# Trade Elasticities in General Equilibrium

Farid Farrokhi and Anson Soderbery*

May 31, 2020

**Abstract**

For a class of widely used general equilibrium trade models, we derive the export supply elasticity and analyze its importance in welfare and policy analysis. We show export supply is disciplined by three key microeconomic channels: 1) internal and external returns to scale for production, 2) the extent of labor mobility across industries, and 3) relative substitutability of exports across destinations (i.e., elasticities of demand). We demonstrate how export supply encapsulates these elasticities sufficiently to perform counterfactual equilibrium analysis. We then develop a structural heteroskedastic estimator of the model that requires only a time series of bilateral trade and production data. Applying our methodology to publicly available trade data from 1994-2017, we estimate the sufficient set of parameters for equilibrium analysis. Our estimates provide insight into countries and industries that are more or less sensitive to economic shocks such as tariffs. We employ our estimated model to analyze recent US protectionist policies to illustrate the role of underlying microeconomic channels and general equilibrium linkages. Finally, we examine the extent to which microeconomic channels of the model shape the gains from trade and home market effects across countries and industries.

*Keywords*: Trade elasticities, Returns to scale, Gains from trade, Tariff pass-through
*JEL Classification*: F12, F14, F59

*Farrokhi: Department of Economics, Krannert School of Management, Purdue University, ffarrokh@purdue.edu. Soderbery: Department of Economics, Krannert School of Management, Purdue University, asoderbe@purdue.edu. This paper has benefited immensely from discussions with David Hummels, Ahmad Lashkaripour, and Chong Xiang. We are additionally grateful to seminar participants at Purdue University and Vanderbilt University. All errors and omissions are our own.

## 1 Introduction

Trade-related policy analyses range from examining traditional questions (e.g., gains from trade) to tracing shocks through economies (e.g., the implications of recent US protectionist policies). These analyses require general equilibrium modeling to evaluate economies as a whole. Analyzing general equilibrium hinges on the elasticities that discipline the interconnections within and across industries, markets, and economies. The literature has worked vigorously to establish the importance of, and estimate elasticities governing; (1) internal and external economies of scale, (2) labor mobility across industries, and (3) market demand across countries and industries. However, existing studies analyze or estimate only a subset of these elasticities in isolation. As a result, estimating these elasticities jointly in a way directly derived from general equilibrium modeling has remained an incomplete task.

In this paper, we develop a method that simultaneously and flexibly models these key channels and applies the structure of the model to consistently estimate the elasticities governing these channels. We apply our results to a number of classic questions in international trade – including gains from trade and home market effects. We also leverage the flexibility of our model and estimates in order to examine microeconomic and macroeconomic implications of recent US protectionist trade policies as a specific application.

The cornerstone of our analysis is our derivation of export supply for a general class of trade models incorporating the three aforementioned microeconomic channels. We demonstrate that elasticities governing the channels of the model can be efficiently combined into a smaller set of parameters constituting elasticities of export supply. Export supply coalesces a wide range of general equilibrium models in a way that is sufficient to predict changes to the full vector of prices and quantities globally in response to shocks. In essence, formalizing export supply characterizes general equilibrium allocations both within and across industries. Put more plainly, we show that export supply encapsulates enough information about general equilibrium allocations that, when combined with import demand, it is sufficient for performing counterfactual analysis of trade models in general equilibrium without calibrating or even observing factor markets empirically. We achieve these results by recasting the global general equilibrium problem as one of supply and demand in product markets rather than the usual treatment of it in factor markets.

Our methodology is designed for flexibility. To illustrate its range, we use the model to structurally estimate elasticities for every observable country and industry in the world. We show that widely accessible data on international trade, industrial production, and tariff data are sufficient for estimation. Specifically, we apply our methodology to data combining CEPII-BACI (trade flows), UNIDO (production), and MacMap (tariffs) from 1994-2017. We then provide a focused application of the model and estimates through an analysis of the recent US China trade war.

Chinese industries are estimated to be particularly resilient to tariffs from the US when evaluated based on partial equilibrium analysis. That is, export supply elasticities of Chinese products to the US are nearly perfectly elastic. This result confirms the recent findings of perfect pass-through of prices onto US consumers (c.f., Amiti et al. (2019) and Fajgelbaum et al. (2019)). However, our general equilibrium analysis provides interesting contrast to the partial equilibrium results. In general equilibrium, pass-through rates are almost complete if tariffs were imposed on an isolated Chinese industry. However, the application of tariffs by the US simultaneously against Chinese manufacturing industries alters the outcome. Our quantitative results yield strong tariff complementarities that are in line with recent theories of optimal policy (e.g., Costinot et al. (2015), Beshkar and Lashkaripour (2016)). Specifically, in general equilibrium when tariffs are applied simultaneously across industries complementarities effectively increase importer market power. Crucially, complementarities rely on the underlying parameters governing the costs of factor reallocation in the exporting country. Our formalization of export supply embodies these channels and allows us to trace reallocations throughout the economy.

Given our estimates, we find tariff complementarities drive down pass-through of tariffs by Chinese exporters from nearly 100% to only around 70% on average. Intuitively, under more comprehensive tariffs simultaneously applied by the US, China cannot readily allocate resources away from targeted industries. This inability to escape the policy effects is then absorbed by the exporter through a lowering of shipped prices in response to tariffs. We thus expect additional distortions stemming from the US China trade war as both countries reallocate resources moving forward.

Our model and estimation reconciles a number of older and emerging strands of the literature that examine the aggregate implications of microeconomic channels, (e.g., Arkolakis et al. (2012), Melitz and Redding (2014)). Specifically, we speak directly to a growing interest in the role of scale economies either external (e.g., Kucheryavyy et al. (2016), Bartelme et al. (2018)) or internal (e.g., Lashkaripour and Lugovskyy (2018)), and the degree of labor mobility (e.g., Galle et al. (2017), Adao et al. (2018)) by incorporating these channels into a unified framework. We spell out the limitations imposed on export supply elasticities across these models as well as benchmark frameworks of Eaton and Kortum (2002) and Krugman (1980). More specifically, export supply elasticities summarize information of microeconomic channels through a combination of two sub-elasticities; one that governs the slope of total supply and the other manages excess supply (i.e., how total supply net of exports to all other markets reacts to prices). We show that these sub-elasticities together with the standard market demand elasticities are sufficient to conduct comparative statics analysis. This result allows us to perform quantitative policy analysis without the entire set of parameters required by the microeconomic channels and contrast our results with more restrictive models from the literature.

Empirically, we contribute a structural estimator building on heteroskedastic methods to identify supply and demand. Heteroskedastic estimators in the international trade literature are lacking a model consistent export supply curve. This limitation is responsible for a gap between policy and welfare analysis in this literature and the tradition of general equilibrium analysis. We show that a model consistent export supply elasticity resembles the ad hoc, albeit intuitive, iso-elastic curve championed by Feenstra (1994) and applied by works such as Broda and Weinstein (2006). However, key differences emerge. First, Feenstra (1994) assumes the export supply elasticity is common across exporters for each good. We show that underlying export supply elasticities are parameters governing returns to scale, the extent of labor mobility across industries, and import demand. The interaction of these parameters demonstrates export supply elasticities are importer-exporter-product specific. In addition, each export supply elasticity is a weighted sum of iso-elastic sub-elasticities where the weights depend on the share of sales by the exporting country across markets that is structurally endogenous and time-varying.

Utilizing these insights, we develop a consistent structural estimator of the model that borrows from Soderbery (2018)'s generalization of Feenstra (1994). We demonstrate how the export supply curve derived from the model introduces functional form restrictions when compared to the existing literature, but relaxes the estimable range of export supply elasticities. Specifically, we do not constrain elasticities to the positive orthant (i.e., where import demand slopes down and export supply slopes up). This is in line with Costinot et al. (2019), who demonstrated a tendency of common trade theories to generate downward sloping export supply. They discuss the impact of this insight on home market effects through an application to international trade in pharmaceuticals. Our analysis is more general and expansive, as the structural method we develop uses only publicly available data on international trade and production and yields estimates for every country and industry present in the data. However, our results echo their findings. We find that home market effects are broadly supported by our estimates, but vary in strength across countries and industries.

More broadly, we complement our examination of the effects from the recent US tariffs on prices and welfare with three other exercises. First, we report the gains from trade as a vehicle to compare our model with benchmarks in the literature. Second, we leverage the channels embodied to export supply to more broadly decompose general equilibrium reallocations and welfare in response to trade liberalization. Third, we compute a series of quantitative exercises to identify industry-country pairs that feature weak or strong home market effects following Costinot et al. (2019).

Finally, we acknowledge that parameters governing the deeper microeconomic channels of the model (i.e., returns to scale and labor mobility) might be of independent interest. We thus demonstrate how to project our unconstrained estimates of export supply and import demand elasticities onto the functional form constraints of the model in order to uncover the underlying parameters of returns to scale and labor mobility. Uncovering these parameters in turn helps us further decompose aggregate effects into their underlying disaggregated channels.

We proceed as follows. Section **2** presents our model. We derive export supply elasticities, demonstrate sufficient elasticities for quantitative analyses, and compare them across several commonly used models nested by our framework. Section **3** shows how to structurally estimate the model. Section **4** applies our estimates to general equilibrium analyses and counterfactuals centered around recent US tariffs. Section **5** concludes.

## 2 Theory

The global economy consists of multiple countries, indexed by $i$ or $n \in N$, and multiple industries, indexed by $k \in K$. Labor is the only factor of production, and every country $n$ is endowed by a given supply of $L_n$ workers. In each industry, goods are differentiated by country of origin, and within each country by firms that produce differentiated varieties. Markets are characterized by monopolistic competition.

### 2.1 Preferences

The representative consumer in country $n$ receives utility $C_n$ as a Cobb-Douglas combination of aggregate industry level goods,

$$C_n = \prod_{k \in K} C_{n,k}^{\beta_{n,k}}$$

where $\beta_{n,k}$ is the expenditure share in $n$ on good $k$ with $\sum_{k \in K} \beta_{n,k} = 1$. Varieties originating from countries indexed by $i$ are bundled via CES to form the product composite $C_{n,k}$ as,

$$C_{n,k} = \Big[ \sum_{i \in N} b_{ni,k}^{\frac{1}{\sigma_{n,k}}} C_{ni,k}^{\frac{\sigma_{n,k}-1}{\sigma_{n,k}}} \Big]^{\frac{\sigma_{n,k}}{\sigma_{n,k}-1}}.$$

The Armington (1969) elasticity of substitution across exporters $i$ within industry $k$ in the eyes of consumers in market $n$ is $\sigma_{n,k}$. We allow for a variety level demand shifter $b_{ni,k}$ that is importer-exporter-product specific. Lastly, firms within $i$ are denoted by $\omega$. The variety level composite $C_{ni,k}$ is a CES aggregation across $C_{ni,k}(\omega)$ as differentiated varieties of product $k$ produced by firms in $i$ exporting to market $n$,

$$C_{ni,k} = \Big[ \int_{\omega \in \Omega_{in,k}} C_{ni,k}(\omega)^{\frac{\eta_{i,k}-1}{\eta_{i,k}}} d\omega \Big]^{\frac{\eta_{i,k}}{\eta_{i,k}-1}}.$$

Here, $\Omega_{ni,k}$ is the set of varieties sold from origin $(i,k)$ to market $n$, and $\eta_{i,k}$ is the elasticity of substitution across varieties within industry $k$ in country $i$. This demand system is standard in the literature. To get a brief sense of its positioning, suppose $\sigma_{n,k} = \eta_{i,k} = \bar{\sigma}_k$, then varieties are differentiated to the same extent across countries and across firms within a country as in a standard multi-sector Krugman (1980) model. Alternatively, when varieties are perfect substitutes, $\eta_{i,k} \to \infty$, the demand system is as in Eaton and Kortum (2002).

### 2.2 Resource Allocation across Industries

Workers are imperfectly mobile across industries and immobile across countries. A worker $z$ in country $i$ is endowed by a vector of efficiency units $(z_1 e_{i,1}, ...z_k e_{i,k}, ..., z_K e_{i,K})$ across industries. $z_k$ is a random variable drawn independently from a Fréchet distribution with dispersion parameter $\varepsilon_i > 1$, and a scale parameter normalized to ensure $\mathbb{E}[z_{i,k}e_{i,k}] = e_{i,k}$. We denote wage per unit of efficiency in industry $(i,k)$ by $w_{i,k}$. The share of workers who select industry $k$ is given by $L_{i,k}/L_i = e_{i,k}w_{i,k}^{\varepsilon_i}\Phi_i^{-\varepsilon_i}$, where

$$\Phi_i \equiv \Big[\sum_{k\in K} e_{i,k}w_{i,k}^{\varepsilon_i}\Big]^{1/\varepsilon_i}. \tag{1}$$

Aggregate efficiency units supplied to industry $(i,k)$ are given by $E_{i,k} = L_i\Phi_i^{1-\varepsilon_i}e_{i,k}w_{i,k}^{\varepsilon_i-1}$. The elasticity of labor mobility across industries with respect to wage per unit of efficiency is governed by $\varepsilon_i$. To provide some insight, if $\varepsilon_i \to \infty$, then the variance of efficiency draws across industries for a worker converges to zero. As such, the model collapses to the one with perfect labor mobility. In the other extreme, as $\varepsilon_i \to 1$, our framework collapses to a specific factor model in which efficiency units employed in every industry is inelastically given. Total income in country $i$ then equals total payments to workers, $\Sigma_k w_{i,k}E_{i,k} = L_i\Phi_i$, and $\Phi_i$ is thus income per capita.

### 2.3 Production, Wedges, and Returns to Scale

Total units of efficiency required to produce $q_{ni,k}(\omega)$ units of $\omega$ of variety $(i,k)$ to be delivered at market $n$ is $f_{ni,k} + d_{ni,k}q_{ni,k}(\omega)/A_{i,k}$, where $d_{ni,k} \geq 1$ is the standard iceberg trade cost, satisfying the triangle inequality and $d_{ii,k} = 1$. Productivity in industry $(i,k)$, $A_{i,k}$, depends on an exogenous productivity shifter $a_{i,k}$, and total efficiency units employed there $E_{i,k}$,

$$A_{i,k} = a_{i,k}E_{i,k}^{\phi_{i,k}}.$$

Here, $\phi_{i,k}$ governs the extent to which the scale of industry $k$ affects productivity of a firm in that industry. We allow this elasticity to vary by industry and country. Since a firm does not internalize the effect of its production on the industry-level aggregates, every firm takes $A_{i,k}$ as given.

International trade is subject to standard iceberg trade costs, $d_{ni,k}$, and import tariffs $t_{ni,k}$. We denote by $\tau_{ni,k} = d_{ni,k}(1 + t_{ni,k})$ the wedge between price at the location of production, $i$, and that of consumption, $n$. The price of a typical variety $(i,k)$ in destination $n$ then equals

$$p_{ni,k} = \frac{\eta_{i,k}}{\eta_{i,k}-1}\frac{\tau_{ni,k}w_{i,k}}{a_{i,k}E_{i,k}^{\phi_{i,k}}}$$

Holding wage $w_{i,k}$ fixed and if $\phi_{i,k} > 0$, price $p_{ni,k}$ is decreasing in the industry-level scale of employed efficiency units $E_{i,k}$, reflecting *external returns to scale.* $E_{i,k}$ itself depends on wage through labor supply.

Combining, we connect the price of a typical variety to wages at the location of production,

$$p_{ni,k} = \frac{\eta_{i,k}/(\eta_{i,k}-1)}{a_{i,k}(L_i\Phi_i^{1-\varepsilon_i}e_{i,k})^{\phi_{i,k}}}\tau_{ni,k}w_{i,k}^{1-(\varepsilon_i-1)\phi_{i,k}}. \tag{2}$$

A higher wage (i) increases prices directly through costs, and (ii) decreases prices indirectly due to external scale economies. The latter dominates the former if and only if $(\varepsilon_i - 1)\phi_{i,k} > 1$. It is thus possible in general equilibrium, with sufficient strength of external returns to scale and labor mobility, for price ($p_{ni,k}$) to be a decreasing function of the wage ($w_{i,k}$).

In general equilibrium, the number of firms producing varieties of ($i$,$k$) is given by

$$M_{i,k} = E_{i,k}/(\eta_{i,k}F_{i,k}),$$

where $F_{i,k} = \Sigma_{n\in N} f_{ni,k}$. A greater number of firms in an industry scales up the aggregate supply of the industry reflecting *internal returns to scale.* These internal returns are stronger within a country-industry pair $(i, k)$ when product varieties are more differentiated. Specifically, industries with lower $\eta_{i,k}$, will exhibit a greater number of varieties ($M_{i,k}$), all else equal.

The value of gross output of industry $k$ in country $i$, $Y_{i,k}$, and its revenue share, $r_{i,k}$, are given by:

$$Y_{i,k} = L_i\Phi_i^{1-\varepsilon_i}e_{i,k}w_{i,k}^{\varepsilon_i} \tag{3}$$

$$r_{i,k} \equiv \frac{Y_{i,k}}{\sum_k Y_{i,k}} = e_{i,k}w_{i,k}^{\varepsilon_i}\Phi_i^{-\varepsilon_i} \tag{4}$$

We can see how aggregate supply side behavior in the economy is disciplined by three key parameters: the elasticity of resource mobility $\varepsilon_i$, internal returns to scale governed by $1/(\eta_{i,k} - 1)$, and external returns to scale governed by $\phi_{i,k}$.

### 2.4 Price Indices and Trade Shares

The price indices associated with consumption aggregates $C_{ni,k}$, $C_{n,k}$ and $C_n$ are:

$$P_{ni,k} = M_{i,k}^{\frac{1}{1-\eta_{i,k}}} p_{ii,k}\tau_{ni,k} \qquad \text{Variety Level Price Index} \tag{5}$$

$$P_{n,k} = \left(\sum_{i\in N} b_{ni,k}P_{ni,k}^{1-\sigma_{n,k}}\right)^{1/(1-\sigma_{n,k})} \qquad \text{Industry Level Price Index} \tag{6}$$

$$P_n = \prod_{k\in K} P_{n,k}^{\beta_{n,k}} \qquad \text{Country Level Price Index} \tag{7}$$

The share of expenditure of destination $n$ on origin $i$ in industry $k$, denoted by $\pi_{ni,k}$, equals

$$\pi_{ni,k} = b_{ni,k}\Big(P_{ni,k}/P_{n,k}\Big)^{1-\sigma_{n,k}}. \tag{8}$$

With price indices and markets shares, the microeconomic structure of the model is in place such that firm and factor allocations within the economy are characterized.

### 2.5 Equilibrium: Recasting to product markets

At this point the standard treatment in the literature would launch into closing the model through factor market clearing conditions in order to study microeconomic and macroeconomic implications of trade. We suspend such analysis for the time being in order to provide an alternative perspective. We now recast the model as one of supply and demand in product markets.

Looking ahead, simultaneously estimating the underlying microeconomic channels of these models through factor market (re)allocations is hindered by the scarcity of reliable disaggregate wage and employment data across industries and countries. In contrast, data on prices and quantities in product markets are abundant in international trade. Moreover, recasting the model as one of product market supply and demand will be shown to be a useful tool to locate sufficient elasticities required to perform comparative statics analysis (Section **2.6.1**). This approach additionally helps us illustrate key interactions between underlying channels of the model that determine aggregate equilibrium outcomes in response to shocks and policies (e.g., tariff changes). We will further highlight the benefits of a supply and demand perspective when our focus turns to estimation. Ultimately, the overarching gain is our ability to simultaneously estimate all required elasticities for general equilibrium analysis rather than piece together estimates from scarce data or isolated results in the literature.

Consider a destination-origin-industry triple $(n, i, k)$. Let $D_{ni,k}$ denote import demand of $n$ from $i$ in industry $k$, and let $S_{ni,k}$ denote export supply of $i$ to $n$ in industry $k$.[1] The model delivers:

$$D_{ni,k} = \pi_{ni,k}\beta_{n,k}X_n \tag{9}$$

$$S_{ni,k} = Y_{i,k} - \sum_{m\neq n} D_{mi,k}. \tag{10}$$

Total expenditure in country $n$, $X_n$, is the sum of wage incomes, tariff revenues, and trade deficits, which is a fraction $\delta_n$ of wage incomes. As such,

$$X_n = L_n\Phi_n(1+\delta_n) + \sum_{i,k} t_{ni,k}D_{ni,k}. \tag{11}$$

[1] Note, our use of the terminology import and export also includes domestic purchases in case of $n = i$.

An equilibrium consists of prices $\mathbf{p} = [p_{ii,k}]_{i=1,k=1}^{N,K}$ such that Equations (1)-(11) hold, and *goods* market clearing conditions hold for all $n$, $i$, $k$,[2]

$$D_{ni,k}(\mathbf{p}) = S_{ni,k}(\mathbf{p}). \tag{12}$$

Throughout the paper, to distinguish between export supply or import demand schedules and their intersections as equilibrium values of trade, we denote by $X_{ni,k}$ the equilibrium values of trade occurring when $X_{ni,k} = S_{ni,k} = D_{ni,k}$.

