The Dynamics of Importer-Exporter Connections*

Victor Gimenez-Perales†

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Abstract

I develop a trade model featuring importer-exporter connections. In the model, importers differ in productivity, final demand elasticity, and substitutability of inputs. Importers invest in expanding the set of potential exporters and choose from which to source. The model delivers three novel predictions. The lower final demand elasticity and the higher substitutability of inputs of an importer: (i) the lower the growth rate in importer's connections, (ii) the more likely are connections to be discontinued, and (iii) the lower the trade value growth per surviving connection. I provide evidence in favor of these predictions by using customs transaction data from Colombia.

JEL: D21, F10, F14, F23, L14

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1 Introduction

There is an exporter and an importer behind every single transaction in international trade. The realization of this simple fact has opened a new area for research in which international trade is a phenomenon shaped by importer-exporter connections. A large body of research is dedicated to understanding how these connections are created, developed, and destroyed. It has already been shown that the survival probability and the trade value growth of importer-exporter connections depend on a variety of factors: on uncertainty (Rauch and Watson, 2003), reputation (Macchiavello and Morjaria, 2015), institutions (Araujo et al., 2016), and switching costs (Monarch, 2021), for example. While this literature has

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†Kiel Institute for the World Economy. E-mail: victor.gimenez-perales@ifw-kiel.de

provided valuable insights into the dynamics of importer-exporter connections, it has so far considered these connections in isolation. Specifically, it has not considered that the survival probability and the trade value growth of connections might depend on other connections that firms might have.

However, importers usually buy more than one product and from more than one exporter. For example, Bernard et al. (2018a) show that Colombian importers buy 14.73 products from 5.26 exporters on average. Given that firms import intermediate inputs to lower their marginal costs, each of an importer's connections is linked to the others through the importer's production function. The contribution of this paper is to study the survival probability and trade value growth of importer-exporter connections taking into account that other connections might be complements or substitutes for the importing firm.

After initially presenting some empirical regularities about importer-exporter connections, my analysis consists of two steps. First, motivated by the empirical regularities, I develop a dynamic theoretical model to analyze how importers decide from which exporters to source their intermediate inputs. An important novel aspect of the model is that an importer's decision to source from an exporter influences its sourcing decision with all other, present and potential, exporters. Second, I test the predictions of the model empirically using detailed transaction-level data from Colombian importers.

In the model, importers are heterogeneous in their productivity, final demand elasticity, and substitutability of inputs. Importers invest in their *supplier list*, defined as the set of exporters they know and can potentially create an active connection and purchase its intermediate input. As each importer's supplier list expands, they choose a set of active connections from their respective supplier list, trading off two effects. While each additional active connection decreases the importer's marginal cost of production, it simultaneously increases the cost of maintaining its connections. The main takeaway of the model is that the relevant determinant of connections' survival and trade value growth is an expression that increases in final demand elasticity and decreases in input substitutability. I label this expression the *elasticity gap*.

The first result of the model is that importers with a smaller elasticity gap increase less the number of connections over time. This result is the outcome of two opposing effects playing out inside the importer's production function. On the one hand, the drop in marginal costs from additional connections is smaller, the higher the substitutability of inputs. On the other hand, a given drop in marginal costs is boosting profits less, the smaller the final demand elasticity. The elasticity gap concisely captures both effects. In other words, the elasticity gap is linked to how profitable is each new connection for the importer. Since importers with a smaller elasticity gap benefit less from new connections, they invest less in meeting new exporters.

The second result is that the connections of importers with a smaller elasticity gap have a lower survival rate. As new exporters are added to the supplier list, importers with a smaller elasticity gap are more prone to drop existing connections. The mechanism for the lower survival rate of importers with a smaller elasticity gap is that, after an increase in the supplier list, they concentrate their intermediate input purchases on high-productivity exporters. This reduces the trade value of their connections with low-productivity exporters, such that relatively more connections fall below the required profitability for maintaining the connection and are consequently destroyed.

The third result concerns connections' trade value growth: conditional on surviving, the connections of importers with a smaller elasticity gap exhibit lower trade value growth. Intuitively, importers facing a low final demand elasticity expand production less after reducing their marginal costs. At the same time, the reduction in marginal costs from connecting to new exporters is smaller when the substitutability of inputs is higher. Again, the elasticity gap captures both effects: the smaller an importer's elasticity gap, the lower the increase in revenues when new exporters become available, which results in a smaller increase in the demand for intermediate inputs from each exporter. Intuitively, for importers with a lower elasticity gap, the increase in their overall demand for intermediate inputs after sourcing from an additional exporter results in a smaller increase in their demand for intermediate inputs from each exporter.

I provide empirical support for the theoretical results of the model using detailed manufacturing and trade data from Colombia. First, I estimate the elasticity gap at the 4-digit ISIC industry level. This is done by estimating the final demand elasticity with the firm-level manufacturing data as in De Loecker (2011) and computing the trade-weighted average of imported inputs elasticity using estimates from Soderbery (2018). Then, I test the model's predictions on number of connections, survival probability, and trade value growth using transaction-level trade data from Colombian importers over more than ten years. The data allow me to control for year, country of the exporter, and imported product at the 6-digit HS level.

I show that, as predicted by the model, importers with a smaller elasticity gap show lower growth in the number of connections, and their connections have lower survival rates and lower trade value growth. Zooming in on the discontinued connections, I provide evidence for the mechanism causing the lower survival of connections in importers with a small elasticity gap. I do so by showing that trade value is a more relevant factor when deciding which connections to keep for importers with a smaller elasticity gap. All empirical results are robust to different methods of estimating the elasticity gap.

My paper connects to various literatures. First, it is related to the literature on two-sided trade, characterized by models featuring individual, possibly heterogeneous, importers and exporters, like Eaton et al. (2011), Eaton et al. (2016), Bernard et al. (2018b), and Eaton et al. (2021). The papers most closely related to the present work in this literature are Eaton et al. (2016) and Eaton et al. (2021). Eaton et al. (2021) investigate how exporters' growth in foreign markets and how search costs affect aggregate exports. In their search and matching model, firms differ in productivity and product attractiveness, which is gradually revealed in a Bayesian manner by finding foreign partners. Their findings indicate that search costs are sizable, up to $50,000 for an expected yield of one customer, but decrease fast after the first connection is formed.

Eaton et al. (2016) features a model with survival and growth of importer-exporter connections in international markets. They find that reductions in search costs can lead to large trade increases and welfare gains: a 30% reduction in search costs leads to a 10% increase in welfare due to the larger number of varieties available. However, Eaton et al. (2016) and Eaton et al. (2021) take connections' survival probability as exogenous while it is endogenous in my model. My contribution to this literature is that, by endogenizing connections' survival, I can reveal a precise mechanism for importer-exporter connections' survival and trade value growth, showing, in particular, how elasticities determine industry differences.

Eaton et al. (2011) and Bernard et al. (2018b) explain features in the data from the perspective of the exporter facing different importers (homogeneous in Eaton et al. (2011) and heterogeneous in Bernard et al. (2018b)) and explore how importer-exporter connections interact with trade costs, but do not consider dynamic connections. I extend their models by adding the time dimension with a reduced form search mechanism on the importer side and introducing different elasticities of substitution in the intermediate input and final good markets. I show that the difference between both elasticities creates an additional dimension along which importer's connections are affected by trade cost shocks.

My paper also relates to the literature on relationships between firms in international markets. Within this literature, it is related to Rauch and Watson (2003), Besedeš and Prusa (2006a), Besedeš (2008), Nitsch (2009), Besedeš and Prusa (2011), Esteve-Pérez et al. (2013), and Cadot et al. (2013), (Macchiavello and Morjaria, 2015), (Araujo et al., 2016), and (Monarch, 2021). Some of these papers investigate the reasons for the large amount of short lived connections (where connections are a country-country or a firm-country pair) with low trade values.[1] One result of this literature is that the trade duration is positively correlated with product differentiation Besedeš and Prusa (2006b) and product elasticity Nitsch (2009). My paper contributes to this literature by providing a theoretical foundation for the correlation between product elasticity and trade duration.

My paper also contributes to the literature on networks and trade, with Oberfield (2018), Bernard et al. (2019b) and Bernard et al. (2019a) being the closest references in this literature. Oberfield (2018) develops a theory of endogenous network formation and shows that small productivity differences can cause large firm size heterogeneity. The factors influencing how productivity differences translate into size heterogeneity are the final good elasticity and the importance of the single intermediate input in the production function. Although I do not explicitly model network formation, my model implies that, when extending the production function to more than one intermediate input, the substitutability of inputs becomes an important determinant for firm size. Bernard et al. (2019b) use a model where firms can perform a task themselves or outsource to a supplier to estimate the impact of reducing search costs on firm performance. They find that after the introduction of a high-speed train in Japan, firms located near a new station connected to more suppliers in more locations and increased the share of inputs outsourced to suppliers. Bernard et al. (2019a) explain heterogeneity in firm size using a model where firms are heterogeneous in productivity and relationship capability. They show that models using only one source of heterogeneity cannot explain that large firms

[1]Most papers use data aggregated at the product-country level in their studies. Only Esteve-Pérez et al. (2013) and Cadot et al. (2013) use product-firm level data.

have more customers, but also sell less to each customer. My paper should be seen as a complement to theirs: they focus on within industry variation in connections' trade value, while my paper explains differences in connections' survival probability and trade value growth across industries.

In terms of the mechanisms shown in this paper, inputs substitutability has been shown relevant in the importers' literature, as in Goldberg et al. (2010) and Halpern et al. (2015). Goldberg et al. (2010) find that there are large productivity losses from an increase in import prices, while the effect depends on the distribution of importers. Halpern et al. (2015) measure a 22% increase in firms' productivity if they import all input varieties. I add to this literature by showing that it is not only inputs substitutability that matters, but rather the interaction between the inputs' substitutability and the final demand elasticity. Including this additional dimension might therefore provide more precise estimates of the productivity gains from importing.

The remainder of this paper is structured as follows. Section 2 shows some empirical regularities present in the Colombian customs data. Section 3 introduces the theoretical model. Section 4 describes the data and develops the empirical strategy used to test the model predictions. Section 5 presents the empirical results. Section 6 concludes.

2 Empirical Regularities

In this section, I describe the data and show some empirical regularities concerning connections' survival probability and trade value growth.

Data - The data presented in this section is the Colombian customs data, provided by the Colombian statistical office (DANE). The data is a transaction-level register of all foreign inputs purchased by Colombian firms during the years 2007 to 2018. In each transaction, there is information about the importing and exporting firms as well as the product at a 10-digit product category and the value traded. All values are expressed in 2014 Colombian pesos. Given the nature of the theoretical model, the set of importing firms is restricted to those industry codes recognized as manufacturing.[2]

Colombian importers are identified with their tax number, which is constant over time.

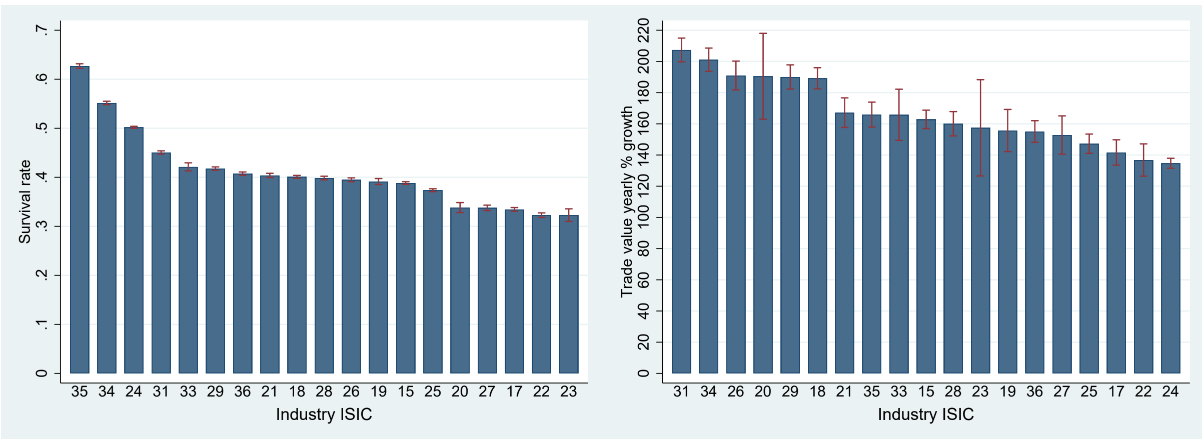
[2] In the Colombian industry codes (CIIU v3), this is 1500 to 3720. Industry codes changed in 2012 to CIIU v4 and have been translated to CIIU v3 using a correspondence table provided by the DANE.

Foreign exporters are identified by the country and the city where they are located. I, therefore, define an exporter as a combination of country, city, and a 10-digit product category. This implies that I consider, as in Armington (1969), Broda et al. (2006) and Soderbery (2018), that the same good from different locations is treated as different varieties. The location, in this case, is cities, instead of countries as in the above-mentioned papers. The procedure for standardizing city names to correct problems related to spelling mistakes or alternative names is explained in Appendix A.

