The Composition of Knowledge and Long-Run Growth

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Abstract

Technologies differ in their scopes of applications. The types of knowledge a country possesses have important implications on its growth. This paper develops a multi-sector model of innovation, trade and growth, in which knowledge in one sector is applicable to innovation in another sector in various degrees and a country's composition of knowledge is endogenously determined. We find that lower trade costs improve aggregate innovation efficiency through the within-country allocation of R&D towards sectors with higher knowledge applicability, demonstrating a "composition effect". We construct measures quantifying the sectoral knowledge *applicability* using cross-sector patent citations. Based on this index, we present cross-country evidence that broadly supports the model's implications.

Keywords: Intersectoral knowledge linkages; Knowledge composition; R&D; Endogenous growth; Technology space; Trade costs

JEL Classification: O30, O40, F43.

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1 Introduction

Long-run economic growth is accompanied and fueled by technological advances. New knowledge creation is often built on prior knowledge from various areas. While some knowledge can be readily adapted to make new products in many other sectors, others are limited in their scope of application. When the interconnections between knowledge from different sectors are intrinsically asymmetric, it is not just the amount of knowledge capital a country possesses that matters for growth, but also its *composition of knowledge*. The latter, however, has been largely absent from the growth literature, especially in theoretical models.

This paper incorporates such a network of knowledge complementarities across sectors into a model of innovation, trade and growth, and develops a tractable framework where a country's composition of knowledge is endogenously determined. The framework is useful to analyze (a) the role of external trade environment in directing the allocation of R&D resources and shaping a country's knowledge composition, and (b) the channels through which the knowledge composition affects growth. Lower trade costs—besides leading to more trade as in conventional models—improve aggregate innovation efficiency and long-term growth through the within-country allocation of R&D investment towards sectors with higher knowledge spillovers. We then present cross-country evidence that broadly supports the model's implications. First, we use data on cross-sector patent citations as an empirical proxy for intersectoral knowledge complementarity, and construct a quantitative measure of *'knowledge applicability'* for each sector. Based on this measure, we show that countries that are geographic remote (higher trade costs) tend to export disproportionately less in highly knowledge applicable sectors. A country's initial knowledge applicability (as revealed through its exports) is also found to bear a statistically and economically significant relationship to subsequent growth differences.

Understanding the economic forces driving a country's composition of knowledge and its implications for growth is informative about the efficacy of policy interventions. Traditional trade theory argues that welfare is maximized when countries specialize in sectors that they can produce relatively less costly. Yet, it is also understood that in the presence of intersectoral externality, goods with large positive externality may be under-developed as the spillovers to the whole economy is not reflected in their own profitability. It has been long debated whether growth-promoting industrial policies should foment sectors with likely externalities.[1] The existing empirical evidence is, however, mixed. Producing technologically sophisticated goods appears to offer growth benefits

[1]It is captured by the debate over the comparative advantage defying strategies (Lin 2009, 2010), referring to government-led industrial policies that direct resources to technologically sophisticated industries without paying attention to their existing comparative advantages. It is based on the observation that economies that are more complex than their level of income would suggest have a tendency to catch up with a spurt of rapid growth.

according to Hausmann, Hwang and Rodrik (2007), and producing goods that have strong synergy with each other and are "close" to potential new products in the product space improves growth in Hidalgo, Klinger, Barasasi and Hausmann (2007), Hausmann and Klinger (2007) and Kali, Reys, McGee and Shirrell (2012). Lederman and Maloney (2012) and Wang, Wei and Wong (2010), on the other hand, provide a dissenting view after considering other omitted variables or alternative characterizations of a country's product structure.

The dispute perhaps is an outcome of the following factors. First, it is difficult to establish causality using the commonly adopted regression-based approaches. After all, growth may actually drive structural changes and provides the means to promote sectors with larger externalities. Second, it is difficult to examine general equilibrium effects of changing technology mix using these approaches. Lastly, it is difficult to categorize sectors and identify those with large externalities in the data. The prevailing approach adopts outcome-based indicators to indirectly infer the interconnections between sectors (and hence, their externalities). For example, previous studies mentioned above assume that two industries have synergy if they are frequently exported together by the same country. The network resulting from this approach is "undirected" (e.g. if i is closely related to j then j must be closely related to i), and more importantly, it can be an outcome of confounding external forces. For example, one sector may appear to have synergy with another if they demand similar infrastructure or resources, even though its development does not inherently benefit the other.

We focus upon a particular source of externality in this paper—the applicability of knowledge embodied in a specific sector in the process of creating new knowledge in others. We develop a theory in which the concept of knowledge interconnections can be made more precise, and identify a novel channel through which trade environment may generate cross-sector variations in knowledge accumulation.[2]

The analytical framework developed in this paper is a variety expanding model with firms innovating simultaneously in multiple sectors to internalize inter-sectoral knowledge complementarities. We interpret the variations in these complementarities across sectors as a result of intrinsic characteristics of technologies or the state of exogenous scientific knowledge at a particular point in time. In the model, the exogenous inter-sectoral knowledge complementarities govern the productivity of research effort when adapting knowledge in one sector to innovate in another. When firms choose R&D optimally in such a setting, the equilibrium value associated with a new innovation is determined not just by its own future profit, but also by its *application value* as a knowledge supplier to

[2]Throughout the paper, we use the terms 'technology' and 'sector' interchangeably. In the model, one sector embodies one type of technology. In the empirical analysis, detailed technological categories are aggregated into larger industrial sectors in order to be combined with the export data.

chains of innovation in downstream technologies. Trade costs reduce profit in highly applicable center sectors more than periphery sectors in the knowledge diffusion network because center sectors export more in equilibrium and thus are more sensitive to trade conditions. The loss of profitability directly discourages innovation in these sectors. Since new knowledge is built on previous knowledge from various sectors, the reduction in the stock of knowledge that can be applied lowers the application value of these upstream technologies to an even greater extent than it affects downstream technologies, further deterring their R&D investment. Therefore, the model predicts that trade costs disproportionately hinder knowledge accumulation in highly applicable technologies. Since these technologies foster subsequent innovations in many different sectors, underinvestment in these sectors generates large multiplier effects in a path-dependent world, impeding overall growth.

The model predicts the following testable outcomes. First, in the presence of asymmetric knowledge complementarities across sectors, an additional "composition effect" of trade costs emerge: Symmetric increase in trade costs in all sectors lead to disproportionately less accumulation of knowledge, and thus less production, in the knowledge highly applicable sectors. Second, countries whose knowledge composition is more applicable tend to grow faster in the balanced growth path equilibrium.

The empirical challenge is how to quantify a sector's knowledge applicability. Technology complementarity is conceptual and not directly observable. We infer the network of knowledge interconnections across sectors based on cross-sector patent citations data provided by U.S. Patent and Trade Office, which contains information related to patents applied by inventors from all over the world. Patent citations, albeit some noises, contain information on which prior technologies are used and in what intensity in the innovation process of other technologies; hence, allow us to directly uncover the intrinsic knowledge linkages between sectors (Hall, Jaffe and Trajtenberg, 2001). By drawing upon methods developed in the complex networks literature (Kleinberg's algorithm), we construct a measure called "applicability" that allows us to evaluate, for each sector, its importance as a knowledge source to its chains of downstream application sectors.[3] These measures are meant to capture technological characteristics of a given sector that are exogenous from the perspective of an individual country or firm and innate to the nature of the knowledge creation process.

Under the further assumption that such technological nature of sectors carry over to all countries, we examine whether the model generated equilibrium outcomes hold true in the data. However, describing a country's knowledge composition is difficult, as it is not possible, with available

[3]Our knowledge applicability measures are both conceptually and empirically distinct from the product sophistication measure of Hidalgo and Hausmann (2009). Although most sectors with lower applicability produce simple products (e.g. food and kindred products, primary ferrous products), some of these sectors actually produce complicated products (e.g. transportation equipment, aircraft, guided missiles and space vehicles) but the knowledge in these sectors could be too specific to have pervasive applications.

data, to directly observe a country's composition of knowledge.[4] What we can observe are only various economic manifestations of the country's progress in knowledge accumulation. For example, it is reasonable to assume that making a product requires specific types of knowledge. Countries can acquire the knowledge to make a product that they did not invent by other kinds of activities, such as learning-by-doing, imitating, or even simply replicating. Therefore, what a country produces captures information regarding what it knows. For this reason, we follow the insight of the previous literature on product space by evaluating a country's knowledge composition by its export composition, for which rich comparable data are available for a large set of countries.[5]

We find that countries that are geographically farther away from the rest of the world (higher trade costs) tend to specialize more on highly applicable sectors, controlling for other factors (such as endowment of human capital, physical capital, natural reserve, etc). We then calculate summary measures of the knowledge applicability of a country's export portfolio, which are used as indicators of how productive it is for a given country to apply its existing knowledge to create new products. We find that a country's initial knowledge applicability is significantly and positively related to subsequent growth differences. This relationship is robust to controlling for a large set of covariates, including initial per capita income, human capital, investment, diversification, institutional quality, etc. This relationship, however, may not reflect an causal effect of knowledge composition on growth since the pattern of specialization itself is endogenous, as predicted by our model. But this empirical finding does provide evidence supporting the mechanism illustrated in our model.

Relating to the Literature The paper contributes to several streams of literature. First, previous empirical studies examine whether a country's *overall* R&D investment affects its subsequent growth, and do not find significant relationships (Klenow and Rodriguez-Clare 2005). This paper argues that if inter-sectoral knowledge complementarities are heterogenous and their distribution is highly skewed—as they are in the data—then even though the average R&D level may not matter, the allocation of innovation effort and knowledge across sectors affects growth. This paper thus contributes to a growing literature emphasizing that a country's product composition plays an important role in its economic performance (e.g. Hausmann et al. 2007; Hidalgo et al. 2007; Koren and Tenreyro 2007; Kali et al. 2009; Nunn and Trefler 2010; Hausmann and Hidalgo 2011). Unlike existing empirical research in the context of innovation and growth—which typically distinguishes

[4] For countries that have patented in the U.S., we observe the distribution of the country's patents over technological classes and can potentially use this data to characterize the technological position of the country. Unfortunately, this information is available for a much smaller number of countries. In addition, for our purpose it is not just a country's innovation output that matters, it is also the knowledge that accumulated through ....XXX.

[5] This approximation was also adopted in previous papers such as Lall et al. (2005), Hidalgo et al. (2007), Hausman and Hidalgo (2011) and Kali et al. (2012).

sectors by their technology intensity[6]—we focus on the role of explicit knowledge linkages between sectors in innovation, yielding new insights on why some countries are substantially richer than others.