### 2.6 Trade Elasticities

Our starting point is to derive export supply as a function of prices and quantities in general equilibrium. Export supply is simple conceptually, but somewhat enigmatic in general equilibrium (especially the off equilibrium supply schedule). Export supply of product $k$ from origin $i$ to destination $n$ is total supply of $(i, k)$ net of sales to all markets other than $n$. We first turn to total supply of product $k$ from origin $i$. Equation (3) gives total supply as a function of wage, $Y_{i,k} = L_i \Phi_i^{1-\varepsilon_i} e_{i,k} w_{i,k}^{\varepsilon_i}$. Replacing wages by prices using Equations (2) and (5), total production as a function of variety-level price index at the location of production $P_{ii,k}$ equals

$$Y_{i,k} = Y_{i,k}^{P} P_{ii,k}^{\frac{\omega_{i,k}^1}{1-\omega_{i,k}^2}}.$$

Here, $Y_{i,k}^P$ is the non-price component of total production.[3] $\omega_{i,k}^1$ is the elasticity of $Y_{i,k}$ with respect to the price of a typical variety at the location of production ($p_{ii,k}$). The elasticity of the variety-level price index at the location of production $P_{ii,k}$ with respect to price of a typical variety there ($p_{ii,k}$) is given by $(1-\omega_{i,k}^2)$. These supply elasticities combine the fundamental elasticities of the model (i.e., internal and external returns to scale and labor mobility) as,

$$\omega_{i,k}^1 \equiv \frac{\partial \ln Y_{i,k}}{\partial \ln p_{ii,k}} = \frac{\varepsilon_i}{1-(\varepsilon_i - 1)\phi_{i,k}} \tag{13}$$

$$\omega_{i,k}^2 \equiv 1 - \frac{\partial \ln P_{ii,k}}{\partial \ln p_{ii,k}} = \frac{(\varepsilon_i - 1)}{1-(\varepsilon_i - 1)\phi_{i,k}} \frac{1}{(\eta_{i,k} - 1)}. \tag{14}$$

[2] An equilibrium in product markets implies that in labor markets and the other way around. Let $\mathbf{w} = [w_{i,k}]_{i=1,k=1}^{N,K}$. We can replace wage and price for each other using equation (2), which is a one-to-one relationship provided that equilibrium is unique. Then, demand and supply for products in country-industry $(i,k)$ are equal if and only if they do so for labor there:

$$D_{ni,k}(\mathbf{p}) = S_{ni,k}(\mathbf{p}) \Leftrightarrow D_{ni,k}(\mathbf{p}) = Y_{i,k}(\mathbf{p}) - \sum_{m \neq n} D_{mi,k}(\mathbf{p}) \Leftrightarrow \sum_{m=1}^{N} D_{mi,k}(\mathbf{p}) = Y_{i,k}(\mathbf{p}) \Leftrightarrow \sum_{m=1}^{N} D_{mi,k}(\mathbf{w}) = Y_{i,k}(\mathbf{w})$$

[3] A detailed derivation of these equations is reported in Appendix **1.3**.

Import demand is more familiar as it falls from the common CES structure. Using Equations (8) and (9), import demand of market $n$ in industry $k$ from producer country $i$ is

$$D_{ni,k} = \underbrace{b_{ni,k}\tau_{ni,k}^{1-\sigma_{n,k}} P_{n,k}{}^{-(1-\sigma_{n,k})}}_{D^P_{ni,k}} P_{ii,k}^{1-\sigma_{n,k}}. \tag{15}$$

Inserting demand, export supply as a function of the price index at the location of exports is then given by

$$\begin{aligned} S_{ni,k} &\equiv Y_{i,k} - \sum_{m\neq n} D_{mi,k} \\ &= Y^P_{i,k} P_{ii,k}^{\frac{\omega^1_{i,k}}{1-\omega^2_{i,k}}} - \sum_{m\neq n} D^P_{mi,k} P_{ii,k}^{1-\sigma_{m,k}}. \end{aligned} \tag{16}$$

Understanding the channels driving export supply is most accessible through the lens of the export supply elasticity. We denote the export supply elasticity by $\omega^S_{ni,k}$ as the partial derivative of $\ln S_{ni,k}$ with respect to $\ln P_{ni,k}$. Since this elasticity is conditional on trade costs $\tau_{ni,k}$, in conjunction with Equation (5), we can derive,

$$\omega^S_{ni,k} \equiv \frac{\partial \ln S_{ni,k}}{\partial \ln P_{ni,k}} = \frac{\frac{\omega^1_{i,k}}{1-\omega^2_{i,k}} Y_{i,k} - \sum_{m\neq n}(1-\sigma_{m,k}) D_{mi,k}}{Y_{i,k} - \sum_{m\neq n} D_{mi,k}}. \tag{17}$$

Equation (17) presents the slope of export supply as a function of price for movements along the curve (potentially off of equilibrium). General equilibrium models of trade deliver export supply in an explicit form only at equilibrium by way of intersecting it with import demand. As such, understanding how export supply operates off the equilibrium point is as valuable as it is challenging.

Interpreting observed data as the baseline equilibrium of our model, we are here deriving the slope of export supply based on an infinitesimal change from the baseline equilibrium point (the intersection of export supply and import demand) to an off-equilibrium point along the export supply curve. To do so, let $\lambda_{ni,k} \equiv S_{ni,k}/Y_{i,k}$ be the share of sales of origin $i$ to destination $n$ in industry $k$, which we refer to as *export penetration.*[4] In the baseline equilibrium, export supply equals import demand, $X_{ni,k} = S_{ni,k} = D_{ni,k}$, and so $Y_{i,k} - \Sigma_{m\neq n} X_{mi,k} = X_{ni,k}$. Therefore,

$$\frac{Y_{i,k}}{(Y_{i,k} - \sum_{m\neq n} X_{mi,k})} = \frac{1}{\lambda_{ni,k}} \quad \text{and} \quad \frac{X_{mi,k}}{(Y_{i,k} - \sum_{m\neq n} X_{mi,k})} = \frac{\lambda_{mi,k}}{\lambda_{ni,k}}.$$

[4]In contrast, $\pi_{ni,k} = \frac{D_{ni,k}}{\sum_i D_{ni,k}}$ denotes the share of expenditures of destination $n$ on origin $i$ in industry $k$, which we refer to as *import penetration.*

Now we can rewrite the export supply elasticity as a function of export penetration and model parameters,[5]

$$\omega^S_{ni,k} = \frac{1}{\lambda_{ni,k}} \frac{\omega^1_{i,k}}{1-\omega^2_{i,k}} - \sum_{m \neq n} \frac{\lambda_{mi,k}}{\lambda_{ni,k}} (1-\sigma_{m,k}) \tag{18}$$

where $\omega^1_{i,k}$ and $\omega^2_{i,k}$ are the sub-elasticities given by (13) and (14). Notice, the export supply elasticity from $i$ to $n$ ($\omega^S_{ni,k}$) depends on the relative importance of market $n$ to $i$'s sales elsewher (i.e., export penetration $\lambda_{ni,k}$). Export supply curves are immediately more elastic in smaller destinations, and perfectly elastic as $\lambda_{ni,k} \to 0$ leads to $\omega^S_{ni,k} \to \infty$. Effectively, this is the relevant assumption underlying a small open economy employed in a large part of the trade literature. Intuitively, if $n$'s consumption of good $k$ is negligible relative to the global consumption of good $k$, shocks to $n$'s imports do not tangibly impact global markets.

Controlling for export penetration ($\lambda_{ni,k}$), the export supply elasticity ($\omega^S_{ni,k}$) contains information about changes to; (1) total production $Y_{i,k}$, whose effect is summarized by $\omega^1_{i,k}$ relative to $(1-\omega^2_{i,k})$ weighted by the inverse of export penetration, and (2) sales elsewhere, which is summarized by a weighted sum of import demand elasticities elsewhere $(1-\sigma_{m,k})$ where weights are relative export shares ($\lambda_{mi,k}/\lambda_{ni,k}$). The former shows that an exporter reacts to a higher price in industry $k$ by reallocating (possibly) more resources to that industry. The latter describes how other markets react to a higher price in $k$ by purchasing less from that industry. In essence, $\omega^1_{i,k}$ governs total supply and $\omega^2_{i,k}$ distributes excess supply such that their ratio embodies the allocations needed to deliver exports of $k$ from $i$ to destination $n$.

This interaction between total supply and sales elsewhere forms and reforms the export supply curve as shocks hit the economy. Equation (18) thus has two immediate implications. First, export supply can be downward sloping due to the interaction between elasticities that govern scale economies and imperfections in resource mobility. Second, the export supply elasticity varies over time in an endogenous manner as export penetrations change over time.

### 2.6.1 Sufficient Elasticities for General Equilibrium Analysis

Notably, recasting the model as one of supply and demand does not detract from general equilibrium analysis. To illustrate the precise set of data and parameters required to perform comparative statics analysis, we show here how supply and demand parameters are sufficient to close the model. This illustration clarifies what elasticities are required from estimation, and the broad usefulness of export supply.

For a generic variable $x$, let $\widehat{x} \equiv x'/x$ denote the ratio of its corresponding value $x'$ in a new equilibrium to that of the baseline equilibrium $x$. Consider a set of shocks, or "policy", as changes to iceberg trade costs $d_{nik,}$ and tariffs $t_{ni,k}$, along with productivity and demand shifters, $\mathscr{P} = \left\{\widehat{d}_{ni,k}, \widehat{t}_{ni,k}, \widehat{a}_{i,k}, \widehat{\beta}_{n,k}, \widehat{b}_{ni,k}\right\}$. We specify baseline equilibrium values as $\mathscr{B} = \left\{X_n, \delta_n, Y_{n,k}, t_{ni,k}, \pi_{ni,k}\right\}$, and tote that the change to trade costs

[5] In addition to our derivation here, we have shown that an approach based on the exact hat algebra yields identical results. We refer the reader to Appendix **1.3** for this supplemental derivation.

are given by $\widehat{\tau}_{ni,k} = \widehat{d}_{ni,k}(1+\widehat{t}_{ni,k}t_{ni,k})/(1+t_{ni,k})$. Then, given a policy $\mathscr{P}$, baseline values $\mathscr{B}$, and parameters $\omega^1_{i,k}$, $\omega^2_{i,k}$, $\sigma_{n,k}$, an *equilibrium in changes* consists of price changes $\widehat{p}_{ii,k}$, such that Equations (19)–(24) hold.

$$\widehat{Y}_{i,k} = \widehat{a}_{i,k}^{\omega^1_{i,k}} \widehat{\Phi}_i^{1-\omega^1_{i,k}} \widehat{p}_{ii,k}^{\omega^1_{i,k}} \qquad \text{(Industry revenue)} \qquad (19)$$

$$\widehat{\Phi}_i = \frac{\sum_{k\in K} \widehat{Y}_{i,k} Y_{i,k}}{\sum_{k\in K} Y_{i,k}} \qquad \text{(Income per capita)} \qquad (20)$$

$$\widehat{X}_n X_n = (1+\delta_n) \sum_k \widehat{Y}_{n,k} Y_{n,k} + \sum_i \sum_k \widehat{t}_{ni,k} t_{ni,k} \widehat{X}_{ni,k} X_{ni,k} \qquad \text{(Total expenditure)} \qquad (21)$$

$$\widehat{P}_{ni,k} = \widehat{a}_{i,k}^{-\omega^2_{i,k}} \widehat{\Phi}_i^{\omega^2_{i,k}} \widehat{p}_{ii,k}^{1-\omega^2_{i,k}} \widehat{\tau}_{ni,k} \qquad \text{(Price index)} \qquad (22)$$

$$\widehat{X}_{ni,k} = \frac{\widehat{b}_{ni,k} \widehat{P}_{ni,k}^{1-\sigma_{n,k}}}{\sum_{\ell\in N} \pi_{n\ell,k} \widehat{b}_{n\ell,k} \widehat{P}_{n\ell,k}^{1-\sigma_{n,k}}} \widehat{\beta}_{n,k} \widehat{X}_n \qquad \text{(Trade flows)} \qquad (23)$$

$$Y_{i,k} \widehat{Y}_{i,k} = \sum_{n\in N} X_{ni,k} \widehat{X}_{ni,k} \qquad \text{(Market clearing)} \qquad (24)$$

Provided that baseline values $\mathscr{B}$ are observed and Equations (19)–(24) have a solution, the set of supply and demand elasticities $\{\omega^1_{i,k}, \omega^2_{i,k}, \sigma_{n,k}\}$ are sufficient for quantifying the full vector of equilibrium changes to prices, trade flows, revenues, and expenditures $\{\widehat{p}_{ii,k}, \widehat{P}_{ni,k}, \widehat{X}_{ni,k}, \widehat{Y}_{i,k}, \widehat{\Phi}_i, \widehat{X}_n\}$ in response to any policy $\mathscr{P}$. In particular, once $\omega^1_{i,k}$ and $\omega^2_{i,k}$ are known, one does not require estimates of the microeconomic elasticities governing labor mobility ($\varepsilon_i$), external ($\phi_{i,k}$) internal ($\eta_{i,k}$) economies of scale to perform counterfactuals.[6]

Given this sufficiency statement, we continue by examining $\omega^1_{i,k}$ and $\omega^2_{i,k}$ across existing models by way of dissecting the elements that form these elasticities. We highlight how differences in underlying channels translate into implications for export supply in general. This analysis informs our empirical methodology to follow. Given their sufficiency, we focus on product markets in order to estimate $\omega^1_{i,k}$, $\omega^2_{i,k}$, $\sigma_{n,k}$. Section **3** will show how to take advantage of our derivation of export supply elasticities to jointly estimate product market supply and demand.

### 2.6.2 Discussion: Across Model Comparisons

To collect general intuition linking export supply to model channels, here we spell out forces at work behind the export supply elasticity by reporting $\omega^1_{i,k}$ and $\omega^2_{i,k}$ in simpler models nested within ours. Table 1 selects a few prominent models from the literature for explicit analysis.[7] We apply the underlying assumptions from

[6] We present equations that define equilibrium in changes with respect to wages $\{w_{i,k}\}$ in Appendix **1.1**. In addition, we have conducted the following numerical cross check. For values $\{\phi_{i,k}, \eta_{i,k}, \varepsilon_i\}$ we calculate $\omega^1_{i,k}$ and $\omega^2_{i,k}$, then once compute equilibrium in changes using equations (A.1)–(A.8) for wages $w_{i,k}$ using $\{\phi_{i,k}, \eta_{i,k}, \varepsilon_i, \sigma_{n,k}\}$, then compute equilibrium in changes using Equations (19)–(24) for prices $\{p_{i,k}\}$ using $\{\omega^1_{i,k}, \omega^2_{i,k}, \sigma_{n,k}\}$. We check that the two exercises produce the exact same set of aggregate variables.

[7] Two comments come in order. First, since we have defined export supply elasticities in terms of *value* of exports with respect to price, an elasticity of unity means that *quantity* of exports remains unchanged with respect to a price change. Second, we can replace EK everywhere in the table with Armington, keeping in mind that $\sigma_{nk}$ is related to the dispersion of Fréchet

these models in order to demonstrate the transparency through which export supply captures the interaction between microeconomic channels that are operative across or within these models. Each of these modeling choices implicitly impose restrictions on the export supply elasticity $\omega^S_{ni,k}$ through their implied restrictions on the total supply elasticity ($\omega^1_{i,k}$) and excess supply elasticity ($\omega^2_{i,k}$).

Table 1: Components of export supply elasticity across trade models

| Model | Parameters | $\omega^1_{i,k}$ | $\omega^2_{i,k}$ |
|---|---|---|---|
| (1) Multi-sector EK[(a)] | $\varepsilon_i \to \infty$, $\eta_{i,k} \to \infty$, $\phi_{i,k} = 0$ | $\infty$ | 0 |
| (2) + ext econ[(b)] | $\varepsilon_i \to \infty$, $\eta_{i,k} \to \infty$, $\phi_{i,k} > 0$ | $\frac{-1}{\phi_{i,k}}$ | 0 |
| (3) + imp mob[(c)] | $\varepsilon_i > 1$, $\eta_{i,k} \to \infty$, $\phi_{i,k} = 0$ | $\varepsilon_i$ | 0 |
| (4) + imp mob + ext econ | $\varepsilon_i > 1$, $\eta_{i,k} \to \infty$, $\phi_{i,k} > 0$ | $\frac{\varepsilon_i}{1-(\varepsilon_i-1)\phi_{i,k}}$ | 0 |
| (5) Multi-sector Krugman | $\varepsilon_i \to \infty$, $\eta_{i,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{i,k} = 0$ | $\infty$ | $\infty$ |
| (6) + nested CES[(d)] | $\varepsilon_i \to \infty$, $\eta_{i,k} = \bar{\eta}_k \neq \sigma_{nk} = \bar{\sigma}_k$, $\phi_{i,k} = 0$ | $\infty$ | $\infty$ |
| (7) + ext econ | $\varepsilon_i \to \infty$, $\eta_{i,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{i,k} > 0$ | $\frac{-1}{\phi_{i,k}}$ | $\frac{-1}{\phi_{i,k}(\bar{\sigma}_k-1)}$ |
| (8) + imp mob | $\varepsilon_i > 1$, $\eta_{i,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{i,k} = 0$ | $\varepsilon_i$ | $\frac{\varepsilon_i-1}{\bar{\sigma}_k-1}$ |
| (9) + imp mob + ext econ | $\varepsilon_i > 1$, $\eta_{i,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{i,k} > 0$ | $\frac{\varepsilon_i}{1-(\varepsilon_i-1)\phi_{i,k}}$ | $\frac{\varepsilon_i-1}{1-(\varepsilon_i-1)\phi_{i,k}}\frac{1}{\bar{\sigma}_k-1}$ |

*Notes*: We abbreviate imperfect labor mobility as "imp mob", and external economies of scale as "ext econ". (a) Costinot et al. (2012) (b) Kucheryavyy et al. (2016). (c) Galle et al. (2017) (d) Lashkaripour and Lugovskyy (2018).

First, consider a class of multi-sector Eaton and Kortum (2002) models in which varieties produced by a country are perfectly substitutable with one another, as $\eta_{ik} \to \infty$. Consequently, there are no internal returns to scale, and the excess supply elasticity ($\omega^2_{i,k}$) is necessarily zero. Put another way, models without variety differentiation do not generate direct export supply linkages across countries. However, extensions of Eaton and Kortum (2002) allow for a flexible range of total supply elasticities ($\omega^1_{i,k}$) through the interaction between parameters governing resource mobility ($\varepsilon_i$) and external returns to scale ($\phi_{ik}$). This interaction is summarized in the elasticity of total supply ($\omega^1_{i,k}$). Scanning down the first four rows of Table 1, we see total supply ($Y_{i,k}$) is more elastic (lower slope) the greater are external returns to scale or when resources are more mobile across industries. Specifically, for a given change to price of products within an industry there is a nonlinear change to the wage in that industry, which induces changes to employment and output. The following spells out the interaction of these forces:

$$\omega^1_{i,k} \equiv \frac{\partial \ln Y_{i,k}}{\partial \ln p_{ii,k}} = \underbrace{\left(\frac{\partial \ln Y_{i,k}}{\partial \ln w_{i,k}}\right)}_{\varepsilon_i} \Big/ \underbrace{\left(\frac{\partial \ln p_{ii,k}}{\partial \ln w_{i,k}}\right)}_{1-(\varepsilon_i-1)\phi_{i,k}}.$$

productivity shocks in EK whereas it is the elasticity of substitution across countries in Armington.

To provide a specific example, the model developed by Galle et al. (2017) analyzes a special case in which labor is imperfectly mobile ($\infty > \varepsilon_i > 1$) and there are no external returns ($\phi_{i,k} = 0$). This model implies a textbook total supply curve with a constant positive slope given by $1/\omega^1_{i,k} = 1/\varepsilon_i$. This result makes clear that production and exports in countries with more mobile resources (i.e., greater $\varepsilon_i$) will be more elastic. A complementary example is Kucheryavyy et al. (2016). There, positive external returns to scale ($\phi_{i,k} > 0$) interact with perfect labor mobility ($\varepsilon_i \to \infty$). Labor supply to any industry is consequently perfectly elastic, which implies there will be no dampening effect on the extent to which external returns to scale lower marginal costs of production. In other words, a rise in wage $w_{i,k}$ directly increases price $p_{ii,k}$ and simultaneously lowers marginal costs as the scale of industry (supply of efficiency units $E_{i,k}$) expands. The indirect force lowering marginal cost dominates the otherwise standard wage effect if resources can be reallocated across industries with sufficient mobility. In the Kucheryavyy et al. (2016) case of perfect mobility and positive external returns, resources are sufficiently mobile to drive the slope of total supply to a negative value. The slope of total supply is then $1/\omega^1_{i,k} = -\phi_{i,k}$, which is negative and steeper in country-industry pairs with stronger external returns to scale.

