary across firms; then, firms with higher capital intensity would be less productive but our results would remain unchanged. We demonstrate in the Appendix section the robustness of the results to the normalization choice.

Firms draw β from a probability distribution $g(\beta)$ with support in $(0, \infty)$ and with cumulative distribution $G(\beta)$ which is assumed to be the same across industries and countries. Let β_i^{*c} be the least value of β such that profits are non negative and whose formal definition will be given below. Only firms with productivity draw larger or equal to β_i^{*c} engage in production and the ex-post distribution of β, $\mu(\beta)$, will be conditional to successful entry

[4] See, e.g., Bernard et. al. (2007b), Verhoogen (2008), Alcalá and Hernández (2009).

and will be truncated at β_i^{*c}:

$$\mu_i^c(\beta) = \begin{cases} \frac{g(\beta)}{1-G(\beta_i^{*c})} & if \ \beta \geq \beta_i^{*c} \\ 0 & Otherwise \end{cases} \tag{4}$$

where $1 - G(\beta_i^{*c})$ is the probability of successful entry. It is convenient at this point to define two averages that will be used in the sequel. First, the harmonic average of β, denoted $\widetilde{\beta}_i^c$,

$$\widetilde{\beta}_i^c(\beta_i^{*c}) = \left[\frac{1}{1 - G(\beta_i^{*c})} \int_{\beta_i^{*c}}^{\infty} \beta^{\sigma-1} g(\beta)\, d\beta\right]^{\frac{1}{\sigma-1}}. \tag{5}$$

Second, the harmonic average of marginal cost, denoted $\widetilde{mc}_i^c(\beta_i^{*c})$,

$$\widetilde{mc}_i^c(\beta_i^{*c}) = \left[\frac{1}{1 - G(\beta_i^{*c})} \int_{\beta_i^{*c}}^{\infty} [mc_i^c(\beta)]^{1-\sigma} g(\beta)\, d\beta\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

Given these definitions, the average factor intensity in industry i and country c, denoted $\overline{\theta}_i^c$, is

$$\overline{\theta}_i^c(\beta_i^{*c}) = \left(\frac{w^c}{r^c}\right)^{\sigma} \left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma} \left[\widetilde{\beta}(\beta_i^{*c})\right]^{\sigma-1} \tag{7}$$

which will be used below in Propositions 1 and 2. It is worth mentioning that the firm whose draw of β is equal to the industry average has marginal cost and factor intensity equal to the industry average, i.e., $mc_i^c\left(\widetilde{\beta}_i^c(\beta_i^{*c})\right) = \widetilde{mc}_i^c(\beta_i^{*c})$ and $\theta_i^c\left(\widetilde{\beta}_i^c(\beta_i^{*c})\right) = \overline{\theta}_i^c(\beta_i^{*c})$.

In Hicks-neutral heterogeneity models the factor in tensity is $\left(\frac{w^c}{r^c}\right)^{\sigma}\left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma}$ for all firms. We see from expression (7) that heterogeneity in RMP results in a factor bias in industry i as long as $\widetilde{\beta}_i^c \neq 1$. The bias is endogenous, it may go in either direction - a *K-bias* (if $\widetilde{\beta}_i^c > 1$) or an *L-bias* (if $\widetilde{\beta}_i^c < 1$) - and the direction may differ in different industries or countries. It is important to keep in mind that none of our results hinges on the direction of the factor bias. In this respect our model is slightly less restrictive than Burstein and Vogel (2009) or Costinot and Vogel (2010) where the existence and direction of the factor-bias are assumed exogenously (though in strong adherence to empirical evidence) and the direction of the bias matters for the results.

Coming to the fixed input technology whether it is homogenous or heterogenous across firms gives qualitatively the same results. For clarity of exposition it is necessary to retain one of the alternatives throughout the paper. We retain the former since it allows focusing on heterogeneity in the production process (which is the heart of the matter).[5] Specifically, we assume that the fix-input technology is a C.E.S. that uses L and K in the same proportions as the industry average. Thus the fixed production cost is $F_i\widetilde{mc}_i^c\left(\beta_i^{*c}\right)$ where F_i is a positive constant.

For clarity of exposition, throughout the paper we assume that country H is $K-abundant$ and that good Y has the $K-intensive$ technology i.e., $\nu_K^H > \nu_L^H$ and $\lambda_Y < \lambda_Z$. Therefore, country H has the comparative advantage in good Y.

2.2 Demand.

The representative consumer's utility function is assumed to be Cobb-Douglas in the C.E.S. aggregates of all varieties of each good produced, i.e., $U = \left(C_Y\right)^{\gamma_Y}\left(C_Z\right)^{\gamma_Z}$,with $\gamma_i \in (0,1)$ and $\gamma_Y + \gamma_Z = 1$ and where C_i is a C.E.S. consumption index for industry i defined over consumption of each variety ξ of industry i, $c_i\left(\xi\right)$, in the set of all varieties of the same industry, Ξ_i, that is: $C_i = \left[\int_{\xi\in\Xi_i}\left[c_i\left(\xi\right)\right]^{\frac{\sigma-1}{\sigma}}d\xi\right]^{\frac{\sigma}{\sigma-1}}$. The dual price index associated with the C.E.S. sub-utility is $P_i^c = \left[\int_{\xi\in\Xi_i}\left[p_i^c\left(\xi\right)\right]^{1-\sigma}d\xi\right]^{\frac{1}{1-\sigma}}$where $p_i^c\left(\xi\right)$ is the price paid in country c for variety ξ of good i.

Utility maximization under the budget constraint and aggregation over consumers gives the demand function emanating from domestic residents, $s_{id}^H\left(\beta\right)$, and from foreign residents, $s_{ix}^H\left(\beta\right)$, for the output of a firm in industry i of country H with draw β (where s stands for sales, d for domestic, and x for foreign):

$$s_{id}^H\left(\beta\right) = \left(\frac{p_{id}^H}{P_i^H}\right)^{1-\sigma}\gamma_i I^H \tag{8}$$

$$s_{ix}^H\left(\beta\right) = \left(\frac{p_{ix}^H}{P_i^F}\right)^{1-\sigma}\gamma_i I^F \tag{9}$$

[5]This assumption is used also in Melitz (2003) and in Bernard, Redding and Schott (2007) among many others. In our model, as in theirs, results are robust to assuming heterogeneity in fixed costs.

Demand depends negatively on the price faced by consumers (respectively p_{id}^{H} and p_{ix}^{H}) and positively on the price index (P_i^c) and national income $\left(I^c = w^c \nu_L^c \overline{L} + r^c \nu_K^c \overline{K},\right)$. Analogous demand functions obtain for the output of a firm with draw β in industry i of country F:

$$s_{id}^{F}(\beta) = \left(\frac{p_{id}^{F}}{P_i^{F}}\right)^{1-\sigma} \gamma_i I^H \tag{10}$$

$$s_{ix}^{F}(\beta) = \left(\frac{p_{ix}^{F}}{P_i^{H}}\right)^{1-\sigma} \gamma_i I^F \tag{11}$$

It is useful to note at this point that relative sales of two firms in the same industry i and in the same market ζ depend solely on the ratio of marginal costs. That is, for any two firms with draws β' and β''we have

$$\frac{s_{i\zeta}^{c}(\beta')}{s_{i\zeta}^{c}(\beta'')} = \left[\frac{mc_i^c(\beta')}{mc_i^c(\beta'')}\right]^{1-\sigma}, \qquad \zeta = d, x. \tag{12}$$

2.3 Production.

Firms maximize profits given the technology and the demand functions described above and given the barriers to international trade. Except in the free trade situation, firms wanting to export face fixed and variable exporting cost. Variable costs are paid in terms of the good transported: for one unit of good shipped only a fraction $\tau_i \in [0,1]$ arrives at destination. The fixed exporting cost is paid in terms of input of both factors in the same proportions as fixed production cost. Thus the fixed exporting cost is $F_{ix}\widetilde{mc}_i^c(\beta_i^{*c})$ where F_{ix} is a positive constant.

In monopolistic competition and under the large-group assumption the profit-maximizing prices for the domestic and the foreign market are:

$$p_{id}^{c}(\beta) = \frac{\sigma}{\sigma-1} mc_i^c(\beta) \tag{13}$$

$$p_{ix}^{c}(\beta) = \frac{\sigma}{\sigma-1}\frac{1}{\tau_i} mc_i^c(\beta) \tag{14}$$

To enter the market, firms have to face a fixed and sunk entry cost which, analogously to the other fixed costs, is paid in terms of inputs of both factors inputted in the same proportions as fixed production cost. Thus, the fixed entry cost is $F_{ie}^{c}\widetilde{mc}_{i}^{c}\left(\beta_{i}^{*c}\right)$ where F_{ei} is a positive constant. After entry, firms draw β from the probability distribution $g\left(\beta\right)$. If the draw is high enough to allow for non-negative profit the firm will produce, otherwise will exit immediately. Profits, if any, in the domestic and foreign market are, respectively,

$$\pi_{id}^{c}\left(\beta\right) = \frac{s_{id}^{c}\left(\beta\right)}{\sigma} - F_{i}\widetilde{mc}_{i}^{c}\left(\beta_{i}^{*c}\right) \tag{15}$$

$$\pi_{ix}^{c}\left(\beta\right) = \frac{s_{ix}^{c}\left(\beta\right)}{\sigma} - F_{ix}\widetilde{mc}_{i}^{c}\left(\beta_{i}^{*c}\right) \tag{16}$$

where we attributed fixed production cost to the domestic profit account and fixed exporting cost to the profit on the foreign market account. The total profit of a firm with draw β is

$$\pi_{i}^{c}\left(\beta\right) = \begin{cases} \pi_{id}^{c}\left(\beta\right) \text{ if it does not export,} \\ \pi_{id}^{c}\left(\beta\right) + \pi_{ix}^{c}\left(\beta\right) \text{ if it exports.} \end{cases} \tag{17}$$

A firm will produce if $\pi_{id}^{c}\left(\beta\right) \geq 0$ and not produce otherwise. Similarly, a firm will export if $\pi_{ix}^{c}\left(\beta\right) \geq 0$ and not export otherwise. Let the zero-profit productivity cut-off, β_{i}^{*c}, be the least value of β such that domestic profits are non negative. By definition, β_{i}^{*c} satisfies $\pi_{id}^{c}\left(\beta_{i}^{*c}\right) = 0$. Using (15), β_{i}^{*c} satisfies the following zero cut-off profit condition:

$$s_{id}\left(\beta_{i}^{*c}\right) = \sigma F_{i}\widetilde{mc}_{i}^{c}\left(\beta_{i}^{*c}\right). \tag{18}$$

Likewise, let the zero-foreign-profit productivity cut-off, β_{ix}^{*c}, be the least value of β such that foreign profits are non negative. By definition, β_{ix}^{*c} satisfies $\pi_{ix}^{c}\left(\beta_{ix}^{*c}\right) = 0$. Using (16), β_{ix}^{*c} satisfies the following zero cut-off profit condition in exports:

$$s_{ix}\left(\beta_{ix}^{*c}\right) = \sigma F_{ix}\widetilde{mc}_{i}^{c}\left(\beta_{i}^{*c}\right). \tag{19}$$

