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METHOD OF FORECASTING FINANCIAL BUBBLES

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Abstract: This article describes the method of forecasting financial bubbles in the financial market. Main results are based on solution of the stochastic differential equation of bubble and evaluation of the expectation function of bubble. We prove the statement that the greater volatility and uncertainty in the model, the faster the bubble occurs and create the technique of working model on real data.

Key words: Mathematical model, financial bubble, exponential growth, volatility, security on the stock market, stochastic differential equation.

Volatility in the financial markets which is often predecessors of financial crises is an important event, which is extremely interesting both for academia and practitioners. For example, if we consider the stock market, it is quite difficult to determine the actual equitable price per share, to identify volatility (in other words a "bubble"), and see whether the trader's strategy is effective. Economists determine a bubble as the difference between the actual equitable price and the market price of security. However, it is rare to find "bubble" term in the context of economic and mathematical models.
Suppose bubbles in the securities market satisfy the equation

$$dB(t) = \mu B^m(t)dt + \sigma B(t)d\omega(t) \tag{1}$$

where
B - market price of security which is different from its actual equitable price, the bubble
μ - coefficient of proportionality expected return of the security and its price
σ - volatility
m - degree of exponential growth
$\omega(t)$ - Wiener process
with the initial condition $B(0)=B_0$. This equation is the simplest stochastic differential equation, the solution of which goes to infinity in a finite time t. Thus, investors expect the super-exponential growth of profits but consider the risk of

return does not depend on investment. The solution of equation (1) depends on 4 parameters: μ, σ, m и B_0.
Make the following change of variables

$$Y = B^{-m+1} = \frac{1}{B^{m-1}} \tag{2}$$

By mathematical transformation of equation (1) obtaining

$$dY = (m-1)(-\mu + 0{,}5m\sigma^2 Y)dt - (m-1)\sigma Y d\omega \tag{3}$$

Expectation function

Let us denote $y(t) = M(Y(t)) = M(1/B^{m-1})$. Then we evaluate expectation function of both sides of (3). For this we assume that $Y(t)$ and $d\omega$ are independent, i.e. the increment of the Wiener process does not depend on the current values of the "bubbles" and therefore $M(\sigma(m-1)Yd\omega) = 0$.
Thus obtain:

$$y = \frac{Ce^{0{,}5m\sigma^2(m-1)t} + \mu}{0{,}5m\sigma^2}$$

We use the initial condition and the fact that $B(0)=B_{0,}$ then

$$y = y_0(e^{0{,}5m\sigma^2(m-1)t}(1-\nu) + \nu) \ , \tag{4}$$

where

$$\nu = \frac{\mu B_0^{m-1}}{0{,}5m\sigma^2}$$

Thus, we have a particular solution of the differential equation, which is the expectation function of Y.

Let us prove that in the situation when volatility is increasing the time period of bubble occurrence will tend to zero, i.e. bubble will appear immediately.

If the condition that $\nu > 1$ is satisfied, the time t_c in which the expectation function $Y(t)$ goes to infinity is determined from the formula

$$e^{0{,}5m\sigma^2(m-1)t}(1-\nu) + \nu = 0$$

Expressing this equation of t, obtaining

$$t_c \overset{\sigma \to 0}{\rightarrow} \frac{1}{Y_0^{m-1}(m-1)\mu} = \widetilde{t}_c \tag{5}$$

Now let’s analyze the dependence of the time goes to infinity from a random factor (σ^2 in model). Thus, if limit $\sigma \to \infty$ in (5) then $t_c \xrightarrow{\sigma \to \infty} 0$. This shows us that the unlimited increase of randomness factors makes the bubble appears immediately. The model assumes that bubbles generate crisis situations. So, the panic on the stock market creates bubbles and subsequent crises.

To understand how this model works on real data, it is possible to simplify the model and make the assumption that in the context of the bubble growth, the price of the asset one time begins to grow super-exponentially. Thus, we can conclude that the solution varies near a hyperbola.

Thus, assume that the solution satisfies the following equation

$$P(t) = \frac{C}{(t^* - t)^m} \qquad (6),$$

where t^*-t – the considered period of time of the closing stock market prices t^* assumes evaluation of different length and time period that could be changed, which increases the accuracy and probability of the forecast of bubbles. Then the parameters m and C should be estimates by LS method.

$$\begin{cases} \hat{m} = \dfrac{Cov(\log(t^* - t), \log P(t))}{Var(\log(t^* - t))} \\ \log \hat{C} = \log \overline{P}(t) - m \log(t^* - \bar{t}) \end{cases}$$

And then errors should be counted at step t and $t + 1$

$$\varepsilon^2{}_t = \frac{(\log P(\bar{t}) - \log \hat{C} - \hat{m} \log(t^* - t))^2}{n-1}, \qquad (7)$$

$$\varepsilon_{t+1}{}^2 = \frac{\log P(t+1) - \log \hat{C} - \hat{m} \log(t^* - t + 1)}{n-1} \qquad (8)$$

where n – the number of approximated values.

According to the results obtained in the previous steps we can make the distribution of errors and calculate the quantile required.

So we could predict forecast growth of bubble in the next step by substituting the coefficients and taking into account falling into the quantile of the distribution.

In the situation then the price dynamics obeys the exponential distribution law and the bubble exists, we can conclude that it may be a kind of signal of a possible start of a crisis on the securities market.

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