Exchange Rates as Trade Frictions: Estimates and Implications for Policy *†

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Abstract

Exchange rate depreciation acts like a tax on imports and subsidy to exports, and fuels suspicions of 'currency manipulation', trade policy in disguise. Obliviously, standard trade policy analysis uses real trade models that neutralize exchange rate effects. In contrast, open economy macro models feature exchange rate effects on trade that assume away the complexities of large and heterogeneous trade costs with many countries that are the focus of modern quantitative trade models. This paper develops and quantifies a structural gravity framework in which exchange rate changes with heterogeneous passthrough are effectively trade frictions. Real effects on country-sector sellers are sometimes substantial. Real national income effects are small but not negligible. Effective exchange Rates with Gravitas (ERGs) are theory-consistent indexes of exchange rate friction effects. ERGs can guide potential policy remedies.

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Exchange rate depreciation acts in partial equilibrium like a tax on imports and subsidy to exports. This intuition fails in the long run when money is neutral. How important are violations of money neutrality? Providing an economically respectable answer requires measuring income effects due to exchange rate changes. An answer is urgent as charges of 'currency manipulation' by China recently became part of Trump's trade war, a precedent likely to endure. This paper develops and quantifies a structural gravity framework in which exchange rate changes passed through to bilateral trade are potentially trade frictions with real effects. Real national income effects of exchange rate movements at annual frequencies are mostly small, but not negligible, and are substantial at the extremes. Sectoral income effects on sellers are sometimes large and may justify compensatory domestic policy. Guides to policy may be based on novel ideal indexes of bilateral exchange rate movements, Effective exchange Rates with Gravitas (ERGs). ERGs measure income effects on buyers and sellers consistent with estimated structural gravity. They differ substantially from the standard atheoretic 'effective exchange rate' indexes.

The structural gravity model setting features big trade costs acting on trade patterns in general spatial equilibrium. The CES version of gravity is applied here because of its simplicity and familiarity, but all the methods developed here can be applied to more general spatial equilibrium models with trade frictions.[1] Bilateral trade costs include *parametric* incomplete exchange rate passthrough to bilateral prices that act like bilateral trade frictions.

Effective exchange rates are atheoretic weighted averages of bilateral exchange rate changes using trade weights for imports and exports separately and are often reported at the sectoral level.[2] Unfortunately, effective exchange rate indexes constructed on these lines

[1]See Anderson and Zhang (2020) for a development of Almost Ideal gravity based on the Almost Ideal Demand System.

[2]Variants include Törnqvist indexes and chain weights. All the indexes suffer from at least four problems. (1) Treating exchange rate changes like price changes does not deal with the well-documented ubiquitous phenomenon of incomplete passthrough of exchange rates to prices. (2) If passthrough is complete and prices are flexible, money is neutral and exchange rates are irrelevant. A proper real exchange rate index should converge on unity as passthrough becomes complete. Typical real effective exchange rate indexes do not have this relationship to incomplete passthrough. (3) Prominent received theory argues that trade costs affect the impact of exchange rate changes (for example, Obstfeld and Rogoff, 2001). There is no role for trade costs in the standard indexes despite abundant evidence from the recent gravity literature that trade costs are

are essentially useless for the quantification of exchange rate changes as trade frictions and dubiously related to the under-valuation question.[3] This paper makes a start on more credible measures, theoretically grounded and empirically implemented with structural gravity modeling. The ERGs we propose here are readily operational.

The application quantifies real effects of exchange rate movements on trade flows at annual frequencies in the period 2000-14 for 17 sectors and 40 countries using the WIOD. Trade shifts are substantial in some sectors. Real national income effects relative to counterfactual long run equilibrium exchange rates are small but not negligible and in some (country-year) cases are substantial. The (average-over-sectors) terms of trade change from this calculation for the top decile ranges around 2% and for the bottom decile ranges around -2%. The global effect of the terms of trade changes (a size-weighted average of the country terms of trade changes) due to yearly exchange rate changes is close to zero (ranging between -0.27% and 0.24%.[4] Exchange rate passthrough friction at the sectoral level drives much wider variation in sectoral 'terms of trade'. This is due to variation in both buyer and seller components. We report swings of 40-50% in some sector-country cases.

Sectoral interest group pressure on exchange rate policy is behind 'currency manipulation' claims, presumably driven by its impact on sectoral incomes. Effective Exchange Rates with Gravitas (ERG) indexes for sectoral buyers and sellers measure exchange rate effects that may in principle be offset with domestic policies. For example, ERGs for sellers quantify the damage to seller incomes from the general equilibrium effects of foreign exchange rate depreciation (thus home appreciation) with incomplete passthrough. Thus seller ERGs

large and vary greatly between trade partners. (4) In a multi-country world, bilateral exchange rates do not appear sufficient to capture all the effects on the home country of the interaction between members of the set of foreign countries. The effective exchange rate concept developed in this paper solves all 4 problems within the framework of the structural gravity model.

[3]The US Treasury Department's guidelines now embeddefubod in NAFTA 2.0 (USMCA) do not use under- or over-valuation measures, but focus on central bank activity and sharing information. Since most central banks intervene in foreign exchange markets for stabilization purposes of various sorts, it is difficult to infer intent from activity. Even with correctly inferred intent, a mutually acceptable remedy requires quantification of the damage that is being offset.

[4]The deviation from zero arises because the exchange rate changes act on the unchanging part of trade frictions. This implies the effects on the world as a whole need not be zero.

provide a basis for seller compensation to mollify interest group pressure that could potentially be consistent with the mutual exchange of market access logic of the WTO and its non-discriminatory MFN principle.[5] Buyer ERGs symmetrically provide a basis for buyer compensation for home depreciation due to appreciation by foreign exporters.

ERGs for buyers and sellers differ significantly from their atheoretic effective exchange rate counterparts, despite relatively high overall correlation. High correlation is unsurprising since identical vectors of exchange rate changes are being aggregated with weights that are themselves positively correlated. More importantly for measuring real impacts, the magnitudes of ERGs and standard indexes differ significantly and for some country-sector-time intervals the correlations are low or negative. Nominal buyer (seller) ERGs have an overall correlation coefficient of 0.9 (0.85) with standard counterparts, with a sectoral low of 0.7 (0.54). For real ERGs the overall correlations and sectoral lows are somewhat lower.

The ERG measures reported here rely heavily on the structural gravity model in its Constant Elasticity of Substitution (CES) representation, extended here to include bilateral exchange rate passthrough shocks that exogenously affect the equilibrium spatial distribution of goods. Structural gravity has become the workhorse model of trade because it fits the data very well, has plausible general equilibrium foundations and accommodates high dimensional heterogeneity of trade frictions. The application permits quantification of the effect on producer and consumer real incomes due to exchange rate changes acting as trade frictions.

The key action in the model is via the multilateral resistance terms estimated in structural gravity and interpreted as buyer's and seller's incidence of all bilateral frictions. Changes in exchange rates shift the system of bilateral frictions directly (a partial equilibrium effect) and induce shifts in the equilibrium buyer and seller incidences. These induce indirect exchange rate effects that act on factor prices. In applications to time series, these factor price effects combine with the effects of other exogenous changes such as technology and endowments

[5] Exchange rate levels and their movements over time induce popular concern and occasional political pressure to engage in countervailing trade policy to offset 'currency manipulators'. (See the *The Economist* July 27, 2019.)

shifts. The direct effect is interpreted as a nominal index of the passthrough of bilateral exchange rate changes, the Effective exchange Rate with Gravitas (ERG, one for the buyer and one for the seller).

In the benchmark case of money neutrality when passthrough is uniform (including complete passthrough as a special case), the multilateral resistance terms of structural gravity absorb all the bilateral effects of exchange rate movements, resulting in no change in any real activity. In the realistic case where passthrough is heterogeneous, multilateral resistances damp but cannot fully offset the real effects of exchange rate movements. Thus the estimated gravity equations reveal trade displacement effects of bilateral exchange rate changes — real friction effects.

The ERGs have a CES functional form where the elasticity is a product of the usual trade elasticity and the *deviation from mean of* passthrough elasticity. In the uniform passthrough case ERG is equal to 1. ERGs reflect general equilibrium forces in distribution set in motion by the bilateral frictions changes due to the heterogenous passthrough of exchange rate changes. The buyer's destination-specific ERG index has weights that are endogenous to the bilateral exchange rate changes passed through to destination prices. The seller's origin-specific ERG index similarly has weights that are endogenous to the bilateral exchange rate changes heterogeneously passed through to seller prices. A separate channel of changes in purchasing power and earnings power acts through the changes in equilibrium seller prices due to heterogeneous passthrough of exchange rate changes combined with technology and endowment changes. Both the direct and indirect components of real changes in purchasing and earnings power are operational with the methods of this paper.

The closest relative to the theoretical ERG here is proposed by Neary (2006). He derives a a theoretically consistent effective exchange rate index that answers the question: given a set of arbitrary changes in external prices or domestic costs, what change in the nominal exchange rate would restore the initial level of output or employment. The question is answered in a small country (price taking) setting where non-neutral money is due to a nominal fixed wage. Both the question and the environments differ here from Neary (2006). Importantly, the setting differs by departing from the small country assumption to deal with many non-price-taking countries in general equilibrium, and modeling non-neutral money as due to parametric incomplete exchange rate passthrough.

The empirical research program applied below takes exchange rate passthrough as exogenous. This simplification is unavoidable given the state of the art in exchange rate modeling. When applied to sectoral trade, as here, the assumption of no causality from trade flows to exchange rates is plausible as well as simplifying. A key aspect is allowance for sector-destination-specific bilateral exchange rate passthrough elasticities. A wide range of pricing-to-market stories justify destination-specific passthrough while empirical confirmation is in Boz et al. (2017, 2019) based on passthrough regressions using bilateral export unit values. Boz et al. (2017) find low passthrough to their definition of bilateral terms of trade. This resembles our finding of low passthrough in gravity models of bilateral trade flows. The structural gravity setting suggests an interpretation of measured heterogeneous passthrough effects as a reflection of rising short run bilateral trade costs due to fixed bilateral 'marketing capital' (Anderson and Yotov, 2019).

The terms of trade theory of the trade policy system (Bagwell and Staiger, 2002) implies that exchange rate changes, manipulated or not, becomes relevant as it affects the terms of trade that the WTO institutions are designed to neutralize. Section 2.3 on implications for policy argues that national *bilateral trade* policies to neutralize real effects on the import side are in principle like countervailing duties, hence potentially consistent with the non-discrimination rules of the WTO and to free trade obligations in free trade areas. On the export side, however, subsidies to offset undervalued exchange rates of partners are banned by the WTO prohibition of export subsidies. On both sides the application of bilaterally discriminatory trade policies that must move over time appears infeasibly complex. An appropriate remedy to offset the real effects of exchange rates acting as trade policy is a system of domestic countervailing subsidy to injured parties. Above a *de minimis* threshold, ERGs measure an offsetting subsidy to qualifying injured parties. The domestic countervailing subsidy system has the added advantage of tending to offset whatever terms of trade manipulation incentives potential 'currency manipulators' may have.

1 Gravity with Exchange Rate Frictions

First we review structural gravity without consideration of exchange rates. Then we introduce exchange rates that are incompletely passed through to prices. Structural gravity assumes perfect spatial arbitrage (any inferred arbitrage profit is due to independent random errors). Exchange rate movements and their passthrough are introduced as an exogenous process like trade cost shocks such as the introduction of free trade agreements. Exogeneity is justified by the extensive literature documenting the superiority of statistical models of exchange rate movements over models with real determinants of exchange rate movements.

Let X_{ij}^k denote the bilateral shipment from origin i to destination j in sector k; Y_i^k denote the world value of shipments from origin i to all destinations, ; and E_j^k denote the value of shipments from all origins to destination j. All shipments are valued at end user prices. Trade requires incurring costs that drive wedge factors between origin and destination captured in cost factors t_{ij}^k. Let $Y^k = \sum_i Y_i^k = \sum_j E_j^k$.

The full structural gravity model is given by:

$$X_{ij}^k = \frac{E_j^k Y_i^k}{Y^k}\left(\frac{t_{ij}^k}{P_j^k \Pi_i^k}\right)^{1-\sigma_k} \tag{1}$$

$$(\Pi_i^k)^{1-\sigma_k} = \sum_j \left(\frac{t_{ij}^k}{P_j^k}\right)^{1-\sigma_k} E_j^k / Y^k \tag{2}$$

$$(P_j^k)^{1-\sigma_k} = \sum_i \left(\frac{t_{ij}^k}{\Pi_i^k}\right)^{1-\sigma_k} Y_i^k / Y^k. \tag{3}$$

The estimation of t_{ij}^k, the bilateral trade friction, is the main object of empirical gravity, while the restrictions of structural gravity imply the two equation systems (2)-(3). It has become standard practice to estimate (1) with importer and exporter fixed effects to control for both the mass variables Y_i^k, E_j^k and the multilateral resistance variables Π_i^k, P_j^k. The latter can be recovered using the mass variables Y_i, E_j and the equation systems. See Anderson and Yotov (2010) for details. The sales and expenditure variables are assumed to be measured at the end user's full price, meaning that the trade flow and the sales and expenditure variables are all measured with error because some user costs are not observable.