Firms with productivity draw $\beta < \beta_{i}^{*}$ will exit immediately, firms with productivity β such that $\beta_{i}^{*} \leq \beta < \beta_{ix}^{*}$ will produce for the domestic market only, and firms with productivity draw $\beta \geq \beta_{ix}^{*}$ will produce for the domestic and the foreign market. After entry, firms face a constant and exogenous probability of death δ due to exogenous and unforeseeable events. Therefore, the value of a firm with draw β is the maximum between 0 (if $\beta < \beta_{i}^{*}$) and the present value of the future stream of profits (if $\beta \geq \beta_i^*$) discounted by the probability of death:

$$\upsilon_i^c(\beta) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t \pi_i^c(\beta)\right\} = \max\left\{0, \frac{\pi_i^c(\beta)}{\delta}\right\}. \tag{20}$$

2.4 Average prices, sales, and profits.

Average prices, sales, and profits can be expressed as functions of average marginal productivity. In addition to the average marginal productivity in the industry denoted $\widetilde{mc}_i^c(\beta_i^{*c})$ and defined in expression (6) we make use of the average marginal productivity of exporting firms, $\widetilde{mc}_i^c(\beta_{ix}^{*c})$, computed as in expression (6) except that β_{ix}^{*c} replaces β_i^{*c} as lower limit of integration. Given the profit-maximizing prices in expressions (13)- (14), the average price and the average export price are, respectively:

$$\widetilde{p}_{id}^c(\beta_i^{*c}) = \frac{\sigma}{\sigma-1}\widetilde{mc}_i^c(\beta_i^{*c}) \tag{21}$$

$$\widetilde{p}_{ix}^c(\beta_{ix}^{*c}) = \frac{1}{\tau_i}\frac{\sigma}{\sigma-1}\widetilde{mc}_i^c(\beta_{ix}^{*c}) \tag{22}$$

Using equations (12) we observe that domestic sales of a firm relative to those of the cut off firm in the domestic market are $[mc_i^c(\beta)/mc_i(\beta_i^*)]^{1-\sigma}$. Analogously, the sales of an exporting firm relative to those of the cut-off firm in the export market are $mc_i^c(\beta)/mc_i(\beta_{ix}^*)$. Using these relationships and equations (18)-(19) we can compute the average (or expected) revenues in each market as follows:

$$\overline{s}_{id}^c \equiv \int_{\beta_i^*}^{\infty} s_{id}^c(\beta)\frac{g(\beta)}{1-G(\beta_i^*)}d\beta = \left[\frac{\widetilde{mc}_i^c(\beta_i^{*c})}{mc_i(\beta_i^*)}\right]^{1-\sigma}\sigma F_i \widetilde{mc}_i^c(\beta_i^{*c}) \tag{23}$$

$$\overline{s}_{ix}^c \equiv \int_{\beta_{ix}^*}^{\infty} s_{ix}^c(\beta)\frac{g(\beta)}{1-G(\beta_{ix}^*)}d\beta = \left[\frac{\widetilde{mc}_i^c(\beta_{ix}^{*c})}{mc_i(\beta_{ix}^*)}\right]^{1-\sigma}\sigma F_{ix} \widetilde{mc}_i^c(\beta_i^{*c}) \tag{24}$$

The overall average (or expected) sales, $\overline{s}_i^c$, is

$$\overline{s}_i^c = \overline{s}_{id}^c + \chi_i^c \overline{s}_{ix}^c, \qquad c = H, F \text{ and } i = Y, Z. \tag{25}$$

where $\chi_i^c \equiv \frac{1-G(\beta_{ix}^{*c})}{1-G(\beta_i^{*c})}$ is the ex-ante probability of exporting conditional to successful entry.

Average profit may be computed from (15)-(16) obtaining

$$\overline{\pi}_i^c = \overline{\pi}_{di}^c + \chi_i^c \overline{\pi}_{xi}^c = \left[\frac{\overline{s}_{id}^c}{\sigma} - F_i \widetilde{mc}_i^c \left(\beta_i^{*c} \right) \right] + \chi_i^c \left[\frac{\overline{s}_{ix}^c}{\sigma} - F_{ix} \widetilde{mc}_i^c \left(\beta_i^{*c} \right) \right]. \tag{26}$$

2.5 Equilibrium.

In addition to profit-maximizing prices and to the zero profit conditions discussed above there are five additional sets of equilibrium conditions: the free entry conditions, the stationarity conditions, equilibrium in goods markets, relationship between domestic and foreign sales, equilibrium in factors markets.

The free entry condition. The expected value of entry, $V_i^c \left(\beta_i^{*c} \right)$, is given by the expected profit stream until death multiplied by the probability of successful entry. Using expression (20) the expected value of entry is: $V_i^c = \left\{ \left[1 - G \left(\beta_i^{*c} \right) \right] \overline{\pi}_i^c \left(\beta_i^{*c} \right) \right\} / \delta$. The presence of an infinity of potential entrants arbitrages away any possible divergence between the expected value of entry and entry cost. Therefore, the free entry condition, is:

$$\left[1 - G \left(\beta_i^{*c} \right) \right] \overline{\pi}_i^c \left(\beta_i^{*c} \right) = \delta F_{ei} \widetilde{mc}_i^c \left(\beta_i^{*c} \right). \tag{27}$$

The stationarity condition Stationarity of the equilibrium requires the mass of potential entrants, M_{ei}^c, be such that at any instant the mass of successful entrants, $\left[1 - G \left(\beta_i^* \right) \right] M_{ei}^c$, equal the mass of incumbent firms who die, δM_i^c, that is:

$$\left[1 - G \left(\beta_i^* \right) \right] M_{ei}^c = \delta M_i^c, \qquad c = H, F \text{ and } i = Y, Z. \tag{28}$$

Equilibrium in goods markets. Using the expressions for average sales as in expression (25) and computing average demand from demand functions (8)-(11) the equilibrium in goods markets requires the following conditions:

$$\overline{s}_i^H = \left(\frac{\widetilde{p}_{id}^H \left(\beta_i^{*H} \right)}{P_i^H} \right)^{1-\sigma} \gamma_i I^H + \chi_i^H \tau_i^{\sigma-1} \left(\frac{\widetilde{p}_{id}^H \left(\beta_{ix}^{*H} \right)}{P_i^F} \right)^{1-\sigma} \gamma_i I^F \tag{29}$$

$$\overline{s}_i^F = \left(\frac{\widetilde{p}_{id}^F \left(\beta_i^{*F} \right)}{P_i^F} \right)^{1-\sigma} \gamma_i I^F + \chi_i^F \tau_i^{\sigma-1} \left(\frac{\widetilde{p}_{id}^F \left(\beta_{ix}^{*F} \right)}{P_i^H} \right)^{1-\sigma} \gamma_i I^H \tag{30}$$

where the price indices are:

$$P_i^H = \left[M_i^H\left(\widetilde{p}_{id}^H\left(\beta_i^{*H}\right)\right)^{1-\sigma} + \chi_i^F M_i^F\left(\widetilde{p}_{ix}^H\left(\beta_{ix}^{*H}\right)\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{31}$$

$$P_i^F = \left[M_i^F\left(\widetilde{p}_{id}^F\left(\beta_i^{*F}\right)\right)^{1-\sigma} + \chi_i^H M_i^H\left(\widetilde{p}_{ix}^F\left(\beta_{ix}^{*F}\right)\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{32}$$

Relationship between foreign and domestic sales. Using the demand functions in expression (8)-(11) and the expressions for average sales (23) and (24) we obtain:

$$\frac{mc_i^H\left(\beta_{ix}^*\right)}{mc_i^H\left(\beta_i^*\right)} = \left[\tau_i^{\sigma-1}\left(\frac{P_i^F}{P_i^H}\right)^{\sigma-1}\frac{I^F}{I^H}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\sigma-1}} \tag{33}$$

$$\frac{mc_i^F\left(\beta_{ix}^*\right)}{mc_i^F\left(\beta_i^*\right)} = \left[\tau_i^{\sigma-1}\left(\frac{P_i^H}{P_i^F}\right)^{\sigma-1}\frac{I^H}{I^F}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\sigma-1}} \tag{34}$$

Empirical evidence shows that that exporting firms are more productive and larger than non exporting firms. We therefore assume that parameter values (in particular τ_i, F_i and F_{ix}) are such that $\beta_i^* < \beta_{ix}^*$ in every country and industry. In addition, this assures that any exporting firm sells also domestically.

Equilibrium in factors market. It requires that factors demand inclusive of all fixed factors inputs, denoted L_i^c and K_i^c, equals supply:

$$L_Y^c + L_Z^c = \nu_L^c \overline{L}, \qquad c = H, F \tag{35}$$

$$K_Y^c + K_Z^c = \nu_k^c \overline{K} \qquad c = H, F \tag{36}$$

Counting equations and unknowns. After replacing (21)-(22) and (31)-(32) into (29)-(30) and (33)-(34) the model counts twenty-three independent equilibrium conditions that, together with one normalization, determine twenty-four endogenous variables. The equilibrium conditions are the four average sales conditions (23)-(24), the four free-entry conditions (27), the four stationarity conditions (28), any three out of the four goods market equilibrium conditions (29)-(30), the four relationships between foreign and domestic sales (33)-(34), and the four factors market equilibrium (35)-(36). The endogenous are the four zero-profit productivity cut-off $\{\beta_i^{*c}\}$, the four zero exporting profits productivity cut-off $\{\beta_{ix}^{*c}\}$, the four factors prices $\{w^c, r^c\}$, the four masses $\{M_i^c\}$ and employment in each industry and country $\{L_Y^c, L_Z^c, K_Y^c, K_Z^c\}$. The equilibrium value of all other endogenous variables can be computed from these.

3 Comparative advantage and relative sales.

In order to study the interaction between comparative advantage and relative sales there is no need to assume fix exporting costs. Therefore, for clarity of exposition we present our results under the assumption $F_{ix} = 0$ so that we do not have to burden the notation and the prose by always distinguishing between domestic and foreign sales. We shall reintroduce fix exporting cost in the Appendix, however, where numerical simulations confirm the analytical results obtained in this section.

The key relationship in our model is between relative sales and relative factor-bias. Relative sales are measured by the sales of a firm relative to the sales of the average firm in the same industry and country $(s_i^c(\beta)/\overline{s}_i^c)$. Relative factor-bias is measured by the factor intensity of a firm relative the factor intensity of the average firm in the same industry and country $\left(\theta_i^c(\beta)/\overline{\theta}_i^c\right)$. We underline the word relative. The absolute bias is irrelevant for our results. We have seen in expression (7) that the average factor intensity, $\overline{\theta}_i^c$, may be $K-biased$ or $L-biased$. The direction of the bias leaves results unchanged as we shall demonstrate in the Appendix.