Facts - The first fact is that the survival rate of connections and the trade value growth present strong differences across the importer's industry. This can be seen in Figure 1a, which illustrates the high variance on connection's survival between industries, even over a long period (2007 to 2018). Survival rates range from 62% in Division 35 (Manufacturing of other transport equipment) to 32% in Division 23 (Manufacture of coke, refined petroleum products, and nuclear fuel). Put differently, while 62% of the connections with importers from Division 35 survive from one year to the next, only 32% of connections with importers from Division 23 survive. The differences in mean trade value growth per connection, in Figure 1b, are very large as well, with values ranging between 134% and 207% per year.[3]

Figure 1: Differences across 2-digit industry

(a) Survival rate (b) Mean trade value growth



The second fact is that the differences between industries are persistent over the connection life-cycle. In Figure 2 I split the sample between those industries above the mean survival rate in the sample and those below. Figure 2a, using those connections

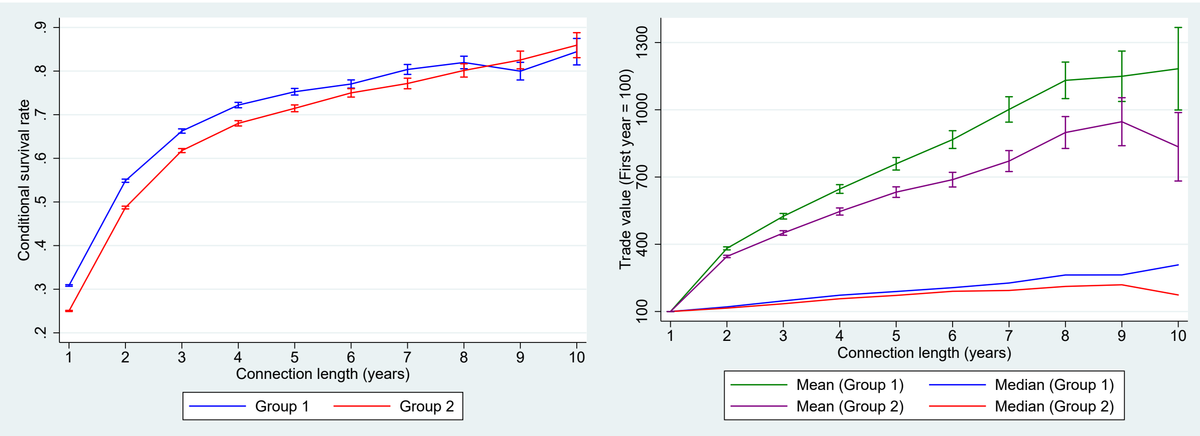
[3]Very similar differences can be observed in the figure on median trade value growth of connections at the 2-digit industry, in Appendix B.

starting after 2007, shows that the survival rate of connections, conditional on having survived the previous year, increases in the number of years survived. It starts from a low level, around 30%, to sharply increase during the first few years and reach 80% after 8 years. The conditional survival of the industries with a higher survival rate than the mean is systematically above that of the industries below the mean, starting with a 6% higher conditional survival (31% vs 25%), and remaining above until the eighth year.[4]

Concerning the trade value growth of connections, those industries with higher survival rates are also those with higher trade value growth at the connection level. Figure 2b shows the mean and median trade value of connections over time for the two groups of industries. Trade values are indexed to 100 in the first year of each connection to make them comparable and have been winsored at the 1st and 99th percentile each year to remove outliers. The median trade value of the connections after 10 years grows to roughly 3 times the initial sales in the industries above the average survival rate, while it is only around 2 times larger in the industries below. The mean trade value of connections follows a similar trend at a larger scale.

Figure 2: Conditional survival and trade value of connections

(a) Conditional survival (b) Trade value



Note: Group 1 includes those industries with a higher survival rate than the sample average, while Group 2 contains the industries with a lower survival rate than the sample average.

In sum, although the growing conditional survival and trade values over time are well known in the international trade literature,[5] the large and persistent differences among

[4]As in Bernard et al. (2018a), the data shows a large turnover of suppliers among Colombian importers in all industries, with only slightly more than one-quarter of the connections lasting more than a year. In absolute numbers, the amount of connections drops from around 750,000 to 50,000 in just four years, and to only 1,000 after eleven years.

[5]For example, Fitzgerald et al. (2016), Ruhl and Willis (2017), and Bernard et al. (2017) show the

industries observed in figures 1 and 2 have not been reported before. This heterogeneity indicates that firms across industries exhibit different behavior towards their connections in the international markets. In the next section, I use a theoretical model with firm-to-firm trade to identify the determinants of these differences.

3 Model

This section develops a dynamic model of partial equilibrium in which importers invest in meeting new exporters and endogenously decide from which exporters they want to source their inputs. The idea of firms actively searching for clients or suppliers has been recurrent in the international trade literature, and the approach taken here is similar to that in Drozd and Nosal (2012) and Fitzgerald et al. (2016), who modeled a *list of customers* and a *customer base*, respectively.[6]

The world consists of two countries,[7] Home and Foreign. As in Bernard et al. (2018b), heterogeneous firms produce either a final good or an intermediate input. The intermediate inputs are used by final good producing firms in their production function. Final good firms, in turn, face a standard CES demand. To simplify notation, I will assume final good producers (the importers) are located only in the Home country, while intermediate input producers (the exporters) are exclusively in the Foreign.

Households - The country Home is populated by a continuum of L consumers, deriving utility from the consumption of varieties of a final good. Their preferences are given by the following CES utility function:

$$U_t = \left[\int_\Omega C_{it}^{\frac{\sigma^F - 1}{\sigma^F}} di \right]^{\frac{\sigma^F}{\sigma^F - 1}}, \tag{1}$$

same patterns, aggregated at the exporter level.

[6] Other forms of investing to reach out to new customers in other countries can be found, for example, in Arkolakis (2010) and Eaton et al. (2011). My approach, as well as that of Drozd and Nosal (2012) and Fitzgerald et al. (2016), can be thought of as a dynamic version of these two papers on the importer side.

[7] The focus on only two countries is a strong simplification, but close to the empirical facts presented in previous literature. Bernard et al. (2018a) report that three-quarters of the importers with multiple suppliers source from a single country, and on average importers source from between two and three countries. The finding of Antràs et al. (2017) is similar using data for the US, with importing firms sourcing on average from only three countries. In my data, the median importer sources from 2 countries and almost 40% source from only one country.

with $\sigma^F > 1$ being the final demand elasticity across varieties of the final good. The set of varieties available to the consumers is given by Ω, with i indicating each differentiated variety of the final good. Each consumer provides one unit of labor and receives the corresponding wage w, which for simplicity is normalized to one. From the consumer's utility maximization problem, the optimal aggregated demand for each variety is $C_{it} = [P_{it}/Q_t]^{-\sigma^F} L/Q_t$, where C_{it} and P_{it} are, respectively, the consumption and price of the final good variety i at time t and Q_t is the price index in the domestic final goods market, defined as

$$Q_t = \left[\int_\Omega P_{it}^{1-\sigma^F} di\right]^{\frac{1}{1-\sigma^F}}. \tag{2}$$

Final goods market - Final goods are produced in Home by importing firms. Importers purchase inputs from exporters and use them in the production of a differentiated variety of the final good, which they then sell to domestic consumers. Importers are identified by the variety they produce (i) and are heterogeneous in two dimensions: first, in their productivity (denoted by Z_i), which is randomly drawn from a distribution $G(Z)$ and constant over time,[8] and second, in the set of intermediate inputs used in production (Λ_{it}). Their production technology is given by

$$Y_{it} = Z_i \left[\int_{\Lambda_{it}} c_{it}(\lambda)^{\frac{\sigma^I-1}{\sigma^I}} d\lambda\right]^{\frac{\sigma^I}{\sigma^I-1}}, \tag{3}$$

where Y_{it} is the quantity of final good produced, $c_{it}(\lambda)$ is the quantity of the intermediate input λ consumed, and $\sigma^I > 1$ is the substitutability of inputs. I refrain from imposing any restriction on the size of Λ_{it}, meaning that the final good producer can always increase production by including more intermediate input varieties to the production function. This assumption is without loss of generality. Restricting Λ_{it} to a range between 0 and 1, as in Bernard et al. (2019b) or Antràs et al. (2017), would not change any of my results.[9]

[8] This assumption could be relaxed leaving results qualitatively unchanged.

[9] In that case one would have to assume a setup in which firms can source these intermediate inputs either within the firm (Bernard et al., 2019b) or within Home country (Antràs et al., 2017) and the increase in imported varieties would then substitute costly domestic intermediate inputs with cheaper foreign ones. The result in the model would be the same: a decrease in the marginal cost from any additional variety imported.

The demand of an importer for an intermediate input λ can be expressed as:

$$c_{it}(\lambda) = \frac{E_{it}}{q_{it}} \left(\frac{p(\lambda)}{q_{it}} \right)^{-\sigma^I}, \tag{4}$$

where E_{it} is the expenditure on inputs of importer i, $p(\lambda)$ is the price of the intermediate input variety λ and q_{it} is the price index of intermediate inputs faced by the importer, which is given by:

$$q_{it} = \left[\int_{\Lambda_{it}} p(\lambda)^{1-\sigma^I} d\lambda \right]^{\frac{1}{1-\sigma^I}}. \tag{5}$$

Given the utility function introduced in equation (1), the price that an importer charge in the final goods market is a constant markup over its marginal costs:

$$P_{it} = \bar{M} \frac{q_{it}}{Z_i}, \tag{6}$$

where $\bar{M} = \frac{\sigma^F}{\sigma^F - 1}$.

Intermediate inputs market - The intermediate inputs market is supplied by a continuum of exporters, each one producing a single differentiated variety of the intermediate input using labor as their only factor of production. They are heterogeneous on their productivity z, randomly drawn from a Pareto distribution with a lower bound z_L: $F(z) = 1 - (z_L/z)^\gamma$, with $\gamma > \sigma^I - 1$. As in the case of importers, the productivity of the exporters does not change over time.

Because all relevant variables of an exporter are determined only by its productivity z, I will denote the exporter producing a variety λ by its productivity z. Given the importers' demand function in equation (4), the price at which an exporter with productivity z sells its variety is a constant markup over their marginal cost:

$$p(z) = \bar{m} \frac{\tau^* w^*}{z}, \tag{7}$$

where w^* is the wage in Foreign country, $\tau^* \geq 1$ is the standard iceberg trade cost and $\bar{m} = \frac{\sigma^I}{\sigma^I - 1}$.

Supplier list - For simplicity, I assume that the amount of firms in both economies is exogenous, with a mass N of importers in Home and a mass n of exporters in Foreign.

Furthermore, importers only know a subset H_{it} of the exporters ($H_{it} \subset n$) at any time t. I label H_{it} the *supplier list*, and it is specific to each importer. The supplier list will be key in determining the set of intermediate inputs used in production by importer i (Λ_{it}), as importers can only source from exporters in their supplier list ($\Lambda_{it} \subset H_{it}$).

Importers can grow their supplier list over time by meeting new exporters. The mass of new exporters that an importer i meets in period t is denoted by A_{it}. To keep the model tractable, I assume that all exporters have the same probability of being met, regardless of their productivity. As a result of this assumption, the subset H_{it} of exporters is a random sample of the population n, and therefore the productivity distribution of exporters in H_{it} is identical to n. The law of motion of H_{it} is given by the following equation:

$$H_{it+1} = \min\{(1-\delta)H_{it} + A_{it}, n\}, \tag{8}$$

where δ is an exogenous death rate of exporters in the economy.[10] Notice that H_{it} has an upper bound in equation (8), equal to the total amount of exporters in Foreign, n. I am assuming for the rest of the analysis that n is large enough, such that the steady state of H_{it} is always smaller than n.[11]

Finally, to add new exporters to the supplier list importers need to incur a cost. The cost of meeting a mass A_{it} of exporters is determined by the cost function $D(A_{it})$:

$$D(A_{it}) = \phi \frac{A_{it}^2}{2}, \tag{9}$$

where ϕ is a parameter governing the cost of meeting new exporters and A_{it} enters the cost function squared to reflect diminishing returns on investments in each period. This investment cost can be interpreted as identifying possible exporters as well as negotiating prices and quantities with them, an activity that reveals the productivity of both firms to each other in the process. Because of the nature of the investment, I assume also irreversibility (i.e. $A_{it} \geq 0$).

Connections - To maintain each connection, i.e. each exporter that the importer is actively sourcing from, firms need to incur some fixed cost f_r. The effect of this cost on the outcomes of the model is the same irrespective of which firm is bearing the cost. Since

[10] Note that this death rate can also be interpreted as the exit rate of foreign firms from exporting.
[11] See Appendix D to see the necessary restriction on n such that $H_{it} < n \; \forall i, t$.

I focus my analysis on the importer side, I assume that importers are bearing the cost.[12]

From equations (4), (6), and (7) the revenue associated with a connection is increasing on importer's and exporter's productivity. This, together with the fixed cost per connection, generates a sorting pattern in which an importer with the characteristics (Z_i,H_{it}) will purchase inputs from all exporters with productivity z above a certain threshold $\underset{\sim}{z}(Z_i, H_{it})$ because all connections with exporters with productivity above $\underset{\sim}{z}(Z_i, H_{it})$ increase the overall profit of the importer. Next, I find this productivity threshold by solving the static profit maximization problem of the importer for any period t.

The static problem of the importer - Given the previous final goods and intermediate inputs markets, the profits of an importer i in period t, without taking into account the investment cost on its supplier list, are given by the following equation:

$$\Pi(Z_i, H_{it}) = \max_{\underset{\sim}{z}(Z_i,H_{it})} \{R(Z_i, H_{it}) - E(Z_i, H_{it}) - F_r(Z_i, H_{it})\},$$

where $R(Z_i, H_{it})$ is importer's revenue, $E(Z_i, H_{it})$ importer's expenditure on intermediate inputs, and $F_r(Z_i, H_{it})$ the total amount of fixed cost an importer needs to pay to keep the connections with its exporters.

Since the price that an importer charges is a constant markup over its marginal costs, I can rewrite $E(Z_i, H_{it}) = \bar{M}^{-1}R(Z_i, H_{it})$. Total revenues is given by equation (10):

$$R(Z_i, H_{it}) = P(Z_i, H_{it})C(Z_i, H_{it}) = L\left[\bar{M}\frac{q(Z_i, H_{it})}{Z_i Q_t}\right]^{1-\sigma^F}. \tag{10}$$

Also, I express the sum of fixed costs as the mass of firms above the threshold multiplied by the fixed cost per connection:

$$F_r(Z_i, H_{it}) = [1 - F(\underset{\sim}{z}(Z_i, H_{it}))]H_{it}f_r = H_{it}z_L^{\gamma}f_r\underset{\sim}{z}(Z_i, H_{it})^{-\gamma}. \tag{11}$$

[12]See Bernard et al. (2018b) for the case where exporters are paying the cost.