Second, recent interest has re-emerged in examining the contribution of inter-sectoral linkages to growth. An earlier literature in development economics has argued that linkages across sectors—the vertical input-output relationships in production, in particular—can be central to economic performance (e.g. Leontief 1936; Hirschman 1958). These insights have recently been incorporated into modern macro models with far-reaching implications (e.g. Ciccone 2002; Jones 2011a, 2011b; Acemoglu et al. 2012; Blonigen 2012). Unlike these studies, we explore the inter-sectoral linkages dictated by the knowledge content of sectors, which is more suitable for understanding the mechanics of technological progress and have not been previously explored in a cross-country study. One exception is Cai and Li (2012), which develops a closed-economy multi-sector model with inter-sectoral knowledge spillovers and pays particular attention to the dynamic decisions of heterogeneous firms in the technology space.[7]

Third, in the broader scheme of things this paper joins the large literature on the growth implications of trade. Recent research explains the gains from trade via reallocations across economic units with heterogenous productivities (i.e. sectors, firms or products) (e.g. Melitz (2002), Arkolakis, et al. (2008) and Arkolakis, et al. (2010)). In this paper we find that once heterogeneous knowledge externalities are taken into account, the reallocation of R&D across sectors affects innovation and growth. Trade costs, therefore, through reallocation effects, have significant growth impact beyond the effects that have been stressed in the previous literature.[8]

The rest of the paper proceeds as follows. Section 2 introduces the model, discusses characteristics of the general equilibrium and solves the model. Section 3 describes the construction of our measure of knowledge applicability and presents the empirical findings using cross-country sectoral trade data. Section 4 concludes and discusses policy implications and potential future research.

[6]The technology intensity of a sector is typically characterized by input measures of innovation—such as research and development as a share of sales and the employment share of scientists and engineers in total—or output measures—such as the number of patents taken out by the sector.

[7]Another exception is the literature of general purpose technologies, such as Jovanovic and Rousseau (2005), Helpman (1998) and Bresnahan and Trajtenberg (1995). However, there are no explicit linkages between different technologies.

[8]Another related literature studies the differential responses of trade components to trade liberalization. For example, Hillberry and Hummels (2002) (2008) show that the trade volume of intermediate inputs at the early stage of the production chain are more sensitive to shipping cost.

2 The Model: R&D Allocation and Growth

This section presents an open-economy model of multi-sector growth to illustrate (a) how trade costs and and institutional factors affect firms' optimal cross-sector allocation of research resources, hence the composition of knowledge in the economy; and (b) how the composition of knowledge matters for growth. Our theoretical framework extends the traditional model of endogenous growth (e.g. Romer 1990; Grossman and Helpman, 1991) to allow for knowledge interconnections between different sectors. It is closely related to Cai and Li (2014), which develops a closed-economy general equilibrium model of heterogeneous firm innovation in multiple sectors. This paper studies an open-economy, and abstracts from heterogeneity across firms to focus on the aggregate implications.

The world is made up of home country and the rest of the world (henceforth, RoW) that each produces and consumes varieties of a finite but expandable number of product categories. Trade is induced by the "love for varieties". Home market is relatively small, thus unable to affect foreign innovation specialization, but prices are flexible and firms optimally determine prices of their exports in the foreign market. Both Home and the RoW each has a given supply of a single primary factor, labor, and engage in two activities—production and R&D, and consumers over the world share the same preference. Since the interest of this paper is innovation, we assume that identical labor productivity in production in every sector. Home and the RoW differ in sizes (population), production productivity and trade costs.

In reality, one country may be intrinsically better at producing some products than others—either due to better relative endowment of certain input factors or differences in contract environment (Nunn 2007), financial systems (Beck 2003, Manova, 2008), and labor market frictions (Helpman and Itskhoki, 2010) which have *direct* disproportionate impact on different sectors. The model, however, abstracts from these existing sources of comparative advantage, and instead focuses on endogenous comparative advantages. We will show, in the following sections, that because knowledge upstream sectors provide large knowledge capital to downstream sectors the effects on knowledge downstream sectors induced by changes in economy-wide factors (such as trade costs, labor productivity, population) accumulate to upstream sectors, generating multiplier effects.

In the following sections, we describe the production and innovation decisions in the home country, given wage, productivity, labor supply and innovation activities in the RoW. Variables with an asterisk are the RoW counterparts of home variables. We will focus on the Balanced Growth Path Equilibrium (hereafter BGP) in which aggregate variables grow at constant rates.

2.1 Goods Demand and Production

A representative household inelastically supplies labor and orders its preference over consumption streams of a single final good according to $U = \sum_{t=0}^{\infty} \beta^t \log C_t$. It has access to a one-period risk-free bond with interest rate r_t and in zero aggregate supply. Optimal intertemporal substitution of consumption implies:

$$1 + r_t = \beta \frac{C_{t+1} P_{t+1}}{C_t P_t}. \tag{1}$$

The final consumption (C_t) is a Cobb-Douglas combination of sectoral products, indexed by $i = 1, 2, ...K$; here K represents the total number of sectors.

$$\log C_t = \sum_{i=1}^{K} s^i \log Q_t^i, \tag{2}$$

where s^i governs the share of income spent in that sector, and Q^i is a CES aggregate of differentiated products denoted by k,

$$Q_t^i = \left(\int_0^{N_t^i + N_t^{i*}} x_t^i(k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

$N_t^i + N_t^{i*}$ is the number of varieties available worldwide, which comprises varieties produced by the home country in sector i (N_t^i), and those produced by the RoW (N_t^{i*}). The elasticity of substitution between any two varieties from the same sector is governed by $\sigma (> 1)$. The corresponding sectoral price index is $P_t^i = \left(\int_0^{n_t^i + n_t^{i*}} p_t^i(k)^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}$. The demand function for varieties within a sector is thus given by

$$x_t^i(k) = \left(\frac{p_t^i(k)}{P_t^i} \right)^{-\sigma} Q_t^i \tag{4}$$

where $Q_t^i = s^i E_t / P_t^i$ and E_t is the country's final consumption expenditure.

There is a continuum of symmetric *multi-sector* firms with a total mass of M_t in Home country. Once enter the economy, the representative firm innovates and produces goods in all sectors and engages in monopolistic competition in the product market in each sector. To focus on the heterogeneity of knowledge applicability across sectors, we assume that sectors within a country do not differ in their production productivity. Home (foreign) firms hire one unit of labor to produce ϕ (ϕ^*) units of goods in each sector. The home production function is given by $y_t^i(k) = \phi l_t$.

Prices can differ across countries due to trade costs, represented by $\tau (> 1)$. Let trade costs take the standard "iceberg" form: for one unit of a variety to arrive in the foreign country, τ units must be shipped. Importantly, we note that τ does not vary across sectors; hence, there is no explicit policy bias towards any sector. Given the wage, w_t, monopolistic competitive prices for the domestic market (p^i_{ht}) and foreign markets (p^i_{ft}) follow the usual fixed-markup pricing rule,

$$p^i_{ht}(k) = p^i_{ht} = \frac{\sigma}{\sigma - 1}\frac{w_t}{\phi} \quad \text{and} \quad p^i_{ft}(k) = \tau p^i_{ht}. \tag{5}$$

Competition in the final-good sectors ensures marginal-cost pricing. Hence, the home sectoral price index is given by $P^i_t = [N^i_t(p^i_{ht})^{1-\sigma} + N^{i*}_t(p^{i*}_{ht})^{1-\sigma}]^{1/(1-\sigma)}$, where $p^{i*}_{ht} = \frac{\sigma}{\sigma-1}\frac{w^*_t}{\phi^*}\tau^*$ is the domestic price of foreign sector-i products. Because preferences across countries are identical, home and foreign country consumers purchase the exact same home and foreign produced varieties, although in different quantities. Therefore, the foreign sectoral price index is $P^{i*}_t = [N^i_t(p^i_{ft})^{1-\sigma} + N^{i*}_t(p^{i*}_{ft})^{1-\sigma}]^{1/(1-\sigma)}$.

The revenue per variety sold in the domestic market is $r^i_{ht} = \left(\frac{p^i_{ht}}{P^i_t}\right)^{1-\sigma} s^i E_t$, and in the foreign market $r^i_{ft} = \left(\frac{p^i_{ft}}{P^{i*}_t}\right)^{1-\sigma} s^i E^*_t$, where E_t (E^*_t) is home (foreign) total consumption expenditure. Based on this, we can derive the profit in real terms (using wage as numeraire) of the home average firm in sector i as

$$\pi^i_t = \frac{(r^i_{ht} + r^i_{ft})n^i_t}{\sigma w_t},$$

where $n^i_t = N^i_t/M_t$ is the average number of varieties per firm in sector i. The firm's demand for production workers in sector i is $L^p_t = (\sigma - 1)\pi^i_t$.

2.2 R&D Allocations Over Multiple Sectors

Economic growth is driven by firms' innovation associated with the development of new blueprints (new varieties). It is reflected in the CES aggregation (3), which introduces a "love-for-variety" effect (as in Ethier, 1982). This section describes the endogenous evolution of the number of varieties over time.

The representative multi-sector firm is defined by a vector of its differentiated products in all K sectors, $\boldsymbol{z}_t = (z^1_t, z^2_t, ..., z^K_t)$, where z^i_t is the number of varieties produced by this firm in sector i. New technologies or new varieties are introduced through an innovation process, and each new variety is then turned into a product under monopolistic competition in the next period. Since only the firm inventing the variety has the right to manufacture it, $\boldsymbol{z}_t$ also characterizes the distribution of the firm's knowledge capital across sectors. For simplicity, we assume that the firm's knowledge capital accumulates in every sector without depreciation:

$$z^i_{t+1} = z^i_t + \Delta z^i_t, \quad \text{for } \forall i. \tag{6}$$

where the new knowledge capital, Δz^i_t, is created by employing researchers and utilizing existing knowledge capital. What marks this paper from the existing literature is that the firm can adapt its accessible knowledge from *all* sectors and fully internalize knowledge spillovers across sectors in their innovation process.[9] Since knowledge spillovers are heterogeneous across sectors, we decompose the firm's R&D investment in a given target sector according to its knowledge source sector.[10] Let R_t^{ij} denote a firm's R&D input when applying sector j's knowledge to generate new knowledge in sector i. The productivity of R&D associated with this activity depends on the (exogenous) knowledge linkages—the knowledge linkage from j to i, A^{ij}. Similar to Klette and Kortum (2004), we assume that new knowledge in sector i is created based on a Cobb-Douglas combination of innovation productivity, R&D investment and the existing stock of knowledge capital:

$$\Delta z_t^i = \sum_{j=1}^{K} A^{ij} \left(n_t^i R_t^{ij}\right)^{\alpha} \left(z_t^j\right)^{1-\alpha}, \tag{7}$$

where α is the share of R&D input in the knowledge creation process. Here, we allow for knowledge externality across firms within the country to some extent: the researchers' R&D efficiency is assumed to be proportional to the average knowledge per firm in the innovating sector, n_t^i.[11] Note that for simplicity and to keep our focus on cross-sector diffusion, we assume that knowledge is not diffused directly across countries in this model.