Our central result is that firms whose relative factor-bias matches up with the comparative advantage of the country receive a boost in their relative sales with respect to firms whose relative factor-bias does not match up with the comparative advantage of the country. Thus, for instance, a particularly $K-intensive$ firm will have higher relative sales if it is in a $K-intensive$ industry of a $K-abundant$ country than otherwise. The mechanisms giving this result hinges on the interaction between factors RMP (reflected in factor intensity) and industry-country characteristics (comparative advantage). Our central result may be formally stated in two propositions. The first proposition relates firms relative sales to the industry technology (λ_i). The second proposition relates firms relative sales to the factor abundance of the country.

Proposition 1 *Consider any two firms in the same country but in different industries and whose $K-intensity$ is in the same proportion ϱ to their respective industry average. The relative sales of the firm in the $K-intensive$ industry are larger than the relative sales of the firm in the $L-intensive$ industry if the proportion ϱ is larger than one and are smaller otherwise.*

Formally, for any β_i' such that $\theta_i^c(\beta_i') = \varrho\overline{\theta}_i^c$ we have:

$$\frac{s_Y^c(\beta_i')}{\overline{s}_Y^c(\beta_Y^{*c})} \gtreqless \frac{s_Z^c(\beta_i')}{\overline{s}_Z^c(\beta_Z^{*c})}, \qquad c = H, F; \quad as\ 0 < \varrho \gtreqless 1. \tag{37}$$

Proof. *See Appendix.* ■

The intuition for this result is simple. Consider two firms in the same country but in different industries and whose $K-intensity$ (or RMP) is ϱ percent higher then their respective industry average. Both firms will have lower marginal cost than the average. However, the higher relative marginal productivity of K (of both firms) makes relative marginal cost lower for the firm in the $K-intensive$ industry because the factor whose relative marginal productivity is higher (K) is used more intensively in this industry. Likewise, consider two firms whose $K-intensity$ (or RMP) is ϱ percent lower than their respective industry average. Both firms will have higher marginal cost than their industry average. However, the relative marginal cost will be higher for the firm in the $K-intensive$ industry because the factor whose relative marginal productivity is lower (K) is intensively used in this industry. Since relative sales of any two firms depend only on relative marginal cost the firm in the $K-intensive$ industry will have higher relative sales if both firms have equiproportionally larger than average $K-intensity$ and will have lower relative sale if both firms have equiproportionally lower than average $K-intensity$; which is Proposition 1. Proposition 1 holds for any level of trade costs, including autarky and free trade, since it hinges only on the technology being different between goods. This proposition shows the interaction between the firm's relative factor intensity (ϱ) and the technology of the industry (λ_i) in determining firm's relative sales.

Proposition 2 *Consider any two firms in the same industry but in different countries and whose $K-intensity$ is in the same proportion ϱ to their respective industry average. The relative sales of the firm in the $K-abundant$ country are larger than the relative sales of the firm in the $L-abundant$ country if the proportion ϱ is larger than one and are smaller otherwise.*

Formally, for any β_i' *such that* $\theta_i^c(\beta_i') = \varrho\overline{\theta}_i^c$ *we have:*

$$\frac{s_i^H(\beta_i')}{\overline{s}_i^H(\beta_i^{*c})} \gtrless \frac{s_i^F(\beta_i')}{\overline{s}_i^F(\beta_i^{*c})}, \qquad i = Y, Z; \quad \text{as } 0 < \varrho \gtrless 1. \tag{38}$$

Proof. *See Appendix.* ∎

To get the intuition consider two firms in the same industry but in different countries and whose $K-intensity$ (or RMP) is ϱ percent higher then their respective industry average. Both firms will have lower marginal cost than the average. However, the higher relative marginal productivity of K makes relative marginal cost lower in the $K-abundant$ country because the factor that both firms use more intensively with respect to the industry average (K) is relatively cheaper in the $K-abundant$ country. Likewise, consider two firms in the same industry but in different countries and whose $K-intensity$ (or RMP) is ϱ percent lower than their respective industry average. Both firms will have higher marginal cost than their industry average. However, the relative marginal cost is higher for the firm in the $K-abundant$ country because the factor that both firms save with respect to the industry average (K) is relatively cheaper in the $K-abundant$ country. Naturally, Proposition 2 does not hold in free trade since in such case factors price equalizes. This proposition shows the interaction between the firm's relative factor intensity (ϱ) and factors proportions as reflected in factors price.

We conclude this section by recalling that the normalization choice is irrelevant for the results. As demonstrated in the Appendix, and as it should be intuitive by now, both propositions remain valid if we assume that $\beta = 1$ and let α vary across firms.

4 Irrelevance of Hicks-neutral heterogeneity on relative sales.

We compare the results in the previous section with those emerging from the model if we assume that the only source of heterogeneity were Hicks-neutral productivity differences. This means assuming $\alpha = \beta = 1$ and by letting ϕ be a random variable distributed according to the density function $h(\phi)$ with cumulative distribution $H(\phi)$. Then, the marginal cost for a firm with productivity ϕ is

$$mc_i^c(\phi) = \frac{1}{\phi}\left[(\lambda_i)^\sigma (w^c)^{1-\sigma} + (1-\lambda_i)^\sigma (r^c)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{39}$$

Replacing expression (3) with expression (39) throughout Section 2 gives the model variant where the only source of heterogeneity is in Hicks-neutral productivity differences represented by different draws of ϕ. Thus, $\widetilde{\phi}_i^c(\phi_i^{*c})$ denotes the harmonic average productivity, $\widetilde{mc}_i^c(\phi_i^{*c})$ denotes average marginal cost, and $\overline{s}_i^c(\phi_i^{*c})$ denotes average sales, all three computed using the ex-post distribution of ϕ. The result of interest is the following:

Proposition 3 *If Hicks-neutral heterogeneity is the only source of heterogeneity relative sales are identical in all industries and countries.*

Formally, for any ϕ_i' such that $\phi_i' = \rho\widetilde{\phi}(\phi_i^{*c})$ we have:

$$\frac{s_i^c(\phi_i')}{\overline{s}_i^c(\phi_i^{*c})} = \rho^{\sigma-1}; \qquad c = H, F; \quad i = Y, Z; \quad \forall\rho > 0; \quad \forall\tau \in [0,1] \,. \tag{40}$$

Proof. *Using expressions (39) the marginal cost of the firm with draw $\phi_i' = \rho\widetilde{\phi}(\phi_i^{*c})$ is equal to $\rho\widetilde{mc}_i^c(\phi_i^{*c})$. From this, using equation (12) we obtain (40).* ■

Comparing Proposition 3 with Propositions 1 and 2 shows how the two sources of heterogeneity (RMP or Hicks-netural productivity) give different results. Heterogeneity in factors RMP interacting with comparative advantage makes relative sales different in different industries within a country and different in different countries within an industry. Hicks-netural heterogeneity has not impact on relative sales regardless of comparative advantage. The reason for this difference in results can be seen by inspection of relative marginal costs. In both cases relative sales are the same function of relative marginal costs as given by expression (12). But, if the source of heterogeneity is in Hicks-neutral differences only the marginal cost of any firm relative to the average depends only on the ratio between the productivity of the firm and the average productivity regardless of country-industry characteristics. That is:

$$\frac{mc_i^c\left(\rho\widetilde{\phi}_i^c\right)}{mc_i^c\left(\widetilde{\phi}_i^c\right)} = \frac{1}{\rho}. \tag{41}$$

Instead, if heterogeneity occurs in factors RMP the ratio of marginal costs is

$$\frac{mc_i^c\left(\varrho\widetilde{\beta}(\beta_i^{*c})\right)}{mc_i^c\left(\widetilde{\beta}(\beta_i^{*c})\right)} = \left[\frac{(\lambda_i)^\sigma\left(\frac{w^c}{\alpha}\right)^{1-\sigma} + (1-\lambda_i)^\sigma\left(\frac{r^c}{\varrho\widetilde{\beta}(\beta_i^{*c})}\right)^{1-\sigma}}{(\lambda_i)^\sigma\left(\frac{w^c}{\alpha}\right)^{1-\sigma} + (1-\lambda_i)^\sigma\left(\frac{r^c}{\widetilde{\beta}(\beta_i^{*c})}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}. \tag{42}$$

which depends not only on the relative factor-bias (ϱ) but also on factors price (w^c, r^c) and industry technology (λ_i). Except in free trade, factors price reflects relative factors endowments which, together with industry technology, determine the comparative advantage of the country. Therefore, heterogeneity in factors RMP makes that comparative advantage and relative factor-bias jointly determine firms relative marginal cost. Hicks-neutral heterogeneity instead makes relative marginal cost independent from comparative advantage and, therefore, relative marginal cost is determined only by relative productivity (ρ).

5 Relating the theoretical findings to the empirical implementation.

We can now assemble the results obtained above in a single estimable equation where total factor productivity and relative marginal factor productivity determine independently the relative sales of firms. A firm with draws $\phi' = \rho\widetilde{\phi}_i^c$ and β' such that $\theta_i^c(\beta') = \varrho\overline{\theta}_i^c$ will have log of relative sales given by[6]

$$\ln\left(\frac{s_i^c}{\overline{s}_i^c}\right) = (\sigma-1)\ln\left(\frac{\phi'}{\widetilde{\phi}_i^c}\right) + \ln\left[a_i^c + (1-a_i^c)\left(\frac{\theta_i^c(\beta')}{\overline{\theta}_i^c}\right)^{\sigma-1}\right], \tag{43}$$

where $\left(\frac{\phi'}{\widetilde{\phi}_i^c}\right)$ represents Hicks-neutral productivity difference, $\left(\frac{\theta_i^c(\beta')}{\overline{\theta}_i^c}\right)$ represents the relative $K - bias$, and a_i^c is

$$a_i^c = \frac{(1-\lambda_i)^\sigma\left(\frac{r^c}{\widetilde{\beta}(\beta_i^{*c})}\right)^{1-\sigma}}{(\lambda_i)^\sigma\left(\frac{w^c}{\alpha}\right)^{1-\sigma} + (1-\lambda_i)^\sigma\left(\frac{r^c}{\widetilde{\beta}(\beta_i^{*c})}\right)^{1-\sigma}} \in (0,1)\,. \tag{44}$$

Equations (43) and (44) summarize what we have learnt so far. First, that Hicks-neutral productivity difference and relative factor bias influence relative sales (this is trivially the result of the assumptions). Second, that the effect of the former is the same for all countries and is not related to industries' factor intensities (Proposition 3) while the effect of the latter depends on country-and-industry characteristics here condensed in a_i^c (Propositions 1

[6] Use (12), (7), (40), and (42) to obtain (43).

and 2). Third that the effect of the $K-bias$ is stronger in $K-intensive$ industries and $K-abundant$ countries (low a_i^c) as we have learnt from Propositions 1 and 2.[7]

Equation (43) is log-linear in the first term but not in the second. We therefore also propose a robustness check based on a second order Taylor expansion about homogeneity in $K-intensity$ ($\varrho=1$) of the second term in (43). We thus obtain a more convenient estimable equation:

$$\begin{aligned}\ln\left(\frac{s_i^c}{\overline{s}_i^c}\right) &= (\sigma-1)\ln\left(\frac{\phi'}{\widetilde{\phi}_i^c}\right)+(1-a_i^c)(\sigma-1)\left(\frac{\theta_i^c(\beta')}{\overline{\theta}_i^c}-1\right) \\ &\quad +\frac{1}{2}(1-a_i^c)(\sigma-1)\left(a_i^c(\sigma-1)-1\right)\left(\frac{\theta_i^c(\beta')}{\overline{\theta}_i^c}-1\right)^2+\varepsilon_i^c,\end{aligned} \tag{45}$$

where ε_i^c, the remainder of the Taylor expansion, can be decomposed into an intercept and a structural error term.