Using equations (10) and (11), the importer's profit can be written as:

$$\Pi(Z_i, H_{it}) = \max_{\underset{\sim}{z}(Z_i,H_{it})} \left\{ \frac{L}{\sigma^F} \left(\frac{Z_i k_1}{\tau^* w^*} \right)^{\sigma^F - 1} (H_{it} k_2)^{\frac{\sigma^F-1}{\sigma^I-1}} \underset{\sim}{z}(Z_i, H_{it})^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}} - H_{it} z_L^{\gamma} f_r \underset{\sim}{z}(Z_i, H_{it})^{-\gamma} \right\}, \tag{12}$$

where $k_1 = \frac{Q_t}{\bar{m}M}$ and $k_2 = \frac{\gamma z_L^{\gamma}}{\gamma-(\sigma^I-1)}$. The first order condition of equation (12) with respect to $\underset{\sim}{z}(Z_i, H_{it})$ implies:

$$\underset{\sim}{z}(Z_i, H_{it}) = \left[\frac{\sigma^F(\sigma^I-1)}{(\sigma^F-1)} \frac{f_r}{L} \right]^{\frac{1}{(\sigma^F-1)(1-\gamma\eta)}} \left(\frac{\tau^* w^*}{k_1 Z_i} \right)^{\frac{1}{1-\gamma\eta}} (k_2 H_{it})^{-\frac{\eta}{1-\gamma\eta}}, \tag{13}$$

where I define the *elasticity gap* (η) as

$$\eta \equiv \frac{1}{\sigma^I - 1} - \frac{1}{\sigma^F - 1}. \tag{14}$$

The elasticity gap decreases if there are fewer complementarities in the importer's production function (high σ^I), and increases if there is more competition in the final good market (high σ^F).

An additional parametrical assumption necessary for the maximization problem to have an interior solution is that $\gamma\eta < 1$.[13] In the cases where $\gamma\eta \geq 1$ the solution to the problem would be one in which importers either purchase intermediate inputs from all known exporters or have zero expenditure on intermediate inputs.

The relationship between the supplier list and the productivity cutoff depends on the elasticity gap and is summarized in lemmas 1 and 2.

Lemma 1. *The elasticity of the productivity cutoff with respect to the supplier list ($\varepsilon_{\underset{\sim}{z},H}$) is increasing on the elasticity gap (η):*

$$\text{If } \varepsilon_{\underset{\sim}{z},H} \equiv \left| \frac{\partial \ln \underset{\sim}{z}(Z_i, H_{it})}{\partial \ln H_{it}} \right|, \text{ then } \frac{\partial \varepsilon_{\underset{\sim}{z},H}}{\partial \eta} > 0.$$

Proof: See the Appendix C.

[13] This might seem a strong assumption, but it is a consequence of the simplicity of the production function and the assumption on the distribution of exporters' productivity. Reducing the importance of intermediate inputs in equation (3) by adding other factors of production or choosing a different exporters' productivity distribution would relax this assumption without influencing the results.

Lemma 2. *The effect of an increase in the supplier list on the productivity cutoff depends on the elasticity gap as follows:*

i) if $\eta < 0$*,* $\underset{\sim}{z}(Z_i, H_{it})$ *is increasing in* H_{it}*.*

ii) if $\eta > 0$*,* $\underset{\sim}{z}(Z_i, H_{it})$ *is decreasing in* H_{it}*.*

iii) if $\eta = 0$*,* $\underset{\sim}{z}(Z_i, H_{it})$ *does not depend on* H_{it}*.*

Proof: See the Appendix C.

Lemma 1 indicates that the larger the elasticity gap (in absolute values) the stronger the elasticity of the productivity cutoff with respect to the supplier list. Specifically, when the importer faces a final demand elasticity that is very different from its substitutability of inputs, any change in the supplier list will have a strong effect on the productivity cutoff. The direction of this effect is given by 2.

Lemma 2 states when exporters are substitutes or complements from the importer's perspective. This result is similar to Antràs et al. (2017), who found that, depending on parametric restrictions, source countries can be substitutes or complements, i.e. an importer might be more or less likely to source from a country if it is sourcing from another country. In the case of Antràs et al. (2017), the parameters determining whether source countries are substitutes or complements are the value of the elasticity of demand faced by the importer and the dispersion of input productivities across countries.

If an importer faces a negative elasticity gap ($\eta < 0$), then exporters are substitutes for the importer: each additional connection decreases the profit that the importer derives from all other connections. In this case, an increase in the number of exporters in the supplier list of the importer causes the importer to substitute away from the less productive exporters towards more productive ones. This can be seen in equation (13) as an increase in the minimum productivity threshold $\underset{\sim}{z}(Z_i, H_{it})$ when the supplier list increases. Notice that this does not imply a reduction in the total number of connections for the importer, but only that the growth rate of connections is smaller than the growth rate of the supplier list.

The case in which exporters are complements follows a similar reasoning. If the elasticity gap of an importer is positive ($\eta > 0$), the importer reduces the minimum exporter's productivity required to establish a connection when its supplier list increases. Moreover, as mentioned above, in the cases where $\gamma\eta > 1$ the solution to the maximization problem degenerates into a corner solution: the complementarity effect is so strong that the importer always connects to all exporters in its supplier list.

Intuitively, an increase in the supplier list implies an increase in the number of exporters above a certain productivity threshold such that, for the same productivity threshold, more varieties of the intermediate input are entering the production function of the importer. The mechanism through which an importer's marginal cost and revenues are affected by an additional connection rests on the following two forces. First, there is a reduction in the importer's marginal cost from adding a new exporter to the production function. But this reduction is larger, the lower the substitutability of inputs σ^I of the importer. Thus, the lower σ^I, the larger the reduction in marginal costs per additional connection. This effect is similar to Halpern et al. (2015), where lower substitutability between domestic and foreign inputs implies larger productivity gains from importing. Second, the reduction in the importer's marginal cost is translated into higher revenues and profits when its final demand elasticity σ^F is higher. This effect is similar to Antràs et al. (2017), where the effect of a marginal cost decrease from importing on firm sales increases on final demand elasticity.

Combining both effects, the reaction of the importer's profit to an increase in connections depends on both elasticities, σ^I and σ^F. In particular, a new connection increases profits more, the lower σ^I and the higher σ^F of the importer. The combination of both effects is reflected in the elasticity gap: those importers with a positive (negative) elasticity gap increase (decrease) the profits of each connection after an increase in their supplier list. As such, Bernard et al. (2018b) can be considered a special case, in which both channels exactly cancel each other (the case $\sigma^I = \sigma^F$) and as a consequence, the number of exporters does not influence the optimal $\underset{\sim}{z}(Z_i, H_{it})$.

Having determined the optimal $\underset{\sim}{z}(Z_i, H_{it})$ for importers, I can calculate now the trade value of the connections. The implied trade value of a connection from importer's demand in equation (4), using the optimal $\underset{\sim}{z}(Z_i, H_{it})$ from equation (13), is:

$$r(z, Z_i, H_{it}) = \left(\frac{p(z)}{q(Z_i, H_{it})}\right)^{1-\sigma^I} E(Z_i, H_{it}) = X_1(z, Z_i) H_{it}^{\frac{\gamma\eta}{1-\gamma\eta}}, \tag{15}$$

where

$$X_1(z, Z_i) = \frac{L}{\bar{M}} \left(\frac{Z_i k_1}{\tau^* w^*}\right)^{\frac{\gamma}{1-\gamma\eta}} z^{\sigma^I - 1} k_2^{\frac{\gamma\eta}{1-\gamma\eta}} \left[\frac{\sigma^F(\sigma^I - 1)}{(\sigma^F - 1)} \frac{f_r}{L}\right]^{\frac{\sigma^I - 1 - \gamma}{(\sigma^I - 1)(1-\gamma\eta)}}.$$

Equation (15) shows that the relationship between the trade value of connections and the supplier list depends on the elasticity gap. This relationship is summarized in lemmas 3 and 4.

Lemma 3. *The elasticity of the trade value of connections with respect to the supplier list ($\varepsilon_{r,H}$) is increasing on the elasticity gap (η):*

$$\textit{If } \varepsilon_{r,H} \equiv \left| \frac{\partial \ln r(z, Z_i, H_{it})}{\partial \ln H_{it}} \right|, \textit{ then } \frac{\partial \varepsilon_{r,H}}{\partial \eta} > 0.$$

Proof: See the Appendix C.

Lemma 4. *The effect of an increase in the supplier list on the trade value of connections depends on the elasticity gap as follows:*

i) if $\eta < 0$, $r(z, Z_i, H_{it})$ is decreasing in H_{it}.

ii) if $\eta > 0$, $r(z, Z_i, H_{it})$ is increasing in H_{it}.

iii) if $\eta = 0$, $r(z, Z_i, H_{it})$ does not depend on H_{it}.

Proof: See the Appendix C.

The intuition for lemmas 3 and 4 is that the reduction in marginal cost from adding new connections after an increase in the supplier list is translated into overall revenue growth, but this growth depends on the importer's elasticity gap. As is the case with the productivity cutoff, the direction of the change in the trade value of connections depends on the sign of the elasticity gap, while the strength of the change depends on its absolute value. In the case of importers with a negative elasticity gap, revenue increases relatively less than the number of new connections, such that the importer's demand from every single exporter is reduced. From an exporter's perspective, the increase in the importer's connections increases the competition within the importer, leading to the importer purchasing less from each exporter. The case is different for importers with a positive elasticity gap, for which the increase in competition within the importer is offset by the increase in inputs demand. This leads to the importer increasing the trade value with all its connections.

The dynamic problem - Up to this point, I have treated the supplier list at time t as exogenous, as it is determined in $t-1$. However, the mass of new exporters added to the supplier list (A_{it}) is chosen endogenously by the importers and determines the evolution of their supplier list over time. In the dynamic maximization problem, an importer i maximizes the expected flow of profits, discounted at a rate β, with $1 > \beta > 0$. The importer does so by choosing at time t the mass of exporters to meet while taking its productivity and supplier list as given, subject to the law of motion described in equation (8), and using the optimal productivity threshold from equation (13). The Bellman equation of the problem is the following:

$$\begin{aligned} V(Z_i, H_{it}) = &\max_{A_{it}\geq 0} \{\Pi(Z_i, H_{it}) - D(A_{it}) + \beta V(Z_i, H_{it+1})\} \\ \text{s.t.} \quad H_{it+1} = &\min\{(1-\delta)H_{it} + A_{it}, n\} \end{aligned} \tag{16}$$

Note that there is no uncertainty in future profits. Uncertainty could easily be added by incorporating shocks to importer's productivity or demand, for example. However, this would not change any of the results presented here and I abstract from such shocks for the sake of simplicity.

Solving the maximization problem in equation (16), provides insights on the evolution of the supplier list over time. The main results are stated in lemmas 5 and 6.

Lemma 5. *The steady state supplier list is given by*

$$H_{iss} = \left(\frac{\beta X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{\phi\delta(1-\beta(1-\delta))(1-\gamma\eta)} \right)^{\frac{1-\gamma\eta}{1-2\gamma\eta}} \tag{17}$$

and depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I). Moreover, a steady state can only exist if $\gamma\eta < 1/2$.

Proof: See the Appendix C.

Lemma 6. *The growth in supplier list (H_{it}) over time depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).*

Proof: See the Appendix C.

Lemma 6 states that the importer's investment on its supplier list depends systematically on the elasticities it faces. As is the case with the elasticity gap, this dependence reflects the higher return on each additional connection. Specifically, the higher its final demand elasticity and the lower its substitutability of inputs, the more an importer invests in its supplier list.

Combining lemma 6 with the results from the static problem maximization, in particular lemmas 1 and 2, lead to the following proposition about the evolution of the number of connections, denoted by $|\Lambda_{it}| = (1 - F(\underline{z}(Z_i, H_{it})))H_{it}$, over time:

Proposition 1. *The growth in the number of connections over time depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).*

Proof: See the Appendix C.

Proposition 1 indicates that importers with high final demand elasticity and low substitutability of inputs will grow more in terms of connections over time. This result is similar to Oberfield (2018), in which a market with high final elasticity causes the emergence of "stars", i.e. firms with many connections. However, in Oberfield (2018) there are no complementarities between suppliers because each firm can choose only one supplier. Furthermore, my model offers a dynamic perspective that is missing in Oberfield (2018).

Survival of connections - The model predicts two manners in which the discontinuation of connections might occur. The first is that the exporter is affected by the death shock δ, which is, as implied by equation (8), independent from exporter's and importer's productivity. The second is the increase in the minimum productivity threshold of the importer. This form of discontinuing connections depends on the importer's elasticities (σ^F and σ^I) as well as on the productivity of the exporter.

The share of connections that importer i destroys between t and $t+1$ due to an increase in its productivity threshold (ψ_{it+1}) is given by:

$$\psi_{it+1} \equiv \frac{[(1 - F(\underline{z}(Z_i, H_{it}))) - (1 - F(\underline{z}(Z_i, H_{it+1})))]H_{it}}{(1 - F(\underline{z}(Z_i, H_{it})))H_{it}} = 1 - \left(\frac{\underline{z}(Z_i, H_{it})}{\underline{z}(Z_i, H_{it+1})}\right)^{\gamma}.$$

The predictions of the model with respect to the survival of connections are summarized in proposition 2.

Proposition 2. *For a given relative increase in the supplier list, importer-exporter connections are more likely to be discontinued if:*

i) the elasticity gap of the importer is low.

ii) the elasticity gap of the importer is low and the productivity of the exporter is low.

Proof: See the Appendix C.

This result follows from lemmas 1 and 2. In lemma 1, a higher elasticity gap (in absolute values) implies a larger movement of the productivity threshold for any given increase in the supplier list. In lemma 2, the direction in which the productivity threshold moves depends on the elasticity gap: a negative elasticity gap implies an increase in the productivity threshold. The larger the increase in the productivity threshold, the larger the share of connections to be discontinued (ψ_{it+1}), which increases the likelihood of a connection being discontinued, especially if the exporter has lower productivity.

Trade value growth - The effect of a change in the supplier list on the trade value of connections is given by lemmas 3 and 4. Combining these two lemmas and lemma 6, the model predicts differences in the trade value growth of connections depending on the elasticities of the importer.

Proposition 3. *The growth in the trade value of importer-exporter connections depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).*

Proof: See the Appendix C.

In this section, I have presented a mechanism that determines the survival rate and the trade value growth of connections as a function of final demand and production technology. Specifically, the mechanism depends on the elasticity gap of the importer: the difference between final demand elasticity and inputs substitutability. When this gap is small, the importer's profit is less sensitive to price changes in the final good market, and there is less complementarity in its production function. This has two implications: first, the importer is more selective in its connections, decreasing the survival rate. Second, its trade value grows less because the final demand is not responding to the price decreases induced by the reduction of marginal costs coming along with adding new connections.

4 Connecting Theory and Empirics

The objective of this section is to define an approach to test the predictions of the theoretical model. This is done in two steps: first, I estimate the two main parameters from the theoretical model, σ^F and σ^I. Then, I present the identification strategy to test propositions 1, 2, and 3.