The theoretical foundation behind (1) supports three different interpretations: (i) a representative user purchasing products differentiated by place of origin, where σ_k is the elasticity of substitution between varieties, (ii) a Ricardian technology producing homogeneous products with national labor productivities generated as random draws from a Frechet distribution where the parameter $1-\sigma_k$ is interpreted as the dispersion parameter of the distribution, and (iii) aggregation heterogeneous users who make discrete choices of country varieties of good k. See Anderson (2011) for details. For present purposes it makes no difference which interpretation is adopted, but for convenience the first interpretation will be used below.

The derivation of (1) under the differentiated demand interpretation begins from the demand equation

$$X_{ij}^k = (\beta_i^k p_i^k t_{ij}^k / P_j^k)^{1-\sigma_k} E_j^k, \tag{4}$$

where p_i^k is the 'factory gate' price or unit cost of the variety of k sold by seller i, β_i^k is a parameter of taste or technology and P_j^k is the CES price index $\sum_i \left[(\beta_i^k p_i^k t_{ij}^k)^{1-\sigma_k}\right]^{1/(1-\sigma_k)}$. Market clearance implies $\sum_j X_{ij}^k = Y_i^k$, permitting substitution in the demand equation for $(\beta_i^k p_i^k)^{1-\sigma_k}$ using the definition of Π_i^k in (2). This same substitution also implies that for sellers shares Y_i^k/Y^k the gravity model implies that it is as if the seller makes all his sales on the world market, making them to a buyer whose CES share is given on the right hand side of the following equation:

$$Y_i^k/Y^k = (\beta_i^k p_i^k \Pi_i^k)^{1-\sigma_k} \tag{5}$$

This is a powerful implication because it permits treating the allocation of resources between sectors in each country as determined by aggregate demand on the world market, the effect of trade costs being aggregated into outward multilateral resistance Π_i^k. Moreover, Π_i^k is recognized as the sellers' incidence of trade costs.

Exchange rate changes passed through to prices are introduced as exogenous trade cost shocks that affect the system (1)-(3). The price wedge shock that results is transitorily a complex object reflecting currency invoicing in contracts and hedging choices along with pricing-to-market behavior.[6] At the annual frequency of standard gravity modeling focused on the value of trade, it seems reasonable to simplify the price wedges to the sector-destination-specific passthrough of bilateral exchange rate changes while also abstracting from dynamic quantity adjustment except for a common cross-border-time fixed effect.[7] We further simplify by abstracting from possible effects of exchange rate risk – volatility plays no role. The system (2)-(3) is shocked when the t_{ij}^k's change. These shocks also change the multilateral resistances, directly and through price changes due to (5) that change the Y_i^k's and E_j^k's at given t_{ij}^k.

Prices in the preceding model are in a numeraire currency. (In the application below the US dollar is the numeraire currency.) Prices in the numeraire currency relate to local currencies via exchange rates. By choice of units, all local currency prices in a base period can be set equal to 1. Exchange rates of currencies defined in numeraire units per unit of currency j appreciate (depreciate) relative to base as $r_j > (<)1$. Exchange rate changes incompletely passed through from origin i to prices in each destination j are represented by $(r_i/r_j)^{\rho_j}$ where $\rho_j \in [0, 1]$ is a destination specific passthrough elasticity. The property of destination-specific passthrough allows for pricing-to-market behavior in a reduced form. Evidence on destination-specific heterogeneous passthrough is provided by Boz et al. (2019).

[6]See Boz et al. (2017) for evidence based on bilateral export unit value comparison data. Focusing on currency invoicing practices, their results suggest low passthrough of bilateral exchange rates to destination prices (local currency invoicing) but substantial separate influence of the dollar exchange rate suggesting the importance of US dollar invoicing.

[7]The US dollar effect on destination prices that is emphasized by Boz et al. is in our gravity model setting absorbed in the cross-border-time fixed effect that also absorbs common globalization effects.

The passthrough of depreciation of j's currency in terms of i's currency (r_i/r_j rises) acts like a tax on imports and subsidy to exports from j's point of view, while from i's point of view the bilateral appreciation of its exchange rate acts like a tax on exports and a subsidy to imports. Drawing on this equivalence, the bilateral trade cost factor $t_{ij}^k = \tau_{ij}^k (r_i/r_j)^{\rho_j^k}$ where τ_{ij}^k is the trade cost factor exclusive of exchange rate passthrough (the usual function of proxy variables such as distance and borders). The passthrough elasticity is taken here and in much of the empirical passthrough literature to be a parameter.

In moving from (4) to the structural gravity equation (1), the market clearance condition is used to substitute for $(\beta_i^k p_i^k r_i)^{1-\sigma_k}$. Thus to analyze the effect of exchange rate changes on the new equilibrium, replace t_{ij}^k in (1)-(3) with $\tau_{ij}^k (r_i/r_j)^{\rho_j^k}$. Suppress for now considerations that changes in exchange rates or relative prices will lead to changes in E_j, Y_i; i.e., analyze conditional general equilibrium. The initial solution of (2)-(3) for multilateral resistances yields $\{\Pi_i^{k0}, P_j^{k0}\}$. With the new bilateral trade costs due to incompletely passed through exchange rate changes the multilateral resistances satisfy:

$$(\Pi_i^k)^{1-\sigma_k} = \sum_j \left(\frac{\tau_{ij}^k (r_i/r_j)^{\rho_j^k}}{P_j^k} \right)^{1-\sigma_k} E_j^k / Y^k \tag{6}$$

$$(P_j^k)^{1-\sigma_k} = \sum_i \left(\frac{\tau_{ij}^k (r_i/r_j)^{\rho_j^k}}{\Pi_i^k} \right)^{1-\sigma_k} Y_i^k / Y^k. \tag{7}$$

Notice first that *money neutrality obtains when passthrough is uniform* ($\rho_j^k = \rho^k, \forall j$), with complete passthrough being a special case. Neutrality follows because, given that $\{\Pi_i^{k0}, P_j^{k0}\}$ solve (2)-(3), the new multilateral resistances must satisfy $P_j^k r_j^{\rho^k} = P_j^{k0}$ and $\Pi_i^k / r_i^{\rho^k} = \Pi_i^{k0}$. Trade flows are unchanged, as the right hand side of (1) is constant. Real purchasing power of its currency is constant for each country j, $r_j^{\rho^k} P_j^k / P_j^{k0} = 1$. That is, the appreciation passthrough factor $r_j^{\rho^k}$ is equal to the factor by which j's price index falls. Real income is likewise constant for each country after combining seller and buyer outcomes. This follows because in (5) the factory gate price p_i remains constant when Π_i^{k0} is replaced by its equal value $\Pi_i^k r_i^{\rho^k}$.

An implication of the money neutrality property is that gravity *estimates* of exchange rate elasticity $\rho_j^k(1-\sigma)$ are actually estimates of $(\rho_j^k - \bar{\rho}^k)(1-\sigma)$ for an arbitrary $\bar{\rho}^k$. Gravity regressions cannot identify $\bar{\rho}^k$, only the destination-specific deviations from $\bar{\rho}^k$.

A possible structural gravity interpretation of the exchange rate influence $(r_i/r_j)^{\rho_j^k(1-\sigma_k)}$ is that it arises from short run increasing bilateral trade costs due to fixed bilateral 'marketing capital'. Destination-specific heterogeneity arises from given capacities that move over time toward long run efficient capacities. See Anderson and Yotov (2019) for details. This interpretation puts structure on the inferred bilateral exchange rate friction that is consistent with the perfect spatial arbitrage assumed in structural gravity models.

The triangular arbitrage condition implies theoretical limits on the variation of exchange rate influence $(r_i/r_j)^{\rho_j^k(1-\sigma_k)}$. A smell test of the logic of the model and its estimator checks whether the condition violated. Henceforth the sector k notation is dropped for simplicity. The limit condition is[8]

$$\frac{\tau_{ij}\tau_{jl}}{\tau_{il}} \geq (r_i/r_j)^{\rho_l - \rho_j}, \forall i, j, l.$$

With a uniform passthrough rate the right hand side of the limit condition reduces to 1, the standard triangular arbitrage condition. The results below show that the estimated bilateral trade costs never violate the triangular arbitrage condition.

2 Effective Exchange Rate Indexes

Section 5 shows that exchange rates have real effects at annual frequencies. These act directly on bilateral trade in (1), a partial equilibrium effect, and through the shifts in equilibrium multilateral resistance that are determined by (6)-(7). This finding suggests a

[8]The condition comes from comparing p_{ij}, p_{il} with the indirect $p_{ij->l}$ yielding

$$\tau_{ij}\tau_{jl}(r_i/r_j)^{\rho_j}(r_j/r_l)^{\rho_l} \geq \tau_{il}(r_i/r_l)^{\rho_l}$$

where the initial inequality is divided through by the common factory gate price p_i.

role for treating exchange rate effects as trade policy – heterogeneous passthrough seen in high frequency price comparison data is not sufficiently transitory or limited in scope to justify abstracting from it in the context of longer run policy making.

For this purpose it is useful to derive and quantify real effective exchange rate indexes for buyers and sellers. These differ from the trade weighted exchange rate indexes exemplified by appendix equation (22) in essential ways due to their general equilibrium treatment of the incidence of trade costs and their emphasis on differential exchange rate passthrough as the source of non-neutrality. Less essentially, the CES structure of ERGs is a particular treatment of substitution effects relative to the variety of *ad hoc* treatments in standard effective exchange rates measures.

2.1 Buyer ERG

The purchasing power of a unit of j's currency rises (falls) as inward multilateral resistance – buyers incidence of trade costs including exchange rate change frictions – falls (rises). That is, purchasing power rises (falls) when inward multilateral resistance in the new equilibrium P_j is lower (higher) than inward multilateral resistance in the base equilibrium. Using (7) yields the key relationship between buyer's multilateral resistances:

$$P_j^{1-\sigma} = (P_j^0)^{1-\sigma} \sum_i \left(\frac{\tau_{ij}(r_i/r_j)^{\rho_j}}{\Pi_i P_j^0} \right)^{1-\sigma} Y_i/Y.$$

Exponentiate on both sides by $1/(1-\sigma)$. On the right hand side, factor out $1/r_j^{\rho_j}$ and then divide both sides by P_j^0. The left hand side is now the real purchasing power term P_j/P_j^0. On the right hand side substitute in the summation term the predicted value of trade in the initial equilibrium from (1), $\widehat{X}_{ij}^0 = (\tau_{ij}/\Pi_i^0 P_j^0)^{1-\sigma} Y_i^0 E_j^0/Y^0$. Rearrange the result to yield the real purchasing power change factor as

$$\frac{P_j}{P_j^0} = \left[\sum_i \frac{\widehat{X}_{ij}^0}{E_j^0} \frac{Y_i/Y}{Y_i^0/Y^0} \left(\frac{\Pi_i^0}{\Pi_i} \right)^{1-\sigma} (r_i/r_j)^{\rho_j(1-\sigma)} \right]^{1/(1-\sigma)}. \tag{8}$$

The real exchange rate with gravitas is the hypothetical exchange rate appreciation $\widetilde{R}_j$ required to offset the decline in purchasing power. It is defined from:

$$\frac{P_j}{\widetilde{R}_j P_j^0} = 1 \Rightarrow \widetilde{R}_j = \frac{P_j}{P_j^0}.$$

The sellers multilateral resistance changes Π_i/Π_i^0 play a key role in modifying the effect of exchange rate changes in (8) and thus in $\widetilde{R}_j$. More simplification and intuition comes by applying the the relationship of Π_i to sellers factory gate price p_i. Use equation (5) to solve

$$\frac{Y_i/Y}{Y_i^0/Y^0}\left(\frac{\Pi_i^0}{\Pi_i}\right)^{1-\sigma} = \left(\frac{p_i}{p_i^0}\right)^{1-\sigma}$$

where p_i is seller i's 'factory gate' price, the ultimate buyers cost less all trade costs. Substitute the right hand side into equation (8) to yield

$$\frac{P_j}{P_j^0} = \left[\sum_i \frac{X_{ij}^0}{E_j^0}\left(\frac{p_i}{p_i^0}\right)^{1-\sigma} (r_i/r_j)^{\rho_j(1-\sigma)}\right]^{1/(1-\sigma)} \tag{9}$$

The right hand side of the equation above is usefully decomposed into an effective real exchange rate index and an average cost effect due to the vector of sellers factory gate price changes $\{p_i/p_i^0\}$. Thus

$$\widetilde{R}_j = \frac{P_j}{P_j^0} = C_j\left(\frac{\tilde{r}_j}{r_j}\right)^{\rho_j} \tag{10}$$

or

$$\frac{P_j/C_j}{P_j^0} = \left(\frac{\tilde{r}_j}{r_j}\right)^{\rho_j}$$

where

$$\tilde{r}_j = \left[\sum_i \widetilde{w}_{ij} r_i^{\rho_j(1-\sigma)}\right]^{1/\rho_j(1-\sigma)}, \tag{11}$$

and

$$\widetilde{w}_{ij} = \frac{\frac{X_{ij}^0}{E_j^0}\left(\frac{p_i}{p_i^0}\right)^{1-\sigma}}{\sum_i \frac{X_{ij}^0}{E_j^0}\left(\frac{p_i}{p_i^0}\right)^{1-\sigma}}.$$

and

$$C_j = \left[\sum_i \frac{X_{ij}^0}{E_j^0} \left(\frac{p_i}{p_i^0} \right)^{1-\sigma} \right]^{1/(1-\sigma)}.$$

The average sellers cost index C_j in practice is the effect on sellers' prices of all the forces of demand, supply and technology along with heterogeneous exchange rate passthrough. Deflating P_j by this cost effect, $\tilde{r}_j/r_j$ is the nominal Effective exchange Rate with Gravitas: country *j's appreciation/depreciation of its exchange rate that maintains initial purchasing power by offsetting the direct effects of the vector of exchange rate appreciation factors.*[9] $(\tilde{r}_j/r_j)^{\rho_j}$ is a CES index function with the base expenditure weights adjusted for general equilibrium effects of sellers price changes. It is operational with structural gravity estimation.