Since each variety is sold in the same quantity and priced at the same level, the profit per variety in sector i is given by $\frac{\pi_t^i}{n_t^i}$. A firm with a knowledge portfolio of $\boldsymbol{z}_t$ in period t thus receives a flow of total profit $\sum_{i=1}^{K} \frac{\pi_t^i}{n_t^i} z_t^i$ in the product market. It chooses an R&D investment portfolio $(R_t^{ij})_{i,j\in J}$ to maximize its present value $V(\mathbf{z}_t)$, given the interest rate $\tilde{r}_t$. The firm needs to hire researchers to conduct R&D, whose wage is the same as that of the production workers. Formally, given the exogenous knowledge diffusion matrix $A = (A^{ij})_{K\times K}$, the firm solves the following optimal R&D investment problem (using wage as the numeraire):

$$\max_{\{R_t^{ij}\}_{i,j\in\{1,2,\ldots,K\}}} V(\boldsymbol{z}_t) = \sum_{j=1}^{K} \frac{\pi_t^j}{n_t^j} z_t^j - \sum_{i=1}^{K}\sum_{j=1}^{K} R_t^{ij} + \frac{V(\boldsymbol{z}_{t+1})}{1+\tilde{r}_t}, \tag{8}$$

subject to the knowledge accumulation equation (6) and the incremental innovation production

[9] This is equivalent to assuming each firm innovates and produces in one sector but the knowledge spillovers *across firms* are complete within a country. In reality, there are all kinds of barriers for this perfect internalization of inter-sectoral knowledge spillovers and countries can differ substantially in their ability to internalize these spillovers. This variation, although interesting, is not the focus of the current paper and is left for future endeavor. A possible extension of the current model is to allow for sectoral entry barriers which might entice countries to also differ in the set of products they produce.

[10] This can be interpreted as firms having to devote a certain amount of time to digesting and adopting knowledge in one sector to apply it to another. Thus, every research activity is source-knowledge-and-target-knowledge-specific.

[11] This assumption keeps the number of researchers constant while the number of varieties increases in the BGP equilibrium.

function (7).

We focus on the BGP equilibrium in which growth rates of aggregate variables remain constant over time and trade is balanced. Since labor supply is fixed, the (real) profit per firm in a given sector in the BGP equilibrium is constant, i.e. $\pi_t^i = \pi^i, \forall i$. Households' time preference pins down the discount factor $\frac{1}{1+\tilde{r}_t} = \frac{w_{t+1}}{w_t}\frac{1}{1+r_t} = \beta$. Define the BGP growth rate of the number of varieties in sector i as $g_t^i \equiv N_{t+1}^i/N_t^i - 1 = n_{t+1}^i/n_t^i - 1$. In Appendix A we show that as long as every sector benefits from knowledge from at least another sector (i.e. $\exists j$ s.t. $A^{ij} > 0, \forall i$) all sectors grow at the same constant rate in the BGP: $g_t^i = g$, $\forall i$. Thus, the relative size of sectors is constant over time: $n_t^i/n_t^j = n^i/n^j, \forall i, j, t$. The number of firms is fixed in the BGP equilibrium: $M_t = M$.

The linear form of the Bellman equation and the constant-return-to-scale knowledge production allow us to derive closed-form solutions for the firm's optimal R&D decisions. Define $\rho \equiv \frac{\beta}{1+g}$. In Appendix A, we prove that in the BGP equilibrium, the firm's value is a *linear* aggregate of the value of its knowledge capital in all sectors: $V(z) = \sum_{i=1}^{K} v_t^i$, where the market value of the firm's knowledge capital in sector i is constant in BGP equilibrium:

$$v_t^i = v^i = \frac{1}{1-\rho}(\pi^i + \sum_{j=1}^{K} \omega^{ji}), \quad \forall i, j \tag{9}$$

and the constant ω^{ji} captures the *application value* of sector i's knowledge to innovation in sector j. It increases with the knowledge abundance in sector i relative to j (n^i/n^j), the value of the target sector (v^j) and the knowledge linkages from i to j, A^{ji}:

$$\omega^{ji} = \frac{1-\alpha}{\alpha}\frac{n^i}{n^j}\left(A^{ji}\alpha\rho v^j\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

Substituting (10) into (9) implies that solving for the equilibrium value of knowledge capital, $(v^i)_{1\times K}$, is an iterative process:

$$v^i = \frac{1}{1-\rho}\left[\pi^i + \sum_{j=1}^{K}\frac{1-\alpha}{\alpha}\frac{n^i}{n^j}\left(A^{ji}\alpha\rho v^j\right)^{\frac{1}{1-\alpha}}\right]. \tag{11}$$

The market value of the firm's sectoral knowledge depends on not only the present value of the future profits in the exact sector, but also on the value of its knowledge application to future innovation in all sectors.

The firm's optimal investment associated with applying sector-j knowledge to i is proportional to its (relative) knowledge capital in the knowledge-source sector $(\frac{z_t^j}{n_t^j})$ and is positively related to the application value of sector j to i (ω^{ij}):

$$R_t^{ij} = \frac{\alpha}{1-\alpha}\omega^{ij}\frac{z_t^j}{n_t^j}. \tag{12}$$

Because firms are symmetric, we have $z_t^j = n_t^j$, which implies that R&D investment is also time-invariant $R_t^{ij} = \frac{\alpha}{1-\alpha}\omega^{ij}$. In addition, firm R&D is independent of firm's scale, consistent with empirical level observations as in Klette and Kortum (2004).

In addition, firms need to make an initial fixed-cost investment in the form of final goods $F > 0$ (using wage as numeraire) to enter. Free entry conditions imply that firms enter until their future discounted profit (after covering their R&D costs in each period) is the same as the entry cost F:

$$\frac{1}{1-\beta}\sum_{i=1}^{K} v^i = F. \tag{13}$$

At BGP, (13) implies $\frac{1}{1-\beta}(\sum_{i=1}^{K}\pi^i - \sum_{i=1}^{K}\sum_{j=1}^{K}R^{ij}) = F$. This equilibrium condition helps to determine the number of firms (M).

2.3 Aggregate Conditions

In this section, we complete the list of equilibrium requirements by adding conditions that stipulate market clearing in factor and goods markets. The population supplies L units of labor in every period which are allocated as production workers, researchers and workers making goods to meet the entry costs: $L = \sum_{i=1}^{K} L_t^{pi}M_t + \sum_{i=1}^{K}\sum_{j=1}^{K} R_t^{ij}M_t + (1-\beta)FM_t$. Substituting (9), (10) and (12) into the labor-market clearing condition, we have

$$L = \sum_{i=1}^{K}\sigma\pi^i M, \tag{14}$$

which implies labor income $w_t L_t$ equals total revenue.

Trade is balanced in every period in the equilibrium as there is no international financial market. That is, the total export value equals the total import value:

$$\sum_{i=1}^{K} r_{ht}^{i*}N_t^{i*} = \sum_{i=1}^{K} r_{ft}^{i}N_t^{i}. \tag{15}$$

2.4 Equilibrium

Given a set of parameters $\{\beta, \alpha, \sigma, (s^i)_{1\times K}, (A^{ij})_{K\times K}\}$ that are common across countries, a set of parameters $\{L, L^*, \tau, \tau^*, \phi, \phi^*\}$ that differ across countries, and time paths of external conditions in the RoW $\{n_t^{i*}, w_t^*, p_{ht}^{i*}, p_{ft}^{i*}\}_{i=1,\ldots K,t=0}^{\infty}$, a balanced growth path is an equilibrium in which output, consumption and innovation grow at constant rates. It is given by: time paths of aggregate quantities and prices $\{C_t, M_t, g_t, w_t, r_t\}_{t=0}^{\infty}$, time paths of sectoral quantities and prices $\{R_t^{ij}, z_t^i, n_t^i, v_t^i, p_{ht}^i, p_{ft}^i\}_{i,j=1,2,\ldots K,t=0}^{\infty}$, such that:

1. Given w_t, r_t, the representative household maximizes life-time utility subject to an intertemporal budget constraint. That is, (1) is satisfied.

2. Given w_t, r_t, the individual firm decides on the quantity and prices of goods produced and production labor needed, and optimal R&D investment. That is, (4), (5), (**??**), (9), (10), (12) and (13) are satisfied.

3. Prices are such that all markets clear. That is, (14) and (15) are satisified.

2.5 Implications for Equilibrium R&D Allocation Across Sectors and Growth

In this section, we derive and discuss reduced-form implications of aggregate R&D allocation across sectors and the relationship between knowledge composition and growth in the model.

Proposition 1 *Define the total R&D expenditure in sector i as $R^i \equiv \sum_{j=1}^{K} R^{ij}$. At the aggregate, R&D resources are allocated according to the sectoral knowledge value in the equilibrium. That is*

$$\frac{R^i}{R^j} = \frac{v^i}{v^j}. \tag{16}$$

Proof. See the Appendix.

Recall from (9) and (10), we have

$$v^i = (1-\rho)^{-1} \left[\pi^i + \sum_{j=1}^{K} \frac{1-\alpha}{\alpha} \frac{n^i}{n^j} (A^{ji} \alpha \rho v^j)^{\frac{1}{1-\alpha}} \right]. \tag{17}$$

The market value of sectoral knowledge capital depends on both the future profit in its sector and its application value in other sectors, which hinges on its relative knowledge abundance to the application sector ($\frac{n^i}{n^j}$), and the market value of the application sector (v^j), as long as $A^{ji} > 0$. For illustrative purpose, consider a stark example of two sectors: the center sector (denoted by c) whose knowledge can be applied to the other sector, and the periphery sector (denoted by p)

whose knowledge is not that applicable (i.e. $A^{pc} > 0$ and $A^{cp} = 0$). When the trade cost, τ, is high, profit in the center sector is low, discouraging its own innovation activity directly and the resulting insufficient knowledge accumulation (low n^i/n^j) further decreases its application value. In contrast, the periphery sector has no application value, and thus its market value depends only on its own profit. Therefore, as will be shown in Section 2.6, with reasonable parameter values, higher τ disproportionately reduces the market value of knowledge capital in the center sector more than that in the periphery sector, leading to a flattening of the distribution of R&D investment: $\partial(R^c/R^p)/\partial\tau < 0$.

Now, how does the change of R&D allocation and composition of knowledge affect growth? The source of growth in this economy comes from increasing varieties. Intuitively, when a country invests disproportionately more in highly applicable sectors, the economy benefits more from the knowledge spillovers from these sectors and creates more varieties at the aggregate. We call this the *composition effect* of R&D. It can be seen from the following proposition.

Proposition 2 *On the BGP, the aggregate innovation rate is an increasing function of* $\frac{\sum_i \sum_j \omega^{ij}}{\sum_i v^i}$, *the fraction of firm's value that is accounted for by its knowledge application value:*

$$g = \left(\beta(1-\alpha)\frac{\sum_i v^i}{\sum_i \sum_j \omega^{ij}} - 1\right)^{-1}. \tag{18}$$

And the aggregate real output growth rate is $g^y \approx \frac{1}{\sigma-1} g$.

