Investment Portfolio Formation for the Pension Fund of Russia

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Abstract In this paper a simulation model of the formation of an effective investment portfolio had estimated for the different conditions and states of the economy on the basis of selected stocks of large and reliable companies with stable rates of earnings in the Russian and American markets. Further work is aimed at assessing the method of simulation time interval during which compiled a portfolio is efficient.

In the Russian Federation during the last 10 years, measures were taken to improve the pension system. However, as part of the pension reform is not achieved long-term financial stability and balance of the Pension Fund. Funded pillar of the pension system requires substantial modernization, especially as it relates to the financial sustainability of pension funds and the creation of an effective system of investment, as well as expanding the list of financial institutions and instruments.

In this article we are going to construct a portfolio of the pension fund based on estimates future profitability of financial assets. Portfolio theory was initiated by economist Harry Markowitz and imagined MPT in 1952.

Modern portfolio theory (MPT) is a theory of investment which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. Although MPT is widely used in

practice in the financial industry and several of its creators won a Nobel memorial prize for the theory.

MPT is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. That this is possible can be seen intuitively because different types of assets often change in value in opposite ways.

More technically, MPT models an asset's return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, MPT seeks to reduce the total variance of the portfolio return. MPT also assumes that investors are rational and markets are efficient.

In other words we take this model because it is a theory on how risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward.

The future values of the assets will are estimated using the Sharpe Model.

The Sharpe Model is based on the method of linear regression analysis to link the two random variables. The independent variable X and the dependent variable Y connected by a linear expression. In the Sharpe Model independent variable is the market index. For the dependent variable takes the profitability of the financial instrument.

A formal statement of the Sharpe Model is given below

$$r_i = \alpha_{iI} + \beta_{iI} \cdot r_I + \varepsilon_{iI}$$

Where r_i is profitability of security i for given period of time, r_I is profitability for market indices I for the same period, ε_{iI} is the disturbance term, α_{iI}, β_{iI} are the coefficients.

To achieve the goal were selected stocks of large, liquid and reliable companies with stable income to the Russian and American markets for the period from 2006 to 2012

Table 1. List of Assets

The Russian Market	**The U.S. Market**
(GAZP)	Apple Inc. (AAPL)
(GMKN)	Cisco Systems, Inc.
(LKOH)	(CSCO)
(SIBN)	Google Inc. (GOOG)
(AFLT)	Microsoft Corporation
	(MSFT)
	Coca-Cola (KO)

The basic assumptions of the model are following.

1. Investors are assessing investment portfolios based on the expected return and standard deviation for the period of ownership.

2. There is a risk-free interest rate at which the investor can lend (invest), or take the money.

3. Taxes and transaction costs are insignificant.

4. Investors are never oversaturated. When you select a portfolio they choose the one that gives the highest expected returns (at equal risk).

5. Investors do not want to risk it. They will select a portfolio that has the lowest standard deviation (with equal expected returns.)

The expected return of the portfolio is

$$r_n = E(R_n) = \sum_{i=1}^{5} \nu_i r_i = \vec{\nu}^T \vec{r}$$

Portfolio risk is defined by the formula

$$\sigma_n = \sqrt{\sum_{i.j=1}^{5} v_i v_j \rho_{ij} \sigma_i \sigma_j} = \sqrt{\vec{v}^T \cdot V \cdot \vec{v}}$$

where ρ_{ij} is correlation coefficient between random variables R_i and R_j, V is covariance matrix of the random variables R_i.

In matrix form, the problem can be written as

$$\begin{cases} \vec{v}^T \cdot V \cdot \vec{v} \to \min \\ \vec{v}^T \cdot (\vec{r} - r_0 \vec{I}) = r^* - r_0 \end{cases}$$

where r_0 is risk-free rate of return of the asset (for the study will take it at 4%), r * - given the expected rate of return (equal to 12%), $\vec{I} = (1,...,1)^T$ is column vector of dimension 5.

The formula for determining the effective interest of the form

$$\vec{v} = \frac{r^* - r_0}{(\vec{r} - r_0 \vec{I})^T V^{-1} (\vec{r} - r_0 \vec{I})} \cdot V^{-1} \cdot (\vec{r} - r_0 I)$$

The effective portions of shares for the portfolio are calculated by this formula. For the investment of pension fund portfolios is important to know the period during which the portfolio will be composed effective. To determine this period were considered different options for the shares for each asset and calculate the mean return and risk.

Results for some periods are presented in Table 2 for the training sample and in Table 3 for the control sample.