In the most general case, allowing for both imperfect labor mobility and external returns, the slope of total supply in principle may take any real-valued number. Relevant to our subsequent empirical analysis, there is no general restriction on the sign of $\omega^1_{i,k}$. In contrast, the most restrictive case removes external returns and assumes perfect labor mobility. These assumptions deliver the standard multi-sector Eaton and Kortum (2002) as in Costinot et al. (2012). Export supply is then perfectly elastic as supply in an industry responds fully to price changes in the face of costless factor reallocation given the linear relationship between wages and prices. Tangentially, this is the implicit assumption underlying identification of import demand in the empirical gravity literature (e.g., Anderson and Wincoop (2003)).

Next, consider a class of multi-sector Krugman (1980) models in which varieties within a country-industry pair are not perfect substitutes. Then, internal returns to scale operate through an endogenous mass (number) of product varieties produced by each country-industry. In contrast, Eaton and Kortum (2002) and its extensions assume the mass of products is fixed within every country-industry pair. Generally, internal returns to scale are reflected in the relationship between the variety level price index and firm level prices. This relationship gives rise to the excess supply elasticity ($\omega^2_{i,k}$), and can be spelled out using the following decomposition:

$$1 - \omega^2_{i,k} \equiv \frac{\partial \ln P_{ii,k}}{\partial \ln p_{ii,k}} = \underbrace{\left(\frac{\partial \ln P_{ii,k}}{\partial \ln p_{ii,k}}\Big| M_{i,k}\right)}_{1} - \underbrace{\left(\frac{\partial \ln P_{ii,k}}{\partial \ln M_{i,k}}\Big| p_{ii,k}\right)}_{\frac{1}{\eta_{i,k}-1}} \underbrace{\left(\frac{\partial \ln M_{i,k}}{\partial \ln p_{ii,k}}\right)}_{\frac{(\varepsilon_i - 1)}{1-(\varepsilon_i-1)\phi_{i,k}}}.$$

Here, by putting the two channels $\left(\partial \ln M_{i,k}/\partial \ln p_{ii,k}\right)$ and $\left(\partial \ln P_{ii,k}/\partial \ln M_{i,k}|p_{ii,k}\right)$ together, $\omega^2_{i,k}$ summarizes the relationship between the marginal cost of a firm and the price index that disciplines the demand behavior.[8]

Consider the elasticity of the mass of firms ($M_{i,k}$) with respect to the price charged by a typical firm ($p_{ii,k}$). An increase in the price $p_{ii,k}$ implies an increase in wage $w_{i,k}$, which in turn implies an increase in the supply of total output as well as prospective profits for a typical firm in the industry. The increase in firm profitability induces entry, and increases the mass of firms. This relationship, $\partial \ln M_{i,k}/\partial \ln p_{ii,k}$, depends on labor mobility and external returns to scale. Although this channel operates by putting together standard mechanisms in the literature, we are not aware of any particular paper that allows for these mechanisms to coexist. Extensions of the Krugman model typically assume zero external returns to scale ($\phi_{i,k} \to 0$) and perfect labor mobility ($\varepsilon_i \to \infty$). As a result, the marginal cost of a firm does not change when the mass of firms $M_{i,k}$ rises. For this reason, the firm level price is perfectly inelastic with respect to the mass of firms (i.e., $\left(\partial \ln M_{i,k}/\partial \ln p_{ii,k}\right)^{-1} = 0$).

In an extension in which labor is imperfectly mobile ($\infty > \varepsilon_i > 1$) and scale economies are purely internal ($\phi_{i,k} = 0$), an increase in the price of a typical firm $p_{ii,k}$ implies an increase in wage $w_{i,k}$. In turn the scale of employment ($E_{i,k}$) and, by relation the mass of firms ($M_{i,k}$), increase with prices. The extent of this relationship is governed by the elasticity of labor mobility as $\partial \ln M_{i,k}/\partial \ln p_{ii,k} = (\varepsilon_i - 1)$. That is to say, countries with more mobile factors of production (higher $\varepsilon_i$) experience larger increases of $M_{i,k}$ in response to price as resources can be more flexibly reallocated to the higher paying industry.

Alternatively, Row 7 of Table 1 demonstrates that if labor is perfectly mobile ($\varepsilon_i \to \infty$), but we allow for positive external returns to scale ($\phi_{i,k} > 0$), an increase in the price that firms charge in an industry has to be accompanied by a decrease in the scale of production in that industry (i.e., the total supply elasticity is downward-sloping as $1/\omega^1_{i,k} = -\phi_{i,k}$). Declining output then means less prospective profits for a typical firm which in turn implies less entry. In this case, $\partial \ln M_{i,k}/\partial \ln p_{ii,k} = (-1/\phi_{i,k}) < 0$, reflecting a negative relationship between the mass of firms ($M_{i,k}$) and price ($p_{ii,k}$). In other words, a larger $\phi_{i,k}$ leads to a more downward-sloping total supply. Consequently, an increase in firm level prices brings about a larger decrease in profits, which discourages potential entrants from paying the fixed costs of entry.

The second channel at work $\left(\partial \ln P_{ii,k}/\partial \ln M_{i,k}|p_{ii,k}\right)$, highlights the well-studied margin of gains from variety. This literature asserts that an increase in the mass of varieties within an exporter-industry pair lowers the associated price index faced by consumers as consumers inherently value a greater set of varieties. This relationship is governed by the degree of differentiation among product varieties within exporter-industry pairs, reflected by $1/(\eta_{i,k} - 1)$. In a typical Krugman (1980) model, the extent to which products are differentiated across countries is the same as that across firms within a country, $\eta_{i,k} = \sigma_{n,k} = \bar{\sigma}_k$. In a more generalized setup, such as Lashkaripour and Lugovskyy (2018), $\eta_{i,k}$ could be larger than $\sigma_{n,k}$ meaning that varieties within a country are differentiated but to a lesser extent than across countries. Hence, consumers' gains from variety is less than what is implied by a typical Krugman (1980) model. This margin will

[8] We refer the reader to Appendix **1.3** for the full derivation.

be reflected in the ratio $\frac{\omega^2_{i,k}-1}{\omega^1_{i,k}}$. In other words, the export supply elasticity ($\omega^S_{ni,k}$), which fundamentally contains the ratio, simultaneously captures internal production tradeoffs and tradeoffs faced by exporters in demand for their varieties across destinations. Explicitly, in the case of Lashkaripour and Lugovskyy (2018) the export supply elasticity collapses to $\omega^S_{ni,k} = \frac{1}{\lambda_{ni,k}}(\bar{\sigma}_k - \bar{\eta}_k) + (1 - \bar{\sigma}_k)$ which is necessarily negative if $\bar{\eta}_k > \bar{\sigma}_k \geq 1$, and smaller than the a standard multi-sector Krugman (1980) which delivers $\omega^S_{ni,k} = (1 - \bar{\sigma}_k)$.

We conclude our discussion by highlighting a takeaway for our estimation. Suppose we were to narrow our focus on one particular channel, say factor mobility $\varepsilon_i$. For instance, Galle et al. (2017) uses the China shock in the US to identify $\varepsilon_i$. Two issues arise. First, expanding their analysis to many countries is infeasible due to data limitations. Second, their estimates and methodology are specific to their model which assumes away returns to scale. The implications for export supply are documented by rows (3) and (8) of Table 1. Using their preferred estimate of $\varepsilon_i = 2$, it is immediate that the implied slope of total supply is $\omega^1_{i,k} = 2$ and the slope of excess supply $\omega^2_{i,k}$ is either 0 or $\frac{1}{\bar{\sigma}_k - 1}$. While we are not particularly interested in model identification exercises, it is worth pointing out that targeting particular channels of the model in isolation inherently limits the scope of the analysis. Put another way, by isolating a particular channel the empirical specification (potentially) implies the model or vice versa.

Interdependence between underlying micro parameters is a key motivator behind our analysis. Since our model nests a wide range of common general equilibrium trade models with different perspectives on the range and magnitudes of supply elasticities, our empirical application will aim to be agnostic as to the precise model generating the data. Specifically, instead of targeting microeconomic parameters in isolation $\{\varepsilon_i, \phi_{i,k}, \eta_{i,k}, \sigma_{i,k}\}$ we aim at estimating supply and demand elasticities $\{\omega^1_{i,k}, \omega^2_{i,k}, \sigma_{i,k}\}$. The benefits of targeting supply and demand elasticities are the abundance of cross country and product data. Our methodology will confront the data flexibly and allow the data to speak to the operable microeconomic channels of the model without imposing *ex ante* restrictions. We have shown supply and demand elasticities are sufficient to conduct policy analysis as they efficiently summarize the interaction between micro parameters. Our estimates will thus readily highlight the channels at work and their magnitudes across countries and industries as we refer back to Table 1.

## 3 Estimation

Our goal is to utilize minimal data and constraints in order to estimate import demand and export supply elasticities. Our strategy will allow the data to speak freely to these *market* elasticities, which we will decompose following the model in order to back out model primitives. For each period $t$ trade flows from exporter $i$ to importer $n$ in industry $k$, our model yields export supply and import demand locally of the form:

$$\ln S_{ni,kt} = \widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt} + \delta_{ni,k} + \delta_{n,kt} + \varphi_{ni,kt} \qquad \text{(Export Supply)}$$
$$\ln D_{ni,kt} = \widetilde{\sigma}^D_{ni,k} \ln p_{ni,kt} + \upsilon_{ni,k} + \upsilon_{n,kt} + \rho_{ni,kt} \qquad \text{(Import Demand)}, \tag{25}$$

where $p_{ni,kt}$ is the price of a typical variety (unit value in trade data), $S_{ni,kt}$ is the value of exports from $i$ to $n$, and $D_{ni,kt}$ is the value of imports by $n$ from $i$. Both supply and demand contain shifters that vary across different dimensions. We denote these shifters as import demand fixed effects $\upsilon$ and export supply fixed effects $\delta$. Generically, supply and demand fixed effects vary at the level of importer-exporter-industry and industry-year. For instance, $\upsilon_{n,kt}$ represents the importer price index and total expenditure for industry $k$ in period $t$. Additionally, supply and demand contain importer-exporter-industry-year shifters $\varphi_{ni,kt}$ and $\rho_{ni,kt}$. For supply, $\varphi_{ni,kt}$ is mainly comprised of unobserved productivity shocks, and, for demand, $\rho_{ni,kt}$ is mainly comprised of unobserved demand shocks. Market elasticities are the export supply ($\widetilde{\omega}^S_{ni,kt}$) and import demand ($\widetilde{\sigma}^D_{ni,k}$) elasticities. Notice that the preceding market elasticities were derived with respect to model consistent price indices $P_{ni,k}$, which are not observed by an econometrician. Turning to estimation requires converting these elasticities with respect to data consistent prices $p_{ni,k}$ (i.e., unit values). Export supply and import demand elasticities with respect to $p_{ni,kt}$ are derived structurally as,

$$\widetilde{\omega}^S_{ni,kt} \equiv \frac{\partial \ln S_{ni,kt}}{\partial \ln p_{ni,kt}} = \frac{1}{\lambda_{ni,kt}} \omega^1_{i,k} - \sum_{m \neq n} \frac{\lambda_{mi,kt}}{\lambda_{ni,kt}} (1 - \sigma_{m,k})(1 - \omega^2_{i,k})$$
$$\widetilde{\sigma}^D_{ni,k} \equiv \frac{\partial \ln D_{ni,kt}}{\partial \ln p_{ni,kt}} = (1 - \sigma_{n,k})(1 - \omega^2_{i,k}), \tag{26}$$

where the sub-elasticities $\omega^1_{i,k}$ and $\omega^2_{i,k}$ are as defined by Equations (13) and (14).

Our supply and demand equations, described by (25)-(26), clarify restrictions imposed on market estimators in the literature. Beyond functional forms, the takeaways from mapping the theory to supply and demand estimation are threefold. First, we should not be constraining demand and supply elasticity estimates to the orthant where demand slopes downward and supply slopes upward. Second, export supply elasticities ($\widetilde{\omega}^S_{ni,kt}$) are structurally composed of two sub-supply elasticities $\omega^1_{i,k}$ and $\omega^2_{i,k}$, the import demand elasticity $\sigma_{n,k}$, and export penetration ratios $\lambda_{ni,kt}$. Third, when using unit values (standard trade data) in place of transaction prices (theoretical prices), the net import demand elasticity ($\widetilde{\sigma}^D_{ni,k}$) is importer-exporter-industry specific and is confounded by the sub-supply elasticity $(1 - \omega^2_{i,k})$. Each of these takeaways drive the admissible range and variation of elasticities. Ultimately, we will estimate the sub-supply elasticities $\omega^1_{i,k} \in (-\infty, \infty)$ and $\omega^2_{i,k} \in (-\infty, \infty)$, that vary by exporter-product, and the elasticity of substitution $\sigma_{n,k} \in (1, \infty)$, that varies by importer-product. In contrast to the literature, common methods for jointly estimating supply and demand (e.g., Broda and Weinstein (2006)) estimate a restricted export supply elasticity $\widetilde{\omega}^S_{ni,kt} = \omega^S_{n,k} \in (0, \infty)$ and an import demand elasticity $\widetilde{\sigma}^D_{ni,k} = 1 - \sigma_{n,k} \in (-\infty, 0)$ that both vary by importer-product.

Precision regarding the variation in market elasticities and their admissible ranges is necessitated by our methods for jointly estimating import demand and export supply. Our model demonstrates a number of hurdles associated with applying standard methodologies. First, the scale of the estimation (many importers and exporters trading many goods) rules out instrumental variable (IV) strategies (e.g., Khandelwal (2010)), since we would need to develop at least two instruments for each importer-exporter-product in our data – exogenous shifters of demand (supply) that trace out supply (demand). Second, these challenges are magnified as the export supply elasticity ($\widetilde{\omega}^S_{ni,kt}$) is time varying. To be clear, one would potentially require an IV that exogenously shifts only demand for every importer-exporter-product-year in order to estimate export supply elasticities. We thus find IV for this class of models infeasible on a large scale.[9]

An alternative to IV in the international trade literature are heteroskedastic market estimators (e.g., Feenstra (1994), Broda and Weinstein (2006) and Soderbery (2015)). Our model lends some support to this method. Specifically, we structurally derive an export supply curve similar in nature to their assumed iso-elastic form. Additionally, one key identifying assumption of the method is that supply and demand "error" terms ($\varphi_{ni,kt}$ and $\rho_{ni,kt}$) are independent over time. Our model supports this assumption provided productivity shocks $a_{i,kt}$ and demand shocks $b_{ni,kt}$ are independent.[10] However, the second key identifying assumption in the literature is that import demand and export supply elasticities are constant over time and homogeneous across exporters in a particular destination, which our model refutes in general. The following will develop a structural heteroskedastic estimator of our more general model and show how to overcome these identification hurdles.

### 3.1 Estimation Procedure

Applying an heteroskedastic estimator is structural in nature. We will allow the data to speak to the range of the elasticity estimates, but apply the restrictions in their variation and functional forms derived from the model. In the spirit of generality, our method is developed such that it can be applied using publically accessible data recording trade flows and domestic production. We integrate bilateral trade data from CEPII-BACI that is based on ComTrade, with production data from UNIDO, and bilateral tariff data

[9]Two uses of instruments that cannot be feasibly applied to multiple countries and industries come to mind. First, an instrument that is a meaningful object in only one industry. For example, Costinot et al. (2019) construct an instrument based on disease-related variables, a strategy that is only applicable to the industry they study – pharmaceuticals. Second, strategies that depend on the availability of firm-level data. For example, the approach taken by Lashkaripour and Lugovskyy (2018) requires detailed data on firm-level imports by origin country (their method is applied to Colombia). Generalizing these approaches to multiple industries or multiple countries is hindered both by the nature of instrumental variable construction and the availability of detailed data on the global level.

[10]The exact conditions that ensure supply and demand error terms are more complicated. We disentangle these error terms and lay out the full set of assumptions for the estimator in Appendix **1.6**. We demonstrate the estimator fundamentally hinges on independence between productivity shocks to the exporter $a_{i,kt}$ and demand shocks in the import market $b_{ni,kt}$, but also note a portion of the error term remains in the standard methodology. We describe how fixed effects will be used to control for this remaining term, as it is effectively the variance of productivity shocks which is an exporter specific constant.

from MacMap. Required by the availability of production records, we merge our data into 16 manufacturing industries and one non-manufacturing. We take also from CEPII-BACI export unit values, which we combine with bilateral tariff data to construct import unit values.

Our challenge is estimating the endogenous system in (25) with an unbalanced panel of values and quantities across importers and exporters. We first convert supply and demand into market shares. This aligns the data with the theoretical model and alleviates potential measurement error in recorded trade flows (c.f., Feenstra (1994)). Let $\pi_{ni,kt}$ denote the share of import value by $n$ captured by exporter $i$. Additionally, supply and demand fixed effects $\delta$ and $\upsilon$ are unobservable in the data so we will use first- and reference-differencing to eliminate them, which yields,

$$\Delta^j \ln \pi_{ni,kt} = \Delta(\widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt}) - \Delta(\widetilde{\omega}^S_{nj,kt} \ln p_{nj,kt}) + \Delta^j \varphi_{ni,kt}$$
$$\Delta^j \ln \pi_{ni,kt} = \widetilde{\sigma}^D_{ni,k} \Delta \ln p_{ni,kt} - \widetilde{\sigma}^D_{nj,k} \Delta \ln p_{nj,kt} + \Delta^j \rho_{ni,kt},$$

where $\Delta$ denotes the first difference and superscript $j$ denotes the reference difference.[11] Disparities between this system and the standard from the literature (e.g., Feenstra (1994)/Broda and Weinstein (2006)) emerge. Notice, export supply elasticities are first differenced as they vary over time with import shares. Time variation in export supply elasticities results from *export penetration* ($\lambda_{ni,k}$) in the model affecting the slope of export supply. Explicitly, exporters divide excess supply to trade partners around the world. As such, export supply elasticities to a given destination are shaped by domestic production and total exports to all other destinations. Each of these characteristics may vary over time, which poses a problem for heteroskedastic identification in general. The following will show how to overcome the limitations of the standard estimators.

Under our assumption of error independence, we can multiply the preceding equations together to begin constructing the estimator. The resulting system is:

$$\begin{aligned}(\Delta^j \ln \pi_{ni,kt})^2 = {} & \Delta\left((\widetilde{\sigma}^D_{ni,k} + \widetilde{\omega}^S_{ni,kt}) \ln p_{ni,kt}\right) \Delta^j \ln \pi_{ni,kt} - \widetilde{\sigma}^D_{ni,k} \Delta \ln p_{ni,kt} \Delta(\widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt}) \\ & - \Delta\left((\widetilde{\sigma}^D_{nj,k} + \widetilde{\omega}^S_{nj,kt}) \ln p_{nj,kt}\right) \Delta^j \ln \pi_{ni,kt} + \widetilde{\sigma}^D_{nj,k} \Delta \ln p_{nj,kt} \Delta(\widetilde{\omega}^S_{nj,kt} \ln p_{nj,kt}) \\ & + \widetilde{\sigma}^D_{ni,k} \Delta \ln p_{ni,kt} \Delta(\widetilde{\omega}^S_{nj,kt} \ln p_{nj,kt}) - \widetilde{\sigma}^D_{nj,k} \Delta \ln p_{nj,kt} \Delta(\widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt}) + \Delta^j \varphi_{ni,kt} \Delta^j \rho_{ni,kt}.\end{aligned} \quad (27)$$

To briefly contrast with the literature, if we were to assume demand and supply elasticities were homogenous and constant such that $\widetilde{\sigma}^D_{ni,k} = \sigma_{i,k}$ and $\widetilde{\omega}^S_{ni,kt} = \omega_{i,k} \ \forall \ n, t$, Equation (27) would reduce precisely to the estimator developed by Feenstra (1994). Since homogeneous import demand and export supply elasticities are not supported by the model, Feenstra (1994)'s identification strategy breaks down. Soderbery (2018) discusses this possibility (with no theoretical basis) in the context of *constant* heterogeneous export supply elasticities (i.e., $\widetilde{\omega}^S_{ni,kt} = \omega^S_{ni,k} \ \forall \ t$), and develops a heteroskedastic estimator that leverages contact of

[11] For instance, $\Delta^j \ln \pi_{ni,kt} \equiv (\ln \pi_{ni,kt} - \ln \pi_{ni,kt-1}) - (\ln \pi_{nj,kt} - \ln \pi_{nj,kt-1})$.

exporters in multiple markets. We will borrow much of the intuition developed by Soderbery (2018) to construct our estimator, but note our additional identification hurdle will be addressing time variation in the export supply elasticities.