Proof. See the Appendix.

Consider the previous illustrative example (two types of sectors), when α is very small (assuming $\alpha \to 0$), it can be easily shown based on (9) and (10), that innovation rate g strictly increases with n^c/n^p. In this case, when higher trade costs hinder the accumulation of knowledge in the center sector, growth also suffers. That is, $\partial g/\partial\tau < 0$.

It is useful to note that the v^i of a center sector (i.e. generally the sector with large positive values of A^{ji}, given j) is sensitive to the knowledge value of its application sectors (v^j) and its relative knowledge abundance ($\frac{n^i}{n^j}$), while the v^i of a periphery sector is more responsive to changes to its own profit (π^i). We will show in Section 2.6 that increasing transport costs disproportionately reduces both the profit and application value of center sectors, deterring R&D investment and endogenous accumulation of knowledge in center sectors, the effects of which permeate the whole economy, hurting the country's aggregate knowledge accumulation.

2.6 Solving the Model with a Star Network of Knowledge Linkages

In this section, we explore how changes in the trade cost (τ) and population (L) affect the knowledge composition and growth in general equilibrium, given reasonable parameter values. Appendix B offers a complete set of equilibrium conditions that are used to solve the model.

Here, we solve the model economy for a special case of two types of sectors: one center sector and $K-1$ identical periphery sectors. More specifically, consider a star-shaped knowledge diffusion network depicted in Figure 1, in which sector 1 is the sole input supplier to all others, where the other $K-1$ sectors are applicable to sector 1 but not to each other. That is, $A^{ii} = a$, $\forall i$, $A^{1j} = A^{j1} = a$, for $j > 1$ and $A^{ij} = 0$, for $i > 1$, $j > 1$. We set a to 0.4 such that the aggregate growth rates fluctuate within a reasonable range when τ varies. As simple as it seems, this star-shaped network captures well the highly skewed distribution of knowledge linkages across sectors estimated using patent citation data (in Section 3.1). In addition, we consider K=10, and the elasticity of substitution between different varieties in the same sector, $\sigma = 6$, which is broadly consistent with the empirical evidence at 3-digit level.[12] We assume that the home country is small relative to the rest of the world: $L^* = 50L$. Let us normalize the ROW wage, labor productivity, population to 1, and the foreign number of varieties per sector to 50. The subjective discount rate $\beta = 0.98$, and the share of researchers in knowledge production $\alpha = 0.4$ according to Cai and Li (2014).

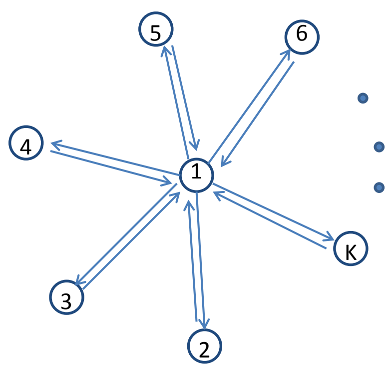


Figure 1: A Star Network of Knowledge Diffusion between K sectors

Figure 2(a) shows the effects of increasing trade cost (τ and τ^*) on home's sectoral profit (π^c for total profit in the center sector and π^p total profit in periphery sectors), equilibrium wage (w), total exports (X), R&D investment in the center sector relative to that in the periphery sector (R^c/R^p), the knowledge composition (n^c/n^p) and the export share of the center sector (X^c/X). Figure 2(b) illustrates the relationship between aggregate innovation rate and the composition of

[12] For estimates of elasticity of substitution, see Anderson and Wincoop (2004).

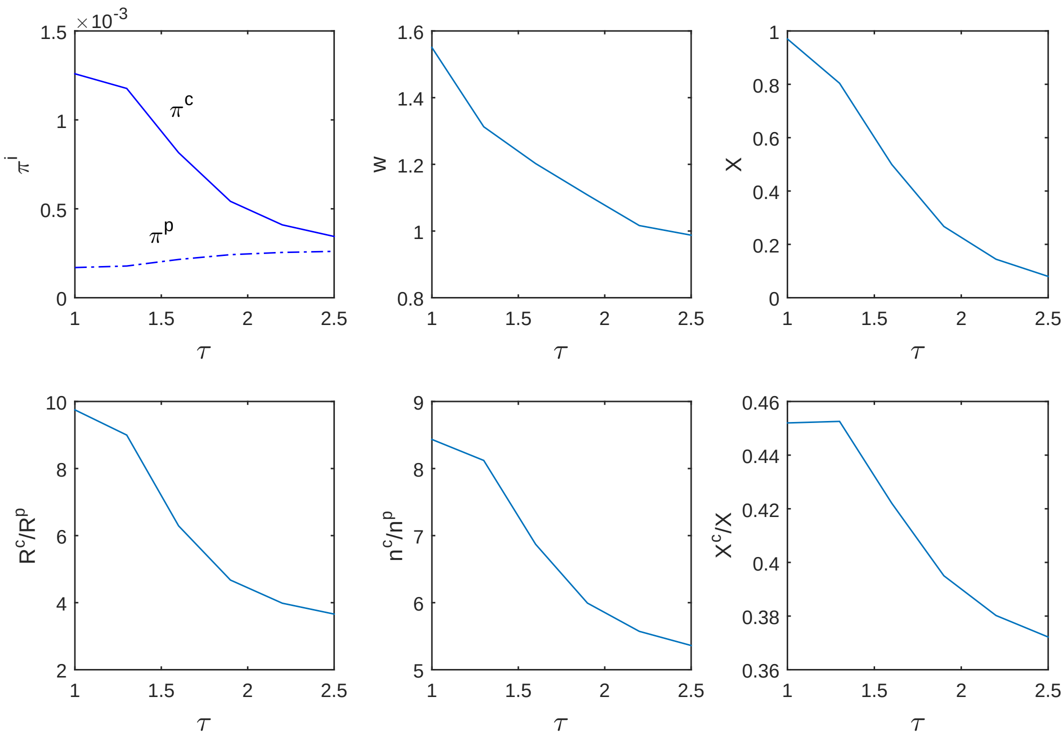
knowledge.

In general equilibrium, when the trade cost rises, wages decrease as the demand for researchers and production workers falls. Nevertheless, the profit in the center sector still suffers due to the greater loss in competitiveness associated with higher trade costs in the export market. Profit in periphery sectors, however, slightly increases, because these sectors have small export markets, and hence are less affected by trade costs. Lower wages associated with rising trade cost implies higher profit in these sectors. As firms have relatively less incentive to innovate in the center sector, the total number of varieties developed in the center sector declines more than in the periphery. The value of knowledge capital in the center sector relative to that of the periphery (v^c/v^p) also drops as a result of (a) decreased profitability and (b) reduced application value as less knowledge is accumulated in this sector (as n^c/n^p falls), further deterring R&D allocation (R^c/R^p) in the center (Proposition 1). Since knowledge in the center is responsible for fostering innovation in many application sectors, lower investment in these sectors hurts innovation and growth in the whole economy, as demonstrated in Figure 2(a).

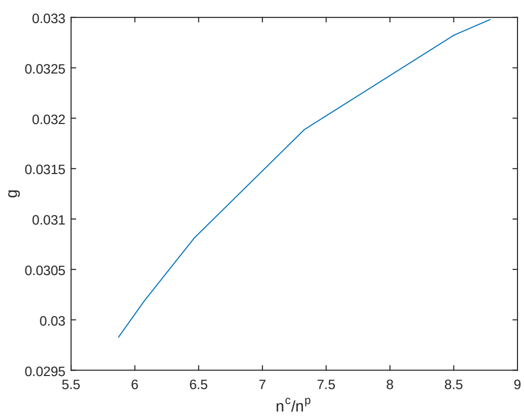
In summary, our model implies that, all other things being equal, countries of different transport costs to potential trading partners and different population sizes innovate and produce a different composition of products. The value of knowledge capital in a given sector consists of two components—its own discounted future profit (in the product market) and its value as knowledge input to create new knowledge in all related sectors. Countries with larger population and lower transport costs specialize more in the center sector—as the differential value between center and periphery sectors is larger—and grow faster. Thus, the model accounts for the empirical findings documented earlier in the paper.

3 Empirical Evidence: Determinants of the Knowledge Composition, and Relationship with Growth

This section provides empirical evidence for the main prediction of the model. We first propose a measure that characterizes for each given sector its importance as knowledge suppliers to all the knowledge-downstream sectors (including both its immediate application sectors and indirectly related chains of downstream sectors). We then proceed to investigate the main predictions of the theory: whether higher trade costs induce less specialization in knowledge applicable sectors, and whether countries which initially specialized in knowledge-applicable sectors grow faster in subsequent decades.



(a) The Effects of Changing Trade Costs, τ.



(b) The Relationship between g and n^c/n^p.

Figure 2: General Equilibrium Effects of Changing Trade Costs

3.1 Data

Proxy for Knowledge Linkages Across Sectors Direct observation on the actual adaption of knowledge across sectors is not available as knowledge flows are invisible. The literature, however, has typically found that patent citations seem to represent an indicator of knowledge spillovers, albeit with some degree of noise (Jaffe et al. 2000, Bottazzi and Peri, 2003, Branstetter, 2006, etc.).[13] One of the advantages of patent citation data, as noted in Hall et al.(2001), is that "the decision regarding which patents to cite ultimately rests with the patent examiner, who is supposed to be an expert in the area and hence to be able to identify relevant prior art that the applicant misses or conceals." Thus, citations are informative of links between innovations. If a single technology is cited in numerous patents, it is apparently involved in many developmental efforts. We thus use patent citations between patents that belong to different sectors to trace the direction and intensity of knowledge flows across sectors.

This paper takes the view of Nelson and Winter (1977) that "innovations follow 'natural trajectories' that have a technological or scientific rationale rather than being fine tuned to changes in demand and cost conditions." Therefore, technology interconnections in this paper are intrinsic characteristics of technologies.

In addition, we assume that the differences in the applicability of knowledge in one sector to another persist across countries, so that we can use a sector's knowledge applicability as identified in the U.S. patent database as a measure of its applicability in other countries. Note that patents applied in U.S. are not necessarily generated by U.S. inventors. According to the territorial principle in U.S. patent laws, anyone intending to claim exclusive rights for inventions is required to file U.S. patents. In fact, about 50 percent of patents applied in the United States in the early 2000s were from foreign inventors. Given that the United States has been the largest technology consumption market in the world over the past few decades, it is reasonable to assume that most important innovations from other countries have been patented in the U.S. Therefore, the knowledge linkages uncovered in the U.S. patent data are reasonably representative of the deep fundamental relationship of technologies. All we really need is that statements of the following sort hold: If knowledge in electronic components sector is potentially useful for develop new products in radio and television receiving equipment in the U.S., similar relationship also holds for inventors in Mexico. Even if the linkages captured by the U.S. patent citation network had not currently been explored in developing countries, they might be in the future. It is precisely this underlying

[13]For example, not all innovations are patented, especially process innovations (which are often protected in other ways such as copyright, trademarks and secrecy). Levin et al (1987) find that secrecy was more effective for process innovations, We implicitly assume that for any sector, the unpatented and patented knowledge utilizes knowledge (patented or unpatented) from other sectors in the same manner, with the same likelihood and intensity.

relationship that predicts future product entry and innovation which leads to growth.