The ERG $\widetilde{r}_j$ on the right hand side of (10) is not directly comparable to the typical effective exchange rate index $\bar{r}_j$ because it uses weights that embed general equilibrium effects, and it is a CES index with elasticity $\rho_j(1-\sigma)$. A decomposition based on local rates of change around equation (9) establishes a direct connection between $\widetilde{R}_j$ and a CES version of $\bar{r}_j$ defined to include home goods and denoted $\bar{r}'_j$. In general the local difference between $\tilde{R}_j$ and $\bar{r}'_j$ is given by differentiating (9):

$$(1-\sigma) d \ln(P_j/P_j^0) = \sum_i \frac{X_{ij}^0}{E_j^0} \rho_j d \ln(r_i/r_j) + \sum_i \frac{X_{ij}^0}{E_j^0} d \ln(p_i/p_i^0)$$

The right hand side can be rewritten as

$$(1-\sigma) d \ln(P_j/P_j^0) = \rho_j [d \ln \bar{r}'_j - d \ln r_j] + \sum_i \frac{X_{ij}^0}{E_j^0} d \ln(p_i/p_i^0).$$

Here $d \ln \bar{r}'_j$ denotes the percentage change in the CES version of the nominal effective exchange rate (including home goods) with elasticity $\rho_j(1-\sigma)$. With no real effects due to

[9] C_j contains indirect effects of exchange rate changes. In principle it is possible to account for these with counterfactual general equilibrium calculations that hold constant all factors other than exchange rate changes. This would be the real counterpart to ERG.

uniform passthrough the second term is equal to zero and the first term would need to be equal to zero to be consistent with the assumed no real effects property – the appreciation of j's currency would equal the appreciation of currencies in the basket of goods that it buys. Non-uniform passthrough has real effects due to the second term on the right hand side, the average sellers' price effect. $d\bar{r}_j$ may be understood as a Laspeyres index that attempts to control for the contribution to inflation of the buyers' price index that is due to exchange rates under partial equilibrium assumptions $p_i = p_i^0$ and disregarding incomplete passthrough. Refinements of $\bar{r}_j$ or $\bar{r}'_j$ such as chain weights to adjust for discrete changes in shares X_{ij}^0/E_j^0 between equilibria cannot be interpreted to approximate $\tilde{r}_j$ because even for infinitesimal changes they necessarily miss real effects associated with the second term. They do adjust for the sellers' price effect on the weights in the first term.[10]

Note that the elasticity parameter in $\tilde{r}_j$ in equation (11) is $\rho_j(1-\sigma)$ where ρ_j is the *level* of destination j's passthrough elasticity. An external value of the average $\bar{\rho}$ and the elasticity σ is required to solve $\tilde{r}_j$ from the inferred $(\tilde{r}_j/r_j)^{\rho_j(1-\sigma)}$. As the level of $\rho_j \to 0$, $\partial \ln \tilde{r}_j / \partial \ln r_i \to \tilde{w}_{ij}$ and thus $\tilde{r}_j \to \bar{r}_j$. For finite but small inferred passthrough elasticity deviation ρ_j, the cross country variation in exchange rate changes and in the effect of sellers' prices on weights $\tilde{w}_{ij}$ makes only small differences from $\bar{r}_j$. Results below thus indicate mostly high correlation between $\tilde{r}_j$ and $\bar{r}_j$ for small ρ_j inferred from annual gravity equations. In contrast, correlation falls dramatically with higher external values of passthrough elasticity $\bar{\rho}$.

[10] Chain weights allow for changes in X_{ij}/E_j. The ratio of new to base shares is given in structural gravity by

$$\frac{X_{ij}/E_j}{X_{ij}^0/E_j^0} = \frac{Y_i/Y}{Y_i^0/Y^0}\left(\frac{\Pi_i P_j}{\Pi_i^0 P_j^0}\right)^{1-\sigma}(r_i/r_j)^{\rho_j(1-\sigma)} = \left(\frac{p_i}{p_i^0}\right)^{1-\sigma}\left(\frac{P_j}{P_j^0}\right)^{1-\sigma}(r_i/r_j)^{\rho_j(1-\sigma)}$$

where the right hand equation uses (5).

2.2 Sellers Effective Exchange Rate

Seller earnings are inversely related to sellers incidence by equation (5), just as the buyers purchasing power is inversely related to buyers incidence. In relative form (5) implies

$$\frac{Y_i/Y}{Y_i^0/Y^0} = \left(\frac{p_i\Pi_i}{p_i^0\Pi_i^0}\right)^{1-\sigma}.$$

For an endowments economy, the relative earnings change is given by[11]

$$\widehat{p_i} = \frac{p_i}{p_i^0} = \left(\frac{Y^0}{Y}\right)^{1/\sigma}\left(\frac{\Pi_i}{\Pi_i^0}\right)^{1/\sigma-1} \tag{12}$$

The effective exchange rate index that is equivalent in sellers' earnings power is based on using equation (6) for Π_i and steps parallel to (8). Relative earnings are inversely proportional to changes in sellers' multilateral resistance, given by the real sellers appreciation

$$\tilde{R}_i^x \equiv \frac{\Pi_i}{\Pi_i^0} = \left[\sum_j \frac{X_{ij}^0}{Y_i^0}\frac{E_j/Y}{E_j^0/Y^0}\left(\frac{P_j^0}{P_j}\right)^{1-\sigma}\right]^{1/(1-\sigma)}\left[\sum_j \widetilde{w}_{ij}^x (r_i/r_j)^{\rho_j(1-\sigma)}\right]^{1/(1-\sigma)}, \tag{13}$$

where

$$\widetilde{w}_{ij}^x = \frac{\frac{X_{ij}^0}{Y_i^0}\frac{E_j/Y}{E_j^0/Y^0}\left(\frac{P_j^0}{P_j}\right)^{1-\sigma}}{\sum_i \frac{X_{ij}^0}{Y_i^0}\frac{E_j/Y}{E_j^0/Y^0}\left(\frac{P_j^0}{P_j}\right)^{1-\sigma}}. \tag{14}$$

The second term on the right hand side of (13) is the passthrough of bilateral exchange rate appreciation (relative to appreciation in the individual seller's destination markets) to sellers incidence. This is the nominal ERG passthrough for sellers, inversely related to sellers' earnings as in the partial equilibrium case. To complete the parallel of nominal ERG for

[11] Allowing for substitutability in supply results in implicit functions for the within-country sectoral shares and their relationship to cross-country shares. The same principle governs the relationship of earnings to seller incidence but is complicated by supply side substitution.

sellers to buyers nominal ERG, define a seller-specific passthrough $\bar{\rho}_i$ as the solution to

$$\left[\sum_j \widetilde{w}_{ij}^x (r_i/r_j)^{\rho_j(1-\sigma)}\right]^{1/(1-\sigma)} = \left[\sum_j \widetilde{w}_{ij}^x (r_i/r_j)^{\bar{\rho}_i(1-\sigma)}\right]^{1/(1-\sigma)}.$$

Then the passthrough to sellers incidence implies a sellers nominal ERG passthrough $\tilde{r}_i^x$ as:

$$\left[\sum_j \widetilde{w}_{ij}^x (r_i/r_j)^{\rho_j(1-\sigma)}\right]^{1/(1-\sigma)} = (r_i/\tilde{r}_i^x)^{\bar{\rho}_i}. \tag{15}$$

In the application below, we report inferred estimates of the left hand side of (15), to be interpreted as the right hand side.[12] An appreciation of i's exchange rate relative to its partners raises $\tilde{r}_i^x$, which is passed through to sellers incidence at rate $\bar{\rho}_i$ and lowers sellers earnings at rate $\bar{\rho}_i(1/\sigma - 1)$ via equation (12).

The first term on the right hand side of sellers real ERG defined in equation (13) is a CES index of relative changes in buyer multilateral resistances, with endogenous weights. Buyers price increases in (13) reduce Π_i/Π_i^0 and hence raise earnings.

The steps above for national income and expenditure carry through to the sectoral level under the common simplifying assumption (in gravity modeling) that the upper level preference/technology aggregator is Cobb-Douglas. Unbalanced trade is handled with the assumption that $E_i = \phi_i Y_i$ subject to $\sum_i \phi_i Y_i = Y = \sum_i Y_i$. At the sector level, the variables in the preceding expression have sector k superscripts and α_i^k is the expenditure share parameter for sector k goods from country i. On the left hand side of (13) for sector k the factor $\phi_i \alpha_i^k$ appears in numerator and denominator, hence it cancels.

Evaluation of (13) for local changes reveals important differences from the purchasing power index. Log-differentiate the sectoral form and suppress variation in Y^{k0}/Y^k.[13] The

[12] The exponent $\bar{\rho}_i$ is implicitly defined, unlike the exponent ρ_j in the nominal buyers ERG $\widetilde{r}_j$. In principle the $\bar{\rho}_i$ values can be solved but this is unnecessary for present purposes, and computationally burdensome. Nominal seller ERG passthrough $\tilde{r}_i^x$ is easily constructed with an estimated gravity model.

[13] In a multi-sector endowments economy, the exchange rate changes would generally induce relative seller price variation.

result is

$$(1-\sigma_k) d\ln \Pi_i^k/\Pi_i^{k0} = \bar{\rho}_{ik}[d\ln \tilde{r}_{ik}] + Cov_{ik}(\overrightarrow{\rho}, \overrightarrow{r}) - \sum_j \frac{X_{ij}^{k0}}{Y_i^{k0}} \widehat{P}_j^k. \tag{16}$$

$\overrightarrow{\rho}$ denotes the vector $(\rho_1, ..., \rho_n)$, $\bar{\rho}_i$ is its i-specific trade weighted mean and $\overrightarrow{r}_j$ denotes the vector $(r_1/r_j, ..., r_n/r_j)$. The covariance term captures the effect on seller i's income of the interaction of destination-specific variation of exchange rate passthrough with destination-specific exchange rate variation. The covariance is seller-specific because the generalized trade weights $\tilde{w}_{ij}^x$ are seller-specific.

Compared to the local evaluation of the purchasing power index (10), (16) requires an origin specific $\bar{\rho}_i$ that is an export (for i) weighted average of the destination passthrough rates in the first term. A second difference is that the general equilibrium effects of sellers prices in (10) are replaced by the general equilibrium effects of buyers price index changes in P_j in (16). The third and more novel difference is the covariance term. Even with partial equilibrium assumptions that shut down the general equilibrium price terms, (16) implies that standard effective exchange rate indexes corrected for country specific passthrough are, in contrast to purchasing power indexes, inadequate to capture sellers income effects due to the variation in destination exchange rate passthrough rates.

By construction, the real ERGs $\widetilde{R}_j^k$ and $\widetilde{R}_i^{x,k}$ are consistent with equilibrium multilateral resistances (6)-(7). They share a close resemblance in structure but they generally diverge and tend to be negatively correlated because they inherit the normally negative correlation of buyer and seller multilateral resistances. Intuition from partial equilibrium applies – appreciation is good for buyers and bad for sellers.

2.3 Policy Implications

The two real ERGs – purchasing power index $\widetilde{R}_j^k$ and earnings power index $\widetilde{R}_i^{x,k}$ – are theory-consistent measures of the real effects of exchange rate movements on buyers and on sellers. By construction the measures are comparable across countries and may be used to indicate desirable directions of change of exchange rates in the 'jawboning' commonly done between national economic policymakers. They do not, however, necessarily give reliable information about long run equilibrium exchange rate changes from current positions. Section 3 below specifies a counterfactual long run general equilibrium simulation that that projects the equilibrium changes for comparison to the ERGs. The two are highly correlated but magnitudes differ and for some country-time intervals the correlation is low or even negative.

Policy making based on an alternative use of earnings power nominal ERG $\tilde{r}_i^{x,k}$ or purchasing power ERG $\tilde{r}_j^k$ appears more promising. Temporary national compensation policies at the sectoral level could be based on movements in the indexes that exceed a threshold. This would be analogous to the producer price support payments or consumption subsidies that are prominent in primary and agricultural products on both production and consumption sides. Compensation in this form is consistent with the *all else equal* structure of the ERGs.