At this point, a brief discussion of these heteroskedastic market estimators is warranted. In the absence of believable instruments, estimating supply and demand suffers from well-known simultaneity bias. Equation (27) highlights these issues – every proposed regressor is endogenous. Feenstra (1994)'s innovation is an extension of Leamer (1981). They demonstrate that while a regression such as Equation (27) is endogenous in levels, it is exogenous under the assumption that $E[\Delta^j \varphi_{ni,kt}, \Delta^j \rho_{ni,kt}] = 0$ after averaging over time. The proposed averaging in essence converts the system into a regression of market share variances on price and market share variances and covariances. Soderbery (2015) calls this procedure a mapping of "Leamer (1981) hyperbolae" into data. Feenstra (1994) demonstrated this mapping yields consistent estimates of import demand and export supply using only a time series from a single importer provided the elasticities do not vary across exporters or over time. Our modeling demonstrates that market elasticities generally are both heterogenous across importer-exporter pairs and and vary over time.

Soderbery (2018)'s estimator is intuitively the same as Feenstra (1994)'s, but shows how to identify heterogenous elasticities. Essentially, the process for identifying more degrees of heterogeneity in elasticity estimates requires introducing another market for the same $(i, k)$ product variety. Equation (27) describes the import market $n$ across origins $(i, k)$. Under heterogeneity, we additionally require an equation that specifies the export market $i$ across destinations $(n, k)$. This market explains export market shares, $\lambda_{ni,kt}$ (i.e., the share of total exports by $i$ destined for $n$). The construction is similar to Equation (27), except our reference differencing will subtract a reference destination $o$. Notice, supply and demand error terms in the export market are comprised of the same productivity and taste shifters as the import market. Multiplying supply and demand across destinations now yields:

$$\begin{aligned}(\Delta^o \ln \lambda_{ni,kt})^2 = {} & \Delta\left((\widetilde{\sigma}^D_{ni,k} + \widetilde{\omega}^S_{ni,kt}) \ln p_{ni,kt}\right) \Delta^o \ln \lambda_{ni,kt} - \widetilde{\sigma}^D_{ni,k} \Delta \ln p_{ni,kt} \Delta(\widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt}) \\ & - \Delta\left((\widetilde{\sigma}^D_{oi,k} + \widetilde{\omega}^S_{oi,kt}) \ln p_{oi,kt}\right) \Delta^j \ln \lambda_{ni,kt} + \widetilde{\sigma}^D_{oi,k} \Delta \ln p_{oi,kt} \Delta(\widetilde{\omega}^S_{oi,kt} \ln p_{oi,kt}) \\ & + \widetilde{\sigma}^D_{ni,k} \Delta \ln p_{ni,kt} \Delta(\widetilde{\omega}^S_{oi,kt} \ln p_{oi,kt}) - \widetilde{\sigma}^D_{oi,k} \Delta \ln p_{oi,kt} \Delta(\widetilde{\omega}^S_{ni,kt} \ln p_{ni,kt}) + \Delta^o \varphi_{ni,kt} \Delta^o \rho_{ni,kt}.\end{aligned} \tag{28}$$

Soderbery (2018) shows that if export supply elasticities are constant over time, jointly estimating Equations (27) and (28) can identify import demand and export supply elasticities. We have shown that export supply is effectively excess supply from the exporter destined for the importer. Consequently, export supply elasticities internalize exports to all destinations, which is embodied by export penetration weights ($\lambda_{ni,kt}$) underlying the super-export supply elasticity ($\widetilde{\omega}^S_{ni,kt}$). Rather than attempting to estimate super-elasticities as written, we will unbundle the preceding equations and estimate the sub-elasticities in Equation (26). This creates additional identification challenges, as we are now requiring the estimator to identify three elasticities; $\sigma_{n,k}$, $\omega^1_{i,k}$ and $\omega^2_{i,k}$. Notice, that the elasticities to be estimated are no longer time varying provided we have data on export penetration.

Treating export penetration as data coalesces the estimator. Since $\lambda_{ni,kt}$ is comprised of the same (endogenous) price and value variables as the regressors in Equations (27) and (28), the intuition of heteroskedastic identification is unchanged. Namely, we are effectively weighting the hyperbolae by export penetration ratios. Additionally, this weighting facilitates separately identifying $\omega^1_{i,k}$ from $\omega^2_{i,k}$. It is convenient to rearrange the super export supply elasticity for estimation as:

$$\widetilde{\omega}^S_{ni,kt} \equiv \frac{\partial \ln S_{ni,kt}}{\partial \ln p_{ni,kt}} = \frac{1}{\lambda_{ni,kt}}\omega^1_{i,k} - \frac{1-\lambda_{ni,kt}}{\lambda_{ni,kt}}(1-\sigma_{n,k})(1-\omega^2_{i,k}) + \sum_{m\neq n}\frac{\lambda_{mi,kt}}{\lambda_{ni,kt}}(\sigma_{m,k}-\sigma_{n,k})(1-\omega^2_{i,k}).$$

The first term highlights the relationship between $\omega^1_{i,k}$ and $\lambda_{ni,kt}$. We will thus be able to identify $\omega^2_{i,k}$ from $\omega^1_{i,k}$ as the second and third terms are tied together with the elasticity of substitution $\sigma_{n,k}$ and interact differently with $\lambda_{ni,kt}$. Put another way, the sub-export supply elasticities are weighted by different variation in the data. In essence this provides the estimator with multiple hyperbolae to achieve identification.

Simultaneously estimating Equations (27) and (28) after averaging each over time yields estimates of the sub-elasticities $\sigma_{n,k}$, $\omega^1_{i,k}$ and $\omega^2_{i,k}$ that are consistent provided supply and demand shocks are independent and hyperbolae across origins and destinations are not asymptotically proportional. Put more simply, the Leamer (1981) and Feenstra (1994) methodology whereby variances and covariances of prices and quantities can be used to consistently estimate supply and demand, as long as supply and demand shocks are heteroskedastic across exporters, is still the basis of the estimator. What we have added to the model leverages the structure of the underlying model to bound the variation and ranges of the elasticity estimates. Additionally, identification comes from jointly estimating the import and export markets and utilizing export penetration data along with the model's constraints as highlighted by the model.

### 3.2 Market Estimates

To get a sense of our market elasticity estimates, Table 2 constructs inverse super-export supply elasticities ($1/\omega^S_{ni,kt}$) and presents their mean and median statistics across exporters and importers in the largest industries in our data.[12] Elasticity of substitution estimates are the most directly comparable to the literature. Ninety percent of our estimates across all products and countries fall between 1.63 and 7.64. The range and the variation presented in Table 2 are in line with a broad literature.

Evidence regarding export supply elasticities is relatively scant.[13] Heteroskedastic estimates of export supply elasticities vary widely across studies. Broda et al. (2008) estimate an average inverse super-export supply elasticity of 75.69 with a median of 1.78 for a handful of developing countries. Those estimates do not

[12] Given the scope of our estimates, we will construct statistics in the full sample but limit our presentation to the seven largest products and countries in our data.

[13] The literature historically focuses on the inverse export supply elasticity ($1/\omega^S_{ni,kt} \equiv \partial p_{ni,kt}/\partial S_{ni,kt}$), which we will maintain for comparison purposes and call the super-export supply elasticity in the following.

allow for heterogeneity, time variation, or negative elasticities in export supply. Soderbery (2018) extends the estimator to allow for heterogeneity, but still produces relatively large estimates with an average of 68.55 and median of 0.69 for all countries and products. The only study we are aware that allows for negative values when estimating export supply is Costinot et al. (2019). Their methodology is only directly applicable to US exports of pharmaceuticals, for which they estimate an inverse export supply elasticity of -0.14. In comparison, our average estimate for Chemicals and Chemical Products (which contains pharmaceuticals) is -0.012 with a relatively large standard deviation of 0.016. Costinot et al. (2019) compare their estimates to Basu and Fernald (1997) and Antweiler and Trefler (2002), which both suggest an export supply elasticity around -0.23 for pharmaceuticals. We will expand on this assertion structurally through the model, but roughly the more negative is the slope of export supply, the stronger are economies of scale. Our estimates suggest that office and computing machinery thus presents the strongest economies of scale amongst our estimated products with an average inverse export supply elasticity of -0.024. The following will delve into the differences between our estimates and some of the isolated estimates from the literature in the context of the model and counterfactuals. However, we find the patterns of our estimates to be broadly comparable to the literature, and the deviations from the literature to be quite intuitive.

Table 2: Market Super-Elasticity Estimates by Product

| | Total Trade | Inverse Export Supply Elasticity ($1/\omega^S_{ni,kt}$) | | | Elasticity of Substitution ($\sigma_{i,k}$) | | |
|---|---|---|---|---|---|---|---|
| Industry | ($Trillions) | Mean | Med | SD | Mean | Med | SD |
| Textiles | 0.836 | −0.014 | −0.009 | 0.024 | 2.749 | 2.789 | 0.787 |
| Chemicals | 2.274 | −0.012 | −0.006 | 0.016 | 3.120 | 3.182 | 1.223 |
| Basic metals | 0.919 | −0.004 | −0.001 | 0.008 | 3.788 | 3.198 | 2.069 |
| Machinery and Equipment | 1.835 | −0.010 | −0.005 | 0.015 | 3.137 | 3.199 | 0.591 |
| Computers and Electronics | 2.574 | −0.024 | −0.012 | 0.031 | 2.476 | 2.658 | 0.617 |
| Motor Vehicles and Trailers | 1.329 | −0.013 | −0.006 | 0.018 | 3.070 | 3.199 | 1.417 |
| Furniture Manufacturing | 0.522 | 0.001 | −0.002 | 0.015 | 3.142 | 3.200 | 1.742 |

Notes: Total Trade is for 2017 in trillions of US dollars. Mean is the average and Med is the median estimate across all goods within the country. SD is the standard deviation.

Generally, heteroskedastic elasticity estimates present with a long right tail and large differences across deciles. Overall, our export supply estimates are considerably more tame than other heteroskedastic estimators, with ninety percent of all products and countries falling between -0.063 and 0.012. We are hesitant to read too much into these broad distributional and level differences with the literature, but at first glance relaxing the constraint that the export supply slopes up is supported by the data. Additionally, the variation we observe in Table 2 within products follows intuitive patterns. Office and computing machinery present the most downward sloping export supply curves. We find this result to be in line with reduced form empirical evidence on the related mechanism of home market effects. For example, Hanson and Xiang (2004) argue that industries with more differentiated products and higher transport costs present with strong home market effects. Office and computing machinery contains computers which likely face high costs of transportation and are highly differentiated (further supported by their low import demand elasticity estimates, which yield an average $\sigma_{i,k}$ of 2.476). Section 4.3 more precisely analyzes home market effects implied by our model and estimates, but more downward sloping export supply curves are suggestive of the strength of this channel. In contrast to computers, furniture manufacturing produces upward sloping export supply curves on average. It is intuitive to us that returns to scale operate more strongly in industries such as computing and automobiles than industries such as as basic metals and furniture manufacturing. This intuition underlies our estimated export supply elasticities which become less negative as we move from more differentiated (e.g., computers and autos) to less differentiated (e.g., metals and furniture) industries.

Table 3: Market Super-Elasticity Estimates by Country

| | Inverse Export Supply Elasticity ($1/\omega^S_{ni,kt}$) | | | Elasticity of Substitution ($\sigma_{n,k}$) | | |
|---|---|---|---|---|---|---|
| Country | Mean | Med | SD | Mean | Med | SD |
| Canada | −0.026 | −0.022 | 0.028 | 2.930 | 3.199 | 0.549 |
| China | −0.004 | −0.002 | 0.006 | 3.439 | 3.197 | 1.920 |
| Germany | −0.006 | −0.004 | 0.006 | 3.138 | 3.200 | 0.529 |
| India | −0.001 | −0.001 | 0.002 | 2.899 | 3.200 | 0.701 |
| Japan | −0.008 | −0.004 | 0.014 | 3.145 | 3.200 | 0.790 |
| UK | −0.007 | −0.005 | 0.007 | 3.295 | 3.195 | 1.999 |
| USA | −0.007 | −0.003 | 0.014 | 3.563 | 3.190 | 1.852 |

Notes: Mean is the average and Med is the median estimate across all goods within the country. SD is the standard deviation.

Not only do countries export products at different intensities, our estimates reveal they also export with different fundamental elasticities. Table 3 presents summary statistics of our super-elasticities across all products imported and exported within each country. Finally to provide a sense of the microeconomic channels implied by our estimates, Table 4 presents summary statistics for our sub-export supply elasticity estimates. Examining the distribution of sub-elasticities within each country (i.e., across industries), the US, China and India appear to be the most responsive when reallocating resources across industries. Specifically, the average sub-export supply elasticity governing aggregate supply ($\omega^1_{i,k}$) in these countries are relatively large on average.

Table 4: Market Sub-Elasticity Estimates by Country

| | Aggregate Supply Elasticity ($\omega^1_{i,k}$) | | | Excess Supply Elasticity ($\omega^2_{i,k}$) | | |
|---|---|---|---|---|---|---|
| Country | Mean | Med | SD | Mean | Med | SD |
| Canada | 2.104 | 2.265 | 0.325 | 1.121 | 1.156 | 0.310 |
| China | 2.261 | 2.292 | 0.162 | 1.051 | 1.176 | 0.417 |
| Germany | 2.119 | 2.379 | 0.420 | 1.189 | 1.172 | 0.108 |
| India | 2.266 | 2.281 | 0.230 | 1.202 | 1.166 | 0.096 |
| Japan | 2.170 | 2.253 | 0.268 | 1.140 | 1.160 | 0.323 |
| UK | 2.167 | 2.296 | 0.365 | 1.190 | 1.173 | 0.093 |
| USA | 2.204 | 2.290 | 0.246 | 1.224 | 1.166 | 0.109 |

Notes: Mean is the average and Med is the median estimate across all goods within the country. SD is the standard deviation.

# 4 Equilibrium Analysis & Applications

Our model and estimates lend themselves to a wide range of applications. The following will focus on three applications in order to highlight the role of microeconomic channels in determining partial and general equilibrium international outcomes: (1) pass-through of recent US tariffs to prices and allocations, (2) gains from trade, and (3) home market effects. We are particularly interested in changes to prices and the allocation that drive welfare implications associated with various shocks to economies.

## 4.1 Application: Tariffs

We begin by providing a deeper look into how countries react to particular shocks through factor market reallocations and product market outcomes by studying the recent US China trade war. This focused application provides useful insight into the microeconomic pinnings of the macroeconomic outcomes (e.g., welfare) driven by specialization. To show how our model and estimates can be used for tariff analysis, we take two steps. First, we derive tariff pass-through rates and report their magnitudes and variation across markets. Second, we examine the general equilibrium implications of recent US tariff policy.

### 4.1.1 Pass-through Rates of Tariffs to Consumer Prices

The effectiveness of unilateral trade policies crucially depends on the extent to which such policies change international prices. Consider an increase in tariffs imposed by importer $n$ on products of industry $k$ from exporter $i$. We define the partial equilibrium pass-through rate ($\mathfrak{z}_{ni,k}$) as the partial derivative of log consumer price index $P_{ni,k}$ in the importing country with respect to log ad-valorem equivalent tariff:

$$\mathfrak{z}_{ni,k} \equiv \frac{\partial \ln P_{ni,k}}{\partial \ln \tau_{ni,k}} = \frac{1}{1 + \left(\frac{1}{\omega^S_{ni,k}}\right)(\sigma_{n,k} - 1)(1 - \pi_{ni,k})}. \tag{29}$$

The pass-through rate ($\mathfrak{z}_{ni,k}$) is a function of the export supply elasticity ($\omega^S_{ni,k}$), import demand elasticity ($\sigma_{n,k}$), and import share ($\pi_{ni,k}$). Additionally, pass-through is bounded between 0 and 1 if $\omega^S_{ni,k} \geq 0$, $\sigma_{n,k} \geq 1$. In this (standard) case, given the import share, the greater are the export supply and import demand elasticities, the greater is the pass-through of tariffs to consumer prices. Given export and import elasticities, the pass-through rate additionally rises with the market share of $(i, k)$ in country $n$.[14]

As a primer to what follows, we subsequently examine recent US tariffs applied against Chinese exports. Applying our estimates to Equation (29), we find pass-through from China to US consumers to be centered around unity with only minor deviations across industries.[15] Recall, Chinese export supply is relatively

[14] See Appendix 1.4 for the derivation.

[15] Column (5) of Table 5 fully reports partial equilibrium pass-through rates of recent US tariffs imposed on Chinese goods.

elastic on average (Table 3). Our estimates thus imply that US tariffs have virtually no impact on Chinese producer prices in partial equilibrium. Consequently, the burden of recent tariffs lie almost entirely onto US consumers. These results are consistent with recent studies from Fajgelbaum et al. (2019) and Amiti et al. (2019) which found near complete pass-through of US tariffs using reduced form empirics applied to partial equilibrium models of trade.

However, it is important to emphasize that pass-through rates from Equation (29) are partial equilibrium in nature. In general equilibrium, interconnections across markets and interdependencies in trade policy may imply different pass-through rates when importer tariffs are substantial enough to induce broad reallocations by the exporter. For this reason, we next consider the general equilibrium impact of the recent US protectionist policies. We will compare partial and general equilibrium pass-through rates given our estimates and highlight the channels driving their differences.

#### 4.1.2 General Equilibrium Impact of Tariffs

While the channels determining pass-through in partial equilibrium also operate in general equilibrium, the shifts to export supply curves in industries facing tariff changes are accompanied by (potentially costly) reallocations by the exporter. These reallocations lead to additional adjustments by exporters to shipped and delivered prices. In order to decompose the mechanisms of tariff responses, we consider a recent example of extreme and unexpected tariffs applied by the US. Over the course of 2018, the United States increased tariffs on a wide range of its imports from China. We study the general equilibrium implications of these increases in US tariffs on Chinese goods. Column (1) of Table 5 records the changes in ad valorem equivalent tariff rates across industries.[16]

As mentioned before, partial equilibrium pass-through rates of US tariffs on the price index of Chinese goods in the US market (i.e., ${}^{\Delta \ln P_{USA,CHN,k}}/{}_{\Delta \ln t_{USA,CHN,k}}$) are reported in Column (5). On average, pass-through of US tariffs are full and range from 97.77% in paper to 107.70% in furniture. In Column (6) we allow for general equilibrium linkages. The resulting pass-through rates range between 60% for textiles and 79.37% for electronics. These pass-through rates are significantly lower than those implied by partial equilibrium rates reported in Column (5). The differences arise due to general equilibrium linkages whereby a tariff on one industry alters resource allocations and costs across all industries in the exporting country.

The results in our general equilibrium exercise echo recent theories of trade policy. These theoretical results as in Costinot et al. (2015) and Beshkar and Lashkaripour (2016) assert that the optimal tariff for the importer is uniform across industries. Additionally, Beshkar and Lashkaripour (2016) demonstrate that optimal tariffs across industries are complementary. These results are obtained in frameworks that

[16]Values are extracted from Fajgelbaum et al. (2019) and applied to only Chinese goods (in case other exporters are also targeted), and to the entire industry (in case a subset of products are targeted within that industry). We hold tariffs elsewhere unchanged and suppose that in our baseline equilibrium there are no tariffs for the sake of clarity.

have shut down certain mechanisms in comparison to our model (e.g., they impose perfect labor mobility). Nonetheless, they illustrate an important margin of optimal unilateral policy which remains operative in our more general model. Intuitively, an importer can exercise a higher degree of market power by imposing tariffs on *all* industries of an exporting country. That is to say, tariffs ranging across exporter industries do not allow the exporter to reallocate resources in order to escape the distortionary effects of the policies.

Table 5: Pass-through rates onto US consumers from US tariffs on Chinese goods

| Industry | Δ Tariff | Import Share | Elasticities $1/\omega^S_{ni,k}$ | Elasticities $\sigma_{n,k}$ | Partial Equilibrium | General Equilibrium Full | General Equilibrium Single |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Food | 0.10 | 0.72% | 0.000 | 3.040 | 100.02% | 59.88% | 98.87% |
| Textile | 0.10 | 47.03% | −0.013 | 3.198 | 101.52% | 59.39% | 91.38% |
| Wood | 0.10 | 3.65% | −0.001 | 3.200 | 100.12% | 59.74% | 100.07% |
| Paper | 0.10 | 1.40% | 0.010 | 3.097 | 97.99% | 63.65% | 97.67% |
| Petroleum | 0.10 | 0.18% | −0.002 | 3.200 | 100.34% | 58.99% | 100.01% |
| Chemical | 0.10 | 2.67% | −0.003 | 3.182 | 100.54% | 59.55% | 99.29% |
| Rubber | 0.11 | 8.16% | −0.020 | 1.964 | 101.84% | 61.57% | 101.16% |
| Mineral | 0.14 | 3.63% | −0.001 | 3.209 | 100.31% | 70.17% | 100.18% |
| Basic Metal | 0.11 | 2.32% | 0.002 | 3.088 | 99.51% | 65.25% | 99.20% |
| Fabricated Metal | 0.15 | 5.21% | −0.003 | 3.199 | 100.62% | 72.08% | 100.38% |
| Machinery | 0.10 | 12.27% | −0.008 | 2.377 | 100.93% | 58.94% | 100.46% |
| Electronics | 0.22 | 30.33% | −0.015 | 2.035 | 101.07% | 79.31% | 98.74% |
| Electric Machinery | 0.20 | 16.14% | −0.003 | 2.789 | 100.52% | 78.06% | 99.56% |
| Vehicle | 0.20 | 1.78% | −0.001 | 6.890 | 100.42% | 78.25% | 99.24% |
| Other Transp | 0.14 | 1.41% | −0.001 | 3.210 | 100.12% | 69.48% | 100.03% |
| Furniture | 0.12 | 23.65% | −0.012 | 8.613 | 107.70% | 70.24% | 93.82% |

*Note:* Column (1) reports percentage changes in USA statutory tariffs against China. We extract these values from Table 2 in Fajgelbaum et al. (2019) and apply them to USA tariffs on Chinese goods and to the entire industry in case a subset of products within an industry are targeted. Column (2) reports partial US import share from China, and Column (3) reports estimates of supply and demand elasticities. In Column (5) we report partial equilibrium pass-through rates according to Equation (29). In Columns (6) and (7) we report the general equilibrium pass-through rates of US tariffs on US consumers for tariff increases that are reported in Column (1). Column (6) reports results when tariffs are imposed on only a single industry at a time, whereas Column (7) reports results when tariffs are imposed on all industries.