Construction of the Knowledge Applicability The method to construct the applicability measure is discussed in detail in Cai and Li (2014) and here we content ourselves by outlining the main idea. We use data from the 2006 edition patent database provided by the U.S. Patent and Trade Office (USPTO).[14] In the dataset, patents are organized by their technical features and each patent belongs to a technology field according to the International Patent Classification (IPC).

We start by adding up citations made from (and to) patents that belong to the same IPC sector to generate a *cross-sector* citation matrix $(c^{ij})_{(i,j)\in J\times J}$, where c^{ij} denotes the number of citations to sector i made by j. For each sector i, we calculate the total number of patent applications, $(s^i)_{i\in J}$. We then apply the iterative algorithm developed by Kleinberg (1998) to the citation matrix and construct an index, called *authority weight* (aw), to capture the 'knowledge applicability' of each sector (i.e. the extent to which they enable the creation of knowledge in all sectors). The algorithm simultaneously generates another index, hub weight (hw), which characterizes the extent to which the sector relies on knowledge from other sectors. Formally,

$$\begin{aligned} aw^i &= \lambda^{-1}\sum_{j\in J} W^{ji} hw^j \\ hw^i &= \mu^{-1}\sum_{j\in J} W^{ij} aw^j \end{aligned} \tag{19}$$

where λ and μ are the norms of vectors $(aw^i)_{i\in J}$ and $(hw^i)_{i\in J}$, respectively. W^{ji} corresponds to the number of citations received by patents in sector i from patents in sector j. We calculate the time-variant aw^i_t for each IPC sector based on rolling window subsamples, pooling citations from the previous 10 years for each year during 1985-2006.[15]

Generally speaking, a sector with high aw provides large knowledge spillover to sectors with highly ranked hw; a sector with high hw utilizes large knowledge flow from sectors with highly ranked aw. Compared to Garfield's "impact factor index" (i.e. the average number of citations received), this algorithm takes into account not only the direct spillovers, but also the indirect subsequent linkages between sectors. In addition, it distinguishes the importance of a sector as a knowledge supplier and as a knowledge user.[16] Overall, its highly efficient at estimating the

[14] The NBER patent database is available at: https://sites.google.com/site/patentdataproject/Home. It contains detailed patent and citation information, including the patent application year, grant year, the technological area to which it belongs, the nationality of patent inventors, the patent assignees, the citations made and received by each patent, etc.

[15] The ranking for most sectors does not change drastically over the sample period, although the quality of our measure decreases close to the end of the sample as a result of citation lags. The average correlation of the ranking across different decades is about 0.90.

[16] See Cai and Li (2014) for detailed explanation on the advantage of Kleinberg's two-level pattern of linkages in

potential knowledge contribution of each sector.

Importantly for our analysis, we find that citation linkages across sectors are highly heterogeneous and the knowledge embodied in a small number of sectors aids a disproportionately large number of subsequent innovations. This observation renders particular importance to the effects of a country's initial knowledge composition on its subsequent growth.

Proxy for A Country's Knowledge Composition As discussed earlier, lack of direct observation of a country's composition of knowledge, we use the its export structure as a proxy. This is based on the premise that making a product requires specific types of knowledge, which can be either created by innovating or acquired elsewhere. The composition of exports thus contains evidence of this process of knowledge acquisition and accumulation.

Countries' export structures are measured based on an updated version of the UN-NBER World Trade Flows dataset (see Feenstra et al, 2005), which harmonizes COMTRADE annual bilateral trade flow data for SITC sectors over the 1978-2013 period. To rank these industrial sectors according to their knowledge applicability, we employ the IPC–SITC concordance provided by WIPO to generate the aw for each 2-digit SITC sector. As not all export sectors fall into the set of innovating sectors, we exclude countries whose significant share of export cannot be mapped into innovating sectors.[17] This yields an unbalanced panel of 218 countries for the period 1985-2006.[18]

Based on the sector-specific knowledge applicability measure, we are now equipped to describe a country's knowledge applicability associated with its export basket. We make use of two different proxies for a country's knowledge applicability. The first indicator, $TA_{c,t}$, is the weighted average of sectoral knowledge applicability (in log) for country c in period t, using the share of export by the country in the respective sectors, $x^i_{c,t} = EX^i_{c,t}/\sum_i EX^i_{c,t}$, as weights, and $EX^i_{c,t}$ is the export value in sector i of country c in period t. That is, $\log(TA_{c,t}) = \sum_i x_{c,t} \log(aw^i_t)$. The second measure, $Perc33_{c,t}$, is the fraction of exports in the most applicable third of all sectors of country c in period t.

Geographic remoteness We use overall geographic remoteness to proxy cross-country variations in trade costs. It is measured as a weighted average of a country's bilateral distance to all other

comparison to the one-level iteration algorithm for influence factor proposed by Pinski and Narin (1976). In addition, they show that this measure of applicability is both conceptually and empirically distinct from other centrality measures in the network literature—such as indegree, or generality index proposed by Hall et al. (1997)—and is better suited to study the question at hand.

[17] Agricultural sectors are excluded from the sample.

[18] We have also explored the United Nations Industrial Development Organization (UNIDO)'s industry statistics database which provides industrial production information at the 4-digit ISIC (rev.3) level. However, the large number of missing observations in the UNIDO data at the 4-digit ISIC level significantly impedes the accuracy of our empirical analysis. Therefore, we focus on the results from trade flow data.

countries in the world, using countries' population share as weights.[19] The reasons that we do not use tariffs as proxy are as follows. First, the level of tariffs in any given country is not exogenous to level of economic development. Second, tariffs are often heterogeneous across sectors within a country, and the degree to which the tariff structure disproportionately favors certain sectors is an endogenous policy decision and can directly impact growth (see Nunn and Trefler, 2010). In our model, sectors only differ, ex ante, in terms of their knowledge complementarities. The uniform trade costs across sectors have differentiated effects in different sectors in equilibrium. It is the distributional effects of the "unbiased" trade costs what we attempt to test here.

Data on real GDP per capita, population, investment, the number of workers, and the measure of openness (exports plus imports divided by GDP) are taken from Penn World Table Mark 7.1 (PWT). Human capital stock is measured using Mincerian non-linear returns to education (the average years of schooling for the population aged 25 years old) as reported by Barro and Lee (2010). Physical capital stocks are constructed using the perpetual inventory method, as explained in Caselli (2005). The measures of the rule of law and regulation quality are from World Bank's Worldwide Governance Indicator (2014). Data on distance from the equator are from Hall and Jones (1999) and other geographic data (i.e. size, bilateral distance) are from Frankel and Romer (1999) and Helpman Melitz and Rubinstein (2008). Total natural resource rents data are from World Bank's World Development Indicator.

3.2 Sector-level Evidence: Geographic Remoteness and Knowledge Composition

In the model outlined in Section 2, changes in trade costs have a disproportionate effect on R&D investment, knowledge creation and production across sectors. Lower trade cost lead to an expansion of varieties in knowledge highly applicable sectors, which in turn generates a positive knowledge externality that increases per capital GDP growth. This implies that at the sector level, countries that are less distant from the world tend to concentrate more in sectors with high knowledge applicability.

To test whether this distributional effects of geography is present in the data, we estimate the following specification:

$$\ln(x^i_{c,t}) = c + \beta_0 \ln(aw^i_t) + \beta_1 \ln(aw^i_t) \times remoteness_c + \alpha \ln(aw^i_t) \times \boldsymbol{Z}_{c,t} + \gamma\ \boldsymbol{Z}_{c,t} + \mu_c + \eta^i + \varepsilon^i_{c,t}, \quad (20)$$

where $x^i_{c,t}$ is the share of sector i's exports in country c's overall exports. $remoteness_c$ is measured

[19] Using GDP share as weights does not change results significantly. Also see Baldwin and Harrigan (2011) for a discussion of alternative measures of remoteness.

as a weighted average of country c's bilateral distance to all other countries in the world, using other countries' population share as weights. $\boldsymbol{Z}_{c,t}$ collects other potential factors that affect a country's knowledge (export) composition. We include the country fixed effects μ_c to control for constant differences across countries (such as culture, language, geography etc), and sector fixed effects η^i to control for country-time-invariant sector characteristics.

Table 1: Determinants of a Country's Knowledge Composition

	(1)	(2)	(3)	(4)	(5)
$\log(aw_t^i)$	1.45	1.28	0.97	1.22	1.36
	(3.27)***	(2.84)***	(2.03)**	(2.49)**	(3.17)***
$\boldsymbol{\ln(aw_t^i) \times remoteness_c}$	-0.16	-0.16	-0.15	-0.14	-0.15
	(-3.13)***	(-3.25)***	(-2.88)***	(-2.57)**	(-3.09)***
$\ln(aw_t^i) \times human\ capital_{c,t}$		0.12			
		(3.00)***			
$\ln(aw_t^i) \times capital\text{-}labor\ ratio_{c,t}$		0.02			
		(1.27)			
$\ln(aw_t^i) \times natural\ resource_c$		-0.00			
		(-2.48)**			
$\ln(aw_t^i) \times IPR_{c,t}$			0.06		
			(7.11)***		
$\ln(aw_t^i) \times dist\ from\ equator_c$				0.01	
				(0.87)	
$\ln(aw_t^i) \times regulation\ quality_{c,t}$					0.06
					(2.25)**
Sector FEs	Yes	Yes	Yes	Yes	Yes
Country FEs	Yes	Yes	Yes	Yes	Yes
Six governance variables	No	No	No	No	Yes
Observations	125,727	54,883	23,578	75,057	65,154
R^2	0.45	0.47	0.47	0.46	0.49

Notes: The dependent variable is the (log) share of export in sectorr i in country c in period t, $\ln(x_{c,t})$. There are 56 sectors and 117 countries over the period 1985-2006. Remoteness is measured as a weighted average of a country's bilateral distance to all other countries in the world, using countries' population share as weights. Equator indicates the country's distance from the equator. World Bank's six governance variables are control of corruption, government effectiveness, political stability and absence of violence, rule of law, regulatory quality, voice and accountability. t-statistics are calculated from robust standard error adjusted for clustering at the country level. *** and ** indicates significance at 1 percent and 5 percent level, respectively.