The next three sections present the data, show the results of structural estimations of equations (43) and (45), and provide a more general test by verifying Propositions 1 and 2 separately.

6 Data

Our empirical verifications combine two sources of data, firm-level balance sheets and country-level capital and labor endowments. Firm-level data are provided by Bureau Van Dijk's Amadeus database.[8] Amadeus compiles balance sheet information for a very large number of companies located in 41 European countries. Its coverage increases progressively. To get the most comprehensive database, we retain the two most recent years available at the time of writing, 2006 and 2007. When companies are present in the database in both years, we simply retain the mean value of the information for 2006 and 2007. We extract from Amadeus the information needed to rely firms' capital intensity to their sales. We proxy capital intensity by the ratio of tangible fixed assets on total employment and sales by the turnover of the firm,

[7] Indeed, a_i^c is a decreasing function of the $K-intensity$ of the industry technology (low λ_i) and of the $K-abundance$ of the country (high w^c/r^c).

[8] http://www.bvdep.com/en/AMADEUS.html

without distinction between exports and domestic sales. Firms in Amadeus are classified according their primary activity. Each company is assigned to a single 3-digit NACE-Rev2 code. We restrict our empirical analysis to manufacturing sectors (including agrofood), i.e. to firms with a primary activity code between 101 and 329.[9] Moreover, we drop all country-industry pairs living us with a too small number of firms to perform robust regressions. We fix an arbitrary limit and retain country-industry pairs with more than 20 firms.

Capital abundance for each country, (K^c/L^c), is built from several sources. We use ILO and United Nations data for active population. Capital stocks are estimated by the perpetual inventory method using investment data from the World Bank and national sources.[10] Industry-level capital intensity is computed directly with our data. For each country and industry, we compute the weighted average firm-level capital-labor ratio. Then, $K-intensity$ for industry i, (K_i/L_i), is the industry-level average of these values across all countries, weighted by countries' output of good i.

The final database is a panel of 445,853 firms in 87 industries and 26 European countries.[11] The country-industry panel is unbalanced because all countries do not have more than 20 firms in all the 87 industries. We have data for 1,419 country-industry pairs, for a total of 2,262 possible combinations. The average number of firms per country-industry pair is 314.2, but the population within each group varies greatly. The median country-industry pair has only 100 firms, and the largest group contains 9,920 observations.[12]

Table 1 shows a variance decomposition analysis for firm-level total factor productivity and capital intensity in our sample.[13] The first column gives the

[9] We also exclude manufacturers of coke and refined petroleum products.

[10] We are indebted to Jean Fouré for giving us these country-level data. See Bénassy-Quéré et al. (2010) for a description of the source data and the methodology.

[11] Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Italy, Latvia, Lithuania, Poland, Romania, Russian Federation, Serbia, Slovakia, Slovenia, Spain, Sweden, Ukraine, United Kingdom.

[12] France-Manufacture of bakery and farinaceous products.

[13] In order to keep the largest possible number of firms in the data, we use a quite rough proxy for TFP which does not require additional firm-level information. We simply regress, for each country and industry separately, the log of firm's turnover on their total employment and fixed assets. As usual, the estimated coefficients are constrained in order to sum to one. Our proxy for TFP is the exponential of the sum of the intercept and the residuals of this estimated equation. Note that we also computed firms' TFP imposing similar technologies in all countries, as assumed in our model. This does not change

total variance of each variable while the four last columns report the share of variance (R^2) in the log of total factors productivity (TFP) and the log capital intensity that is explained respectively by different set of fixed effects. Column 2 introduces industry level (nace3) fixed effects, column 3 reports the explanatory power of country level fixed effects, we use the two sets of fixed effects together in column 4 and country-industry pairs fixed effect in column 5. It appears first that firms are much more heterogeneous in terms of capital intensity than in terms of productivity. More importantly for the premise of the paper, the different set of fixed effects explain systematically a larger share of variance in TFP than in capital-intensity. The fist R^2 reported in column 5 establishes that 58 percent of the total TFP variance results from countries and industries common characteristics. In other words, 42 percent of firm-level heterogeneity in terms of TFP is within countries and industries. This is quite a lot, but still relatively low compared to capital intensity's variance. The R^2 reported in column 5 for this variable is a bit less than 0.33, which means that about 66 percent of the observed firm-level heterogeneity is within country-industry groups. This finding clearly confirms that the assumption of homogeneous factor intensity within industries, largely adopted in the literature, contrasts with actual observations.

Table 1: Variance decomposition of firm's TFP and Capital intensitiy: Explanatory power (R^2) of different set of fixed effects

	Total Variance	Fixed Effects: Industry	Country	Country and Industry	Country-Industry pairs
TFP	1.5681	0.0762	0.5127	0.5470	0.5777
K/L	2.8773	0.0735	0.2361	0.3134	0.3297
Nb. obs	445853	445853	445853	445853	445853

7 Structural estimations

Table (2) reports estimates corresponding to the equation (43) and its Taylor expansion (45). In both cases, the dependant variable, $\ln(s_i^c/\bar{s_i^c})$, is the log of

significantly our final results.

firms' total sales, s_i^c, relative to the corresponding country-industry average, $\bar{s_i^c}$, in country c and industry i. The right-hand side variables are the total factor productivity and capital intensity of this firm, relative to the country-industry averages. In both equations the model imposes strict predictions for the structural estimated coefficients σ and a: $\sigma > 1$ and $a \in (0, 1)$. Moreover, a must be relatively lower for $K - abundant$ countries and $K - intensive$ industries and larger for $L - abundant$ countries and $L - intensive$ industries. Columns (1)-(3) show the estimates resulting from equation (43). Columns (4)-(6) report the estimates when an intercept is introduced to give the model some flexibility. The intercept is not in the estimable equation but may be introduced nevertheless to alleviate the consequence of a possible systematic measurement error in any variable. Finally, columns (7)-(9) show results obtained by estimating the Taylor expansion (45) as a robustness check. Since the Taylor expansion approximates better the true function the closer the independent variable is to the expansion point (i.e., relative $K - intensity$ close to 1) we estimate expansion (45) on a restricted sample of firm. Within each country-industry pair, we retain firms with a relative $K - intensity$ between the 10th and the 90th percentile. In all cases, we perform non-linear least squared and impose all the constraints on σ and a given by the model.

We first estimate the model pooling all the industries and countries, results are reported in columns (1), (4), and (7). Then, we restrict the sample to country-industry pairs that exhibit the prerequisite for comparative advantage. Columns (2), (5) and (8) retain countries whose $K - abundance$ is above the median and industries whose $K - intensity$ is above the median. We shall refer to this sample as the *KK-group*. Similarly, columns (3), (6) and (9) retain countries with lower-than-median $K - abundance$ and industries with lower-than-median $K - intensity$ industries. We shall refer to this sample as the *LL-group*. Propositions 1 and 2 (and equation 43), predict a smaller value of coefficient a for the *KK-group* than for the *LL-group*.

The results bring clear supportive evidence in favor of our framework. Both coefficients σ and a always range significantly in the expected intervals: σ is larger than one and a is positive and less than one. Our estimates for σ appear to be very robust across the different estimations. They vary between 1.96 and 2.11, but they are never significantly different from each other. These values of σ are relatively small according to some of the estimates proposed by the existing literature. For instance, Anderson and Van Wincoop (2004), surveying several empirical trade analysis, consider that a reasonable range for σ is between 5 and 10. But we are very close to Broda and Weinstein

Table 2: Impact of relative TFP and $K-intensity$ on relative sales: structural estimations

Dependant variable: ln firms' relative sales ($\ln(s_i^c/\overline{s}_i^c)$)			
Countries	All	$K-abundant$	$L-abundant$
Industries	All	$K-intensive$	$L-intensive$
	Eq. (44) - without intercept		
	(1)	(2)	(3)
σ	1.964^a	1.980^a	1.981^a
	(0.013)	(0.021)	(0.031)
a	0.094^a	0.055^a	0.146^a
	(0.005)	(0.008)	(0.013)
R^2	0.5037	0.504	0.519
Observations	445853	142618	64605
	Eq. (44) - with intercept		
	(4)	(5)	(6)
σ	1.973^a	1.875^a	2.043^a
	(0.014)	(0.020)	(0.030)
a	0.687^a	0.383^a	0.913^a
	(0.016)	(0.029)	(0.019)
R^2	0.608	0.601	0.623
Observations	445853	142618	64605
	Taylor expansion		
	(7)	(8)	(9)
σ	2.096^a	2.111^a	2.069^a
	(0.017)	(0.023)	(0.031)
a	0.402^a	0.244^a	0.398^a
	(0.042)	(0.034)	(0.052)
R^2	0.325	0.306	0.348
Observations	359207	114842	52091

Notes: Non-linear least squared. Starting values: $\sigma = 6$, $a = 0.5$. Robust standard errors adjusted for country-industry clusters in parentheses. Columns (1)-(6) report the estimates of the log of equation (43). Columns (7)-(9) report the estimated of equation (45), dropping firms with a $K-intensity$ beyond their respective country-industry 10th and 90th percentile. Significance level: ^{a} $p < 0.01$.

(2006) who report a median value for σ of 2.2, when they conduct their estimations using a 3-digit product classification. Our result is also in line with Imbs and Méjean (2010) who find a value of σ ranging from 2.5 to 3 when they force the elasticity to be equal across sectors as we do.

Conversely, our estimates of the coefficient a vary a lot. Considering all the observations, the non-linear estimations of equation (43) give values around 0.094, but this estimate jumps to 0.687 when one introduces an intercept. The coefficient provided by the Taylor expansion is close to the latter value. This finding, along with the higher values of R^2 reported in columns (4)-(6) than the ones in (1)-(3), confirm that the use of the intercept helps the model fit the data. Coming to the heart of the matter, we systematically observe a lower value of coefficients a for the *KK-group* than for the *LL-group*. The estimated value of parameter a reported in column (3) is about 2.7 times bigger than the one in column (2). We observe a comparable proportion between the estimates reported in columns (5) and (6): 2.4. These differences are statistically different at conventional confidence levels. This is confirmed by the estimates resulting from the Taylor expansion (columns 8 and 9) though the difference between estimated coefficients is smaller and significant only at the 10% level.

These structural estimations undoubtedly reveal that comparative advantages magnifies the consequences of firm-level heterogeneity in $K-intensity$, as predicted by our Propositions 1 and 2, while it has not influence on the relationship between firms' relative TFP and firms' relative sales.

8 Non-structural estimations

Equations (43) and (45) impose strict structural constraints on the key parameters of the model. In this section we abandon structural estimations and focus on verifying empirically the validity of the relationships stated in Propositions 1 and 2. This is important since it provides an empirical assessment not only of our model but, potentially, of an entire class of model exhibiting heterogeneity in factor intensity, such as Burstein and Vogel (2009) for instance.