Short run real trade displacement effects of heterogeneous exchange rate passthrough indicate possible harm to producer and consumer interests. In principle the harm can be offset by targeted compensation based on nominal earnings power ERG $\tilde{r}_i^{x,k}$ and purchasing power ERG $\tilde{r}_j^k$.[14] The temporary domestic compensation policies could be made subject to WTO rules and dispute settlement: allowed when justified by findings of harm, similar to the current WTO treatment of 'safe-guards' and anti-dumping cases.

This potential extension of 'adjustment assistance' might bleed off the political pressure associated with claims of 'currency manipulation', as it does with anti-dumping. A further advantage is that this setup would tend to neutralize countries' incentives to use exchange rate policy for temporary advantage, as the prohibition of export subsidies does in current WTO law.

[14] The real ERGs move over time due to many other factors with effects embedded in indexes $C_j^{x,k}$ and C_j^k. A policy aimed at compensation for exchange rate frictions should not compensate for the latter general equilibrium forces.

3 Equilibrium ERG Projection

The long run equilibrium obtains when money is neutral. Given the endowments and trade imbalances of a particular year in the data, the bilateral appreciation/depreciation elements r_i/r_j for that year are set equal to 1. The full general equilibrium solution is calculated, yielding a set of seller and buyer incidences $\{\Pi_i^{k*}, P_j^{k*}\}$. The ratios of base year incidences to counterfactual long run equilibrium incidences form the set $\{\Pi_i^k/\Pi_i^{*k}, P_j^k/P_j^{*k}\}$. The decomposition steps used to separate direct and indirect effects of exchange rate changes in Sections 2.1 and 2.2 also apply here to yield long run ERGs.

The full general equilibrium solution required to project the effect of non-uniform exchange rate changes is completed by specifying a supply side of the model and closing the model with a relationship between expenditure and income. Assume to begin with that demand for all goods is aggregated in a single CES expenditure function. Supply is modeled as a vector of endowments.

For each origin i the value of sales at world currency prices is $Y_i = p_i y_i$ where y_i is the units of output of origin i and p_i is its 'factory gate' price in world currency units. Then $Y_i/Y_i^0 = p_i/p_i^0$. Using equation (5)

$$\frac{p_i}{p_i^0} = \left(\frac{\Pi_i}{\Pi_i^0}\right)^{(1-\sigma)/\sigma} \left(\frac{Y}{Y^0}\right)^{\sigma}.$$

Sellers prices change in spatial equilibrium due to the shifting incidence of trade costs induced by non-uniform exchange rate passthrough. A full general equilibrium solution is found as a fixed point of (2)-(3), (5) with Y_i replaced by $p_i y_i$. Standard practice to resolve the indeterminacy of price levels in general equilibrium is to normalize the price vector $\{p_i\}$ for non-base projections by $\sum_i p_i y_i = \sum_i y_i$ where $p_i^0 = 1$ by choice of units. The general equilibrium counterfactual projection of equilibrium exchange rate changes can be used to better guide exchange rate policy or to inform jawboning about currency manipulation.

For more intuition, begin from the short run model estimated for some end year t using

(6)-(7). The solution generates a set of inward multilateral resistances (equal in the setup to buyers' price indexes). For the same underlying data, the counterfactual long run equilibrium is based on solving system (2)-(3) for the long run multilateral resistances $\{\Pi_i^*, P_j^*\}$, taking away the effect of incomplete and non-uniform passthrough. The sellers' factory gate prices (in world currency units) in the endowments model case are solved from (5). The normalization is $\sum_i p_i y_i = \sum_i y_i$ where p_i is the factory gate price, y_i is the endowment (both in year t implicitly) and the year t sellers prices are set to 1 by units choice.

The full general equilibrium solution requires closure of the model with an assumption connecting expenditures to incomes. The simplest closure consistent with unbalanced trade (which is always observed) is $E_i = \phi_i Y_i$ where ϕ_i is observed in the benchmark equilibrium and assumed constant in moving to the counterfactual equilibrium.[15] The adding up condition for world equilibrium requires $\sum_i E_i = \sum_i Y_i \Rightarrow E_j/Y = \phi_j Y_j / \sum_j \phi_j Y_j$ for counterfactual equilibria. With these added structures in place, the counterfactual multilateral resistances can be computed.

In the long run there are no real effects of exchange rates. Given the endowments in year t, solve for the long run counterfactual equilibrium. The vector of consumer price indexes P_j^* gives the purchasing power of a unit of the world endowment (subject to the normalization) in country j in the long run equilibrium. The long run equilibrium exchange rate change vector given the endowments and exchange rates of year t is:

$$r_i^* = \frac{P_i^t}{P_i^*}, \ \forall i. \tag{17}$$

Vector r_i^* has several potentially important uses. Most obviously, it serves as the benchmark for deducing over- or under-valuation based on it relation to effective exchange rates as

[15] An intuitive justification for constant ϕ_is is that a counterfactual income deviation in one period would be intertemporally smoothed so that the marginal utility of external borrowing/lending remained equal to the marginal utility of wealth. The exact amount of smoothing depends on many details. Constant ϕs imply that deficit countries borrow more (less) as wealth rises (falls) due to income changes in the counterfactual period. The direction of change is intuitive with constant ϕs justified as simplification in a model focused on static equilibrium.

measured by

$$\frac{r_i}{r_i^*}.$$

r_i/r_i^* is a counterfactual concept that holds all variables constant except for the exchange rate. In contrast $\widetilde{R}_i$ compares a base year with a subsequent year using actual exchange rates, supply vectors and expenditure data for both base year and subsequent year. A second use of the counterfactual and implicitly of r^* is in calculating the terms of trade effects of going from the estimated actual equilibrium in each year to the counterfactual long run equilibrium. The details are covered in Section 4.

3.1 Mult-sector ERGs

The extension from the one sector case to multiple sectors is simple under a standard (in the recent literature) Cobb-Douglas aggregation. For each sector k, the multilateral resistance systems and the sellers' price equations hold as in the 1 good per country case. Thus all the steps leading to (10) hold at the sectoral level:

$$\widetilde{R}_j^k = \frac{r_j^{\rho_j^k} P_j^k}{P_j^{k0}} = C_j^k (\tilde{r}_j^k / r_j)^{\rho_j^k}.$$

The aggregate ERG is the Cobb-Douglas aggregator of the sectoral ERGs:

$$\mathcal{R}_j = \prod_k (\widetilde{R}_j^k)^{\alpha_k}.$$

The second equation can be decomposed into

$$\mathcal{R}_j = \mathcal{C}_j \mathbf{r}_j^{\tilde{\rho}_j}$$

where $\mathcal{C}_j = \prod_k (C_j^k)^{\alpha_k}$, $\tilde{\rho}_j = \sum_k \alpha_k \rho_j^k$ and $\mathbf{r}_j^{\tilde{\rho}_j} = \prod_k (\tilde{r}_j^k)^{\rho_j^k} / r_j)^{\alpha_k}$.

Full general equilibrium in the endowments model aggregates sectors in similar fashion.

Aggregate incomes are the sum of sectoral incomes $Y_i = \sum_k Y_i^k$. Cobb-Douglas demand systems imply $E_j^k = \alpha_k E_j$; where $\alpha_k \in (0,1)$, $\sum_k \alpha_k = 1$. As in the 1 sector case, trade imbalance is modeled with a fixed ratio of expenditure to income ϕ_i, hence in combination with the requirement that global income equals global expenditure, $E_i = \phi_i Y_i / \sum_i \phi_i Y_i$. The normalization of sellers' prices is $\sum_{i,k} p_i^k y_i^k = \sum_{i,k} y_i^k$. Closure is either the simple trade balance $E_j = Y_j$, or a somewhat more general $E_j = \phi_j Y_j$ subject to $\sum_j \phi_j Y_j = \sum_j Y_j = Y$.

The counterfactual long run equilibrium calculation yields a set of buyers' sectoral price indexes $\{P_j^{*k}\}$. The Cobb-Douglas aggregator of these is the economy wide price index in the long run. The short run price index for period t implied by gravity is similarly a Cobb-Douglas aggregate of the sectoral inward multilateral resistances. Then

$$r_i^* = \prod_k \left(\frac{P_i^{*k}}{P_i^{tk}} \right)^{\alpha_k}, \ \forall i. \tag{18}$$

4 Terms of Trade and Exchange Rates

The terms of trade in the one sector case equal the real earnings of country j given by $r_j p_j / P_j$.[16] The relative change in real earnings is given by

$$\hat{T}_j = \frac{r_j p_j / p_j^0}{P_j / P_j^0}$$

Use the market clearance equation (5) evaluated at the two equilibria to solve for

$$r_j p_j / p_j^0 = \frac{\Pi_j^0}{\Pi_j} \left(\frac{Y_j / Y}{Y_j^0 / Y^0} \right)^{1/(1-\sigma)}.$$

[16] This usage of 'terms of trade' is somewhat eccentric because in the numerator is the sellers' price of tradables (including sales to the home market) while in the denominator is the buyers' price of tradables (including purchases in the home market). The local rate of change of real income is equal to the local rate of change of the terms of trade because the income effect of local sales price changes is equal to zero. For discrete changes, the real income measure is preferred to the usual terms of trade measure approximation.

Substitute into the change in real earnings to yield:

$$\hat{T}_j = \frac{\Pi_j^0 P_j^0}{\Pi_j P_j} \left(\frac{Y_j/Y}{Y_j^0/Y^0} \right)^{1/(1-\sigma)} = \frac{1}{\widetilde{R}_j \widetilde{R}_j^x} \left(\frac{Y_j/Y}{Y_j^0/Y^0} \right)^{1/(1-\sigma)}. \tag{19}$$

$\hat{T}_j$ can be calculated using estimated gravity coefficients and data to construct bilateral trade costs and solving system (2)-(3). The second equation expression of $\hat{T}_j$ in (19) in terms of real ERGs decomposes the real income effects of non-uniform passthrough. For the money neutrality case when all other variables are constant, $\hat{T}_j = 1$: the terms of trade are constant.

4.1 Multi-sector Terms of Trade

Terms of trade more generally refers to an aggregate of sectors. The aggregate terms of trade for multiple sectors follows the technique of Anderson and Yotov (2016). Resuscitating the sector index k, (19) gives a terms of trade index for each sector k, T_i^k. Rather than mechanically forming an average of the sectoral indexes, it is preferable to build from sellers' and buyers' price indexes separately, then form their ratio as the terms of trade index.

For the sellers' price index we follow Anderson and Yotov in building upon an endowment economy. Thus $Y_i^k = r_i p_i^k y_i^k$ where y_i^k is the endowment of country i's variety of the good in sector k (the resources used in both production and distribution). Because of the endowment assumption, $y_i^k = y_i^{k0}$. It is convenient to choose units such that $p_i^{k0} = 1, \forall i, k$. The price index for sellers is defined with the intuitive normalization $\sum_{i,k} r_i p_i^k y_i^k / \sum_{i,k} y_i^k = 1$, implying that the value of the world endowment is constant. This normalization along with the homogeneity restrictions of the model turns out to imply (Anderson and Yotov, 2016) a sector-by-sector restriction $\sum_i r_i p_i^k y_i^k = \sum_i y_i^k$. For any country i, the seller's price index relative to its initial value of 1 is given by $\sum_k r_i p_i^k y_i^k / \sum_k y_i^k$. Solving the effective market clearing condition (5) for the new price in the endowment economy, $r_i p_i^k = \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k}$. Then $Y_i^k/Y^k = \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k} y_i^k / \sum_i y_i^k$. For conducting counterfactual long run equilibrium experiments, $r_i p_i^k = r_i^*,\ \forall i, k$.

For buyers, the price index is formed by aggregating the sectoral indexes P_i^k. The Cobb-Douglas price index $\mathcal{P}_i = \prod_k (P_i^k)^{\alpha_k}$. In the present application evaluating the change in terms of trade, P_i^k is replaced by its relative change P_i^k/P_i^{k0}. In the counterfactual long run equilibrium experiment, P_i^k is the long run counterfactual value.

The terms of trade for country i is given by

$$\hat{T}_i = \frac{\sum_k \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k} y_i^k / \sum_k y_i^k}{\prod_k (P_i^k/P_i^{k0})^{\alpha_k}}. \tag{20}$$

For the one good economy (20) reduces to (19). For the counterfactual long run equilibrium experiment, $\hat{T}_i = T_i^*$ and the multilateral resistances with superscript 0 denote the inferred values for the base year.

The form of (20) is based on the endowments economy structure, but the same value of $\hat{T}_i$ results from the Ricardian economy model of Eaton and Kortum (2002) extended to multiple sectors by Costinot, Komunjer and Donaldson (2012). Under this interpretation the terms of trade change factor is interpreted as the real wage change factor.

The full general equilibrium solution for $\{\hat{T}_i\}$ requires values for the multilateral resistances. These in turn require closing the model to obtain global sales and expenditure shares $\{Y_i^k/Y^k, E_j^k/Y^k\}$. It was shown above that in the endowment economy, $Y_i^k/Y^k = \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k} y_i^k / \sum_i y_i^k$. The global expenditure shares are obtained from the budget constraints that relate income to expenditure. Due to the Cobb-Douglas preferences assumption, $E_i^k/E_i = \alpha_k, \ \forall i, k$. Allow for trade imbalance that is parametric: $E_i = \phi_i Y_i, \sum_i \phi_i Y_i = Y$, where $\phi_i > 0$ is the parametric ratio of expenditure to income for country i. In this case $E_i^k/Y^k = \alpha_k \sum_k \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k} \phi_i y_i^k / \sum_i \phi_i y_i^k$. Substitute this expression and $Y_i^k/Y^k = \left(\Pi_i^{k0}/\Pi_i^k\right)^{1-1/\sigma_k} y_i^k / \sum_i y_i^k$ into (2)-(3) and solve for the multilateral resistances $\{\Pi_i^k, P_j^k\}$.