Table 1 reports the estimates of Equation (20) from various specifications. Since all specifications control for sector-specific effects and country-specific effects, the only effects that are identified are those relative to variables that vary both across countries and across sectors. Thus, we report only the coefficient of the sector's knowledge applicability and the coefficients of the interaction between knowledge applicability and different country-specific characteristics. We are particularly interested in the coefficient of the interaction between knowledge applicability and geographic remoteness, β_1. A negative and statistical significant β_1 implies that knowledge applicable sectors tend to have larger shares in exports in countries that are less remote from the rest of the world, as predicted by the model.

Column (1) shows the baseline result where only geographic remoteness is considered. In the rest of the table, we control for other potential factors that may also affect a country's knowledge (export) composition. A country's factor endowments determine its comparative advantage in traditional trade theory, and thus, are controlled for in the second column. In addition, firms may be able to better internalize spillovers from highly applicable sectors and hence have more incentive to innovate in these sectors, when the protection of their intellectual property rights (IPR) is stronger. For this reason, we add the Ginarte and Park index of IPR in the third column. Lastly, favorable social infrastructure and conducive institutions may also lead to more development in highly knowledge applicable sectors. We thus control for the distance from the equator which typically is an exogenous historical determinant for social infrastructure (Hall and Jones, 1999) in the fourth column, and the World Bank's six governance quality indexes in the last column. In our exercise, we also consider other factors that may intuitively play a role in determining country's knowledge composition, namely, population, area and whether the country is landlocked. However, none of these factors enter in a significant way and are thus not reported in the table.

Table 1 shows that throughout all specifications, the coefficients of the interaction term between geographic remoteness and sector-specific applicability measure are significantly negative, consistent with the implication of the model. Unsurprisingly, an educated workforce and lower endowment of natural resources are significantly associated with more specialization in sectors with high knowledge applicability, pointing to another channel through which human capital and natural resource curse can affect growth—by changing the knowledge (export) composition. Consistent with our intuition, stronger IPR protection is also associated with a distribution of exports biased towards the highly knowledge applicable sectors. Distance to the equator does not appear to have significant impact on a country's knowledge composition. Better institutional quality—regulation quality in particular—fosters more production and exports in highly applicable sectors.

3.3 Country-level Evidence: Knowledge Composition and Long-term Growth

Our model has a clear implication on the relationship between a country's knowledge composition and its long-term level of growth. When trade costs are reduced, more R&D resources are shifted to sectors whose knowledge is highly applicable to later innovations, increasing the overall application value of knowledge capital (relative to its production value) in the economy. Consequently, growth increases according to (18). This section provides suggestive evidence on the positive relationship between a country's knowledge applicability revealed through its exports and its subsequent growth.

We note that the growth regressions examined below reveal correlations, not causality.[20] The causal mechanisms are established in the model. We examine the following cross-country regression specification:

$$(\ln y_{c,t} - \ln y_{c,0})/t = \beta_1 + \beta_2 \log TA_{c,0} + \boldsymbol{\delta X}_{c,0} + \varepsilon_c. \tag{21}$$

The left-hand-side measures the annual growth rate of per capita GDP (y_c) for country c from year 1985 to 2010. $\log TA_{c,0}$ (constructed in Section 3.1) is the proxy for the applicability of country c's knowledge composition in 1985. We choose 1985 because its the first year that we can calculate the rolling-window $\{aw_t^i\}$ for. $\boldsymbol{X}_{c,0}$ includes standard controls for initial country characteristics from the cross-country growth literature.

Estimates of Equation (21) are reported in Table 2 and Table 3. All specifications control for (log of) initial GDP per capita, initial human capital and initial investment-to-GDP ratio. Column (1) shows that the coefficient on $\log TA$ is positive and significant, suggesting that specializing in sectors with large knowledge applicability brings higher growth in the future, consistent with the implication of our model. In Column (2), we introduce nine region fixed effects, which is more than what is typical in the literature. Once the region fixed effects are controlled for, the initial investment and human capital are no longer significant. As noted in Nunn and Trefler (2011), it is usually very hard to find a statistically significant country-specific characteristic when so many region fixed effects are included, because it is the growth variation within narrowly defined regions that we try to explain in this case (an average of 8.4 observations per region with 10 regions and 84 countries). Despite of the presence of these fixed effects, the initial knowledge applicability is still found to be significantly positively associated with subsequent growth, making our results for the role of knowledge composition more compelling.

Moreover, the size of the estimated effect is large. The estimated coefficient is about 0.5, implying that a 1 percent increase in TA_0, which is approximately what Thailand achieved between 1985 and 1995 and what Poland achieved between 2000 and 2006, on average enhances a country's subsequent annual growth by half a percentage point.

In Table 3, we consider other covariates for growth regressions. Column (1) shows that despite its (arguable) role of causing income differences, de facto trade openness (measured by sum of export and import as a ratio of GDP) does not seem to have a significant impact on subsequent growth differences. Column (2) considers other characteristics of a country's export structure. In

[20] In general, growth regressions are plagued with omitted variables and reverse causality issues. One way to gain traction on the causal mechanism in the literature is to use sectoral observations. This is, however, not an option for our mechanism, because the positive externality transmitted through knowledge linkages accrue to the whole economy, not to the particular sectors. In fact, in the model all sectors grow at the same rate on the balanced growth path, but countries with different composition of knowledge (which is also endogenous) enjoy different growth rates.

Table 2: Country-level per capita GDP Growth Regressions.

	(1)		(2)	
	β	t-statistic	β	t-statistic
Knowledge applicability				
Initial TA	0.42	(2.19)**	0.52	(2.12)**
Country Characteristics				
Initial income	-0.52	(-1.81)	-0.80	(-2.05)**
Initial investment share	0.05	(2.22)**	0.02	(1.04)
Initial human capital	0.80	(1.63)	0.19	(0.47)
Region fixed effects				
Latin America			-0.55	(-1.05)
West Africa			-2.39	(-2.13)**
East Africa			-2.05	(-1.46)
South Central Africa			-3.25	(-2.37)**
East Asia			0.90	(0.90)
South East Asia			0.09	(0.09)
South West Asia			-0.42	(-0.35)
North Africa, Middle East			-0.91	(-1.31)
Eastern Europe			-0.19	(-0.37)
Number of obs	84		84	
R^2	0.16		0.41	

Notes: The dependent variable is the average annual log change in GDP per capita in country c over the period 1985-2010, $(\log y_{c,t} - \log y_{c,0})/t$. t-statistics are calculated based on robust clustered standard errors. ** indicates significance at 5 percent level.

particular, countries also differ in terms of their positions on the global supply chains (Andres, Chor, fally and Hillberry, 2012) and in terms of the levels of export diversification. Specializing in some stages of the global production process may be associated with better growth opportunities than specializing in others. However, when adding the Andres et al. (2012) summary measure of the upstreamness of a country's exports, the relationship between the initial knowledge applicability and growth is essentially intact. In addition, when replicating the results of Hausmann et al. (2007), Lederman and Maloney (2012) find that, once diversification is controlled for, the export sophistication measure no longer contributes to growth, suggesting that it is the concentration, and not the sectors per se, that matters.[21] Our model assumes a fixed number of sectors that a country produces (and exports) in, allowing no variations in diversification. However, conceptually a more diversified product structure would allow countries to better internalize inter-sectoral knowledge spillovers and promote growth. Thus, we also control for the standard measures of export concentration (Herfindahl-Hirschman index) in our regression. A higher index implies less diversification. Column (2) in Table 3 shows that the diversification measure enters with a correct sign but is not significant, and does not greatly affect the significance of $\log TA$.

A long list of literature has documented the important effects of geography (which determines

[21]Their argument is that countries with low levels of export diversification may experience higher export price volatility and hence increased macroeconomic volatility.

Table 3: Country-level per capita GDP Growth Regressions—Robustness

	(1)		(2)		(3)		(4)	
	β	t-statistic	β	t-statistic	β	t-statistic	β	t-statistic
Knowledge applicability								
Initial TA	0.52	(2.09)**	0.63	(2.84)***	0.66	(2.98)***		
Initial Perc33							2.80	(3.48)***
Country Characteristics								
initial GDP per capita	-0.82	(-2.08)**	-0.91	(-2.40)**	-1.10	(-2.82)***	-1.09	(-2.73)***
Initial investment share	0.02	(1.04)	0.03	(1.31)	0.04	(1.82)	-0.05	(2.04)**
Initial human capital	0.20	(0.49)	0.11	(0.27)	0.39	(0.83)	0.32	(0.69)
Add Openness								
Initial Openness	0.00	(0.34)	0.00	(0.77)	0.00	(0.64)	0.00	(0.66)
Add Other Export Structure								
Initial Diversification			-1.65	(-1.66)	-1.09	(-1.17)	-0.56	(-0.61)
Initial Upstreamness			0.54	(2.00)**	0.52	(1.71)	0.53	(1.81)
Add Geography								
Landlocked					-1.04	(-2.16)**	-1.12	(-2.44)**
Area					0.04	(0.46)	0.03	(0.29)
Distance from the equator					-0.66	(-0.43)	-0.93	(-0.62)
Remoteness					-1.23	(-1.02)	-1.36	(-1.13)
9 region fixed effects	Yes		Yes		Yes		Yes	
Number of obs	84		84		82		82	
R^2	0.42		0.45		0.49		0.50	

Notes: The dependent variable is the average annual log change in GDP per capita in country c over the period 1985-2010, $(\log y_{c,t} - \log y_{c,0})/t$. t-statistics are calculated based on robust clustered standard errors. In Column (1)-(2), knowledge applicability is measured by the export share weighted average of $\log(aw^i)$ in country c. In Column (3), knowledge applicability is measured by the share of export in top third sectors with highest $\log(aw^i)$. ** and *** indicates significance at 5 percent and 1 percent level. Diversification is measured using standard Herfindahl-Hirschman index for export structure. Landlock is a dummy variable which equals one if the country is enclosed by land. Equator is the distance from the equator, and remoteness is measured as a weighted average of a country's bilateral distance to all other countries in the world, using countries' population share as weights.

the amount of trade and institutions) on growth in the very long run (e.g. Hall and Jones, 1999; Acemoglu et al. 2001, 2005; Dollar and Kraay, 2003; Rodrik et al. 2004). Column (3) adds the standard geographic variables (i.e. Landlocked dummy, area, distance from the equator and remoteness) into the growth regression and shows that these variables do not seem to enter in a robustly significant way except for the landlocked dummy.

In Column (4), we replace the $\log(TA)$ variable with the alternative measure of knowledge applicability associated with a country's exports—$Perc33$—and re-estimate all the regressions. Again, the coefficient on $Perc33$is positive and statistically significant. The magnitude of the coefficient is also larger, almost 2.5. This implies that a 10 percent increase in $Perc33$ boosts annual growth by 1/4 percent, which is substantial.