4.1 Estimation of Elasticities

Final demand elasticity - The estimation of final demand elasticity has been one of the objectives of the literature on trade and markups,[14] and I therefore draw from this literature to estimate σ^F in my model. Specifically, I calculate the final demand elasticity (σ^F) at the industry 2-digit ISIC level, using the Colombian Annual Manufacturing Survey from 2000 to 2014, following closely De Loecker (2011). The description of the procedure and the results by sector can be found in Appendix E.

Substitutability of inputs - To estimate the parameter for the substitutability of inputs I make use of the literature on the impact of new varieties on the gains from trade.[15] This literature estimates the import demand elasticities based on trade data. To be able to transfer their approach to my estimation of σ^I, I assume that the only demand for foreign goods comes from the production function of the final good producers, such that the import demand is given by the aggregation of the demand of final good producers following the production technology in equation (3). Given that these elasticities are estimated at the 4-digit HS product level, I mapped the elasticities to each 4-digit ISIC by calculating the trade-weighted average of the 4-digit HS elasticities. This approach is very similar to that of Alfaro et al. (2019), who use the same method to calculate the input elasticities at the 4-digit ISIC in the US market. I use the import demand elasticities from Soderbery (2018), which are calculated for South America at the 4-digit HS level.[16] A more detailed explanation on the construction of σ^I can be found in Appendix E.

Elasticity gap - Both elasticity parameters can be combined, following the theoretical model, into the structural parameter $\eta = 1/(\sigma^I - 1) - 1/(\sigma^F - 1)$. The summary statistics for the estimated elasticities as well as the elasticity gap can be seen in table 1.

[14] See De Loecker (2011), De Loecker and Warzynski (2012), and De Loecker et al. (2016), among others.

[15] Examples of this literature are Feenstra (1994), Broda and Weinstein (2006), Broda et al. (2006) and Soderbery (2018).

[16] As a robustness check, I also use the import demand elasticities from Broda et al. (2006), calculated for Colombia at the 3-digit HS level. The elasticities reported by Broda et al. (2006) are for the period 1994-2003 while those in Soderbery (2018) are for the period 1991-2007. The results using these alternative

Table 1: Summary statistics

Parameter	Description	Mean	Std.Dev.	Min.	Max.	Obs.
σ^I	Input elasticity	3.16	0.60	2.06	12.69	1,425,147
σ^F	Demand elasticity	3.76	3.79	1.39	48.26	1,425,147
$1/(\sigma^I - 1)$	1/(Input ela.-1)	0.48	0.09	0.09	0.94	1,425,147
$1/(\sigma^F - 1)$	1/(Demand ela.-1)	0.65	0.60	0.02	2.56	1,425,147
η	Elasticity gap	-0.17	0.63	-2.15	0.88	1,425,147

The difference in the level of detail at which σ^F and σ^I are calculated (2-digit and 4-digit ISIC) is exclusively due to data limitations. Final demand elasticities are calculated at the 2-digit level in the literature (De Loecker (2011) and De Loecker and Warzynski (2012) for example), and estimations at lower level industries can result in unreliable estimates. Import demand elasticity for South America is available at the 4-digit HS and can be aggregated as a weighted average to the 4-digit ISIC, the most detailed information on firms' industry available in the Colombian customs data. The difference in the level at which the elasticities are measured must be kept in mind when interpreting the results. In particular, it has been shown in Broda and Weinstein (2006) that elasticity is increasing with the level of disaggregation. An additional aspect to consider when working with those elasticity estimates is that both are time-invariant, and the only heterogeneity that can be exploited is across industries.

4.2 Identification Strategy

My empirical analysis estimates how the survival probability and trade value growth of connections depend on the elasticities faced by the importer. Because the theory has also implications at the importer level, I estimate in addition how the growth in the number of connections and the trade value of importers depend on these elasticities. Hence, a set of regressions uses data at the connection level and another uses data at the importer level.

Note that although propositions 1 and 3 are expressed in terms of the individual elasticities (σ^F and σ^I), the predicted effect is highly non-linear. This is because the effect of σ^F and σ^I on the supplier list growth and steady-state is mainly through the elasticity gap, which is a non-linear combination of the individual elasticities. Furthermore, the elasticities $\varepsilon_{z,H}$ and $\varepsilon_{r,H}$, which also influence propositions 1 and 3, depend on the elasticity gap rather than on σ^F and σ^I individually. To account for this non-linearity, I

import demand elasticities are in Appendix F.

focus the empirical approach on the elasticity gap, rather than the final demand elasticity or the substitutability of inputs individually.

To control for possible confounders, I use a full set of fixed effects at the HS6/country/year level.[17] These fixed effects capture supply shocks at the product-country-year level that could cause connections to be destroyed. I am therefore comparing only the survival of those connections with the same HS6 product, country, and year, but in which importing firms have different elasticity gaps. It is important to note that there is still some variation remaining in the product dimension since a connection is defined at the 10-digit code level.

The only dimension I cannot control for with fixed effects is the industry of the importer because, although the estimated elasticity gap is a variable at the 4-digit ISIC level, the demand elasticity is estimated at the 2-digit industry level. However, the final demand elasticity could be correlated to industry growth, and industry growth then is correlated to the survival probability and the trade value growth of connections within an industry. To control for the effect of industry growth on the variables of interest, I include the increase of log of total imports from the industry as well as the increase of log of the total number of connections in the industry. This should account for any increase in survival probability or trade value growth due to increases in import demand from a specific industry, allowing me to disentangle the effect of industry growth and final demand elasticity.

5 Empirical Results

Growth in the number of connections - Proposition 1 predicts that importers' growth in connections is positively correlated with its final demand elasticity and negatively correlated with its substitutability of inputs. As explained above, this relationship is better approximated with the elasticity gap rather than with each of the elasticities. Hence, the regression equation to bring proposition 1 to the data is the following:

$$\Delta \log Connections_{it} = \beta_1 \eta_s + \mu_t + \mathbf{X}_{st} + \epsilon_{it} \tag{18}$$

[17] Results do not change if I use HS4 or HS10 instead.

where i refers to an importer, s to a sector and t to a year. The coefficient of interest is β_1, that indicates whether importers in sectors with different elasticity gaps differ in their growth rate of connections. The results of the regression are in table (2).

Table 2: Growth in connections

	(1) $\Delta \log Con$	(2) $\Delta \log Con$	(3) $\Delta \log Con$	(4) $\Delta \log Con$
$1/(\sigma^I - 1)$	0.017		-0.031	
	(0.49)		(-1.14)	
$1/(\sigma^F - 1)$	-0.009**		-0.013***	
	(-2.51)		(-6.10)	
η		0.009***		0.011***
		(3.06)		(5.25)
Controls	No	No	Yes	Yes
R^2	0.010	0.010	0.030	0.030
Observations	40,092	40,092	40,092	40,092

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year and importing length fixed effects included.

The main result is the coefficient in column (4): an increase of one in the elasticity gap of an importer leads to an increase in the growth rate of connections of 1.1 percentage points per year. Most of this effect comes from the differences in final demand elasticity, with the effect from the substitutability of inputs being statistically insignificant. However, table (2) shows that the search mechanism in the model can be seen in the data: importers with a large elasticity gap tend to have higher growth in the number of connections over time, consistent with them investing more into searching for exporters.

Survival of connections - The model delivers a clear prediction about connection's survival in proposition 2: those importers with a low η should show lower survival rates, especially among connections with low productivity exporters. The reason behind this difference is that in industries with lower elasticity gaps, as the importers invest in their supplier list, new connections substitute old ones (i.e. the productivity threshold increases). Furthermore, this substitution effect should be larger the smaller (or more negative) η is.

To test this result from the model, I follow closely Cadot et al. (2013), Albornoz et al. (2016) and Egger et al. (2019). They used transaction-level data to study the survival of firms in the international markets with a binary choice model that can be estimated using a linear probability model or a probit.[18] Specifically, I regress the survival of a connection on the structural parameter η as follows:

$$Surv_{ijt} = \beta_1 \eta_s + \mu_{hct} + \mathbf{X}_{st} + \epsilon_{ijt} \tag{19}$$

where i refers to an importer, j to an exporter, s to a sector, h to a product, c to a sourcing country, t to a year, $Surv$ is a dummy variable that takes the value 1 if the connection continues in $t+1$ and 0 otherwise.

However, proposition 2 implies not only a lower survival probability of connections for importers in industries with lower elasticity gaps but also tells us that the connections with low productivity exporters are the ones with lower survival probability. As such, for importers with a lower elasticity gap, the survival of connections should depend on the exporter's productivity, that is, whether the connection is close to the productivity threshold $\underline{z}$, while this productivity should be less important for a connection's survival for importers with larger elasticity gap. I use the trade value of a connection as a proxy for the exporter's productivity. Hence, the coefficient of the interaction between the trade value of a connection and the elasticity gap is expected to be negative. The equation to be estimated in this case is:

$$Surv_{ijt} = \beta_1 \eta_s + \beta_2 \log sales_{ijt} + \beta_3 \log sales_{ijt} \eta_s + \mu_{hct} + \mathbf{X}_{st} + \epsilon_{ijt} \tag{20}$$

where $sales$ is connection's trade value, measured in thousands of 2014 Colombian pesos.

I estimate equations (19) and (20) with a linear probability model and a probit.[19] The results of the linear probability model are shown in table 3. The results of the probit model are in table 4.

First, I regress both components of the elasticity gap on the survival dummy in column (1). The results show that the surviv of a connection depends on the estimated elasticities with the sign predicted by proposition 2: the probability of survival is increasing on the inverse of the substitutability of inputs and decreasing on the inverse of the final demand elasticity. That the coefficient of the inverse substitutability of inputs is significant points in the direction indicated by Goldberg et al. (2010) and Halpern et al. (2015), who

[18] Another segment of the literature uses a Cox hazard model, for example, Besedeš (2008), Nitsch (2009) and Esteve-Pérez et al. (2013).

[19] I use a reduced amount of fixed effects when estimating a probit, see notes in table 4.

Table 3: Survival probability, OLS regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Surv	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*
$1/(\sigma^I - 1)$	0.166**		0.152*					
	(2.01)		(1.87)					
$1/(\sigma^F - 1)$	-0.0198		-0.0223*					
	(-1.59)		(-1.84)					
η		0.028**		0.030**	0.025	0.109**	0.026	0.110***
		(2.15)		(2.34)	(1.45)	(2.53)	(1.56)	(2.62)
$\log sales$					0.066***	0.065***	0.065***	0.064***
					(39.69)	(49.82)	(39.85)	(50.18)
$\log sales \times \eta$						-0.009***		-0.009***
						(-3.05)		(-3.13)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.216	0.216	0.218	0.217	0.280	0.280	0.281	0.281
Observations	1,155,950	1,155,950	1,155,821	1,155,821	1,155,950	1,155,950	1,155,821	1,155,821

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. $\times$ country $\times$ HS6 fixed effects included. Estimated with OLS. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

underline how the degree of substitutability of inputs is relevant for importers' sourcing decisions. Column (1) shows that importers with lower substitutability of inputs are more likely to keep their connections, in line with these importers profiting more from each additional imported intermediate input.

The coefficient for the elasticity gap in column (4) is positive and statistically significant. Considering that the standard deviation of the elasticity gap is 0.63 (see table 1), table 3 predicts that an increase of one standard deviation in the elasticity gap of an importer increases the survival probability of its connections by roughly 1.8 percentage points per year. This is a substantial increase in the survival probability of connections, considering that the average survival rate in the sample is 42.7% and in the first year of the connection as low as 28.8%. Note that columns (1) to (4) do not take into account the productivity of the exporter, and are just looking at differences in the average survival probabilities across importers.

The main specification of table 3 is in column (8), which includes all controls, the proxy for exporter's productivity, and its interaction with the elasticity gap. The result shows that the effect of the elasticity gap on the survival probability of a connection decreases with the exporter's productivity. That is, the elasticity gap is more important for survival probability in connections with lower exporter productivity.

Table 3 highlights the importance of the theory, which delivers the specific form in which final demand elasticity and intermediate input elasticity affect the survival of connections. Furthermore, all results remain significant in table 4, which are estimated using a probit model.

Table 4: Survival probability, probit regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Surv	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*	*Surv*
$1/(\sigma^I - 1)$	0.601**		0.565**					
	(2.46)		(2.35)					
$1/(\sigma^F - 1)$	-0.065**		-0.071**					
	(-2.02)		(-2.23)					
η		0.093**		0.096***	0.090	0.393**	0.093	0.395**
		(2.53)		(2.69)	(1.53)	(2.13)	(1.62)	(2.19)
$\log sales$					0.137***	0.134***	0.137***	0.134***
					(18.52)	(19.02)	(18.54)	(19.09)
$\log sales \times \eta$						-0.033**		-0.032**
						(-2.36)		(-2.40)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Pseudo R^2	0.028	0.027	0.030	0.029	0.077	0.078	0.079	0.080
Observations	1,309,249	1,309,249	1,309,099	1,309,099	1,309,249	1,309,249	1,309,099	1,309,099

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year, country, and HS Section fixed effects included. Estimated using a probit model. $Surv$ takes value 1 at time t if the connection is active in $t + 1$.

Trade value growth - Proposition 3 implies that importers with a larger elasticity gap should exhibit larger trade value growth in each of their connections. This is because for these importers the trade value of their connections grows more when they increase the number of connections. I test this prediction with equation (21):

$$\Delta \log sales_{ijt} = \beta_1 \eta_s + \sum_{k=1}^{K} \mathbb{1}[age_{ijt} = k] + \mu_{hct} + \epsilon_{ijt} \tag{21}$$

where $\Delta \log sales_{ijt}$ is growth rate in trade value of a connection between t and $t + 1$ and age_{ijt} is the age of the connection at time t. To be able to compare across connections, I include also fixed effects at the HS6/country/year level.

I test also the implications of proposition 3 in combination with proposition 1. That is, for importers with larger elasticity gaps, not only the trade value of each connection grows faster, but also the overall trade value grows faster. Therefore, I regress equation (22) after aggregating the data to the importer level, such that the left-hand side variable is the total trade value of a given importer in a year.

$$\Delta \log sales_{it} = \beta_1 \eta_s + \sum_{k=1}^{K} \mathbb{1}[age_{it} = k] + \mu_t + \epsilon_{it} \tag{22}$$

Analogously to equation (21), $\Delta \log sales_{it}$ is growth rate of the trade value of an importer between t and $t+1$ and age_{it} is the number of years a firm has been importing at time t.