Table 2. The results on the learning sample

Period of time	**Profit**		**Risk**	
	Russia	U.S.	Russia	U.S.
Before the Crisis	12%	12%	0.0021	0.0019
During the Crisis	12%	12%	0.0014	0.0028
After the Crisis	12%	12%	0.0012	0.0014

Table 3. The results for the control sample

Russia				U.S.			
Before the Crisis							
the Learning Sample							
Period of time	**Risk**	**Daily Profit**	**Annual Profitabiliy**	**Period of time**	**Risk**	**Daily Profit**	**Annual Profitabiliy**
12 months	0.0021	0.033%	12%	12 months	0.0019227	0.033%	12%
the Control Sample							
1 month	0.00201	0.136%	49.495%	1 month	0.00158	0.008%	2.891%
2 months	0.00199	0.083%	30.241%	2 months	0.00198	0.002%	0.850%
3 months	0.00196	0.081%	29.427%	3 months	0.00189	0.018%	6.586%
6 months	0.00202	0.047%	17.026%	6 months	0.00189	0.021%	7.501%
9 months	0.00190	0.047%	17.055%	9 months	0.00199	0.026%	9.665%
12 months	0.00191	0.031%	11.175%	12 months	0.00202	0.023%	8.270%
During the Crisis							
the Learning Sample							
12 months	0.00135	0.033%	12%	12 months	0.00284	0.033%	12%
the Control Sample							
1 month	0.00250	0.016%	5.971%	1 month	0.00230	-0.010%	-3.563%
2 months	0.00190	-0.008%	-2.794%	2 months	0.00228	0.030%	10.835%
3 months	0.00182	-0.007%	-2.622%	3 months	0.00227	-0.014%	-5.233%
6 months	0.00161	-0.001%	-0.208%	6 months	0.00188	-0.018%	-6.578%
9 months	0.00149	-0.005%	-1.842%	9 months	0.00168	-0.016%	-5.741%
12 months	0.00144	-0.008%	-2.749%	12 months	0.00153	-0.020%	-7.145%

After the Crisis							
the Learning Sample							
12 months	0.0012	0.033%	12%	12 months	0.0014	0.033%	12%
the Control Sample							
1 month	0.0012	-0.051%	-18.51%	1 month	0.0011	0.002%	0.85%
2 months	0.0018	-0.027%	-9.76%	2 months	0.0014	0.029%	10.46%
3 months	0.0016	-0.010%	-3.49%	3 months	0.0014	0.027%	9.89%
6 months	0.0014	0.000%	0.14%	6 months	0.0013	0.032%	11.65%
9 months	0.0015	0.003%	1.08%	9 months	0.0014	0.033%	11.93%
12 months	0.0017	-0.011%	-4.00%	12 months	0.0015	0.025%	9.28%

As can be seen from these results, in both cases, a portfolio designed for the control sample was efficient. It should be noted that if an investor wants to get the same rate of return, which he used in the construction of the portfolio using the training set, it must create a portfolio of U.S. stocks and keep it 6 months. In compiling the portfolio at 12 months in the U.S. investor gets a lower yield (9.283%).

Based on the results, we can draw the following **conclusions:**

Investing in U.S. stocks, to obtain the desired yield (12%) during the crisis and after the crisis, the investor will have to take on more risk than investing in Russian equities.

In Russia before the crisis increased the expected return for periods 1,3,6 and 9 months (an increase of 5% to 37%). Only for a period of 12 months, this indicator decreased (-1% - looming crisis). The risk for the entire control sample decreased. This is due primarily to the growth of the entire economy of Russia (2006-2007 years). Therefore, the series in question do not pass the test for stationary. However, the portfolio remained effective throughout the control sample.

In the United States before the crisis there was a decline, both risk and expected return. As well as reducing yield was low (-5% to -4%), the prepared portfolio still remains effective. Thus, the article discussed the construction of

efficient portfolio can be used for economic growth for the period of 9 months in Russia, and up to a year in the U.S.

During the crisis, nearly all occasions portfolio was ineffective. There was a negative return of -19% to -12%. Thus, during the crisis portfolio can only be used for purchase of shares for 1 month in the U.S. market.

In the post-crisis period, almost all of the time series were stationary. In almost all cases the Russian market portfolio was ineffective because instability has affected the Russian economy.

The reverse situation was observed in the United States. It is important to note that for a period of 6 and 9 months the portfolio return was 12% per annum, that is the same as in the training set. Thus, if an investor wants to get the same rate of return, which he used in constructing the portfolio, the training set, it must create a portfolio of U.S. stocks and keep it 6 or 9 months. The portfolio must be created in a steady-state period.