Propositions 1 and 2 both relate firms' relative sales to their relative capital intensity. Whatever the country and the industry, the relative capital intensity of a firm should systematically increases its sales relative to those of the average firm. But Proposition 1 implies that this relationship between firms' relative sales and firms' relative capital intensity should be steeper in capital intensive industries, *ceteris paribus*. Similarly, Proposition 2 implies a steeper relationship in capital abundant countries. These predictions are tested with a two-steps procedure. The first step consists in estimating the following non-structural form of equation (43):

$$\ln\left(\frac{s_i^c}{\bar{s}_i^c}\right) = F + \psi \ln\left(\frac{\theta_i^c(\beta')}{\overline{\theta}_i^c}\right) + \eta \ln\left(\frac{\phi'}{\widetilde{\phi}_i^c}\right) + \epsilon_i^c, \tag{46}$$

where F is an intercept and ϵ_i^c is an error term. This specification is much more flexible and comprehensive than the structural equation and should provide more robust results. We estimate this equation separately for each of the 1,419 countries-pairs and collect the corresponding estimated coefficients on firms' $K-intensity$, ψ. The second step consists in testing whether these coefficients, now specific to each country c and industry i, $\widehat{\psi}_i^c$, vary with the industry-level $K-intensity$ and the country-level $K-abundance$.

The first step gives extremely robust results. Table 3 reports the estimates of this non-structural equation obtained on the pooled dataset.

Table 3: Impact of relative TFP and K-intensity on relative sales: non-structural log-linear model

	Dependant Variable: Ln firms' relative sales				
Country	All	All	All	K-abund.	L-abund.
Industry	All	All	All	K-intens.	L-intens.
	(1)	(2)	(3)	(4)	(5)
Ln Rel. K-intensity	0.2684^a	0.2684^a	0.0844^a	0.3413^a	0.2197^a
	(0.008)	(0.008)	(0.030)	(0.014)	(0.016)
Ln Rel. TFP		1.1177^a	1.1177^a	1.1297^a	1.1008^a
		(0.014)	(0.014)	(0.020)	(0.029)
Ln Rel. K-intensity			0.0117^a		
$\times \ln(K^c/L^c) \times \ln(K_i/L_i)$			(0.002)		
Observations	445853	445853	445853	142618	64605
R^2	0.050	0.349	0.350	0.342	0.364

Notes: Country-Industry fixed effects for all columns. Robust standard errors adjusted for country-industry clusters in parentheses. Within R^2 are reported. Significance levels: ^{a} $p < 0.01$

Table 3 confirms the results obtained from the structural specification (cf. Table 2). Column (1) omits the total factor productivity term. The positive and very significant coefficient confirms that firms with higher relative $K-intensity$ are significantly bigger. A firm with a $K-intensity$ 10% above the country-industry mean would have a market share 2.7% larger than the average firm. Column (2) introduces firms' relative TFP. Not surprisingly, productivity as a great influence on firms' performances. The coefficient on TFP is highly significant and very large in magnitude. The introduction of this variable also improves greatly the global fit of the regression, raising the R^2 by a factor of 7. More importantly, controlling for TFP does not effect the coefficient on relative capital abundance, suggesting that these two variables can be reasonably considered as orthogonal. Indeed, regressing relative $K-intensity$ on relative TFP with a full set of country-industry fixed effects fails to reveal a significant relationship between the two variables.[14]

Column (3) introduces an additional variable interacting firms' relative $K-intensity$ with its respective country-level $K-abundance$ and industry-level $K-intensity$. This interaction term attracts a positive coefficient which confirm that firm-level $K-intensity$ has a greater impact on firms' sales when it belongs to a $K-intensive$ industry and is located in a $K-abundant$ country. Finally, columns (4) and (5) replicate the tests shown in table 2. Column (4) reports the results obtained on the sample restricted to relatively $K-abundant$ countries and $K-intensive$ industries (the *KK-group*) while column (5) shows the coefficient obtained when considering $L-abundant$ countries and $L-intensive$ industries (the *LL-group*). Consistently with the theoretical results the estimated coefficient on relative $K-intensity$ is significantly larger for the *KK-group* than for the *LL-group* and the TFP coefficients in columns (4) and (5) are not significantly different from each other. The differences between coefficients on relative $K-intensity$ reported in columns (4) and (5) is not only statistically significant, but also important in magnitude. The slope of the relationship between relative $K-intensity$ and relative sales is 50% larger in *KK-group* than in the *LL-group*: a $K-intensity$ 10% above the country-industry mean results in a relative sales 22% larger in the *LL-group*, but more than 34% in the *KK-group*. All together, these results corroborate our theoretical predictions. They also support the fact that the log-linear equation (46) is a reasonable approximation of our

[14]The correlation between the two variables is only 0.014, and the regression coefficient is $1.60e-11$ with a student's t of 0.12.

model.

When estimating equation (46) separately for each of the 1,419 country-industry pairs, we obtain quite robust results. The coefficient on relative firms' $K - intensity$, $\widehat{\psi_i^c}$ is negative in only 99 regressions (less than 7 percent). For most of these unexpected results, the estimates are not significant at the 1 percent level. Only 9 country-industry pairs (0.63 percent) show a significantly negative coefficient. In 342 cases (24.1 percent), the coefficient is not significantly different from zero (90 are negative, and 252 are positive). Finally, we obtain a strictly positive coefficients for a huge majority of country-industry pairs (1,068 cases, representing 75.3 percent of the sample).[15]

Figure (1) illustrates the relationships between $\widehat{\psi}_i^c$ coefficients and the determinants of comparative advantages. Panel (a) plots the mean values of $\widehat{\psi}_i^c$ for each industry i, with the corresponding mean standard deviations, against industry's capital intensity. Panel (b) relates the country means of $\widehat{\psi}_i^c$ and its standard deviations to countries' capital abundance. While it is barely significant in panel (b), the two graphs exhibit the positive slope predicted by our model. This is confirmed by the regression results shown in Table (4).

The top panel of Table 4, i.e. columns (1)-(4), tests the validity of Proposition 1. Here, we regress the estimated slope of the relationship between firms' relative $K - intensity$ and firms' relative sales, $\widehat{\psi}_i^c$, on industry-level capital intensity and country fixed effects. The positive coefficient reported in column (1) explicitly validates Proposition 1. It says that, in a given country, the payoff, in terms of relative sales, of having a higher relative capital-labor ratio is bigger in relatively $K - intensive$ industries, and lower in relatively $L - intensive$ industries. This is exactly what Proposition 1 claims. This regression only considers the estimated coefficients $\widehat{\psi}_i^c$ without controlling for their significance level or economic relevance. Regressions reported in columns (2), (3) and (4) make use of information we have on the precision of each estimate. In columns (2), we keep significantly positive coefficients $\widehat{\psi}_i^c$ only. Saxonhouse (1976) advocates that regressions using estimated parameters as dependant variables are likely to be affected by heteroschedasticity. He suggests to weight the observations in order to give more importance to

[15]The coefficients $\widehat{\psi_i^c}$ range between -0.60 and 1.74, with a mean of 0.37 and a median of 0.33.

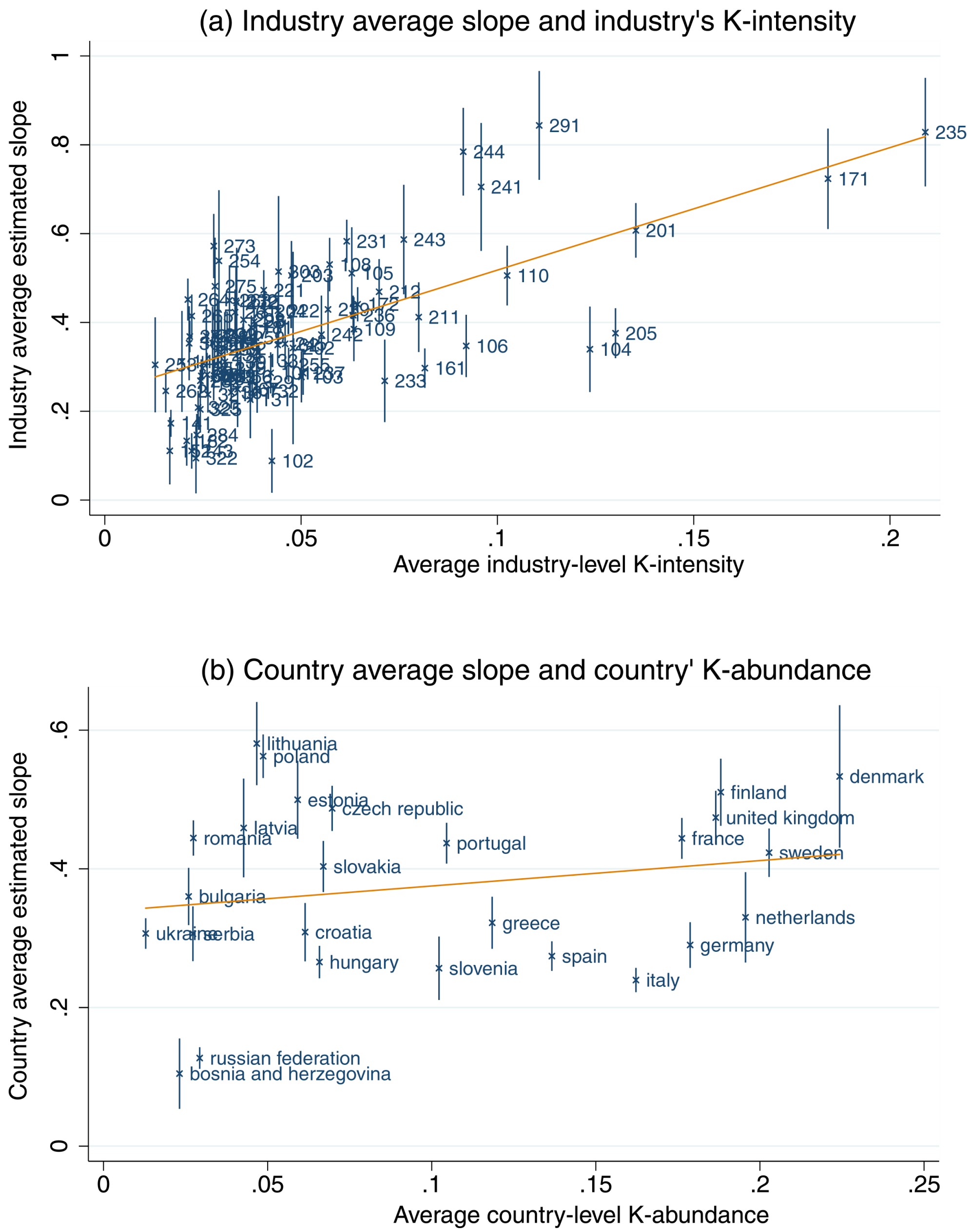

Figure 1: Average $\widehat{\psi}_i^c$, indystry's $K-intensity$ and country's $K-abundance$

more significant estimates. In column (3), the weight we give is the inverse of the standard error reported for each $\widehat{\psi}_i^c$. A second possible weight we can use to control for the significance of the estimates is the degree of freedom in the first step regressions. Regressions in column (4) is performed giving a weight equal to the square root of the number of firms within each country-industry group minus 3. The result shown in column (1) appears to be very robust. Dropping negative and non-significant values of $\widehat{\psi}_i^c$ has almost no impact on the second step regression. The two weighted regressions give a slightly lower coefficient on industry-level $K - intensity$. It remains positive however, and very significant.