I report the results of the trade value growth regressions in table 5. Columns (1) to (4) show the results of the regressions at the connection level, as defined in equation (21). The empirical pattern is in line with the model predictions: the coefficient of η is significant and positive, implying that in importers with a smaller elasticity gap the trade value of connections grows less over time. Similarly, columns (5) to (8), show that the same pattern is true if the unit of observation is the importer, as in equation (22). In this case, the trade value of importers with a smaller elasticity gap has a smaller growth rate.

Table 5: Trade value growth

	$\Delta \log Sales$ (Connection level)				$\Delta \log Sales$ (Importer level)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$1/(\sigma^I - 1)$	0.115***		0.010**		-0.0004		-0.040	
	(2.63)		(2.51)		(-0.01)		(-0.96)	
$1/(\sigma^F - 1)$	-0.021**		-0.023**		-0.022***		-0.026***	
	(-2.16)		(-2.43)		(-3.17)		(-4.99)	
η		0.026***		0.027***		0.021***		0.023***
		(2.62)		(2.88)		(3.39)		(5.05)
Industry Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.203	0.203	0.204	0.204	0.016	0.016	0.034	0.033
Observations	458,838	458,838	458,838	458,838	40,092	40,092	40,092	40,092

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Columns (1)-(3) include year × country × HS6 and connection length fixed effects. Columns (4)-(6) include year and importing length fixed effects.

5.1 Robustness

Here, I address some of the possible concerns about the results presented in the last section. Specifically, I estimate the elasticity gap using alternative values, at different levels of aggregation, and using a subset of products. The detailed regression tables of the different robustness checks can be found in Appendix F.

First, I calculate the substitutability of inputs using the import demand elasticities from Broda et al. (2006) instead of those from Soderbery (2018). The idea of this robustness check is that although both measurements are highly correlated, as shown in Soderbery (2018)), they are not equivalent. The results of the alternative measure for substitutability of inputs are presented in table 6. They show that the method used to estimate the substitutability of inputs does not affect my main results, with all relevant coefficients being of the same sign and statistically significant.

Second, I calculate the substitutability of inputs at the importer level instead of at the 4-digit ISIC level. By doing so, I obtain a value of the elasticity gap at the importer level, allowing me to exploit a larger degree of heterogeneity in my main explanatory variable. The drawback of this approach is that it opens endogeneity concerns due to the small number of connections used to calculate the substitutability of inputs. The results of this exercise are in table 6 and are all in line with the results presented above.

Third, because the theoretical model relies on importers sourcing their intermediate inputs from abroad, I limit my data to only connections with products categorized as intermediate inputs under the classification of the Broad Economic Categories (BEC). The idea is that the effect should be present also in this subset of products if the predictions of the model are correct and the importers are indeed the ones behind the effect of the elasticity gap observed in the data. Again, all coefficients of the elasticity gap are significant and with the predicted sign. Moreover, using only intermediate inputs increases the effect of the elasticity gap on survival and trade value growth. This indicates that the effect is particularly strong for connections in this category of products, supporting the theoretical predictions of the model.

Table 6: Results of robustness checks

Using import demand elasticities from Broda et al. (2006)					
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log Con$	$Surv$	$Surv$	Δ log Sales	Δ log Sales
η	0.010***	0.022*	0.097***	0.024**	0.021***
	(4.58)	(1.68)	(2.63)	(2.57)	(5.63)
log Sales			0.064***		
			(48.99)		
log Sales $\times$ η			-0.009***		
			(-3.34)		
Estimating substitutability of inputs at the importer level					
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log Con$	$Surv$	$Surv$	Δ log Sales	Δ log Sales
η	0.026**	0.013***	0.049***	0.030***	0.023***
	(2.26)	(5.98)	(2.69)	(2.84)	(4.36)
log Sales			0.065***		
			(50.94)		
log Sales $\times$ η			-0.009***		
			(-3.18)		
Using only products categorized as intermediate inputs					
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log Con$	$Surv$	$Surv$	Δ log Sales	Δ log Sales
η	0.006***	0.106***	0.142**	0.034***	0.028***
	(2.97)	(2.64)	(2.36)	(3.37)	(6.22)
log Sales			0.066***		
			(48.86)		
log Sales $\times$ η			-0.011**		
			(-2.45)		

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Column (4) estimated at the connection level. Column (5) estimated at the importer level. $Surv$ takes value 1 at time t if the connection is active in $t + 1$.

6 Conclusion

In this paper, I analyze the reason behind the differences in survival rates and trade value growth of importer-exporter connections. To explain those differences, I develop a dynamic model of importer-exporter trade, in which importers endogenously decide from which exporters to source. The model features importers that are heterogeneous in their productivity, in their final demand elasticity, and in the substitutability of their inputs. Consequently, they differ in the intensity of their search for exporters and their choice on which connections to keep over time.

My main theoretical result is that the benefits of connecting to new exporters depend on the difference in the elasticities that importers face on their final good and their intermediate inputs. The intuition is that each of these elasticities governs one side of the importer decision: the elasticity in the final good market determines the increase in profits from a reduction in the price the importer charges to consumers, while the substitutability of inputs governs how important a new exporter is to decrease the importer's marginal costs and hence its price. This effect is captured by a term that I label the elasticity gap, which decreases in final demand elasticity and increases in substitutability of inputs.

This result delivers three testable implications on importer-exporter connections. First, importers with a larger elasticity invest more in finding new suppliers and, therefore, have a larger growth rate in the number of connections. Second, importers with a larger elasticity gap are less likely to drop old connections as new ones are established, creating a difference in survival rates across sectors. Third, the sales-boosting effect of incorporating new exporters into the production function causes importers with a larger elasticity gap to also increase more the trade value of their existing connections as well as their trade value overall.

I provide empirical evidence for these theoretical predictions using transaction-level data from Colombian importers from 2007 to 2018. The empirical results show that connections of importers with a larger elasticity gap grow more in terms of the number of connections, and these connections have a higher survival rate and trade value growth. I further document empirical patterns consistent with the predicted channel: the effect of the elasticity gap on connection's survival rate decreases on exporter productivity. This is because the connections affected in importers with a small elasticity gap are those with low productivity exporters.

Future work should focus on testing further theoretical predictions, such as the relationship between the elasticity gap and search intensity. Doing so would open the door to investigate how the elasticity gap shapes the life-cycle of the importer.

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A Data

In this section, I describe in detail the processing of the data used in the paper. As mentioned in the main text, the Colombian customs data identifies Colombian importers with their tax number, which is unique and constant over time.

Foreign exporters lack a unique identifier and are identified with a combination of country, city, and a 10-digit product category. Because cities in the data are stored as text, and this text might contain spelling mistakes or different names to identify the same city, I use a string matching method to standardize the name of the cities. For this, I match the cities in the data to the cities in the GeoNames dataset,[20] which contains all world cities with a population above 500 as well as alternative names for these cities.

Identifying exporters based on country-city-product is likely to cause a difference with respect to the results that would be obtained if the exporters were also identified by name for two possible reasons. The first possibility is that the same product is imported from two different cities, which is then counted as two connections but belongs to the same exporter. The second possibility is that an importer purchases the same product from the same city, but from two different exporters, which would be counted as only one exporter.

To assess the severity of the possible bias induced by this different definition of the exporter, I compare the mean of products imported by the importer and the mean of exporter-product per importer in my data set in the year 2014 to that of Bernard et al. (2018a), which reports that information for Colombian importers for the same year.[21] The mean exporter-products per importer are 20.01 in my data and 23.3 in Bernard et al. (2018a), while the mean (median) products per importer are 13.22 (4) and 14.73 (4) respectively. This indicates a slight divergence in both data sets, probably because of the two possible issues mentioned previously. The difference, however, is only between 10 and 20%.

B Additional Figures

[20]The Geonames dataset is downloaded from http://download.geonames.org/.

[21]Bernard et al. (2018a) reports mean exporters per importer (4.43), mean products per importer-exporter (5.26), and mean and median products per importer (14.73 and 4).

Figure B.1: Survival rate by 2-digit industry, individual years

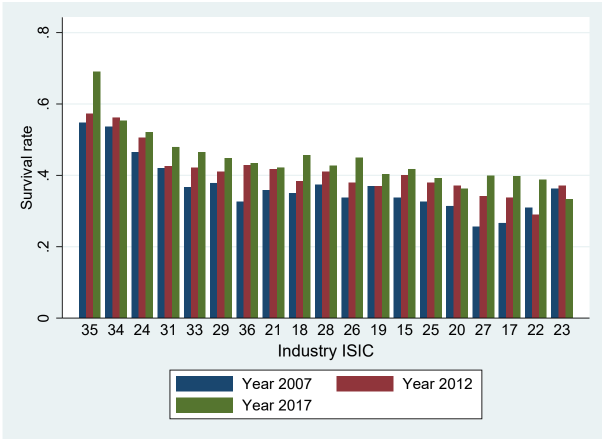
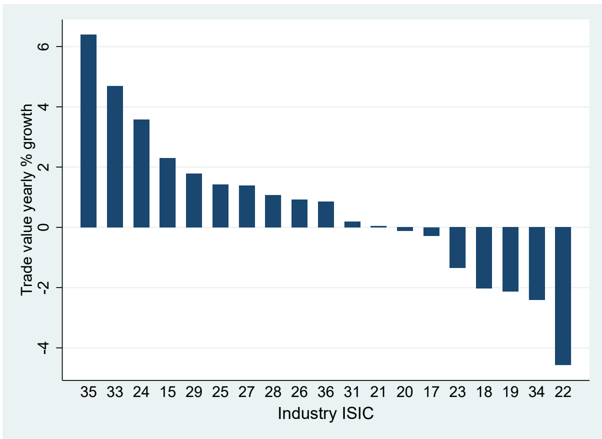


Figure B.2: Median trade volume growth by 2-digit industry



C Proofs

Lemma 1 - The elasticity of the productivity cutoff with respect to the supplier list ($\varepsilon_{\underset{\sim}{z},h}$) is increasing on the elasticity gap (η):

$$\text{If } \varepsilon_{\underset{\sim}{z},h} \equiv \left| \frac{\partial \ln \underset{\sim}{z}(Z_i, H_{it})}{\partial \ln H_{it}} \right|, \text{ then } \frac{\partial \varepsilon_{\underset{\sim}{z},h}}{\partial \eta} > 0.$$

Proof of lemma 1 - Start from equation (13) and take logs:

$$\ln \underset{\sim}{z}(Z_i, H_{it}) = \frac{1}{(\sigma^F - 1)(1 - \gamma\eta)} \ln \left(\frac{\sigma^F(\sigma^I - 1)}{(\sigma^F - 1)} \frac{f_r}{L} \right) + \frac{1}{1 - \gamma\eta} \ln \left(\frac{\tau^* w^*}{k_1 Z_i} \right) - \frac{\eta}{1 - \gamma\eta} \ln(k_2 H_{it}).$$

Then

$$\varepsilon_{\underset{\sim}{z},h} \equiv \left| \frac{\partial \ln \underset{\sim}{z}(Z_i, H_{it})}{\partial \ln H_{it}} \right| = \left| \frac{\eta}{1 - \gamma\eta} \right|.$$

Taking the derivative with respect to η:

$$\frac{\partial \varepsilon_{\underline{z},h}}{\partial \eta} = \frac{1}{(1-\gamma\eta)^2} > 0.$$

Hence, $\varepsilon_{\underline{z},h}$ is increasing in η.

Lemma 2 - The effect of the supplier list on the productivity cutoff depends on the elasticity gap as follows:

i) if $\eta < 0$, $\underline{z}(Z_i, H_{it})$ is increasing in H_{it}.

ii) if $\eta > 0$, $\underline{z}(Z_i, H_{it})$ is decreasing in H_{it}.

iii) if $\eta = 0$, $\underline{z}(Z_i, H_{it})$ does not depend on H_{it}.

Proof of lemma 2 - Start from equation (13) and take the derivative of $\underline{z}(Z_i, H_{it})$ with respect to H_{it}:

$$\frac{\partial \underline{z}(Z_i, H_{it})}{\partial H_{it}} = -\frac{\eta}{1-\gamma\eta} \underbrace{\left[\frac{\sigma^F(\sigma^I-1)}{(\sigma^F-1)}\frac{f_r}{L}k_2^{\frac{\sigma^I-\sigma^F}{\sigma^I-1}}\right]^{\frac{1}{(\sigma^F-1)(1-\gamma\eta)}}}_{>0} \underbrace{\left(\frac{\tau^* w^*}{k_1 Z_i}\right)^{\frac{1}{1-\gamma\eta}}}_{>0} \underbrace{H_{it}^{-\frac{\eta}{1-\gamma\eta}-1}}_{>0}$$

Since all terms are positive, the sign of the derivative depends only on the sign of $-\frac{\eta}{1-\gamma\eta}$. Given the parametric restriction $\gamma\eta < 1$, the denominator of this expression is positive. Hence, if $\eta < 0$ ($\eta > 0$) the derivative is positive (negative), and if $\eta = 0$ the derivative is zero.

Lemma 3 - The elasticity of the trade value of connections with respect to the supplier list ($\varepsilon_{r,h}$) is increasing on the elasticity gap (η):

$$\text{If } \varepsilon_{r,h} \equiv \left|\frac{\partial \ln r(z, Z_i, H_{it})}{\partial \ln H_{it}}\right|, \text{ then } \frac{\partial \varepsilon_{r,h}}{\partial \eta} > 0.$$

Proof of lemma 3 - Start from equation (15) and take logs:

$$\ln r(z, Z_i, H_{it}) = \ln X_1(z, Z_i) + \frac{\gamma\eta}{1-\gamma\eta} \ln H_{it},$$

Then,

$$\varepsilon_{r,h} \equiv \left| \frac{\partial \ln r(z, Z_i, H_{it})}{\partial \ln H_{it}} \right| = \frac{\gamma\eta}{1-\gamma\eta},$$

and taking the derivative with respect to η:

$$\frac{\partial \varepsilon_{r,h}}{\partial \eta} = \frac{\gamma}{(1-\gamma\eta)^2} > 0.$$

Hence, $\varepsilon_{r,h}$ is increasing in η.

Lemma 4 - The effect of the supplier list on the trade value of connections depends on the elasticity gap as follows:

i) if $\eta < 0$, $r(z, Z_i, H_{it})$ is decreasing in H_{it}.

ii) if $\eta > 0$, $r(z, Z_i, H_{it})$ is increasing in H_{it}.

iii) if $\eta = 0$, $r(z, Z_i, H_{it})$ does not depend on H_{it}.