To shed more light on the importance of interconnectedness across industries, Column (7) reports the general equilibrium results from tariff increases on the reported Chinese industries if they had happened one at a time. The resulting pass-through rates are almost the same as the partial equilibrium ones. Intuitively, since the exporting country reallocates resources to non-targeted industries at the time of the tariff, the general equilibrium wage effects remain negligible if US tariffs had been staggered across the Chinese economy.

### 4.2 Welfare Analysis: Gains from Channels of Trade and Specialization

Given the strong general equilibrium reallocations highlighted by our tariff application, we now turn to more broadly quantifying the welfare effects of trade and trade liberalization. We characterized in Section **2.6.1** that for any given policy $\mathscr{P}$ Equations (19)–(24) solve for equilibrium in changes. Additionally, general equilibrium analysis only requires baseline data $\mathscr{B}$, and minimal set of parameters $\{\sigma_{n,k}, \omega^1_{n,k}, \omega^2_{n,k}\}$ for every country and industry in the world. The resulting change to welfare for every country $n$ is then,[17]

$$\widehat{W}_n = \underbrace{\prod_k \widehat{\pi}_{nn,k}^{-\frac{\beta_{n,k}}{\sigma_{n,k}-1}}}_{\widehat{TR}} \underbrace{\prod_k \widehat{r}_{n,k}^{\frac{\beta_{n,k}(\omega^2_{n,k}-1)}{\omega^1_{n,k}}}}_{\widehat{SP}}. \tag{30}$$

Given expenditure shares $\beta_{n,k}$ as well as changes to $\widehat{\pi}_{nn,k}$, $\widehat{r}_{n,k}$, trade elasticities $(\sigma_{nk} - 1)$ and $(\omega^2_{n,k}-1)/\omega^1_{n,k}$ are sufficient statistics for welfare analysis. The first set of terms, $\widehat{TR}$, governed by $\widehat{\pi}_{nn,k}$ and $\beta_{n,k}/(\sigma_{n,k} - 1)$ have been studied extensively in the literature beginning with Arkolakis et al. (2012). We call this portion of welfare the *trade channel*. The second set of terms, $\widehat{SP}$, governed by $\widehat{r}_{n,k}$ and $\beta_{n,k}(\omega^2_{n,k} - 1)/\omega^1_{n,k}$, which we call the *specialization channel*, has been relatively less studied.

The key elasticity for analysis is the ratio $(\omega^2_{n,k}-1)/\omega^1_{n,k}$, which equals the inverse negative of the elasticity of total supply $Y_{i,k}$ with respect to variety-level price index at the location of production $P_{ii,k}$. In other words, how total supply responds in relation to the industry price index governs the impact of resource reallocations on welfare. We refer to this ratio of supply elasticities $(\omega^2_{n,k}-1)/\omega^1_{n,k}$ as the *specialization elasticity*, because it reflects the combined effects from elasticities that govern scale economies and labor mobility. We can see these two effects clearly if we shut down the other.

First consider the case of no scale economies, then $(\omega^2_{n,k}-1)/\omega^1_{n,k} = -1/\varepsilon_n < 0$. In this case, controlling for the trade channel, the industry to which more resources will be allocated decreases welfare through the *adjustment costs of reallocation* channel. Consider the thought experiment where productivity in services increases relative to manufacturing industries due to a technological shock. Consequently, some fraction of high efficiency workers in manufacturing reallocate from the contracting manufacturing sector to the expanding service sector. This selection margin compresses the average efficiency of workers in services inducing a negative contribution to welfare through the specialization channel. Specialization effects are more pronounced in countries with less labor mobility (i.e., where $\varepsilon_i$ is smaller).

Convoluting the adjustment cost channel are economies of scale. Consider the same thought experiment where manufacturing is contracting, but shut down the adjustment cost channel by assuming workers are perfectly mobile (i.e., $\varepsilon_n \to \infty$). Then, $(\omega^2_{n,k}-1)/\omega^1_{n,k} = \frac{1}{\eta_{n,k}-1} + \phi_{n,k} > 0$. Resources allocated to an industry positively contribute to welfare through scale economies. In general, the interplay between the adjustment costs of reallocation and benefits from scale economies give rise to the combined elasticity $(\omega^2_{n,k}-1)/\omega^1_{n,k}$. Passing the combined channels to welfare depends on the response to the vector of industry-level revenue shares $(\widehat{r}_{n,k})$ and the specialization elasticity weighted by the expenditure share on the industry $(\beta_{n,k})$. Table 6 reports the specialization elasticity across the models considered in Table 1.

Similar to our discussion around Table 1, models based on Eaton and Kortum (2002) or Krugman (1980) impose restrictions on the sign and magnitude of the specialization channel. In comparison, our framework

[17] We refer the reader Appendix **1.2** for a full derivation.

allows for combinations of the mechanisms that have been studied in isolation. To be specific, our estimates suggest economies are generally characterized by imperfect mobility along with internal and external returns to scale of varying degrees. Each of these channels appear to operate in the data, but economies of scale seem to dominate in general as 97% of our estimates of the specialization elasticity lie between 0.039 and 0.209.[18]

Table 6: Components of combined mobility-scale elasticity across trade models

| Model | Parameters | $\frac{\omega^2_{n,k}-1}{\omega^1_{n,k}}$ |
|---|---|---|
| Multi-sector EK$^{(a)}$ | $\varepsilon_n \to \infty$, $\eta_{n,k} \to \infty$, $\phi_{n,k} = 0$ | 0 |
| + ext econ$^{(b)}$ | $\varepsilon_n \to \infty$, $\eta_{n,k} \to \infty$, $\phi_{n,k} > 0$ | $\phi_{n,k}$ |
| + imp mob$^{(c)}$ | $\varepsilon_n > 1$, $\eta_{n,k} \to \infty$, $\phi_{n,k} = 0$ | $-\frac{1}{\varepsilon_n}$ |
| + imp mob + ext econ | $\varepsilon_n > 1$, $\eta_{n,k} \to \infty$, $\phi_{n,k} > 0$ | $-\frac{1}{\varepsilon_n} + \frac{\varepsilon_n-1}{\varepsilon_n}\phi_{n,k}$ |
| Multi-sector Krugman | $\varepsilon_n \to \infty$, $\eta_{n,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{n,k} = 0$ | $\frac{1}{\bar{\sigma}_k-1}$ |
| + nested CES$^{(d)}$ | $\varepsilon_n \to \infty$, $\eta_{n,k} = \bar{\eta}_k \neq \sigma_{nk} = \bar{\sigma}_k$, $\phi_{n,k} = 0$ | $\frac{1}{\bar{\eta}_k-1}$ |
| + ext econ | $\varepsilon_n \to \infty$, $\eta_{n,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{n,k} > 0$ | $\frac{1}{\bar{\sigma}_k-1} + \phi_{n,k}$ |
| + imp mob | $\varepsilon_n > 1$, $\eta_{n,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{n,k} = 0$ | $-\frac{1}{\varepsilon_n} + \frac{\varepsilon_n-1}{\varepsilon_n}\frac{1}{\bar{\sigma}_k-1}$ |
| + imp mob + ext econ | $\varepsilon_n > 1$, $\eta_{n,k} = \sigma_{nk} = \bar{\sigma}_k$, $\phi_{n,k} > 0$ | $-\frac{1}{\varepsilon_n} + \frac{\varepsilon_n-1}{\varepsilon_n}\left(\frac{1}{\bar{\sigma}_k-1} + \phi_{n,k}\right)$ |

*Notes*: (a) Costinot et al. (2012). (b) Kucheryavyy et al. (2016). (c) Galle et al. (2017) (d) Lashkaripour and Lugovskyy (2018)

A multi-sector Krugman (1980) model highlights the interplay of these channels on welfare. There we can see the benefits from scale economies and adjustment costs of specialization interact in interesting ways. The former drives up gains from trade as exporters scale up production in traded industries, while the latter drives down welfare as the exporter finds it costly to allocate additional resources to these growing industries. Throughout our analysis the specialization elasticity $(\omega^2_{i,k}-1)/\omega^1_{i,k}$ has played a central role. This elasticity effectively governs (re)allocations within and across industries in general equilibrium. As such, its prominence in welfare is quite intuitive.

Table 7 presents our estimates of the specialization elasticity across countries for each manufacturing industry in our data. We record the mean, median, minimum and maximum of our estimates to provide a sense of the variation across countries. The first thing that stands out from our estimates is an occasionally wide distribution across countries. Textiles, for instance, produce a specialization elasticity that ranges from -0.601 for Indonesia to 0.178 for India. Roughly, the larger is this elasticity the stronger are returns to scale. Conversely, lower elasticities indicate less (more costly) factor mobility. Values below zero indicate the costs of factor reallocations exceed the gains from returns to scale.

[18] The 3% of our estimates below zero are spread across four industries in China, Japan, Canada and Indonesia.

Table 7: Specialization Elasticities

| Industry | Specialization Elasticity $\left(\frac{\omega^2_{n,k}-1}{\omega^1_{n,k}}\right)$ Estimates Across Countries: Mean | Min | Med | Max | Literature: LL | BCDR |
|---|---|---|---|---|---|---|
| Food, Beverages and Tobacco | 0.050 | 0.042 | 0.051 | 0.066 | 0.265 | 0.16 |
| Textiles | 0.102 | -0.601 | 0.155 | 0.178 | 0.207 | 0.12 |
| Wood Products | 0.052 | 0.040 | 0.053 | 0.056 | 0.270 | 0.11 |
| Paper Products | 0.115 | -0.448 | 0.151 | 0.181 | 0.397 | 0.11 |
| Coke/Petroleum Products | 0.064 | 0.042 | 0.063 | 0.106 | 1.758 | 0.07 |
| Chemicals | 0.084 | 0.071 | 0.082 | 0.098 | 0.212 | 0.20 |
| Rubber and Plastics | 0.156 | 0.118 | 0.160 | 0.173 | 0.162 | 0.25 |
| Mineral products | 0.067 | 0.055 | 0.068 | 0.072 | 0.186 | 0.13 |
| Basic metals | -0.020 | -0.449 | 0.045 | 0.051 | 0.189 | 0.11 |
| Fabricated Metals | 0.077 | 0.068 | 0.078 | 0.084 | 0.189 | 0.13 |
| Machinery and Equipment | 0.068 | 0.061 | 0.069 | 0.073 | 0.100 | 0.13 |
| Computers and Electronics | 0.172 | 0.116 | 0.172 | 0.210 | 0.453 | 0.09 |
| Electrical Machinery | 0.072 | 0.055 | 0.075 | 0.082 | 0.453 | 0.09 |
| Motor Vehicles and Trailers | 0.078 | 0.054 | 0.078 | 0.164 | 0.133 | 0.15 |
| Other Transport Equipment | 0.118 | 0.090 | 0.117 | 0.172 | 0.133 | 0.16 |
| Furniture Manufacturing | -0.046 | -0.565 | 0.057 | 0.176 | – | – |

*Notes*: LL refers to Lashkaripour and Lugovskyy (2018). BCDR refers to Bartelme et al. (2018).

The final two columns of Table 7 present the specialization elasticity implied by recent estimates from Lashkaripour and Lugovskyy (2018) and Bartelme et al. (2018). Comparing our estimates with the literature requires some caution. Lashkaripour and Lugovskyy (2018) assume perfectly mobile factors and only internal returns to scale in an application to Columbia. Bartelme et al. (2018) assume perfectly mobile factors and only external returns to scale. In both cases, the resulting elasticities that map to the specialization elasticity are assumed to be industry-specific, and are necessarily positive due to assuming perfect labor mobility. Generally, both alternative methodologies generate larger estimates than ours. This might be expected as the model suggests that perfect labor mobility drives up the combined elasticity. However, a direct comparison is difficult because, in addition to methodological differences, our data and variation leveraged for estimation are fundamentally different. Conditional on those caveats, we believe the differences that emerge are suggestive evidence that imperfect labor mobility is a real friction (level differences) in the data and external returns to scale dominate internal returns to some extent (correlation patterns).

#### 4.2.1 Quantifying the Gains from Trade and Trade Liberalization

In this section, we quantify the gains from trade as the amount of welfare loss generated from a move to autarky. Additionally, we further the analysis by developing the gains from trade liberalization as the amount of welfare generated by elimination of tariffs in all manufacturing industries between all countries.

We first apply Equation (30) to obtain the gains from trade formula. Since in autarky $\pi^A_{nn,k} = 1$ and $r^A_{n,k} = \beta_{n,k}$, it follows that $\widehat{\pi}_{nn,k} = 1/\pi_{nn,k}$ and $\widehat{r}_{n,k} = \beta_{n,k}/r_{n,k}$. Consistent with the literature, we define the gains from trade as the loss of welfare when a country moves from the observed equilibrium to autarky,

$$GFT_n \equiv \frac{W_n - W_n^A}{W_n} = 1 - \Delta W_n = 1 - \Delta_n^{TR}\Delta_n^{SP}, \tag{31}$$

where the "trade channel" ($\Delta TR_n$), and the "specialization channel" ($\Delta SP_n$) are given by,

$$\Delta TR_n = \prod_k \pi_{nn,k}^{\frac{\beta_{n,k}}{\sigma_{nk}-1}} \quad \text{and} \quad \Delta SP_n = \prod_k \left(\beta_{n,k}/r_{n,k}\right)^{\frac{\beta_{n,k}(\omega_{n,k}^2-1)}{\omega_{n,k}^1}}. \tag{32}$$

Given model parameters, $\Delta_n^{TR}$ is larger if the observed domestic expenditure share $\pi_{nn}^k$ is smaller. That is to say, more open economies would lose more in a shift to autarky. These are the classic Arkolakis et al. (2012) gains from trade. The specialization channel ($\Delta_n^{SP}$) comprises forces that can operate with opposing tension. Deviation between the expenditure share ($\beta_{n,k}$) and the revenue share ($r_{n,k}$) in an industry $k$ necessarily implies deviation in at least one other industry in that country. This specialization channel thus captures enhancement and depression of efficiency in allocations of productive resources induced by trade. The contribution of this *divergence ratio* ($\beta_{n,k}/r_{n,k}$) to welfare is raised to the specialization elasticity, which summarizes the tensions in the model driven by the degree of labor mobility and returns to scale. The aggregate effect of the specialization channel on welfare is then given by,

$$\log \Delta SP_n = \sum_{k \in K} \log \Delta SP_{n,k} \quad \text{where} \quad \log \Delta SP_{n,k} \equiv \frac{\beta_{n,k}(\omega_{n,k}^2-1)}{\omega_{n,k}^1} \log\left(\frac{\beta_{n,k}}{r_{n,k}}\right). \tag{33}$$

Accordingly, we refer to $(-\log \Delta SP_{n,k}) \approx 1 - \Delta SP_{n,k}$ as the contribution of industry $k$ to gains from trade in country $n$ through the specialization channel when controlling for the trade channel.

When $\beta_{n,k} < r_{n,k}$, resources would move to industry $k$ as the country transitions from autarky to the baseline trade equilibrium. If in addition $(\omega_{n,k}^2-1)/\omega_{n,k}^1 > 0$, the contribution of industry $k$ to welfare is positive as $-\log \Delta SP_{n,k} > 0$. In this case, the country is reallocating resources to the industry exploiting economies of scale. Economies of scale dominate costly reallocations in this example, which boosts gains from trade. Conversely, if $\beta_{n,k} > r_{n,k}$, resources would move out of industry $k$ in the transition from autarky to the baseline. If in addition $(\omega_{n,k}^2-1)/\omega_{n,k}^1 > 0$, then again the contribution of industry $k$ to welfare is positive. Here, resources are being reallocated away from the industry $k$ where the costs of reallocation exceed the gains from returns to scale, which agains boosts the gains from trade through specialization.

The interaction of the forces underlying production and exports are a result of the general flexibility in the export supply elasticities ($\omega_{n,k}^1$ and $\omega_{n,k}^2$) we allow for relative to existing models in the literature. For example, Table 6 demonstrates the model developed by Galle et al. (2017) yields $(\omega_{n,k}^2-1)/\omega_{n,k}^1 < 0$, such that the specialization channel always boosts welfare for $\beta_{i,k} > r_{i,k}$. In contrast, Kucheryavyy et al. (2016) and Lashkaripour and Lugovskyy (2018) generate a positive specialization elasticity, such that the specialization channel always reduces welfare for $\beta_{i,k} > r_{i,k}$. In summary, when the specialization elasticity is positive (negative), an industry from which resources are removed necessarily contributes to dampen (boost) gains from trade through the specialization channel. By allowing for scale economies and imperfect labor mobility, we provide and quantify new insight into the microfoundations of gains from trade.

Applying our estimates and data, of Table 8 reports the gains from trade for a handful of countries. Additionally, we decompose the contribution of the trade channel and the specialization channel in determining overall gains from trade across countries. The contribution of specialization channel is rather small reflecting the competing welfare effects of industries that expand with those that shrink. Overall, the gains from trade generated by our model are in line with multi-industry models in the literature (c.f., Costinot and Rodríguez-Clare (2014)).

Table 8: Gains from Trade and Trade Liberalization

| | Gains from Trade | | | Gains from Trade Liberalization | | |
|---|---|---|---|---|---|---|
| | Welfare | Trade | Specialization | Welfare | Trade | Specialization |
| Country | $1-\Delta W$ | $1-\Delta TR$ | $1-\Delta SP$ | $\widehat{W}$ | $\widehat{TR}$ | $\widehat{SP}$ |
| Canada | 8.97% | 9.12% | −0.16% | 0.53% | 0.55% | −0.02% |
| China | 1.60% | 1.52% | 0.08% | 0.20% | 0.20% | 0.00% |
| Germany | 9.40% | 9.58% | −0.19% | 0.49% | 0.48% | 0.01% |
| India | 1.98% | 2.07% | −0.08% | 0.22% | 0.21% | 0.01% |
| Japan | 2.06% | 2.11% | −0.05% | 0.24% | 0.26% | −0.03% |
| UK | 11.92% | 12.19% | −0.32% | 0.36% | 0.37% | −0.01% |
| USA | 2.04% | 2.27% | −0.23% | 0.19% | 0.28% | −0.09% |

*Note:* Panel (A) reports the gains from trade with $1-\Delta W$, $1-\Delta TR$, $1-\Delta SP$ calculated according to equations (31) and (32). Gains from liberalization reports welfare changes from counterfactual elimination of all international manufacturing tariffs. We obtain $\widehat{W}$, $\widehat{TR}$, and $\widehat{SP}$ by computing equilibrium in changes as described by Equation (30).

Of additional and related interest, we next consider a counterfactual policy that eliminates all tariffs in the manufacturing industries. Our most recent year of data are from 2017, which will serve as our baseline trade equilibrium levels including welfare and tariffs. We compute the equilibrium in changes predicted by our model and estimates described in Section **2.6.1** in a counterfactual world where tariffs are reduced to zero globally. The resulting gains represent the extent to which welfare in every country reacts to the general equilibrium reallocations induced by global free trade.

The final three columns of Table 8 report the gains from trade liberalization. Gains from liberalization are modest compared to the gains from trade. This result is expected since tariffs themselves are historically low in the baseline and all countries decreasing tariffs simultaneously brings with it countervailing effects. In order to further decompose these effects, we apply Equation (30) to decompose the gains from liberalization into trade and specialization channels. Gains from the specialization channel are not necessarily positive or negative, and their absolute values are smaller than those from the trade channel. The relative magnitudes of the trade and specialization channels broadly echo the estimates from gains from trade. However, in certain countries the patterns of reallocation differ between the two exercises. For instance, specialization negatively impacts welfare in Germany with trade. With liberalization this pattern is reversed, suggesting the current structure of global tariffs is leading to misallocations. That is to say, by reallocating production in response to liberalization Germany realizes efficiency gains through returns to scale. In contrast, specialization in the US across both exercises negatively impacts welfare. However, moving from autarky to trade the negative impact of specialization is merely one tenth of the total welfare effect, while in the move to free trade accounts for nearly one third of the overall welfare effect. This suggests that it is particularly costly to reallocate resources in the USA in order to adjust to the new free trade equilibrium.