Empirical tests of Proposition 2 are shown in the bottom panel of Table 4. These tests are the same as for Proposition 1, but exploit the variance of $\widehat{\psi}_i^c$ across countries rather than across industries. Here, the second step consists in regressing $\widehat{\psi}_i^c$ on countries' $K - abundance$ and industry fixed effects. While much smaller than those reported in the top panel, the positive coefficient on $K - abundance$ in column (5) corroborates Proposition 2. In a given industry, differences in relative firm-level capital intensity generate greater heterogeneity in relative sales in capital-abundant countries. Robustness checks shown in columns (6)-(7) confirm this result. Considering only significantly positive $\widehat{\psi}_i^c$ or weighting the observations lowers the estimated coefficient on capital-abundance but the significance remains, at the 5 percent level.

In conclusion these results strongly confirm the empirical validity of Propositions 1 and 2. This confirmation is particularly interesting since it is the result of a non-structural analysis and, as such, may give empirical validity to an entire class of models exhibiting heterogeneity in factor intensity.

9 Conclusion.

What determines the relative performance of firms? In this paper we have shown that comparative advantage jointly with differences in factors relative marginal productivity explain the differences in relative firms sales across industries and countries. Two firms with identical relative factor intensity have different relative sales if they belong to different industries or countries. The firm whose relative factor intensity matches up with the comparative advantage of the country has larger relative sales than the firm whose relative factor intensity does not match up with the comparative advantage of the

Table 4: Tests of propositions 1 and 2

Dependant Variable: $\widehat{\psi}_i^c$				
	Test of proposition 1			
	(1)	(2)	(3)	(4)
Industry K-intensity	2.749^a	2.768^a	2.513^a	2.570^a
	(0.376)	(0.350)	(0.195)	(0.203)
Observations	1419	1068	1419	1419
R^2	0.119	0.173	0.342	0.321
Fixed effects	Country			
	Test of proposition 2			
	(5)	(6)	(7)	(8)
Country K-abundance	0.351^a	0.246^b	0.208^b	0.222^b
	(0.106)	(0.108)	(0.087)	(0.088)
Observations	1419	1068	1419	1419
R^2	0.009	0.006	0.244	0.241
Fixed effects	Industry			

Notes: Robust standard errors in parentheses. Significance levels: ^{b} $p < 0.05$, ^{a} $p < 0.01$. Within R^2 are reported. Regressions in columns (2) and (6) only retain significantly positive values of $\widehat{\psi}_i^c$. Regressions in columns (3) and (7) are performed with weight = 1/s.e.($\widehat{\psi}_i^c$). Regressions in columns (4) and (8) are performed with weight = degree of freedom in the first step regression.

country. This result is due to two separate effects: the interaction between relative factor intensity and the industry technology (Proposition 1) and the interaction between relative factor intensity and factor endowments (Proposition 2). These results do not require any assumption about the direction of the technology bias (if any) or about the relationship between productivity and factor intensity (the normalization choice).

Our study contributes to the literature in three ways. First, it show that factor intensity is an importance source of heterogeneity across firms. This source is found to be relevant in determining firms relative sales. Second, heterogeneity in factor intensity makes that comparative advantage matters for within-industry relative sales. Differences in factors relative marginal productivity across firms show up as magnified or dampened by the interaction with comparative advantage. Third, the empirical evidence of this interaction provides - to our knowledge - the first firm-level verification of the Heckscher-Ohlin model.

We have verified empirically the predictions of the model using firm-level data for a large number of countries and industries. The data contains information on capital intensities and total sales for a panel of 445,853 European firms in 87 industries and 26 countries. The econometric analyses compare the influence of firms' capital intensity on their sales across countries-industry pairs characterized by different comparative advantages. We find, as expected, that comparative advantage clearly interacts with firms relative factor intensities in explaining the observed heterogeneity in relative sales. This result is robust to different empirical specifications. The structural estimates corroborate our theoretical conclusions and support our modelling choices. The non-structural estimates confirm that firm-level relative capital intensity is associated with greater market shares in most country-industry pairs. More interestingly, the non structural-estimates dissect the impact of comparative advantage into its two constitutive elements, i.e., industry technology and relative factors proportions. Within a given country, the premium in terms of firms' relative sales of having higher ratio of capital per worker increases sharply with average capital intensity at the industry-level. Whereas the evidence is less striking, we also confirm that the premium is larger in capital abundant countries, within a typical industry.

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10 Appendix.

In this section we provide analytical results and numerical solutions. Analytical result are derived for the model without fix exporting cost whereas we resort to numerical solutions for the model with fix exporting cost.

10.1 Analytical Results.

In this section $F_{ix} = 0$, which implies $\chi_i^c = 1$. Further, to isolate the effect of comparative advantage we eliminate any cross-industry differences in fix cost and trade cost: i.e., $F_i = F$, $F_{ei} = F_e$, and $\tau_i = \tau$ for $i = Y, Z$.

10.1.1 Ranking of cut off values.

We begin by establishing the ranking of cut off values. This will serve in the proof of Propositions 1 and 2. The ranking of cut-off values obtains from the free entry and zero cut-off profit conditions alone. Replacing expressions (23)-(24) and (26) into equation (27) we obtain a single equation which combines the free entry and the zero cut-off profit condition. This condition, henceforth referred to as the free entry zero cut-off profit condition, or FE-ZCP, is

$$\int_{\beta_i^{*c}}^{\infty} \left\{ \left[\frac{mc_i^c(\beta)}{mc_i^c(\beta_i^{*c})} \right]^{1-\sigma} - 1 \right\} g(\beta)\, d\beta = \delta \frac{F_e}{F} \tag{47}$$

To save space in the mathematical passages it is useful to define the integral on the left hand side as:

$$\Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c) \equiv \int_{\beta_i^{*c}}^{\infty} \left\{ \left[\frac{mc_i^c(\beta)}{mc_i^c(\beta_i^{*c})} \right]^{1-\sigma} - 1 \right\} g(\beta)\, d\beta \tag{48}$$

where the notation recalls that the integral is function of the cut-off value of β, of the industry technology (λ_i), and of country relative factors price ($\omega^c \equiv w^c/r^c$) since the marginal cost depends on these three variables. It is clear that $\Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)$ is a monotonic transformation of the value of entry. We note here for future reference the sign of the three partial derivatives of Υ_i^c.

First, $\Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)$ is decreasing in β_i^{*c}:

$$\frac{\partial \Upsilon_i^c(\beta_i^{*c}, \lambda_i, w^c, r^c)}{d\beta_i^{*c}} = \frac{\partial mc(\beta_i^{*c})}{\partial \beta_i^{*}} \frac{(\sigma-1)}{mc(\beta_i^{*})} \Upsilon < 0 \tag{49}$$

where the inequality is due to the fact that the marginal cost is declining in β, i.e., $\frac{\partial mc(\beta_i^*)}{\partial \beta_i^*} < 0$.

Second, $\Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)$ is decreasing λ_i:

$$\frac{\partial \Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)}{d\lambda} = -A\sigma\left[\left(\widetilde{\beta}_i^c\right)^{\sigma-1} - \left(\beta_i^{*c}\right)^{\sigma-1}\right]\left[1 - G\left(\beta_i^{*c}\right)\right] < 0 \qquad (50)$$

where $A = \frac{[\lambda_i(1-\lambda_i)]^{\sigma-1}(\omega^c)^{1-\sigma}}{\left[(\lambda_i)^{\sigma}(\omega^c)^{1-\sigma} + (1-\lambda_i)^{\sigma}(\beta_i^{*c})^{\sigma-1}\right]^2} > 0$ and the sign is due to the fact that $\widetilde{\beta}_i^c > \beta_i^{*c}$.

Third, $\Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)$ is increasing ω_i:

$$\frac{\partial \Upsilon_i^c(\beta_i^{*c}, \lambda_i, \omega^c)}{d\omega} = A(\sigma-1)\left[\left(\widetilde{\beta}_i^c\right)^{\sigma-1} - \left(\beta_i^{*c}\right)^{\sigma-1}\right]\left[1 - G\left(\beta_i^{*c}\right)\right] > 0 \quad (51)$$

We can now establish two lemmas.

Lemma 4 *The $K-$intensive industry has the highest zero-profit productivity cut off. In our notation:*

$$\beta_Y^{*c} > \beta_Z^{*c} \qquad \forall \tau \in [0,1]\ . \qquad (52)$$

Proof. Applying the implicit function theorem to equation (47) gives:

$$\frac{d\beta_i^{*c}}{d\lambda_i} = -\left(\frac{\partial \Upsilon_i^c}{\partial \beta_i^*}\right) / \left(\frac{\partial \Upsilon_i^c}{\partial \lambda}\right) < 0, \qquad (53)$$

which proves the lemma. ■

Lemma 5 *Except in free trade, the $K-abundant$ country has higher zero-profit productivity cut-off in both industries. Further, each cut-off value of the $K-abundant$ country is larger in costly trade than in free trade whereas each cut-off value of the $L-abundant$ country is smaller in free trade than in autarky. In our notation:*

$$\left(\beta_i^{*H}\right)_{Costly\ Trade} \geqslant \left(\beta_i^*\right)_{Free\ Trade} \geqslant \left(\beta_i^{*F}\right)_{Costly\ Trade} \quad \forall i, \qquad (54)$$

with equality holding only in free trade.

Proof. Applying the implicit function theorem to equation (47) gives:

$$\frac{d\beta_i^{*c}}{d\omega_i} = -\left(\frac{\partial \Upsilon_i^c}{\partial \beta_i^*}\right) / \left(\frac{\partial \Upsilon_i^c}{\partial \omega}\right) > 0. \tag{55}$$

Recalling that the $K - abundant$ country has the highest relative price of L (i.e., $\omega^H > \omega^F$) proves the lemma.[16] ■

10.1.2 Proof of Propositions 1 and 2.

Proof of Proposition 1. Using equation (12) into inequalities (37) we obtain

$$\left[\frac{mc_Y^C\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}\right)}{mc_Y^C\left(\widetilde{\beta}_Y^C\right)}\right]^{1-\sigma} \gtrless \left[\frac{mc_Z^C\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}\right)}{mc_Z^C\left(\widetilde{\beta}_Y^C\right)}\right]^{1-\sigma} \quad \text{as } 0 < \varrho \gtrless 1. \tag{56}$$

Replacing the expressions for marginal costs into (56) and rearranging we obtain

$$(\lambda_Y)^\sigma (1-\lambda_Z)^\sigma (\varrho - 1)\left(\widetilde{\beta}_Z^C\right)^{\sigma-1} \lesseqgtr (\lambda_Z)^\sigma (1-\lambda_Y)^\sigma (\varrho - 1)\left(\widetilde{\beta}_Y^C\right)^{\sigma-1} \\ \text{as } 0 < \varrho \gtrless 1 \tag{57}$$

which is satisfied since $\widetilde{\beta}_Y^C > \widetilde{\beta}_Z^C$ from Lemma 4 and $\lambda_Y < \lambda_Z$ from the assumption on factor intensity; therefore, $\frac{\widetilde{\beta}_Z^C}{\widetilde{\beta}_Y^C} < \frac{\lambda_Z}{\lambda_Y}\frac{1-\lambda_Y}{1-\lambda_Z}$.