Proof of lemma 4 - Start from equation (15) and take the derivative of $r(z, Z_i, H_{it})$ with respect to H_{it}:

$$\frac{\partial r(z, Z_i, H_{it})}{\partial H_{it}} = \frac{\gamma\eta}{1-\gamma\eta} \underbrace{X_1(z, Z_i)}_{>0} \underbrace{H_{it}^{-\frac{1}{1-\gamma\eta}}}_{>0}$$

Since all terms are positive, the sign of the derivative depends only on the sign of $\frac{\gamma\eta}{1-\gamma\eta}$. Given the parametric restriction $\gamma\eta < 1$, the denominator of this expression is positive. Hence, if $\eta < 0$ ($\eta > 0$) the derivative is negative (positive), and if $\eta = 0$ the derivative is zero.

Lemma 5 - The steady-state supplier list (H_{iss}) depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I). Moreover, a steady-state can only exist if $\gamma\eta < 1/2$.

Proof of lemma 5 - For the solution of the steady-state value of H_{it}, start in the static problem of the importer by substituting equation (12) into equation (13):

$$\Pi(Z_i, H_{it}) = \frac{L}{\sigma^F}\left(\frac{Z_i k_1}{\tau^* w^*}\right)^{\sigma^F-1}(H_{it}k_2)^{\frac{\sigma^F-1}{\sigma^I-1}}$$
$$\times\left(\left[\frac{\sigma^F(\sigma^I-1)}{(\sigma^F-1)}\frac{f_r}{L}\right]^{\frac{1}{(\sigma^F-1)(1-\gamma\eta)}}\left(\frac{\tau^* w^*}{k_1 Z_i}\right)^{\frac{1}{1-\gamma\eta}}(k_2 H_{it})^{-\frac{\eta}{1-\gamma\eta}}\right)^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}}$$
$$- H_{it} z_L^{\gamma} f_r\left(\left[\frac{\sigma^F(\sigma^I-1)}{(\sigma^F-1)}\frac{f_r}{L}\right]^{\frac{1}{(\sigma^F-1)(1-\gamma\eta)}}\left(\frac{\tau^* w^*}{k_1 Z_i}\right)^{\frac{1}{1-\gamma\eta}}(k_2 H_{it})^{-\frac{\eta}{1-\gamma\eta}}\right)^{-\gamma}$$

Rearranging the terms and factoring out Z_i and H_{it}, the profit can be rewritten as:

$$\Pi(Z_i, H_{it}) = X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}} H_{it}^{\frac{1}{1-\gamma\eta}}, \tag{23}$$

where

$$X_2 = \left(\frac{k_1}{\tau^* w^*}\right)^{\frac{\gamma}{1-\gamma\eta}}\left(\frac{L(\sigma^F-1)}{\sigma^F(\sigma^I-1)f_r}\right)^{\frac{\gamma}{(1-\gamma\eta)(\sigma^F-1)}} f_r k_2^{\frac{1}{1-\gamma\eta}}(\sigma^I-1)\frac{1-\gamma\eta}{\gamma}.$$

Using the expression for profits from equation (23) and the investment cost in equation (9), the Bellman equation in (16) is then:

$$V(Z_i, H_{it}) = \max_{A_{it}}\left\{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}} H_{it}^{\frac{1}{1-\gamma\eta}} - \phi\frac{A_{it}^2}{2} + \beta V(Z_i, H_{it+1})\right\}$$
$$s.t.\ H_{it+1} = (1-\delta)H_{it} + A_{it}$$

Rewriting $A_{it} = H_{it+1} - (1-\delta)H_{it}$ and substituting in:

$$V(Z_i, H_{it}) = \max_{H_{it+1}}\left\{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}} H_{it}^{\frac{1}{1-\gamma\eta}} - \phi\frac{(H_{it+1}-(1-\delta)H_{it})^2}{2} + \beta V(Z_i, H_{it+1})\right\}$$

The first order condition with respect to H_{it+1}:

$$-\phi(H_{it+1}-(1-\delta)H_{it}) + \beta\frac{\partial V(Z_i, H_{it+1})}{\partial H_{it+1}} \overset{!}{=} 0$$

The envelope condition:

$$\frac{\partial V(Z_i, H_{it})}{\partial H_{it}} = \frac{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{1-\gamma\eta} H_{it}^{\frac{\gamma\eta}{1-\gamma\eta}} + \phi(1-\delta)(H_{it+1} - (1-\delta)H_{it})$$

Advance one period:

$$\frac{\partial V(Z_i, H_{it+1})}{\partial H_{it+1}} = \frac{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{1-\gamma\eta} H_{it+1}^{\frac{\gamma\eta}{1-\gamma\eta}} + \phi(1-\delta)(H_{it+2} - (1-\delta)H_{it+1})$$

Substitute into the first order condition:

$$\beta\frac{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{1-\gamma\eta} H_{it+1}^{\frac{\gamma\eta}{1-\gamma\eta}} = \phi(H_{it+1} - (1-\delta)H_{it}) - \beta\phi(1-\delta)(H_{it+2} - (1-\delta)H_{it+1}) \qquad (24)$$

Assume that there exist a steady-state, such that $H_{it} = H_{it+1} = H_{it+2} = H_{iss}$. This assumption excludes the case $\eta \geq 0$. The previous equation then reduces to:

$$\beta\frac{X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{1-\gamma\eta} H_{iss}^{\frac{\gamma\eta}{1-\gamma\eta}} = \phi\delta H_{iss}(1-\beta(1-\delta)).$$

Rearranging:

$$H_{iss} = \left(\frac{\beta X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}}}{\phi\delta(1-\beta(1-\delta))(1-\gamma\eta)}\right)^{\frac{1-\gamma\eta}{1-2\gamma\eta}},$$

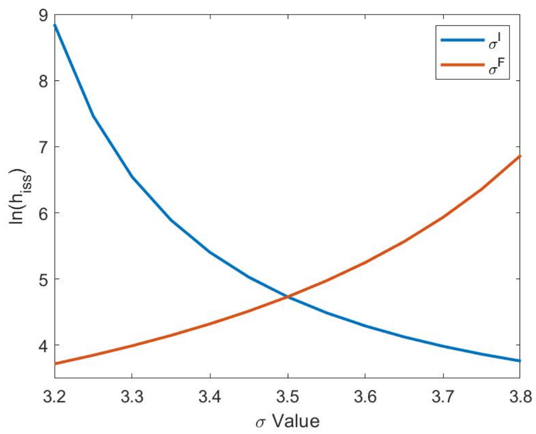
where $\beta X_2 > 0$ and $\phi\delta(1-\beta(1-\delta))(1-\gamma\eta) > 0$.

Note that the exponent $\frac{1-\gamma\eta}{1-2\gamma\eta}$ is positive only if $1-2\gamma\eta > 0$, because I have assumed before that $1-\gamma\eta > 0$. This creates an additional parameter restriction to the model. The restriction $1-\gamma\eta > 0$ was necessary for the maximization problem of the importer to have an interior solution. Now, for the supplier list to have a steady-state, $1-2\gamma\eta > 0$ is also required. If $\gamma\eta > 1/2$, importers will keep growing their supplier list infinitely.

As shown in figure C.1, H_{iss} is increasing in σ^F and decreasing in σ^I.

Lemma 6 - The growth in supplier list (H_{it}) over time depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).

Figure C.1: Value of the supplier list.



Note: $\tau^* = w^* = f_r = z_L = Q = 1$, $L = 50$, $\gamma = 5$, $Z = 2$, $\sigma^F = 3.5$ (blue line), and $\sigma^I = 3.5$ (red line).

Proof of lemma 6 - Define $x_{it} = \frac{H_{it}}{H_{iss}}$ and $v(x_{it}) = \frac{V(x_{it}H_{iss})}{H_{iss}}$. Then: $V(H_{it}) = H_{iss}v(\frac{H_{it}}{H_{iss}})$. I can rewrite the Bellman equation as:

$$v(Z_i, x_{it}) = \max_{x_{it+1}} \left\{ X_2 Z_i^{\frac{\gamma}{1-\gamma\eta}} x_{it}^{\frac{1}{1-\gamma\eta}} - \phi\frac{(x_{it+1} - (1-\delta)x_{it})^2}{2} + \beta v(Z_i, x_{it+1}) \right\}$$

The first order condition:

$$\phi(x_{it+1} - (1-\delta)x_{it}) = \beta\frac{\partial v(Z_i, x_{it+1})}{\partial x_{it+1}}$$

If x_{it} increases, LHS decreases. Then, RHS must also decrease. Since $v(Z_i, x_{it+1})$ is concave, $\frac{\partial v(Z_i, x_{it+1})}{\partial x_{it+1}}$ decreases if x_{it+1} increases. Hence, x_{it+1} is increasing in x_{it}.

Assume two importers have different σ^F, with $\sigma^F_H > \sigma^F_L$, but the same supplier list at time t, $H_{Ht} = H_{Lt}$. Since $H_{Hss} > H_{Lss}$, this implies $x_{Ht} = \frac{H_{Ht}}{H_{Hss}} < \frac{H_{Lt}}{H_{Lss}} = x_{Lt}$. Because x_{it+1} is increasing in x_{it}, then $x_{Ht+1} < x_{Lt+1}$ and $\frac{\partial v(Z_H, x_{Ht+1})}{\partial x_{Ht+1}} > \frac{\partial v(Z_L, x_{Lt+1})}{\partial x_{Lt+1}}$. Investment at time t:

$$A_{it} = H_{iss}(x_{it+1} - (1-\delta)x_{it}) = H_{iss}\frac{\beta}{\phi}\frac{\partial v(Z_i, x_{it+1})}{\partial x_{it+1}}.$$

Since $H_{Hss} > H_{Lss}$ and $\frac{\partial v(Z_H, x_{Ht+1})}{\partial x_{Ht+1}} > \frac{\partial v(Z_L, x_{Lt+1})}{\partial x_{Lt+1}}$, then $A_{Ht} > A_{Lt}$. The same proof can be used to show that A_{it} is decreasing in σ^I.

Proposition 1 - The growth in the number of connections $((1 - F(\underline{z}(Z_i, H_{it})))H_{it})$ over time depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).

Proof of proposition 1 - This result follows from lemmas 1, 2, and 6. Assume two importers with a different final demand elasticity ($\sigma_A^F > \sigma_B^F$) but otherwise equal. Then, it holds that $\eta_A > \eta_B$ and hence $\varepsilon^A_{\underset{\sim}{z},H} > \varepsilon^B_{\underset{\sim}{z},H}$ (lemma 1). Together with lemma 2, it also means that $\underset{\sim}{z}(Z_A, H_{At+1}) < \underset{\sim}{z}(Z_B, H_{Bt+1})$. Note that this is independent of whether the elasticity gap is positive or negative. The growth in the number of connections between time t and time $t+1$:

$$
\begin{aligned}
&(1 - F(\underset{\sim}{z}(Z_i, H_{it+1})))H_{it+1} - (1 - F(\underset{\sim}{z}(Z_i, H_{it})))H_{it} = \\
&z_L^\gamma \underset{\sim}{z}(Z_i, H_{it+1})^{-\gamma} H_{it+1} - z_L^\gamma \underset{\sim}{z}(Z_i, H_{it})^{-\gamma} H_{it}.
\end{aligned}
$$

Because $\underset{\sim}{z}(Z_A, H_{At+1})^{-\gamma} > \underset{\sim}{z}(Z_B, H_{Bt+1})^{-\gamma}$ and lemma 6 implies $H_{At+1} > H_{Bt+1}$, the increase in the number of connections is increasing on final demand elasticity. The same proof can be used to show that the increase in the number of connections is decreasing in σ^I.

Proposition 2 - For a given increase in the supplier list, importer-exporter connections are more likely to be discontinued if:

i) the elasticity gap of the importer is low.

ii) the elasticity gap of the importer is low and the productivity of the exporter is low.

Proof of proposition 2 - This result follows from lemmas 1 and 2. Assume two importers with different, negative, elasticity gap ($0 > \eta_A > \eta_B$) and both face the same relative increase in their supplier list. Then, it holds that $\varepsilon^A_{\underset{\sim}{z},H} < \varepsilon^B_{\underset{\sim}{z},H}$ (lemma 1). Because the elasticity gap is negative, the productivity threshold of both importers increases, i.e. $\underset{\sim}{z}(Z_A, H_{At+1}) > \underset{\sim}{z}(Z_A, H_{At})$ and $\underset{\sim}{z}(Z_B, H_{Bt+1}) > \underset{\sim}{z}(Z_B, H_{Bt})$ (lemma 2).

Combining both lemmas:

$$
\frac{\underset{\sim}{z}(Z_A, H_{At+1})}{\underset{\sim}{z}(Z_A, H_{At})} < \frac{\underset{\sim}{z}(Z_B, H_{Bt+1})}{\underset{\sim}{z}(Z_B, H_{Bt})}. \tag{25}
$$

Use the definition of the share of connections that importer i destroys between t and

$t+1$ due to an increase in its productivity threshold (ψ_{it+1}):

$$\begin{aligned}\psi_{it+1} &= \frac{[(1-F(\underline{z}(Z_i,H_{it}))) - (1-F(\underline{z}(Z_i,H_{it+1})))]H_{it}}{(1-F(\underline{z}(Z_i,H_{it})))H_{it}} \\ &= \frac{\underline{z}(Z_i,H_{it})^{-\gamma} - \underline{z}(Z_i,H_{it+1})^{-\gamma}}{\underline{z}(Z_i,H_{it})^{-\gamma}} = 1 - \left(\frac{\underline{z}(Z_i,H_{it})}{\underline{z}(Z_i,H_{it+1})}\right)^{\gamma}.\end{aligned}$$

From equation (25), the last term is smaller for the importer with higher η. Then it holds that $\psi_{Bt+1} > \psi_{At+1}$.

For part ii) of the proposition, one has to notice that the connections being destroyed are the ones with exporter productivity between $\underline{z}(Z_i,H_{it})$ and $\underline{z}(Z_i,H_{it+1})$, that is, the connections with the lowest productivity at time t.

Proposition 3 - The growth in the trade value of importer-exporter connections depends positively on final demand elasticity (σ^F) and negatively on the substitutability of inputs (σ^I).