### 4.3 Home Market Effects

In this section, we use our model and estimates to shed light on the strength of scale economies reflected in home market effects. To do so, we examine the sign and magnitudes of elasticities of exports and imports in every country-industry pair with respect to home demand. For instance, for electronics industry in China, we specifically answer this question: If Chinese demand for electronics increased exogenously by one percent, then by how many percents Chinese exports and imports of electronics would change? Generally speaking, elasticities of exports and imports with respect to home demand are meant to measure the percentage change in exports and imports in an industry when home demand for that industry exogenously rises.

We compute these elasticities as follows. First, we rewrite Cobb-Douglas demand shifters $\beta_{n,k}$ as $\beta_{n,k} = b_{n,k}/(b_{n,1} + ... + b_{n,K})$, where $b_{n,k}$ is home demand for industry $k$. Let $b'_{n,k}$ denote a new set of home demand parameters, where $b'_{n,k} = b_{n,k} + \Delta$ for $k = j$, and $b'_{n,k} = b_{n,k}$ for $k \neq j$. This process generates a new set of Cobb-Douglas parameters $\beta'_{n,k}$ in which the share of spending on $k = j$ is higher than that of $k \neq j$. Export and import elasticities with respect to home demand for country-industry pair $(n, j)$ are then defined by,

$$\beta^X_{n,j} \equiv \frac{S'_{n,j}/S_{n,j} - 1}{b'_{n,j}/b_{n,j} - 1}, \quad \text{and} \quad \beta^M_{n,j} \equiv \frac{D'_{n,j}/D_{n,j} - 1}{b'_{n,j}/b_{n,j} - 1}. \tag{34}$$

The literature distinguishes between weak and strong home market effects, as discussed by Costinot et al. (2019). Weak home market effects present when $\beta_X > 0$, and strong home market effects when $\beta_X > \beta_M$.

We conduct $N \times K$ counterfactual exercises in which for every country $n$ and industry $j$, one at a time, we numerically compute a new equilibrium for a local change to $b_{n,j}$. Comparing exports and imports in the baseline and new equilibrium allows us to calculate $\beta^X_{n,j}$ and $\beta^M_{n,j}$ for every country-product pair. We report the full table of our results in Appendix (Tables A.1, A.2), and plot them in Figure 1 to connect with a broad literature studying home market effects.

Several observations stand out. First, across all estimates $\beta^X_{n,j}$ is more likely to be large when $\beta^M_{n,j}$ is small. The reason lies in the forces that affect exports and imports in opposite directions. An increase in exports due to home demand originates from an increase in total supply. Since a sufficient rise of total supply helps satisfy total demand, there will be less demand for foreign goods and less imports.

In addition, there is a substantial heterogeneity in our estimated elasticities across industries and countries. Despite this heterogeneity, weak home market effects (i.e. cases with $\beta^M > \beta^X > 0$) are a prevalent outcome. In contrast, no home market effects or strong home market effects are rare outcomes across industries and countries.

The closest comparison to our estimates is with Costinot et al. (2019) who find evidence for strong home market effects in the pharmaceutical industry. Their baseline estimates for the pharmaceutical industry are $\beta^X = 0.93$ and $\beta^M = 0.54$. In our data, the closest we have to pharmaceuticals is the chemicals which includes pharmaceuticals along with several non-pharmaceutical industries. For chemicals in the US, we obtain $\beta^X = 0.14$ and $\beta^M = 0.89$ which support the assertion of weak home market effects in the US chemical industry.

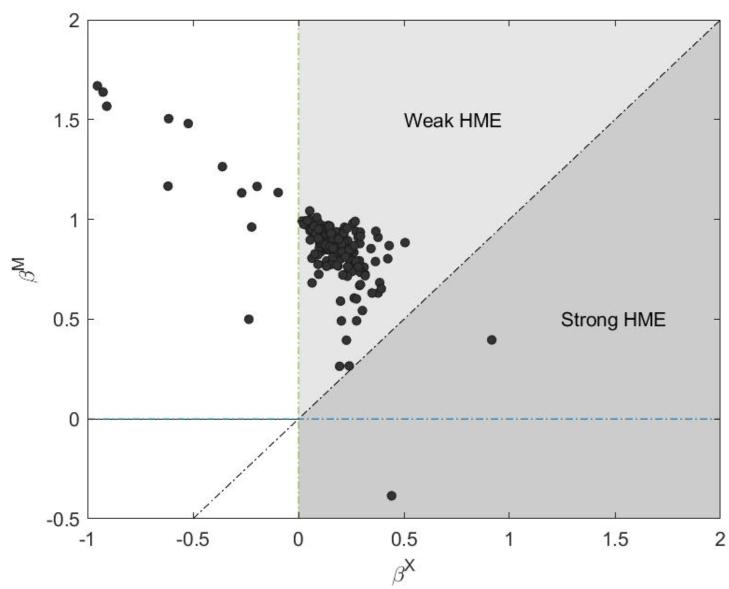


Figure 1: $\beta^X$ and $\beta^M$

*Notes:* $\beta^X$ and $\beta^M$ are elasticities of exports and imports with respect to home demand for every industry-country pair. Weak home market requires $\beta^X > 0$ and strong home market effect requires $\beta^X > \beta^M$.

### 4.4 Channel Estimates

We have shown how to utilize market elasticity estimates in a range of applications from evaluating policy to estimating aggregate outcomes such as gains from trade. Here we take the model and estimates a bit further by dissecting the microeconomic channels of the model. Doing so requires taking a precise stand on the underlying model. Here we will show how to use our (general) market elasticity estimates to uncover the (specific) underlying parameters of our model.

Our goal here is to exploit the estimated differences in sub-export supply elasticities across exporters and products they export in order to uncover the deeper model parameters. Export supply sub-elasticities govern how exporters supply domestic versus foreign markets and across foreign markets. The first sub-elasticity ($\omega^1_{i,k}$) governs aggregate supply. The second sub-elasticity ($\omega^2_{i,k}$) governs how exporters divide excess supply across destinations with different demand elasticities. In combination, $\omega^1_{i,k}$ relative to $\omega^2_{i,k}$ will provide insight into the internal frictions the exporter faces when reallocating resources across industries. Specifically, we can write the ratio of $\omega^2_{i,k}$ and $\omega^1_{i,k}$ depending solely on microeconomic model parameters:

$$\frac{\omega^2_{i,k}}{\omega^1_{i,k}} = \frac{\epsilon_i - 1}{\epsilon_i}\frac{1}{\eta_{i,k} - 1}. \tag{35}$$

The ratio of $\omega^2_{ni,k}$ and $\omega^1_{ni,k}$ is thus a nonlinear combination of only two parameters from the model, $\varepsilon_i$ and $\eta_{i,k}$. Furthermore, the model bounds the sign and variation of the ratio. Notice the parameter governing the labor supply elasticity $\varepsilon_i > 1$ is exporter specific and does not vary across goods, while the parameter governing external returns $\eta_{i,k} > 1$ is exporter-product specific.

Table 9: Model Parameter Estimates

| | Labor Mobility | Internal Returns ($\eta_{i,k}$) | | | External Returns ($\phi_{ni,k}$) | | |
|---|---|---|---|---|---|---|---|
| Country | ($\epsilon_i$) | Mean | Med | SD | Mean | Med | SD |
| Canada | 1.897 | 1.837 | 1.953 | 0.238 | 0.136 | 0.184 | 0.185 |
| China | 1.798 | 2.057 | 1.952 | 0.833 | 0.284 | 0.276 | 0.056 |
| Germany | 2.020 | 1.927 | 1.954 | 0.229 | 0.107 | 0.148 | 0.114 |
| India | 1.658 | 1.824 | 1.954 | 0.304 | 0.459 | 0.417 | 0.116 |
| Japan | 1.913 | 1.930 | 1.954 | 0.343 | 0.215 | 0.187 | 0.102 |
| UK | 1.784 | 1.995 | 1.952 | 0.867 | 0.312 | 0.322 | 0.146 |
| USA | 2.118 | 2.111 | 1.950 | 0.803 | 0.053 | 0.068 | 0.086 |

*Notes:* Mean is the average and Med is the median estimate across all goods within the country. SD is the standard deviation.

Extracting these deeper parameters, however, requires a normalization to achieve identification. We thus impose a proportionality assumption between each exporter's elasticity of substitution ($\sigma_{i,k}$) and the elasticity governing external returns ($\eta_{i,k}$). We find it plausible that the elasticity of substitution across firms producing product $k$ ($\eta_{i,k}$) is related to the substitutability across varieties of the product ($\sigma_{i,k}$).[19] Given this normalization, we can jointly estimate $\varepsilon_i$ and $\eta_{i,k}$ by applying nonlinear least squares. Then with $\varepsilon_i$ and $\eta_{i,k}$ in hand, we can further back out internal returns to scale ($\phi_{i,k}$) implied by $\omega^1_{i,k}$ and $\omega^2_{i,k}$ themselves. Table 9 reports our results.

[19] Explicitly we nonlinearly estimate $\varepsilon_i$ and a proportionality parameter call it $\kappa$ such that $\eta_{i,k} = \kappa\sigma_{i,k}$ after taking logs of in Equation (35). This delivers the following nonlinear relationship, $\log(\mu_{i,k}) = \log(\frac{\epsilon_i - 1}{\epsilon_i}) + \log(\frac{1}{\kappa\sigma_{i,k} - 1})$, which can be estimated via nonlinear least squares.

## 5 Conclusion

We first developed a general equilibrium model that incorporates key channels from widely used models of international trade. We recast these models as one of supply and demand in product markets through a derivation of export supply from model primitives. Export supply was shown to contain unique information regarding the microeconomic channels underlying general equilibrium models of trade. Specifically, we demonstrated that export supply summarizes the interaction between elasticities that govern scale economies, labor mobility, and demand for products.

Our derivation highlights three sub-elasticities underlying export supply; (1) the elasticity of total supply with respect to product-level prices, (2) the elasticity of the industry level price index with respect to product level prices, and (3) elasticity of demand with respect to price index. We showed how to estimate these three channels by developing a heteroskedastic estimator for international product markets. Identification of these three sub-elasticities required publicly available trade and production data. We further showed that these three sub-elasticities are sufficient for quantitative predictions of aggregate outcomes (e.g., tariff passthrough and gains from trade). That is, we not only estimate the model by projecting it into product markets consisting of goods prices and quantities, but we also solve the model using that projection.

This exercise, whereby we developed a flexible model of trade, recast it as a general equilibrium model of product markets and estimated the parameters driving aggregate outcomes, equipped us with tools to address aggregate implications of trade-related shocks and policies. In particular, we examined the effects of recent US protectionist policies on prices and welfare. Crucial for this exercise was general equilibrium analysis and a flexible model and parameter estimates. We applied our model and estimates to shed light on the importance of both of these dimensions when analyzing trade policy shocks (or any shock to international trade). Rounding out the analysis, we conducted a final exercise dissecting the microeconomic channels implied by our estimates as the meet the model. These channels were shown to crucially operate through scale economies and labor mobility as they shape the aggregate behavior of the model.

## References


Adao, R., Arkolakis, C., and Esposito, F. (2018). Spatial linkages, global shocks, and local labor markets: Theory and evidence.

Amiti, M., Redding, S. J., and Weinstein, D. (2019). The Impact of the 2018 Trade War on U.S. Prices and Welfare. *National Bureau of Economic Research Working Paper Series*.

Anderson, J. and Wincoop, E. (2003). Gravity with Gravitas: A Solution to the Border Puzzle. *The American Economic Review*, 93(1):170–192.

Antweiler, W. and Trefler, D. (2002). Increasing Returns and All That: A View from Trade. *The American Economic Review*, 92(1):93–119.

Arkolakis, C., Costinot, A., and Rodriguez-Clare, A. (2012). New Trade Models, Same Old Gains? *American Economic Review*, 102(1):94–130.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers - International Monetary Fund*, 16(1):159–178.

Bartelme, D., Costinot, A., Donaldson, D., and Rodriguez-Clare, A. (2018). External economies of scale and industrial policy: A view from trade.

Basu, S. and Fernald, J. G. (1997). Returns to Scale in U.S. Production: Estimates and Implications. *Journal of Political Economy*, 105(2):249–283.

Beshkar, M. and Lashkaripour, A. (2016). Interdependence of Trade Policies in General Equilibrium. *SSRN Electronic Journal*.

Broda, C., Limao, N., and Weinstein, D. (2008). Optimal tariffs and market power: the evidence. *The American Economic Review*, 98(5):2032–2065.

Broda, C. and Weinstein, D. (2006). Globalization and the gains from variety. *Quarterly Journal of Economics*, 121(2):541–585.

Costinot, A., Donaldson, D., and Komunjer, I. (2012). What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas. *The Review of Economic Studies*, 79(2):581–608.

Costinot, A., Donaldson, D., Kyle, M., and Williams, H. (2019). The More We Die, The More We Sell? A Simple Test of the Home-Market Effect*. *The Quarterly Journal of Economics*, 134(2):843–894.

Costinot, A., Donaldson, D., Vogel, J., and Werning, I. (2015). Comparative Advantage and Optimal Trade Policy *. *The Quarterly Journal of Economics*, 130(2):659–702.

Costinot, A. and Rodríguez-Clare, A. (2014). Trade theory with numbers: Quantifying the consequences of globalization. In *Handbook of international economics*, volume 4, pages 197–261. Elsevier.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.

Fajgelbaum, P. D., Goldberg, P. K., Kennedy, P. J., and Khandelwal, A. K. (2019). *The Return to Protectionism*.

Feenstra, R. C. (1994). New product varieties and the measurement of international prices. *The American Economic Review*, 84(1):157–177.

Galle, S., Rodriguez-Clare, A., and Yi, M. (2017). Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade. *National Bureau of Economic Research Working Paper Series*.

Hanson, G. H. and Xiang, C. (2004). The Home-Market Effect and Bilateral Trade Patterns. *The American Economic Review*, 94(4):1108–1129.

Khandelwal, A. (2010). The long and short (of) quality ladders. *Review of Economic Studies*, 77(4):1450–1476.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review*, 70(5):950–959.

Kucheryavyy, K., Lyn, G., and Rodríguez-Clare, A. (2016). Grounded by gravity: A well-behaved trade model with industry-level economies of scale.

Lashkaripour, A. and Lugovskyy, V. (2018). Scale Economies and the Structure of Trade and Industrial Policy.

Leamer, E. (1981). Is it a demand curve, or is it a supply curve? Partial identification through inequality constraints. *The Review of Economics and Statistics*, 63(3):319–327.

Melitz, M. J. and Redding, S. J. (2014). Heterogeneous Firms and Trade. pages 1–54. Elsevier.

Soderbery, A. (2015). Estimating import supply and demand elasticities: Analysis and implications. *Journal of International Economics*, 96(1):1–17.

Soderbery, A. (2018). Trade elasticities, heterogeneity, and optimal tariffs. *Journal of International Economics*, 114:44–62.

# Appendix A Technical Notes

## 1.1 Equilibrium in Changes – Using wages and labor market clearing

Given "baseline" values $L_{i,k}$, $\pi_{ni,k}$, $X_{ni,k}$, and a "policy" as changes to iceberg trade costs $d_{n,ik}$ for all $i$, $n$, $k$, an equilibrium in changes consists of $\widehat{w}_{i,k}$ for all $i$, $k$ such that equations A.1-A.8 hold. Below, $L_i = \sum_k L_{i,k}$ and $\alpha_{ni,k} \equiv (\sigma_{nk}-1)((\eta_{i,k}-1)^{-1}+\phi_{i,k})$.

$$\widehat{L}_{i,k} = \widehat{\Phi}_i^{-\varepsilon_i} \widehat{w}_{i,k}^{\varepsilon_i} \tag{A.1}$$

$$\widehat{\Phi}_i = \left[\sum_k \frac{L_{i,k}}{L_i} \widehat{w}_{i,k}^{\varepsilon_i}\right]^{1/\varepsilon_i} \tag{A.2}$$

$$\widehat{E}_{i,k} = \widehat{\Phi}_i^{1-\varepsilon_i} \widehat{w}_{i,k}^{\varepsilon_i - 1} \tag{A.3}$$

$$\widehat{Y}_{i,k} = \widehat{\Phi}_i^{1-\varepsilon_i} \widehat{w}_{i,k}^{\varepsilon_i} \tag{A.4}$$

$$\widehat{\pi}_{ni,k} = \frac{\widehat{E}_{i,k}^{\alpha_{ni,k}} (\widehat{d}_{ni,k} \widehat{w}_{i,k})^{-(\sigma_{nk}-1)}}{\sum_\ell \pi_{n\ell,k} \widehat{E}_{\ell,k}^{\alpha_{ni,k}} (\widehat{d}_{n\ell,k} \widehat{w}_{\ell,k})^{-(\sigma_{nk}-1)}} \tag{A.5}$$

$$\widehat{P}_{n,k} = \left[\sum_\ell \pi_{n\ell,k} \widehat{E}_{\ell,k}^{\alpha_{ni,k}} (\widehat{d}_{n\ell,k} \widehat{w}_{\ell,k})^{-(\sigma_{nk}-1)}\right]^{\frac{1}{1-\sigma_{nk}}} \tag{A.6}$$

$$\widehat{X}_{ni,k} = \widehat{\pi}_{ni,k} \widehat{\Phi}_n \tag{A.7}$$

$$\sum_n \widehat{X}_{ni,k} X_{ni,k} = \widehat{Y}_{n,k} \tag{A.8}$$

## 1.2 Welfare Gains

The change to indirect utility $W_n \equiv I_n/P_n = \gamma_n L_n \Phi_n / P_n$ is given by

$$\widehat{W}_n = \frac{W_n'}{W_n} = \frac{\widehat{\Phi}_n}{\widehat{P}_n}$$

where $\widehat{P}_n = \prod_k \widehat{P}_{nk}$, where $\widehat{P}_{n,k}$ is given by equation (A.6). Using equations A.3, A.5, and A.6,

$$\widehat{P}_{n,k} = \widehat{\pi}_{nn,k}^{\frac{1}{\sigma_{nk}-1}} \widehat{E}_{n,k}^{\frac{\alpha_{nn,k}}{1-\sigma_{nk}}} \widehat{w}_{n,k} = \widehat{\pi}_{nn,k}^{\frac{1}{\sigma_{nk}-1}} \left(\widehat{\Phi}_n^{1-\varepsilon_n} \widehat{w}_{n,k}^{\varepsilon_n - 1}\right)^{\frac{\alpha_{nn,k}}{1-\sigma_{nk}}} \widehat{w}_{n,k}$$

Replacing for $\widehat{P}_{n,k}$ from the above equation, and since $\sum_k \beta_{n,k} = 1$,

$$\begin{aligned}
\widehat{W}_n &= \frac{\widehat{\Phi}_n}{\prod_k \widehat{\pi}_{nn,k}^{\frac{\beta_{n,k}}{\sigma_{nk}-1}} \left(\widehat{\Phi}_n^{-1} \widehat{w}_{n,k}\right)^{\frac{\beta_{n,k}\alpha_{nn,k}(\varepsilon_n - 1)}{1-\sigma_{nk}}} \widehat{w}_{n,k}^{\beta_{n,k}}} \\
&= \prod_k \widehat{\pi}_{nn,k}^{-\frac{\beta_{n,k}}{\sigma_{nk}-1}} \prod_k \left(\widehat{\Phi}_n^{-1} \widehat{w}_{n,k}\right)^{\frac{\beta_{n,k}\alpha_{nn,k}(\varepsilon_n - 1)}{\sigma_{nk}-1}} \left(\widehat{\Phi}_n \widehat{w}_{n,k}^{-1}\right)^{\beta_{n,k}}
\end{aligned}$$

Given $\widehat{\Phi}_n^{-1} \widehat{w}_{n,k} = \widehat{r}_{n,k}^{1/\varepsilon_n}$, and since $\alpha_{nn,k} \equiv (\sigma_{nk}-1)((\eta_{n,k}-1)^{-1} + \phi_{n,k})$,

$$\widehat{W}_n = \prod_k \widehat{\pi}_{nn,k}^{-\frac{\beta_{n,k}}{\sigma_{nk}-1}} \prod_k \widehat{r}_{n,k}^{\frac{\beta_{n,k}}{\varepsilon_n}\left[-1+(\varepsilon_n - 1)\left((\eta_{n,k}-1)^{-1}+\phi_{n,k}\right)\right]}$$

Using equations 13-14 which define $\omega^1_{i,k}$ and $\omega^2_{i,k}$, we can rewrite the GFT formula as:

$$\widehat{W}_n = \prod_k \widehat{\pi}_{nn,k}^{-\frac{\beta_{n,k}}{\sigma_{nk}-1}} \prod_k \widehat{r}_{n,k}^{\frac{\beta_{n,k}(\omega^2_{n,k}-1)}{\omega^1_{n,k}}}$$

This reproduces equation (31) in the main text. Given $\widehat{\pi}_{nn,k}$, $\widehat{r}_{n,k}$ and Cobb-Douglas shares $\beta_{n,k}$, sufficient statistics for gains from trade are trade elasticity $\sigma_{nk}-1$ and $(\omega^2_{n,k}-1)/\omega^1_{n,k}$.