5 ERGs in Practice

This section presents inferred ERGs and their implications based on structural gravity estimates of the effect of exchange rate changes on trade flows. First we detail the gravity equation to be estimated, then briefly describe the results with a focus on the exchange rate change term. The estimated exchange rate change term is used to calculate ERGs and their implications.

First we examine the empirical relationship between the ERGs and the standard measures of effective exchange rates. Correlations are fairly high, but quantitatively the two measures differ significantly. Importantly, for some time periods and countries, the correlation is negative.

Next we perform counterfactual exercises to compare the hypothetical money neutrality equilibrium differs from each year's estimated outcomes. The first use of the counterfactual is to compare the inferred real ERGs with the counterfactual long run Purchasing Power Parity (PPP) outcomes. Again, the correlation is high but quantitatively there are significant differences.

The second use of the counterfactual is to calculate the implied terms of trade effects of each year's deviation from long run money neutrality. Real income (terms of trade) effects are mostly small, but for the top and bottom deciles the average (within decile, across all years) terms of trade effect averages around 2% and −2% respectively.

5.1 Data

We require a data set capable of yielding internal trade along with cross border trade in multiple sectors.[17] The WIOD dataset concords production data with international trade data, hence it is convenient for this purpose. The data used here is:

[17] Observations on internal trade empower the gravity regression to distinguish exchange rate change effects from from the origin-time and destination-time fixed effects required to control for multilateral resistance. See the discussion of equation (21) below.

- Structural gravity is estimated from the WIOD data (covering 2000-2014, 17 sectors, 43 countries) that includes sectoral production for each country, and bilateral trade data.
- Standard trade cost proxies like distance, FTAs, etc. are from the CEPII dataset.
- Exchange rate data is from WIOD dataset.

5.2 Specification

The gravity estimator is based on the CES structural gravity model applied to the bilateral trade, including internal trade, for all countries in each sector. The percentage of zero trade flows is shown in Table 1, justifying our PPML estimator for any sector k as:

$$\begin{aligned} X_{ijt} = e^{\widetilde{\rho}_j ln\left(\frac{r_{it}}{r_{jt}}\right)} e^{\beta_{1t} INTR\_BRDR_{ij} * \delta_{t>2000} + \beta_2 RTA_{ijt} + \beta_3 comcur_{ijt}} * \\ e^{\beta_4 ln\_distw_{ij} + \beta_5 CNTG_{ij} + \beta_6 CLNY_{ij} + \beta_7 LANG_{ij} + \beta_8 INTR\_BRDR_{ij}} * \\ e^{\alpha_{it}} e^{\eta_{jt}} e^{\alpha} \epsilon_{ijt}. \end{aligned} \tag{21}$$

Here ϵ_{ijt} is a Poisson distributed random error term, α_{it} is an origin-time fixed effect, η_{jt} is a destination-time fixed effect, α is a constant, superscript k is omitted to reduce clutter, and the remaining terms are standard controls for the effects of trade costs, including in our case the effect of exchange rate movements on bilateral trade costs in the first term of the first line of equation (21). The second term is a cross-border-time fixed effect that controls for time-varying investments in cross-border marketing capital (Anderson-Yotov, 2019). The remaining cost controls are standard – respectively controlling for implementation of a regional trade agreement (RTA), common currency (*commcurr*), distance (*distw*), contiguity (CNTG), former colonial tie (CLNY), common language (LANG) and a time invariant cross border fixed effect (INTR_BRDR). The origin-time and destination-time fixed effects absorb the effects of country size along with the effects of multilateral resistances. The presence of internal trade flows on the left hand side of regression estimator (21) permits distinguishing exchange rate effects from the origin-time and destination-time fixed effects. Without internal trade, the exchange rate effects are absorbed by the fixed effects. Data on country-time production and expenditure in each sector combine with the theoretical interpretation of the estimated fixed effects to imply estimates of the multilateral resistances.

Table 1: Percentage of zero trade flows by sector (Averaged over years)

Sector	Percent of Zero Trade flows	
	mean	s.e.
Agriculture	0.4	0.12
Mining	0.96	0.12
Manufacturing		
Food	0.07	0.07
Textile	0.02	0.05
Wood	0.38	0.24
Paper	0.12	0.11
Petroleum	6.95	0.89
Chemicals	0.05	0.07
Plastic	0.02	0.03
Minerals	0.05	0.04
Basic metals	0.44	0.2
Metal products	2.31	0.05
Machinery	0.05	0.07
Electrical	2.3	0.03
Auto	0.14	0.09
Other Transport	3.07	0.38

Continued on next page

Sector	Percent of Zero Trade flows	
	mean	s.e.
Other	0.02	0.04

5.3 Results

5.3.1 Gravity Coefficients

The estimated sectoral gravity equation results have no elements of novelty except in the estimated exchange rate effects, so that is the focus of the discussion. As context, the equations fit the data well, bilateral distance is important, globalization effects (upward trending cross-border-time fixed effects, as in Anderson and Yotov, 2019) are revealed and the usual list of bilateral friction proxies performs as usual.

The estimated exchange rate effects $\tilde{\rho}_j$ in (21) are generally statistically significantly different from zero. Recall that the theoretical interpretation of $\tilde{\rho}_j$ is $(1-\sigma)(\rho_j - \bar{\rho})$ where $\bar{\rho}$ is the (unknown) mean value of the ρ_js. A t-test that cannot reject the null means that for the given sector, passthrough is close to uniform and exchange rates have no real effect. For 17 sectors and 40 countries we find 70% (81%) of cases where we cannot reject the null at the 5% (1%) significance level. Passthrough uniformity requires that all destinations taken as a group fail to reject the null. The joint test rejects the null in all sectors.

Moving from econometric inference of $\tilde{\rho}_j$s to construction of the ρ passthrough elasticities uses the theoretical structure $\rho_j = \tilde{\rho}_j/(1-\sigma)+\bar{\rho}$. The right hand side of the equation requires external estimates of average $\bar{\rho}$ and trade elasticity $1-\sigma$. The constructed ρs are used to calculate the ERGs.

5.3.2 Constructed Estimates of ρ

We apply the passthrough rate for the USA equal to 0.51 (Burstein and Gopinath, 2014) and apply the estimate of the sectoral trade elasticities from Caliendo and Parro (2015). The table reports the resulting mean and standard deviation of the sector-country point estimates of $\rho_j = \tilde{\rho}_j/(1-\sigma) + \bar{\rho}$. (We do not report standard errors because the external parameters are taken from different data and models than our estimate of $\tilde{\rho}_j$.) In two sectors, Auto and Other Transport, the constructed mean is above 1 and the standard deviation is above 2. These cases arise due to estimated trade elasticity < 1 reported by Caliendo and Parro (0.49 for Autos and 0.90 for Transport), with big standard errors (0.91 and 1.61). This suggests a measurement error issue for the constructed ρ reported for the Auto and Other Transport sectors, and perhaps for other sectors.[18] $\rho > 1$ is theoretically possible, depending on how passthrough is modeled. The passthrough literature assumes exogenous passthrough.

Table 2: Summary of ρ estimates

Sector	ρ	
	mean	s.e.
Agriculture	0.34	0.17
Mining	0.52	0.15
Manufacturing		
Food	0.57	0.71
Textile	0.5	0.19
Wood	0.47	0.15
Paper	0.56	0.17
Petroleum	0.52	0.04
Chemicals	0.77	0.42
Plastic	0.25	1.46
Minerals	0.3	0.35
Basic metals	0.33	0.17

Continued on next page

[18]More precisely estimated trade elasticities (ideally based on the same data and model) are needed to improve the quality of constructed ρ.

Year	ρ	
	mean	s.e.
Metal products	0.27	0.26
Machinery	0.43	0.38
Electrical	0.4	0.14
Auto	1.88	2.84
Other Transport	1.24	2.42
Other	0.42	0.31

5.3.3 Relation of ERG with typical effective exchange rate

The results show that ERGs differ significantly from standard effective exchange rates – magnitudes are quantitatively different and for some country-sector-time intervals are negatively correlated. The standard effective exchange rate measure requires an adjustment to make it comparable to the inclusion of domestic sales in the ERGs. Thus the standard effective exchange rate is modified to include domestic sales in the index: $\bar{r}_j = \sum_i w_{ij}(r_i/r_j)$ where the w_{ij}s are the expenditure share weights in j.

The overall correlation of the ERGs (nominal and real) with effective exchange rate counterparts is fairly high, in the range of 0.8 to 0.9. This is because the indexes differs mainly in the weights, which locally are positive and sum to 1.[19] See the online Appendix for details on overall correlations.

Nominal ERGs and their effective exchange rates counterparts diverge over time by significant amounts. The divergence is greater for the sellers index than for the buyers index. The overall US case illustrates, chosen as likely *a priori* to minimize divergence due to high diversification of both imports and exports. Differences emerge of around 8% for the buyers case and close to 15% for the sellers case. The cases also illustrate high correlation between

[19]The ERGs also differ by an origin or destination specific passthrough exponent that has no counterpart in the standard formula.

$\tilde{r}_{US}$ and $\bar{r}_{US}$.

Time series plots for the US are in the charts below. The movement of $\bar{r}$ and $\tilde{r}$ on the buyers index and $\bar{r}^x$ and $\tilde{r}^x$ on the sellers side is relative to 1 in the base year 2000 for all countries. For the US, the yearly changes of ERGs are damped considerably compared to their effective exchange rate counterparts. See Figures 1 and 2. Figure 3 displays an overall inverse relationship between buyer $\tilde{r}$ and seller $\tilde{r}^x$. This arises because of the influence of the multilateral resistances in the respective buyer weights $\tilde{w}$ and seller weights $\tilde{w}^x$. Buyer wieghts $\tilde{w}$ depend on seller multilateral resistances Π, seller weights $\tilde{w}^x$ depend on buyer multilateral resistances P, and the the overall movements of buyer and seller multilateral resistances tend to be negatively correlated.

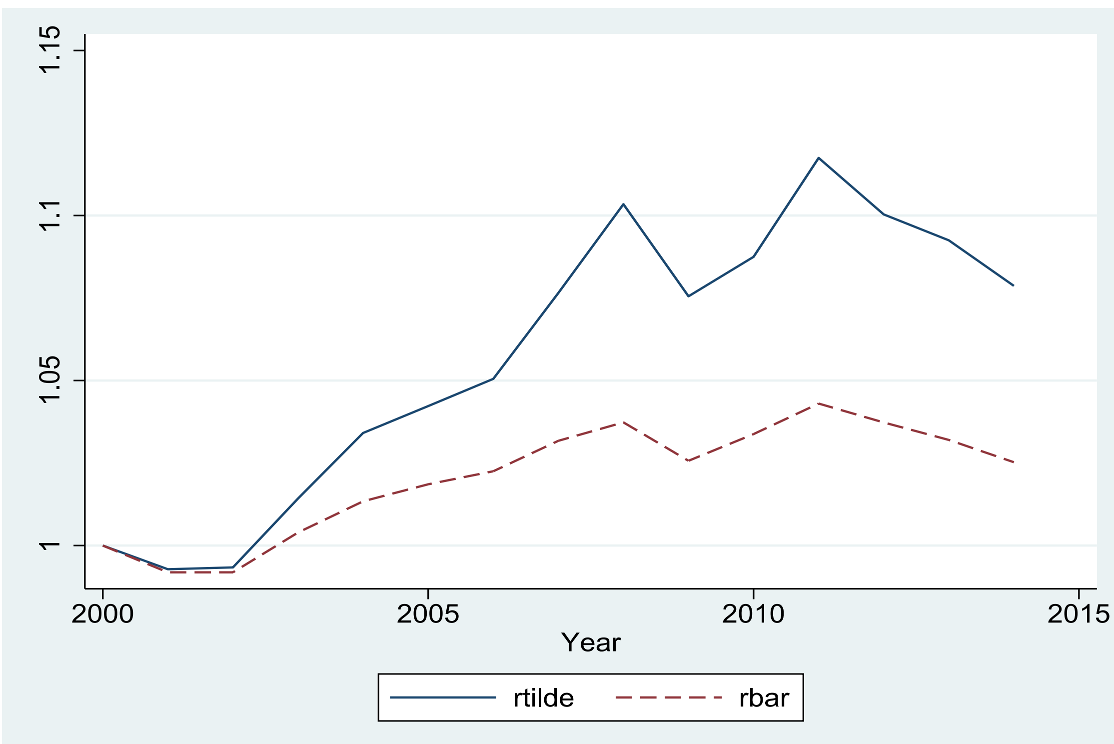

Figure 1: $\tilde{r}$ Vs $\bar{r}$ for United States (Aggregate)