Proof of Proposition 2. Using equation (12) into inequalities (38) we obtain

$$\left[\frac{mc_i^H\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}_i^H\right)}{mc_i^H\left(\widetilde{\beta}_i^F\right)}\right]^{1-\sigma} \gtrless \left[\frac{mc_i^F\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}_i^F\right)}{mc_i^H\left(\widetilde{\beta}_i^F\right)}\right]^{1-\sigma} \quad \text{as } 0 < \varrho \gtrless 1. \tag{58}$$

Replacing the expressions for marginal costs into (58) and rearranging we obtain

[16] Here we should demonstrate that for any positive level of trade cost the relative price of a factor is higher in the country where that factor is relatively scarce. This may be demonstrated through a few pages of mathematical passages but since it is a rather intuitive and standard result we omit the proof for reason of space.

$$\left(\omega^{F}\right)^{\sigma-1}(\varrho-1)\left(\widetilde{\beta}_{i}^{F}\right)^{\sigma-1} \lesseqgtr \left(\omega^{H}\right)^{\sigma-1}(\varrho-1)\left(\widetilde{\beta}_{i}^{F}\right)^{\sigma-1} \text{ as } 0<\varrho \gtreqless 1 \quad (59)$$

which is satisfied since in costly trade we have $\omega^{H}>\omega^{F}$ and $\widetilde{\beta}_{i}^{H}>\widetilde{\beta}_{i}^{F}$ from Lemma 5; Therefore: $\left(\frac{\widetilde{\beta}_{i}^{F}}{\widetilde{\beta}_{i}^{H}}\right)^{\sigma-1}<\left(\frac{\omega^{H}}{\omega^{F}}\right)^{\sigma-1}$.

10.1.3 Robustness to normalization.

We replace the normalization choice in the text with its alternative to show that results are the same. Let $\beta=1$ and $\alpha \in(0, \infty)$. Combining the free entry and the zero cut-off profit conditions we have an expression similar to equation (47) where the difference is in that α replaces β, that is:

$$\int_{\alpha_{i}^{*}}^{\infty}\left\{\left[\frac{mc_{i}^{c}(\alpha)}{mc_{i}^{c}\left(\alpha_{i}^{*c}\right)}\right]^{1-\sigma}-1\right\} g(\alpha)\, d\alpha=\delta \frac{F_{e}}{F}. \quad (60)$$

Applying the same differentiations as in Lemmas 4 and 5 it may be shown that

$$\begin{aligned} \alpha_{Z}^{c*} &> \alpha_{Y}^{c*}, \qquad \forall \tau \in[0,1] \quad (61)\\ \alpha_{i}^{H*} &< \alpha_{i}^{F*}, \qquad \forall \tau \in(0,1). \quad (62)\end{aligned}$$

Let $\underline{\theta}_{i}^{c}(\alpha) \equiv\left[\theta_{i}^{c}(\alpha)\right]^{-1}$ denote firm $L-intensity$. Then, the average $L-intensity$ in the industry is $\overline{\underline{\theta}}_{i}^{c}\left(\alpha_{i}^{*}\right)=\left(\frac{r^{c}}{w^{c}} \frac{\lambda_{i}}{1-\lambda_{i}}\right)^{\sigma}\left(\widetilde{\alpha}_{i}^{c}\right)^{\sigma-1}$

where $\widetilde{\alpha}_{i}^{c}=\left[\frac{1}{1-G\left(\alpha_{i}^{c*}\right)} \int_{\alpha_{i}^{c*}}^{\infty} \alpha^{\sigma-1} g(\alpha)\, d\alpha\right]^{\frac{1}{\sigma-1}}$.

Robustness of Proposition 1. Proposition 1 requires that for any $\alpha_{i}^{\prime}$ such that $\underline{\theta}_{i}^{c}\left(\alpha_{i}^{\prime}\right)=\varrho \overline{\underline{\theta}}_{i}^{c}\left(\widetilde{\alpha}_{i}\right)$ we have

$$\frac{s_{Y}^{c}\left(\alpha^{\prime}\right)}{s_{Y}^{c}\left(\widetilde{\alpha}_{Y}\right)} \gtreqless \frac{s_{Z}^{c}\left(\alpha^{\prime}\right)}{s_{Z}^{c}\left(\widetilde{\alpha}_{Z}\right)} \text{ as } 0<\varrho \lesseqgtr 1. \quad (63)$$

which is satisfied since $\alpha_{Z}^{c*}>\alpha_{Y}^{c*}$ and $\lambda_{Y}<\lambda_{Z}$. To see this it suffices to follow the same steps as in the proof of Proposition 1. Inequality (63) says that for any two firms with same larger-than-average $K-intensity$ the firm in the $K-intensive$ industry has larger relative sales (analogously for $L-intensity$ and $L-intensive$), which is Proposition 1.

Robustness of Proposition 2. Robustness of 2 requires that for any α_i' such that $\underline{\theta}_i^c(\alpha_i') = \varrho\overline{\underline{\theta}}_i^c(\widetilde{\alpha}_i^c)$ we have

$$\frac{s_i^H(\alpha')}{s_i^H(\widetilde{\alpha}_i^c)} \gtrless \frac{s_i^F(\alpha')}{s_i^F(\widetilde{\alpha}_i^c)} \text{ as } 0 < \varrho \lesseqgtr 1. \tag{64}$$

which is satisfied since $\alpha_i^{*H} < \alpha_i^{*F}$and $\omega^H > \omega^F$. To see this it suffices to follow the same steps as in the proof of Proposition 2. Inequality (64) says that for any two firms with same larger-than-average $K - intensity$ the firm in the $K - abundant$ country has larger relative sales (analogously for $L - intensive$ and $L - abundant$), which is Proposition 2.

10.1.4 No average factor intensity reversal.

Lemma 4 implies no average factor intensity reversal as a corollary. From equation (7) we see that the average $K - intensity$ is higher in the industry whose technology is $K - intensive$:

$$\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{1-\lambda_Y}{\lambda_Y}\right)^{\sigma} \left(\frac{1-\lambda_Z}{\lambda_Z}\right)^{-\sigma} \left(\frac{\widetilde{\beta}_Y}{\widetilde{\beta}_Z}\right)^{\sigma-1} > 1. \tag{65}$$

In fact, if firms were homogenous or heterogeneity Hicks-neutral the ratio of $K - intensities$ would simply be $\left(\frac{1-\lambda_Y}{\lambda_Y}\right)^{\sigma}\left(\frac{1-\lambda_Z}{\lambda_Z}\right)^{-\sigma} > 1$ since $\lambda_Y < \lambda_Z$. With heterogeneity in factors RMP the no-factor-intensity-reversal holds *a fortiori* since Lemma 4 establishes that $\widetilde{\beta}_Y^{*c} > \widetilde{\beta}_Z^{*c}$.

10.1.5 The Four Core Theorems.

The four core-theorems of international trade (Stolper-Samuelson, Rybczynski, Factor Price Equalization, Heckscher-Ohlin) remain valid when heterogeneity is in factors RMP but - compared to a model where heterogeneity is Hicks-neutral - their intensity is affected. Recall that, with regard to the four core theorems, a Hicks-neutral heterogeneity model is equivalent to a homogenous firms model due to the fact that the cut off values depend neither on factors price nor on factor intensity. The effect of RMP heterogeneity on the Stolper-Samuelson and Rybczynski theorems depends on the direction of the bias of the average factor intensity whereas the effect on the size of the

FPE set and on Heckscher-Ohlin specialization does not.[17] Writing the closed economy (or integrated equilibrium) system in the canonical Jones' (1965) form and by applying "Jones Algebra" we obtain the following results:

(1). The Stolper-Samuelson and Rybczynski magnification effects are attenuated (amplified) if the ex-post average factor intensity is $K-biased$ $(L-biased)$.

(2). The FPE set is expanded by heterogeneity in RMP. This can bee seen in inequality (65) which shows that the diversification cone is larger if heterogeneity results from differences in factors RMP than if it results from differences in TFP since $\left(\frac{\widetilde{\beta}_Y}{\widetilde{\beta}_Z}\right)^{\sigma-1} > 1$.[18] The expansion of the FPE does not depend on the normalization choice or on the direction of the factor bias. Changing the normalization we have $a_Z^* > a_Y^*$ and $\frac{\overline{\underline{\theta}}_Y^c}{\overline{\underline{\theta}}_Z^c} = \left(\frac{1-\lambda_Y}{\lambda_Y}\frac{\lambda_Z}{1-\lambda_Z}\right)^{\sigma}\left(\frac{\widetilde{\alpha}_Z}{\widetilde{\alpha}_Y}\right)^{\sigma-1} > \left(\frac{1-\lambda_Y}{\lambda_Y}\frac{\lambda_Z}{1-\lambda_Z}\right)^{\sigma} > 1$.

(3). The Heckscher-Ohlin specialization occurring when moving from autarky to free trade is attenuated regardless of the ex-post bias. The attenuation is asymmetric: it is stronger (weaker) for the $L-abundant$ $(K-abundant)$ country when the ex-post average factor intensity is *K-biased*, vice versa when the ex-post average factor intensity is *L-biased*.

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[17] If heterogeneity is Hicks-neutral, the average factor intensity is $\left(\frac{w^c}{r^c}\right)^{\sigma}\left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma}$ $\forall i, c$. If heterogeneity is in factors RMP the average factor intensity is as given in expression (7) and exhibits a bias even if the technology is assumed to be neutral on ex-ante average factor intensity; i.e., if $\int_0^{\infty}(\beta)^{\sigma-1} g(\beta)\, d\beta = 1$. In such case and if all firms could survive in the market the average factor intensity would be exactly $\left(\frac{w^c}{r^c}\right)^{\sigma}\left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma}$ $\forall i, c$. Yet, because of selection into entry a factor bias emerges ex post (a $K-bias$ in this case) since $\widetilde{\beta}_i^c > 1$ even if $\int_0^{\infty}(\beta)^{\sigma-1} g(\beta)\, d\beta = 1$. Naturally, one could impose such a low average value of β that the resulting $\widetilde{\beta}_i^c$ are all smaller than 1. In such case average factor intensity would be $L-biased$. The results in our model do not depend on the average value of β and, therefore, do not depend on the bias of the technology. Nor they depend on the normalization choice. The average factor-bias, however, determines in which way heterogeneity influences the intensity of the Stolper-Samuelson and Rybczynski magnification effects.