4 Final Remarks

This paper provides a systematic analysis to investigate the relationship between the composition of knowledge and growth. There are two contributions. First, we incorporate heterogeneous inter-sectoral knowledge linkages into a formal model of trade and endogenous growth. The model provides a theoretical interpretation of how exogenous trade costs can account for differences in countries' composition of knowledge accumulation, which in turn impacts growth through its implication of knowledge spillovers. Traditionally, lower trade costs increase growth by granting firms a larger market and higher incentive to innovate. We describe in this paper that lower trade costs are also associated with "composition effect", in that it encourages countries to allocate more R&D resources towards highly knowledge-applicable sectors, which provide ample knowledge spillovers and growth opportunities to the rest of the economy. Second, using cross-sector patent citation data, we propose an measure of knowledge applicability for each specific sector and use this measure to describe the applicability of knowledge a country possesses. We then document empirical observations that are broadly consistent with the predictions of the model.

The mechanism highlighted in our model implies that knowledge specialization can be another source of economic prosperity. The natural questions to ask are: How do countries increase the amount of applicable knowledge? What are potential barriers to this process? Can policy changes such as trade liberalization help to improve the knowledge structure of the economy? The preceding theoretical analysis in Section 2.6 is well-suited to answer some of the questions. In particular it can trace the equilibrium outcome of an economy undergoing a change in trade costs resulting either from reductions in real cost levels or from multilateral agreements to reduce tariffs (changes in τ) or non-tariff barriers to trade (changes in exchange rate captured by relative wage w in the model). The main impact of lower trade barriers—besides leading to more exposure to trade as in conventional trade models—is an increase in aggregate innovation productivity generated by a reallocation of R&D resources towards sectors with higher knowledge applicability. Our empirical studies, however, focus on exploring how geography-induced technology specialization affects a country's subsequent growth. Thus, it does not directly examine whether countries with lower policy-induced barriers grow more quickly. More empirical studies in this direction will be helpful in providing a vigorous answer to these questions.

Moreover, it is worth pointing out that to keep the discussion focused on the composition of knowledge rather than the diversification of knowledge, we have assumed that the total number of sectors in which a country innovates and produces is constant and fixed in the model. Intuitively, a country producing in a larger range of sectors can internalize more inter-sectoral knowledge spillovers and can thus enjoy more growth opportunities.Our example with a star-shaped knowledge diffusion network implies that a country with a higher degree of diversification would specialize more in highly applicable knowledge, since the model predicts that R&D resources are allocated according to sectoral knowledge value. In practice, the ability of a country to diversify and move into new sectors depends on its existing knowledge structure and other country characteristics. This dynamic entry decision is absent in our model, but can be explored once allowing for sectoral entry barriers. This kind of extension can provide valuable insights, as policy interventions can affect the fixed cost of doing business (i.e. license, regulation fees). This could be a promising venue for future research.

References

[1] Acemoglu, D., S. Johnson, and J. Johnson. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation". American Economic Review, 91(5): 1369-1401.

[2] Acemoglu, D., S. Johnson, and J. Johnson. 2005. "The Rise of Europe: Atlantic Trade, Institutional Change and Economic Growth." American Economic Review, 95(3): 546-79.

[3] Acemoglu, D., V. M. Carvalho, A. Ozdaglar and A. Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations". Econometrica 80(5):1977-2016.

[4] Aghion, P., M. Dewatripont, L. Du, A. Harrison and P. Legros. 2012. "Industrial Policy and Competition". NBER Working Paper No. 18048.

[5] Antras, P. D. Chor, T. Fally and R. Hillberry. 2012. "Measuring the Upstreamness of Production and Trade Flows. " American Economic Review, 102(3): 412-16.

[6] Barro, R. J., and J.W. Lee. 2010. "International Data on Educational Attainment: Updates And Implications". Oxford Economic Papers, 53: 541-63.

[7] Beck, T., Demirgüç-Kunt, A. and R. Levine. 2000. "A New Database on Financial Development and Structure."World Bank Economic Review, 597-605.

[8] Blonigen, Bruce. 2013. "Industrial Policy and Downstream Export Performance". NBER working paper No. 18964.

[9] Bresnahan, T. and M. Trajtenberg. 1995. "General Purpose Technologies 'Engines of Growth'?", Journal of Econometrics, 65: 83-108.

[10] Cai, J. and N. Li. 2012. "Growth Through Intersectoral Knowledge Linkages". mimeo, Ohio State University.

[11] Ciccone, A. 2002. "Input Chains and Industrialization". The Review of Economic Studies, 69(3): 565-587.

[12] Dollar, D. and A. Kraay. 2004. "Trade, Growth and Poverty". The Economic Journal, 114(493): 22-49.

[13] Feenstra, R. C., R.E. Lipsey, Deng, H., Ma, A.C. and Hengyong Mo. 2005. "World Trade Flows." 1962-2000, NBER Working Paper 11040.

[14] Frankel, J. A. and D. Romer. 1999. "Does Trade Cause Growth?" American Economic Review, 89(3): 379-399.

[15] Feyrer, J. 2009. "Trade and Income – Exploiting Time Series in Geography". mimeo, Dartmouth College.

[16] Grossman, G. and E. Helpman. 1991. "Innovation and Growth in the Global Economy." MIT Press, Cambridge, MA.

[17] Grossman, G. and E. Helpman. 1990. "Comparative Advantage and Long-Run Growth". American Economic Review, 80 (4): 796-815.

[18] Hall, R. E., & Jones, C. I. 1999. "Why Do Some Countries Produce So Much More Output Per Worker Than Others?" Quarterly Journal of Economics, 114(1): 83–116.

[19] Hall, B, A. Jaffe, and M. Trajtenberg. 2001. "The NBER Patent Citations Data File: lessons, Insights, and Methodological Tools". NBER Working Paper 8498.

[20] Harrison, A. and A. Rodriguez-Clare. 2010. "Trade, Foreign Investment, and Industrial Policy", in Handbook of Development Economics, Vol 5, edited by D.Rodrik and M. Rosenzweig.

[21] Hausmann, R., J. Hwang and D. Rodrik. 2007. "What You Export Matters". Journal of Economic Growth, 12(1), 1-25.

[22] Hausmann, R. and C. A. Hidalgo. 2011. "The Network Structure of Economic Output". Journal of Economic Growth. 16: 309-342.

[23] Hausmann R., and B. Klinger. 2007. "The Structure of the Product Space and the Evolution of Comparative Advantage". CID working paper.

[24] Helpman, E., M. Melitz and Y. Rubinstein. 2008. "Estimating Trade Flows: Trading Partners and Trading Volumes". Quarterly Journal of Economics, 123(2): 441-487.

[25] Hidalgo, C. A. and Hausmann, R. 2009. "The Building Blocks of Economic Complexity". Proceedings of the National Academy of Sciences of the United States of America, 106(26): 10570-10575

[26] Hidalgo, C.A., B. Klinger, A.L. Barabasi and R. Hausman. 2007. "The Product Space Conditions the Development of Nations". Science, 317: 482-487.

[27] Jaffe, A., M. Trajtenberg and M. S. Fogarty. 2000. "Knowledge Spillovers and Patent Citations: Evidence from a Survey of Inventors". American Economic Review 90(2), 215-218.

[28] Jones, C. I. 2011a. "Intermediate Goods and Weak Links in the Theory of Economic Development". American Economic Journal: Macroeconomics, 3(2): 1-28.

[29] Jones, C. I. 2011b. "Misallocations, Economic Growth, and Input-Output Economics". In Proceedings of Econometric Society World Congress (D. Acemoglu, M. Arellano, and E. Dekel, eds.), Cambridge University Press.

[30] Jones, C. I. 1995. "Time Series Tests of Endogenous Growth models". Quarterly Journal of Economics, 110:495-525.

[31] Kali, R., J. Reyes, J. McGee and S. Shirrell. 2009. "Growth Networks". Journal of Development Economics, 101: 216-227.

[32] King, R. and R. Levine.1993. "Finance and Growth: Schumpeter Might Be Right." Quarterly Journal of Economics, 58(3): 717-737.

[33] Klenow P. and A. Rodriguez-Clare. 2005. "Externalities and Growth", Handbook of Economic Growth, Vol 1A, P. Aghion and S. Durlauf, eds. 817-861.

[34] Koren M. and S. Tenreyro. 2007. "Volatility and Development". Quarterly Journal of Economics, 122(1): 243-287.

[35] Kleinberg, R. 1998. “Authoritative Sources in a Hyperlinked Environment” in Proceedings of ACM-SIAM Symposium on Discrete Algorithms, 668-677.

[36] Klette, T. J. and S. Kortum. 2004. “Innovating Firms and Aggregate Innovation.” Journal of Political Economy, 112: 986-1018.

[37] La Porta, R., Lopez-de-Silanes, F., Shleifer, A. and R. Vishny. 1998. “Law and Finance.” Journal of Political Economy, 106: 1113-55.

[38] Melitz, M. 2002. “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”. Econometrica, 71(6): 1695-1725.

[39] Newman, M. E. J. 2003. “The Structure and Function of Complex Networks”. SIAM Review 45, 167-256.

[40] Ngai R. and R. Samaniego. 2011. “Accounting for Research and Productivity Growth Across Industries.” Review of Economic Dynamics, 14(3): 475-495.

[41] Nunn, N. and D. Trefler. 2010. “The Structure of Tariffs and Long-Term Growth.” American Economic Journal: Macroeconomics, 2(4): 158-94.

[42] Park, W. G. 2008. “International Patent Protection: 1960-2005”. Research Policy, 37:761-766.

[43] Rodriguez, F. and D. Rodrik. 2001. “Trade Policy and Economic Growth: A Sceptic’s Guide to the Cross-National Evidence?”NBER Macroeconomics Annual 2000. Cambridge, MA: MIT Press, pp. 261-324.

[44] Romer, P. and C. Jones. 2010. “The New Kaldor Facts: Ideas, Institutions, Population and Human Capital”. Forthcoming, American Economic Journal: Macroeconomics.

[45] Rose, A. K. 2006. “Size Really Doesn’t Matter: In Search of a National Scale Effect”. NBER Working Paper No. 12191.

[46] Sachs, J. and A. Warner. 1995. “Economic Reform and the Process of Global Integra-tion”, Brookings Papers on Economic Activity, 1: 1-118.

[47] Schmoch, U. F. Laville, P. Patel, R. Frietsch. 2003. “Linking Technology Areas to Industrial Sectors”. European Commision, DG Research.

[48] Wang, Z. and S.J. Wei, 2008. “What Accounts For the Rising Sophistication of China’s Exports?” NBER working paper No. 13771.

[49] Wang, Z., S. J. Wei and A. Wong, 2010. “Does a Leapfrogging Growth Strategy Raise Growth Rate? Some International Evidence”. NBER working paper No. 16390.