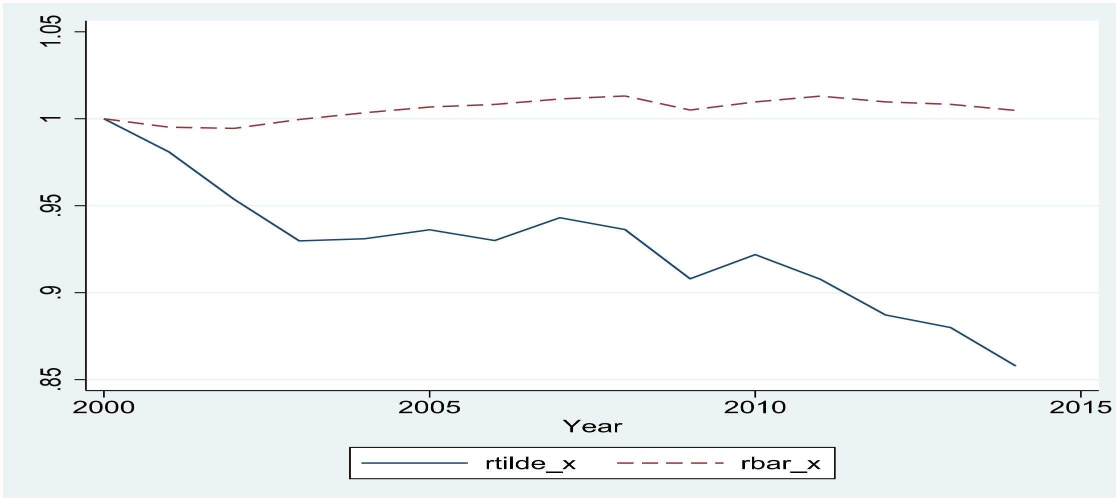

Figure 2: $\tilde{r}^x$ Vs $\bar{r}^x$ for United States (Aggregate)

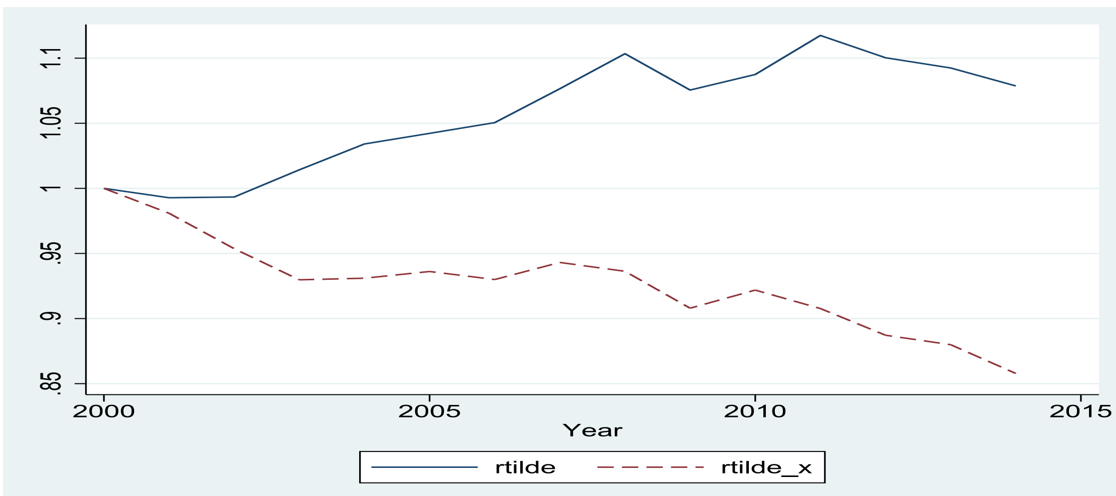

Figure 3: $\tilde{r}$ Vs $\tilde{r}^x$ for United States (Aggregate)

Figures 4 and 5 plot the time series of US aggregate real and nominal ERGs along with the aggregate price indexes C_{US} for the buyer and C^x_{US} for the seller. The price indexes combine the general equilibrium effects of exchange rate movements with the many other time varying forces that drive the changing pattern of world production. In some intervals $\tilde{r}^x$ and C^x are negatively correlated. For example, in Figure 5 the real seller ERG $\tilde{R}^x$ appreciates nearly 15% due to the general equilibrium effects of C^x, partly offset by the 15% depreciation of nominal seller ERG $\tilde{r}^x$. Some less dramatic negative correlation is also visible in Figure 4 in the interval around the Great Recession.

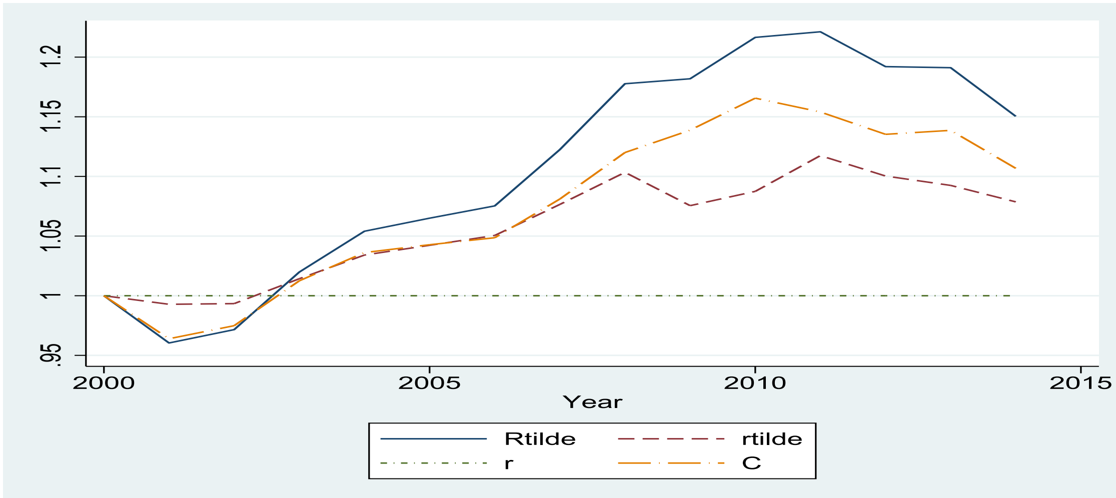

Figure 4: $\tilde{r}$ and $\tilde{R}$ for United States (Aggregate)

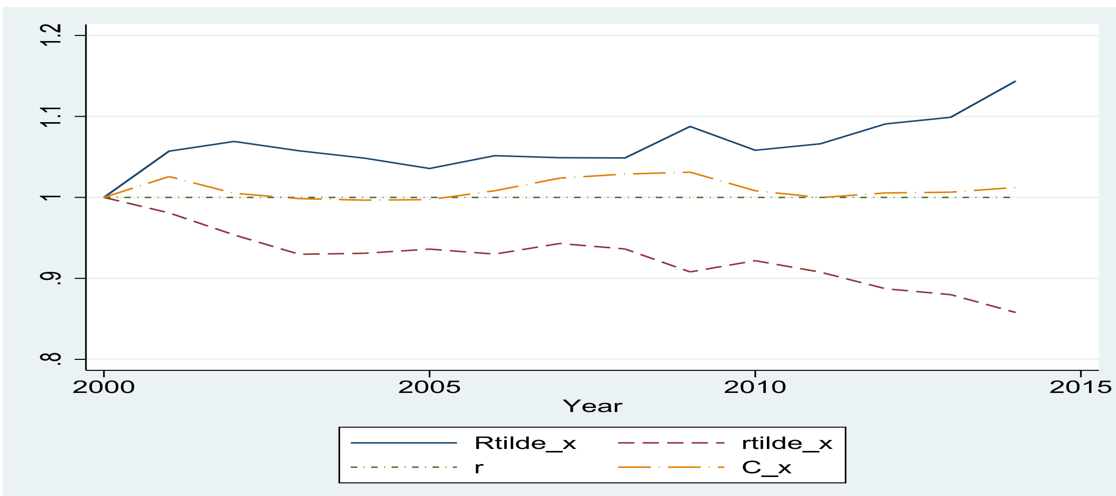

Figure 5: $\tilde{r}^x$ and $\tilde{R}^x$ for United States (Aggregate)

At the sectoral level there is even wider variation of the plots, dramatically different for some country-sector-time interval selections. Policy relevance is suggested in a few sector-country cases chosen for the size of the deviation between $\tilde{r}_i^x$ the effective rate seller index $\bar{r}_i^x$ and good fit of the model. Sweden and Switzerland in Electrical Products have $\tilde{r}^x$ fall to half of $\bar{r}^x$ between 2000 and 2014, with some intervals of negative correlation between them. Belgium in Chemicals has $\tilde{r}^x$ fall to half of $\bar{r}^x$ between 2000 and 2014, with a bit of negative correlation in some time intervals. US agriculture experiences a 6% fall in $\tilde{r}^x$ relative to $\bar{r}^x$ with some intervals of negative correlation. See Figures 6,7, 8 and 9. In these cases the fall in sellers nominal ERG $\tilde{r}^x$ raises seller earnings in the source country.

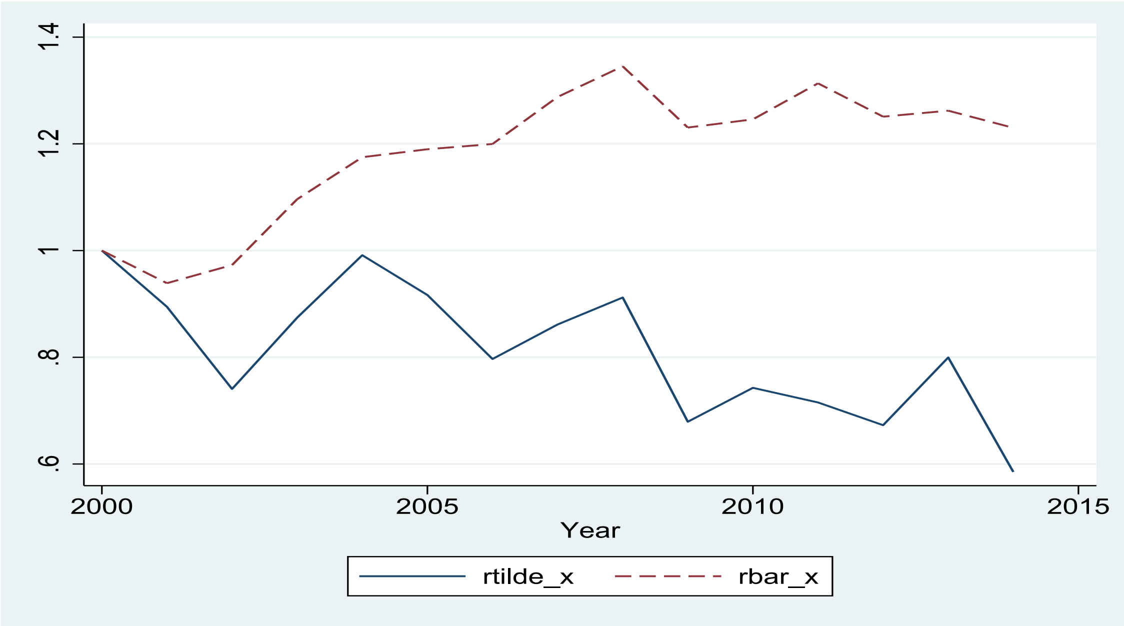

Figure 6: $\tilde{r}^x$ Vs $\bar{r}^x$ for Sweden (Sector Electrical data)

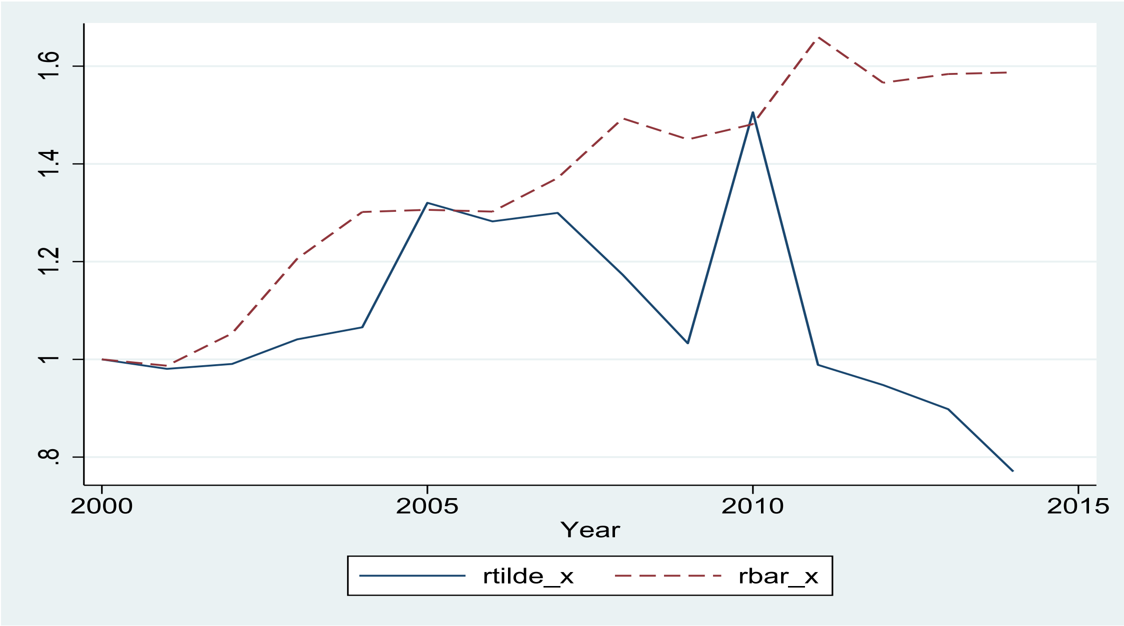

Figure 7: $\tilde{r}^x$ Vs $\bar{r}^x$ for Switzerland (Sector Electrical data)