[18] In a two-by-two setting the size of the FPE set is increasing with the size of the diversification cone.

10.1.6 Robustness of Proposition 3 to $F_X > 0$

Proposition 3 remains valid when $F_x > 0$. This is proven by observing that by use of expressions (12) we obtain

$$\frac{s_{i\zeta}^{c}(\phi')}{s_{i\zeta}^{c}\left(\widetilde{\phi}_i^c\right)} = \rho^{\sigma-1} \qquad \zeta = d, x; \quad c = H, F; \quad i = Y, Z. \tag{66}$$

10.2 Numerical simulations with $F_x > 0$.

In this section we solve the model numerically in order to verify the validity of Propositions 1 and 2 in the presence of fix exporting cost. There are fifteen parameter values to be assigned in order to solve the model numerically and, of course, a large number of possible combinations. The only requirement on parameters is that they must be such that no firm is an exporting firm without also selling in the domestic market. This condition means that the resulting zero exporting profit productivity cut off must be no less than the zero profit productivity cut off. This is hardly restrictive given the large number of parameters. The only guidance to the choice of parameters concerns σ. In accordance to empirical estimates of the substitution elasticity and to our own results we assign to σ values that range between 2 and 6. Concerning size, preferences, and factors proportions we have chosen to assume symmetry: goods are equally liked ($\gamma_Y = \gamma_Z = 1/2$) and countries have symmetric differences in endowments with H being the $K-$*abundant* country ($\nu_K^H = \nu_L^F$, and $\nu_L^H = \nu_K^F$, with $\nu_K^H = 0.55$ and $\nu_L^H = 0.45$). Good Y is $K-$*intensive* and we have chosen symmetry in technology, $\lambda_Y = (1 - \lambda_Z) = 0.4$. World endowments are $\overline{K} = 2200$ and $\overline{L} = 2200$. Variable trade cost τ take values that range from 0 to 1 at interval of 0.2, that is: $\tau = \{0,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1\}$.[19] There is no empirical guidance as to the value of the three types of fix costs. As a representative example of many simulations we show the results for $F = 0.6$, $F_x = 0.4$, and $F_e = 0.2$. Lastly we assume $g(\beta)$ to be Pareto with lower bound β_M and shape parameter $k > 1$:

$$g(\beta) = \frac{k\beta_M^k}{\beta^{k+1}}, \quad \beta \in [\beta_M, \infty] \tag{67}$$

[19] In passing we mention that $\tau = 0$ corresponds to autarky while $\tau = 1$ does not correspond to free trade since there are fix exporting cost $F_x > 0$.

The parameter k is chosen consistently with σ in such a way that all the integrals in the model converge. The value of β_M is irrelevant but we may choose it in such a way that the ex-ante average factor intensity is the same as if there where no heterogeneity. This is done by assuming that the ex-ante harmonic average of β is equal to $\alpha^{\sigma-1}$, i.e., $\int_{\beta_M}^{\infty} (\beta)^{\sigma-1} g(\beta)\, d\beta = \alpha^{\sigma-1}$ which gives endogenously the value of β_M. For instance, with $\sigma = 3$ and $k = 4$ we obtain $\beta_M = 0.7071067810$. In this way if all firms were able to survive in the market the average $K - intensity$ would be the same as if there were no heterogeneity, that is, equal to $\left(\frac{w^c}{r^c}\right)^{\sigma} \left(\frac{1-\lambda_i}{\lambda_i}\right)^{\sigma}$. Naturally, nothing hinges on this particular parametrization. A final check is that all the zero profit productivity cut off resulting from the simulations must be at least as large of β_M.

With positive fix export cost we have to distinguish between domestic and total sales. To decide which of them is relevant for our purposes we should recall the logic of our propositions. Both propositions come from the result that the comparative advantage influences the relative marginal cost of production. Incidentally, this is apparent from inequalities 56 and 58 which restate Propositions 1 and 2 in terms of relative marginal cost. Indeed each of these inequalities written in terms of relative marginal costs implies and is implied by the corresponding inequality written in terms of relative domestic sales. This can be seen by simply replacing equation (12) for domestic sales into inequalities (37) and (38) to obtain, respectively:

$$\begin{gathered}\left[\frac{mc_Y^c\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}\right)}{mc_Y^c\left(\widetilde{\beta}_Y^c\right)}\right]^{1-\sigma} \gtrless \left[\frac{mc_Z^c\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}\right)}{mc_Z^c\left(\widetilde{\beta}_Y^c\right)}\right]^{1-\sigma} \\ \Leftrightarrow \frac{s_{Yd}^c(\beta')}{s_{Yd}^C\left(\widetilde{\beta}_Y^c\right)} \gtrless \frac{s_{Zd}^c(\beta')}{s_{Zd}^C\left(\widetilde{\beta}_Z^c\right)}\end{gathered} \tag{68}$$

$$\begin{gathered}\left[\frac{mc_i^H\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}_i^H\right)}{mc_i^H\left(\widetilde{\beta}_i^F\right)}\right]^{1-\sigma} \gtrless \left[\frac{mc_i^F\left(\varrho^{\frac{1}{\sigma-1}}\widetilde{\beta}_i^F\right)}{mc_i^H\left(\widetilde{\beta}_i^F\right)}\right]^{1-\sigma} \\ \Leftrightarrow \frac{s_{id}^H(\beta')}{s_{id}^H\left(\widetilde{\beta}_i^c\right)} \gtrless \frac{s_{id}^F(\beta')}{s_{id}^F\left(\widetilde{\beta}_i^c\right)}\end{gathered} \tag{69}$$

Therefore, to verify that when $F_x > 0$ the effect of comparative advantage on relative marginal cost is as predicted by the model we have to verify Propositions 1 and 2 written in terms of domestic sales.

We show here a representative example of the many simulations. Figure 2 relates to Proposition 1. It shows domestic sales relative to industry average for industry Y and Z, i.e., $\frac{s^C_{Yd}(\beta')}{s^C_{Yd}\left(\widetilde{\beta}\left(\beta^{*c}_Y\right)\right)} \gtrless \frac{s^C_{Zd}(\beta')}{s^C_{Zd}\left(\widetilde{\beta}\left(\beta^{*c}_Z\right)\right)}$: Panel (a) shows it for country H and panel (b) does it for country F. As predicted by Proposition 1 the relative sales of industry Y are above those of industry Z in any country when $\rho > 1$ while they are below when $\rho < 1$. Figure 3 refers to Proposition 2. It shows domestic sales relative to the industry average for country H and F, i.e., $\frac{s^H_{id}(\beta')}{s^H_{id}\left(\widetilde{\beta}(\beta^{*c}_i)\right)} \gtrless \frac{s^F_{id}(\beta')}{s^F_{id}\left(\widetilde{\beta}(\beta^{*c}_i)\right)}$: Panel (a) does it for industry Y and panel (b) does it for industry Z. As predicted by Proposition 2 the relative sales in country H are larger than those in country F in any industry when $\rho > 1$ while they are below when $\rho < 1$. In conclusion, numerical simulations confirm that the comparative advantage interacts with firms characteristics in determining relative marginal cost of production (and relative domestic sales) giving a relative advantage to firms whose factor intensity matches up with the comparative advantage of the country.

Figure 2: Domestic sales relative to country average for industries Y and Z

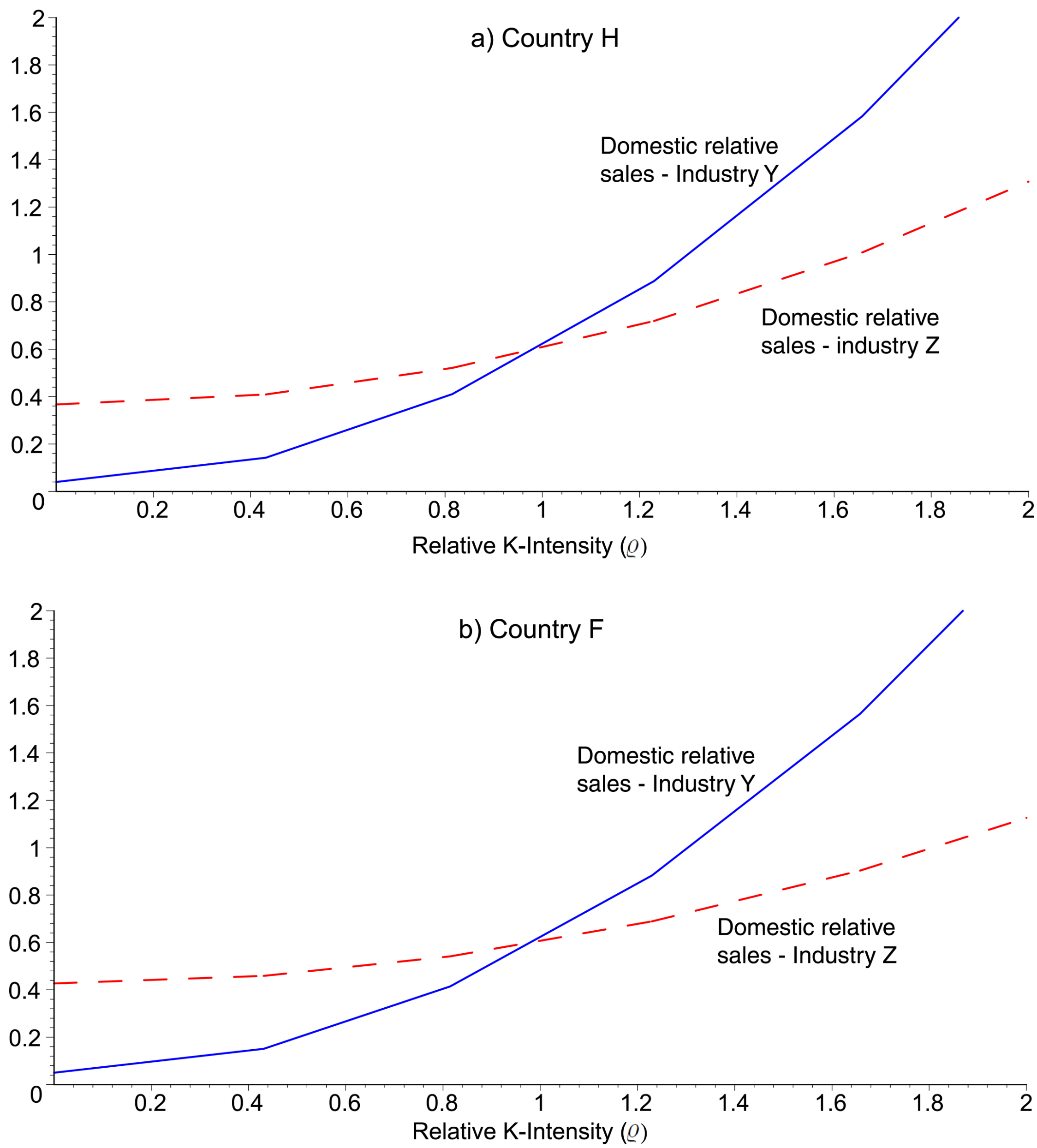

Figure 3: Domestic sales relative to country average for countries H and F