### 1.3 Components of Export supply elasticities

We restate equations 18 and 25 that report export supply elasticity with respect to price index and price of a typical variety,

$$\omega^S_{ni,k} \equiv \frac{\ln S_{ni,k}}{\ln P_{ni,k}} = \frac{1}{\lambda_{ni,k}} \frac{\omega^1_{i,k}}{1-\omega^2_{i,k}} - \sum_{m\neq n} \frac{\lambda_{mi,k}}{\lambda_{ni,k}}(1-\sigma_{m,k})$$

$$\widetilde{\omega}^S_{ni,k} \equiv \frac{\ln S_{ni,k}}{\ln p_{ni,k}} = \frac{1}{\lambda_{ni,k}}\omega^1_{i,k} - \sum_{m\neq n} \frac{\lambda_{mi,k}}{\lambda_{ni,k}}(1-\sigma_{m,k})(1-\omega^2_{i,k})$$

These two equations are connected through this relationship

$$\partial \ln P_{ii,k}/\partial \ln p_{ii,k} = (1-\omega^2_{i,k})$$

In the next subsection, we provide detailed derivations of this relationship and other equations in Section **2.6**. Then, we present an alternative derivation of export supply elasticity based on exact hat algebra.

#### 1.3.1 Derivations in Section 2.6

Using equation (2), we write wage $w_{i,k}$ as a function of price of a typical variety at the location of production $p_{ii,k}$,

$$w_{i,k} = p_{ii,k}^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left(L_i\Phi_i^{1-\varepsilon_i}e_{i,k}\right)^{\frac{\phi_{i,k}}{1-(\varepsilon_i-1)\phi_{i,k}}} \tag{A.9}$$

Replacing equation (A.9) into equation (3) we express total production $Y_{i,k}$ as a function of price of a typical variety at the location of production $p_{ii,k}$,

$$\begin{aligned}
Y_{i,k} &= \left(L_i\Phi_i^{1-\varepsilon_i}e_{i,k}\right) w_{i,k}^{\varepsilon_i} \\
&= \left(L_i\Phi_i^{1-\varepsilon_i}e_{i,k}\right)\left[p_{ii,k}^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left(L_i\Phi_i^{1-\varepsilon_i}e_{i,k}\right)^{\frac{\phi_{i,k}}{1-(\varepsilon_i-1)\phi_{i,k}}}\right]^{\varepsilon_i} \\
&= \left(L_i\Phi_i^{1-\varepsilon_i}e_{i,k}\right)^{\frac{1+\phi_{i,k}}{1-(\varepsilon_i-1)\phi_{i,k}}} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{\frac{\varepsilon_i}{1-(\varepsilon_i-1)\phi_{i,k}}} p_{ii,k}^{\frac{\varepsilon_i}{1-(\varepsilon_i-1)\phi_{i,k}}} \\
&= \underbrace{\left(L_i e_{i,k}\right)^{\frac{1+\phi_{i,k}}{\varepsilon_i}\omega^1_{i,k}} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{\omega^1_{i,k}} \Phi_i^{\frac{1-\omega_{i,k}}{\omega_{i,k}}}}_{Y^p_{i,k}} p_{ii,k}^{\omega^1_{i,k}}
\end{aligned} \tag{A.10}$$

which delivers equation (13),

$$\omega^1_{i,k} \equiv \frac{\partial \ln Y_{i,k}}{\partial \ln p_{ii,k}} = \frac{\varepsilon_i}{1-(\varepsilon_i-1)\phi_{i,k}}$$

In order to write $Y_{i,k}$ as a function of price index at the location of exports $P_{ii,k}$, first we write mass of firms $M_{i,k}$ as a function of price $p_{ii,k}$. To do so, we replace $E_{i,k} = L_i\Phi_i^{1-\varepsilon_i}e_{i,k}w_{i,k}^{\varepsilon_i-1}$ into $M_{i,k} = E_{i,k}/(\eta_{i,k}F_{i,k})$, and use equation (A.9) to replace wages by prices,

$$\begin{aligned}
M_{i,k} &= \frac{L_i \Phi^{\varepsilon_i - 1} e_{i,k} w_{i,k}^{\varepsilon_i - 1}}{\eta_{i,k} F_{i,k}} \\
&= \frac{L_i \Phi^{\varepsilon_i - 1}}{\eta_{i,k} F_{i,k}} \left[ p_{ii,k}^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{\frac{\phi_{i,k}}{1-(\varepsilon_i-1)\phi_{i,k}}} \right]^{\varepsilon_i - 1} \\
&= (\eta_{i,k} F_{i,k})^{-1} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{\frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} p_{ii,k}^{\frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} \qquad \text{(A.11)}
\end{aligned}$$

Replacing this relationship into equation (5),

$$\begin{aligned}
P_{ii,k} &= M_{i,k}^{\frac{1}{1-\eta_{i,k}}} p_{ii,k} \\
&= \left[ (\eta_{i,k} F_{i,k})^{-1} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{\frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{\frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} p_{ii,k}^{\frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} \right]^{\frac{1}{1-\eta_{i,k}}} p_{ii,k} \\
&= (\eta_{i,k} F_{i,k})^{\frac{1}{\eta_{i,k}-1}} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{-\frac{1}{\eta_{i,k}-1} \frac{1}{1-(\varepsilon_i-1)\phi_{i,k}}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{-\frac{1}{\eta_{i,k}-1} \frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} p_{ii,k}^{1-\frac{1}{\eta_{i,k}-1} \frac{\varepsilon_i - 1}{1-(\varepsilon_i-1)\phi_{i,k}}} \\
&= (\eta_{i,k} F_{i,k})^{\frac{1}{\eta_{i,k}-1}} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{-\frac{\omega_{i,k}^2}{\varepsilon_i - 1}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{-\omega_{i,k}^2} p_{ii,k}^{1-\omega_{i,k}^2} \qquad \text{(A.12)}
\end{aligned}$$

which delivers equation (14),

$$\frac{\partial \ln P_{ii,k}}{\partial \ln p_{ii,k}} = 1 - \omega_{i,k}^2, \quad \omega_{i,k}^2 = \frac{1}{(\eta_{i,k}-1)} \frac{(\varepsilon_i - 1)}{1-(\varepsilon_i-1)\phi_{i,k}}$$

By inverting equation (A.12),

$$p_{ii,k} = \left[ (\eta_{i,k} F_{i,k})^{\frac{1}{\eta_{i,k}-1}} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{-\frac{\omega_{i,k}^2}{\varepsilon_i - 1}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{-\omega_{i,k}^2} \right]^{-\frac{1}{1-\omega_{i,k}^2}} P_{ii,k}^{\frac{1}{1-\omega_{i,k}^2}} \qquad \text{(A.13)}$$

Replacing equation (A.13) into equation (A.10),

$$\begin{aligned}
Y_{i,k} &= \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{\frac{1+\phi_{i,k}}{\varepsilon_i} \omega_{i,k}^1} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{\omega_{i,k}^1} \left[ (\eta_{i,k} F_{i,k})^{\frac{1}{\eta_{i,k}-1}} \left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{-\frac{\omega_{i,k}^2}{\varepsilon_i - 1}} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{-\omega_{i,k}^2} \right]^{-\frac{\omega_{i,k}^1}{1-\omega_{i,k}^2}} P_{ii,k}^{\frac{\omega_{i,k}^1}{1-\omega_{i,k}^2}} \\
&= \underbrace{\left( L_i \Phi_i^{1-\varepsilon_i} e_{i,k} \right)^{\omega_{i,k}^1 \left[ \frac{1+\phi_{i,k}}{\varepsilon_i} + \frac{\omega_{i,k}^2}{(1-\omega_{i,k}^2)(\varepsilon_i - 1)} \right]} \left( a_{i,k} \frac{\eta_{i,k}-1}{\eta_{i,k}} \right)^{\frac{\omega_{i,k}^1}{\omega_{i,k}^2}} \left( \eta_{i,k} F_{i,k} \right)^{-\frac{\omega_{i,k}^1}{\eta_{i,k}-1(1-\omega_{i,k}^2)}}}_{Y_{i,k}^P} P_{ii,k}^{\frac{\omega_{i,k}^1}{1-\omega_{i,k}^2}} \qquad \text{(A.14)}
\end{aligned}$$

which reproduces the first term in the RHS of equation (16).

### 1.3.2 Deriving export supply elasticity using exact hat algebra

Consider an exogenous increase in demand, $b_{ni,k}$, for exporter $i$, importer $n$, and good $k$. We have defined export supply elasticity as the partial derivative of log exports value with respect to log price. Illustrated by Figure (A.1), the inverse of export supply elasticity is given by

$$(\widetilde{\omega}_{ni,k}^S)^{-1} = \tan(\theta) = \frac{\Delta_1}{d \ln b_{ni,k}} = \frac{d \ln p_{ni,k}}{d \ln S_{ni,k}} \qquad \text{(A.15)}$$

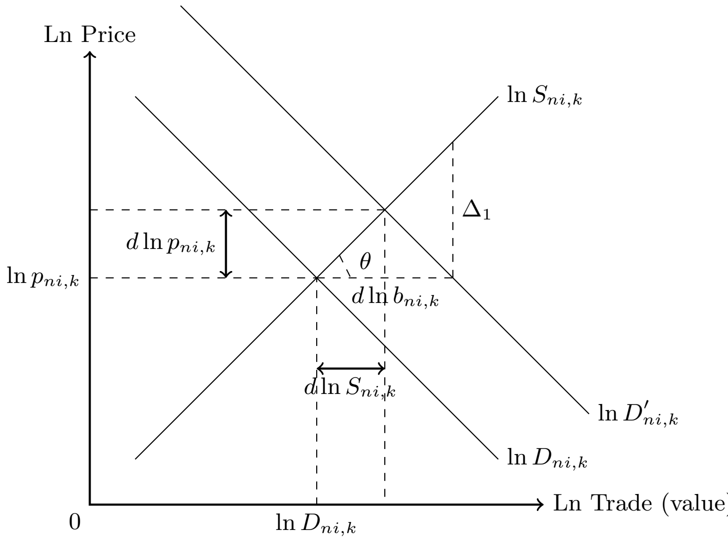


Figure A.1: Export Supply Elasticity

In order to find $\widetilde{\omega}_{ni,k}$, we use the exact hat algebra to calculate responses to a demand shock, $\widehat{b}_{ni,k} > 1$. All other exogenous parameters remain unchanged. In our calculations, consistent with taking into account only the partial derivatives, we ignore the second order effects of a change in $b_{ni,k}$ on factor rewards and aggregate income, so that $\widehat{w}_{\ell,k} = 1$ for all $\ell \neq i$, and $\widehat{\Phi}_n = 1$ for all $n$. The numerator of equation (A.5) equals

$$\widehat{b}_{ni,k}\widehat{w}_{i,k}^{\mu_{ni,k}}$$

where

$$\mu_{ni,k} \equiv (\sigma_{n,k} - 1)\Big((\varepsilon_i - 1)((\eta_{i,k} - 1)^{-1} + \phi_{i,k}) - 1\Big)$$

The change to price index (A.6) is

$$\widehat{P}_{n,k}^{1-\sigma_{n,k}} = (1 - \pi_{ni,k}) + \pi_{ni,k}\widehat{b}_{ni,k}\widehat{w}_{i,k}^{\mu_{ni,k}}$$

Replacing the above expressions into equations (A.5), (A.6), (A.7), and ignoring second order effects, $dxdy$, for generic $x$ and $y$,

$$\widehat{D}_{ni,k} = \frac{\widehat{b}_{ni,k}\widehat{w}_{i,k}^{\mu_{ni,k}}}{(1 - \pi_{ni,k}) + \pi_{ni,k}\widehat{b}_{ni,k}\widehat{w}_{i,k}^{\mu_{ni,k}}}$$

Using $\widehat{x} = 1 + d\ln x$ for a generic variable $x$,

$$\begin{aligned}
1 + d\ln D_{ni,k} &= \frac{(1 + d\ln b_{ni,k})(1 + \mu_{ni,k} d\ln w_{i,k})}{(1 - \pi_{ni,k}) + \pi_{ni,k}(1 + d\ln b_{ni,k})(1 + \mu_{ni,k} d\ln w_{i,k})} \\
d\ln D_{ni,k} &= \frac{1 + d\ln b_{ni,k} + \mu_{ni,k} d\ln w_{i,k}}{1 + \pi_{ni,k}(d\ln b_{ni,k} + \mu_{ni,k} d\ln w_{i,k})} - 1 \\
d\ln D_{ni,k} &= \frac{(1 - \pi_{ni,k})(d\ln b_{ni,k} + \mu_{ni,k} d\ln w_{i,k})}{1 + \pi_{ni,k}(d\ln b_{ni,k} + \mu_{ni,k} d\ln w_{ni,k})} \qquad \text{(A.16)}
\end{aligned}$$

In addition, using equations (A.7) and (A.4),

$$d\ln D_{mi,k} = \mu_{mi,k} d\ln w_{i,k} \qquad \text{(A.17)}$$

$$d\ln Y_{i,k} = \varepsilon_i d\ln w_{i,k} \qquad \text{(A.18)}$$

Using market clearing equation (A.8) before and after the change to $b_{ni,k}$,

$$D_{ni,k} d\ln D_{ni,k} + \sum_{m \neq n} D_{mi,k} d\ln D_{mi,k} = Y_{i,k} d\ln Y_{i,k}$$

We now replace (A.16), (A.17), (A.18) into the above equation,

$$D_{ni,k}\left[\frac{(1-\pi_{ni,k})(d\ln b_{ni,k}+\mu_{ni,k}d\ln w_{i,k})}{1+\pi_{ni,k}(d\ln b_{ni,k}+\mu_{ni,k}d\ln w_{i,k})}\right]+\sum_{m\neq n}D_{mi,k}\mu_{mi,k}d\ln w_{i,k}=Y_{i,k}\varepsilon_i d\ln w_{i,k}$$

Rearranging the above equation and ignoring second order effects, the wage response, $d\ln w_{i,k}$, to the demand shock, $d\ln b_{ni,k}$, is given by

$$d\ln w_{i,k}=\frac{1-\pi_{ni,k}}{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}-(1-\pi_{ni,k})\mu_{ni,k}}d\ln b_{ni,k} \tag{A.19}$$

Replacing $d\ln w_{i,k}$ from (A.19) into (A.16), and since $d\ln D_{ni,k}=d\ln S_{ni,k}$,

$$d\ln S_{ni,k}=(1-\pi_{ni,k})\Big[\frac{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}}{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}-(1-\pi_{ni,k})\mu_{ni,k}}\Big]d\ln b_{ni,k} \tag{A.20}$$

In addition, equation (2) implies that $d\ln p_{ni,k}=(1-(\varepsilon_i-1)\phi_{i,k})d\ln w_{i,k}$. Replacing $d\ln w_{i,k}$ from (A.19),

$$d\ln p_{ni,k}=\frac{(1-(\varepsilon_i-1)\phi_{i,k})(1-\pi_{ni,k})}{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}-(1-\pi_{ni,k})\mu_{ni,k}}d\ln b_{ni,k} \tag{A.21}$$

Using equations (A.20)-(A.21), the definition of export supply elasticity by equation (A.15), and $\mu_{ni,k}\equiv(\sigma_{n,k}-1)((\varepsilon_i-1)((\eta_{i,k}-1)^{-1}+\phi_{i,k})-1)$,

$$\begin{aligned}\widetilde{\omega}^S_{ni,k} &= \frac{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}(\sigma_{m,k}-1)\Big((\varepsilon_i-1)((\eta_{i,k}-1)^{-1}+\phi_{i,k})-1\Big)}{1-(\varepsilon_i-1)\phi_{i,k}}\\ &= \frac{1}{\lambda_{ni,k}}\omega^1_{i,k}-\sum_{m\neq n}\frac{\lambda_{mi,k}}{\lambda_{ni,k}}(1-\sigma_{m,k})(1-\omega^2_{i,k})\end{aligned}$$

where $\omega^1_{i,k}$ and $\omega^2_{i,k}$ are given by equations (13) and (14).

## 1.4 Pass-through Rates

Consider a change to $\tau_{ni,k}$, and that this is the only change to exogenous variables. Then, the change to price index is:

$$\widehat{P}^{1-\sigma_{n,k}}_{ni,k}=\widehat{E}^{\alpha_{ni,k}}_{i,k}\widehat{\tau}^{1-\sigma_{n,k}}_{ni,k}\widehat{w}^{1-\sigma_{n,k}}_{i,k}$$

Using the aboves relationship, $\widehat{E}_{ni,k}=\widehat{\Phi}^{1-\varepsilon_i}_i\widehat{w}^{\varepsilon_i-1}_{i,k}$, and ignoring second order effects,

$$d\ln P_{ni,k}=d\ln\tau_{ni,k}+\frac{\mu_{ni,k}}{1-\sigma_{n,k}}d\ln w_{i,k} \tag{A.22}$$

where

$$\mu_{ni,k}\equiv(\sigma_{n,k}-1)\Big((\varepsilon_i-1)((\eta_{i,k}-1)^{-1}+\phi_{i,k})-1\Big)$$

Similar to derivation of equation (A.19), we calculate $d\ln w_{i,k}$ in response to $d\ln\tau_{ni,k}$,

$$d\ln w_{i,k}=\underbrace{\frac{1-\pi_{ni,k}}{\varepsilon_i\frac{Y_{i,k}}{D_{ni,k}}-\sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}-(1-\pi_{ni,k})\mu_{ni,k}}}_{\Upsilon_{ni,k}}(1-\sigma_{n,k})d\ln\tau_{ni,k} \tag{A.23}$$

Replacing (A.22) into (A.23),

$$
\begin{aligned}
d\ln P_{ni,k} &= \left[1+\Upsilon_{ni,k}\mu_{ni,k}\right] d\ln\tau_{ni,k} \\
&= \frac{\varepsilon_i \frac{Y_{i,k}}{D_{ni,k}} - \sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k}}{\varepsilon_i \frac{Y_{i,k}}{D_{ni,k}} - \sum_{m\neq n}\frac{D_{mi,k}}{D_{ni,k}}\mu_{mi,k} - (1-\pi_{ni,k})\mu_{ni,k}} d\ln\tau_{ni,k}
\end{aligned}
$$

Dividing the numerator and denominator by $(1-\omega^2_{i,k})(1-(\varepsilon_i-1)\phi_{i,k})$, and rearranging terms gives the pass-through of trade cost wedge to consumer price index, i.e. equation (29) in the main text,

$$
\mathfrak{z}_{ni,k} \equiv \frac{d\ln P_{ni,k}}{d\ln\tau_{ni,k}} = \frac{\omega^S_{ni,k}}{\omega^S_{ni,k} + (\sigma_{n,k}-1)(1-\pi_{ni,k})}
$$

The pass-through rate defined for the price of a typical variety can be then obtained as follows:

$$
\begin{aligned}
d\ln P_{ni,k} &= d\ln P_{ii,k} + d\ln\tau_{nik} \\
\mathfrak{z}_{ni,k} d\ln\tau_{ni,k} &= (1-\omega^2_{i,k}) d\ln p_{ii,k} + d\ln\tau_{nik} \\
d\ln p_{ii,k} &= -\frac{1-\mathfrak{z}_{ni,k}}{1-\omega^2_{i,k}} d\ln\tau_{ni,k}
\end{aligned}
$$

It then follows that

$$
\begin{aligned}
d\ln p_{ni,k} &= d\ln p_{ii,k} + d\ln\tau_{ni,k} \\
d\ln p_{ni,k} &= -\frac{1-\mathfrak{z}_{ni,k}}{1-\omega^2_{i,k}} d\ln\tau_{ni,k} + d\ln\tau_{ni,k} \\
\widetilde{\mathfrak{z}}_{ni,k} &\equiv \frac{d\ln p_{ni,k}}{d\ln\tau_{ni,k}} = \frac{\mathfrak{z}_{ni,k} - \omega^2_{i,k}}{1-\omega^2_{i,k}}
\end{aligned}
$$

If $\omega^2_{i,k}=0$, then $\widetilde{\mathfrak{z}}_{ni,k} = \mathfrak{z}_{ni,k}$. If $\mathfrak{z}_{ni,k}=1$, then $\widetilde{\mathfrak{z}}_{ni,k}=1$ as well.