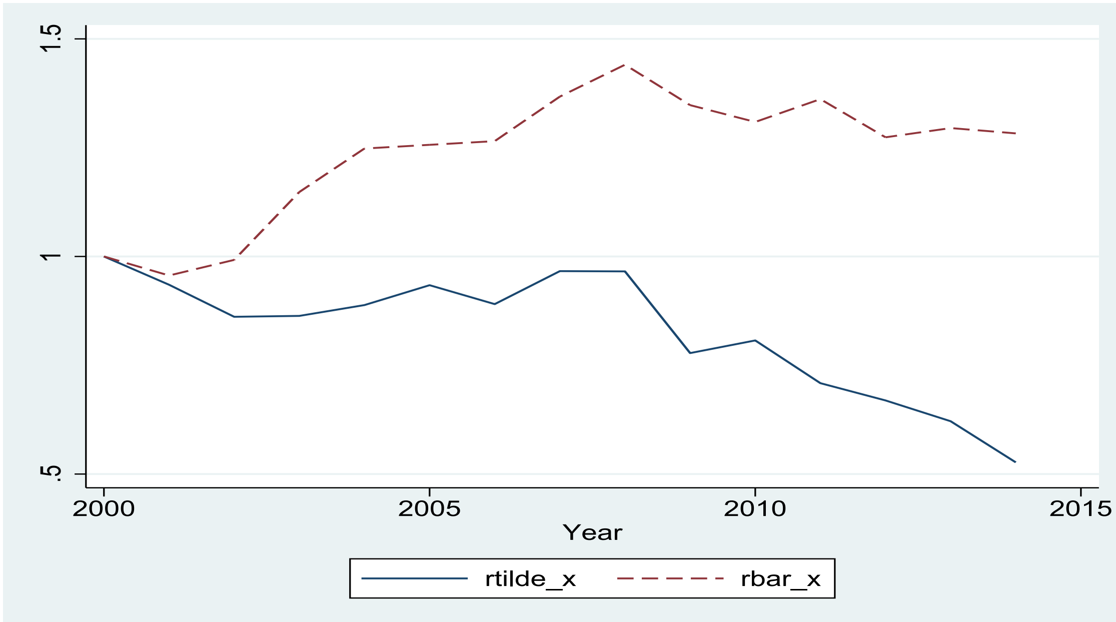

Figure 8: $\tilde{r}^x$ Vs $\bar{r}^x$ for Belgium (Sector Chemicals data)

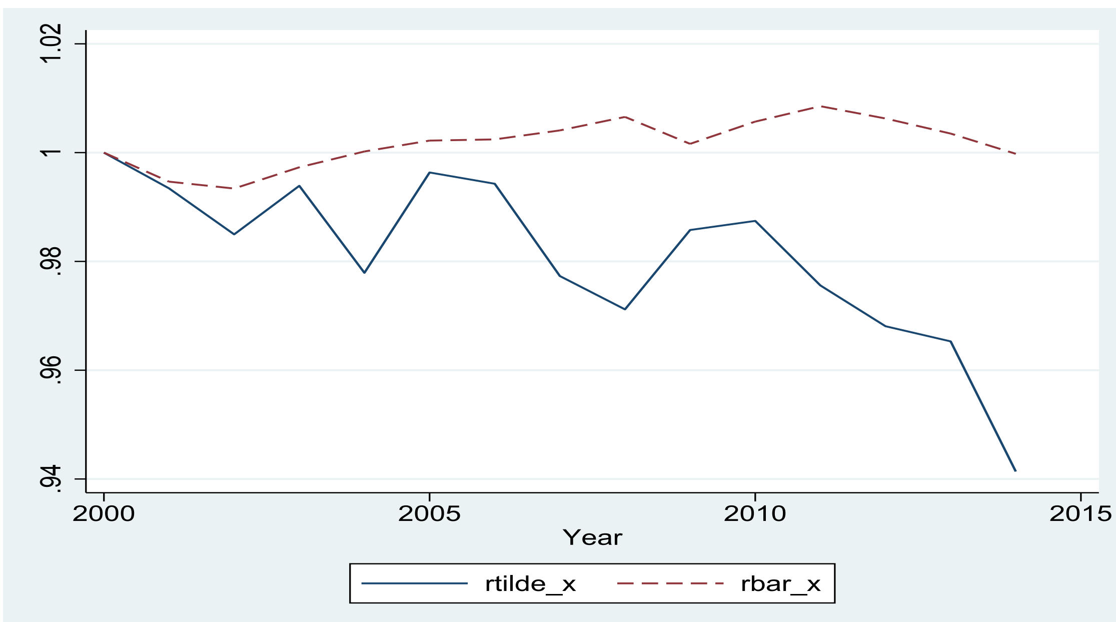

Figure 9: $\tilde{r}^x$ Vs $\bar{r}^x$ for United States (Sector Agriculture data)

In contrast, a number of country-sector cases show quantitatively significant passthrough of sellers nominal ERG appreciation, implying downward pressure on earnings. Cases like these might justify policy intervention to compensate the damage due to exchange rate passthrough frictions. In some such cases $\bar{r}^x$ tracks $\tilde{r}^x$ closely. Two cases exhibiting sharp divergence are Germany in Textiles and Canada in Paper, charted in Figures 10 and 11.

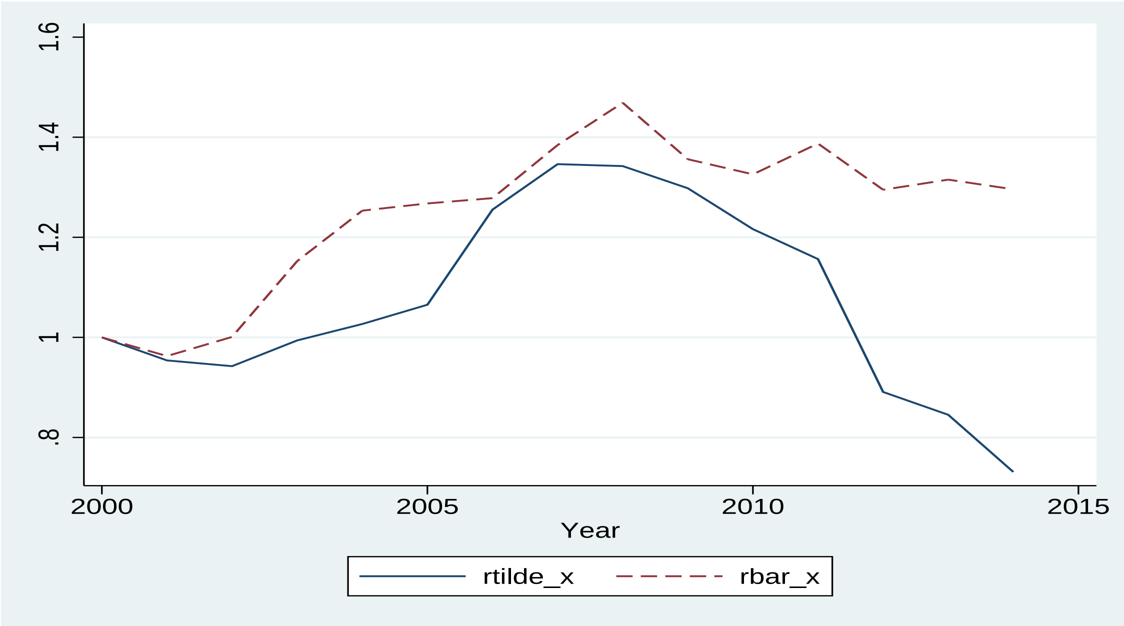

Figure 10: $\tilde{r}^x$ Vs $\bar{r}^x$ for Germany (Sector Textile data)

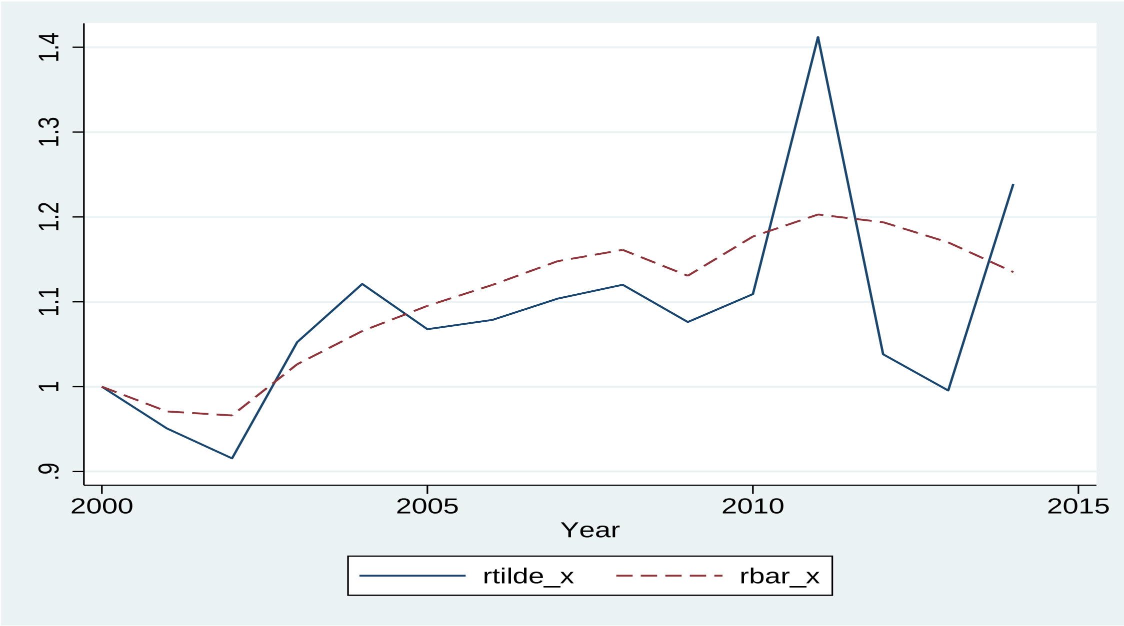

Figure 11: $\tilde{r}$ Vs $\tilde{r}^x$ for Canada (Sector Paper data)

5.3.4 ERG Vs Long Run ER

The estimated gravity model is usefully deployed to examine the counterfactual long run equilibrium in which money is neutral, the deviations from uniform passthrough are removed. Two separate objectives suggest two variations on "long run" equilibrium. The first exercise examines how informative the ERGs are about the "long run" exchange rate. Given the focus on sector level effects due to treating exchange rates as trade policy, it makes sense to treat each sector as a "world" and examine the "long run" equilibrium of this sectoral "world economy". This implies a set of long run exchange rate changes r_i^{*k} for each country i in sector k. These are compared to the ERGs.

The correlation between r^* and both nominal and real ERGs for buyers is very high with the exception of Plastics, Auto and Other Transport. The latter two sectors are suspect due to possible mis-specification (because their passthrough elasticities are greater than 1). Thus real ERG for buyers promises to be a usefully accurate indicator of long run exchange rates.

In contrast the nominal ERG $\tilde{r}^x$ for sellers is much less highly correlated with r^*. The real ERG for sellers $\tilde{R}^x$ restores the high correlation with r^* observed for sellers ERG, with the same 3 exceptions.

Table 3 shows the cross-sectional correlation between buyer nominal and real effective exchange rate with gravitas ($\widetilde{r}$) with the long run exchange rate (r^*), averaged over time.

Table 3: Correlation of Buyer ERG with Long Run ER

Sector	corr($\widetilde{r}, r^*$)				corr($\widetilde{R}, r^*$)			
	All Countries		Non EU Members		All Countries		Non EU Members	
	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.
Agriculture	-0.86	0.31	-0.72	0.5	0.99	0	0.99	0.01
Mining	-0.76	0.32	-0.63	0.46	0.99	0	1	0

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Sector	corr($\widetilde{r}, r^*$)				corr($\widetilde{R}, r^*$)			
	All Countries		Non EU Members		All Countries		Non EU Members	
	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.
Manufacturing								
Food	-0.81	0.4	-0.73	0.49	0.99	0	1	0
Textile	-0.76	0.33	-0.62	0.48	0.98	0.01	0.99	0.01
Wood	-0.77	0.4	-0.56	0.57	0.99	0.01	0.99	0
Paper	-0.78	0.31	-0.62	0.4	0.99	0	1	0
Petroleum	-0.72	0.25	-0.58	0.37	1	0	1	0
Chemicals	-0.79	0.38	-0.61	0.52	1	0.01	1	0
Plastic	-0.47	0.56	-0.31	0.7	0.94	0.02	0.98	0.01
Minerals	-0.63	0.38	-0.54	0.58	0.95	0.02	0.97	0.01
Basic metals	-0.73	0.39	-0.56	0.54	0.97	0.02	0.97	0.01
Metal products	-0.79	0.36	-0.61	0.52	0.95	0.02	0.98	0.01
Machinery	-0.7	0.47	-0.42	0.63	0.97	0.01	0.98	0.01
Electrical	-0.69	0.49	-0.44	0.7	0.99	0	0.99	0
Auto	-0.6	0.52	-0.25	0.73	0.44	0.24	0.62	0.19
Other Transport	-0.47	0.65	-0.23	0.77	0.84	0.04	1	0
Other	-0.77	0.33	-0.59	0.47	0.98	0.01	0.99	0
Aggregate	-0.81	0.31	-0.67	-0.67	0.96	0.02	0.99	0.01

Table 4 shows the cross-sectional correlation between between seller nominal and effective exchange rate with gravitas ($\widetilde{r}^x$) with the long run exchange rate (r^*), averaged over time.