Proof of proposition 3 - This result follows from lemmas 3 and 6. Assume two importers with a different final demand elasticity ($\sigma^F_A > \sigma^F_B$) but otherwise equal. Then, it holds that $\eta_A > \eta_B$ and hence $\varepsilon^A_{r,H} > \varepsilon^B_{r,H}$ (lemma 3). Moreover, lemma 6 implies that $H_{At+1}/H_{At} > H_{Bt+1}/H_{Bt}$. Combining both results, immediately follows that:

$$\frac{r(z,Z_A,H_{At+1})}{r(z,Z_A,H_{At})} > \frac{r(z,Z_B,H_{Bt+1})}{r(z,Z_B,H_{Bt})}. \tag{26}$$

The same proof can be used to show that the growth in the trade value is decreasing in σ^I.

D Theory

Derivation of equation (12) - The static profit of the importer can written as:

$$\begin{aligned}
\Pi(Z_i, H_{it}) &= \frac{L}{\sigma^F}\left(\bar{M}\frac{q(Z_i,H_{it})}{Z_iQ}\right)^{1-\sigma^F} - [1-F(\underset{\sim}{z}(Z_i,H_{it}))]H_{it}f_r \\
&= \frac{L}{\sigma^F}\left(\frac{\bar{M}}{Z_iQ}\right)^{1-\sigma^F}\left[H_{it}\int_{\underset{\sim}{z}(Z_i,H_{it})}^{\infty} p(z)^{1-\sigma^I} f(z)dz\right]^{\frac{1-\sigma^F}{1-\sigma^I}} - [1-F(\underset{\sim}{z}(Z_i,H_{it}))]H_{it}f_r \\
&= \frac{L}{\sigma^F}\left(\frac{Z_ik_1}{\tau^*w^*}\right)^{\sigma^F-1}\left[H_{it}\int_{\underset{\sim}{z}(Z_i,H_{it})}^{\infty} z^{\sigma^I-1} f(z)dz\right]^{\frac{\sigma^F-1}{\sigma^I-1}} - [1-F(\underset{\sim}{z}(Z_i,H_{it}))]H_{it}f_r \\
&= \frac{L}{\sigma^F}\left(\frac{Z_ik_1}{\tau^*w^*}\right)^{\sigma^F-1}(H_{it}k_2)^{\frac{\sigma^F-1}{\sigma^I-1}}\,\underset{\sim}{z}(Z_i,H_{it})^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}} - H_{it}z_L^{\gamma}f_r\underset{\sim}{z}(Z_i,H_{it})^{-\gamma}
\end{aligned}$$

where $k_1 = \frac{Q}{\bar{m}\bar{M}}$ and $k_2 = \frac{\gamma z_L^{\gamma}}{\gamma-(\sigma^I-1)}$. In the second equality I have used that $q(Z_i, H_{it}) = \left[H_{it}\int_{\underset{\sim}{z}_{it}}^{\infty} p(z)^{1-\sigma^I} f(z)dz\right]^{\frac{1}{1-\sigma^I}}$. The third equality makes use of the intermediate input producer optimal price from equation (7). Finally, in the forth equality I use the assumption that $F(z)$ is Pareto distributed.

Using the fact that importers select the productivity threshold $\underset{\sim}{z}(Z_i, H_{it})$ to maximize profits yields equation (12).

Derivation of equation (13) - Starting from the static profit maximization problem in equation (12), solve for the optimal $\underset{\sim}{z}_{it}$:

$$\begin{aligned}
\frac{\partial\Pi(Z_i,H_{it})}{\partial \underset{\sim}{z}_{it}} = {} & \frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}\frac{L}{\sigma^F}\left(\frac{Z_ik_1}{\tau^*w^*}\right)^{\sigma^F-1}(H_{it}k_2)^{\frac{\sigma^F-1}{\sigma^I-1}}\,\underset{\sim}{z}(Z_i,H_{it})^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}-1} \\
& + \gamma H_{it}z_L^{\gamma}f_r\underset{\sim}{z}(Z_i,H_{it})^{-\gamma-1} \overset{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\rightarrow \frac{\underset{\sim}{z}(Z_i,H_{it})^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}-1}}{\underset{\sim}{z}(Z_i,H_{it})^{-\gamma-1}} &= -\frac{\gamma H_{it}z_L^{\gamma}f_r}{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)}{\sigma^I-1}\frac{L}{\sigma^F}\left(\frac{Z_ik_1}{\tau^*w^*}\right)^{\sigma^F-1}(k_2H_{it})^{\frac{\sigma^F-1}{\sigma^I-1}}} \\
\underset{\sim}{z}(Z_i,H_{it})^{\frac{(\sigma^I-1-\gamma)(\sigma^F-1)+\gamma(\sigma^I-1)}{\sigma^I-1}} &= \frac{\sigma^F(\sigma^I-1)f_r}{L(\sigma^F-1)}\left(\frac{\tau^*w^*}{k_1Z_i}\right)^{\sigma^F-1}(k_2H_{it})^{\frac{\sigma^I-\sigma^F}{\sigma^I-1}}
\end{aligned}$$

Rearranging and using $\eta \equiv \frac{1}{\sigma^I - 1} - \frac{1}{\sigma^F - 1}$:

$$
\begin{aligned}
\underline{z}(Z_i, H_{it}) &= \left[\frac{\sigma^F(\sigma^I - 1) f_r}{L(\sigma^F - 1)} \left(\frac{\tau^* w^*}{k_1 Z_i} \right)^{\sigma^F - 1} \right]^{\frac{\sigma^I - 1}{\gamma(\sigma^I - 1) - [\gamma - (\sigma^I - 1)](\sigma^F - 1)}} (k_2 H_{it})^{\frac{\sigma^I - \sigma^F}{\gamma(\sigma^I - 1) - [\gamma - (\sigma^I - 1)](\sigma^F - 1)}} \\
&= \left[\frac{\sigma^F(\sigma^I - 1)}{(\sigma^F - 1)} \frac{f_r}{L} \right]^{\frac{1}{(\sigma^F - 1)(1 - \gamma\eta)}} \left(\frac{\tau^* w^*}{k_1 Z_i} \right)^{\frac{1}{1 - \gamma\eta}} (k_2 H_{it})^{-\frac{\eta}{1 - \gamma\eta}},
\end{aligned}
$$

Derivation of equation (15) - The sales from an exporter with productivity z to an importer with productivity Z_i and a supplier list H_{it}:

$$
\begin{aligned}
r(z, Z_i, H_{it}) &= \left(\frac{p(z)}{q(Z_i, H_{it})} \right)^{1 - \sigma^I} E(Z_i, H_{it}) = \left(\frac{p(z)}{q(Z_i, H_{it})} \right)^{1 - \sigma^I} \left(\frac{q(Z_i, H_{it}) \bar{M}}{Z_i Q_t} \right)^{1 - \sigma^F} \frac{L}{\bar{M}} \\
&= \frac{L}{\bar{M}} \left(\frac{\bar{M}}{Z_i Q} \right)^{1 - \sigma^F} \left(\bar{m} \frac{\tau^* w^*}{z} \right)^{1 - \sigma^I} \left[H_{it} \int_{\underline{z}(Z_i, H_{it})}^{\infty} p(z)^{1 - \sigma^I} f(z) dz \right]^{\frac{\sigma^I - \sigma^F}{1 - \sigma^I}} \\
&= \frac{L}{\bar{M}} \left(\frac{\tau^* w^*}{Z_i k_1} \right)^{1 - \sigma^F} z^{\sigma^I - 1} k_2^{\frac{\sigma^I - \sigma^F}{\sigma^I - 1}} \underline{z}(Z_i, H_{it})^{\frac{(\sigma^I - 1 - \gamma)(\sigma^F - 1)}{\sigma^I - 1}} H_{it}^{\frac{\sigma^I - \sigma^F}{1 - \sigma^I}} \\
&= \frac{L}{\bar{M}} \left(\frac{\tau^* w^*}{Z_i k_1} \right)^{(1 - \sigma^F) + \frac{(\sigma^I - 1 - \gamma)(\sigma^F - 1)}{(\sigma^I - 1)(1 - \gamma\eta)}} z^{\sigma^I - 1} k_2^{\frac{\sigma^I - \sigma^F}{\sigma^I - 1}(1 + \frac{\sigma^I - 1 - \gamma}{(\sigma^I - 1)(1 - \gamma\eta)})} \\
&\times \left[\frac{\sigma^F(\sigma^I - 1)}{(\sigma^F - 1)} \frac{f_r}{L} \right]^{\frac{\sigma^I - 1 - \gamma}{(\sigma^I - 1)(1 - \gamma\eta)}} H_{it}^{\frac{\eta}{1 - \gamma\eta} \frac{(\sigma^I - 1 - \gamma)(\sigma^F - 1)}{\sigma^I - 1} - \frac{\sigma^I - \sigma^F}{\sigma^I - 1}} \\
&= X_1(z, Z_i) H_{it}^{\frac{\gamma\eta}{1 - \gamma\eta}}
\end{aligned}
$$

where $X_1(z, Z_i) = \frac{L}{\bar{M}} \left(\frac{Z_i k_1}{\tau^* w^*} \right)^{\frac{\gamma}{1 - \gamma\eta}} z^{\sigma^I - 1} k_2^{\frac{\gamma\eta}{1 - \gamma\eta}} \left[\frac{\sigma^F(\sigma^I - 1)}{(\sigma^F - 1)} \frac{f_r}{L} \right]^{\frac{\sigma^I - 1 - \gamma}{(\sigma^I - 1)(1 - \gamma\eta)}}$. In the second equality I have used the definitions of $p(z)$ and $q(Z_i, H_{it})$ and in the fourth equality I have used the optimal $\underline{z}(Z_i, H_{it})$ from equation (13).

Assumption on n - The assumption that the steady state value of the supplier list, H_{iss}, is smaller than the total amount of firms, n, for any value of Z_i is:

$$
H_{iss} = \left(\frac{\beta X_2 Z_i^{\frac{\gamma}{1 - \gamma\eta}}}{\phi\delta(1 - \beta(1 - \delta))(1 - \gamma\eta)} \right)^{\frac{1 - \gamma\eta}{1 - 2\gamma\eta}} < n \; \forall i.
$$

Hence, it can only hold for values of $\gamma\eta < 1/2$, even if n tends to infinity. It is important to keep in mind that I did not assume any productivity distribution of importers $G(Z)$. In the case of some specific distributions, like Pareto for example, some additional assumptions would be necessary such that $H_{iss} < n \; \forall i$ holds. See Bernard et al. (2018b) and Oberfield (2018) for how to deal with the specific case where $G(Z)$ follows a Pareto distribution.

E Elasticity Estimation

Demand Elasticity

The demand elasticity is calculated following De Loecker (2011), which has been used in Halpern et al. (2015) as well.

Production - The first step is to transform the production function in the main text, such that it includes other factors of production in a way that can be estimated in the data. I do this with the following definitions:

$$Z_{it} \equiv \exp(\theta_{it} + u_{it}) L_{it}^{\alpha_l} K_{it}^{\alpha_k}$$
$$\left[\int_{\Lambda_{it}} c_{it}(\lambda)^{\frac{\sigma^I - 1}{\sigma^I}} d\lambda\right]^{\frac{\sigma^I}{\sigma^I - 1}} \equiv X_{it}^{\alpha_x}$$

where L is labor, K is capital, and X intermediate inputs. θ is a firm-specific productivity shock and u a measurement error. Notice that the productivity term in the main text, Z, is now time varying and contains four components. Given the Cobb-Douglas production function assumption[22], simplifying all other production factors in the production function to a single productivity parameter leaves all results unaffected. The inclusion of these four components in the theoretical model would only unnecessarily complicate the model since the mechanism of the model plays out in the intermediate inputs part of the production function.

Using these two definitions, equation (3), expressed for an importer i instead of a variety ω, can be rewritten as

$$Y_{it} = \exp(\theta_{it} + u_{it}) L_{it}^{\alpha_l} K_{it}^{\alpha_k} X_{it}^{\alpha_x} \tag{27}$$

Demand - Similar to the case of the production function, to bring the demand assumed in the model closer to the data, I include shocks to the demand of the final goods, denoted

[22]The Cobb-Douglas production function implies constant expenditure shares, limiting the interaction between intermediate inputs and the rest of the production factors.

by ξ. Starting from the final goods demand equation, I allow the aggregate demand L and the price index of the final good, Q_t to vary across sectors s:

$$Y_{it} = Y_{st} \left[\frac{P_{it}}{Q_{st}} \right]^{-\sigma_s^F} \exp(\xi_{it}) \tag{28}$$

where I have used the identity $C_{it} = Y_{it}$ and defined $Y_{st} \equiv L_{st}/Q_{st}$ as the total demand in the sector, i.e. total expenditure in the sector over the price index of the sector.

Revenue production function - Solving for price in equation (28) and using the definition of revenues:

$$R_{it} = P_{it} Y_{it} = Y_{it}^{\frac{\sigma_s^F - 1}{\sigma_s^F}} Y_{st}^{1/\sigma_s^F} Q_{st} (\exp(\xi_{it}))^{1/\sigma^F}. \tag{29}$$

Finally, I get to the equation to be estimated by substituting (27) into equation (29) and taking logs

$$\log \tilde{R}_{it} = \beta_l \log L_{it} + \beta_k \log K_{it} + \beta_x \log X_{it} + \beta_y \log Y_{st} + \xi_{it}^* + \theta_{it}^* + u_{it}, \tag{30}$$

where $\tilde{R}_{it}$ is the deflated revenue of firm i at time t (i.e. R_{st}/Q_{st}). similar to De Loecker (2011), the parameters estimated are $\beta_j = \alpha_j \frac{\sigma_s^F - 1}{\sigma_s^F}$ for $j = \{l, k, x\}$ and $\beta_y = 1/\sigma_s^F$. The elasticity of demand is therefore the inverse of the aggregate demand coefficient: $\beta_s = \frac{1}{|\sigma_s^F|}$. Moreover, I decompose the demand shock ξ_{it} into observable and unobservable components: $\xi_{it} = \xi_{st} + \tilde{\xi}_i t$.

I follow De Loecker (2011) and Halpern et al. (2015) to measure the aggregate demand of each industry Y_{st} as the total sales of in the Colombian region where the firm is located, discounting own sales. With respect to the observable demand shock, ξ_{st}, I approximate it as the sales growth of the 2-digit industry each year.