A The Firm's Optimal R&D Decision and General Equilibrium Conditions

We solve the firm's R&D decision in the Balanced Growth Path (BGP) equilibrium. We adopt the guess-and-verify method to solve the all-sector firm's problem. Guess that the value of a firm is a linear combination of its accessible knowledge capital in all the sectors:

$$V(z_t) = \sum_{j=1}^{K} \left(v_t^j \frac{z_t^j}{n_t^j}. \right)$$

Substituting it back to the Bellman equation (8), we get

$$V(z_t) = \sum_{j=1}^{K} (\pi_t^j \frac{z_t^j}{n_t^j}) - \sum_{i=1}^{K} \sum_{j=1}^{K} \left(R_t^{ij} \right) + \beta \sum_{j=1}^{K} (v_{t+1}^j \frac{z_t^j + \sum_{i=1}^{K} \left[A^{ji} \left(\bar{z}_t^j R_{f,t}^{ji} \right)^{\alpha} \left(z_t^i \right)^{1-\alpha} \right]}{n_{t+1}^j}). \tag{22}$$

The first order condition with respect to $R_{f,t}^{ij}$ is:

$$R_t^{ij} = \frac{n_t^j}{n_t^i} \left(A^{ij} \alpha \rho_t^i v_{t+1}^i \right)^{\frac{1}{1-\alpha}} \frac{z_t^j}{n_t^j}. \tag{23}$$

where $\rho_t^j = \beta \frac{n_t^j}{n_{t+1}^j}$. Substituting the optimal R&D in (23) back to (22), we get:

$$\begin{aligned} \sum_{j=1}^{K} \left(v_{t+1}^j \frac{z_t^j}{n_t^j} \right) &= \sum_{j=1}^{K} \left(\pi^j \frac{z_t^j}{n_t^j} \right) - \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{n_t^j}{n_t^i} \left(A^{ij} \alpha \rho_t^i v_{t+1}^i \right)^{\frac{1}{1-\alpha}} \left(\frac{z_t^j}{n_t^j} \right) \\ &+ \beta [\sum_{j=1}^{K} \frac{v_{t+1}^j z_t^j}{n_{t+1}^j} + \sum_{j=1}^{K} \sum_{i=1}^{K} \frac{v_{t+1}^i}{n_{t+1}^i} A^{ij} \left(A^{ij} \alpha \rho_t^i v_{t+1}^i \right)^{\frac{\alpha}{1-\alpha}} \left(z_t^j \right)]. \end{aligned}$$

Comparing the coefficients of z_t^j on both sides, we have

$$\frac{v_t^j}{n_t^j} = \frac{\pi_t^j}{n_t^j} - \sum_{i=1}^{K} \frac{n_t^j}{n_t^i} \left(A^{ij} \alpha \rho_t^i v_{t+1}^i \right)^{\frac{1}{1-\alpha}} \frac{1}{n_t^j} + \beta \sum_{i=1}^{K} A^{ij} \left(A^{ij} \alpha \rho_t^i v_{t+1}^i \right)^{\frac{\alpha}{1-\alpha}} \frac{v_{t+1}^i}{n_{t+1}^i} + \beta \frac{v_{t+1}^j}{n_{t+1}^j}.$$

The transversality condition takes the form

$$\lim_{T \to \infty} \prod_{t=0}^{T} \beta^T \frac{v_T^i}{n_T^i} = 0, \forall i.$$

In a stationary BGP equilibrium, the sectoral knowledge values and the application value of knowledge j to i are all constant, i.e. $v_t^i = v^i, u_t^i = u^i$, (to be proved later). Now we get:

$$v^j = (1 - \rho_t^j)^{-1} [\pi^j + \frac{1-\alpha}{\alpha} \sum_{i=1}^{K} \frac{n_t^j}{n_t^i} \left(A^{ij} \alpha \rho_t^i v^i \right)^{\frac{1}{1-\alpha}}]$$

To simplify the notations, define the value of sector-j knowledge in contributing to innovations in sector i for the representative firm as

$$\omega^{ij} = \frac{1-\alpha}{\alpha}\frac{n_t^j}{n_t^i}\left(A^{ij}\alpha\rho_t^i v^i\right)^{\frac{1}{1-\alpha}}$$

Substituting it back, we have

$$v^j = (1-\rho_t^j)^{-1}(\pi^j + \sum_{i=1}^{K}\omega_t^{ij}),$$

and given that firms are identical with measure one

$$R_t^{ij} = \frac{\alpha}{1-\alpha}\omega_t^{ij}\frac{z_t^j}{n_t^j} = \frac{\alpha}{1-\alpha}\omega_t^{ij}.$$

To prove that $\rho_t^j, v_t^j, u_t^j, \omega_t^{ij}$ are all constant, we first need to show that the innovation rates across sectors are the same on the BGP; therefore, we need to show $\frac{n_t^i}{n_t^j} = \frac{n^i}{n^j}, \forall t$.

The evolution of the number of varieties in sector i is given by:

$$\begin{aligned} n_{t+1}^i &= n_t^i + \triangle z_t^i && (24) \\ &= n_t^i + \sum_{j=1}^{K}(A^{ij})^{\frac{1}{1-\alpha}}\left(\alpha\rho_t^i v^i\right)^{\frac{\alpha}{1-\alpha}} z_t^j \\ &= n_t^i + \sum_{j=1}^{K}\left[(A^{ij})^{\frac{1}{1-\alpha}}\left(\frac{\alpha\beta v^i}{g_t+1}\right)^{\frac{\alpha}{1-\alpha}}\right] n_t^j. \end{aligned}$$

The innovation rate (the growth rate of varieties) in sector i is $g_t^i = n_{t+1}^i/n_t^i - 1$. Rearranging the terms, we have

$$g_t^i(1+g_t^i)^{\frac{\alpha}{1-\alpha}} = \left(\alpha\beta v^i\right)^{\frac{\alpha}{1-\alpha}}\sum_{j=1}^{K}\left(A^{ij}\right)^{\frac{1}{1-\alpha}}\left(\frac{n_t^j}{n_t^i}\right), \quad (25)$$

On the BGP, the number of goods in every sector grows at the same speed, because inter-sector knowledge spillovers keep all sectors on the same track. To see tihs, suppose sector i had been growing more slowly than other sectors for a lengthy period, its number of varieties would be extremely small relative to other sectors. As long as $\exists j$, such that $A^{ij} > 0$, for $\forall i$, (25) implies that the cross-sector knowledge spillovers would increase g_t^i tremendously through a large ratio n_t^j/n_t^i until g_t^i is the same as the innovation rates in other sectors. And vice versa for sectors starting with a faster growth rate. Therefore, in the BGP equilibrium, $g^i = g^j = g$, and the distribution of the sector is stable and rank-preserving. Therefore, $\frac{n_t^i}{n_t^j} = \frac{n^i}{n^j}, \forall t$, and $i, j = 1, 2, ..., K$.

This result implies that $\rho_t^j = \beta/g \equiv \rho$ and $\omega_t^{ij} \equiv \omega^{ij}$ are both constants, consistent with our original guess. Therefore, we have (9), (10) and (12). Now we can verify our previous guess that the all-sector firm's value is a linear constant-coefficient combination of its knowledge in all sectors.

Re-arranging (18) implies the common innovation rate as

$$g = \frac{1}{(1-\alpha)\rho}\sum_{j=1}^{K}\frac{\omega^{ij}}{v^i}. \quad (26)$$

Based on (9), we can rewrite the equation above as

$$g = \frac{1-\rho}{(1-\alpha)\rho} \frac{\sum_{i=1}^{K} \sum_{j=1}^{K} \omega^{ij}}{\sum_{i=1}^{K} \pi^i + \sum_{i=1}^{K} \sum_{j=1}^{K} \omega^{ij}}, \tag{27}$$

Substituting out $\rho = \beta/g$ leads to (27). After rearranging the terms, we get (18).

The sectoral research effort is given by: $R^i = \sum_{j=1}^{K} R^{ij}$. Substitute the optimal R&D expenditure (12) and (18) into the equation, we have

$$R^i = \alpha\rho g v^i.$$

Therefore, we get (16): $R^i/R^j = v^i/v^j$ (Proposition 1).

B Solving the Model: Collated GE Conditions

The model specified previously offers closed-form solutions at the BGP equilibrium. This section provides the collated list of equilibrium conditions that are used to compute the model economy in Section 2.6. Given the parameter values $\{\beta, \alpha, \sigma, (s^i)_{1\times K}, (A^{ij})_{K\times K}, L, L^*, \tau, \tau^*, \phi, \phi^*\}$, we solve for $3(K+1)$ number of unknowns: $(v^i)_{K\times 1}$, $(\pi^i)_{K\times 1}, (n^i/n^{i*})_{K\times 1}$, M, g and w/w^* in the general equilibrium. They are determined by the exact same numbers of equations, including K numbers of equations determining the per-firm knowledge value in each sector,

$$v^j = (1-\rho)^{-1} \left[\pi^j + \sum_{i=1}^{K} \frac{1-\alpha}{\alpha} \frac{n^j}{n^i} \left(A^{ij} \alpha\rho v^i \right)^{\frac{1}{1-\alpha}} \right],$$

where $\rho = \beta/g$, and K equations specifying the sectoral aggregate profit, based on (5) and (**??**):

$$\pi^i = \frac{s^i}{\sigma M} \left[\frac{L}{1 + \frac{n^{i*}}{n^i} \left(\frac{\phi w^*}{\phi^* w} \right)^{1-\sigma} (\tau^*)^{1-\sigma}} + \frac{\frac{w^*}{w} L^*}{1 + \frac{n^{i*}}{n^i} \left(\frac{\phi w^*}{\phi^* w} \right)^{1-\sigma} \tau^{\sigma-1}} \right],$$

K number of equations on aggregate growth rate, based on (10) and (26):

$$g = 1 + \left(\alpha\rho v^i \right)^{\frac{\alpha}{1-\alpha}} \sum_{j=1}^{K} (A^{ij})^{\frac{1}{1-\alpha}} \frac{n^j}{n^i},$$

and three aggregate equilibrium conditions specifying the balance of trade condition:

$$\sum_{i=1}^{K} \frac{\frac{w^*}{w} L^*}{1 + \frac{n^{i*}}{n^i} \left(\frac{\phi}{\phi^*} \frac{w^*}{w} \frac{1}{\tau} \right)^{1-\sigma}} = \sum_{i=1}^{K} \frac{L}{1 + \frac{n^i}{n^{i*}} \left(\frac{\phi^*}{\phi} \frac{w}{w^*} \frac{1}{\tau^*} \right)^{1-\sigma}},$$

the labor market clearing condition:

$$L = \sum_{i=1}^{K} \sigma \pi^i M,$$

and the free entry condition:

$$\frac{1}{1-\beta}[\sum_{i=1}^{K}\pi^{i} - \alpha\rho(g-1)\sum_{i=1}^{K}v^{i}] = F.$$