## 1.5 Uniqueness Condition

We connect our setup to Kucheryavyy et al. (2016) in order to reproduce their uniqueness condition for the setting of our model. Using the employment allocation equation, we can write wage per unit of efficiency as:

$$
w_{i,k} = L_{i,k}^{1/\varepsilon_i}(L_i e_{i,k})^{-1/\varepsilon_i}\Phi_i
$$

Then, using employment equation and $E_{i,k} = L_i \Phi_i^{1-\varepsilon_i} e_{i,k} w_{i,k}^{\varepsilon_i - 1}$, we can write aggregate supply of efficiency units as

$$
E_{i,k} = (L_i e_{i,k})^{1/\varepsilon_i} L_{i,k}^{(\varepsilon_i-1)/\varepsilon_i}
$$

where $\alpha_{ni,k} \equiv (\sigma_{n,k}-1)\psi_{i,k}, \quad \psi_{i,k} \equiv (\eta_{i,k}-1)^{-1} + \phi_{i,k}$. Combining the two above equations,

$$
\begin{aligned}
E_{i,k}^{\alpha_{ni,k}} w_{i,k}^{1-\sigma_{n,k}} &= \gamma_i^{\alpha_{ni,k}} (L_i e_{i,k})^{(\alpha_{ni,k}+\sigma_{n,k}-1)/\varepsilon_i} \Phi_i^{1-\sigma_{n,k}} L_{i,k}^{(\varepsilon_i-1)\alpha_{ni,k}/\varepsilon_i + (1-\sigma_{n,k})/\varepsilon_i} \\
&= \bar{\gamma}_i \Phi_i^{1-\sigma_{n,k}} L_{i,k}^{(1-\sigma_{n,k})(1-\omega^2_{i,k})/\omega^1_{i,k}},
\end{aligned}
$$

where $\bar{\gamma}_i \equiv \gamma_i^{\alpha_{ni,k}} (L_i e_{i,k})^{(\alpha_{ni,k}+\sigma_{n,k}-1)/\varepsilon_i} \Phi_i^{1-\sigma_{n,k}}$. Plugging the above expression into trade share equation,

$$
\begin{aligned}
\pi_{ni,k} &= \frac{h_{ni,k} E_{i,k}^{\alpha_{ni,k}} (\tau_{ni,k} w_{i,k})^{1-\sigma_{n,k}}}{\sum_\ell h_{n\ell,k} E_{\ell,k}^{\alpha_{n\ell,k}} (\tau_{n\ell,k} w_{\ell,k})^{1-\sigma_{n,k}}} \\
& \frac{h_{ni,k} L_{i,k}^{(1-\sigma_{n,k})(1-\omega^2_{i,k})/\omega^1_{i,k}} (\tau_{ni,k}\Phi_i)^{1-\sigma_{n,k}}}{\sum_\ell h_{n\ell,k} L_{\ell,k}^{(1-\sigma_{n,k})(1-\omega^2_{\ell,k})/\omega^1_{\ell,k}} (\tau_{n\ell,k}\Phi_\ell)^{1-\sigma_{n,k}}}
\end{aligned}
\tag{A.24}
$$

where $h_{ni,k} \equiv b_{ni,k}(\eta_{i,k}F_{i,k})^{-\frac{\sigma_{n,k}-1}{\eta_{i,k}-1}}\left(\frac{\eta_{i,k}}{\eta_{i,k}-1}\right)^{1-\sigma_{n,k}} a_{i,k}^{\sigma_{n,k}-1}$ is a composite exogenous shifter. Equation A.24 connects our model to equation (6) in Kucheryavyy et al. (2016) by noting that

$$\alpha_{ni,k}^{KLR} = (1-\sigma_{n,k})(1-\omega_{i,k}^{2})/\omega_{i,k}^{1}, \quad w_{i}^{KLR} = \Phi_i, \quad \epsilon_{n,k}^{KLR} = \sigma_{n,k}-1$$

This mapping then translates their key uniqueness condition to the following inequality:

$$(1-\sigma_{n,k})(1-\omega_{i,k}^{2})/\omega_{i,k}^{1} \leq 1.$$

Note that this inequality is a necessary condition for uniqueness, whereas its violation is sufficient for multiplicity. We refer the interested reader to Propositions 1–5 in Kucheryavyy et al. (2016).

### 1.6 Supplementary material for the estimation

We begin by rewriting import demand $D_{ni,k}$ and export supply $S_{ni,k}$ as functions of consumer price index $P_{ni,k}$,

$$D_{ni,k} = D_{ni,k}^{P} P_{ni,k}^{1-\sigma_{n,k}} \tag{A.25}$$

$$S_{ni,k} = Y_{i,k} - \sum_{m\neq n} D_{mi,k} = Y_{i,k}^{P} P_{ii,k}^{\frac{\omega_{i,k}^{1}}{1-\omega_{i,k}^{2}}} - \sum_{m\neq n} D_{mi,k}^{P} P_{ii,k}^{1-\sigma_{m,k}} \tag{A.26}$$

Here, replacing $p_{ii,k}$ from the following

$$P_{ii,k} = \Lambda_{i,k} p_{ii,k}^{1-\omega_{i,k}^{2}} \tag{A.27}$$

we can obtain $D_{ni,k}$ and $S_{ni,k}$ as functions of price of a typical variety $p_{ni,k}$. The shifters used in these equations are:

$$D_{ni,k}^{P} = b_{ni,k}\tau_{ni,k}^{1-\sigma_{n,k}} P_{n,k}{}^{-(1-\sigma_{n,k})} \tag{A.28}$$

$$Y_{i,k}^{P} = \left(\gamma_i L_i \Phi_i^{1-\varepsilon_i} e_{i,k}\right)^{\omega_{i,k}^{1}\left[\frac{1+\phi_{i,k}}{\varepsilon_i} + \frac{\omega_{i,k}^{2}}{(1-\omega_{i,k}^{2})(\varepsilon_i-1)}\right]} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{\frac{\omega_{i,k}^{1}}{\omega_{i,k}^{2}}} \left(\eta_{i,k}F_{i,k}\right)^{-\frac{\omega_{i,k}^{1}}{\eta_{i,k}-1(1-\omega_{i,k}^{2})}} \tag{A.29}$$

$$\Lambda_{i,k} = (\eta_{i,k}F_{i,k})^{\frac{1}{\eta_{i,k}-1}} \left(\gamma_i L_i \Phi_i^{1-\varepsilon_i} e_{i,k}\right)^{-\frac{\omega_{i,k}^{2}}{\varepsilon_i-1}} \left(a_{i,k}\frac{\eta_{i,k}-1}{\eta_{i,k}}\right)^{-\omega_{i,k}^{2}} \tag{A.30}$$

Without loss of generality, and for the sake of exposition, we suppose that the supply shifter that changes over time is the productivity shifter $a_{i,k}$ (that is, we normalize $e_{i,k} = 1$ and $f_{i,k} = 1$ with the understanding that by a possible renaming $a_{i,k}$ can incorporate employment shifters $e_{i,kt}$ and entry costs $f_{i,k}$), and the demand shifter that changes over time is $b_{ni,k}$ (that is, we normalize $\tau_{nik} = 1$ with the understanding that by a possible renaming of $b_{ni,k}$, it can incorporate trade cost $\tau_{ni,k}$).

We now write import demand $D_{ni,k}$ and export supply $S_{ni,k}$ in log-linear terms, for local changes in price $p_{ni,k}$, as well as local changes in productivity and demand shifters, $a_{ni,k}$ and $b_{ni,k}$. Starting with import demand, and incorporating the time subscript,

$$\begin{aligned} \Delta \ln D_{ni,k,t} &= \widetilde{\sigma}_{ni,k}^{D}\Delta \ln p_{ni,k,t} + \widetilde{\sigma}_{ni,k}^{a}\Delta \ln a_{i,k,t} + \widetilde{\sigma}_{ni,k}^{b}\Delta \ln b_{ni,k,t} + \delta_{ni,k}^{D} \\ \text{where} \quad & \widetilde{\sigma}_{ni,k}^{D} = (1-\sigma_{n,k})(1-\omega_{i,k}) \\ & \widetilde{\sigma}_{ni,k}^{a} = (1-\sigma_{n,k})(-\omega_{i,k}) \\ & \widetilde{\sigma}_{ni,k}^{b} = 1 \end{aligned} \tag{A.31}$$

and, $\delta_{ni,k}^{D}$ is a possible fixed effect that is not time-variant. The log-linearization of changes to export supply requires more work, which delivers the following:

$$
\begin{aligned}
\Delta \ln S_{ni,k,t} \quad &= \quad \widetilde{\omega}^S_{ni,k,t} \Delta \ln p_{ni,k,t} + \widetilde{\omega}^a_{ni,k,t} \Delta \ln a_{i,k,t} + \sum_{m \neq n} \widetilde{\omega}^b_{mi,k,t} \Delta \ln b_{mi,k,t} + \delta^S_{ni,k} \qquad \text{(A.32)} \\
\text{where} \qquad & \widetilde{\omega}^S_{ni,k,t} = \frac{1}{\lambda_{ni,kt}} \omega^1_{i,k} - \sum_{m \neq n} \frac{\lambda_{mi,kt}}{\lambda_{ni,kt}} (1 - \sigma_{m,k})(1 - \omega^2_{i,k}) \\
& \widetilde{\omega}^a_{ni,k,t} = \frac{1}{\lambda_{ni,kt}} \left( \frac{\omega^1_{i,k}}{\omega^2_{i,k}} - \frac{\omega^1_{i,k} \omega^2_{i,k}}{1 - \omega^2_{i,k}} \right) - \sum_{m \neq n} \frac{\lambda_{mi,kt}}{\lambda_{ni,kt}} (1 - \sigma_{m,k})(-\omega^2_{i,k}) \\
& \widetilde{\omega}^b_{mi,k,t} = \frac{1 - \lambda_{ni,kt}}{\lambda_{ni,kt}}
\end{aligned}
$$

and, $\delta^S_{ni,k}$ is a possible fixed effect that is not time-variant. Note that unlike the import demand, the coefficients of local changes to prices and shifters are time-dependent in export supply.

The residuals of import demand and export supply are then given by:

$$
\Delta \rho_{ni,kt} \quad = \quad \widetilde{\sigma}^a_{ni,k} \Delta \ln a_{i,k,t} + \widetilde{\sigma}^b_{ni,k} \Delta \ln b_{ni,k,t} \qquad \text{(A.33)}
$$

$$
\Delta \varphi_{ni,kt} \quad = \quad \widetilde{\omega}^a_{ni,k,t} \Delta \ln a_{i,k,t} + \sum_{m \neq n} \widetilde{\omega}^b_{mi,k,t} \Delta \ln b_{mi,k,t} \qquad \text{(A.34)}
$$

Our identification assumption relies on (after controlling for possible fixed effects, including the referencing)

1. Productivity shocks and demand shocks are independent over time, $\mathbb{E}[\Delta \ln a_{i,k,t} \Delta \ln b_{ni,k,t}] = 0$.
2. Demand shocks across countries are independent over time, $\mathbb{E}[\Delta \ln b_{mi,k,t} \Delta \ln b_{ni,k,t}] = 0$, for $m \neq n$.

Putting together assumptions (1) and (2), and the equations describing residuals of import demand and export supply,

$$
\begin{aligned}
\mathbb{E}[\Delta \rho_{ni,k,t} \Delta \varphi_{ni,k,t}] = \mathbb{E}[\widetilde{\sigma}^a_{ni,k} \widetilde{\omega}^a_{ni,k,t} (\Delta \ln a_{i,k,t})^2] + \mathbb{E}[\widetilde{\sigma}^b_{ni,k} \widetilde{\omega}^a_{ni,k,t} (\Delta \ln a_{i,k,t})(\Delta \ln b_{i,k,t})] \qquad \text{(A.35)} \\
+ \mathbb{E}[\widetilde{\sigma}^b_{ni,k} \Delta \ln a_{i,k,t}) \sum_{m \neq n} \widetilde{\omega}^b_{mi,k,t} \Delta \ln b_{mi,k,t}] + \mathbb{E}[\widetilde{\sigma}^b_{ni,k} \Delta \ln b_{ni,k,t} \sum_{m \neq n} \widetilde{\omega}^b_{mi,k,t} \Delta \ln b_{mi,k,t}]
\end{aligned}
$$

The resulting expression depends on the second moment of changes to log productivity shock, multiplied by the shift it generates in import demand i.e. $\widetilde{\sigma}^a_{ni,k}$ and that in export supply i.e. $\widetilde{\omega}^a_{ni,k,t}$. Ultimately, there is a positive component remaining in the expectation term which we control for in our estimation. Although we derive our estimable equation more closely from the theory compared to previous studies, this remainder term presents in a similar fashion to the measurement error discussed by Feenstra (1994) and Broda and Weinstein (2006). We will thus follow Broda and Weinstein (2006) methodology and include the term $T^{\frac{3}{2}}_{ni,k} \left( \frac{1}{X_{ni,kt}} + \frac{1}{X_{ni,k,t-1}} \right)^{-\frac{1}{2}} \delta_{i,k}$, where $T$ is the duration of the trade relationship for $ni, k$, as a right-hand side control, and $\delta_{i,k}$ is an exporter-industry fixed effect. Here, the duration term and lagged trade flows absorb the variation in $\lambda$ presented in $\widetilde{\omega}^a_{ni,k,t}$, as well as the remaining term $\widetilde{\sigma}^a_{ni,k}$. Controlling for fixed effect $\delta_{i,k}$ is meant to capture the variance in productivity term $((\Delta \ln a_{i,k,t})^2)$.

# Appendix B Additional Results

## 2.1 Import and Export Elasticities w.r.t. Home Demand

Table A.1: Export elasticity of home demand

| | Food | Textile | Wood | Paper | Petr | Chem | Rubber | Mineral | B Metal | F Metal | Mach | Elect | E Mach | Vehicle | Other T | Furniture |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AUS | 0.10 | 0.27 | 0.12 | 0.39 | 0.14 | 0.19 | 0.22 | 0.15 | 0.08 | 0.14 | 0.13 | 0.22 | 0.19 | 0.43 | 0.26 | 0.37 |
| BRA | 0.24 | 0.30 | 0.09 | 0.23 | 0.20 | 0.29 | 0.20 | 0.19 | 0.09 | 0.17 | 0.13 | 0.17 | 0.14 | 0.05 | 0.20 | 0.26 |
| CAN | 0.15 | 0.29 | 0.09 | 0.31 | 0.10 | 0.10 | 0.11 | 0.15 | 0.04 | 0.14 | 0.03 | 0.18 | 0.09 | 0.38 | 0.23 | -1.59 |
| CHN | 0.20 | 0.10 | 0.11 | -0.62 | 0.20 | 0.13 | 0.15 | 0.12 | -0.95 | 0.12 | 0.09 | 0.10 | 0.13 | 0.21 | 0.21 | 0.09 |
| DEU | 0.26 | 0.21 | 0.10 | 0.29 | 0.09 | 0.12 | 0.23 | 0.15 | 0.05 | 0.11 | 0.08 | 0.10 | 0.10 | 0.12 | 0.10 | 0.05 |
| FRA | 0.14 | 0.16 | 0.11 | 0.36 | 0.15 | 0.09 | 0.27 | 0.12 | 0.05 | 0.14 | 0.06 | 0.13 | 0.13 | 0.10 | 0.09 | 0.15 |
| GBR | 0.24 | 0.27 | 0.08 | 0.29 | 0.11 | 0.13 | 0.27 | 0.13 | 0.07 | 0.12 | 0.08 | 0.12 | 0.09 | 0.27 | 0.21 | 0.21 |
| IDN | -0.46 | -0.52 | 0.07 | 0.26 | 0.42 | 0.09 | 0.22 | 0.11 | 0.10 | 0.13 | 0.10 | 0.29 | 0.22 | 0.05 | 0.19 | 0.15 |
| IND | 0.38 | 0.28 | 0.12 | 0.39 | 0.13 | 0.23 | 0.23 | 0.20 | 0.13 | 0.18 | 0.16 | 0.25 | 0.16 | 0.19 | 0.24 | 0.27 |
| ITA | -0.10 | 0.14 | 0.09 | 0.29 | 0.15 | 0.10 | 0.28 | 0.14 | -0.61 | 0.14 | 0.08 | 0.11 | 0.13 | 0.19 | 0.22 | 0.09 |
| JPN | -0.20 | 0.29 | 0.14 | 0.35 | 0.20 | 0.15 | 0.21 | 0.12 | 0.11 | 0.10 | 0.07 | 0.08 | 0.15 | 0.07 | 0.27 | -0.91 |
| KOR | 0.03 | 0.24 | 0.11 | 0.32 | 0.07 | 0.06 | 0.07 | 0.13 | 0.06 | 0.09 | 0.09 | 0.06 | 0.13 | 0.09 | 0.08 | -0.93 |
| MEX | 0.13 | 0.26 | 0.12 | 0.29 | 0.16 | 0.23 | 0.14 | 0.15 | 0.05 | 0.17 | 0.02 | 0.07 | 0.02 | 0.50 | 0.13 | 0.23 |
| RUS | 0.17 | 0.92 | 0.11 | 0.32 | 0.08 | 0.17 | 0.31 | 0.07 | 0.08 | 0.06 | 0.16 | 0.18 | 0.18 | 0.26 | 0.15 | 0.29 |
| USA | -0.24 | 0.34 | 0.10 | 0.25 | 0.13 | 0.14 | 0.13 | 0.15 | 0.07 | 0.10 | 0.08 | 0.13 | 0.11 | 0.23 | 0.16 | 0.44 |

Table A.2: Import elasticity of home demand

| | Food | Textile | Wood | Paper | Petr | Chem | Rubber | Mineral | B Metal | F Metal | Mach | Elect | E Mach | Vehicle | Other T | Furniture |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AUS | 0.96 | 0.94 | 0.91 | 0.68 | 0.88 | 0.94 | 0.96 | 0.86 | 0.98 | 0.85 | 0.95 | 0.85 | 0.91 | 0.87 | 0.98 | 0.94 |
| BRA | 0.80 | 0.54 | 0.91 | 0.73 | 0.80 | 0.67 | 0.59 | 0.77 | 0.91 | 0.82 | 0.86 | 0.87 | 0.90 | 1.00 | 0.84 | 0.87 |
| CAN | 0.86 | 0.94 | 0.92 | 0.76 | 0.91 | 0.96 | 0.95 | 0.94 | 0.97 | 0.87 | 0.99 | 0.90 | 0.96 | 0.91 | 0.84 | 1.14 |
| CHN | 0.49 | 0.95 | 0.84 | 1.17 | 0.26 | 0.90 | 0.78 | 0.87 | 1.67 | 0.97 | 0.82 | 0.95 | 0.77 | 0.93 | 0.84 | 0.95 |
| DEU | 0.83 | 0.93 | 0.91 | 0.67 | 0.91 | 0.94 | 0.76 | 0.87 | 0.94 | 0.86 | 0.95 | 0.96 | 0.93 | 0.89 | 0.95 | 0.98 |
| FRA | 0.94 | 0.89 | 0.90 | 0.79 | 0.86 | 0.94 | 0.78 | 0.89 | 0.98 | 0.85 | 0.98 | 0.93 | 0.87 | 0.90 | 0.91 | 0.96 |
| GBR | 0.28 | 0.92 | 0.93 | 0.74 | 0.93 | 0.92 | 0.79 | 0.89 | 0.98 | 0.88 | 0.96 | 0.94 | 0.99 | 0.86 | 0.88 | 0.93 |
| IDN | 2.10 | 1.48 | 0.94 | 0.77 | 0.80 | 0.77 | 0.89 | 0.92 | 0.73 | 0.94 | 0.95 | 0.88 | 0.85 | 1.04 | 0.90 | 0.93 |
| IND | 0.63 | 0.49 | 0.90 | 0.65 | 0.87 | 0.89 | 0.71 | 0.93 | 0.91 | 0.94 | 0.89 | 0.85 | 0.91 | 0.83 | 0.85 | 0.99 |
| ITA | 1.13 | 0.85 | 0.92 | 0.73 | 0.85 | 0.95 | 0.76 | 0.87 | 1.50 | 0.90 | 0.94 | 0.93 | 0.92 | 0.90 | 0.81 | 0.90 |
| JPN | 1.17 | 0.76 | 0.96 | 0.63 | 0.85 | 0.97 | 0.72 | 0.85 | 0.92 | 0.86 | 0.93 | 0.83 | 0.86 | 0.97 | 0.60 | 1.57 |
| KOR | 0.99 | 0.83 | 0.90 | 0.72 | 0.95 | 0.90 | 0.97 | 0.94 | 0.81 | 0.95 | 0.92 | 0.95 | 0.93 | 1.01 | 0.93 | 1.64 |
| MEX | 0.79 | 0.78 | 0.94 | 0.79 | 0.89 | 0.85 | 0.97 | 0.77 | 0.98 | 0.90 | 0.99 | 0.97 | 0.98 | 0.88 | 0.90 | 0.96 |
| RUS | 0.93 | 0.40 | 0.92 | 0.72 | 0.94 | 0.82 | 0.71 | 0.99 | 0.96 | 0.68 | 0.81 | 0.77 | 0.86 | 0.61 | 0.94 | 0.92 |
| USA | 0.50 | 0.85 | 0.92 | 0.74 | 0.89 | 0.88 | 0.90 | 0.89 | 0.96 | 0.88 | 0.97 | 0.95 | 0.96 | 0.40 | 0.86 | -0.39 |