For the estimation of equation (30), I follow Ackerberg et al. (2015) to properly identify the coefficients of interest in two stages. Using lowercase letters denote logs, I assume that firm's intermediate input demand is given by $x_{it} = \tilde{f}_t(l_{it}, k_{it}, y_{st}, \xi_{st}, \theta_{it})$. Assuming that the intermediate input demand is invertible, I rewrite it as $\theta = \tilde{f}_t^{-1}(l_{it}, k_{it}, x_{it}, y_{st}, \xi_{st})$

and substitute it in equation (30):

$$\tilde{r}_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_y y_{st} + \beta_\xi \xi_{st} + \tilde{f}_t^{-1}(l_{it}, k_{it}, x_{it}, y_{st}, \xi_{st}) + u_{it} = \tilde{\Phi}_t(l_{it}, k_{it}, x_{it}, y_{st}, \xi_{st}) + u_{it}. \tag{31}$$

For the estimation of equation (31) I use a third-degree polynomial of k, l, x, y and ξ_s. Finally, the parameters are obtained by generalized method of moments (GMM) with the following moment conditions:

$$E\Bigg[(\tilde{r}_{it} - \beta_l l_{it} - \beta_k k_{it} - \beta_x x_{it} - \beta_y y_{st} - \beta_\xi \xi_{st} - \Phi_{t-1}^{-1}(l_{it-1}, k_{it-1}, x_{it-1}, y_{st-1}, \xi_{st-1})$$
$$- \beta_l l_{it-1} - \beta_k k_{it-1} - \beta_x x_{it-1} - \beta_y y_{st-1}) \otimes \begin{pmatrix} k_{it} \\ l_{it-1} \\ x_{it-1} \\ y_{st-1} \\ \xi_{st} \end{pmatrix} \Bigg] = 0$$

The estimated parameters (α_k, α_l, α_x and σ^F) for each sector are presented in table 7.

Table 7: Factor and demand elasticities by sector

Sector	α_k	α_l	α_x	σ^F
15	0.10	0.20	0.54	4.06
17	0.05	0.38	0.41	3.85
18	0.02	0.36	0.20	1.39
19	0.02	0.20	0.48	2.01
20	0.07	0.32	0.39	2.46
21	0.07	0.48	0.38	3.67
22	0.13	1.70	-0.00	2.22
23	0.04	0.11	0.89	48.26
24	0.08	0.21	0.50	3.31
25	0.06	0.23	0.45	2.53
26	0.18	0.53	0.19	3.67
27	0.05	0.25	0.48	2.57
28	0.08	0.25	0.41	2.36
29	0.04	0.32	0.38	2.45
31	0.05	0.20	0.49	2.74
33	0.06	0.66	0.19	3.49
34	0.06	0.40	0.48	6.08
35	0.18	0.15	0.60	16.63
36	0.04	0.25	0.50	3.13
Average	0.07	0.33	0.42	6.15

Substitutability of Inputs

Based on the import demand elasticities from Soderbery (2018), estimated for South America at the HS4 product level. The elasticity of each 4-digit ISIC is calculated as the trade-weighted average of the HS4 product elasticity.

F Robustness Checks

Import Demand Elasticities from Broda, Greenfield, and Weinstein (2006)

Tables 8, 9, 10, and 11 show the same regressions as in the main text, with the input elasticity parameter (σ^I) calculated using Broda et al. (2006) estimates of import demand elasticity at 3-digit HS for Colombian. All results remain statistically significant.

Table 8: Growth in connections, σ^I from Broda et al. (2006)

	(1) $\Delta \log Con$	(2) $\Delta \log Con$	(3) $\Delta \log Con$	(4) $\Delta \log Con$
$1/(\sigma^I - 1)$	-0.015		-0.006	
	(-0.61)		(-0.35)	
$1/(\sigma^F - 1)$	-0.011***		-0.013***	
	(-3.05)		(-5.54)	
η		0.007**		0.010***
		(2.48)		(4.58)
Controls	No	No	Yes	Yes
R^2	0.010	0.010	0.030	0.030
Observations	40,092	40,092	40,092	40,092

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year and importing length fixed effects included.

Import Demand Elasticities at the Importer Level

Tables 12, 13, 14, and 15 show the same regressions as in the main text, with the input elasticity parameter (σ^I) calculated using the import demand elasticities from Soderbery (2018), but at the importer level instead of at the 4-digit ISIC.

Table 9: Survival probability, OLS regressions, σ^I from Broda et al. (2006)

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	-0.008		-0.006					
	(-0.14)		(-0.10)					
$1/(\sigma^F - 1)$	-0.024**		-0.026**					
	(-1.98)		(-2.22)					
η		0.020		0.022*	0.016	0.096**	0.018	0.097***
		(1.49)		(1.68)	(1.03)	(2.54)	(1.16)	(2.63)
log *sales*					0.066***	0.064***	0.065***	0.064***
					(39.40)	(48.50)	(39.58)	(48.99)
log *sales* $\times\ \eta$						-0.009***		-0.009***
						(-3.28)		(-3.34)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.215	0.215	0.217	0.217	0.279	0.280	0.281	0.281
Observations	1,155,950	1,155,950	1,155,821	1,155,821	1,155,950	1,155,950	1,155,821	1,155,821

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. $\times$ country $\times$ HS6 fixed effects included. Estimated with OLS. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Table 10: Survival probability, probit regressions, σ^I from Broda et al. (2006)

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	0.049		0.051					
	(0.28)		(0.30)					
$1/(\sigma^F - 1)$	-0.077**		-0.082***					
	(-2.49)		(-2.72)					
η		0.073**		0.078**	0.064	0.380**	0.069	0.384**
		(2.03)		(2.19)	(1.16)	(2.13)	(1.25)	(2.18)
log *sales*					0.137***	0.132***	0.137***	0.132***
					(18.04)	(18.31)	(18.07)	(18.40)
log *sales* $\times\ \eta$						-0.034**		-0.034**
						(-2.40)		(-2.44)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Pseudo R^2	0.027	0.027	0.029	0.029	0.077	0.078	0.078	0.080
Observations	1,309,249	1,309,249	1,309,099	1,309,099	1,309,249	1,309,249	1,309,099	1,309,099

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year, country, and HS Section fixed effects included. Estimated using a probit model. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Restricted Sample, Only Intermediate Inputs

Tables 16, 17, 18, and 19 show the same regressions as in the main text, but with the sample restricted to products categorized as intermediate inputs under the classification of

Table 11: Trade value growth, σ^I from Broda et al. (2006)

	$\Delta \log Sales$ (Connection level)				$\Delta \log Sales$ (Importer level)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$1/(\sigma^I - 1)$	0.022		0.025		-0.020		-0.0029	
	(0.58)		(0.73)		(-0.65)		(-0.14)	
$1/(\sigma^F - 1)$	-0.022**		-0.023***		-0.023***		-0.025***	
	(-2.38)		(-2.64)		(-3.35)		(-4.45)	
η		0.022**		0.024**		0.017***		0.021***
		(2.29)		(2.57)		(3.73)		(5.63)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.203	0.203	0.204	0.204	0.016	0.016	0.034	0.033
Observations	458,838	458,838	458,838	458,838	40,092	40,092	40,092	40,092

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Columns (1)-(3) include year $\times$ country $\times$ HS6 and connection length fixed effects. Columns (4)-(6) include year and importing length fixed effects.

Table 12: Growth in connections, σ^I at the importer level

	(1)	(2)	(3)	(4)
	$\Delta \log Con$	$\Delta \log Con$	$\Delta \log Con$	$\Delta \log Con$
$1/(\sigma^I - 1)$	0.017		0.019	
	(0.96)		(1.17)	
$1/(\sigma^F - 1)$	-0.009***		-0.012***	
	(-2.71)		(-5.30)	
η		0.010***		0.013***
		(3.06)		(5.98)
Controls	No	No	Yes	Yes
R^2	0.001	0.001	0.030	0.030
Observations	40,057	40,057	40,057	40,057

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year and importing length fixed effects included.

the Broad Economic Categories (BEC). The input elasticity parameter (σ^I) is calculated using the import demand elasticities from Soderbery (2018) at the 4-digit ISIC.

Table 13: Survival probability, OLS regressions, σ^I at the importer level

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	0.054		0.046					
	(1.30)		(1.15)					
$1/(\sigma^F - 1)$	-0.022*		-0.025**					
	(-1.74)		(-1.98)					
η		0.025**		0.026**	0.021	0.106**	0.022	0.106***
		(2.06)		(2.26)	(1.37)	(2.59)	(1.50)	(2.69)
$\log sales$					0.066***	0.065***	0.065***	0.065***
					(39.57)	(50.57)	(39.72)	(50.94)
$\log sales \times \eta$						-0.009***		-0.009***
						(-3.10)		(-3.18)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.216	0.215	0.217	0.217	0.280	0.280	0.281	0.281
Observations	1,155,845	1,155,845	1,155,716	1,155,716	1,155,845	1,155,845	1,155,716	1,155,716

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. × country × HS6 fixed effects included. Estimated with OLS. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Table 14: Survival probability, probit regressions, σ^I at the importer level

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	0.278**		0.257**					
	(2.44)		(2.33)					
$1/(\sigma^F - 1)$	-0.073**		-0.078**					
	(-2.15)		(-2.36)					
η		0.087**		0.091***	0.083	0.384**	0.087	0.386**
		(2.56)		(2.73)	(1.53)	(2.18)	(1.62)	(2.24)
$\log sales$					0.137***	0.134***	0.137***	0.134***
					(18.40)	(19.26)	(18.42)	(19.32)
$\log sales \times \eta$						-0.032**		-0.032**
						(-2.41)		(-2.46)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Pseudo R^2	0.027	0.027	0.029	0.029	0.077	0.078	0.079	0.080
Observations	1,309,128	1,309,128	1,308,978	1,308,978	1,309,128	1,309,128	1,308,978	1,308,978

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year, country, and HS Section fixed effects included. Estimated using a probit model. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Table 15: Trade value growth, σ^I at the importer level

	$\Delta \log Sales$ (Connection level)				$\Delta \log Sales$ (Importer level)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$1/(\sigma^I - 1)$	0.125***		0.116***		0.005		0.011	
	(3.32)		(3.27)		(0.12)		(0.29)	
$1/(\sigma^F - 1)$	-0.021**		-0.022**		-0.021***		-0.024***	
	(-2.11)		(-2.36)		(-3.09)		(-4.37)	
eta		0.029***		0.030***		0.020***		0.023***
		(2.60)		(2.84)		(3.22)		(4.36)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.203	0.203	0.204	0.204	0.016	0.016	0.034	0.034
Observations	458,805	458,805	458,805	458,805	40,057	40,057	40,057	40,057

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Columns (1)-(3) include year × country × HS6 and connection length fixed effects. Columns (4)-(6) include year and importing length fixed effects.

Table 16: Growth in connections, only intermediate inputs

	(1)	(2)	(3)	(4)
	$\Delta \log Con$	$\Delta \log Con$	$\Delta \log Con$	$\Delta \log Con$
$1/(\sigma^I - 1)$	0.039		-0.009	
	(1.41)		(-0.44)	
$1/(\sigma^F - 1)$	-0.007**		-0.006***	
	(-2.59)		(-3.30)	
η		0.008***		0.006***
		(3.91)		(2.64)
Controls	No	No	Yes	Yes
R^2	0.009	0.009	0.031	0.031
Observations	35,012	35,012	35,012	35,012

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year and importing length fixed effects included.

Table 17: Survival probability, OLS regressions, only intermediate inputs

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	0.183**		0.162**					
	(2.34)		(2.10)					
$1/(\sigma^F - 1)$	-0.040**		-0.041***					
	(-2.55)		(-2.69)					
η		0.049***		0.049***	0.041*	0.145**	0.041*	0.142**
		(2.88)		(2.97)	(1.90)	(2.34)	(1.94)	(2.36)
$\log sales$					0.067***	0.067***	0.067***	0.066***
					(40.94)	(48.45)	(41.25)	(48.86)
$\log sales \times \eta$						-0.012**		-0.011**
						(-2.44)		(-2.45)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.202	0.202	0.204	0.204	0.269	0.270	0.271	0.271
Observations	819,395	819,395	819,297	819,297	819,395	819,395	819,297	819,297

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. $\times$ country $\times$ HS6 fixed effects included. Estimated with OLS. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Table 18: Survival probability, probit regressions, only intermediate inputs

	(1) *Surv*	(2) *Surv*	(3) *Surv*	(4) *Surv*	(5) *Surv*	(6) *Surv*	(7) *Surv*	(8) *Surv*
$1/(\sigma^I - 1)$	0.592***		0.540**					
	(2.71)		(2.50)					
$1/(\sigma^F - 1)$	-0.105***		-0.108***					
	(-2.66)		(-2.78)					
η		0.131***		0.131***	0.139*	0.583**	0.139*	0.579**
		(2.88)		(2.95)	(1.84)	(2.24)	(1.87)	(2.25)
$\log sales$					0.146***	0.144***	0.146***	0.144***
					(20.40)	(22.37)	(20.44)	(22.44)
$\log sales \times \eta$						-0.047**		-0.046**
						(-2.37)		(-2.37)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Pseudo R^2	0.024	0.023	0.025	0.025	0.080	0.081	0.082	0.083
Observations	917,039	917,039	916,929	916,929	917,039	917,039	916,929	916,929

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Year, country, and HS Section fixed effects included. Estimated using a probit model. *Surv* takes value 1 at time t if the connection is active in $t + 1$.

Table 19: Trade value growth, only intermediate inputs

	$\Delta \log Sales$ (Connection level)				$\Delta \log Sales$ (Importer level)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$1/(\sigma^I - 1)$	0.129***		0.094**		0.047		-0.017	
	(2.95)		(2.30)		(0.97)		(-0.43)	
$1/(\sigma^F - 1)$	-0.029***		-0.030***		-0.033***		-0.031***	
	(-2.74)		(-2.89)		(-6.66)		(-7.62)	
η		0.036***		0.034***		0.034***		0.028***
		(3.25)		(3.37)		(6.82)		(6.22)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
R^2	0.195	0.195	0.196	0.196	0.018	0.018	0.034	0.034
Observations	347,101	347,101	347,101	347,101	35,012	35,012	35,012	35,012

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. t statistics in parentheses. Standard errors clustered at the 4-digit industry level. Columns (1)-(3) include year × country × HS6 and connection length fixed effects. Columns (4)-(6) include year and importing length fixed effects.