Table 4: Correlation of Seller ERG with Long Run ER

Sector	**corr**$(\widetilde{r}^x, r^*)$				**corr**$(\widetilde{R}^x, r^*)$			
	All Countries		Non EU Members		All Countries		Non EU Members	
	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.
Agriculture	-0.86	0.17	-0.81	0.24	-0.95	0.02	-0.96	0.02
Mining	-0.07	0.43	-0.3	0.49	-0.93	0.03	-0.95	0.03
Manufacturing								
Food	-0.78	0.33	-0.82	0.22	-0.81	0.09	-0.85	0.09
Textile	-0.66	0.29	-0.64	0.41	-0.92	0.03	-0.93	0.04
Wood	-0.53	0.36	-0.66	0.39	-0.94	0.03	-0.95	0.03
Paper	-0.39	0.42	-0.53	0.45	-0.93	0.03	-0.93	0.04
Petroleum	0.7	0.33	0.62	0.5	-0.96	0.01	-0.96	0.02
Chemicals	-0.32	0.51	-0.54	0.61	-0.83	0.07	-0.87	0.06
Plastic	-0.5	0.53	-0.36	0.65	-0.53	0.11	-0.62	0.15
Minerals	-0.65	0.35	-0.56	0.53	-0.82	0.04	-0.83	0.05
Basic metals	-0.8	0.25	-0.7	0.38	-0.94	0.02	-0.95	0.02
Metal products	-0.84	0.23	-0.73	0.33	-0.9	0.04	-0.93	0.04
Machinery	-0.77	0.32	-0.63	0.51	-0.82	0.06	-0.85	0.06
Electrical	-0.42	0.37	-0.53	0.49	-0.94	0.02	-0.96	0.02
Auto	-0.55	0.51	-0.26	0.72	-0.42	0.09	-0.44	0.15
Other Transport	-0.54	0.59	-0.43	0.68	-0.6	0.1	-0.64	0.11
Other	-0.78	0.26	-0.68	0.39	-0.91	0.03	-0.9	0.05
Aggregate	-0.75	0.24	-0.68	-0.68	-0.89	0.04	-0.91	0.05

5.3.5 Real Income Effects

Real income effects of exchange rate changes with passthrough frictions can be quantified by calculating the real income changes due to removing the frictions in the estimated model, simulation of the counterfactual long run equilibrium. The counterfactual yields the terms of trade effects of removing exchange rate passthrough frictions in the world economy consisting of 17 sectors and 40 countries. The calculation is based on each year's endowments and the yearly changes of exchange rates over the preceding year for the actual equilibrium, compared to the counterfactual long run equilibrium with the same endowments, tastes and trade costs except for removal of the exchange rate frictions.

The US is a representative case. the US terms of trade over the period 2000 to 2014 move within a band of around 0.5% up and down. Figure 12 plots the time series.

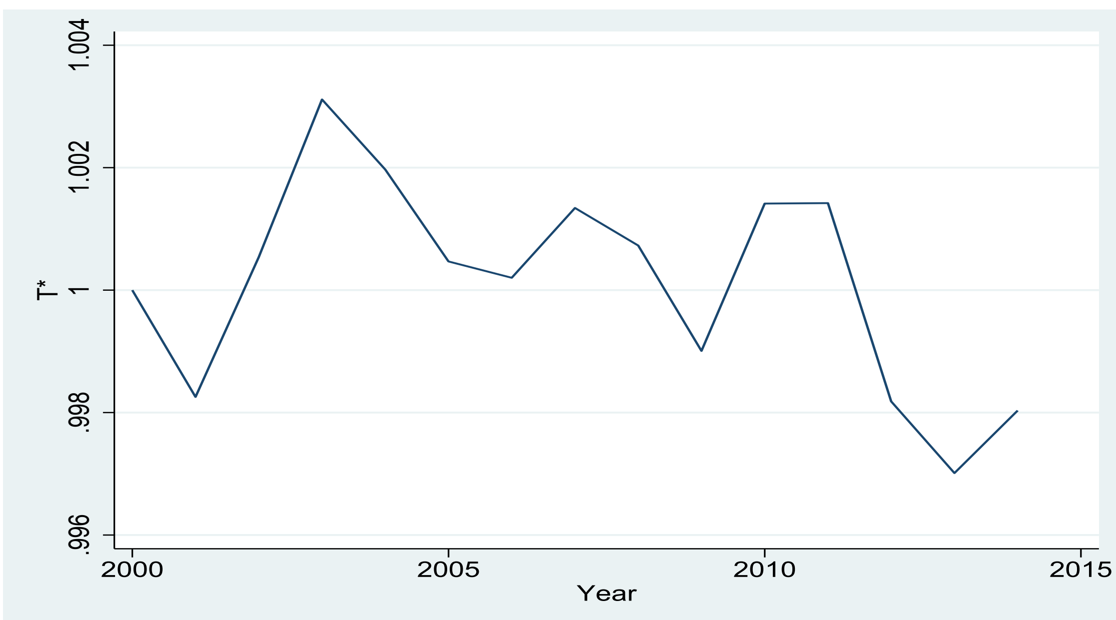

Figure 12: Tstar for United States (Aggregate)

Table 5 reports cases of larger changes in terms of trade (T^*). Max and min columns show the means of the top and bottom deciles respectively. There is no obvious pattern to the countries in the top and bottom deciles of each year's terms of trade effects. Membership changes by year and includes both large and small economies. Some are commodity exporters but some are highly diversified exporters. Deeper exploration awaits future work.

Table 5 also reports the world efficiency effect of exchange rate passthrough frictions calculated as the size-weighted average of the country level terms of trade (T^{**}).

Table 5: Terms of trade changes

Year	T^{*}		T^{**}
	(Max)	(Min)	
2000	1	1	1
2001	1.0428	0.9898	1.0007
2002	1.0236	0.988	1.0002
2003	1.0296	0.9782	0.9975
2004	1.0182	0.9902	0.9987
2005	1.0131	0.9833	0.9997
2006	1.0051	0.9864	0.9996
2007	1.0111	0.9859	0.9981
2008	1.0213	0.9871	1.0016
2009	1.0301	0.986	1.0028
2010	1.0235	0.9857	1.0006
2011	1.0075	0.9929	0.9999
2012	1.0193	0.9968	1.0036
2013	1.0079	0.986	1
2014	1.0165	0.991	1.0004

6 Conclusion

Structural gravity is applied in the paper to quantify real effects of heterogeneous exchange rate passthrough. We define theory consistent operational indexes of bilateral exchange rates suitable for evaluating the real effects on buyers and sellers. The results reveal quantitatively significant real effects at the sectoral level, with much smaller but still non-negligible effects at the aggregate level.

We suggest potential policy implications in the form of domestic subsidies to politically significant losers. Domestic policies on these lines would relieve incoherent political pressure to act against 'currency manipulation' and could be consistent with WTO principles.

More speculatively, the gravity model connection to exogenously determined exchange rates here may be step toward a re-connection of real trade to exchange rate determination. The gravity model estimated here can be interpreted as a short run model in which bilateral 'marketing capital' capacities are fixed, and adjust slowly toward long run zero profit values (Anderson and Yotov, 2019). This setting suggests a structural dynamic channel from real trade to exchange rate movements.

References

Afriat, S. N. (1977), *The Price Index*, Cambridge: Cambridge University Press.

Anderson, J.E. and J.P. Neary (2005), *Measuring the Restrictiveness of International Trade Policy*, Cambridge: MIT Press.

Anderson, J.E. and E. van Wincoop (2004), "Trade Costs", *Journal of Economic Literature*, 42, 691-751.

Anderson, J. E. and Yoto V. Yotov (2016), "Terms of Trade and Global Efficiency Effects of Free Trade Agreements, 1990-2002", *Journal of International Economics*, 99, 279-98.

Anderson, J.E., Mykyta Vesselovsky and Yoto V. Yotov (2016), "Gravity with Scale Effects", *Journal of International Economics*, 100, 174-193.

Anderson, J.E. and Yoto V. Yotov (2020), "Short Run Gravity", *Journal of International Economics* 126, revised version of NBER WP 23458.

Boz, Emine, Gita Gopinath and Mikkel Plagborg-Møller (2017), "Global Trade and the Dollar", NBER Working Paper No. 23988.

———————————————————— (2019), "Dollar Invoicing and the Heterogeneity of Exchange Rate Passthrough", *American Economic Association Papers & Proceedings*, 109: 527-532.

Chinn, Menzie D. (2006), "A Primer on Real Effective Exchange Rates: Determinants, Overvaluation, Trade Flows and Competitive Devaluation", *Open Economies Review*, 17: 115-43.

Costinot, Arnaud, David Donaldson and Ivana Komunjer (2012), "What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas" *Review of Economic Studies*, 79(2), 581-608.

Eaton, Jonathan and Samuel Kortum (2002), "Technology, Geography and Trade", *Econometrica*, 70: 1741-1779.

Neary, J. Peter (2006), "Measuring Competitiveness", *The Economic and Social Review*, 37(2), 197-213.

Obstfeld, Maurice and Kenneth Rogoff, 2001. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," *NBER Macroeconomics Annual 2000*, 15, 339-412.

Appendix: Effective Exchange Rates in Practice

A typical effective exchange rate index is calculated as:

$$\bar{r}_j = \sum_{i \neq j} r_i \frac{X_{ij}^0}{\sum_{i \neq j} X_{ij}^0}. \tag{22}$$

where X_{ij}^0 denotes the value of bilateral trade shipped from i to j in base period 0. Often r_j and $\bar{r}_j$ are in logs, in which case the levels are obtained by exponentiating. Sometimes (22) is calculated for exports as well as imports and sometimes for disaggregated trade. Recognizing that (22) is a Laspeyres index, some practitioners use Tornqvist indexes (for backward looking studies) or Laspeyres chain weights to replace the simple Laspeyres weights in (22).

The apparent intent of index definition (22) is to measure the impact on the buyer's purchasing power of the vector of bilateral exchange rate changes – $\bar{r}_j/r_j > (<)1$ implies that j's currency has lost (gained) purchasing power. An appreciation (depreciation) of r_j would be needed to restore the base purchasing power of a unit of j's currency over a trade weighted basket of other currencies. Changes in actual purchasing power are measured by buyer price indexes P_j/P_j^0 where P_j is the current period local currency price index (for the bundle of goods imported) at j and P_j^0 is the base price index in local currency prices. Real purchasing power change in j's currency is measured by

$$\frac{\bar{r}_j/r_j}{P_j/P_j^0} = \frac{\bar{r}_j/P_j}{r_j/P_j^0}, \tag{23}$$

the hypothetical appreciation of j's currency needed to restore purchasing power parity with the base period.

Effective exchange rate indexes are also frequently calculated from the seller's point of view. Mechanically, sum over j rather than i in (22) to define seller i's effective exchange rate index of appreciation $\bar{r}_i^x$. Appreciation tending to drive down sellers' prices, the intent is to measure the effect of exchange rate appreciation on real earnings of sellers. The real effective exchange rate for sellers deflates by a sellers' price index in parallel to (23). Finally, while the most commonly reported effective exchange rate indexes are for aggregate trade, sectoral effective exchange rates are also often reported.

It is well recognized that effective exchange rate index (22) and the real exchange rate index (23) based on it are unsatisfactory for several reasons. Whether for buyers purchasing power or sellers earnings, aggregated or sectoral, here are the key problems:[20]

1. The price index structure does not specify links to incomplete exchange rate passthrough.

2. Theory suggests that trade costs affects the operation of exchange rates. Trade cost links to (22) are unspecified.

3. In a multi-country world, cross effects necessarily act on prices of goods to and from partners of j, affecting the trade shares in (22).

4. The preceding three problems all point to missing general equilibrium links of $\{r_j\}$ to $\{P_j\}$.

This paper provides a real effective exchange rate index that appropriately treats all 4 problems within the restrictions of the structural gravity model, Effective exchange Rate with Gravitas (ERG). The structurally based real exchange rate index differs from (22) deflated by the price index deflator for all cases in which exchange rates matter; i.e., when money is not neutral.

[20]There are many other purposes for which differing real exchange rates have been implemented. See Chinn (2006) for a useful survey. All the indexes surveyed there share the fundamental problems analyzed here: partial equilibrium assumptions that ignore trade costs and ignore incomplete passthrough.